Chiral Hamiltonians, charge distributions, and the search for $\mu \to e$ conversion in nuclei

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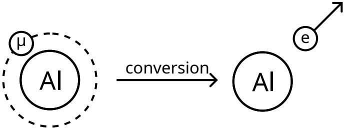
INT program on

Nuclear Hamiltonians for Advancing Nuclear Physics and Beyond

Noël, MH JHEP 08 (2024) 052

Heinz, MH, Miyagi, Noël, Schwenk PLB 871 (2025) 139975, to appear

What is $\mu \to e$ conversion?



- What is $\mu \to e$ conversion? A theorist's perspective:
  - Muon bound in 1s level of nucleus
  - Muon converts to electron within Coulomb field of nucleus

$$\bar{e}\sigma^{\alpha\beta}\mu F_{\alpha\beta}, \qquad \bar{e}\mu\,\bar{q}q, \qquad \ldots$$

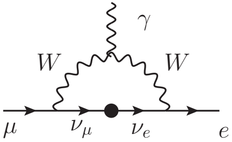
  $\hookrightarrow$ both long- and short-range BSM mechanisms possible

  - Electron ejected with energy of muon–electron mass difference (minus binding energy)

  $\hookrightarrow$ **very clean experimental signature**

- How to get accurate predictions for the matrix elements?
  - Fits into the "Beyond" part of this program
  - Challenges for chiral Hamiltonians, in particular **charge distributions**

Why lepton flavor violation?



- **Lepton flavor symmetry**
  - Lepton flavor conserved in SM with massless neutrinos
  - **Neutrino oscillations** sign of lepton flavor violation (LFV) in neutral sector
  - Propagates to charged sector via **mass insertions** in loops, but, e.g.,

$$\text{Br}[\mu \to e\gamma] \simeq \left(\frac{\Delta m_\nu^2}{M_W^2}\right)^2 \simeq 10^{-50}$$

  $\hookrightarrow$ unobservably small in SM!
  - Lepton flavor **"accidental" symmetry** of SM

  $\hookrightarrow$ LFV expected to occur for a wide range of BSM scenarios
  - In practice: LFV highly sensitive null test

Why $\mu \to e$ conversion in nuclei?

LFV process	current limit on Br	(planned) experiments
$\mu \to e\gamma$	$< 4.2 \times 10^{-13}$ MEG	MEG II
$\mu \to 3e$	$< 1.0 \times 10^{-12}$ SINDRUM	Mu3e
$\tau \to \ell\gamma, 3\ell, \ell P, \ldots$	$\lesssim 10^{-8}$ Belle, LHCb, ...	Belle 2, ...
$K \to \mu e, \mu e\pi, \mu e\pi\pi$	$\lesssim 10^{-11}$ KTeV, NA62, BNL	KOTO, LHCb
$\pi^0 \to \bar{\mu}e$	$< 3.6 \times 10^{-10}$ KTeV	
$\eta \to \bar{\mu}e$	$< 6 \times 10^{-6}$ SPEC	JEF, REDTOP (?)
$\eta' \to \bar{\mu}e$	$< 4.7 \times 10^{-4}$ CLEO II	
Au $\mu^- \to$ Au e^-	$< 7 \times 10^{-13}$ SINDRUM II	
Ti $\mu^- \to$ Ti e^-	$< 6.1 \times 10^{-13}$ SINDRUM II	
Al $\mu^- \to$ Al e^-	$\lesssim 10^{-17}$ (projected)	Mu2e, COMET

$\hookrightarrow$ major experimental improvements expected in coming years!

Why effective field theory?

