

Charged-weak currents in pion β decay and $\tau \rightarrow \pi\pi\nu_\tau$

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Martin Hoferichter

Albert Einstein Center for Fundamental Physics,
Institute for Theoretical Physics, University of Bern

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Testing the Standard Model in charged-weak decays

INT-26-95W, Seattle

Colangelo, Cottini, MH, Holz, [arXiv:2510.26871](#), [arXiv:2511.07507](#)

Cirigliano, MH, Valori, to appear

Radiative corrections to neutron β decay

- Start from **LEFT Lagrangian**

$$\mathcal{L}_{\text{LEFT}} = -2\sqrt{2}G_F \bar{e}_L \gamma_\rho \mu_L \bar{\nu}_{\mu L} \gamma^\rho \nu_{eL} - 2\sqrt{2}G_F V_{ud} \mathbf{C}(a, \mu) \bar{e}_L \gamma_\rho \nu_{eL} \bar{u}_L \gamma^\rho d_L + \text{h.c.} + \dots$$

\hookrightarrow scheme for G_F defined by muon decay

- Wilson coefficient for the semileptonic operator in $\overline{\text{MS}}$ + NDR for γ_5 [Cirigliano et al. 2023](#)

$$\mathbf{C}(a, \mu) = 1 + \frac{\alpha}{\pi} \log \frac{M_Z}{\mu} + \frac{\alpha}{\pi} \underbrace{\left(\frac{a}{6} - \frac{3}{4} \right)}_{\equiv B(a)} - \frac{\alpha \alpha_s}{4\pi^2} \log \frac{M_W}{\mu} + \mathcal{O}(\alpha \alpha_s, \alpha^2)$$

$$\gamma^\alpha \gamma^\rho \gamma^\beta P_L \otimes \gamma_\beta \gamma_\rho \gamma_\alpha P_L = 4[1 + a(4 - d)] \gamma^\rho P_L \otimes \gamma_\rho P_L + E(a)$$

- Dependence on a and μ needs to cancel in observables (at the order considered)
- Matching to ChPT**: relevant low-energy constant is $g_V(\mu_\chi) = 1 + \mathcal{O}(\alpha)$, and lots of work to determine these corrections, now at NLL accuracy [Gorbahn et al. 2025](#)

Radiative corrections to neutron β decay

Master formula Cingolano et al. 2023

$$g_V(\mu_\chi) = \bar{C}(\mu) \left[1 + \bar{\square}_{\text{had}}^V(\mu_0) - \frac{\alpha(\mu_\chi)}{2\pi} \left(\frac{5}{8} + \frac{3}{4} \log \frac{\mu_\chi^2}{\mu_0^2} + \left(1 - \frac{\alpha_s(\mu_0)}{4\pi} \right) \log \frac{\mu_0^2}{\mu^2} \right) \right]$$

$$C(a, \mu) \equiv \bar{C}(\mu) \left(1 + \frac{\alpha(\mu)}{\pi} B(a) \right)$$

$$\bar{\square}_{\text{had}}^V(\mu_0) \equiv -ie^2 \int \frac{d^4 q}{(2\pi)^4} \frac{\nu^2 + Q^2}{Q^4} \left[\frac{T_3(\nu, Q^2)}{2m_N \nu} - \frac{2}{3} \frac{1}{Q^2 + \mu_0^2} \left(1 - \frac{\alpha_s(\mu_0)}{\pi} \right) \right]$$

- $T_3(\nu, Q^2)$ two-current matrix element of the nucleon, μ_0 another factorization scale
- Dependence on a, μ, μ_χ, μ_0 drops out from decay rate at the considered order
- **This talk:** $C(a, \mu)$ also mediates $\tau^\pm \rightarrow \pi^\pm \pi^0 \nu_\tau$ and pion β decay $\pi^\pm \rightarrow \pi^0 e^\pm \nu_e$
 \hookrightarrow recent advances for neutron decay should prove valuable
 - Pion β decay: further improve theory for PIONEER
 - $\tau \rightarrow \pi\pi\nu_\tau$: measure pion form factor $\gamma^* \rightarrow \pi\pi$ (via isospin rotation) for a_μ^{HVP}

