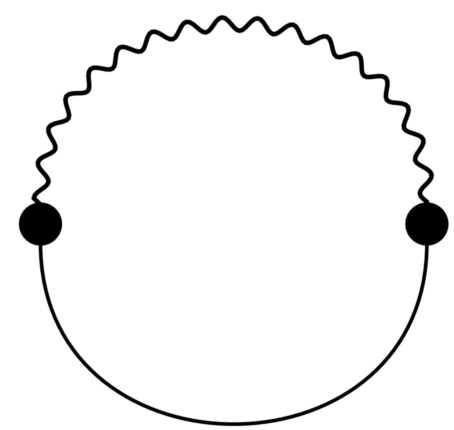
Soft electromagnetic radiation from critical fluid in the vicinity of QCD critical point

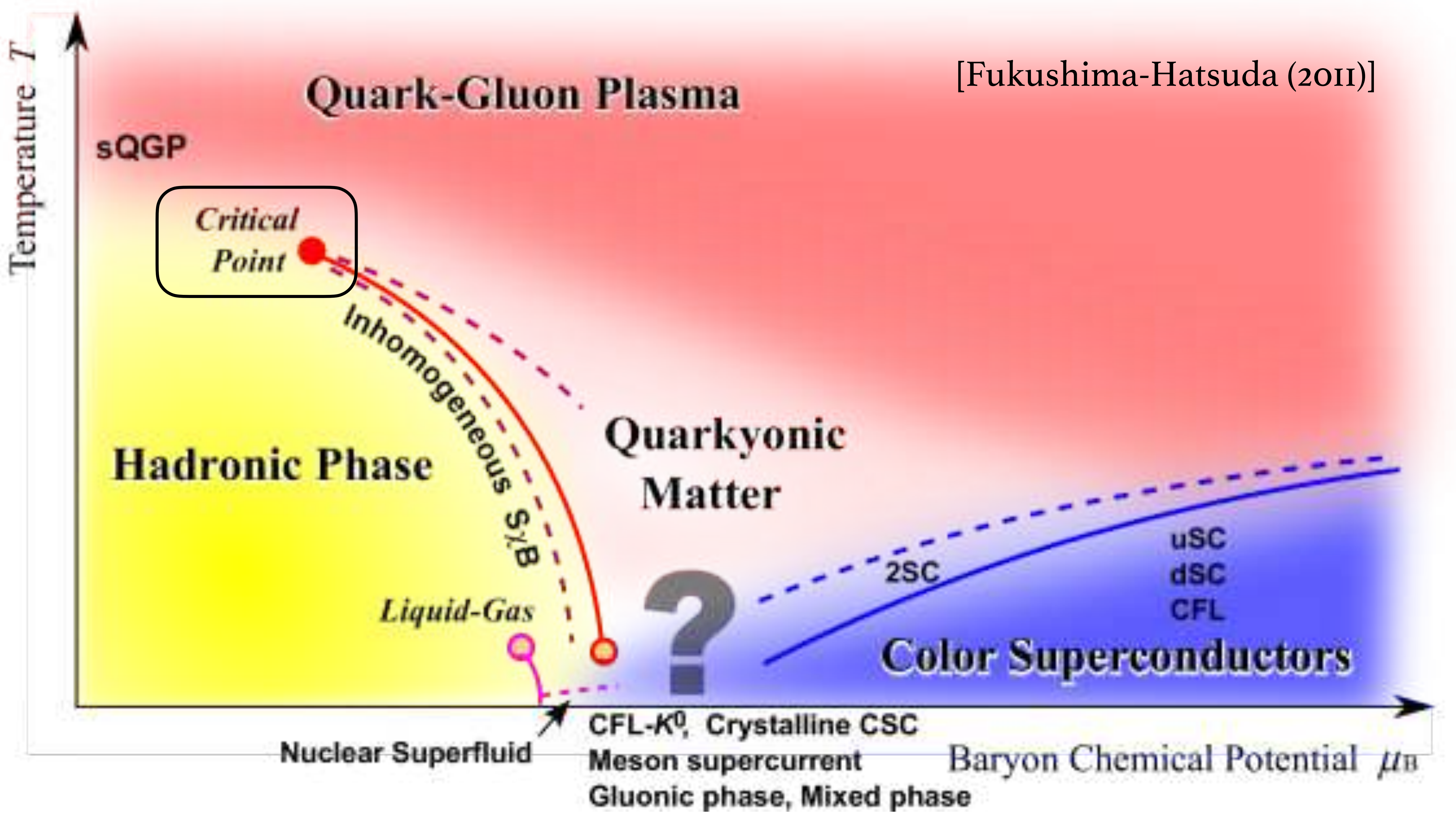


Masaru Hongo (Niigata Univ./RIKEN iTHEMS)

2023/08/22, INT Workshop on Chirality and Criticality

Based on the ongoing work with Akamatsu, Asakawa, Stephanov, and Yee

QCD Critical point



Properties and Signals of QGP

Temperature?

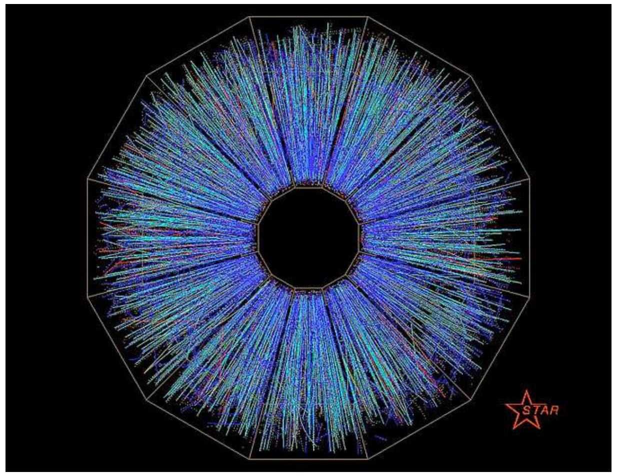
Hadron spectra, thermal photon⋯

Equation of state?

Collective flow

Viscosity?

Elliptic flow, triangular flow⋯



Chiral magnetic effect

Charge-dependent correlation

Diffusion constant?

Nuclear modification factor for heavy quark

Stopping power?

Nuclear modification factor for jet

Electric conductivity?

Dilepton spectra, charge difference in v_1

QCD Critical Point (QCD CP)

Fluctuation of conserved charges

Other possible experimental signals?

Outline

Motivation:

Search for a potential signal of
a QCD CP using photons

Approach:

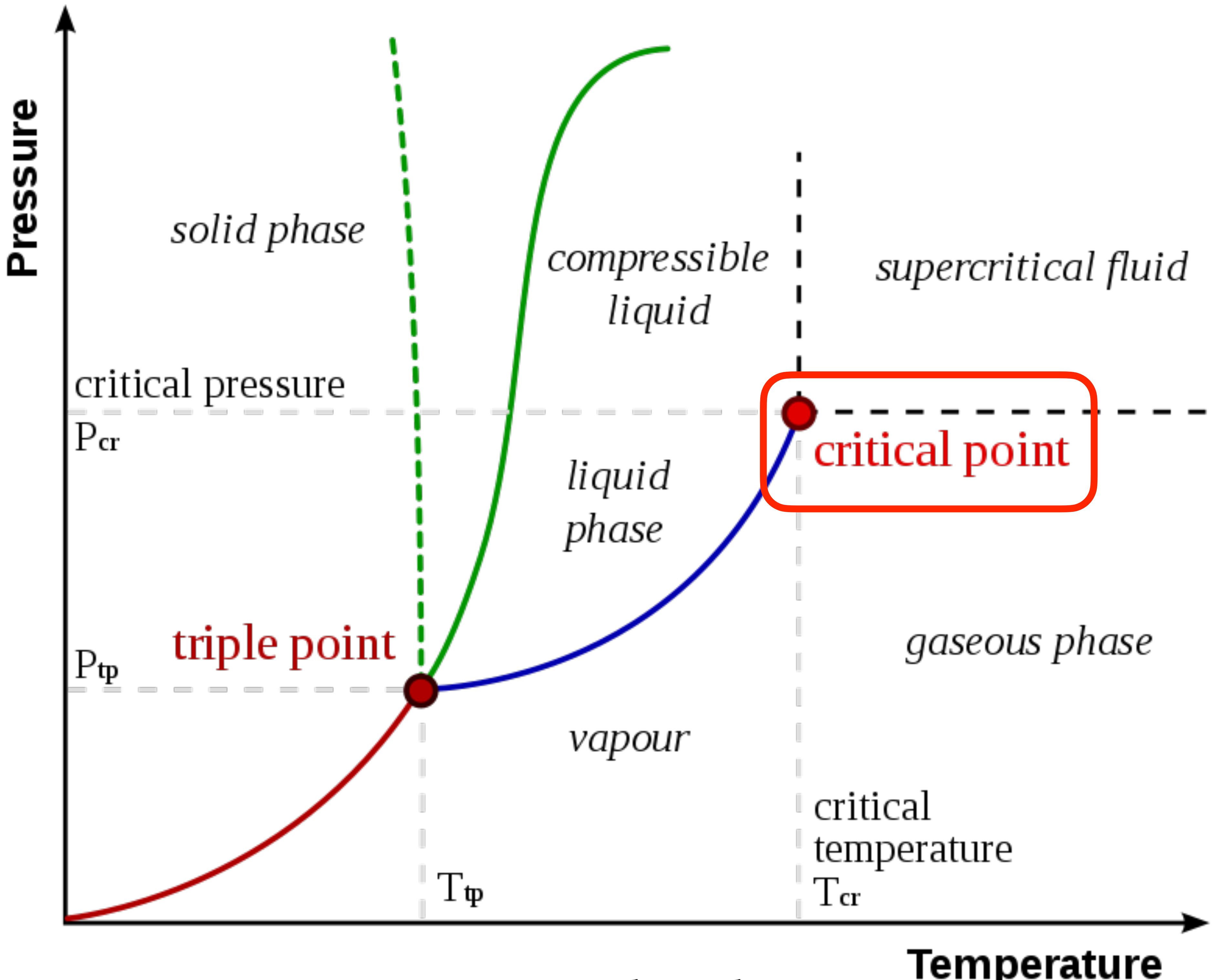
Stochastic dynamics with the Model H
with one-loop approximation

