

Towards a more precise description of the neutrinoless double beta decay

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Plan of talk

- Neutrinoless double beta decay: Brief introduction
- Nuclear matrix elements
 - Shell model approach
 - Statistical analysis for ¹³⁶Xe
- Future work: Interference effects
- Conclusions and outlook





Classical Double Beta Decay Problem



Neutrino $\beta\beta$ effective mass

arxiv:1507.08204

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KamLAND – Zen, PRL 117, 082503 (2016): ¹³⁶*Xe*





Other models: Left-Right Symmetric Model (LRSM) and SUSY R-parity violation



(e)

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M. Horoi, A. Neacsu, PRD 93, 113014 (2016) M. Horoi CMU



$0v\beta\beta$ decay mass mechanism

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Effective field theory after hadronization

 $e_{L/R}$



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 e_L

d

 $e_{L/R}$

W

и

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The Black Box Theorems

Black box I (electron neutrino)

J. Schechter and J.W.F Valle, PRD 25, 2951 (1982)

E. Takasugi, PLB 149, 372 (1984)

J.F. Nieves, PLB 145, 375 (1984)

0vββ observed	\Leftrightarrow	(i) Lepton number conservation is violated by 2 units.					
at some level		(ii) Electron neutrinos are Majorana fermions (with $m > 0$).					

$$\overline{\nu_{e}} \quad e^{-} \quad \overline{\nu_{e}} \quad e^{-} \quad \overline{\nu_{e}}$$



M. Duerr et al, JHEP 06 (2011) 91

 $\left(\delta m_{v_e}\right)_{BB} \sim 10^{-24} \, eV << \sqrt{\left|\Delta m_{32}^2\right|} \approx 0.05 \, eV$

Black box II (all flavors + oscillations)

M. Hirsch, S. Kovalenko, I. Schmidt, PLB 646, 106 (2006)

0νββ observed ⇔(i) Lepton number conservation is violated by 2 units.
(i) Neutrinos are Majorana fermions. (i) Lepton number conservation is violated by 2 units.(ii) Neutrinos are Majorana fermions. (i) Neutrinos are Majorana fermions.

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Rep. Prog. Phys. 80 (2017) 046301





QRPA-En M. T. Mustonen and J. Engel, Phys. Rev. C 87, 064302 (2013).

QRPA-Jy J. Suhonen, O. Civitarese, Phys. NPA **847** 207–232 (2010).

QRPA-Tu A. Faessler, M. Gonzalez, S. Kovalenko, and F. Simkovic, arXiv:1408.6077

ISM-Men J. Menéndez, A. Poves, E. Caurier, F. Nowacki, NPA 818 139–151 (2009).

SM M. Horoi et. al. PRC 88, 064312 (2013), PRC 89, 045502 (2014), PRC 89, 054304 (2014), PRC 90, 051301(R) (2014), PRC 91, 024309 (2015), PRL 110, 222502 (2013), PRL 113, 262501(2014).

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Light left-handed neutrino-exchange NME update arxiv:2207.01085





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PHYSICAL REVIEW C 107, 045501 (2023)

Predicting the neutrinoless double- β -decay matrix element of ¹³⁶Xe using a statistical approach

M. Horoi¹, A. Neacsu,^{2,3} and S. Stoica²

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PHYSICAL REVIEW C 106, 054302 (2022)

Statistical analysis for the neutrinoless double- β -decay matrix element of ⁴⁸Ca

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IOP Publishing

Journal of Physics G: Nuclear and Particle Physics

J. Phys. G: Nucl. Part. Phys. 49 (2022) 120502 (30pp)

https://doi.org/10.1088/1361-6471/aca03e

Towards precise and accurate calculations of neutrinoless double-beta decay

V Cirigliano¹, Z Davoudi², J Engel^{3,*}, R J Furnstahl⁴, G Hagen^{5,6}, U Heinz⁴, H Hergert⁷, M Horoi⁸, C W Johnson⁹, A Lovato^{10,11,12}, E Mereghetti¹³, W Nazarewicz^{14,*}, A Nicholson^{3,15}, T Papenbrock^{5,6}, S Pastore¹⁶, M Plumlee^{17,18}, D R Phillips^{19,*}, P E Shanahan²⁰, S R Stroberg¹⁰, F Viens²¹, A Walker-Loud¹⁵, K A Wendt²² and S M Wild^{18,23}



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 $e_{eff}^{p} = 1.5 e_{eff}^{n} = 0.5$

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B. Pritycenko, J. Choquette, M. Horoi, B. Karamy, B. Singh, Atomic Data and Nuclear Data Tables, **107**, 1 (2016)

Shell-model Calculations





Shell Model Spaces



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Statistical Model: why ¹³⁶Xe?

