Statistical evaluation of $0\nu\beta\beta$ Nuclear Matrix Elements uncertainties

Mihai Horoi

Department of Physics, Central Michigan University, Mount Pleasant, Michigan 48859, USA

Support from DOE grant DE-SC0022538 is acknowledged

CENTRAL MICHIGAN UNIVERSITY

Plan of talk

- Neutrinoless double beta decay: Brief introduction
- Nuclear matrix elements
  - Shell model approach
  - Statistical analysis for ^{136}Xe
  - Statistical analysis for ^{76}Ge
- Conclusions and outlook

U.S. DEPARTMENT OF ENERGY Office of Science

Two-Neutrino Double Beta Decay

SEPTEMBER 15, 1935 PHYSICAL REVIEW VOLUME 48

Double Beta-Disintegration

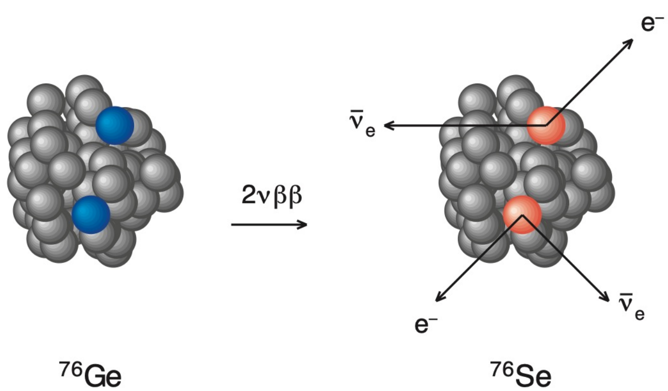
M. GOEPPERT-MAYER, *The Johns Hopkins University*

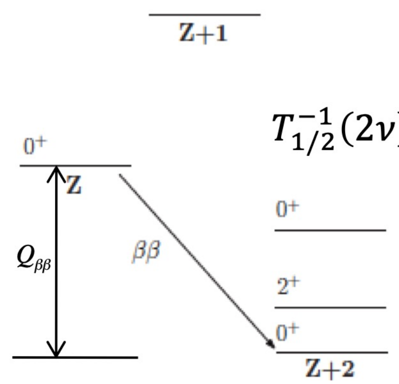
(Received May 20, 1935)

From the Fermi theory of β-disintegration the probability of simultaneous emission of two electrons (and two neutrinos) has been calculated. The result is that this process occurs sufficiently rarely to allow a half-life of over 10^{17} years for a nucleus, even if its isobar of atomic number different by 2 were more stable by 20 times the electron mass.

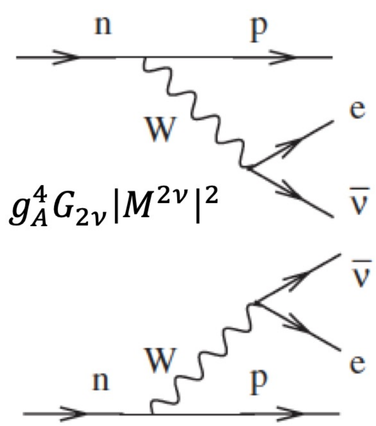
$$P = 1.15\times10^{-35}F(\epsilon-2)\ \text{sec.}^{-1}$$
$$= 3.6\times10^{-28}F(\epsilon-2)\ \text{year}^{-1}. \qquad (12)$$

$\epsilon=$	4	6	8	10
$F(\epsilon+2)=$	0.37×10^2	9.2×10^4	3.4×10^6	4.2×10^7
$\epsilon=$	12	20		
$P(\epsilon-2)=$	3.3×10^8	1×10^{11}		





$$T^{-1}_{1/2}(2\nu) = g_A^4 G_{2\nu}|M^{2\nu}|^2$$



Neutrinoless Double Beta Decay

DECEMBER 15, 1939 PHYSICAL REVIEW VOLUME 56

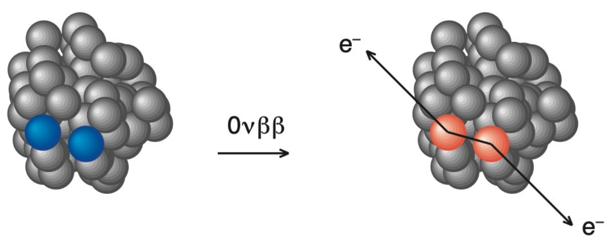
On Transition Probabilities in Double Beta-Disintegration

W. H. Furry
Physics Research Laboratory, Harvard University, Cambridge, Massachusetts
(Received October 16, 1939)

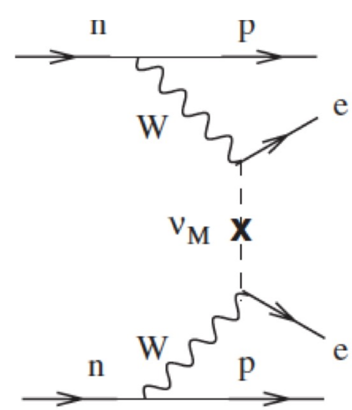
THE probability of double β-disintegration was calculated some years ago by Goeppert-Mayer[1] on the basis of the Fermi theory.[2, 3] The result obtained was extremely small, corresponding to a lifetime of the order of 10^{25} years in the case of two isobars whose masses differ by 0.002

IV. Conclusion

We have seen that the phenomenon of double beta-disintegration is one for which there is a decided difference between the results of the Majorana theory and those of the older theory of the neutrino. According to the older theory it seemed certain that double beta-disintegration could never be capable of observation because of its extremely minute probability, but the Majorana theory indicates that this is by no means necessarily the case. Indeed, if the inter-



$$\langle m_{\beta\beta} \rangle = \left| \sum_k m_k U_{ek}^2 \right|$$

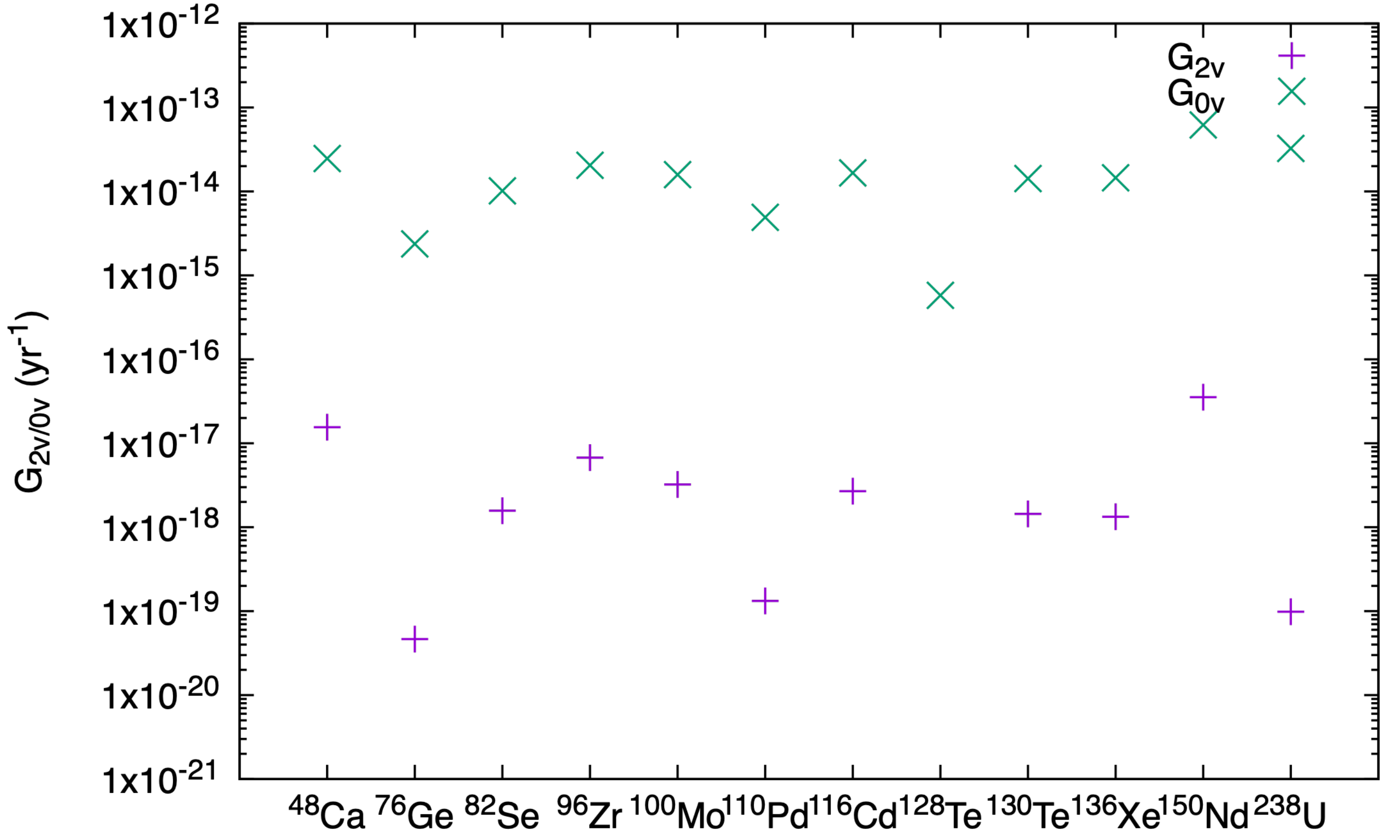


$$T_{1/2}^{-1}(0\nu) = G^{0\nu}(Q_{\beta\beta})\left[M^{0\nu}(0^+)\right]^2 \left(\frac{< m_{\beta\beta} >}{m_e} \right)^2$$

$$\nu_e(x) = \sum_i U_{ei} \nu_i(x)$$

$$\left| \nu_e \right\rangle = \sum_i U_{ei}^* \left| \nu_i \right\rangle$$

DBD Phase Space Factors



$$T_{1/2}^{-1}(2\nu) = g_A^4 G_{2\nu} |M^{2\nu}|^2$$

$$T_{1/2}^{-1}(0\nu) = G^{0\nu}(Q_{\beta\beta})\left[M^{0\nu}(0^+)\right]^2 \left(\frac{< m_{\beta\beta} >}{m_e}\right)^2$$

First Searches of $0\nu\beta\beta$

Phys. Rev 75, 323 (1949) Letter to the Editor

A Measurement of the Half-Life of Double Beta-Decay from $_{50}Sn^{124}$ *

E. L. Fireman
Department of Physics, Princeton University, Princeton, New Jersey
November 29, 1948

Table II. $L(A)$ gives the coincidences from specimen A between counters L, and $R(B)$ gives the coincidences from B between counters R. Holder 0° and 180° are the two holder positions. Positions 1 and 2 are positions for the specimens in the holder.

Pos. 1	Holder 0° Coin. counts/hr.	$L(A)$ 16.4±0.3	$R(B)$ 14.3±0.3
	Holder 180° Coin. counts/hr.	$L(B)$ 14.4±0.3	$R(A)$ 15.9±0.3
Pos. 2	Holder 0° Coin. counts/hr.	$L(B)$ 14.6±0.3	$R(A)$ 16.4±0.3
	Holder 180° Coin. counts/hr.	$L(A)$ 16.4±0.3	$R(B)$ 13.9±0.3

A detailed report of this work is being prepared for publication in the Physical Review.

The author is grateful to Professor R. Sherr for accepting the supervision of this research and to Professor E. P. Wigner for many profitable discussions.

* This work is assisted by the Office of Naval Research.
** These isotopes were obtained from Oak Ridge.
[1] Maria Goeppert-Mayer, Phys. Rev. **48**, 512 (1935).
[2] W. H. Furry, Phys. Rev. **56**, 1184 (1939).

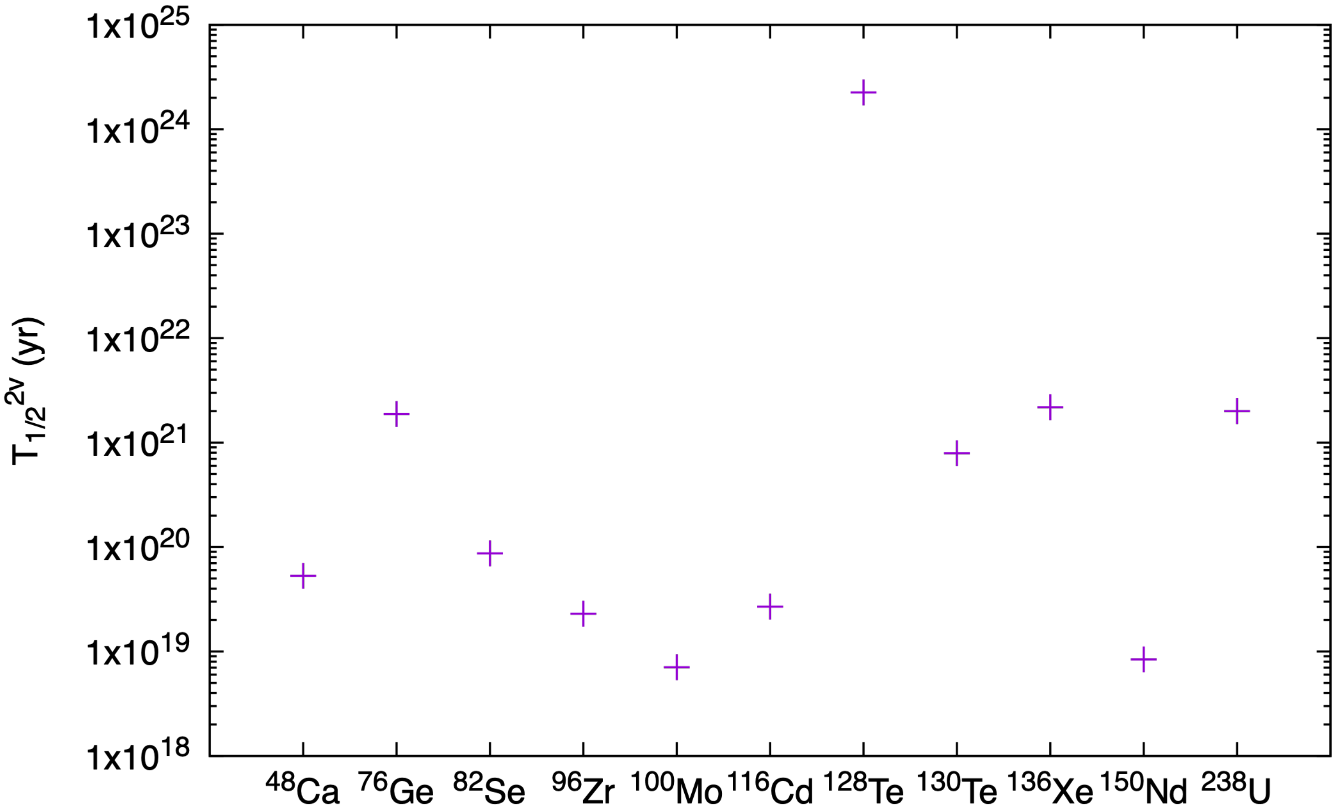
PHYSICAL REVIEW VOLUME 86, NUMBER 4 MAY 15, 1952