Result:

Critical enhancement
and scaling behavior of soft photons

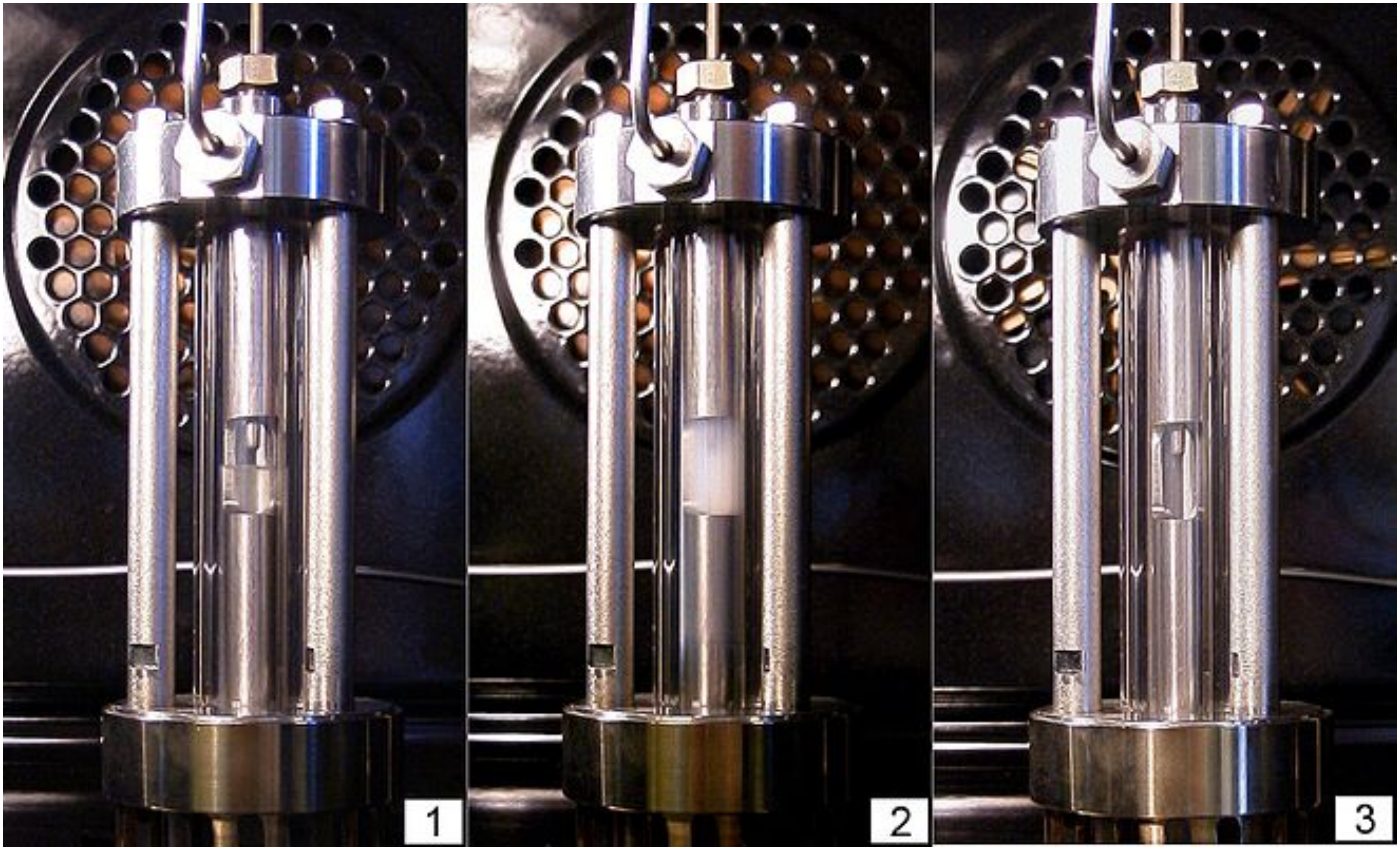
Lesson from stat-phys in 1970s

Critical point for H_2O



(From Wikipedia)

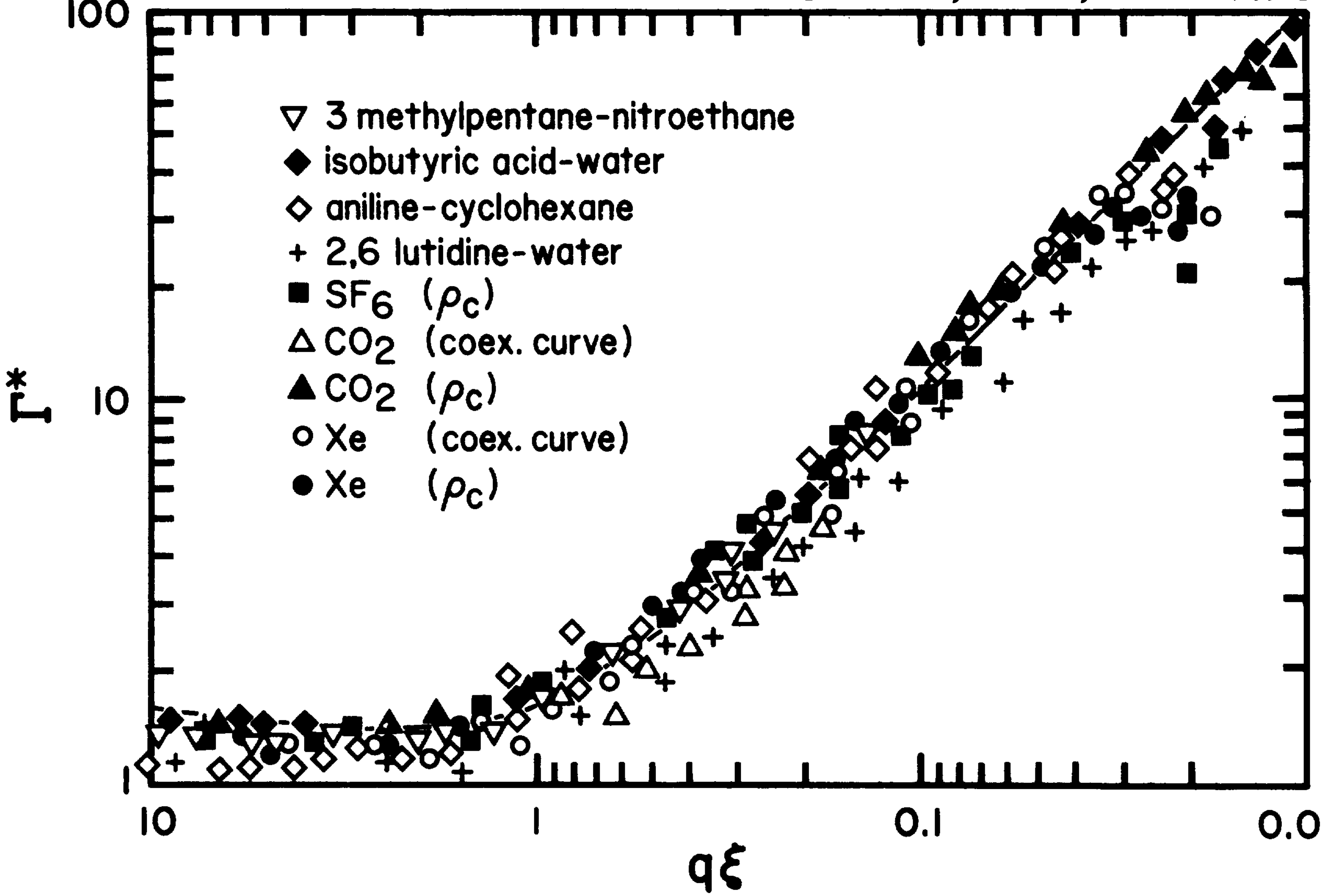
Critical opalescence



$\xi \ll \lambda_{\text{light}}$ $\xi \sim \lambda_{\text{light}}$ (From Wikipedia)

Old lesson from stat-phys

[Swinney-Henry PRA (1973)]



Scaling behavior for the dynamic light scattering!

