Summary of weeks 1–3: Perspective from effective field theory

UNIVERSITÄT BERN

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INT program on

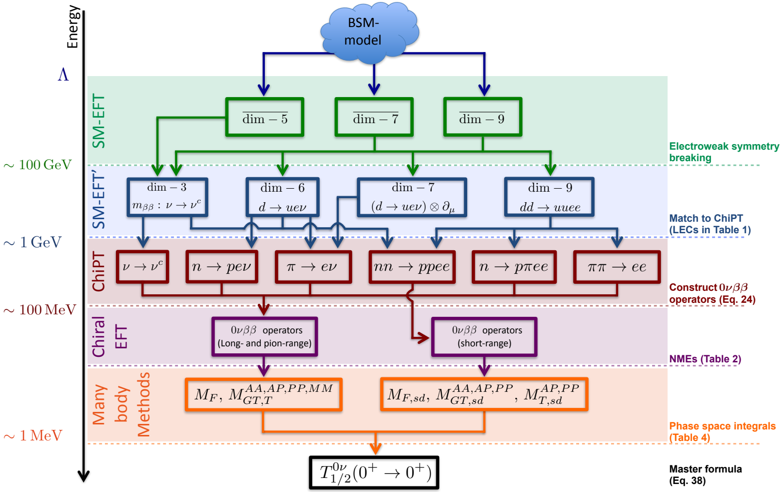
Recommended Values for Neutrinoless Double-Beta Decay

Nuclear Matrix Elements

Outline

1. Effective field theory: general strategy
2. Light Majorana mechanism
   - Momentum scaling
   - Contact term
   - Higher-order corrections
3. Beyond the light Majorana exchange
4. Decomposition of nuclear matrix elements?

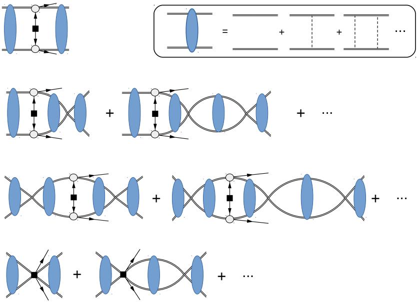
Effective field theory: from BSM to nuclear scales



Cirigliano et al. 1806.02780

see talk by W. Dekens

Light Majorana mechanism: chiral EFT



Cirigliano et al. 1802.10097

- Counting of diagrams (expansion by region):
  1. **Soft**: $q^0 \simeq |\mathbf{q}| \simeq M_\pi \simeq k_F$
  2. **Potential**: $q^0 \simeq |\mathbf{q}|^2/m_N$, $|\mathbf{q}| \simeq M_\pi \simeq k_F$
  3. **Ultrasoft**: $q^0 \simeq |\mathbf{q}| \ll M_\pi \simeq k_F$

Leading potential

$$V_{\nu,0} = \frac{\tau^{(1)+}\tau^{(2)+}}{\mathbf{q}^2}\bigg[1 - g_A^2\,\boldsymbol{\sigma}^{(1)}\cdot\boldsymbol{\sigma}^{(2)} + g_A^2\,\boldsymbol{\sigma}^{(1)}\cdot\mathbf{q}\,\boldsymbol{\sigma}^{(2)}\cdot\mathbf{q}\,\frac{2M_\pi^2+\mathbf{q}^2}{(\mathbf{q}^2+M_\pi^2)^2}\bigg]$$

Light Majorana mechanism: spectral representation

- Can write the $0\nu\beta\beta$ amplitude in a **spectral representation**:
  1. Loop integral:

$$\mathcal{A}_{0\nu} = 4G_F^2 V_{ud}^2 m_{\beta\beta} \bar{u}_L(p_1) u_L^c(p_2)\, \bar{\mathcal{A}}_{0\nu} \qquad \bar{\mathcal{A}}_{0\nu} = 2g^{\mu\nu} \int \frac{d^4q}{(2\pi)^4} \frac{\mathcal{T}_{\mu\nu}(q, p_{\text{ext}}, 0)}{q^2 + i\epsilon}$$