A Re-Investigation of the Double Beta-Decay from Sn^{124}

E. L. Fireman and D. Schwarzer
Brookhaven National Laboratory, Upton, New York
(Received February 5, 1952)

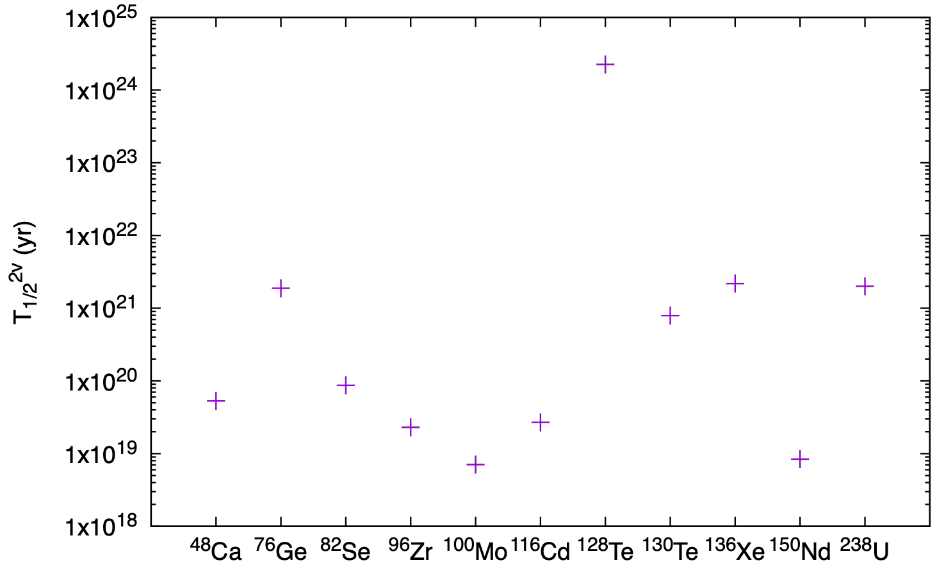
Radiations from natural tin and tin enriched with the 124 isotope are examined in a magnetic field with a helium filled cloud chamber that is triggered by internal counters. Only three pictures out of more than four thousand photographs are pictures of two electrons coming out of the same point in the tin and entering the counters, and even these may be pictures of multiply scattered electrons passing through the tin. However, one may set a lower limit to the half-life of double beta-decay from Sn^{124} as 10^{17} years. This is a decay rate less than one-tenth of a value previously reported by one of the authors.

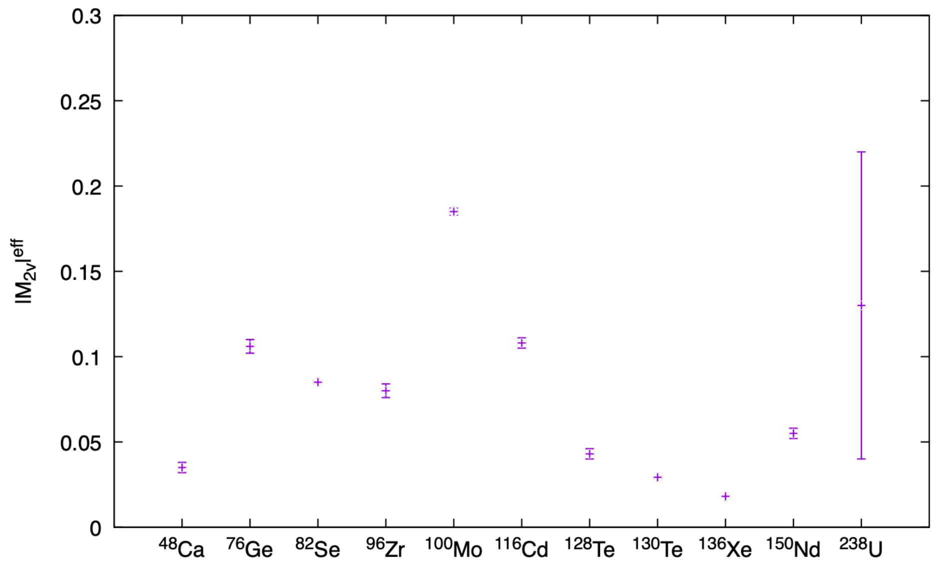
2νββ Half-Lives



2νββ Half-Lives and NME

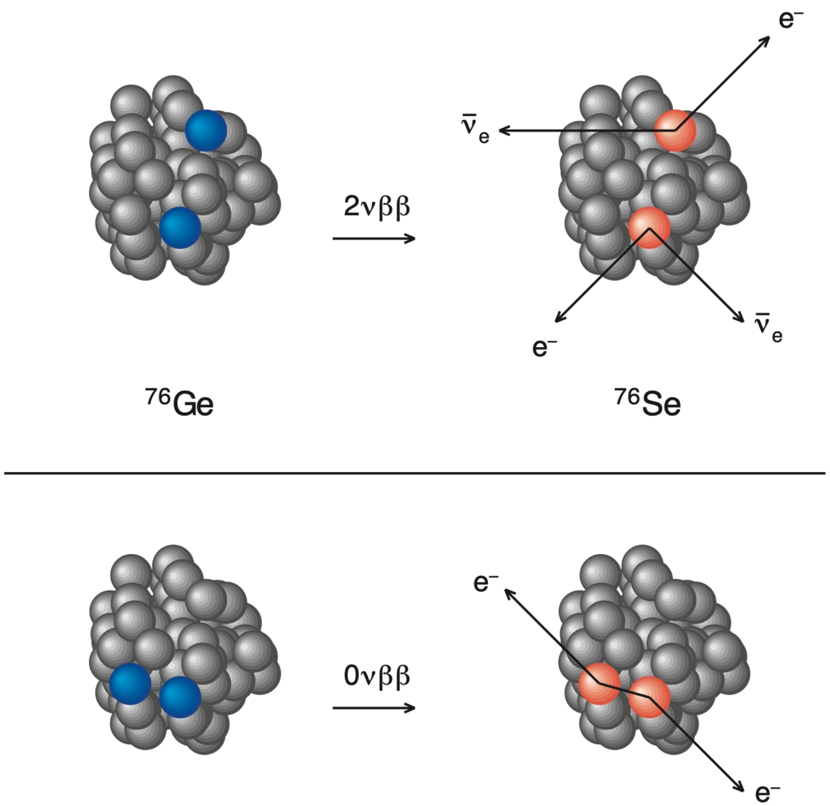
$$\left[T^{2\nu}_{1/2}\right]^{-1} = G_{2\nu} \cdot \left[g_A^2 \left(m_e c^2 \cdot M_{2\nu}\right)\right]^2 \equiv G_{2\nu} \left(M^{eff}_{2\nu}\right)^2$$

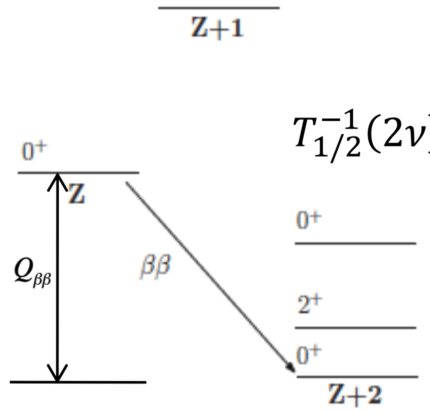




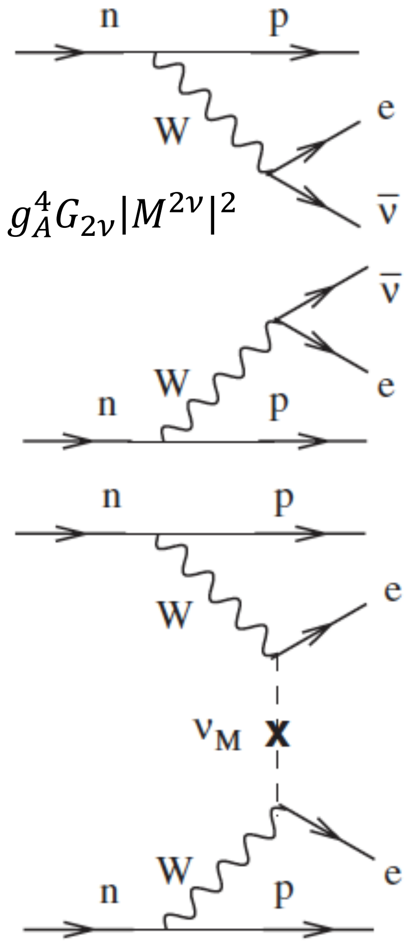
A. Barabash, Universe 6, 159 (2020)

Classical Double Beta Decay Problem





$$T_{1/2}^{-1}(2\nu) = g_A^4 G_{2\nu} |M^{2\nu}|^2$$



$$\langle m_{\beta\beta} \rangle = \left| \sum_k m_k U_{ek}^2 \right|$$

$$T_{1/2}^{-1}(0\nu) = G^{0\nu}(Q_{\beta\beta}) \left[M^{0\nu}(0^+) \right]^2 \left(\frac{< m_{\beta\beta} >}{m_e} \right)^2$$

$$\nu_e(x) = \sum_i U_{ei} \nu_i(x)$$

$$|\nu_e\rangle = \sum_i U_{ei}^* |\nu_i\rangle$$

Neutrino Masses

$$|\nu_\alpha\rangle = \sum_i U^*_{\alpha i}\,|\nu_i\rangle$$
$$|\nu_i\rangle = \sum_\alpha U_{\alpha i}\,|\nu_\alpha\rangle$$

PMNS – matrix

$$U = \begin{bmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{bmatrix} = \begin{bmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{bmatrix} \begin{bmatrix} e^{i\alpha_1/2} & 0 & 0 \\ 0 & e^{i\alpha_2/2} & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$c_{12} \equiv \cos\theta_{12},\ s_{12} = \sin\theta_{12},\ etc$$

- Tritium decay:

$$^3H \rightarrow {}^3He + e^- + \bar{\nu}_e$$

$$m_{\nu_e} = \sqrt{\sum_i |U_{ei}|^2 m_i^2} < 2.2eV\,(Mainz\ \exp.)$$

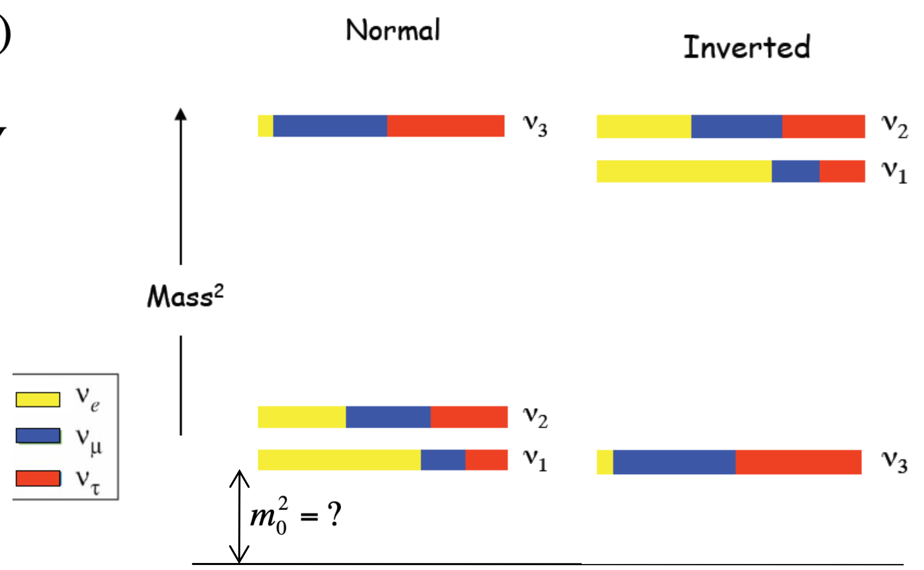
$$KATRIN\ (to\ take\ data):\ goal\ \ m_{\nu_e} < 0.3eV$$

- Cosmology: CMB power spectrum, BAO, etc,

$$\sum_{i=1}^{3} m_i < 0.23eV$$

$$Goal: 0.01eV\ (5-10\ y)$$

$$\Delta m^2_{21} \approx 7.5\times 10^{-5}\ eV^2\,(solar)$$
$$|\Delta m^2_{32}| \approx 2.4\times 10^{-3}\ eV^2\,(atmospheric)$$

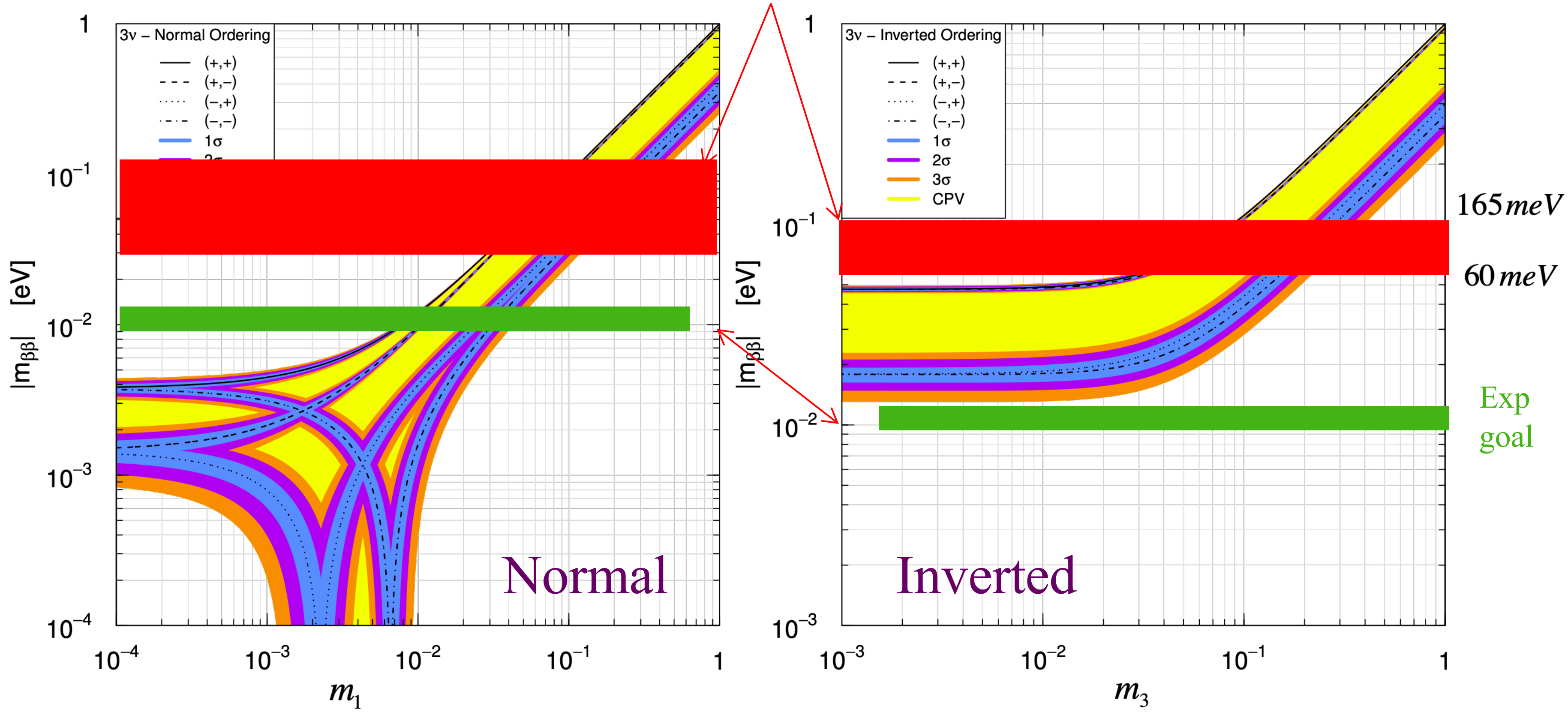


Two neutrino mass hierarchies

Neutrino ββ effective mass

arxiv:1507.08204

KamLAND – Zen, PRL 117, 082503 (2016) : ^{136}Xe



$$\left|m_{\beta\beta}\right| = \left|\sum_{k=1}^{3} m_k U_{ek}^2\right| = \left|c_{12}^2 c_{13}^2 m_1 + c_{13}^2 s_{12}^2 m_2 e^{i\phi_2} + s_{13}^2 m_3 e^{i\phi_3}\right|$$

$$\Leftarrow T_{1/2}^{-1}(0\nu) = G^{0\nu}(Q_{\beta\beta})\left[M^{0\nu}(0^+)\right]^2 (\eta_{0\nu})^2$$

$$\phi_2 = \alpha_2 - \alpha_1 \qquad \phi_3 = -\alpha_1 - 2\delta$$

$$\eta_{0\nu} = \frac{\left|m_{\beta\beta}\right|}{m_e}$$

NSAC 2023 Long Range Plan

1 EXECUTIVE SUMMARY

ial pursuits. The workforce that enables discovery in nuclear science also makes breakthroughs in technologies with tremendous impact on the nation's

RECOMMENDATION 1
The highest priority of the nuclear science community is to capitalize on the extraordinary opportunities for scientific discovery made possible by the substantial and sustained investments of the United States. We must draw on the talents of all in the nation to achieve this goal.