Divergence of conductivity

◆ Kubo formula for conductivity

$$\lambda = \frac{1}{2T}\langle \widetilde{J}^x(-k)\widetilde{J}^x(k)\rangle_{k=0}$$ with a current operator J^i

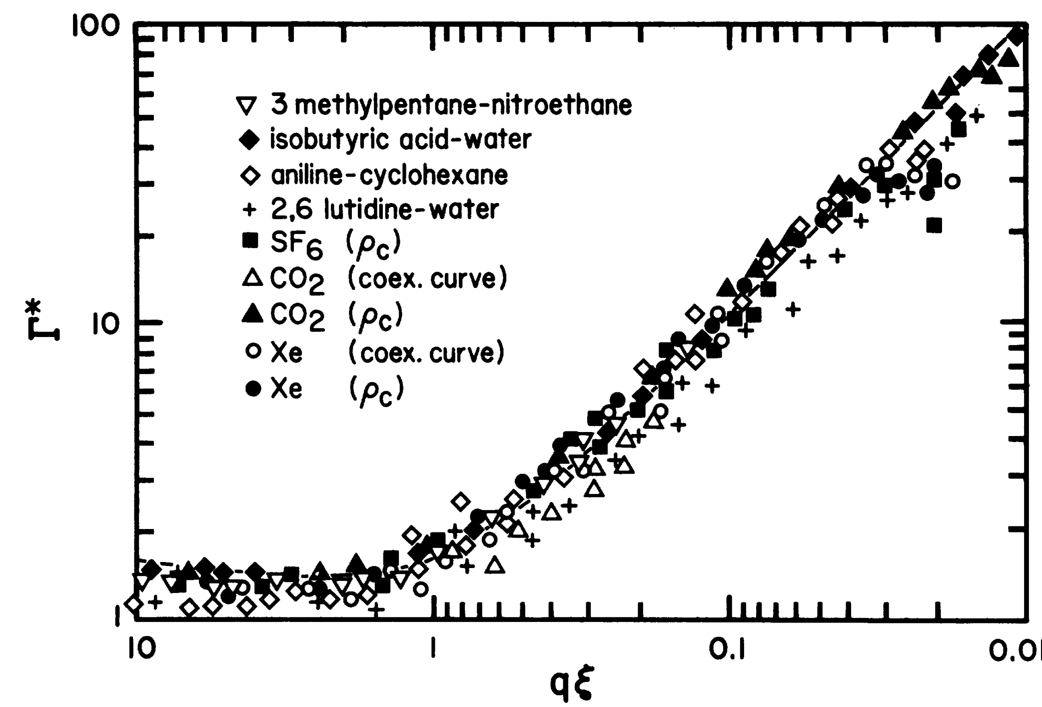
Photon self-energy probed by light scattering!

◆ Experimental result on conductivity around CP

Conductivity diverges as

$$\lambda \sim \xi \to \infty$$

(ξ: correlation length)



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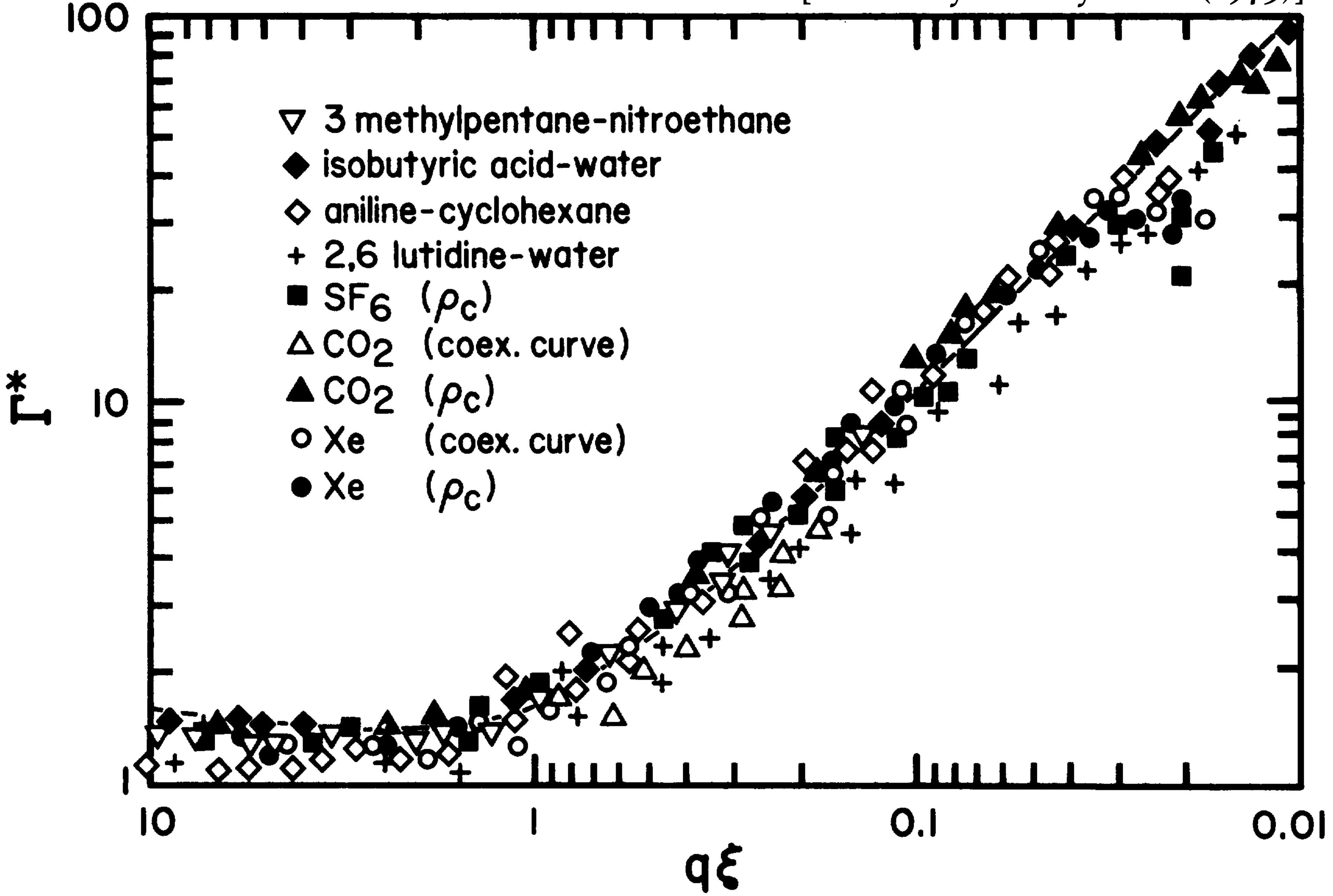


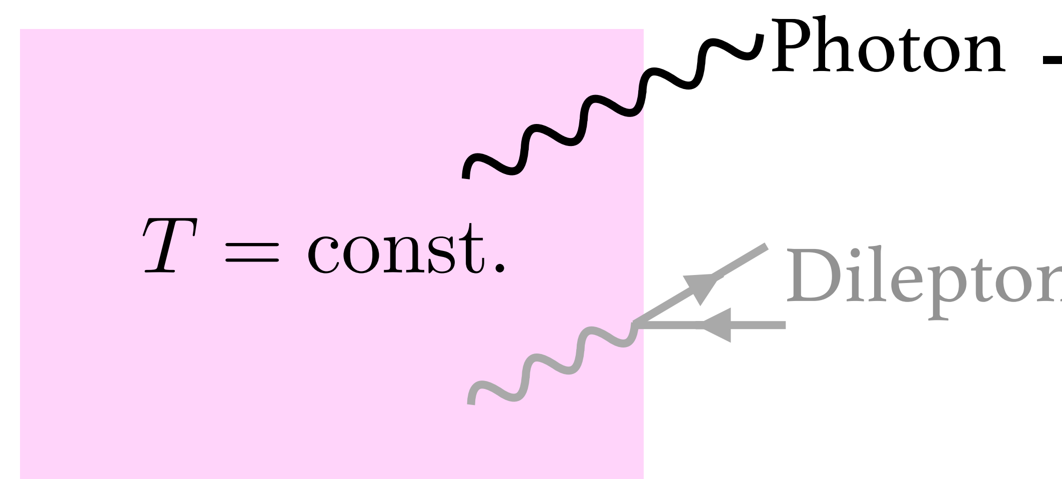
Scaling behavior for the dynamic light scattering!