  2. Definition of correlator + translational invariance:

$$\hat{\Pi}_{\mu\nu}^{LL}(q,x) = \int d^4r\, e^{iq\cdot r}\, T\Big\{J_\mu^L\Big(x+\frac{r}{2}\Big) J_\nu^L\Big(x-\frac{r}{2}\Big)\Big\} \qquad J_\mu^L = \bar{u}_L \gamma_\mu d_L$$

$$\mathcal{T}_{\mu\nu}(q, p_{\text{ext}}, x) = \langle f(p_f)|\hat{\Pi}_{\mu\nu}^{LL}(q,x)|i(p_i)\rangle = e^{ix\cdot(p_f-p_i)}\mathcal{T}_{\mu\nu}(q, p_{\text{ext}}, 0)$$

  3. Spectral representation for correlator:

$$\bar{\mathcal{A}}_{0\nu} = 2g^{\mu\nu} \sum_n \int \frac{d^4q}{(2\pi)^4} \frac{i}{q^2+i\epsilon} \bigg[\frac{\langle f|J_\mu^L(0)|n(q_+)\rangle\langle n(q_+)|J_\nu^L(0)|i\rangle}{q^0 + \tilde{E} - E_n + i\epsilon} + \frac{\langle f|J_\mu^L(0)|n(q_-)\rangle\langle n(q_-)|J_\nu^L(0)|i\rangle}{-q^0 + \tilde{E} - E_n + i\epsilon}\bigg]$$

$$q_\pm^\mu = \tilde{p}^\mu \pm q^\mu \qquad \tilde{p}^\mu = \frac{1}{2}(p_i^\mu + p_f^\mu)$$

  4. Perform q^0 integral by picking up the poles

Light Majorana mechanism: spectral representation

Spectral representation

$$\bar{\mathcal{A}}_{0\nu} = -g^{\mu\nu} \sum_n \int \frac{d^3q}{(2\pi)^3} \frac{1}{|\mathbf{q}|} \left[\frac{\langle f|J^L_\mu(0)|n(q_+)\rangle\langle n(q_+)|J^L_\nu(0)|i\rangle}{|\mathbf{q}| + E_n - \tilde{E} - i\epsilon} + \frac{\langle f|J^L_\mu(0)|n(q_-)\rangle\langle n(q_-)|J^L_\nu(0)|i\rangle}{|\mathbf{q}| + E_n - \tilde{E} - i\epsilon} \right]$$

Cirigliano et al. 2102.03371

- If $|n\rangle$, E_n, J^L_μ are full (QCD/SM) states and operators, this is an **exact result**

  $\hookrightarrow$ includes (pion-loop) corrections at subleading orders in chiral EFT
- Can be expanded by method of regions
- In potential region $|\mathbf{q}| \gg E_n - \tilde{E}$, and due to $\sum_n |n\rangle\langle n| = \mathbb{1}$, we get back $V_{\nu,0}$

  $\hookrightarrow$ **dependence on intermediate states disappears**!
- In ultrasoft region $|\mathbf{q}| \simeq E_n - \tilde{E}$

  $\hookrightarrow$ results depends on intermediate states
- Corrections to “closure approximation” suppressed by two chiral orders

Comparison to $2\nu\beta\beta$ decay

Spectral representation

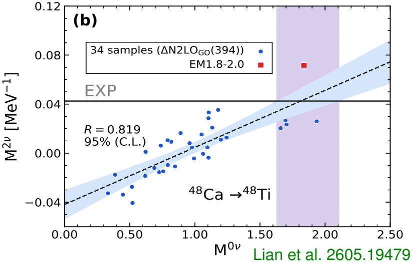
$$\mathcal{A}_{2\nu} = -\frac{4}{3} G_F^2 V_{ud}^2 \Big(M^K L^i_{11} L^i_{22} - M^L L^i_{12} L^i_{21} \Big) \qquad L^\mu_{ij} = \bar{e}_L(p_{e,i})\gamma^\mu \nu_L(p_{\nu,j})$$