Master formula

$$\Gamma[\pi^+ \rightarrow \pi^0 e^+ \nu_e(\gamma)] = \frac{G_F^2 |V_{ud}|^2 M_\pi^5 |f_+^\pi(0)|^2}{64\pi^3} (1 + \Delta_{RC}^{\pi\ell}) I_{\pi\ell} \quad I_{\pi\ell} = 7.385(4) \times 10^{-8}$$

- $|f_+^\pi(0) - 1| \lesssim 10^{-5}$ due to the Behrends–Sirlin–Ademollo–Gatto theorem
- Current status:

$$V_{ud}^\pi = 0.97386(281)_{\text{BR}}(9)_{\tau_\pi}(14)_{\Delta_{RC}^{\pi\ell}}(28)_{I_{\pi\ell}}[283]_{\text{total}}$$

↪ second-largest uncertainty actually from pion-mass difference

- Theory error due to $\Delta_{RC}^{\pi\ell}$ already small, but can now improve/validate further
 - NLL resummation (same Wilson coefficient $C(a, \mu)$)
 - Improved matching to ChPT via LEC X_ℓ (takes role of g_V)

↪ show that the scheme dependence again disappears in the end

Matching relation

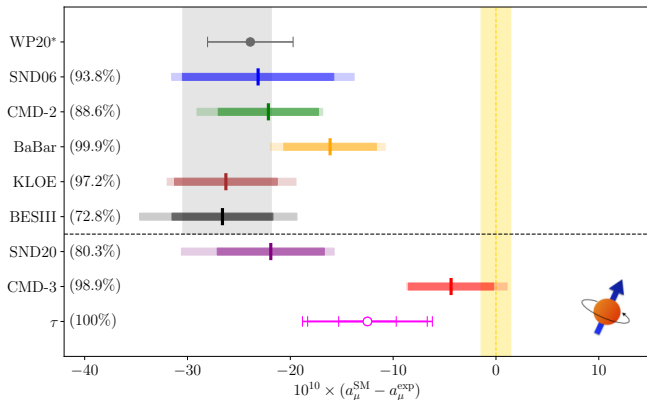
$$1 - \frac{e^2}{2} X_\ell(\mu_\chi, \mu) \equiv \left[1 + \bar{\square}_\pi^V(\mu_0) - \frac{\alpha(\mu_\chi)}{2\pi} \left(\frac{5}{8} + \frac{3}{4} \log \frac{\mu_\chi^2}{\mu_0^2} + 2B(a) + \left(1 - \frac{\alpha_s(\mu_0)}{4\pi} \right) \log \frac{\mu_0^2}{\mu^2} \right) \right]$$

$$\bar{\square}_\pi^V(\mu_0) \equiv -ie^2 \int \frac{d^4 q}{(2\pi)^4} \frac{\nu^2 + Q^2}{Q^4} \left[\frac{T_3^\pi(\nu, Q^2)}{2M_\pi \nu} - \frac{2}{3} \frac{1}{Q^2 + \mu_0^2} \left(1 - \frac{\alpha_s(\mu_0)}{\pi} \right) \right]$$

- Exact same form as for neutron, **universality**
- Derivation 1: matching with spurion method [Descotes-Genon, Moussallam 2005](#)
 - $X_\ell \equiv X_6^r - 4K_{12}^r + \frac{4}{3}X_1^r$, derive representation in terms of correlation functions for each of them
 - X_1 has $\langle 0 | T\{VV\} | \pi \rangle$ correlator and correct a dependence
 - Soft-pion theorem to make connection to T_3^π [Feng et al. 2020](#), [Yoo et al. 2023](#)
- Derivation 2: matching at the amplitude level [Abers, Dicus, Norton, Quinn 1968](#)
 - General arguments: Ward identity, current algebra, IR structure
 - IR singularities independent of spin, thus universality