- One of the longest half-life limit measured, $\sim 2x10^{26}$ years
- One of the potential isotopes chosen for the next "tonne experiment"
- Relatively low shell model dimensions in the *jj55* $(0g_{7/2}, 2s, 1d, 0h_{11/2})$ model space





Statistical Model: Shell Model Effective Hamiltonians

- Isospin conserving effective Hamiltonians in *jj55* model space $(0g_{7/5}, 2s1d, 0h_{11/2})$
- 327 TBME of starting Hamiltonians ±10% uniformly random changes
- Three starting effective Hamiltonians:
 => SVD, PRC 86 044323 (2012)
 => GCN5082, PRC 82 064304 (2010)
 => jj55t, PRL 110 222502 (2013)





Statistical Model: Observables (24)

- Onbb NME (1): M_{0v} , short-range correlator CD-Bonn
- 2nbb NME (1): M_{2v} , q = 0.7
- Energies 2+, 4+, 6+ (6): PE₂₊, PE₄₊, PE₆₊, DE₂₊, DE₄₊, DE₆₊
- B(GT) to 1+1 state (2): PGT, DGT, q = 0.7
- B(E2) 2⁺ -> 0⁺ (2): PBE2, DBE2 $e_{eff}^{p} = 1.5 e_{eff}^{n} = 0.5$
- Proton occupation (8): POPg₇, POPs₁, POPd, POPh₁₁, DOPg₇, DOPs₁, DOPs₁, DOPd, DOPh₁₁
- Neutron vacancies (4): DVNg₇, DVNs₁, DVNd, DVNh₁₁
- Muon Capture rate (0)
- B(M1) (0)





Overview

Observable	Data	Error	svd_s	gcn_s	$j5t_s$	μ_{svd}	σ_{svd}	μ_{gcn}	σ_{gcn}	μ_{j5t}	σ_{j5t}
$M_{0\nu}$	N/A	N/A	1.763	2.645	2.314	1.749	0.111	2.632	0.135	2.306	0.156
$M_{2\nu}$	0.018	0.001	0.025	0.069	0.060	0.022	0.003	0.061	0.007	0.052	0.007
$\mathbf{P}GT$	0.150	0.021	0.163	0.545	0.512	0.141	0.059	0.457	0.105	0.333	0.220
PBE2	0.286	0.081	0.154	0.121	0.096	0.153	0.009	0.122	0.013	0.099	0.012
$\mathbf{P}E_{2^+}$	1.313	0.150	1.498	1.363	1.513	1.494	0.089	1.352	0.089	1.507	0.098
$\mathrm{P}E_{4^+}$	1.694	0.150	2.073	1.747	2.012	2.070	0.089	1.740	0.107	2.011	0.107
$\mathbf{P}E_{6^+}$	1.892	0.150	2.178	1.892	2.254	2.192	0.088	1.884	0.125	2.212	0.096
$POPg_7$	2.930	0.100	2.705	2.716	3.143	2.702	0.187	2.705	0.209	3.082	0.267
$POPs_1$	0.057	0.006	0.089	0.025	0.020	0.090	0.018	0.025	0.006	0.021	0.006
$POPh_{11}$	0.400	0.040	0.190	0.375	0.265	0.189	0.020	0.373	0.050	0.265	0.045
$\operatorname{POP}d$	0.520	0.030	1.016	0.884	0.572	1.019	0.180	0.896	0.197	0.632	0.250
$\mathrm{D}GT$	0.012	0.005	0.001	0.009	0.004	0.001	0.000	0.008	0.003	0.003	0.003
DBE2	0.413	0.011	0.342	0.194	0.158	0.337	0.023	0.195	0.026	0.163	0.028
$\mathrm{D}E_{2^+}$	0.819	0.150	0.662	0.842	0.917	0.660	0.067	0.836	0.056	0.919	0.049
$\mathrm{D}E_{4^+}$	1.867	0.150	1.389	1.873	2.113	1.403	0.131	1.861	0.116	2.087	0.082
$\mathrm{D}E_{6^+}$	2.207	0.150	2.157	2.196	2.502	2.171	0.151	2.197	0.090	2.507	0.117
$DVNg_7$	0.000	0.150	0.102	0.174	0.130	0.100	0.010	0.172	0.014	0.132	0.023
$DVNs_1$	0.080	0.020	0.271	0.251	0.415	0.286	0.117	0.255	0.058	0.407	0.110
$\mathrm{DVN}h_{11}$	1.680	0.130	1.205	0.726	0.347	1.177	0.237	0.724	0.132	0.385	0.162
DVNd	0.240	0.050	0.423	0.850	1.108	0.437	0.132	0.850	0.118	1.076	0.158
$DOPg_7$	3.860	0.100	3.189	3.475	4.145	3.187	0.209	3.477	0.249	4.078	0.436
$DOPs_1$	0.200	0.020	0.263	0.083	0.049	0.264	0.047	0.084	0.020	0.052	0.017
$\mathrm{DOP}h_{11}$	0.620	0.060	0.264	0.658	0.625	0.269	0.049	0.658	0.093	0.613	0.121
$\mathrm{DOP}d$	1.290	0.080	2.285	1.785	1.181	2.280	0.227	1.781	0.265	1.258	0.447