- For theory description, many different scales matter:
  - **BSM scale**: LFV operators $\Rightarrow$ Standard Model EFT (SMEFT)
  - **Electroweak scale**: integrate out W, Z $\Rightarrow$ low-energy EFT (LEFT)
  - **Hadronic scales**: $\Lambda_\chi \simeq 4\pi F_\pi \simeq m_N$ $\Rightarrow$ chiral perturbation theory
  - **Nuclear scales**: $M_\pi \simeq k_F \simeq \gamma$ $\Rightarrow$ chiral EFT, pionless EFT
  - **Atomic scales**: atomic binding $\Rightarrow$ Dirac equation, Coulomb distortions
- Objectives of EFT approach:
  - **Compare different probes of LFV**

    $\hookrightarrow$ example: $\mu \to e$ conversion in nuclei vs. $P \to \bar{\mu} e$ decays, $P = \pi^0, \eta, \eta'$
  - Discriminate among different underlying BSM operators
  - **Control theoretical uncertainties**

    $\hookrightarrow$ hadronic matrix elements, nuclear responses, Coulomb distortions
  - RG corrections
- Here: focus on the "nuclear responses" part, in particular the **challenges for chiral Hamiltonians**

Master formulae

Very schematic master formula for $\mu \to e$ conversion

$$\Gamma[\mu(A,Z) \to e(A,Z)] \simeq \text{BSM Wilson coefficients} \otimes \text{hadronic matrix elements} \otimes \text{nuclear responses} \otimes \text{bound-state solution}$$

- Latter two traditionally combined into **overlap integrals** Kitano et al. 2002

$$S^{(N)} = \frac{\#N}{2\sqrt{2}} \int_0^\infty dr\, \rho_N(r)\big[g^{(e)}_{-1}(r) g^{\mu}_{-1}(r) - f^{e}_{-1}(r) f^{\mu}_{-1}(r)\big] \qquad V^{(N)} = \frac{\#N}{2\sqrt{2}} \int_0^\infty dr\, \rho_N(r)\big[g^{(e)}_{-1}(r) g^{\mu}_{-1}(r) + f^{e}_{-1}(r) f^{\mu}_{-1}(r)\big]$$

$$D = \frac{-4m_\mu}{\sqrt{2}} \int_0^\infty dr\, E(r)\big[g^{e}_{-1}(r)\, f^{\mu}_{-1}(r) + f^{e}_{-1}(r)\, g^{\mu}_{-1}(r)\big]$$

  $\hookrightarrow$ covers coherently enhanced **spin-independent** responses, $\Gamma_{\text{SI}} \propto \#N^2$

- Similarly, dominant **spin-dependent** contribution from nuclear responses finite for $q = 0$, e.g., for axial-vector operators $\Gamma_{\text{SD}} \propto g_A^2$ Davidson et al. 2018
- Further subleading responses from **multipole decomposition** Serot 1978

  $\hookrightarrow$ need to combine with solution of Dirac equation

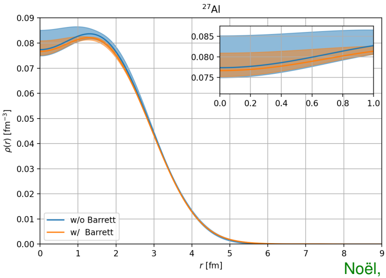
Uncertainty quantification for nuclear responses

- Uncertainty estimates difficult, especially for neutron responses due to lack of data comparisons
  $\hookrightarrow$ parity-violating electron scattering
- **Ab-initio approaches** promise uncertainty quantification, but:
  - Often uncertainties dominated by chiral Hamiltonian, not by many-body solution
  - Often correlations between different responses much more stable
    Hagen et al. 2015, Payne et al. 2019, Hu et al. 2022
- Need for **charge distributions with quantified uncertainties**
  - Solution of the Dirac equation
  - Input for correlation analysis in ab-initio approaches
- Charge distributions extracted from electron scattering **without uncertainties (!)**
  $\hookrightarrow$ **Fourier–Bessel expansions** Dreher et al. 1974, de Vries et al. 1987

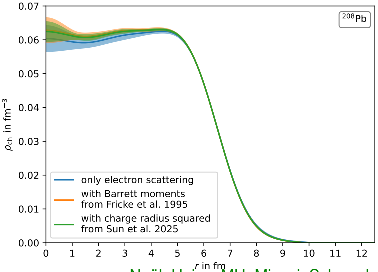
$$\rho(r) = \begin{cases} \sum_{n=1}^{N} a_n j_0(q_n r) & r \leq R \\ 0 & r > R \end{cases} \quad \text{with} \quad q_n = \frac{\pi n}{R}$$