$$M^{K,L} = \sum_n \left[\frac{\langle f|J^L_i|n\rangle\langle n|J^L_i|i\rangle}{E_n - \tilde{E} - \epsilon_{K,L}} + \frac{\langle f|J^L_i|n\rangle\langle n|J^L_i|i\rangle}{E_n - \tilde{E} + \epsilon_{K,L}} \right]$$

$$\tilde{E} = \frac{E_i + E_f}{2} \qquad \epsilon_K = \frac{E_{22} - E_{11}}{2} \qquad \epsilon_L = \frac{E_{12} - E_{21}}{2} \qquad E_{ij} = E_{ei} + E_{\nu j}$$

- Already the leading-order result is sensitive to the intermediate states
- From EFT perspective, **no reason to expect $0\nu\beta\beta$ and $2\nu\beta\beta$ matrix elements to be correlated**
- Some correlation recently observed in ab-initio approach

el Morabit et al. 2412.14160



Lian et al. 2605.19479

Light Majorana mechanism: contact term

- Singular part of $0\nu\beta\beta$ potential

$$V_\nu^{\text{sing}} = \frac{\tau^{(1)+}\tau^{(2)+}}{\mathbf{q}^2}\left[1 - \frac{2}{3}g_A^2\,\boldsymbol{\sigma}^{(1)}\cdot\boldsymbol{\sigma}^{(2)}\right]$$

leads to divergence, e.g., for the $nn \to ppe^-e^-$ amplitude in $\overline{\text{MS}}$

$$\mathcal{A}^{\text{div}} = -\left(\frac{m_N}{4\pi}\right)^2(1+2g_A^2)\left[L_{\mathbf{p},\mathbf{p}'}(\mu) + \frac{1}{4-d} + \cdots\right] \qquad L_{\mathbf{p},\mathbf{p}'}(\mu) = \frac{\log\frac{\mu^2}{-(|\mathbf{p}|+|\mathbf{p}'|)^2+i\epsilon}+1}{2}$$

$\hookrightarrow$ in EFT this is absorbed by a **contact term** see talk by B. van Kolck

$$V_\nu^{\text{CT}} = -2g_\nu^{NN}\tau^{(1)+}\tau^{(2)+}$$

- Renormalization group equation ensures scale independence of $\mathcal{A}_{0\nu}$

$$\mu\frac{d\tilde{g}_\nu^{NN}}{d\mu} = \frac{1+2g_A^2}{2} \qquad \tilde{g}_\nu^{NN} = \left(\frac{4\pi}{m_N\tilde{C}}\right)^2 g_\nu^{NN} \qquad V_{NN} = \tilde{C} - \frac{g_A^2}{4F_\pi^2}\frac{M_\pi^2}{M_\pi^2+\mathbf{q}^2}$$

- The same sensitivity to short-range physics is present in all approaches using $V_{\nu,0}$

$\hookrightarrow$ general consensus in discussions during the program

How to determine the contact term?

- $0\nu\beta\beta$ contact related to $I = 2$ component of two electromagnetic currents

  $\hookrightarrow$ relation to **charge independence breaking**
- Unfortunately, two independent operators

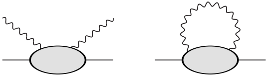
$$\mathcal{L} = \frac{e^2}{4}\big(C_1\mathcal{O}_1 + C_2\mathcal{O}_2\big) \qquad \mathcal{O}_1 = \bar{N}\mathcal{Q}_L N\,\bar{N}\mathcal{Q}_L N - \frac{\mathrm{Tr}[\mathcal{Q}_L^2]}{6}\bar{N}\boldsymbol{\tau}N\cdot\bar{N}\boldsymbol{\tau}N + \{L\leftrightarrow R\}$$