Table I. All relevant data and statistics for all selected observables.



Statistical Model: Heat Map

samples=1,000

M ₀ v	1	0.8	0.78	0.74	0.72	0.67	0.65	0.64	0.61	0.55	0.53	0.38	0.37	0.25	0.22	0.21	0.045	-0.093	-0.097	-0.18	-0.24	-0.26	-0.34	-0.42		
M ₂ v	0.8	1	0.47	0.43	0.42	0.43	0.4	0.39	0.44	0.26	0.28	0.52	0.2	0.44	-0.079	0.066	-0.064	0.052	0.03	-0.049	-0.055	-0.1	-0.094	-0.24		
DE₂+	0.78	0.47	1	0.95	0.96	0.68	0.69	0.55	0.53	0.75	0.39	0.53	0.75	0.34	0.72	0.62	0.36	-0.46	-0.37	-0.54	-0.63	-0.6	-0.8	-0.79		
DE ₆ +	0.74	0.43	0.95	1	0.99	0.75	0.75	0.66	0.57	0.83	0.44	0.5	0.81	0.32	0.72	0.66	0.42	-0.38	-0.45	-0.56	- 0.54	-0.68	-0.83	-0.84		
DE₄+ '	0.72	0.42	0.96	0.99	1	0.7	0.71	0.59	0.52	0.81	0.41	0.55	0.82	0.35	0.77	0.66	0.39	-0.43	-0.41	-0.55	-0.61	-0.65	-0.86	-0.88		
PE4+	0.67	0.43	0.68	0.75	0.7	1	0.98	0.9	0.49	0.52	0.3	0.16	0.39	0.15	0.27	0.5	0.5	-0.26	-0.53	-0.52	-0.17	-0.54	-0.38	-0.52		
PE2+	0.65	0.4	0.69	0.75	0.71	0.98	1	0.87	0.49	0.52	0.27	0.17	0.42	0.16	0.31	0.55	0.56	-0.32	-0.58	-0.58	-0.21	-0.57	-0.41	-0.55		
PE ₆₊	0.64	0.39	0.55	0.66	0.59	0.9	0.87	1	0.51	0.5	0.35	0.0029	0.3	-0.043	0.15	0.33	0.37	-0.05	-0.42	-0.37		-0.42	-0.26	- 0.37		
VNg ₇	0.61	0.44	0.53	0.57	0.52	0.49	0.49	0.51	1	0.5	0.45	0.3	0.44	0.28	0.16	0.28	0.17	0.062	-0.23	-0.19		-0.36	-0.37	-0.28		
OPh ₁₁ D	0.55	0.26	0.75	0.83	0.81	0.52	0.52	0.5	0.5	1	0.74	0.38	0.77	0.19	0.66	0.59	0.38	-0.27	-0.45	-0.47	-0.41	-0.67	-0.77	-0.7		
OPh ₁₁ D	0.53	0.28	0.39	0.44	0.41	0.3	0.27	0.35	0.45	0.74	1	0.085	0.23	-0.036	0.19	0.016	-0.087	0.07		0.044	-0.088	-0.16	- 0.24	-0.19		
DGT P	0.38	0.52	0.53	0.5	0.55	0.16	0.17	0.0029	0.3	0.38	0.085	1	0.66	0.76	0.4	0.39	0.08	-0.29	-0.064	-0.23	-0.47	-0.34	-0.58	-0.58		
PNNd	0.37	0.2	0.75	0.81	0.82	0.39	0.42	0.3	0.44	0.77	0.23	0.66	1	0.5	0.76	0.82	0.54	-0.35	-0.55	-0.63	-0.5	-0.82	-0.95	-0.84		