→ Electromagnetic probes for the QCD CP search?

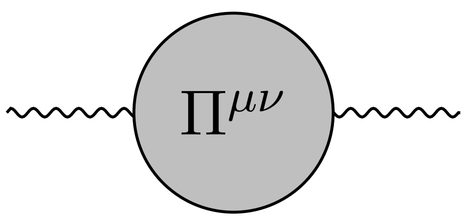
Photon from QCD plasma

◆ Photon production rate

$$k^0 \frac{d\Gamma}{d^3k} \propto n_B(k^0)\eta_{\mu\nu}\frac{k^0}{2T}\langle \tilde{J}^\mu(-k)\tilde{J}^\nu(k)\rangle$$



We can evaluate this correlator (photon self-energy) using field theory!



- Finite-temperature perturbation [see, e.g., Kapusta-Gale (2006)]
- Transport peak with Israel-Stewart hydro [Akamatsu-Hamagaki-Hatsuda-Hirano (2012)]
- Nambu-Jona-Lasinio model for QCD CP [Nishimura-Kitazawa-Kunihiro (2022)]

◆ The crucial observation

This is the same correlator exhibiting diverging conductivity!!

$$\lambda = \frac{1}{2T}\langle \tilde{J}^x(-k)\tilde{J}^x(k)\rangle_{k=0} \xrightarrow{\text{near CP}} \xi \to \infty$$

➡ **There should be critical enhancement of soft photon emission!**

Outline

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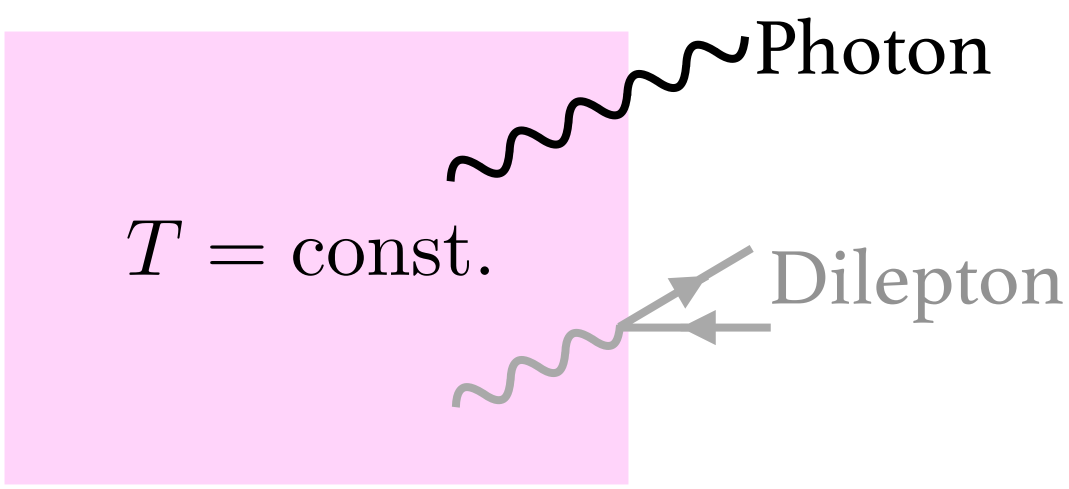
Search for a potential signal of a QCD CP using photons



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Critical dynamics with Model H

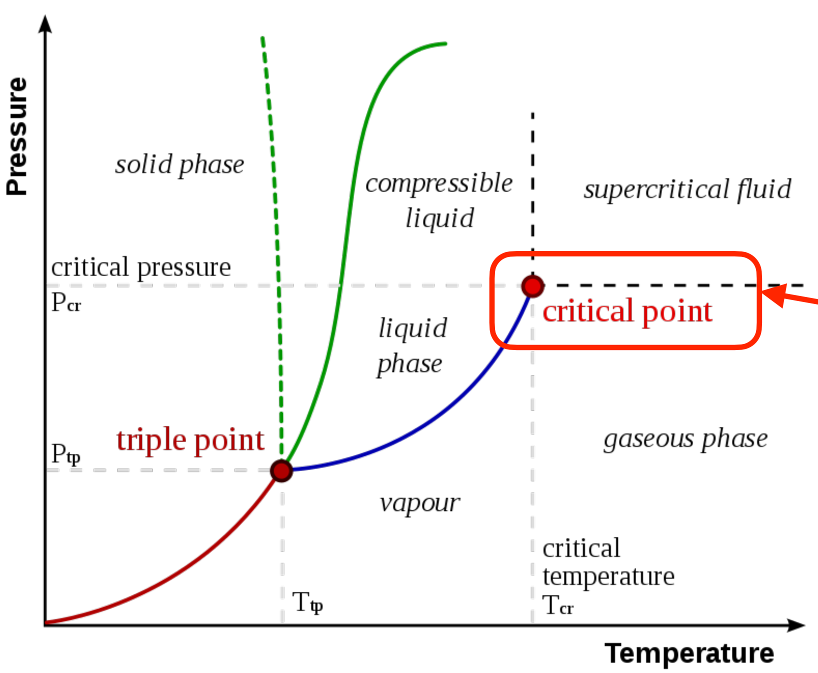
Universality at critical point

Why a critical point is so special?

Critical point is a special point at which systems acquire the emergent scale (or possibly conformal) symmetry!

Thanks to the enhanced symmetry,

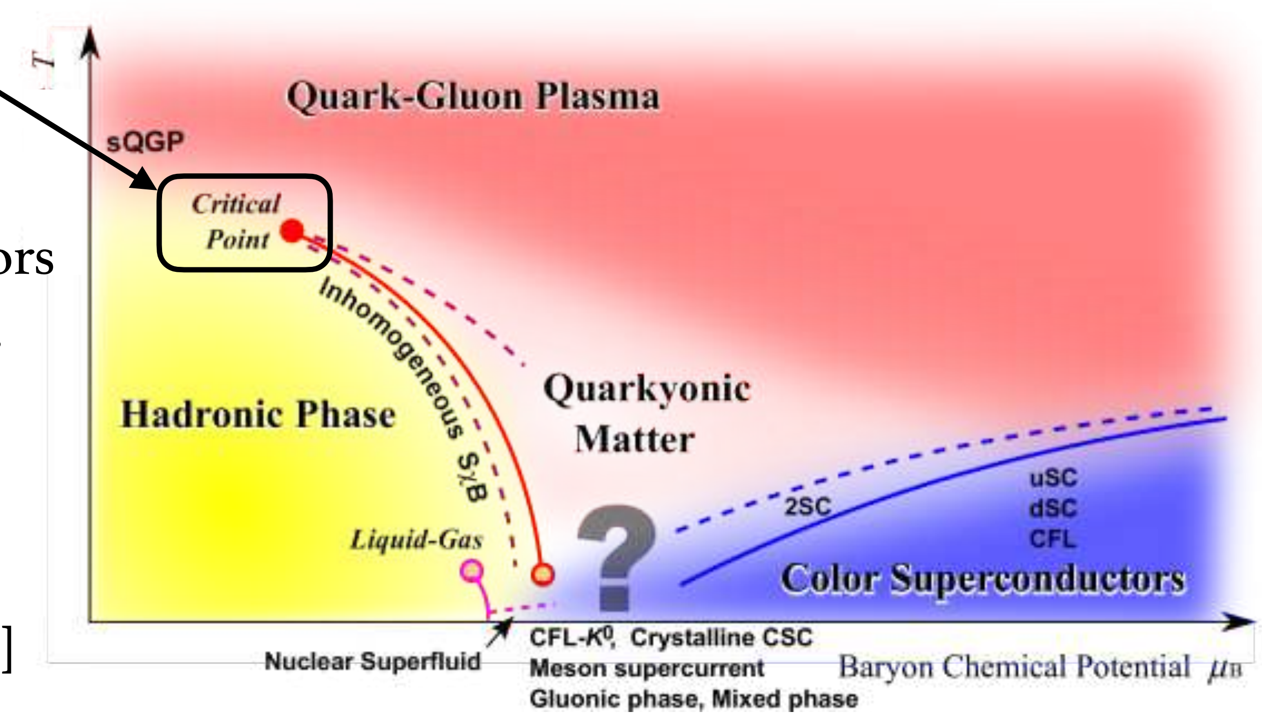
the resulting low-energy dynamics is universal!



If QCD critical point exists, critical dynamics near that point shows essentially same behaviors as that of critical fluids.