Extracting charge distributions from electron scattering

- Practical challenges of re-analysis:
  - Most data taken in the 70s+80s
  - Many data sets not available at all ("private communication"), or only published in PhD theses
  - Documentation of uncertainties rudimentary
- Propagation of uncertainties computationally intensive
  $\hookrightarrow$ truncation errors in R, N
- Carried this program out for ^{27}Al, ^{40,48}Ca, ^{48,50}Ti, and **^{208}Pb**
- Results available as `python` notebook
  2406.06677



Noël, MH 2024



Noël, Heinz, MH, Miyagi, Schwenk to appear

Solving the Coulomb problem

- For a realistic description, need to resum **Coulomb phase shifts**

  $\hookrightarrow$ phase shift model

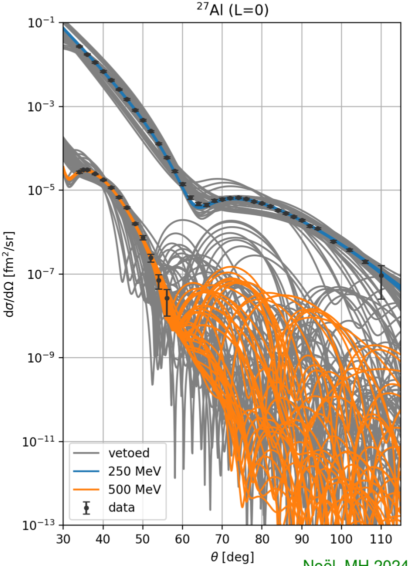
- Fits carried out over large grid of (N, R), using veto on oscillations and asymptotics to prevent overparameterization

- Constraints from **Barrett moments** measured in $2p \to 1s$ transitions of muonic atoms

$$\left\langle r^k e^{-\alpha r}\right\rangle = \frac{4\pi}{Z}\int_0^\infty dr\, r^{k+2}\rho(r) e^{-\alpha r}$$

- Similar approach also applies to Coulomb corrections elsewhere

  $\hookrightarrow$ parity-violating electron scattering



Noël, MH 2024

Results for dipole overlap integrals

Dipole overlap integrals

$$D\left({}^{27}\text{Al}\right) = 0.0359(2) \qquad D\left({}^{40}\text{Ca}\right) = 0.07531(5) \qquad D\left({}^{48}\text{Ca}\right) = 0.07479(10)$$

$$D\left({}^{48}\text{Ti}\right) = 0.0864(1) \qquad D\left({}^{50}\text{Ti}\right) = 0.0870(3)$$

- Dipole overlap integrals

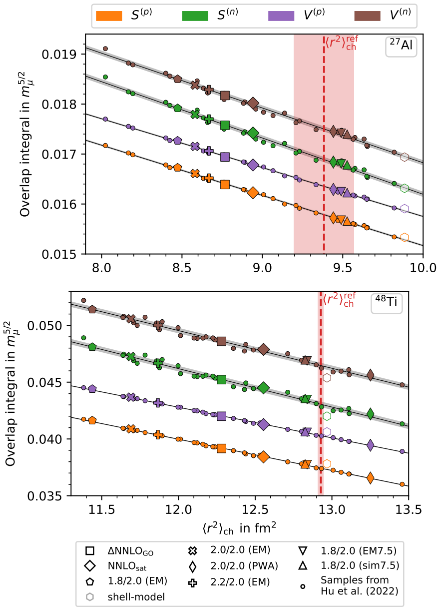
$$D = \frac{-4m_\mu}{\sqrt{2}} \int_0^\infty dr\, E(r) \left[g_{-1}^e(r)\, f_{-1}^\mu(r) + f_{-1}^e(r)\, g_{-1}^\mu(r)\right]$$

  $\hookrightarrow$ only depends on charge distributions, electric field $E(r) = \frac{\sqrt{4\pi\alpha}}{r^2} \int_0^r dr'\, r'^2 \rho(r')$