- With an EFT formulation for the decay rate, we find:
 - Dependence on a cancels between Wilson coefficient and LEC
 - Dependence on μ_χ cancels when combined with ChPT calculation of long-range part of the matrix element [Cirigliano et al. 2003](#)
- Can now be combined with NLL calculation for $C(a, \mu)$ [Gorbahn, Moretti, Jäger 2025](#)
- Numerics to be finalized, expect similar error (dominated by $\bar{\chi}_\pi^V(\mu_0)$), but likely some shift in central value
- One more universality feature: **lepton flavor universality**
 \hookrightarrow same X_ℓ also required for $\tau \rightarrow \pi\pi\nu_\tau$!
- ChPT calculation was done 25 years ago [Cirigliano, Ecker, Neufeld 2001, 2002](#), but matching to short-distance contributions and long-range effects missing so far
 \hookrightarrow relies on lessons learned in the context of neutron decay [Aliberti et al. 2025](#)

Data-driven determinations of HVP



- Confusing situation in $e^+e^- \rightarrow \pi^+\pi^-$
- Can we get $\tau \rightarrow \pi\pi\nu_\tau$ theory under control to justify **τ -based HVP evaluations?**

Master formula for $\tau \rightarrow \pi\pi\nu_\tau(\gamma)$

$$\frac{1}{K_\Gamma(s)} \frac{d\Gamma}{ds} [\tau \rightarrow \pi\pi\nu_\tau(\gamma)] = \underbrace{S_{EW}^{\pi\pi}}_{\text{short distance}} \times \underbrace{[\beta_{\pi\pi^0}]^3}_{\text{phase space}} \times \underbrace{|f_+(s)|^2}_{\langle \pi\pi^0 | j_W^\mu | 0 \rangle} \times \underbrace{G_{EM}(s)}_{\text{radiative corrections}}$$

$$K_\Gamma(s) = \frac{\Gamma_e |V_{ud}|^2}{2m_\tau^2} \left(1 - \frac{s}{m_\tau^2}\right)^2 \left(1 + \frac{2s}{m_\tau^2}\right)$$

- Alternative approach via **hadronic τ decays**: $\tau \rightarrow h\nu_\tau$, $h = 2\pi, 4\pi, \dots$ related to $I = 1$ part of $e^+e^- \rightarrow h$ cross section [Alemany et al. 1998](#)
- Experimental status**: LEP and Belle, new data from Belle II
- Relation exact in limit of **isospin symmetry**
 \hookrightarrow need to control corrections, especially in $\underbrace{\langle \pi^+\pi^- | j_{em}^\mu | 0 \rangle}_{F_\pi^V(s)}$ vs. $\underbrace{\langle \pi^\pm\pi^0 | j_{W^\mp}^\mu | 0 \rangle}_{f_+(s)}$
- Isospin breaking (IB)**: corrections to CVC important, especially $f_+(s)$ vs. $F_\pi^V(s)$

Isospin-breaking corrections to $\tau \rightarrow \pi\pi\nu_\tau$: basics

Master formula for $\tau \rightarrow \pi\pi\nu_\tau(\gamma)$

$$\frac{1}{K_F(s)} \frac{d\Gamma}{ds}[\tau \rightarrow \pi\pi\nu_\tau(\gamma)] = \underbrace{S_{EW}^{\pi\pi}}_{\text{short distance}} \times \underbrace{\beta_{\pi\pi^0}^3}_{\text{phase space}} \times \underbrace{|f_+(s)|^2}_{\langle \pi\pi^0 | j_W^\mu | 0 \rangle} \times \underbrace{G_{EM}(s)}_{\text{radiative corrections}}$$