This recommendation requires

- Increasing the research budget that advances the science program through support of theoretical and experimental research across the country, thereby expanding discovery potential, technological innovation, and workforce development to the benefit of society.
- Continuing effective operation of the national user facilities ATLAS, CEBAF, and FRIB, and completing the RHIC science program, pushing the frontiers of human knowledge.
- Raising the compensation of graduate researchers to levels commensurate with their cost of living—without contraction of the workforce—lowering barriers and expanding opportunities in STEM for all, and so boosting national competitiveness.
- Expanding policy and resources to ensure a safe and respectful environment for everyone, realizing the full potential of the US nuclear workforce.

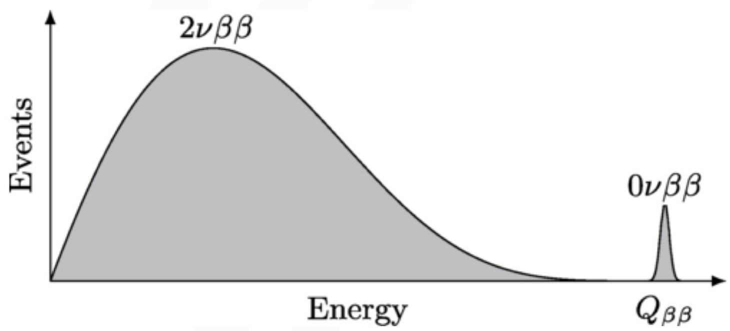
RECOMMENDATION 2
As the highest priority for new experiment construction, we recommend that the United States lead an international consortium that will undertake a neutrinoless double beta decay campaign, featuring the expeditious construction of ton-scale experiments, using different isotopes and complementary techniques.

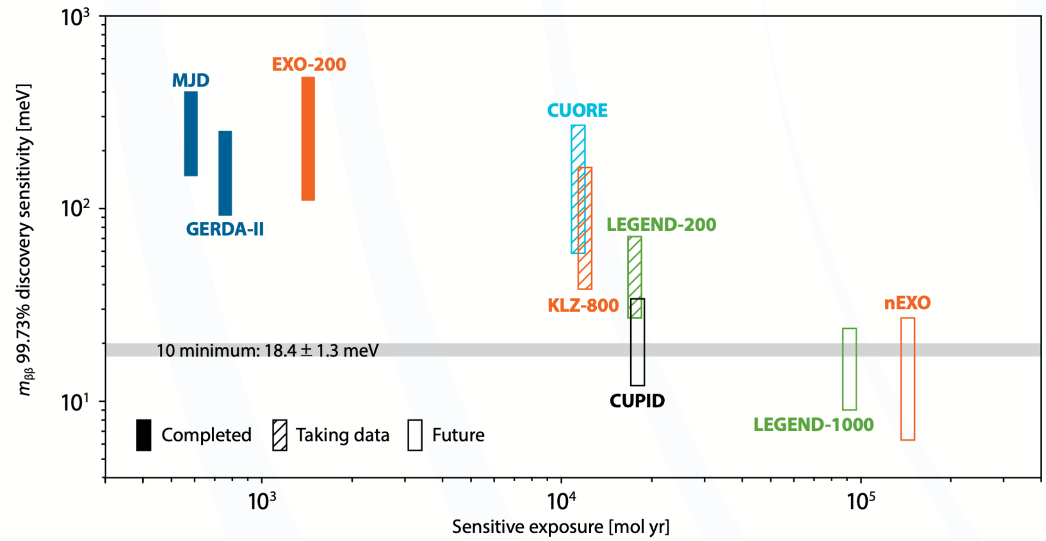
One of the most compelling mysteries in all of science is how matter came to dominate over antimatter in the universe. Neutrinoless double beta decay, a process that spontaneously creates matter, may hold the key to solving this puzzle. Observation of this rare nuclear process would unambiguously demonstrate that neutrinos are their own antiparticles and would reveal the origin and scale of neutrino mass. The nucleus provides the only laboratory through which this fundamental physics can be addressed.

The importance of the physics being addressed by neutrinoless double beta decay has resulted in worldwide excitement and has catalyzed the international cooperation essential to carrying out a successful campaign. An extraordinary discovery of this magnitude requires multiple experiments using different techniques for a select set of isotopes. Such measurements demand unprecedented sensitivity and present unique challenges. Since the 2015 Long Range Plan, the US-led CUPID, LEGEND, and nEXO international collaborations have made remarkable progress with three distinct technologies. An independent portfolio review committee has deemed these experiments ready to proceed now.

Neutrinoless double beta decay is sensitive to new physics spanning very different scales and physical mechanisms. The identification of the underlying physics will pose a grand challenge and opportunity for theoretical research. An enhanced theoretical effort is an integral component of the campaign and is essential for understanding the underlying physics of any signal.

NSAC 2023 Long Range Plan



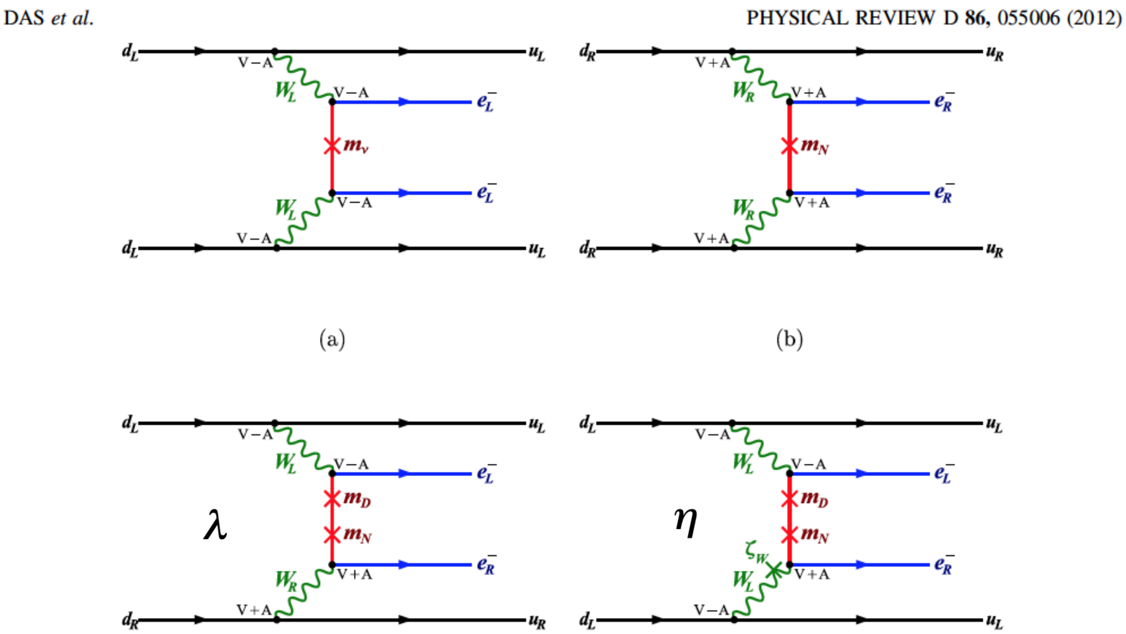


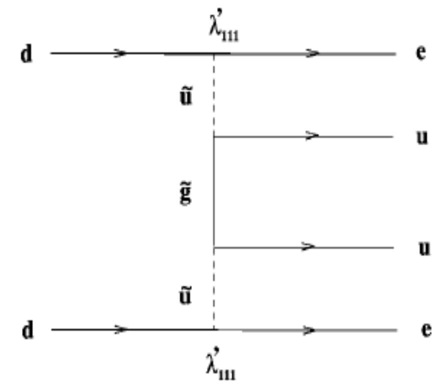
$$T_{1/2}^{-1}(0\nu) = G^{0\nu}(Q_{\beta\beta})\left[M^{0\nu}(0^+)\right]^2\left(\frac{< m_{\beta\beta} >}{m_e}\right)^2$$

$$\left|m_{\beta\beta}\right| = \left|\sum_{k=1}^{3} m_k U_{ek}^2\right| = \left|c_{12}^2 c_{13}^2 m_1 + c_{13}^2 s_{12}^2 m_2 e^{i\phi_2} + s_{13}^2 m_3 e^{i\phi_3}\right|$$

Experiment	Isotope	Half-life limit (10^{26} years)	mββ limit (meV)
MAJORANA	Germanium-76	0.83	113–269
GERDA	Germanium-76	1.8	79–180
EXO-200	Xenon-136	0.35	93–286
KamLAND-Zen	Xenon-136	2.3	36–156
CUORE	Tellurium-130	0.22	90–305

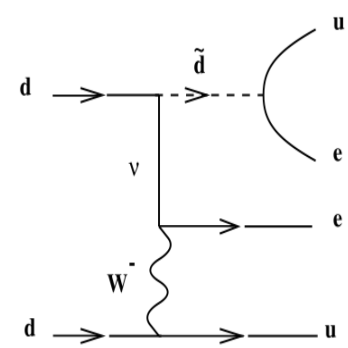
Other models: Left-Right Symmetric Model (LRSM) and SUSY R-parity violation





Gluino exchange

$$\left[T^{0\nu}_{1/2}\right]^{-1} = G_{01} g_A^4 \left| \eta_{0\nu} M_{0\nu} + \left(\eta^L_{N_R} + \eta^R_{N_R}\right) M_{0N} + \eta_{\tilde{q}} M_{\tilde{q}} + \eta_{\lambda'} M_{\lambda'} + \eta_\lambda X_\lambda + \eta_\eta X_\eta \right|^2 .$$



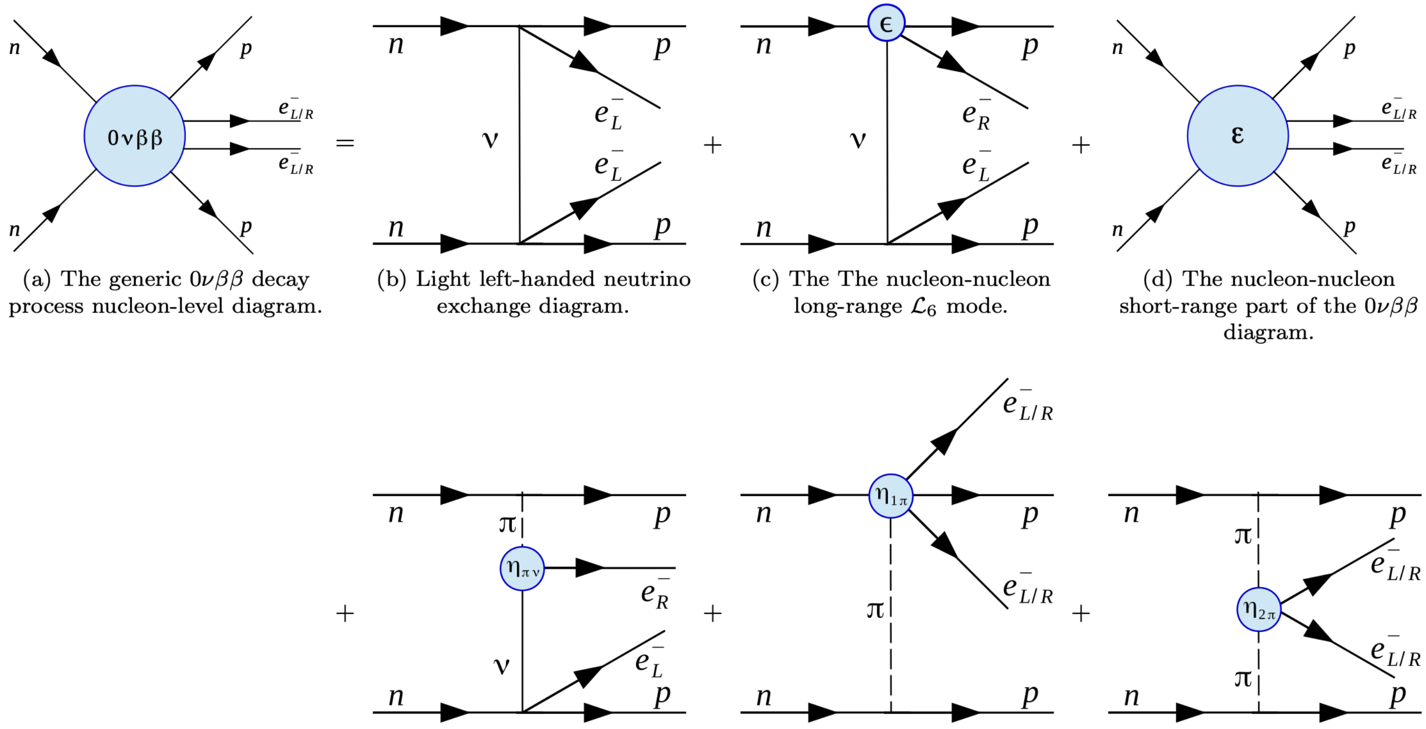
Squark exchange

(e)

M. Horoi, A. Neacsu, PRD 93, 113014 (2016)

Central Michigan University

Effective field theory after hadronization



(a) The generic $0\nu\beta\beta$ decay process nucleon-level diagram.