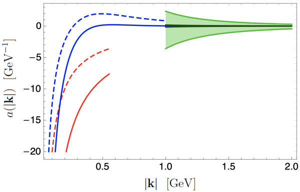
$$\mathcal{O}_2 = 2\bigg(\bar{N}\mathcal{Q}_L N\,\bar{N}\mathcal{Q}_R N - \frac{\mathrm{Tr}[\mathcal{Q}_L\mathcal{Q}_R]}{6}\bar{N}\boldsymbol{\tau}N\cdot\bar{N}\boldsymbol{\tau}N\bigg) \qquad \mathcal{Q}_L = u^\dagger Q_L u \qquad \mathcal{Q}_R = uQ_R u^\dagger$$

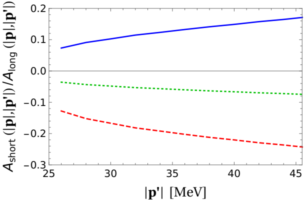
$$u = \exp\frac{i\boldsymbol{\tau}\cdot\boldsymbol{\pi}}{2F_\pi} \qquad Q_L^{\mathrm{EM}} = Q_R^{\mathrm{EM}} = \frac{\tau_3}{2} \qquad Q_L^{0\nu\beta\beta} = \tau^+ \qquad Q_R^{0\nu\beta\beta} = 0$$

- Conclusions from **chiral symmetry**
  - $0\nu\beta\beta$ depends on $C_1 = g_\nu^{NN}$
  - Charge independence breaking depends on $C_1 + C_2$
  - To differentiate: need to look at processes with pions, difficult
  - Large-N_c suggests $C_1 \simeq C_2$ Richardson et al. 2102.02184, but at which scale? Size of $1/N_c$ corrections?

Cottingham estimate







Cirigliano et al. 2012.11602, 2102.03371

- Estimate in analogy to **Cottingham approach**
  $\hookrightarrow$ relation between current–current matrix elements and scattering amplitudes
  - Construct "full amplitude" from elastic *NN* intermediate states
  - Includes information from form factors and *NN* amplitude
  - Match to chiral EFT
  - Provide results in terms of amplitude for $nn \to ppe^-e^-$
  - Use in ab-initio calculations by matching in *NN* system Wirth et al. 2105.05415
    $\hookrightarrow$ size (and even sign) of g_ν^{NN} depend on scheme and scale
  - Uncertainties from higher intermediate states ($NN\pi$ reasonably small van Goffrier 2412.08638) and lack of strict dispersive definition $\Rightarrow$ **remaining model dependence**

Lattice QCD

- Ideally: calculation of the contact term from first principles

  ↪ **lattice QCD** see talk by A. Grebe and talk by J. Moscoso

- Contact term for light Majorana exchange very challenging

  ↪ time scale a few years at physical point

- Lattice QCD gives correlation functions, **need strategy to feed results to ab initio**
  - "Road A: match to EFT": compute simple matrix element to fix low-energy constants
  - "Road B: direct extraction": compute the hadron multi-hadron matrix element itself

- Cautionary tale contact term in $2\nu\beta\beta$ see talk by L. Graf: lattice QCD result available

  NPLQCD 1701.03456, 1702.02929, but not enough information to match to chiral EFT

- Results available for short-range mechanism in $\pi^- \to \pi^+ e^- e^-$, see below

A proposal based on charge independence breaking

Isobaric multiplet mass equation

$$B(A, I; I_3) = a(A, I) + b(A, I) I_3 + c(A, I) I_3^2 \qquad I_3 = \frac{Z - N}{2}$$

- Matching to Cottingham results for $nn \to ppe^- e^-$ (or lattice QCD) works for ab initio, but difficult for phenomenological approaches

  $\hookrightarrow$ how to control short-range behavior?
- Proposal by B. van Kolck to look at **isobaric multiplet mass equation** Wigner 1954
  - $a(A, I)$ (isospin conserving): strong interactions + isoscalar Coulomb
  - $b(A, I)$ (charge symmetry breaking): isovector Coulomb + short-range CSB
  - $c(A, I)$ (charge independence breaking): isotensor Coulomb + pion-mass splitting + **short-range CIB**