- Short-distance corrections

$$S_{EW}^{\pi\pi} = 1 + \frac{2\alpha}{\pi} \log \frac{M_Z}{m_\tau} + \dots$$

- In isospin limit: $f_+(s)$ same as $F_\pi^V(s)$ (matrix element from $e^+e^- \rightarrow \pi^+\pi^-$)
- Radiative corrections subsumed into $G_{EM}(s)$ (similar to $\eta(s)$ in $e^+e^- \rightarrow \pi^+\pi^-$)

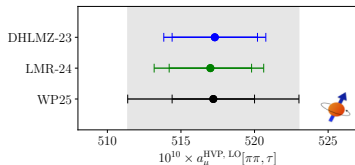
$$\sigma_{e^+e^- \rightarrow \pi^+\pi^-(\gamma)}(s) = \frac{1}{\mathcal{N}(s)\Gamma_e} \frac{d\Gamma_{\tau \pm \rightarrow \pi^\pm \pi^0 \nu_\tau(\gamma)}}{ds} \times \frac{1 + \frac{\alpha}{\pi} \eta(s)}{S_{EW}^{\pi\pi} G_{EM}(s)} \frac{[\beta_{\pi\pi}(s)]^3}{[\beta_{\pi\pi^0}(s)]^3} \left| \frac{F_\pi^V(s)}{f_+(s)} \right|^2$$

- Procedure:

- 1 Remove τ -specific IB corrections: $S_{EW}^{\pi\pi} G_{EM}(s)$ and phase space
- 2 Apply corrections to matrix element to get from $f_+(s)$ to $F_\pi^V(s)$
- 3 Add e^+e^- specific IB corrections ($\eta(s)$ and ρ - ω mixing)

Isospin-breaking corrections to $\tau \rightarrow \pi\pi\nu_\tau$: status

	Refs. [168, 196]	Ref. [211]	Refs. [239, 249]	Our estimate
Phase space	-7.88	-7.52	-	-7.7(2)
S_{EW}	-12.21(15)	-12.16(15)	-	-12.2(1.3)
G_{EM}	-1.92(90)	$(-1.67)^{+0.60}_{-1.39}$	-	-2.0(1.4)
FSR	4.67(47)	4.62(46)	4.42(4)	4.5(3)
ρ - ω mixing	4.0(4)	2.87(8)	3.79(19)	3.9(3)
ΔM_ρ	$0.20(^{+27}_{-19})(9)$	$1.95^{+1.56}_{-1.55}$	-	-
$\Delta\Gamma_\rho(\Delta M_\pi)$	4.09(0)(7)	3.37	-	-
$\Delta\Gamma_\rho(\pi\pi\gamma)$	-5.91(59)(48)	-6.66(73)	-	-
$\Delta\Gamma_\rho(g_{\rho\pi\pi})$	-	-	-	-
Total	-1.62(65)(63)	$(-1.34)^{+1.72}_{-1.71}$	-	-1.5(4.7)
Sum	-14.9(1.9)	$(-15.20)^{+2.26}_{-2.63}$	-	-15.0(5.1)



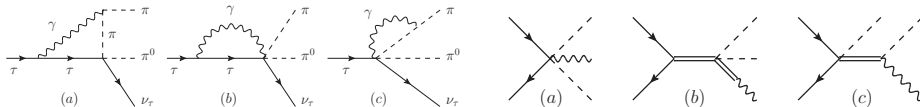
• Status of IB corrections

- FSR(s) = $1 + \frac{\alpha}{\pi}\eta(s)$ and ρ - ω mixing from $e^+e^- \rightarrow \pi^+\pi^-$
 \hookrightarrow reasonably well under control
- Otherwise, uncertainty currently difficult to quantify, attempt made in WP25

• Main challenges:

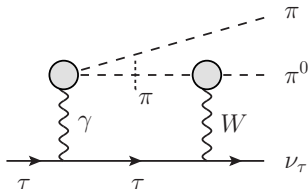
- 1 **Short-distance matching:** $\mathcal{O}(\frac{\alpha}{\pi})$ uncertainty beyond LL Analogy to neutron decay
- 2 **Long-range radiative corrections:** structure-dependent effects in $G_{EM}(s)$ See below
- 3 **IB in matrix elements:** $f_+(s)$ vs. $F_\pi^V(s)$ Colangelo, Cottini, Ruiz de Elvira 2025, Bruno et al.