(b) Light left-handed neutrino exchange diagram.

(c) The The nucleon-nucleon long-range $\mathcal{L}_6$ mode.

(d) The nucleon-nucleon short-range part of the $0\nu\beta\beta$ diagram.

(e) The pion-neutrino long-range diagram.

(f) The one-pion long-range diagram.

(g) The two-pion long-range diagram.

$$\left[T^{0\nu}_{1/2}\right]^{-1} = g_A^4 \left[\sum_i |\mathcal{E}_i|^2 \mathcal{M}_i^2 + \mathrm{Re}\left[\sum_{i\neq j} \mathcal{E}_i \mathcal{E}_j \mathcal{M}_{ij}\right]\right]$$

$$\mathcal{E}_{2-7} = \{\epsilon^{V+A}_{V-A},\ \epsilon^{V+A}_{V+A},\ \epsilon^{S+P}_{S\pm P},\ \epsilon^{TR}_{TR},\ \eta_{\pi\nu}\}$$

$$\mathcal{E}_{8-15} = \{\varepsilon_1,\ \varepsilon_2,\ \varepsilon_3^{LLz(RRz)},\ \varepsilon_3^{LRz(RLz)},\ \varepsilon_4,\ \varepsilon_6,\ \eta_{1\pi},\ \eta_{2\pi}\}$$

M. Horoi, A. Neacsu, PRC 98, 035502 (2018)

U.S. Department of Energy Office of Science

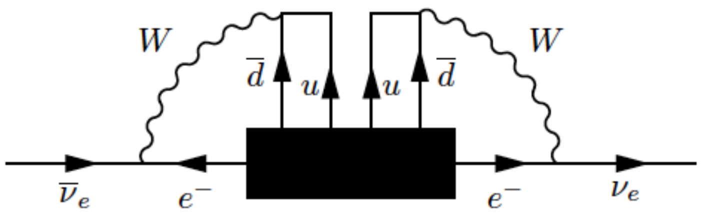
The Black Box Theorems

Black box I (electron neutrino)

J. Schechter and J.W.F Valle, PRD 25, 2951 (1982)

E. Takasugi, PLB 149, 372 (1984)

J.F. Nieves, PLB 145, 375 (1984)



0νββ observed ⇔ at some level

(i) Lepton number conservation is violated by 2 units.

(ii) Electron neutrinos are Majorana fermions (with m > 0).

However:

M. Duerr et al, JHEP 06 (2011) 91

$$\left(\delta m_{\nu_e}\right)_{BB} \sim 10^{-24}\, eV << \sqrt{\left|\Delta m_{32}^2\right|} \approx 0.05\, eV$$

Black box II (all flavors + oscillations)

M. Hirsch, S. Kovalenko, I. Schmidt, PLB 646, 106 (2006)

0νββ observed ⇔ at some level

(i) Lepton number conservation is violated by 2 units.

(ii) Neutrinos are Majorana fermions.

$$(iii)\ \ \langle m_{\beta\beta}\rangle = \left|\sum_{k=1}^{3} m_k U_{ek}^2\right| = \left|c_{12}^2 c_{13}^2 m_1 + c_{13}^2 s_{12}^2 m_2 e^{i\phi_2} + s_{13}^2 m_3 e^{i\phi_3}\right| > 0$$

Regardless of the dominant 0νββ mechanism!

Central Michigan University

NME: mass mechanism

Rev. Mod. Phys. 95, 025002 (2023)

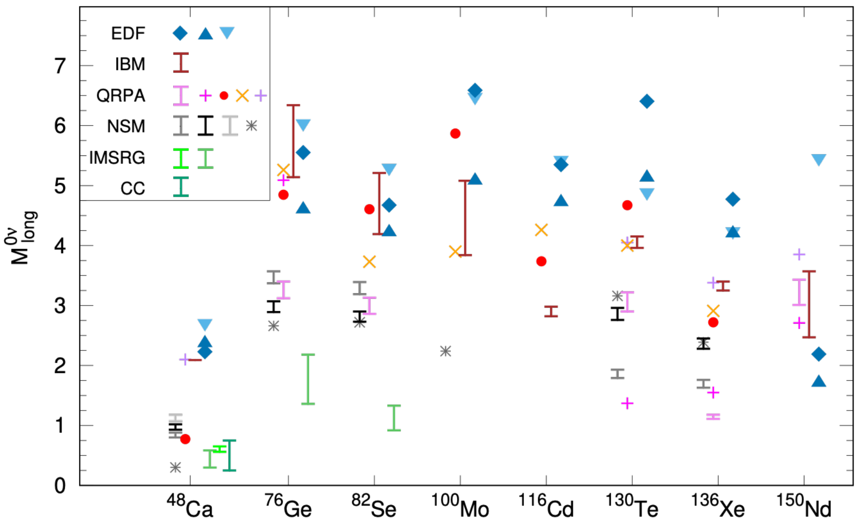
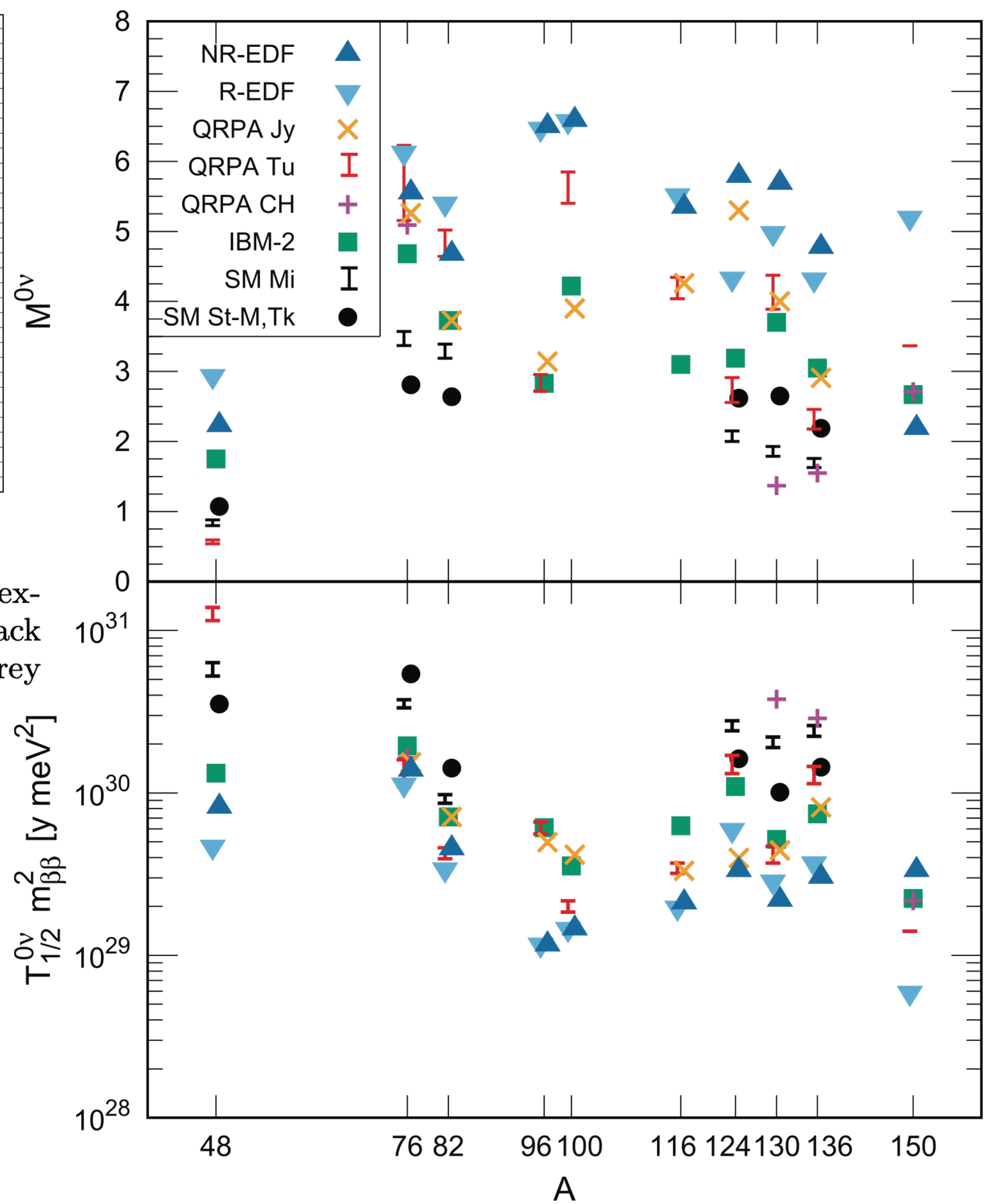


FIG. 9 Nuclear matrix elements $M^{0\nu}$ for light-neutrino exchange from different many-body methods. NSM: black (Menéndez, 2018), grey (Horoi and Neacsu, 2016b), light-grey

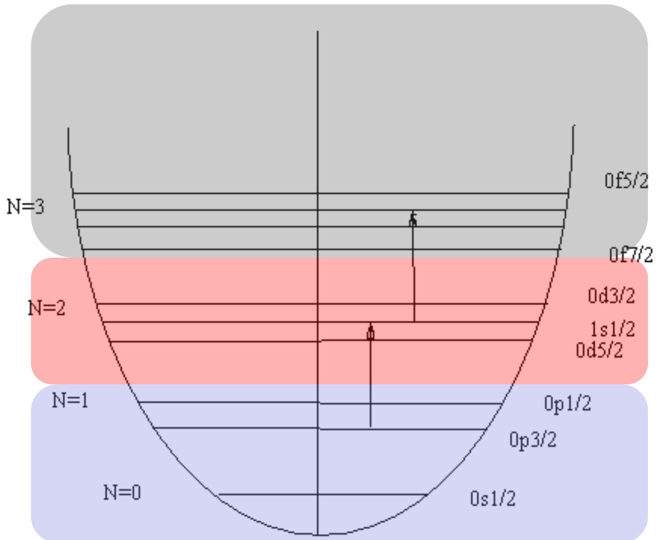
$$M_{0\nu} = M_{GT} + M_F + M_T + \ldots$$

$$\left(T^{0\nu}_{1/2}\right)^{-1} = G_{01} g_A^4 |M_{0\nu}|^2 \left|m_{\beta\beta}\right|^2$$

Rep. Prog. Phys. **80** (2017) 046301



Shell Model Effective Hamiltonians



empty

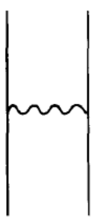
valence

frozen core

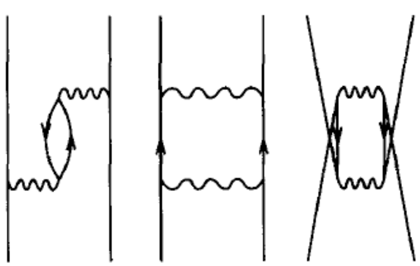
$$H_{valence} = H_{2-body}$$

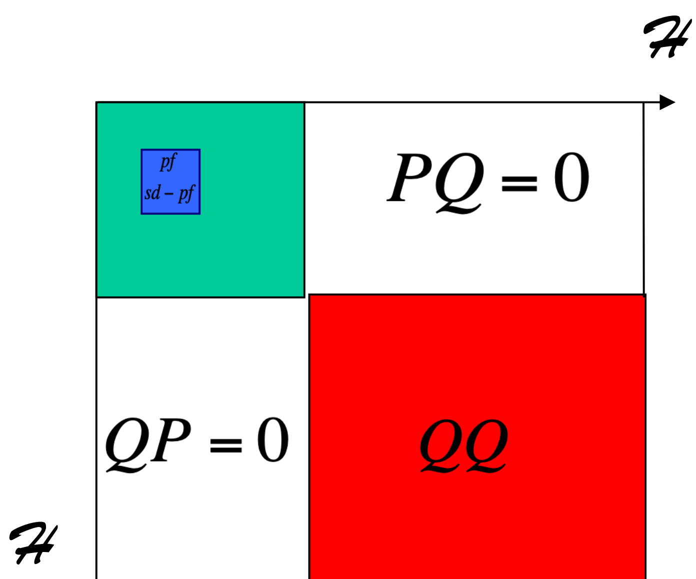
can describe most correlations around the Fermi surface!

$$H_{valence}\Psi = E_n\Psi$$

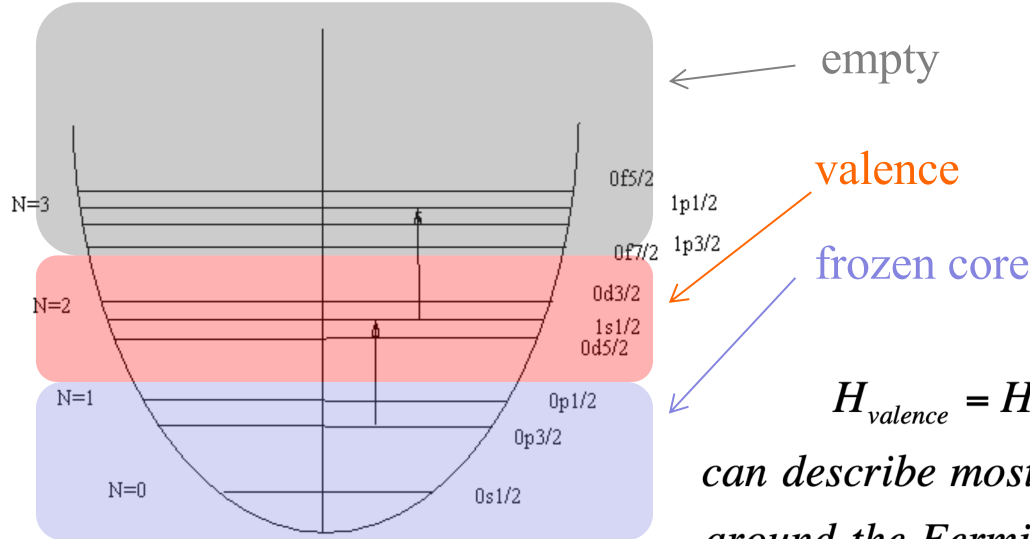


core polarization: Phys.Rep. **261**, 125 (1995)





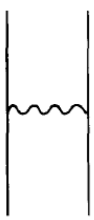
Shell Model Effective Hamiltonians



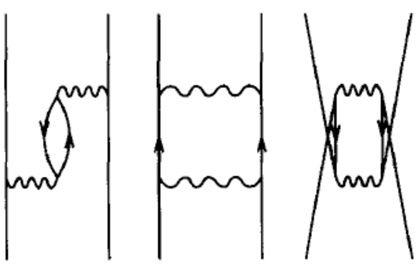
$$H_{valence}\Psi = E_n\Psi$$

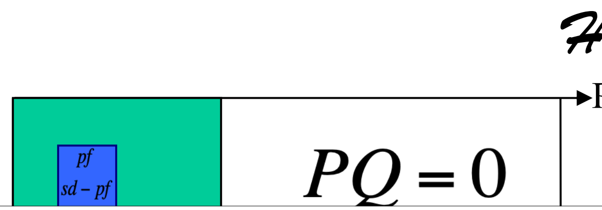
$$H_{valence} = H_{2-body}$$

can describe most correlations around the Fermi surface!