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- Proposal by B. van Kolck to look at **isobaric multiplet mass equation** Wigner 1954
  1. Calculate $c(A,I)$ with Coulomb and pion-mass splitting in one-pion exchange in first-order perturbation theory

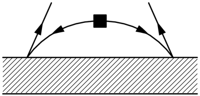
     $\hookrightarrow$ spread among many-body methods indicates long-range model dependence
  2. Add $C_1 + C_2$ Gaussian-regulated short-range interaction, fit regulator parameter to data
  3. Assume $C_1 \simeq C_2$ (or take Cottingham estimate for similar regulator parameter value)
  4. Calculate $0\nu\beta\beta$ matrix element including C_1 in first-order perturbation theory with the same regulator parameter

     $\hookrightarrow$ spread gives an estimate of remaining (hopefully reduced) model dependence

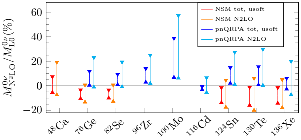
Ultrasoft contribution

$$\mathcal{A}_{0\nu}^{\text{usoft}} = -G_F^2 V_{ud}^2 m_{\beta\beta} \bar{u}_L(p_1) u_L^c(p_2) \sum_n \int \frac{d^3q}{(2\pi)^3} \frac{1}{|\mathbf{q}|} \left[\frac{\langle f|J_\mu|n\rangle\langle n|J^\mu|i\rangle}{|\mathbf{q}| + E_2 + E_n - E_i - i\epsilon} + \frac{\langle f|J_\mu|n\rangle\langle n|J^\mu|i\rangle}{|\mathbf{q}| + E_1 + E_n - E_i - i\epsilon} \right]$$

- Corrections beyond closure approximation from **ultrasoft modes** depend on nuclear intermediate states Cirigliano et al. 1710.01729



- Explicit calculations presented during program
  - NSM and QRPA see talk by D. Castillo and talk by L. Jokiniemi
    $\hookrightarrow$ broadly consistent with $\simeq 10\%$ EFT expectation
  - Ab initio (VS-IMSRG) see talk by A. Remnant
    $\hookrightarrow$ dependence on ultrasoft RG scale to be clarified



Castillo et al. 2408.03373

- In low-energy EFT, the relevant short-distance operators are:

$$\mathcal{O}_{1+}^{++} = \bar{q}_L^\alpha \tau^+ \gamma^\mu q_L^\alpha \, \bar{q}_R^\beta \tau^+ \gamma_\mu q_R^\beta \qquad \mathcal{O}_{1+}^{\prime ++} = \bar{q}_L^\alpha \tau^+ \gamma^\mu q_L^\beta \, \bar{q}_R^\beta \tau^+ \gamma_\mu q_R^\alpha$$

$$\mathcal{O}_{2+}^{++} = \bar{q}_R^\alpha \tau^+ q_L^\alpha \, \bar{q}_R^\beta \tau^+ q_L^\beta + \bar{q}_L^\alpha \tau^+ q_R^\alpha \, \bar{q}_L^\beta \tau^+ q_R^\beta \qquad \mathcal{O}_{2+}^{\prime ++} = \bar{q}_R^\alpha \tau^+ q_L^\beta \, \bar{q}_R^\beta \tau^+ q_L^\alpha + \bar{q}_L^\alpha \tau^+ q_R^\beta \, \bar{q}_L^\beta \tau^+ q_R^\alpha$$

$$\mathcal{O}_{3+}^{++} = \bar{q}_L^\alpha \tau^+ \gamma^\mu q_L^\alpha \, \bar{q}_L^\beta \tau^+ \gamma_\mu q_L^\beta + \bar{q}_R^\alpha \tau^+ \gamma^\mu q_R^\alpha \, \bar{q}_R^\beta \tau^+ \gamma_\mu q_R^\beta$$

