Perspectives on muon $g-2$ and the Cabibbo angle anomaly

Martin Hoferichter

Albert Einstein Center for Fundamental Physics,
Institute for Theoretical Physics, University of Bern

b
UNIVERSITÄT
BERN

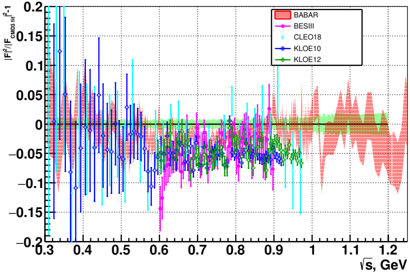
AEC
ALBERT EINSTEIN CENTER
FOR FUNDAMENTAL PHYSICS

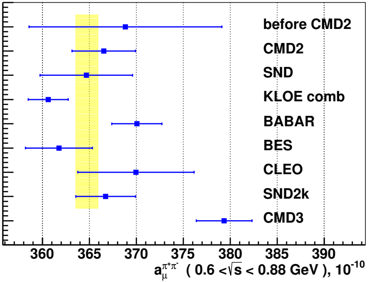
May 11, 2023

INT program on

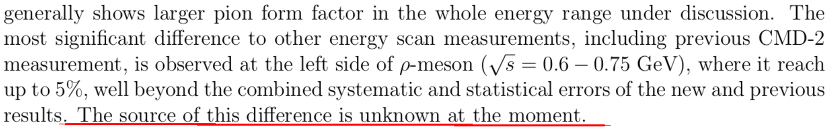
New Physics Searches at the Precision Frontier

A new puzzle: $e^+e^- \to \pi^+\pi^-$ from CMD-3





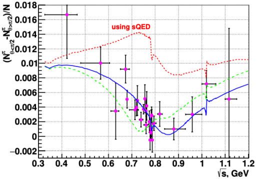
CMD-3, 2302.08834



generally shows larger pion form factor in the whole energy range under discussion. The most significant difference to other energy scan measurements, including previous CMD-2 measurement, is observed at the left side of ρ-meson ($\sqrt{s} = 0.6 - 0.75$ GeV), where it reach up to 5%, well beyond the combined systematic and statistical errors of the new and previous results. The source of this difference is unknown at the moment.

Radiative corrections: forward–backward asymmetry

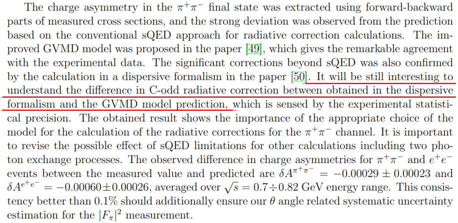
Discrepancy with Calculation of Radiative Corrections



Measured forward-backward asymmetry in
e^+e $^+$ disagrees with standard sQED code

https://indico.cern.ch/event/1204084 CMD-3 Collaboration, arXiv:2302.08834

John Ellis, "The future of particle physics", ALPS 2023

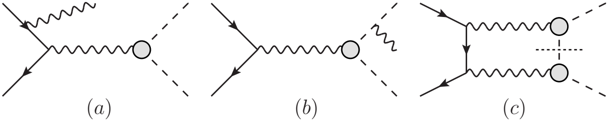


The charge asymmetry in the $\pi^+\pi^-$ final state was extracted using forward-backward parts of measured cross sections, and the strong deviation was observed from the prediction based on the conventional sQED approach for radiative correction calculations. The improved GVMD model was proposed in the paper [49], which gives the remarkable agreement with the experimental data. The significant corrections beyond sQED was also confirmed by the calculation in a dispersive formalism in the paper [50]. It will be still interesting to understand the difference in C-odd radiative correction between obtained in the dispersive formalism and the GVMD model prediction, which is sensed by the experimental statistical precision. The obtained result shows the importance of the appropriate choice of the model for the calculation of the radiative corrections for the $\pi^+\pi^-$ channel. It is important to revise the possible effect of sQED limitations for other calculations including two photon exchange processes. The observed difference in charge asymmetries for $\pi^+\pi^-$ and e^+e^- events between the measured value and predicted are $\delta A^{\pi^+\pi^-} = -0.00029 \pm 0.00023$ and $\delta A^{e^+e^-} = -0.00060 \pm 0.00026$, averaged over $\sqrt{s} = 0.7 \div 0.82$ GeV energy range. This consistency better than 0.1% should additionally ensure our θ angle related systematic uncertainty estimation for the $|F_\pi|^2$ measurement.