- For the first time, **fully quantified uncertainties**
- For $S^{(N)}$, $V^{(N)}$: charge distribution no longer sufficient

  $\hookrightarrow$ need additional nuclear-structure input

Ab-initio calculations for $\mu \to e$ conversion



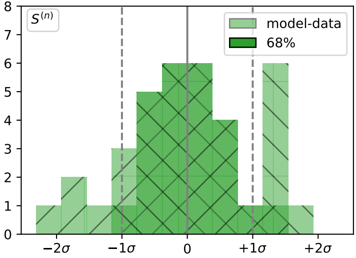
Heinz, MH, Miyagi, Noël, Schwenk 2025

- Ab-initio calculations done using IMSRG
  see talks by Matthias, Takayuki, Achim
- Almost perfect correlations for proton overlap integrals, stringent ones for neutron ones

Ab-initio calculations for $\mu \to e$ conversion

	I_i	this work	Kitano et al. 2002
	D	0.0359(2)	0.0362
	$S^{(p)}$	0.01579(2)(19)	0.0155
^{27}Al	$S^{(n)}$	0.01689(5)(21)	0.0167
	$V^{(p)}$	0.01635(2)(18)	0.0161
	$V^{(n)}$	0.01750(5)(21)	0.0173

	D	$S^{(p)}$	$S^{(n)}$	$V^{(p)}$	$V^{(n)}$
D	1.0000	0.7205	0.7030	0.7210	0.7028
$S^{(p)}$		1.0000	0.9656	1.0000	0.9645
$S^{(n)}$			1.0000	0.9664	1.0000
$V^{(p)}$				1.0000	0.9654
$V^{(n)}$					1.0000



- Distribution of residuals looks reasonable
- **Uncertainties and correlations** for (spin-independent) overlap integrals
- Important to assess **discriminatory power** of subleading responses

Ab-initio calculations for parity-violating electron scattering

Parity-violating asymmetry in electron–nucleus scattering

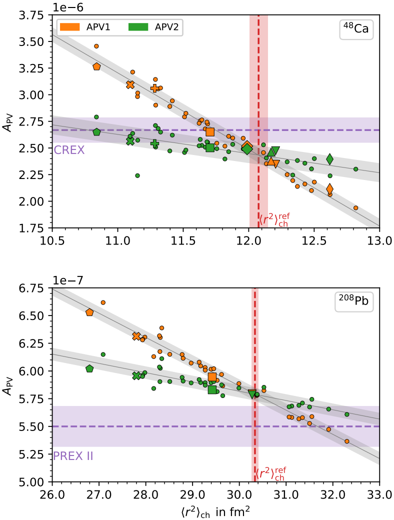
$$A_{\rm PV}=\frac{\left(\frac{d\sigma}{d\Omega}\right)_R-\left(\frac{d\sigma}{d\Omega}\right)_L}{\left(\frac{d\sigma}{d\Omega}\right)_R+\left(\frac{d\sigma}{d\Omega}\right)_L}\simeq-\frac{G_F q^2}{4\pi\alpha_{\rm el}\sqrt{2}}\frac{Q_{\rm w}F_{\rm w}(q)}{ZF_{\rm ch}(q)}$$

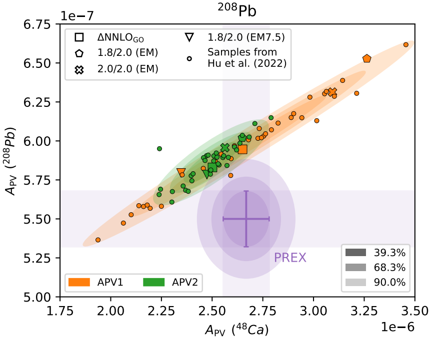
- **Weak form factor** $F_{\rm w}(q)$ requires same responses as overlap integrals $S^{(N)}$, $V^{(N)}$

  $\hookrightarrow$ similar strategy should apply
- Same basic idea as for weak/neutron radius Hagen et al. 2015, Payne et al. 2019, Hu et al. 2022
- Cleanest comparison to CREX/PREX directly at the level of the measured $A_{\rm PV}$
  - Coulomb distortions Horowitz 1998
  - Experimental acceptance function, angular average