related to "competing" basis $\mathcal{O}_i$ by

$$\langle \mathcal{O}_{1+}^{++} \rangle = \langle \mathcal{O}_4 \rangle \qquad \langle \mathcal{O}_{2+}^{++} \rangle = 2\langle \mathcal{O}_2 \rangle \qquad \langle \mathcal{O}_{3+}^{++} \rangle = 2\langle \mathcal{O}_1 \rangle \qquad \langle \mathcal{O}_{1+}^{\prime ++} \rangle = \langle \mathcal{O}_5 \rangle \qquad \langle \mathcal{O}_{2+}^{\prime ++} \rangle = 2\langle \mathcal{O}_3 \rangle$$

- Should we also "recommend values" for the corresponding 15 nuclear matrix elements? From EFT perspective, yes! see talk by A. Todd
- In addition, need hadronic matrix elements of those operators:

$$g_i^{\pi\pi} \simeq \langle \pi | \mathcal{O}_i | \pi \rangle \qquad g_i^{\pi N} \simeq \langle N | \mathcal{O}_i | N\pi \rangle \qquad g_i^{NN} \simeq \langle NN | \mathcal{O}_i | NN \rangle$$

- How to get those:
  - $SU(3)$ symmetry: relations to $K \to \pi\pi$ ($g_1^{\pi\pi}$) and kaon bag parameters Savage nucl-th/9811087, Cirigliano et al. 1701.01443
  - Lattice QCD 1805.02634, 2208.05322, 2508.01900

$0\nu\beta\beta$ beyond light Majorana exchange

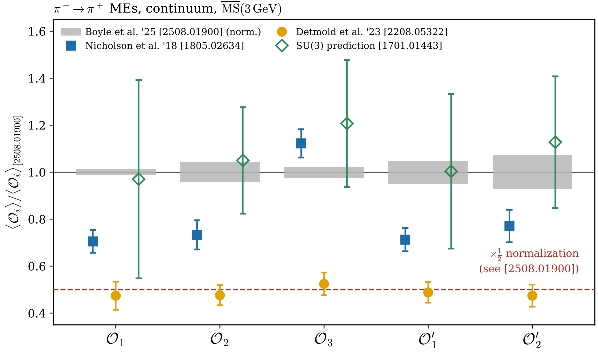


figure courtesy J. Moscoso

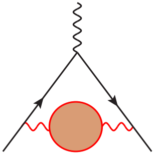
- 2508.01900 consistent with $SU(3)$ symmetry

  $\hookrightarrow$ supports normalization factor 2 in 2208.05322
- Not much known about $g_i^{\pi N}$ or g_i^{NN}:
  - Dominant uncertainty in sterile neutrino models see talk by T. Shickele and talk by A. Todd
  - $SU(3)$ estimates from hyperon decays for $g_i^{\pi N}$ conceivable, but very difficult
  - RG estimates for $g_i^{\pi N}$ for $i > 1$ (the less relevant cases)

Decomposition of nuclear matrix elements?

- Detour to hadronic contributions to the anomalous magnetic moment of the muon a_μ
  - **Hadronic vacuum polarization**: depends on matrix element

$$\Pi_{\mu\nu} \propto \int d^4x\, e^{iq\cdot x}\langle 0|T\{j_\mu^{\text{EM}}(x)j_\nu^{\text{EM}}(0)\}|0\rangle = (q^2 g_{\mu\nu} - q_\mu q_\nu)\Pi(q^2)$$

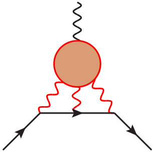


weighted over energy

$$a_\mu^{\text{HVP, LO}} \propto \int ds\, K(s)\, \text{Im}\,\Pi(s)$$

  - Can introduce additional weight functions $\theta(s)$ (“windows”) to focus on specific regions RBC/UKQCD 1801.07224

    $\hookrightarrow$ valuable tool to compare and combine different methods



  - **Hadronic light-by-light scattering**: similar decomposition used to combine dispersive evaluations with model insights on subleading contributions Aliberti et al. 2505.21476

Decomposition of nuclear matrix elements?