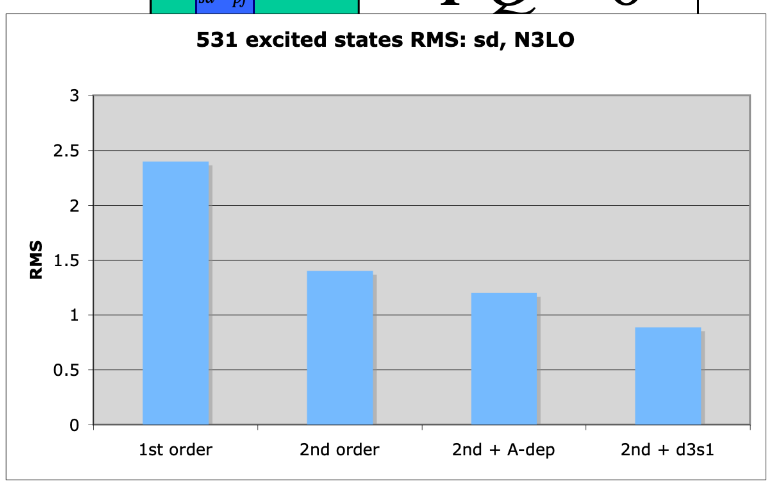


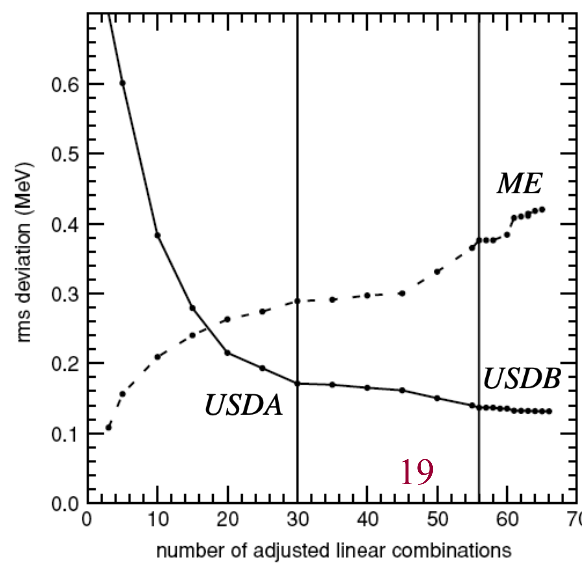
core polarization: Phys.Rep. **261**, 125 (1995)



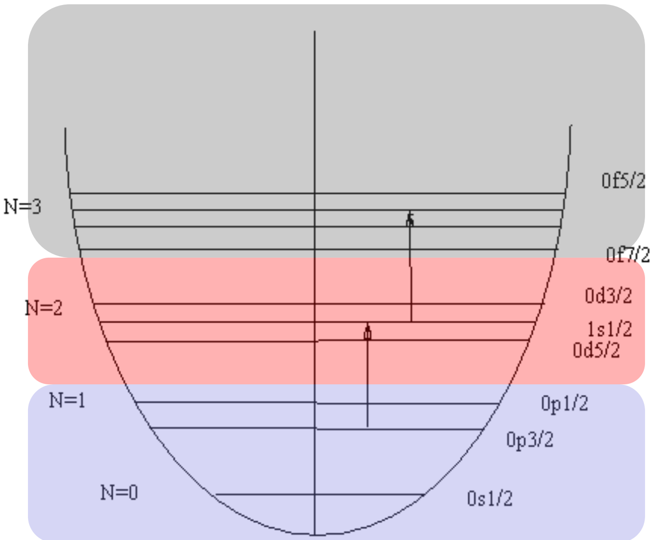


PRC 74, 34315 (2006), 78, 064302 (2008)





Shell Model Effective Hamiltonians



$$H_{valence}\Psi = E_n\Psi$$

$$g_A\sigma\tau \xrightarrow{quenched} g_A 0.77\sigma\tau$$

empty

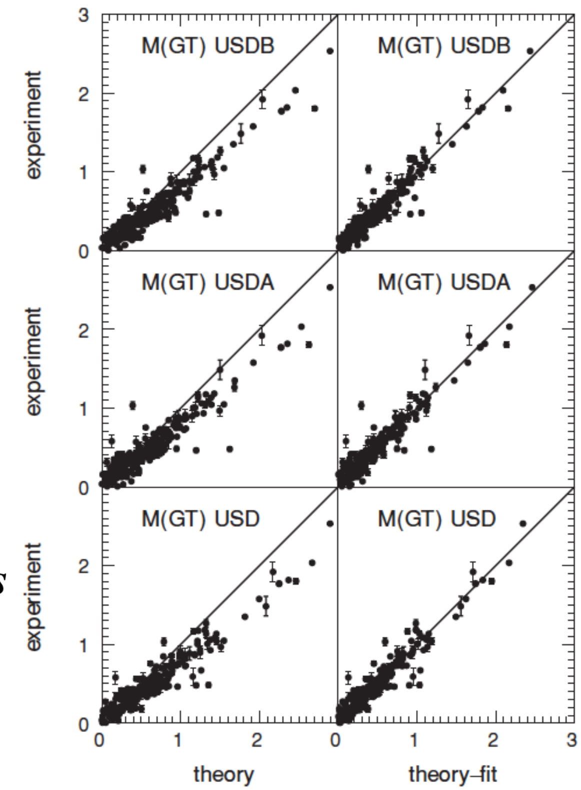
q

valence

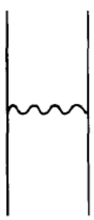
frozen core

$$H_{valence} = H_{2-body}$$

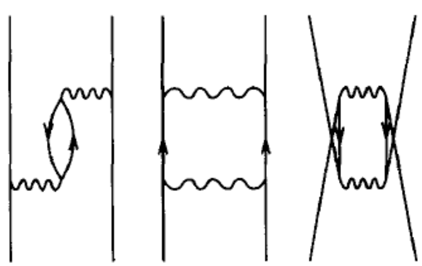
can describe most correlations around the Fermi surface!



$\mathcal{H}$

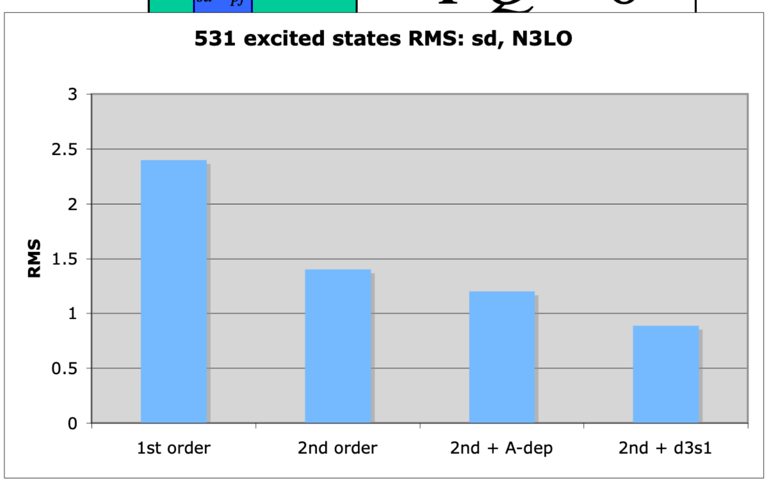


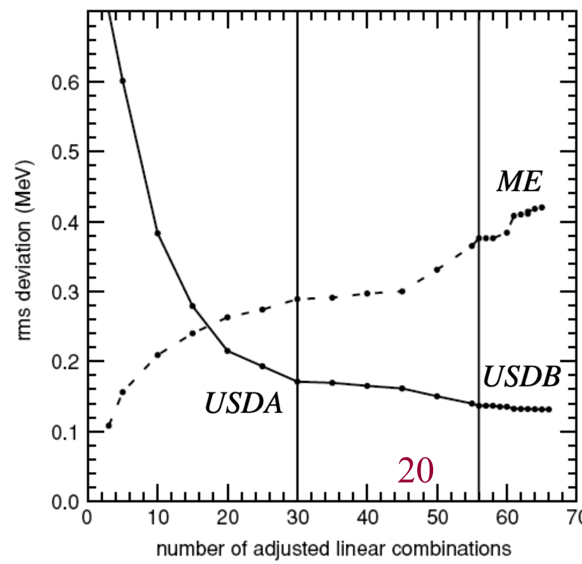
core polarization: Phys.Rep. **261**, 125 (1995)



$$PQ = 0$$

PRC 74, 34315 (2006), 78, 064302 (2008)





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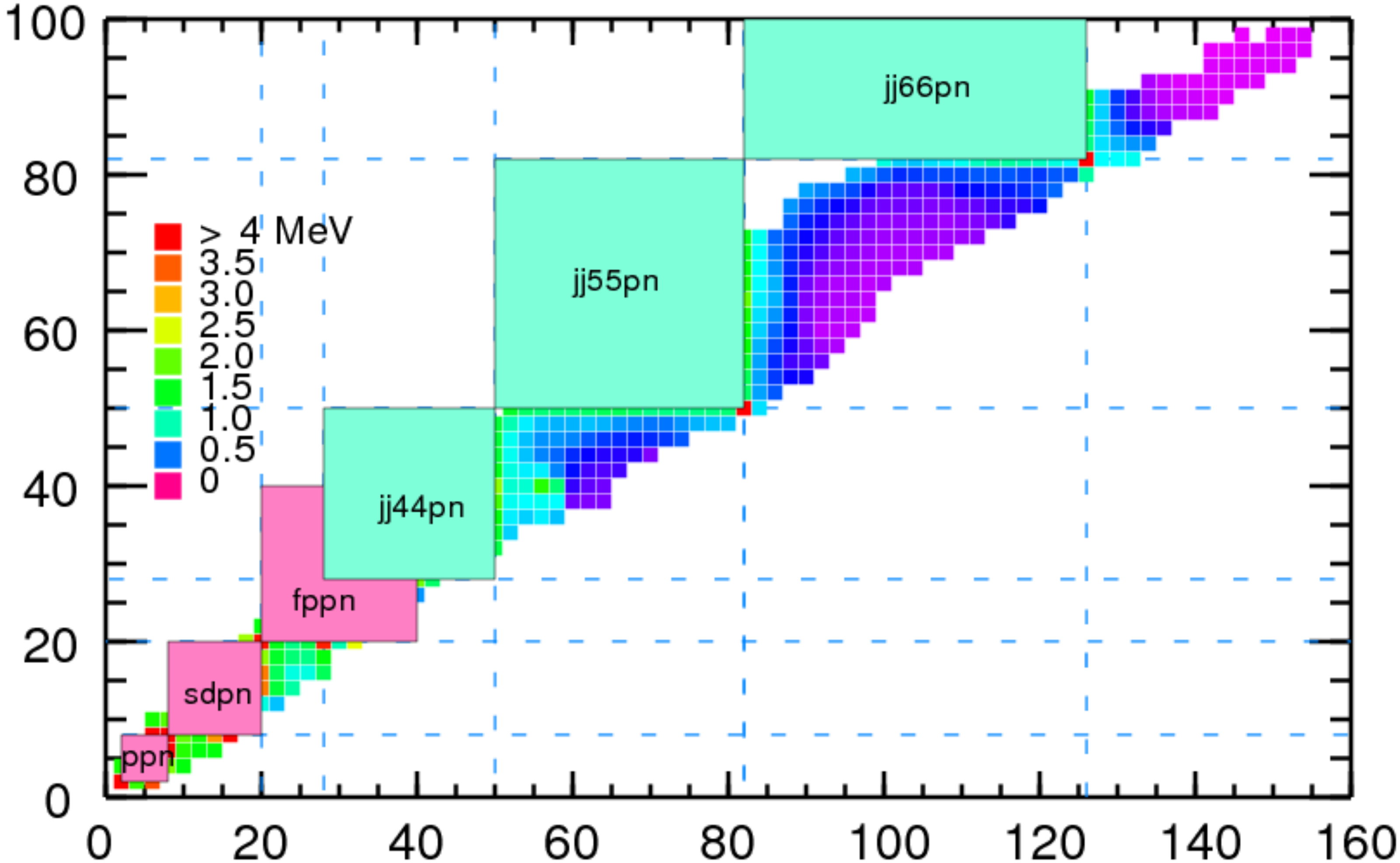
Shell Model Spaces



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Statistical analysis of the NME

PHYSICAL REVIEW C **107**, 045501 (2023)

Predicting the neutrinoless double-β-decay matrix element of ^{136}Xe using a statistical approach

M. Horoi,[1] A. Neacsu,[2,3] and S. Stoica[2]

[1]*Department of Physics, Central Michigan University, Mount Pleasant, Michigan 48859, USA*

[2]*International Center for Advanced Training and Research in Physics (CIFRA), Magurele 077125, Romania*

[3]*Horia Hulubei National Institute of Physics and Nuclear Engineering (IFIN-HH), Magurele 077125, Romania*

PHYSICAL REVIEW C **106**, 054302 (2022)

Statistical analysis for the neutrinoless double-β-decay matrix element of ^{48}Ca

M. Horoi,[1] A. Neacsu,[2,3] and S. Stoica[2]

[1]*Department of Physics, Central Michigan University, Mount Pleasant, Michigan 48859, USA*

[2]*International Center for Advanced Training and Research in Physics (CIFRA), Magurele 077125, Romania*

[3]*Horia Hulubei National Institute for Physics and Nuclear Engineering (IFIN-HH), Magurele 077125, Romania*

IOP Publishing — Journal of Physics G: Nuclear and Particle Physics

J. Phys. G: Nucl. Part. Phys. **49** (2022) 120502 (30pp) https://doi.org/10.1088/1361-6471/aca03e

Towards precise and accurate calculations of neutrinoless double-beta decay

V Cirigliano[1], Z Davoudi[2], J Engel[3,*], R J Furnstahl[4], G Hagen[5,6], U Heinz[4], H Hergert[7], M Horoi[8], C W Johnson[9], A Lovato[10,11,12], E Mereghetti[13], W Nazarewicz[14,*], A Nicholson[3,15], T Papenbrock[5,6], S Pastore[16], M Plumlee[17,18], D R Phillips[19,*], P E Shanahan[20], S R Stroberg[10], F Viens[21], A Walker-Loud[15], K A Wendt[22] and S M Wild[18,23]

Statistical Model: why ^{136}Xe?