Calculation in ChPT



- UV divergences removed by LECs, exactly our X_ℓ from before!
- Scheme ambiguity $\mathcal{O}(\frac{\alpha}{\pi})$ dominant uncertainty in $S_{EW}^{\pi\pi}$
 \hookrightarrow only product $S_{EW}^{\pi\pi} G_{EM}(s)$ is scheme independent
- Some (related) confusion in previous work: local ChPT contribution dropped
 \hookrightarrow includes numerically sizable term $\Delta X_\ell|_{SD} = -\frac{1}{4\pi^2} \log \frac{m_\tau^2}{M_\rho^2}$
- ChPT calculation applies at low energies, to go beyond
 - Resonance approximation for real emission
 \hookrightarrow uncertainty dominated by F_A
 - Dispersive approach to virtual diagrams

Dispersive calculation



- Dominant correction from **pion pole** in $\langle \pi \pi^0 | j_{\text{em}}^\mu j_W^\nu | 0 \rangle$

↪ reduces to ChPT for point-like form factors

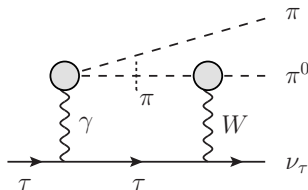
- **Matching at low energies**

$$f_{\text{loop}}^{\text{full}}(s, t) = f_{\text{loop}}^{\text{disp}}(s, t) - f_{\text{loop}}^{\text{disp}}(0, 0) + f_{\text{loop}}^{\text{ChPT}}(0, 0)$$

↪ ensures that IR structure and chiral logs are correct

- Checked that limits match onto ChPT, including narrow-width limit and $M_\rho \rightarrow \infty$ for UV divergence

Some technicalities of the box diagram



- Use an unsubtracted dispersion relation

$$f_+(s) = \frac{1}{\pi} \int_{4M_\pi^2}^{\infty} ds' \frac{\text{Im } f_+(s')}{s' - s}$$

↪ result UV finite

- Interpret Cauchy kernel as loop propagator
- Express the integral in terms of standard Passarino–Veltman functions

$$f_{\text{loop}}^{\text{disp}}(s, t) = \alpha \int_{4M_\pi^2}^{\infty} ds' \int_{4M_\pi^2}^{\infty} ds'' \text{Im } f_+(s') \text{Im } f_+(s'') \sum_{k \in \{B_0, C_0, D_0\}} \mathcal{M}_k(s, t, s', s'')$$

- IR and endpoint singularities need to be treated carefully

Real emission: bremsstrahlung off τ and π

- **Leading Low term**

- Cancels IR divergence
- Logarithmic divergence at threshold

- **Remaining radiation off τ and π**

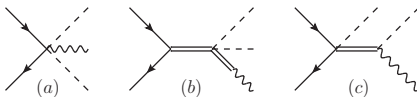
- Exhibits threshold divergence $G_{\text{EM}}(s) \propto 1/(s - 4M_\pi^2)$
- Numerically largest effect
- Delicate to evaluate close to threshold
- Developed parameterization via suitably chosen angles that allows for stable evaluation down to threshold

- **Threshold enhancement** makes certain $\mathcal{O}(e^4)$ effects relevant, otherwise, always work at $\mathcal{O}(e^2)$, ensure correct threshold by mapping

$$G_{\text{EM}}(s) \rightarrow G_{\text{EM}}[\tilde{s}(s)]$$

$$\tilde{s}(s) = \frac{(m_\tau^2 - 4M_\pi^2)s + [4M_\pi^2 - (M_\pi + M_{\pi 0})^2]m_\tau^2}{m_\tau^2 - (M_\pi + M_{\pi 0})^2} \quad \tilde{s}[(M_\pi + M_{\pi 0})^2] = 4M_\pi^2 \quad \tilde{s}(m_\tau^2) = m_\tau^2$$