- Need to subtract singular tail to have chance to get something well defined

$$\mathcal{A}_{0\nu} = -\int d^3r\, \psi^-_{\mathbf{p}'}(\mathbf{r})^* \Big[V_{\nu,0}(\mathbf{r}) - V_\nu^{\text{sing}}(\mathbf{r})\Big]\psi^+_{\mathbf{p}}(\mathbf{r}) - \int d^3r\, \psi^-_{\mathbf{p}'}(\mathbf{r})^* \Big[V_\nu^{\text{sing}}(\mathbf{r}) + V_\nu^{\text{CT}}(\mathbf{r})\Big]\psi^+_{\mathbf{p}}(\mathbf{r})$$
$$\equiv \mathcal{A}_{0\nu}^{\text{LD}} + \mathcal{A}_{0\nu}^{\text{SD}}$$

- This separation no longer depends on the RG scale, each term is separately finite
- Could be modified by introducing **weight functions** to probe different regions

$$V_\nu^{\text{sing}}(\mathbf{r}) \to V_\nu^{\text{sing}}(\mathbf{r})\theta_\Lambda(\mathbf{r}) \qquad \lim_{|\mathbf{r}|\to 0} \theta_\Lambda(\mathbf{r}) = 1 \qquad \text{e.g.} \quad \theta_\Lambda(\mathbf{r}) = 1 - \tanh[|\mathbf{r}|\Lambda]$$

Decomposition of nuclear matrix elements?

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$$\mathcal{A}_{0\nu} = -\int d^3r\, \psi^-_{\mathbf{p}'}(\mathbf{r})^* \Big[V_{\nu,0}(\mathbf{r}) - V_\nu^{\text{sing}}(\mathbf{r})\Big]\psi^+_{\mathbf{p}}(\mathbf{r}) - \int d^3r\, \psi^-_{\mathbf{p}'}(\mathbf{r})^* \Big[V_\nu^{\text{sing}}(\mathbf{r}) + V_\nu^{\text{CT}}(\mathbf{r})\Big]\psi^+_{\mathbf{p}}(\mathbf{r})$$
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- Can such a decomposition be useful for the comparison (and combination) of different methods, to arrive at "recommended values"?
  - The singular part including the contact term represents **short-distance physics**
    $\hookrightarrow$ difficult to resolve, compare $\mathcal{A}_{0\nu}^{\text{SD}}(\Lambda)$ as a whole for suitable choice of Λ
  - As long as $\Lambda \ll \Lambda_{\text{NME}}$ (Λ_{NME} cutoff in many-body method), decompositions of $\mathcal{A}_{0\nu}^{\text{LD}}(\Lambda)$ should be meaningful, can define "windows" $\mathcal{A}_{0\nu}^{\text{LD}}(\Lambda_1, \Lambda_2)$ to isolate different regions
  - Attempt to go beyond comparison of $\mathbf{r}$- (or $\mathbf{q}$-)space distributions

Conclusions

- **EFT expansion** of $0\nu\beta\beta$ amplitude basis for ab-initio approach
- At leading order, no sensitivity to nuclear excitations

  $\hookrightarrow$ stark contrast to $2\nu\beta\beta$
- Corrections to closure approximation in line with expectation from chiral expansion
- **Contact term** required at leading order to ensure renormalization

  $\hookrightarrow$ matches onto short-range uncertainties in phenomenological approaches
- Currently, estimate from **Cottingham approach** used, improved input from **lattice QCD** highly anticipated
- More contact operators for other (dim-7 and dim-9) $0\nu\beta\beta$ mechanisms

  $\hookrightarrow$ $\pi\pi$ ones known, little for πN and NN
- Can one find a decomposition of the $0\nu\beta\beta$ amplitude that allows for better combination of different methods?