(Currently, no one knows whether there is critical points in QCD)

[→BEST Collaboration]



Divergence of conductivity

◆ Kubo formula for conductivity

$$\lambda = \frac{1}{2T} \langle \widetilde{J}^x(-k) \widetilde{J}^x(k) \rangle_{k=0}$$ with a current operator J^i

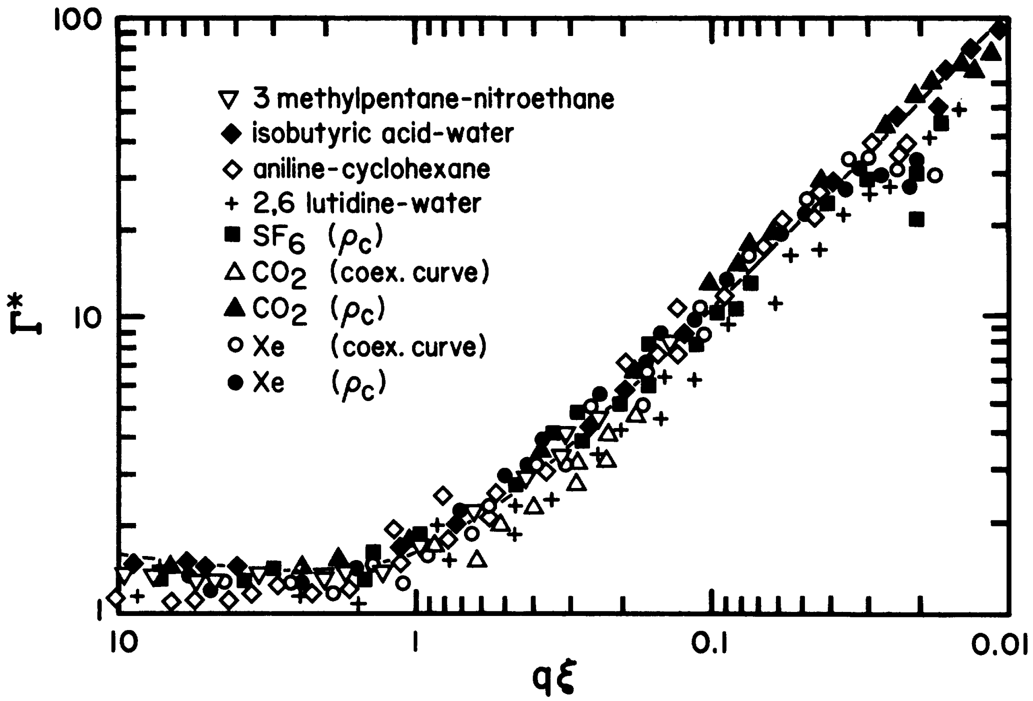
Photon self-energy probed by light scattering!

◆ Experimental result on conductivity around CP

Conductivity diverges as

$$\lambda \sim \xi \to \infty$$

(ξ: correlation length)



Q. What is the origin of diverging conductivity at CP?

Simple understanding

Kubo formula: $\lambda = \frac{1}{2T}\langle \widetilde{J}^x(-k)\widetilde{J}^x(k)\rangle_{k=0} = \frac{1}{2T}\int \mathrm{d}t\mathrm{d}^3x \langle J^x(x)J^x(0)\rangle$

◆ **Decomposition of current operator**

$$J^x(x) = a\delta\hat{s}(x)v_\perp^x(x) + j^x_{\mathrm{micro}}(x) \quad \text{with} \quad \begin{cases} \text{critical fluctuation } \delta\hat{s} \\ \text{transverse velocity } v_\perp^i \end{cases}$$

$$\lambda = \lambda_{\mathrm{micro}} + \frac{a^2}{2T}\int \mathrm{d}t\mathrm{d}^3x \langle \delta\hat{s}(x)v_\perp^x(x)\delta\hat{s}(0)v_\perp^x(0)\rangle$$

$$\simeq \lambda_{\mathrm{micro}} + \frac{a^2}{2T}\int \mathrm{d}t\mathrm{d}^3x \langle \delta\hat{s}(x)\delta\hat{s}(0)\rangle\langle v_\perp^x(x)v_\perp^x(0)\rangle$$

$$\sim \gamma_T^{-1}\xi^2 \times \xi^3 \quad \sim \frac{\mathrm{e}^{-r/\xi}}{r} \sim \xi^{-1} \quad \sim t^{-3/2} \sim (\xi^2)^{-3/2} \sim \xi^{-3}$$

$$\sim \lambda_{\mathrm{micro}} + \frac{a^2}{T\gamma_T}\xi$$

Reproduce divergence of conductivity!

➡ **Hydrodynamic mode coupling plays a crucial role!**

Systematic understanding

◆ Dynamic universality class

Critical dynamics is classified by symmetry!

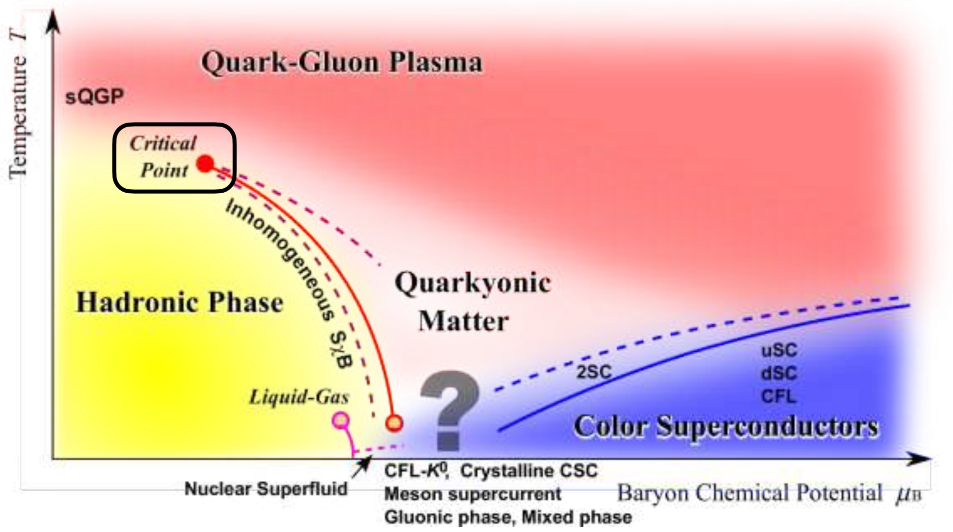
[Hohenberg-Halperin, RMP (1977)]

Symmetry tells us what are relevant slow variables

(e.g., critical fluctuation, Hydro & NG mode)

Model H describe critical dynamics w/

- Conserving critical scalar mode
- Transverse momentum (hydro) mode



Liquid-gas and QCD critical points belongs model H!

[Son-Stephanov, PRD (2004)]

Model H

◆ Dynamical variables = Softest modes in macroscopic scales

- Critical fluctuation: $\delta\hat{s} := n_0 \delta\left(\frac{s}{n}\right) = \frac{1}{T_0}\delta e - \frac{e_0 + p_0}{n_0 T_0}\delta n$

- Transverse momentum fluctuation: g_T^i (satisfying $\nabla \cdot g_T = 0$)

[※ Pressure fluctuation and longitudinal velocity is omitted in the model H]

◆ Langevin equation for model H

- EoM:
$$\begin{cases} \partial_t \delta\hat{s} = -\frac{1}{w}\boldsymbol{g}_T \cdot \boldsymbol{\nabla}\delta\hat{s} + \lambda \boldsymbol{\nabla}^2 \left[(r - C\boldsymbol{\nabla}^2)\delta\hat{s} + u\delta\hat{s}^3\right] + \zeta_s \\ \partial_t \boldsymbol{g}_T = -\left[C(\boldsymbol{\nabla}^2\delta\hat{s})\boldsymbol{\nabla}\delta\hat{s}\right]_T + \gamma_\perp \boldsymbol{\nabla}^2 \boldsymbol{g}_T + \boldsymbol{\zeta}_T \end{cases}$$

-Noise properties:
$$\begin{cases} \langle \zeta_s(x)\zeta_s(x')\rangle = -2T\lambda\boldsymbol{\nabla}^2\delta^{(4)}(x-x') \\ \langle \zeta_{T,i}(x)\zeta_{T,j}(x')\rangle = -2T\eta(\delta_{ij}\boldsymbol{\nabla}^2 - \partial_i\partial_j)\delta^{(4)}(x-x') \end{cases}$$

[See, e.g., Hohenberg-Halperin, RMP (1977)]

➡ **The model explains the divergence of transport coefficients!**

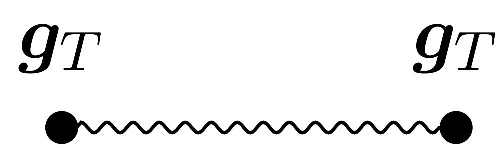
Feynman Diagram for Model H

◆ Langevin equation for model H

- EoM:
$$
\begin{cases}
\partial_t \delta\hat{s} = -\dfrac{1}{w}\boldsymbol{g}_T \cdot \boldsymbol{\nabla}\delta\hat{s} + \lambda \boldsymbol{\nabla}^2 \left[(r - C\boldsymbol{\nabla}^2)\delta\hat{s} + u\delta\hat{s}^3 \right] + \zeta_s \\
\partial_t \boldsymbol{g}_T = -\left[C(\boldsymbol{\nabla}^2\delta\hat{s})\boldsymbol{\nabla}\delta\hat{s} \right]_T + \gamma_\perp \boldsymbol{\nabla}^2 \boldsymbol{g}_T + \boldsymbol{\zeta}_T
\end{cases}
$$