- One of the longest half-life limit measured, ~$3x10^{26}$ years
- One of the potential isotopes chosen for the next "tonne experiment"
- Relatively low shell model dimensions in the *jj55* ($0g_{7/2}, 2s, 1d, 0h_{11/2}$) model space

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Statistical Model: Shell Model Effective Hamiltonians

- Isospin conserving effective Hamiltonians in *jj55* model space ($0g_{7/2}$, $2s1d$, $0h_{11/2}$)
- 327 TBME of starting Hamiltonians $\pm$10% uniformly random changes
- Three starting effective Hamiltonians:

=> SVD, PRC 86 044323 (2012)

=> GCN5082, PRC 82 064304 (2010)

=> jj55t, PRL 110 222502 (2013)

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Statistical Model: Observables (24)

- 0nbb NME (1): $M_{0\nu}$, short-range correlator CD-Bonn
- 2nbb NME (1): $M_{2\nu}$, $q = 0.7$
- Energies 2+, 4+, 6+ (6): PE_{2+}, PE_{4+}, PE_{6+}, DE_{2+}, DE_{4+}, DE_{6+}
- B(GT) to 1+1 state (2): PGT, DGT, $q = 0.7$
- B(E2) $2^+ \to 0^+$ (2): PBE2, DBE2 $e^p_{eff} = 1.5$ $e^n_{eff} = 0.5$
- Proton occupation (8): $POPg_7$, $POPs_1$, POPd, $POPh_{11}$, $DOPg_7$, $DOPs_1$, DOPd, $DOPh_{11}$
- Neutron vacancies (4): $DVNg_7$, $DVNs_1$, DVNd, $DVNh_{11}$
- Muon Capture rate (0)
- B(M1) (0)

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Overview

Observable	Data	Error	svd_s	gcn_s	$j5t_s$	μ_{svd}	σ_{svd}	μ_{gcn}	σ_{gcn}	μ_{j5t}	σ_{j5t}
$M_{0\nu}$	N/A	N/A	1.763	2.645	2.314	1.749	0.111	2.632	0.135	2.306	0.156
$M_{2\nu}$	0.018	0.001	0.025	0.069	0.060	0.022	0.003	0.061	0.007	0.052	0.007
PGT	0.150	0.021	0.163	0.545	0.512	0.141	0.059	0.457	0.105	0.333	0.220
P$BE2$	0.286	0.081	0.154	0.121	0.096	0.153	0.009	0.122	0.013	0.099	0.012
PE_{2^+}	1.313	0.150	1.498	1.363	1.513	1.494	0.089	1.352	0.089	1.507	0.098
PE_{4^+}	1.694	0.150	2.073	1.747	2.012	2.070	0.089	1.740	0.107	2.011	0.107
PE_{6^+}	1.892	0.150	2.178	1.892	2.254	2.192	0.088	1.884	0.125	2.212	0.096
POPg_7	2.930	0.100	2.705	2.716	3.143	2.702	0.187	2.705	0.209	3.082	0.267
POPs_1	0.057	0.006	0.089	0.025	0.020	0.090	0.018	0.025	0.006	0.021	0.006
POPh_{11}	0.400	0.040	0.190	0.375	0.265	0.189	0.020	0.373	0.050	0.265	0.045
POPd	0.520	0.030	1.016	0.884	0.572	1.019	0.180	0.896	0.197	0.632	0.250
DGT	0.012	0.005	0.001	0.009	0.004	0.001	0.000	0.008	0.003	0.003	0.003
D$BE2$	0.413	0.011	0.342	0.194	0.158	0.337	0.023	0.195	0.026	0.163	0.028
DE_{2^+}	0.819	0.150	0.662	0.842	0.917	0.660	0.067	0.836	0.056	0.919	0.049
DE_{4^+}	1.867	0.150	1.389	1.873	2.113	1.403	0.131	1.861	0.116	2.087	0.082
DE_{6^+}	2.207	0.150	2.157	2.196	2.502	2.171	0.151	2.197	0.090	2.507	0.117
DVNg_7	0.000	0.150	0.102	0.174	0.130	0.100	0.010	0.172	0.014	0.132	0.023
DVNs_1	0.080	0.020	0.271	0.251	0.415	0.286	0.117	0.255	0.058	0.407	0.110
DVNh_{11}	1.680	0.130	1.205	0.726	0.347	1.177	0.237	0.724	0.132	0.385	0.162
DVNd	0.240	0.050	0.423	0.850	1.108	0.437	0.132	0.850	0.118	1.076	0.158
DOPg_7	3.860	0.100	3.189	3.475	4.145	3.187	0.209	3.477	0.249	4.078	0.436
DOPs_1	0.200	0.020	0.263	0.083	0.049	0.264	0.047	0.084	0.020	0.052	0.017
DOPh_{11}	0.620	0.060	0.264	0.658	0.625	0.269	0.049	0.658	0.093	0.613	0.121
DOPd	1.290	0.080	2.285	1.785	1.181	2.280	0.227	1.781	0.265	1.258	0.447

Table I. All relevant data and statistics for all selected observables.

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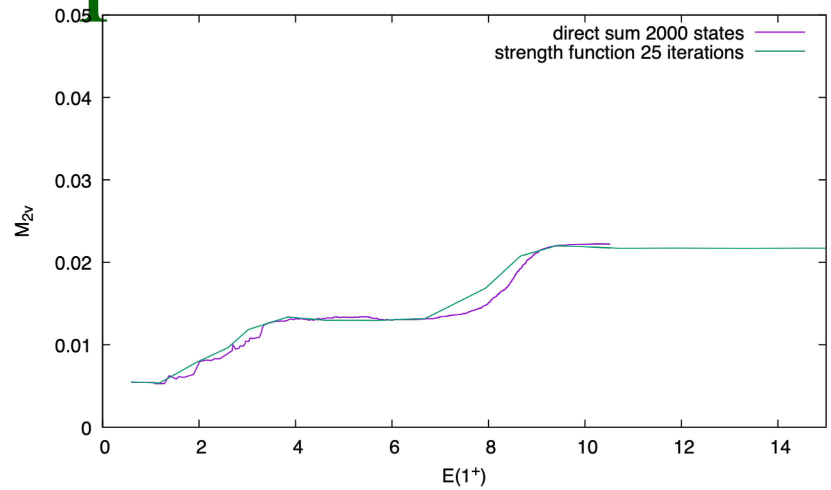
The Late Bump in ^{136}Xe

$$M_{2v}(E_k) = \sum_{n=1}^{k} \frac{\langle\Psi_f|q\sigma\tau^-|1_n^+\rangle\langle 1_n^+|q\sigma\tau^-|\Psi_i\rangle}{E(1_n^+) + \frac{Q_{\beta\beta}}{2} - Q_\beta}$$



^{136}Xe – SVD

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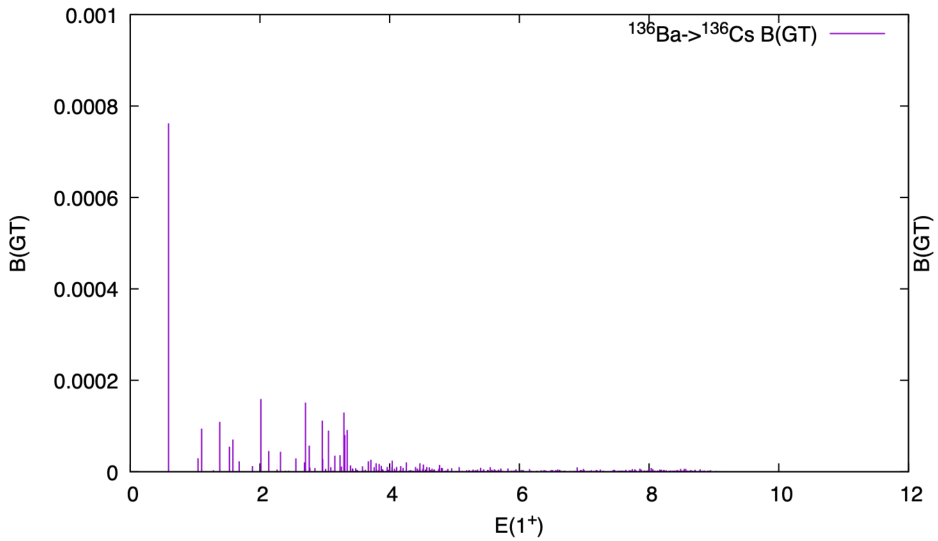

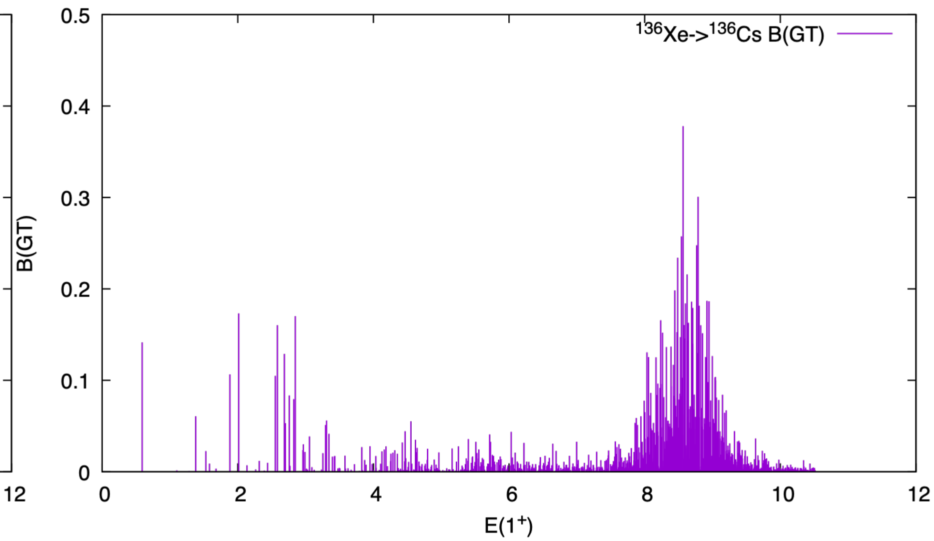

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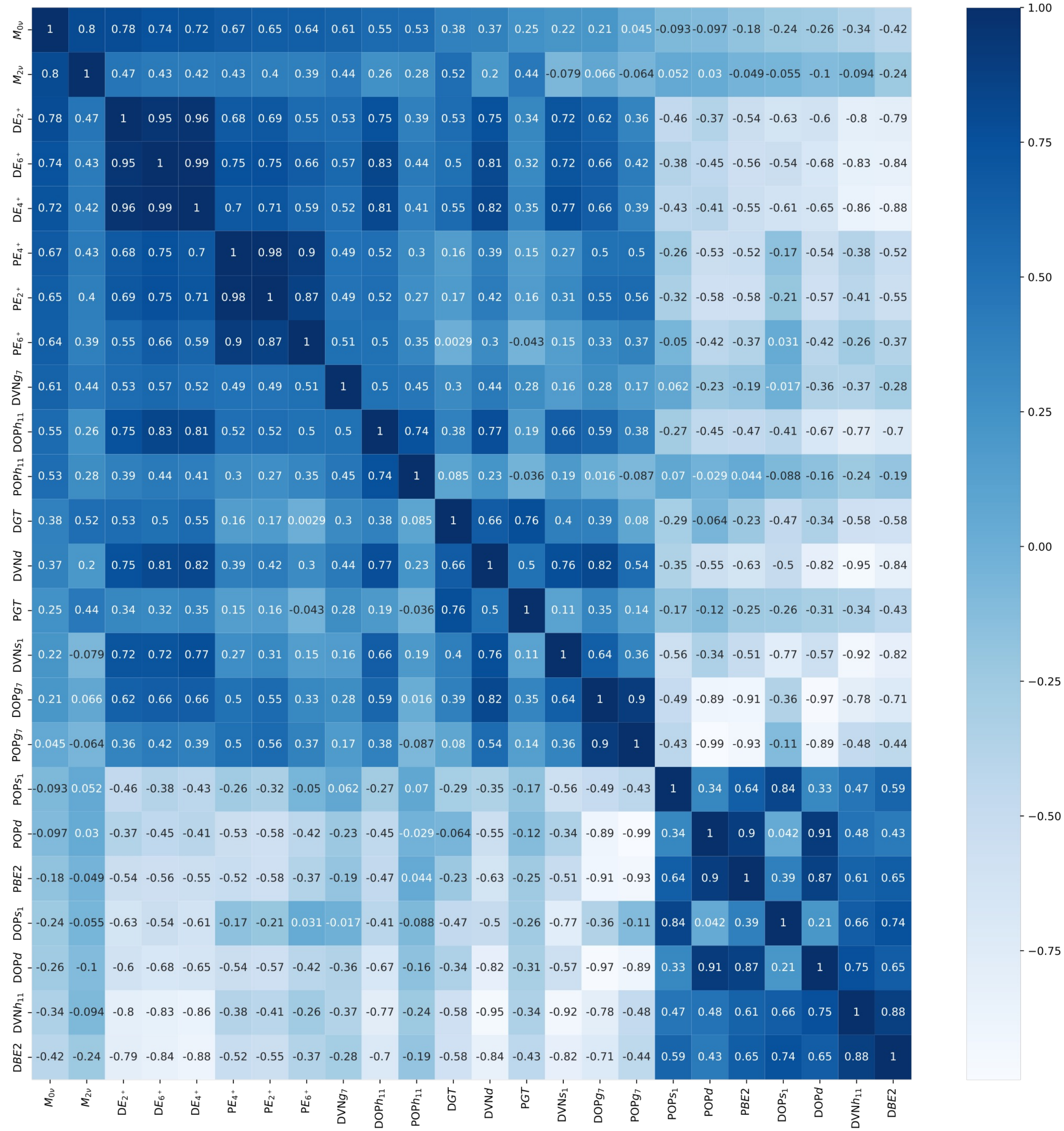
Statistical Model: Heat Map