Real emission: resonance diagrams



- Keep states required for **resonance saturation** of $L_9, L_{10} + \text{WZW}$ anomaly
 \hookrightarrow free parameters F_V, G_V, F_A

- Short-distance constraints**

$$F_V = \sqrt{2}F_\pi \simeq 0.13 \text{ GeV} \quad G_V = \frac{F_\pi}{\sqrt{2}} \simeq 0.065 \text{ GeV} \quad F_A = F_\pi \simeq 0.092 \text{ GeV}$$

- Phenomenological determinations** from $\rho \rightarrow e^+e^-, \pi\pi, K^* \rightarrow K\pi, a_1 \rightarrow \pi\gamma$

$$F_V \simeq 0.16 \text{ GeV} \quad G_V \simeq 0.065 \text{ GeV} \quad F_A \simeq 0.12 \text{ GeV}$$

- We tried a new estimate $a_1 \rightarrow \pi\rho \rightarrow \pi\gamma$, yielding $F_A = (0.07 \dots 0.13) \text{ GeV}$
- Uncertainty in F_A dominant effect, little motivation to include higher multiplets

Dispersive representation of pion form factor

- Need input for $\text{Im } f_+(s)$, should be consistent with $\tau \rightarrow \pi\pi\nu_\tau$ spectrum
- Follow strategy from Colangelo, MH, Stoffer 2018 ..., supplemented by ρ' , ρ''

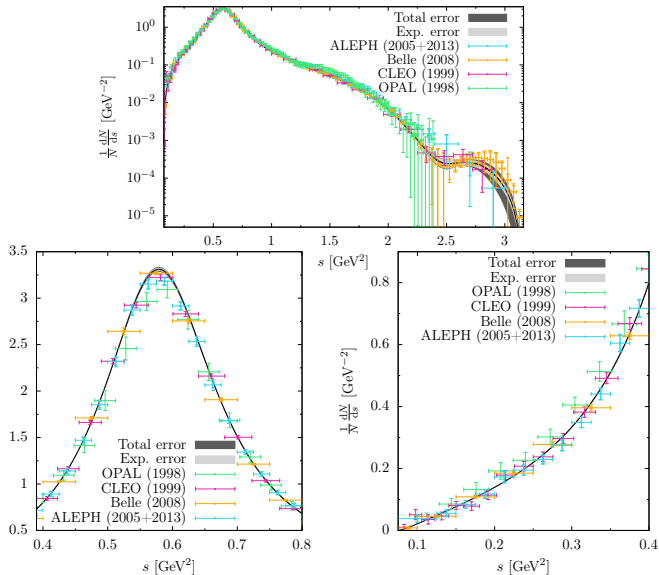
$$f_+(s) = \left[1 + G_{\text{in}}^N(s) + \sum_{V=\rho', \rho''} c_V \mathcal{A}_V(s) \right] \Omega_1^1(s) \quad \Omega_1^1(s) \equiv \exp \left\{ \frac{s}{\pi} \int_{4M_\pi^2}^{\infty} ds' \frac{\delta_1^1(s')}{s'(s' - s)} \right\}$$

- P -wave phase shift $\delta_1^1(s)$ from Roy equations, free parameters: $\delta_1^1(s_0)$, $\delta_1^1(s_1)$
- Conformal polynomial with $\pi\omega$ threshold, constrained as P wave and by $f_+(0) = 1$
- Resonance terms