1. Fluctuation correlators:

(Symmetric Green's function)

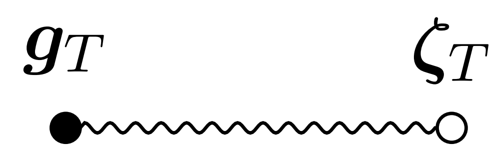


$$= \frac{2T\eta \boldsymbol{k}^2}{(k^0)^2 + \gamma_T^2 \boldsymbol{k}^4}$$

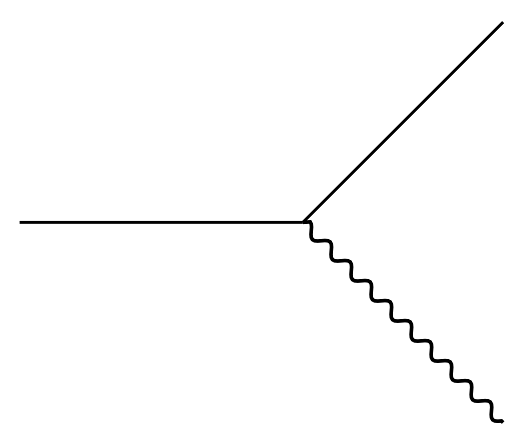
$$= \frac{2T\lambda \boldsymbol{k}^2}{(k^0)^2 + \lambda^2 \boldsymbol{k}^4 (r + C\boldsymbol{k}^2)^2}$$

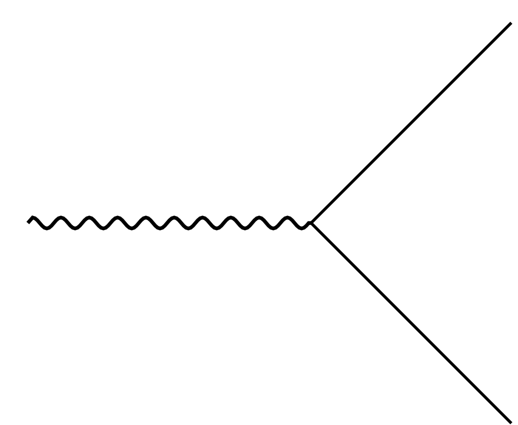
2. Noises as sources:

(Retarded Green's function)



3. Three-point vertices:



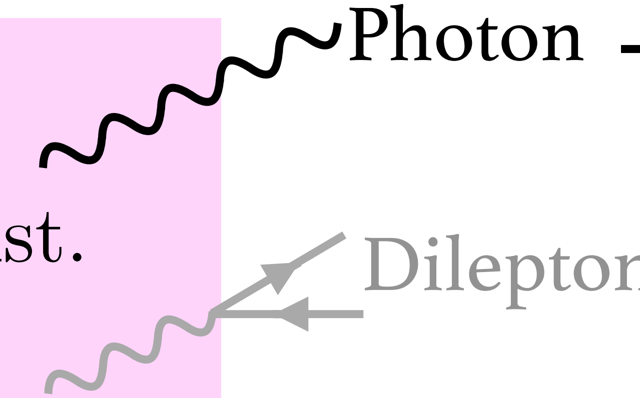


Current operators in Model H

◆ Photon production rate

$$k^0 \frac{\mathrm{d}\Gamma}{\mathrm{d}^3 k} \propto n_B(k^0)\eta_{\mu\nu}\frac{k^0}{2T}\langle \widetilde{J}^\mu(-k)\widetilde{J}^\nu(k)\rangle$$

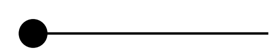
$T = \text{const.}$



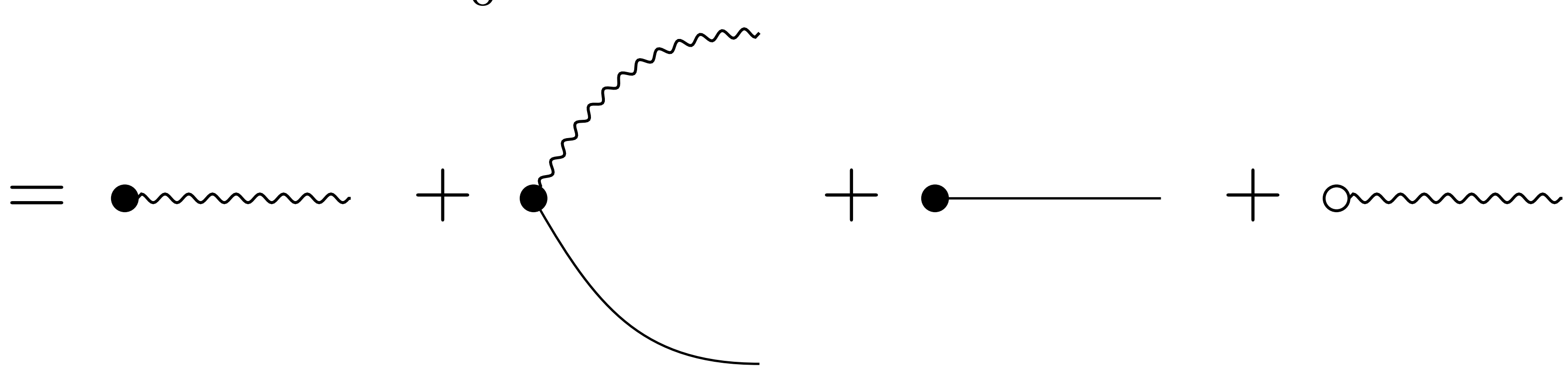
※ Isospin fluctuation is irrelevant [Son-Stephanov (2004)] and thus $J_B^\mu \propto J^\mu$

◆ Current operators in Model H

$$J^0 = \langle J^0 \rangle - \frac{n_0 T_0}{w_0}\delta\hat{s} \quad =$$

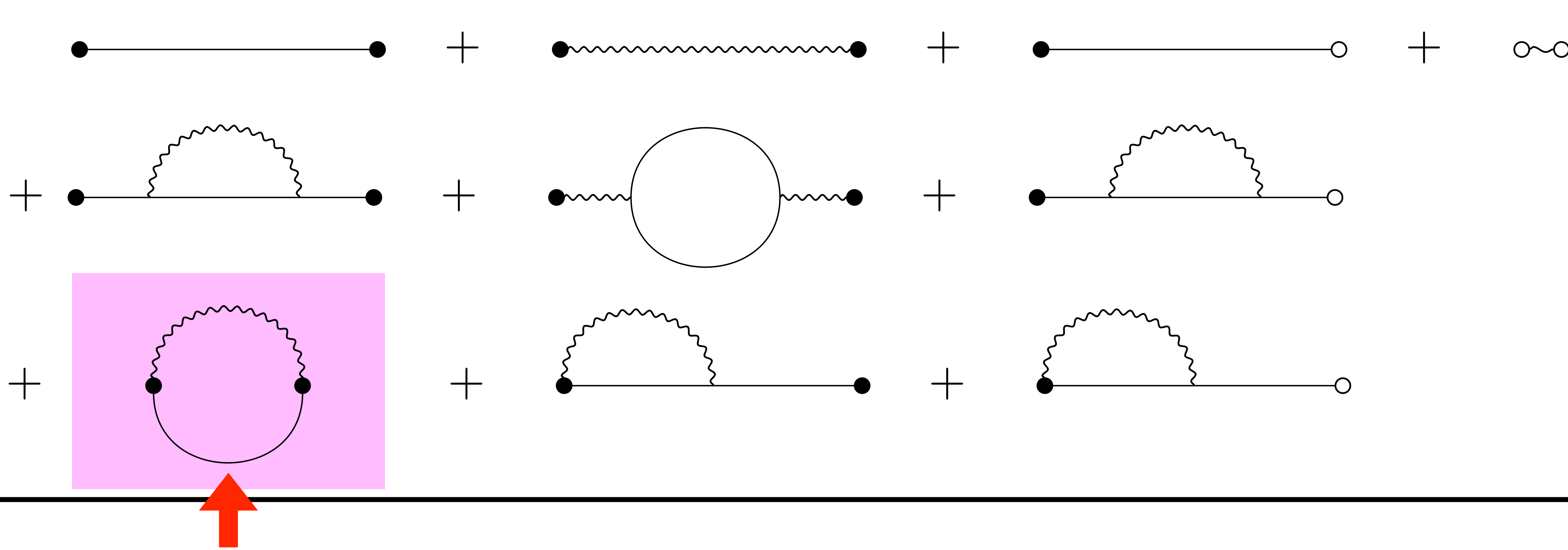


$$\boldsymbol{J} = \frac{n_0}{w_0}\boldsymbol{g}_T - \frac{n_0 T_0}{w_0^2}\delta\hat{s}\boldsymbol{g}_T + \frac{n_0 T_0}{w_0}\lambda\boldsymbol{\nabla}\left[(r - C\boldsymbol{\nabla}^2)\delta\hat{s}\right] + \boldsymbol{\zeta}$$