samples=1,000



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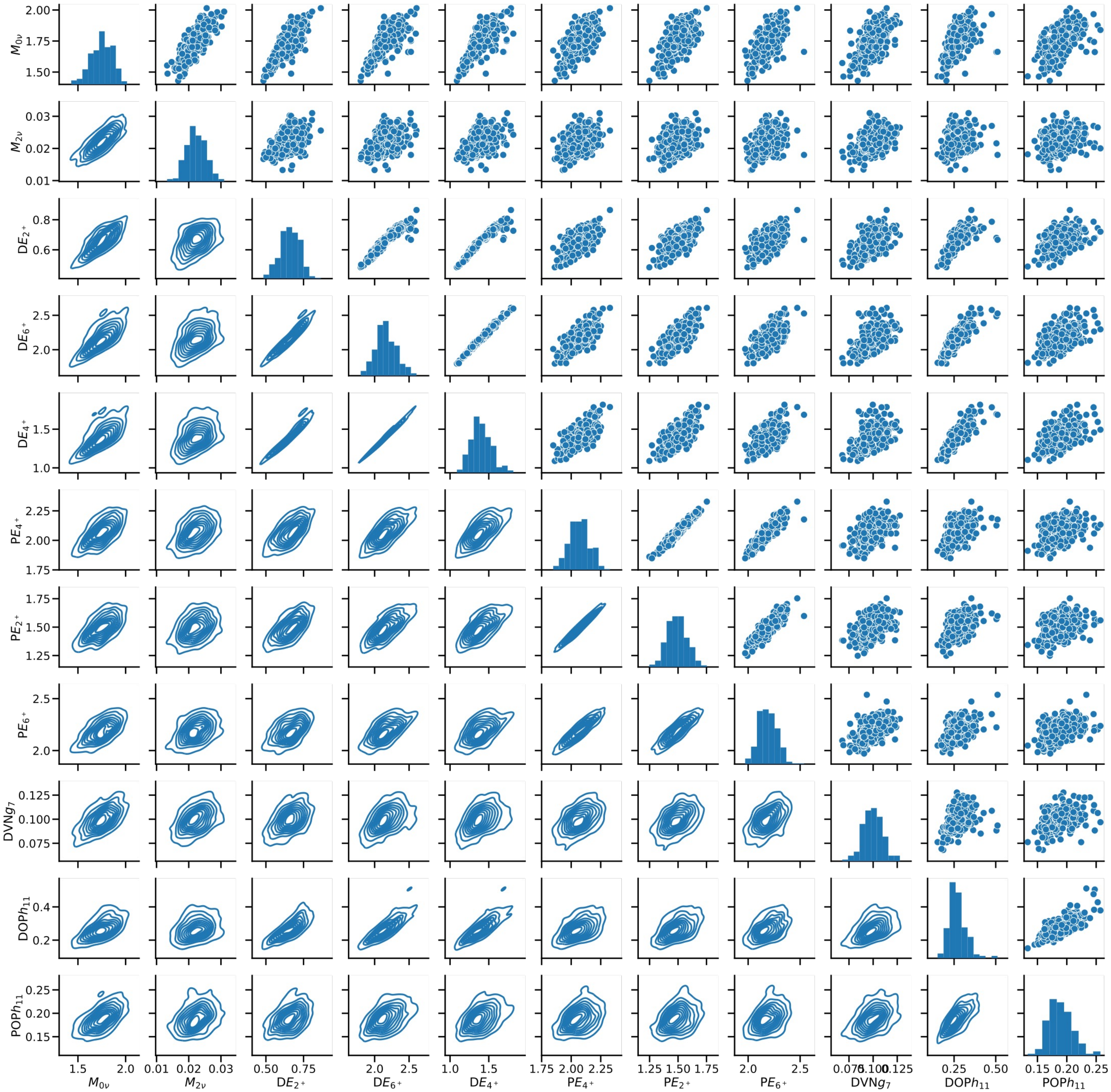
Statistical Model: Correlations Map



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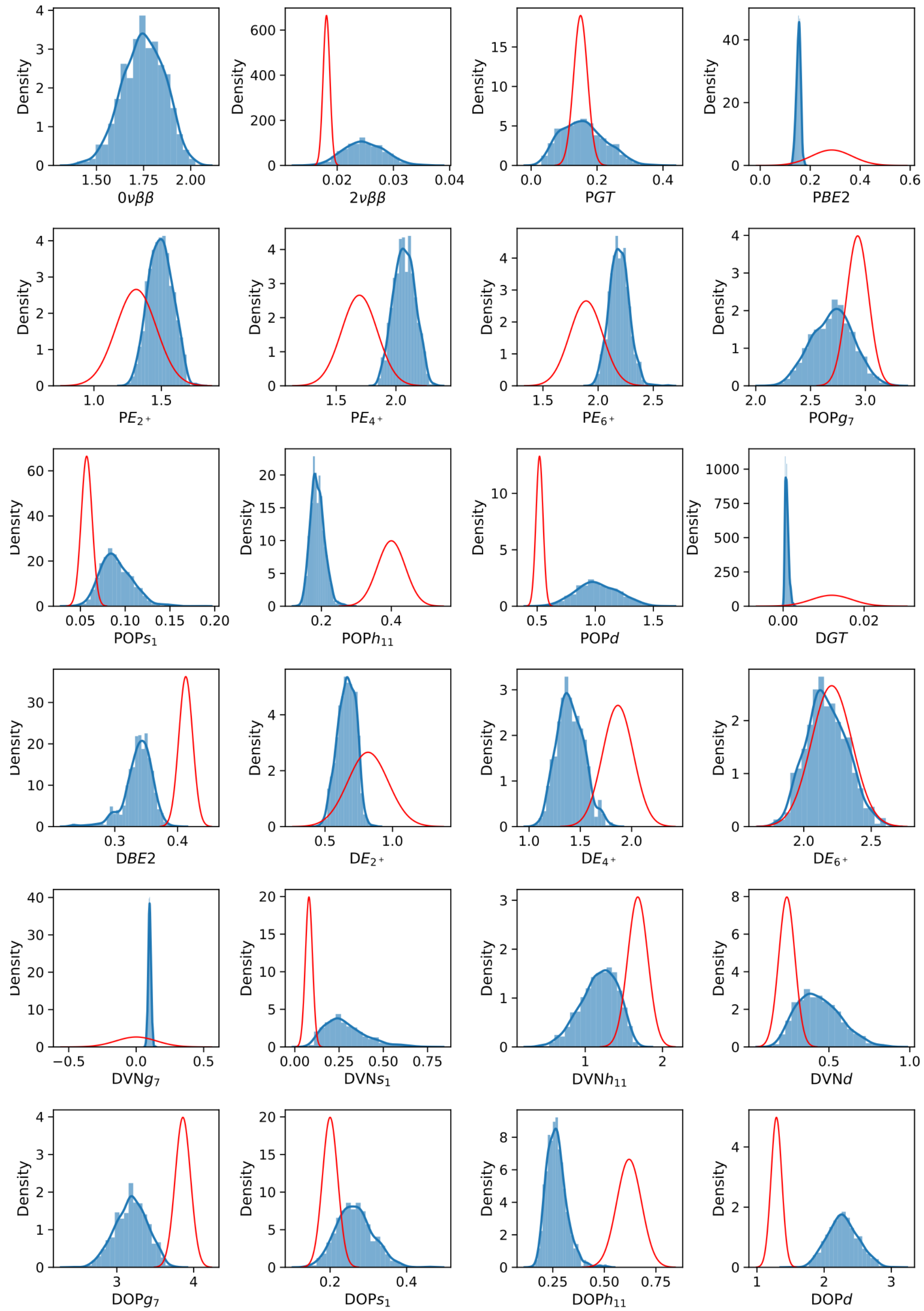
Statistical Model: Density Distributions



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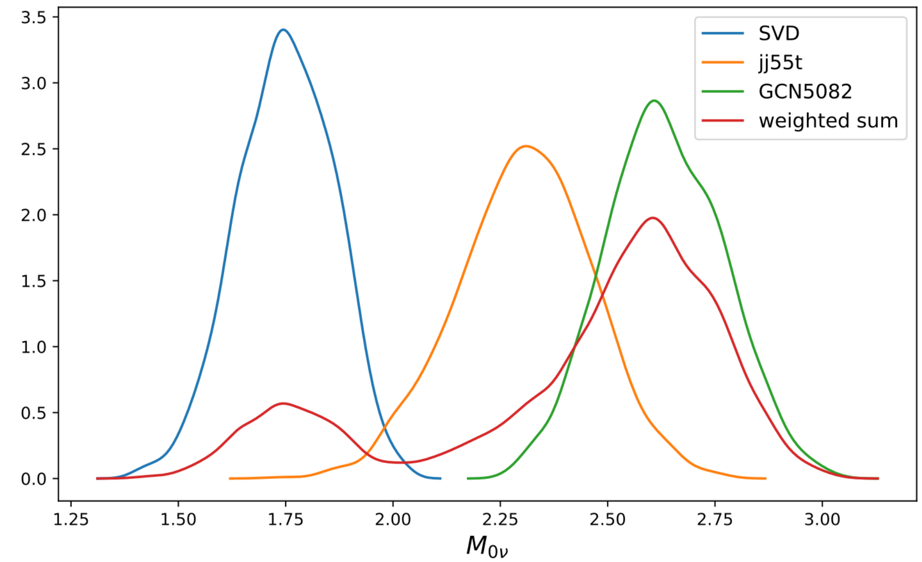
M. Horoi, Universe 10, 252 (2024)



Statistical Model: joint distribution

$$P(x=M_{0\nu}) = W_{svd}P_{svd}(x) + W_{gcn}P_{gcn}(x) + W_{j5t}P_{j5t}(x) \ ,$$

Statistical Model: Bayesian Model Averaging

$M_{0\nu} = 2.2 \pm 0.48$

$$p(x=M_{0\nu}|y_e,\sigma_e) = \sum_{k=svd}^{j5t} p(x=M_{0\nu}|y_e,\sigma_e,\mathcal{M}_k)p(\mathcal{M}_k|y_e,\sigma_e),$$

$$p(\mathcal{M}_k|y_e,\sigma_e) = \frac{p(y_e,\sigma_e|\mathcal{M}_k)\pi(\mathcal{M}_k)}{\sum_{k=svd}^{j5t} p(y_e,\sigma_e|\mathcal{M}_k)\pi(\mathcal{M}_k)}$$

W_H

$$p(y_e,\sigma_e|\mathcal{M}_k) = \int \prod_i^{N_{obs}} dy_i p_{y_e,\sigma_e}(y_i)\left[\int dq \prod_j^{N_{TBME}} d\theta_j p(y_i|\theta_j,q,\mathcal{M}_k)\pi(\theta_j,q|\mathcal{M}_k)\right],$$

Evidence integrals

$$p_{y_e,\sigma_e}(y_i) \propto \prod_i^{N_{obs}} exp[-(y_i - y_{e_i})^2/(2\sigma_{e_i}^2)]$$

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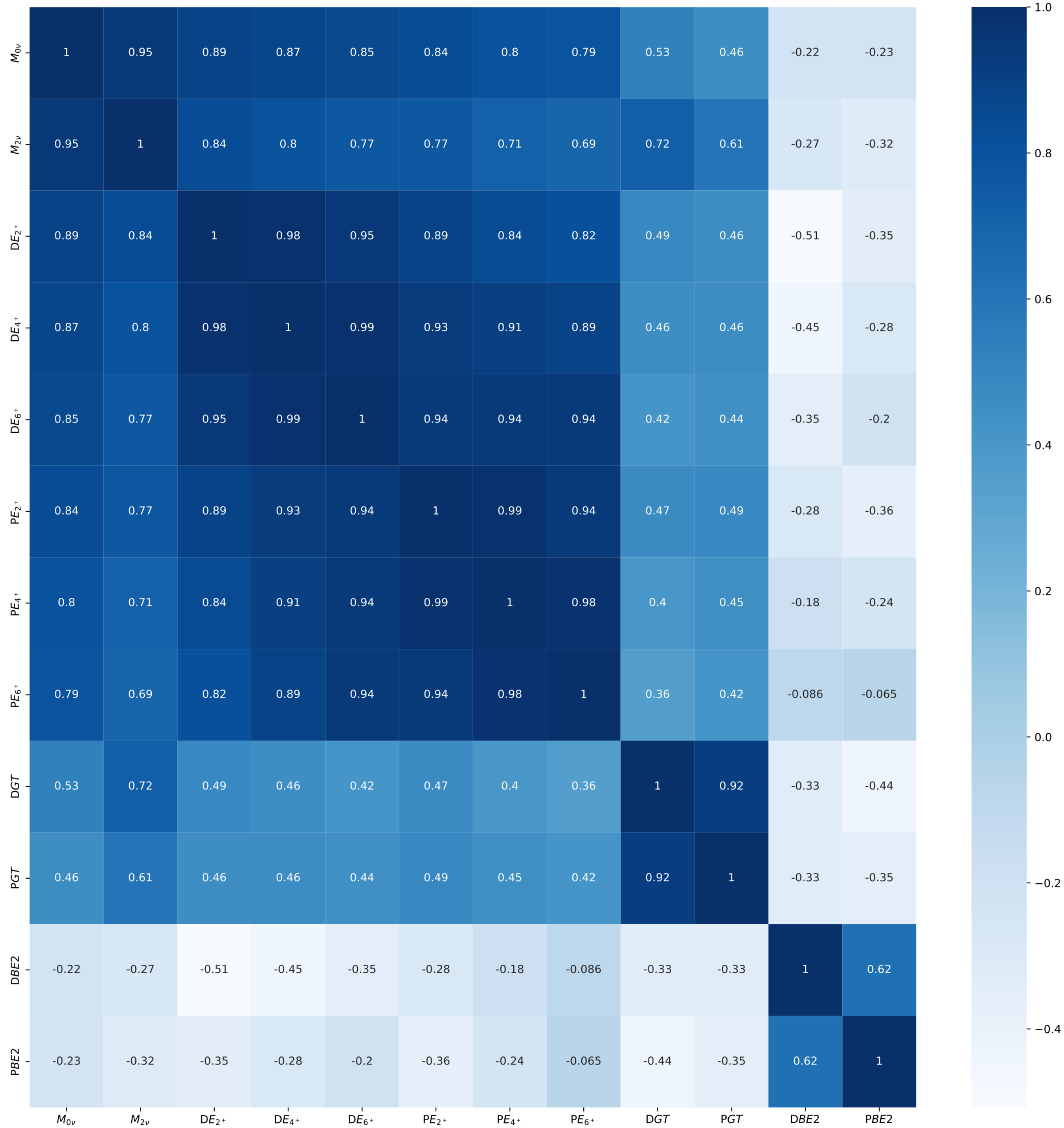
^{76}Ge: Heat Map

samples=200



2νββ NME: Direct Sum vs Strength Function Approach

Direct sum:

$$M_{2\nu} = \sum_k \frac{\left\langle 0_f^+ \mid (\sigma\tau^-)^{eff} \mid 1_k^+ \right\rangle \left\langle 1_k^+ \mid (\sigma\tau^-)^{eff} \mid 0_i^+ \right\rangle}{E_k - E_0}$$

$$(\sigma\tau^-)^{eff} = q\tau^-\sigma$$

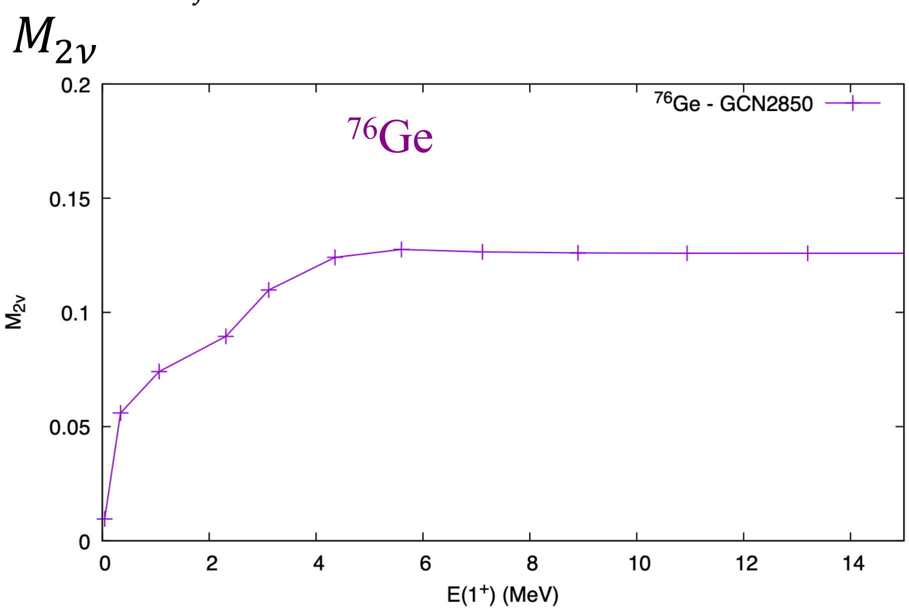
Strength function (Physics 2022, 4, 1135) :