  $\hookrightarrow$ same machinery as developed for the $\mu\to e$ overlap integrals applies
- Two possible correlation strategies
  - "APV1": use experimental charge distributions (for $F_{\rm ch}(q)$ and Coulomb corrections)
  - "APV2": always use the respective ab-initio charge distributions

Ab-initio calculations for parity-violating electron scattering

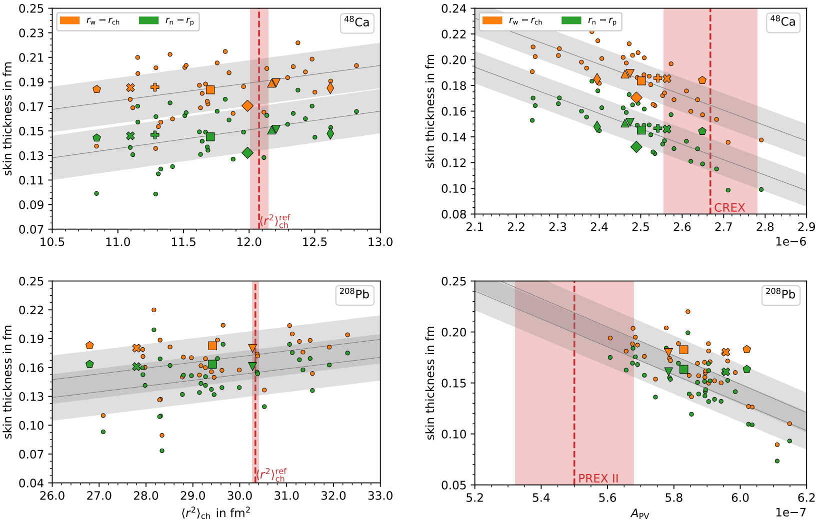




Noël, Heinz, MH, Miyagi, Schwenk to appear

- APV1 and APV2 consistent at r^{ref}_{ch}
- Difference to CREX/PREX $\simeq 1.5\sigma$ (in opposite directions), combined “tension” similar
- Same picture for weak/neutron radii and skin

Correlations for skin thickness: forward and backward



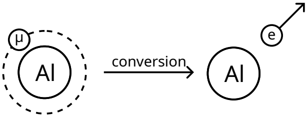
- Determinations from A_{PV}^{exp} and r_{ch}^{ref} consistent at $\lesssim 1.5\sigma$

Challenges for chiral Hamiltonians

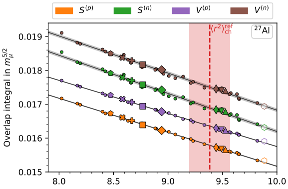
- Is there a way to go **beyond correlation analyses**?
  - Order-by-order convergence in a given scheme (w/ or w/o Δ, given set of regulators)
  - Propagation of uncertainties in the LECs, not scan over “non-implausible” ones
  - Natural proof-of-principle would be charge distributions

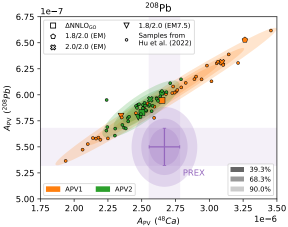
    $\hookrightarrow$ systematic “pure” chiral calculations for nuclei of interest?
- **Spin-dependent contributions to $\mu \to e$** work in progress
  - Master formula more complicated, but can still be reduced to overlap integrals
  - Many-body calculations much more costly, correlation analysis difficult
  - Even if not, to which observables should one correlate?
  - Similar comments apply to the direct-detection of dark matter and CEνNS
- Uncertainty quantification important to assess which BSM mechanisms could realistically be differentiated

Conclusions



- **EFT for $\mu \to e$ conversion in nuclei**
  - Requires input for nuclear responses with quantified uncertainties
  - Spin-independent case “solved” using correlations with charge distributions/radii
- **Parity-violating electron scattering**
  - Direct correlation of A_{PV} including Coulomb distortions
  - Remaining tension with CREX and PREX small
- **Challenges for chiral Hamiltonians**
  - How to do better than correlation analyses?
  - What to do for spin-dependent responses?