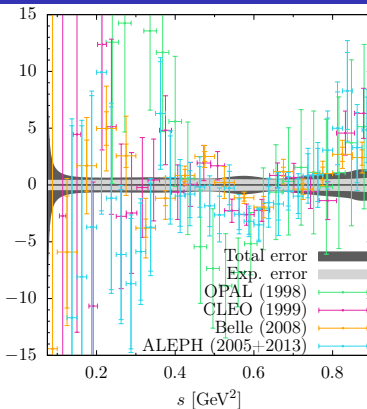
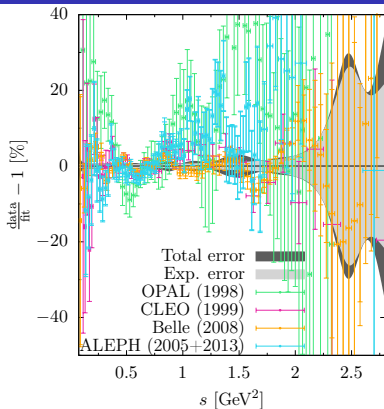
$$\mathcal{A}_V(s) = \frac{s}{\pi} \int_{s_{\text{thr}}}^{\infty} ds' \frac{\text{Im } \mathcal{A}_V(s')}{s'(s' - s)} \quad \text{Im } \mathcal{A}_V(s) = \text{Im} \frac{1}{M_V^2 - s - i\sqrt{s}\Gamma_V(s)}$$

- Total number of parameters: $2 + 3 \times 2 + N - 2 = 6 + N$
- Calculate first approximation for $G_{\text{EM}}(s)$ with $f_+(s) = \Omega_1^1(s)$, iterate until convergence (few steps)

Fits to the τ spectrum



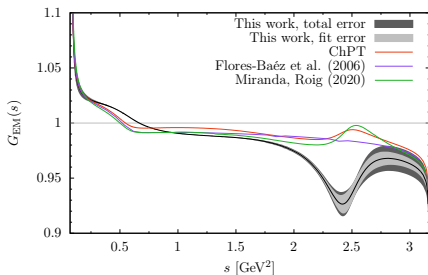
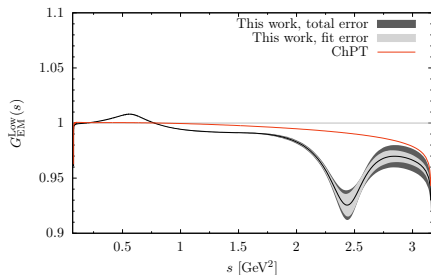
Fits to the τ spectrum



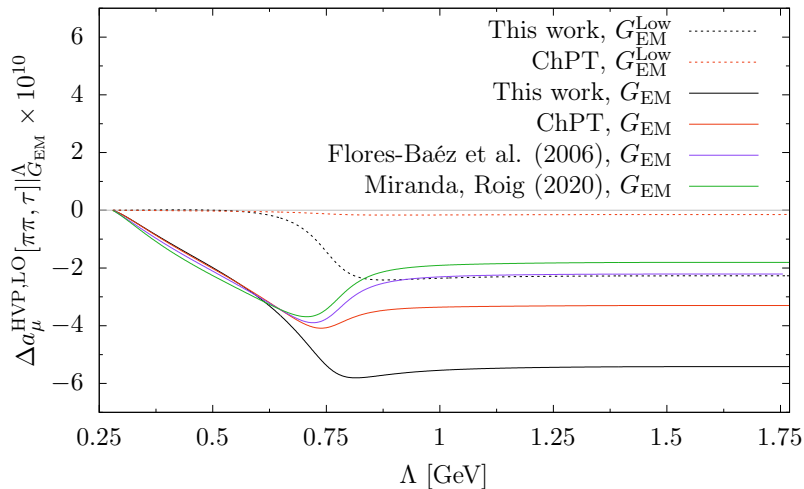
● Observations

- Fits are not perfect, tensions among the data sets do exist
- Tension between threshold and $\rho(770)$ region upon imposing analyticity/unitarity constraints
- **New data from Belle II would be extremely valuable!**