- **Forward–backward asymmetry**:

$$A_{\text{FB}}(z) = \frac{\frac{d\sigma}{dz}(z) - \frac{d\sigma}{dz}(-z)}{\frac{d\sigma}{dz}(z) + \frac{d\sigma}{dz}(-z)} \qquad \frac{d\sigma}{dz}\bigg|_{C\text{-odd}} = \frac{d\sigma_0}{dz}\Big[\delta_{\text{soft}}(\lambda^2,\Delta) + \delta_{\text{virt}}(\lambda^2)\Big] + \frac{d\sigma}{dz}\bigg|_{\text{hard}}(\Delta)$$

Radiative corrections: forward–backward asymmetry



- δ_{soft} in point-like approximation for final-state photon in (b), but pion VFF always included otherwise

  ↪ **FsQED**

- Previously, (c) evaluated in sQED, not FsQED

  ↪ CMD-3 use generalized vector meson dominance instead Ignatov, Lee 2022

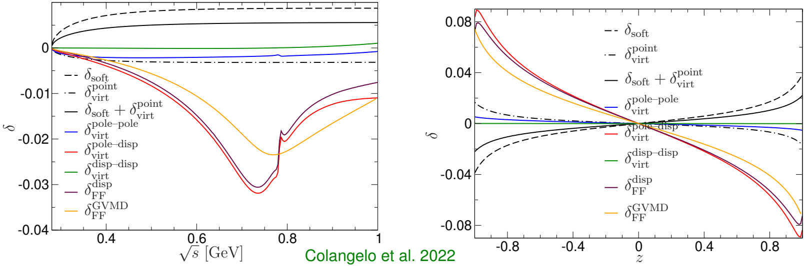
- Problem: unphysical imaginary parts below 2π threshold in loop integral

- Our approach: use **dispersive representation of pion VFF**

$$\frac{F_\pi^V(s)}{s} = \frac{1}{s} + \frac{1}{\pi}\int_{4M_\pi^2}^{\infty} ds' \frac{\operatorname{Im} F_\pi^V(s')}{s'(s'-s)} \to \frac{1}{s-\lambda^2} - \frac{1}{\pi}\int_{4M_\pi^2}^{\infty} ds' \frac{\operatorname{Im} F_\pi^V(s')}{s'} \frac{1}{s-s'}$$

↪ captures all the structure-dependent, infrared-enhanced effects

Radiative corrections: forward–backward asymmetry



Colangelo et al. 2022

- Actually good agreement between dispersive formulation and GVMD!

  ↪ why do the unphysical imaginary parts not matter more?
- FsQED describes the data well, actually confirms common lore
- Are there relevant effects being missed in the *C*-even contributions?

CMD-3 with dispersive constraints

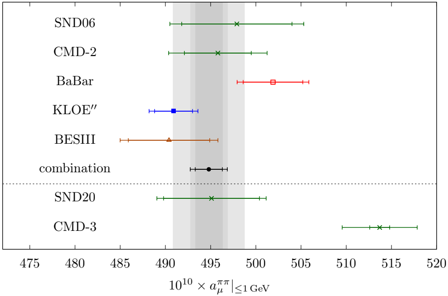
The pion form factor from dispersion relations

$$F_\pi^V(s) = \underbrace{\Omega_1^1(s)}_{\text{elastic } \pi\pi \text{ scattering}} \times \underbrace{G_\omega(s)}_{\text{isospin-breaking } 3\pi \text{ cut}} \times \underbrace{G_{\text{in}}(s)}_{\text{inelastic effects: } 4\pi, \ldots}$$

- $e^+e^- \to \pi^+\pi^-$ cross section subject to strong constraints from **analyticity**, **unitarity**, **crossing symmetry**, leading to dispersive representation with few parameters Colangelo, MH, Stoffer, 2018, 2021, 2022, work in progress
  - **Elastic $\pi\pi$ scattering**: two values of phase shifts
  - **ρ–ω mixing**: ω pole parameters and residue
  - **Inelastic states**: conformal polynomial

$\hookrightarrow$ cross check on data, functional form for all $s \leq 1\,\text{GeV}^2$

CMD-3 with dispersive constraints

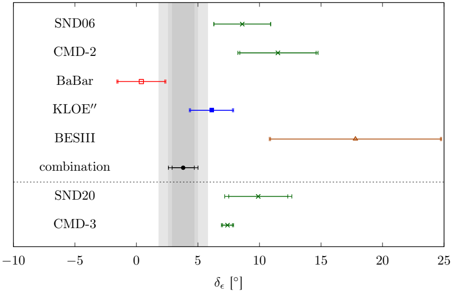


	$a_\mu^{\pi\pi}\big\vert_{\leq 1\,\text{GeV}}$	$a_\mu^{\pi\pi}\big\vert_{[0.60,0.88]\,\text{GeV}}$	$a_\mu^{\pi\pi}\big\vert_{\text{win}}$
SND06	1.7σ	1.8σ	1.7σ
CMD-2	2.0σ	2.3σ	2.1σ
BaBar	2.9σ	3.3σ	3.1σ
KLOE″	4.8σ	5.6σ	5.4σ
BESIII	2.8σ	3.0σ	3.1σ
SND20	2.1σ	2.2σ	2.2σ
comb	3.7σ [5.0σ]	4.2σ [6.1σ]	3.8σ [5.7σ]