Application: indirect limits for $P \to \bar{\mu} e$

- Same operators probed in SD $\mu \to e$ conversion and $P \to \bar{\mu} e$ decays Gan et al. 2022

$$\mathcal{L}_{\text{eff}} \supset \frac{1}{\Lambda^2} \sum_{\substack{Y=L,R \\ q=u,d,s}} \Big[C_Y^{P,q} (\overline{e_Y} \mu)(\bar{q}\gamma_5 q) + C_Y^{A,q} (\overline{e_Y}\gamma^\mu \mu)(\bar{q}\gamma_\mu \gamma_5 q) \Big] + \frac{i\alpha_s}{\Lambda^3} \sum_{Y=L,R} C_Y^{G\tilde{G}} (\overline{e_Y}\mu) G^a_{\mu\nu} \tilde{G}^{\mu\nu}_a + \text{h.c.}$$

$\hookrightarrow$ **axial-vector, pseudoscalar, $G\tilde{G}$ operators**

- Can use $\mu \to e$ conversion limits to derive indirect limits for $P \to \bar{\mu} e$
- In general, not the same linear combinations appear

  $\hookrightarrow$ consider first a single operator at a time, account for matrix elements

$\mu \to e$ (exp)	$P \to \bar{\mu} e$ (derived)	current limit
	$\text{Br}[\pi^0 \to \bar{\mu} e] \lesssim 4 \times 10^{-17}$	$< 3.6 \times 10^{-10}$
$\text{Br}[\mu\text{Ti} \to e\text{Ti}] < 6.1 \times 10^{-13}$	$\text{Br}[\eta \to \bar{\mu} e] \lesssim 5 \times 10^{-13}$	$< 6.0 \times 10^{-6}$
	$\text{Br}[\eta' \to \bar{\mu} e] \lesssim 7 \times 10^{-14}$	$< 4.7 \times 10^{-4}$

- General sensitivity orders of magnitude better, but could there be cancellations?

Indirect limits: impact of cancellations

- For a rigorous limit one needs to **scan over all Wilson coefficients**

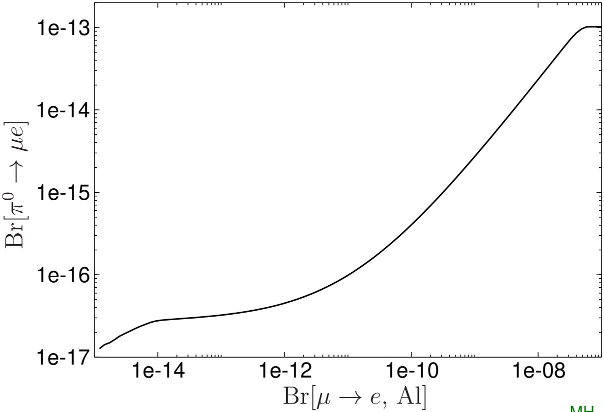
  $\hookrightarrow$ there are (fine-tuned) scenarios in which the $\mu \to e$ rate vanishes exactly
- For $\pi^0 \to \bar{\mu}e$: decay rate vanishes as well!

  rigorous limit: $\quad \text{Br}[\pi^0 \to \bar{\mu}e] < 1.0 \times 10^{-13} \quad (\text{direct limit:} < 3.6 \times 10^{-10})$
- For $\eta, \eta' \to \bar{\mu}e$: in principle, no strict limits, but required cancellations easily lifted by RG corrections
- RG produces SI operators, even when only starting with SD ones at high scale

  Cirigliano et al. 2017, Crivellin et al. 2017

$$C_Y^{V,q} \simeq -3Q_q \frac{\alpha}{\pi} \log \frac{M_W}{m_N} C_Y^{A,q}$$

Future projection for $\pi^0 \to \bar{\mu}e$



MH, Menéndez, Noël 2023

- Combining Al and Ti limits, the single-operator sensitivity is restored

  ↪ **complementarity of different target nuclei**