D D	0.25	0.44	0.34	0.32	0.35	0.15	0.16	- 0.043	0.28	0.19	-0.036	0.76	0.5	1	0.11	0.35	0.14	-0.17	-0.12	-0.25	-0.26	-0.31	-0.34	-0.43		
VNs1 F	0.22	-0.079	0.72	0.72	0.77	0.27	0.31	0.15	0.16	0.66	0.19	0.4	0.76	0.11	1	0.64	0.36	-0.56	-0.34	-0.51	-0.77	-0.57	-0.92	-0.82		
DPg7 D'	0.21	0.066	0.62	0.66	0.66	0.5	0.55	0.33	0.28	0.59	0.016	0.39	0.82	0.35	0.64	1	0.9	-0.49	-0.89	-0.91	-0.36	-0.97	-0.78	-0.71		
P <i>g</i> ⁷ DC	0.045	-0.064	0.36	0.42	0.39	0.5	0.56	0.37	0.17	0.38	-0.087	0.08	0.54	0.14	0.36	0.9	1	-0.43	-0.99	-0.93	-0.11	-0.89	-0.48	-0.44		
PS1 PC	-0.093	0.052	-0.46	-0.38	-0.43	-0.26	-0.32	-0.05	0.062	-0.27	0.07	-0.29	-0.35	-0.17	-0.56	-0.49	-0.43	1	0.34	0.64	0.84	0.33	0.47	0.59		
Pd PC	-0.097	0.03	-0.37	-0.45	-0.41	-0.53	-0.58	-0.42	-0.23	-0.45	-0.029	-0.064	-0.55	-0.12	-0.34	-0.89	-0.99	0.34	1	0.9	0.042	0.91	0.48	0.43		
JEZ PC	-0.18	-0.049	-0.54	-0.56	-0.55	-0.52	-0.58	-0.37	-0.19	-0.47	0.044	-0.23	-0.63	-0.25	-0.51	-0.91	-0.93	0.64	0.9_	1	0.39	0.87	0.61	0.65		
PS ₁ PE	-0.24	-0.055	-0.63	-0.54	-0.61	-0.17	-0.21	0.031	-0.017	-0.41	-0.088	-0.47	-0.5	-0.26	-0.77	-0.36	-0.11	0.84	0.042	0.39	1	0.21	0.66	0.74		
DD DO	-0.24	.0.1	-0.6	-0.69	-0.65	-0.54	-0.57	.0.42	-0.36	.0.67	-0.16	.0.34	.0.82	.0.31	-0.57	.0 97	-0.80	0.33	0.91	0.95	0.21	1	0.75	0.65-		
h11 DC	-0.24	-0.004	-0.0	-0.00	-0.05	_0.24	-0.47	-0.42	-0.50	-0.77	-0.24	-0.54	-0.02	-0.31	-0.02	-0.37	-0.49	0.55	0.91	0.61	0.21	0.75	1	0.05		
E2 DVN	-0.54	0.094	-0.0	-0.03	0.00	0.50	0.55	0.20	0.37	-0.77	0.10	0.50	0.93	0.42	-0.92	0.70	0.46	0.47	0.40	0.01	0.00	0.65	0 00	1		
DBI	-0.42	-0.24	-0.79	-0.84	-0.88 <u>+</u>	-0.52	-0.55	-0.37	-0.28	-0.7	-0.19	-0.58 - Lt	-0.84 -	-0.43 - L5	-0.82	-0.71	-0.44	-0.59 - ¹ S	-0.43	-0.65 -	-0.74 - ¹ 5	-0.65 - p	-0.88			
	M,	W	DE_2	DE6	DE_4	ΡE4	PE_2	ΡE ₆	DVNG	DOPh	POPh	DG	DVN	PG	DVN	DOPç	POPG	POP	POF	PBE	DOP	DOF	DVNh	DBE		