Results



	DHLMZ-23	LMR-24	WP25	This work
Phase space	−7.88	−7.52	−7.7(2)	−7.74(5)
$S_{EW}^{\pi\pi}$	−12.21(15)	−12.16(15)	−12.2(1.3)	−12.2(1.3)
G_{EM}^{full}	−1.92(90)	$(-1.67)^{+0.60}_{-1.39}$	−2.0(1.4)	−5.4(5)
Sum	−22.01(91)	$(-21.35)^{+0.62}_{-1.40}$	−21.9(1.9)	−25.3(1.4)
Full	—	—	—	−24.8(1.4)



- Now that we have the long-distance parts under control (ChPT + dispersion relations), can go **back to matching**

$$G_{\text{EM}}(s)|_{X_\ell(\mu_\chi)=0} = G_{\text{EM}}(s)|_{X_\ell(\mu_\chi)=\bar{X}_\ell(\mu_\chi)} + e^2 \bar{X}_\ell(\mu_\chi)$$

\hookrightarrow linear in X_ℓ , easy to adjust

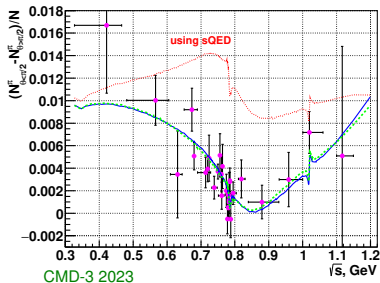
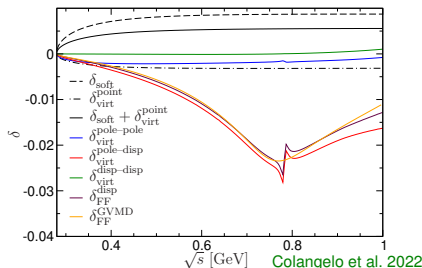
- Standard short-distance factor

$$S_{\text{EW}}^{\pi\pi} = S_{\text{EW}} - \Delta S_{\text{EW}}^{\text{sub, lep}} \quad \Delta S_{\text{EW}}^{\text{sub, lep}} = \frac{\alpha}{2\pi} \left(\frac{25}{4} - \pi^2 \right)$$

- Can replace this via $X_\ell(\mu, \mu_\chi)$ and $\bar{C}(\mu)$
 - Dependence on a cancels between Wilson coefficient and LEC
 - Dependence on μ_χ cancels between LEC and $G_{\text{EM}}(s)$
 - Can again combine with NLL calculation for $C(a, \mu)$

- Recent advances in the theory of neutron decay relevant for other charged-weak decays
- **Pion β decay**
 - Matching to ChPT: express chiral LEC X_ℓ in terms of T_3^π , using $\overline{\text{MS}}$ (LEFT) and $\overline{\text{MS}}_\chi$ (ChPT), respectively
 - In this form, allows for combination with NLL resummation
- **Isospin-breaking corrections to $\tau \rightarrow \pi\pi\nu_\tau$**
 - Extended validity of $G_{\text{EM}}(s)$ beyond low-energy region via dispersion relations
 - Structure-dependent virtual corrections are large, enhanced by $\rho(770)$
 - Improved matching to ChPT and short-distance corrections again in terms of X_ℓ
 - Removes another key source of uncertainty
- Are there relevant effects in strangeness-changing decays?

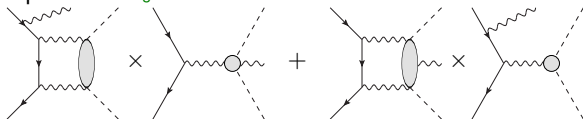
Structure-dependent radiative corrections in $e^+e^- \rightarrow \pi^+\pi^-$



- CMD-3 found large deviations from MC result for **forward-backward asymmetry**
 \hookrightarrow understood from **resonance enhancement of virtual corrections** Ignatov, Lee 2022

- Potentially relevant for ISR experiments Ignatov 2021

\hookrightarrow **ISR-FSR interference:**



- Under investigation RadioMonteCarLow 2