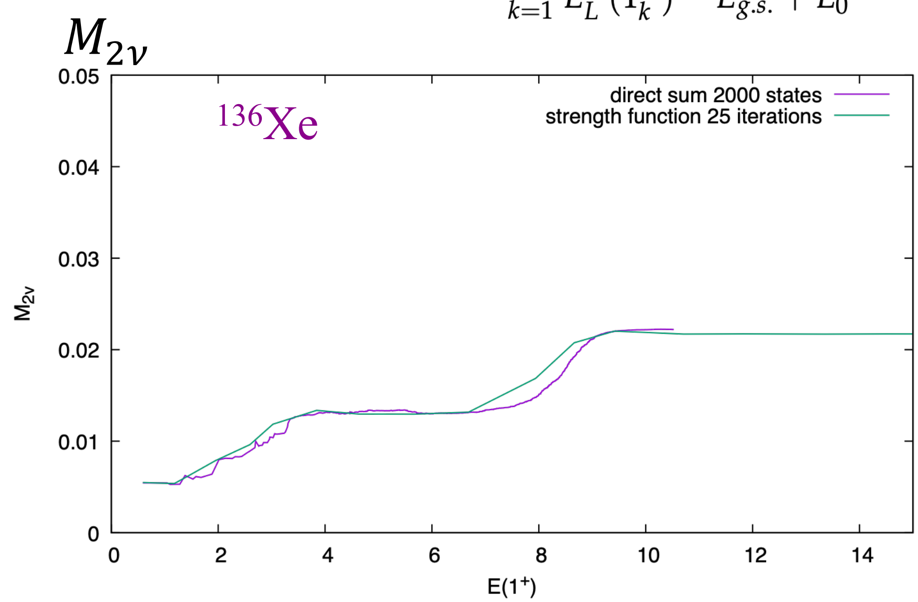
$$|\sigma\tau^- 0_i^+ >= c_-|dw_- >\equiv c_-|L_1^- >$$

$$M_{2\nu}(0^+) \approx 3c_+c_- \sum_m g_m^- < dw_+|L_m^- >$$

$$|\sigma\tau^+ 0_f^+ >= c_+|dw_+ >\equiv c_+|L_1^+ >$$

$$g_m^{\pm} = \sum_{k=1}^{N} \frac{V_{1k}^{\pm} V_{mk}^{\pm}}{E_L^N(1_k^+) - E_{g.s.} + E_0}$$





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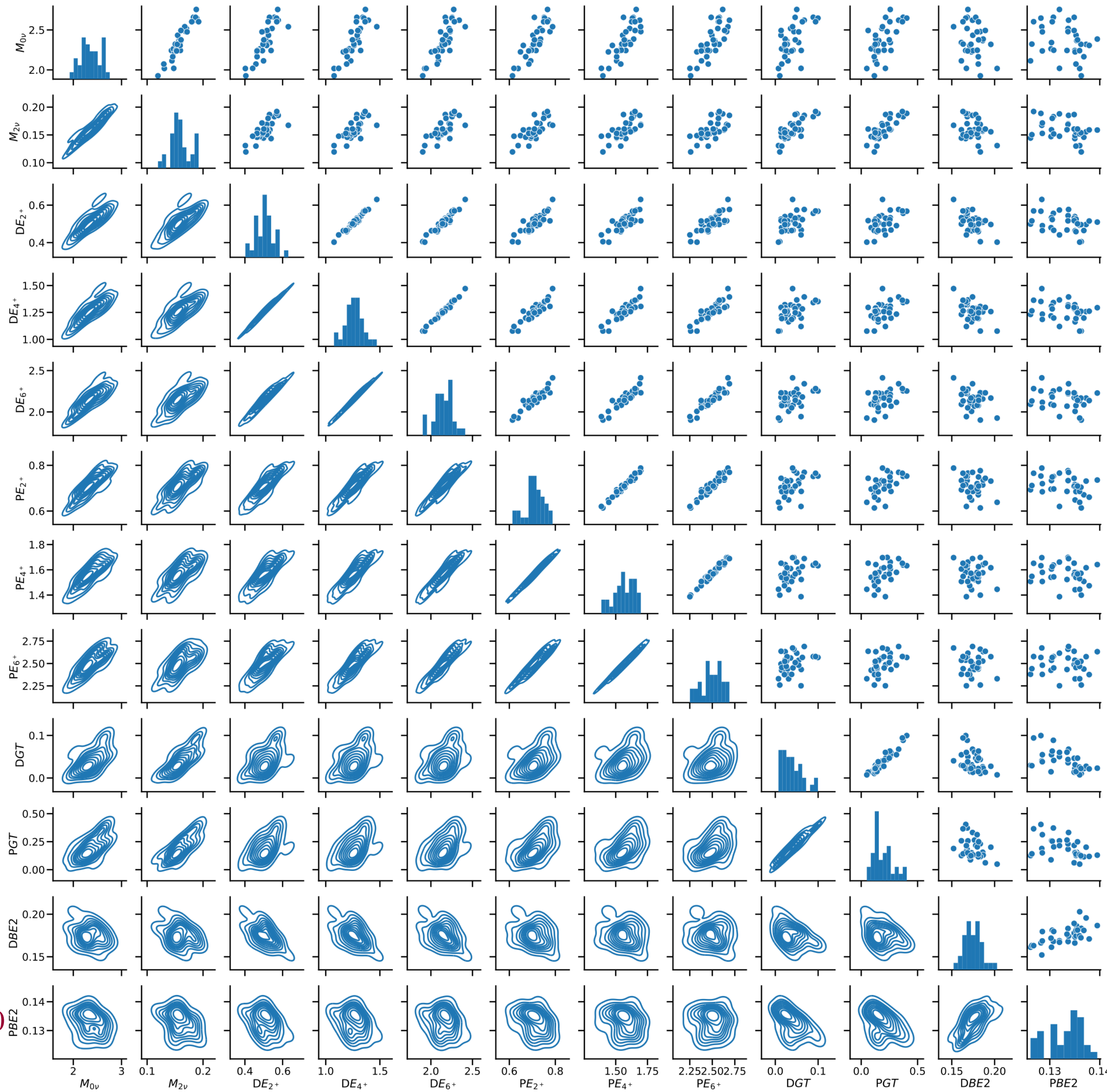
^{76}Ge: Correlations Map



INT-26-2a, 7/13/20

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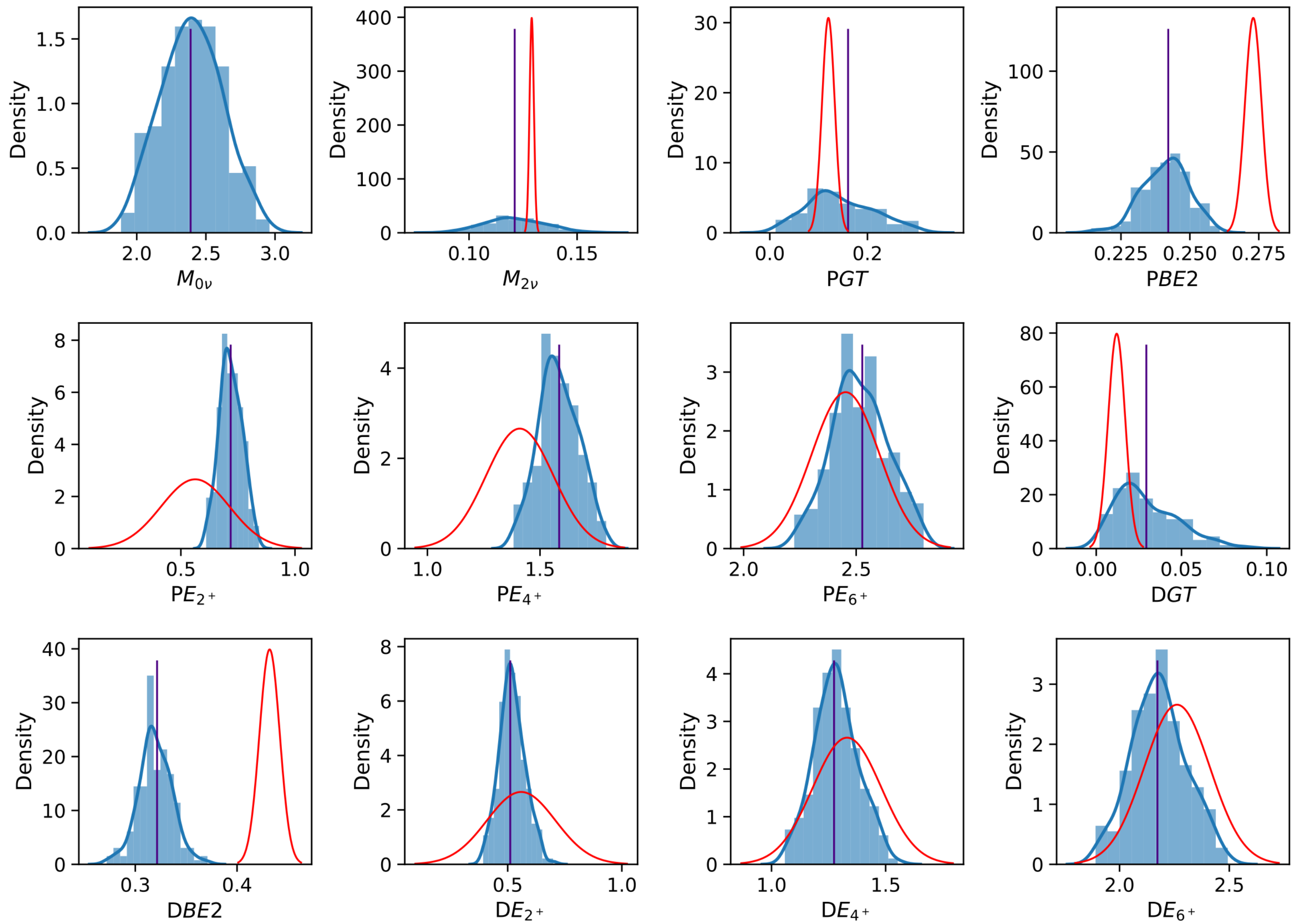
^{76}Ge: Density Distributions



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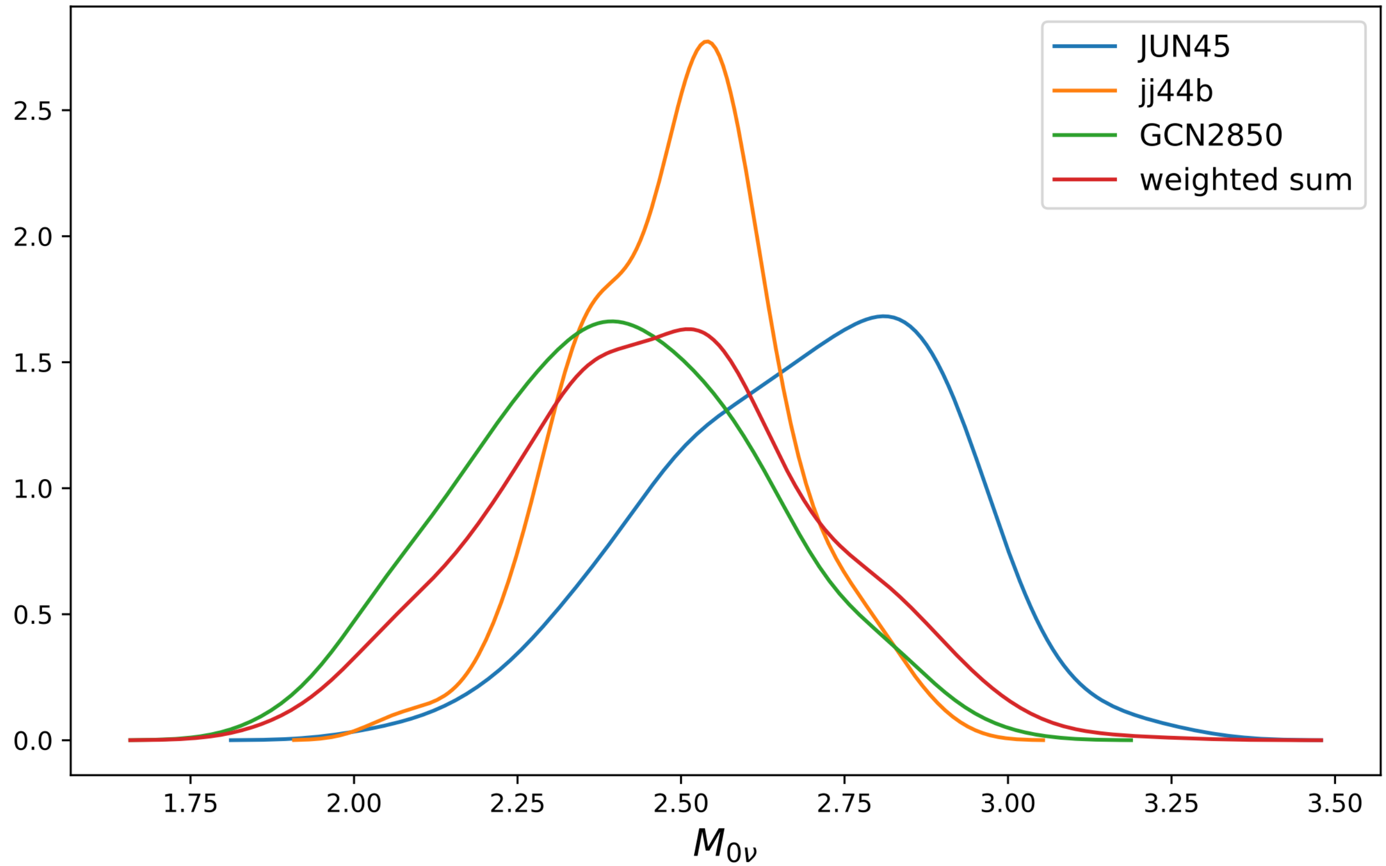
^{76}Ge: Joint Distributions

M. Horoi and A. Neacsu, arXiV:2605.21657



$M_{0\nu}$ =2.46 $\pm$ 0.26

1.93-3.02 @90%

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Statistical Model: Results

- Shell model observables show robustness relative to small random changes of the effective Hamiltonian
- $M_{0\nu}$ shows significant correlations with about half of the observables considered
- The outcome of our **Bayesian Averaging model** is a joint distribution of the ^{76}Ge $M_{0\nu}$ with a range of 1.93 – 3.02 at 90% confidence level
- We propose a $M_{0\nu}$ NME of 2.46 $\pm$ 0.26 for ^{76}Ge

Future work:

- Statistical investigation of the robustness of ^{76}Ge results
- Statistical investigation of the maximal interference effects

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