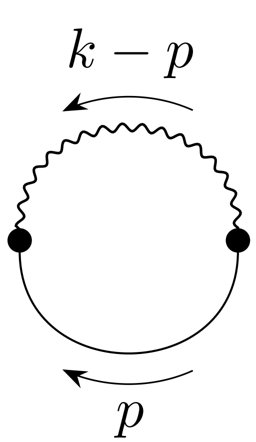


Current correlator at one-loop

◆ One-loop diagrams for $\eta_{\mu\nu}\langle \widetilde{J}^{\mu}(-k)\widetilde{J}^{\nu}(k)\rangle$



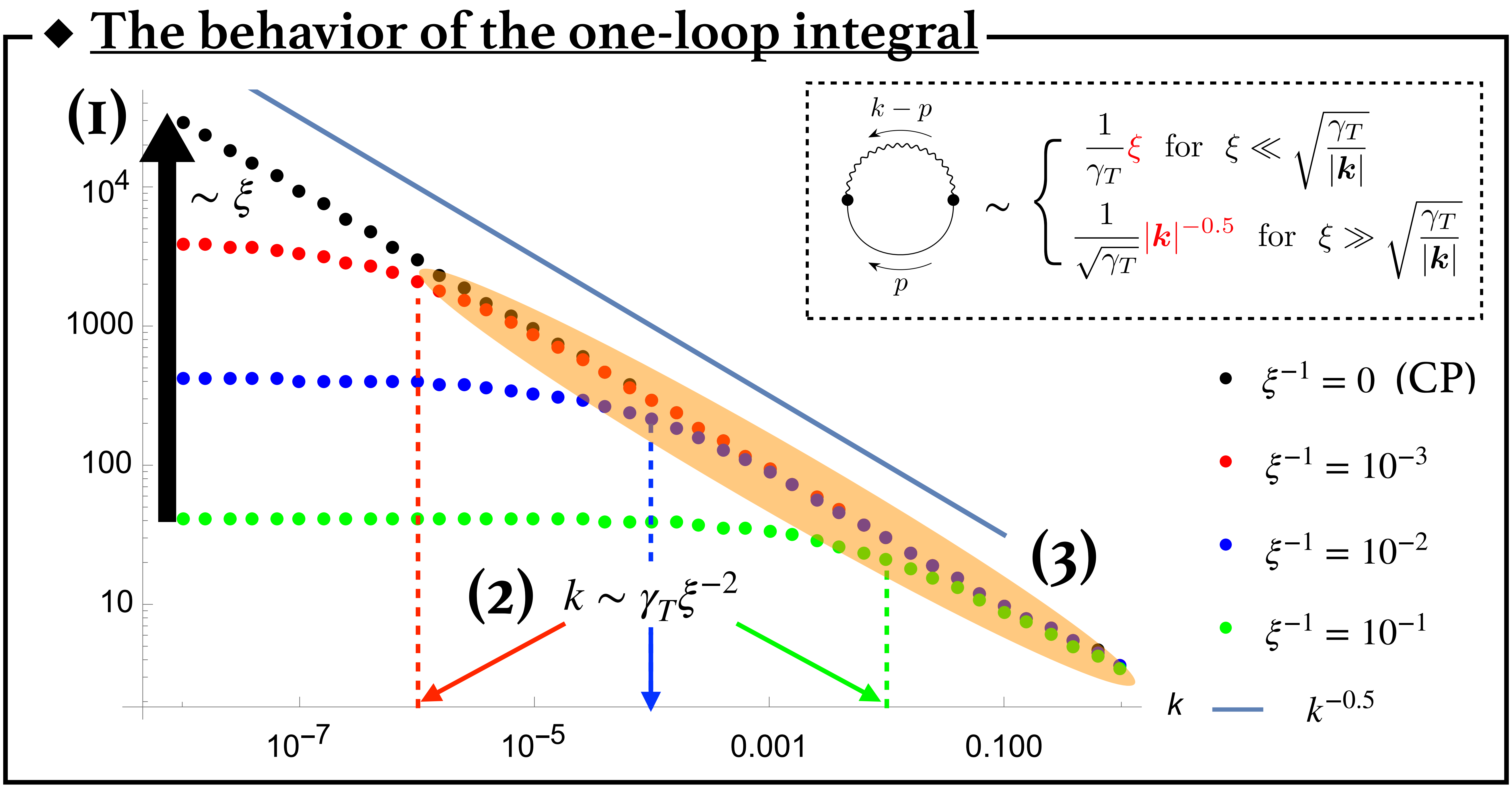
This diagram gives the dominant contribution for on-shell photon!
(This is the diagram responsible for divergence of conductivity)



$$\sim \int \frac{\mathrm{d}^4 p}{(2\pi)^4} G_S^{ss}(p^0,\boldsymbol{p})\, G_S^{ij}(k^0-p^0,\boldsymbol{k}-\boldsymbol{p}) \sim \begin{cases} \dfrac{1}{\gamma_T}\xi & \text{for} \quad \xi \ll \sqrt{\dfrac{\gamma_T}{|\boldsymbol{k}|}} \\ \dfrac{1}{\sqrt{\gamma_T}}|\boldsymbol{k}|^{-0.5} & \text{for} \quad \xi \gg \sqrt{\dfrac{\gamma_T}{|\boldsymbol{k}|}} \end{cases}$$

Critical enhancement of photon

◆ The behavior of the one-loop integral



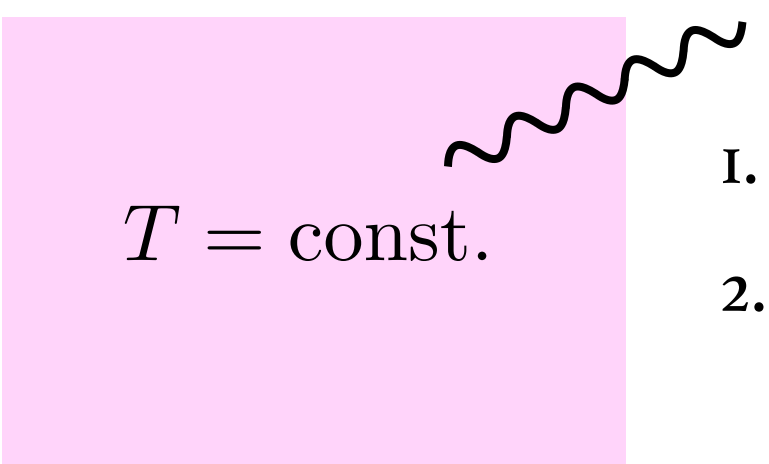
(1) Soft limit at $k \ll \gamma_T \xi^{-2}$ shows critical divergence proportional to ξ

(2) Transition to the scaling regime takes place around $k \sim k_* = \gamma_T \xi^{-2}$

(3) Scaling $\propto k^{-1/2}$ is observed at $\gamma_T \xi^{-2} \ll k (\ll \xi^{-1})$

Physical interpretation

◆ **Two relevant length scales**



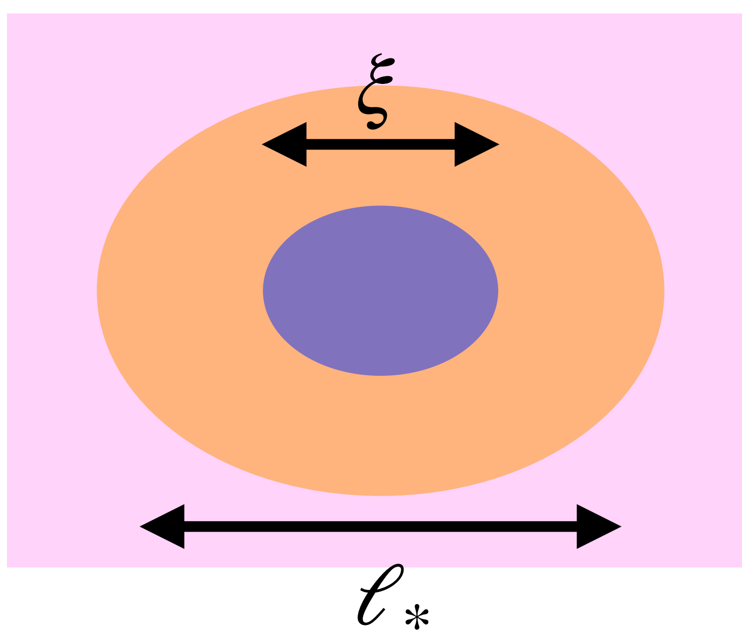
On-shell photon with $k^\mu = (|\boldsymbol{k}|, \boldsymbol{k})$