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1.00

- 0.75

- 0.50

- 0.25

- 0.00

- -0.25

- -0.50

- -0.75



Statistical Model: Correlations Map



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Statistical Model: Density Distributions



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Statistical Model: joint distribution

$$P(x = M_{0\nu}) = W_{svd}P_{svd}(x) + W_{gcn}P_{gcn}(x) + W_{j5t}P_{j5t}(x) ,$$

Statistical Model: Bayesian Model Averaging



i

$$\mathcal{M}_{k}) = \int \prod_{i} dy_{i} p_{y_{e},\sigma_{e}}(y_{i}) \left[\int \prod_{j} d\theta_{j} p(y_{i}|\theta_{j},\mathcal{M}_{k}) \pi(\theta_{j}|\mathcal{M}_{k}) \right] \qquad \text{Evidence integrals}$$

$$p_{y_{e},\sigma_{e}}(y_{i}) \propto \prod_{i=1}^{N_{obs}} exp[-(y_{i}-y_{e_{i}})^{2}/(2\sigma_{e_{i}}^{2})]$$

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Neutrino $\beta\beta$ effective mass

arxiv:1507.08204

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KamLAND – Zen, PRL 117, 082503 (2016): ¹³⁶*Xe*





Mass mechanism interference effects





Mass mechanism interference effects





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LRSM interference effects $\left[T_{1/2}^{0\nu}\right]^{-1} \approx 2g_A^4 G_1^{0\nu} |\eta|^2 |M|^2 [1 - \epsilon \cos \phi]$



Interference between mass mechanism and eta mechanism: F. Ahmed, and M. Horoi, PRC 101, 035504 (2020).

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Interference between mass mechanism and heavy neutrino mechanism: F. Ahmed, A. Neacsu, and M. Horoi, Phys. Lett. B 769, 299 (2017).

In LRSM:

$$\begin{split} [T_{1/2}^{0\nu}]^{-1} &= \left| \mathcal{M}_{GT}^{0\nu} \right|^2 \Big[C_m |\eta_m|^2 + C_N |\eta_N|^2 + C_\lambda |\eta_\lambda|^2 + C_\eta |\eta_\eta|^2 \\ &+ C_{mN} |\eta_m| |\eta_N| \cos \left(\phi_m - \phi_N\right) + C_{m\lambda} |\eta_m| |\eta_\lambda| \cos \left(\phi_m - \phi_\lambda\right) \\ &+ C_{m\eta} |\eta_m| |\eta_\eta| \cos \left(\phi_m - \phi_\eta\right) + C_{N\lambda} |\eta_N| |\eta_\lambda| \cos \left(\phi_N - \phi_\lambda\right) \\ &+ C_{N\eta} |\eta_N| |\eta_\eta| \cos \left(\phi_N - \phi_\eta\right) + C_{\lambda\eta} |\eta_\lambda| |\eta_\eta| \cos \left(\phi_\lambda - \phi_\eta\right) \Big] \end{split}$$



FIG. 5. Coefficient of maximum interference $\epsilon_{m\eta}(1)$ plotted against $Q_{\beta\beta}$ values.







Statistical Model: Results

- Shell model observables show robustness relative to small random changes of the effective Hamiltonian
- $M_{0\nu}$ shows significant correlations with about half of the observables considered
- The outcome of our **Bayesian Averaging model** is a joint distribution of the 136 Xe $M_{0\nu}$ with a range of 1.55 2.65 at 90% confidence level
- We propose a $M_{0\nu}$ NME of 1.99 \pm 0.37 for ¹³⁶Xe

Future work:

- Statistical investigation of the maximal interference effects
- Investigate other random changes, e.g. 15%, GOA, etc

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