1. Correlation length for critical fluctuation: ξ
2. Kinetic regime for momentum fluctuation: $\ell_* \sim \sqrt{\dfrac{\gamma_T}{|\boldsymbol{k}|}}$

$$\gamma_T \ell_*^{-2} \sim k^0 = |\boldsymbol{k}| \;\Leftrightarrow\; \ell_* \sim \sqrt{\frac{\gamma_T}{|\boldsymbol{k}|}}$$

Case 1:

$\xi \ll \ell_*$

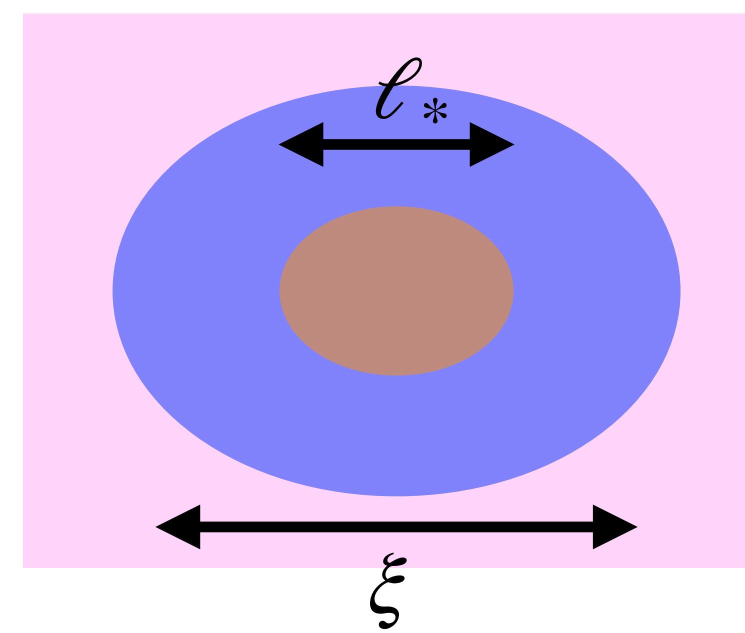


The photon probes ξ-size structure!

$$k^0 \frac{\mathrm{d}\Gamma}{\mathrm{d}^3 k} \sim \frac{1}{\gamma_T} \ell_{\mathrm{typ}} = \frac{1}{\gamma_T} \xi$$

Case 2:

$\xi \gg \ell_*$



The photon probes ℓ_*-size structure!

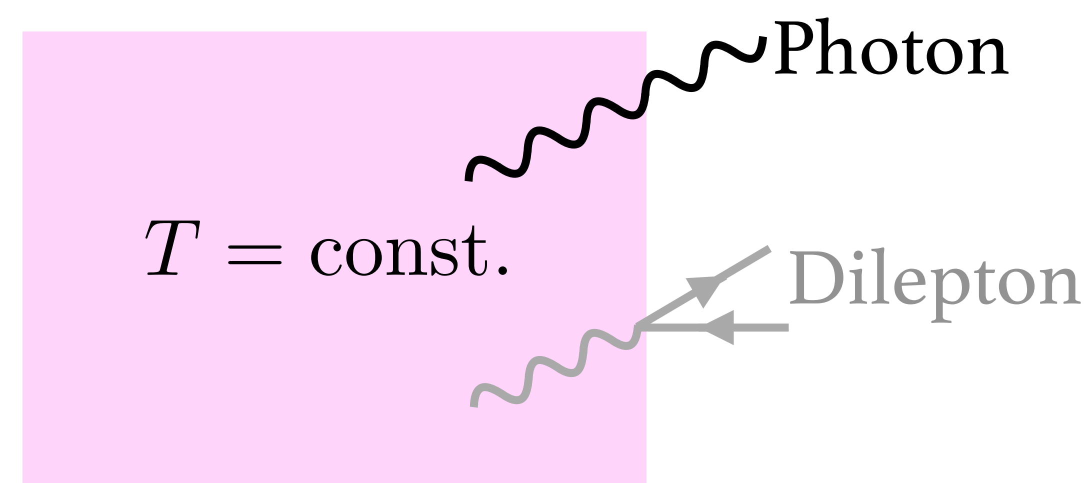
$$k^0 \frac{\mathrm{d}\Gamma}{\mathrm{d}^3 k} \sim \frac{1}{\gamma_T} \ell_{\mathrm{typ}} = \frac{1}{\sqrt{\gamma_T |\boldsymbol{k}|}}$$

Summary & Discussion

Summary

Motivation:

Search for a potential signal of a QCD CP using photons



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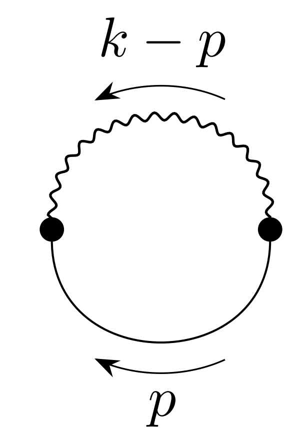
$\langle J^i J^j \rangle \sim$



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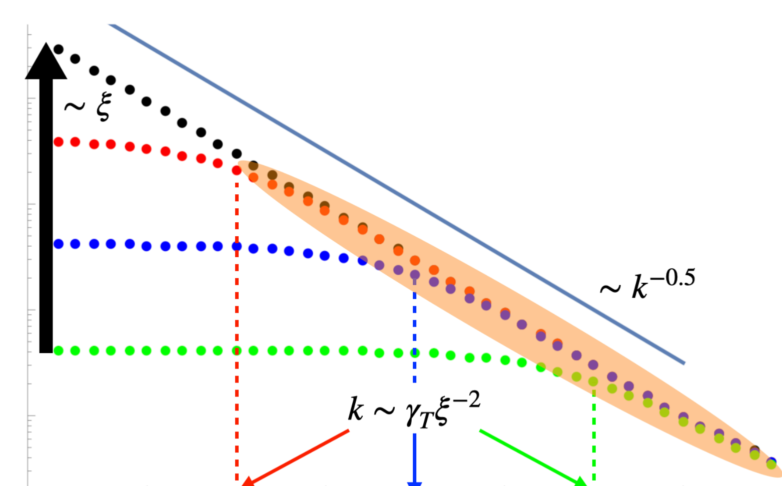
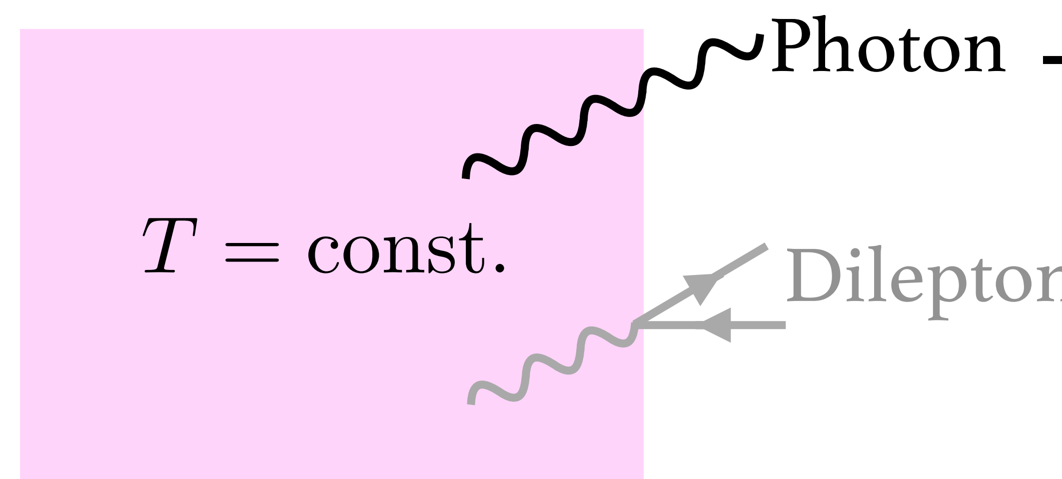
Critical enhancement and scaling behavior of soft photons



Discussion

◆ **Photon production rate**

$$k^0 \frac{\mathrm{d}\Gamma}{\mathrm{d}^3 k} \propto n_B(k^0)\eta_{\mu\nu}\frac{k^0}{2T}\langle \widetilde{J}^\mu(-k)\widetilde{J}^\nu(k)\rangle$$



While we consider the on-shell photon with $k^0 = |\boldsymbol{k}|$,

the Model H does not contain the sound mode!

Q. Could the sound mode be relevant?

$$\Gamma_{\mathrm{sound}}(\boldsymbol{k}) = \frac{1}{e_0 + p_0}\left(\zeta_R + \frac{4}{3}\eta_R\right)\boldsymbol{k}^2 \sim \xi^3 \boldsymbol{k}^2$$

The sound mode shows a fast decay around QCD CP! [Minami (2011)]

→ **The sound mode is probably irrelevant**