- Tensions in $a_\mu^{\pi\pi}\big\vert_{\leq 1\,\text{GeV}}$ compared to CMD-3:
  - Inner/outer error: experiment/total (also shown: combination + BaBar/KLOE error)
  - Theory error dominated by order in conformal polynomial N
- No red flags for CMD-3 so far, but:
  - Large systematic error from N, correlated/anticorrelated for BaBar/other experiments
  - $\pi\pi$ phase shifts remain reasonable, main change in conformal polynomial

$\hookrightarrow$ further constraints from inelastic channels, $e^+e^- \to 4\pi, \pi\omega, \ldots$?

Phase of the ρ–ω mixing parameter



- Can also study consistency of hadronic parameters
  $\hookrightarrow$ **phase of the ρ–ω mixing parameter δ_ε**
  - δ_ε observable, since defined as a phase of a residue
  - δ_ε vanishes in isospin limit, but can be non-vanishing due to $\rho \to \pi^0\gamma, \eta\gamma, \pi\pi\gamma, \ldots \to \omega$
  - Combined-fit $\delta_\varepsilon = 3.8(2.0)[1.2]^\circ$ agrees well with narrow-width expectation $\delta_\varepsilon = 3.5(1.0)^\circ$, but **considerable spread among experiments**
  - Mass of the ω systematically too low compared to $e^+e^- \to 3\pi$

Moving on to $e^+e^- \to 3\pi$

- Some indications that 3π cross section from CMD-3 is also “too high” by $\simeq 4\%$

  $\hookrightarrow$ need to wait for dedicated analysis

- Peak cross section

  $$\sigma_{3\pi}(M_\omega^2) \propto \text{Br}[\omega \to e^+e^-]\text{Br}[\omega \to 3\pi]$$

- Compare to 3π HVP contributions, units of 10^{-10}

  MH, Hoid, Kubis 2019, work in progress

$$a_\mu^{3\pi}[\text{CMD-2,1/2}] = 46.3(7)\,/\,45.4(7)$$

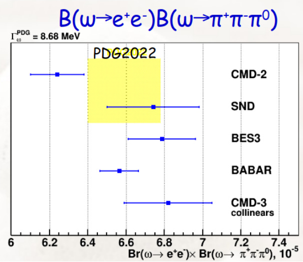
$$a_\mu^{3\pi}[\text{SND}] = 47.0(9)$$

$$a_\mu^{3\pi}[\text{BESIII}] = ?$$

$$a_\mu^{3\pi}[\text{all}] = 46.2(6)$$

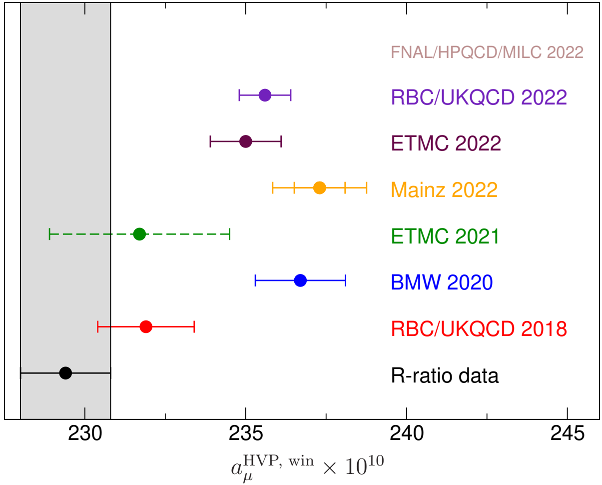
$$a_\mu^{3\pi}[\text{BaBar 2021}] = 45.6(4)$$

$\hookrightarrow$ pattern does not quite match $\sigma_{3\pi}(M_\omega^2)$



Ignatov 2023

On to the next puzzle: e^+e^- vs. lattice QCD in the intermediate window



RBC/UKQCD 2022 supersedes RBC/UKQCD 2018

ETMC 2022 supersedes ETMC 2021

FNAL/HPQCD/MILC 2022 agrees for *ud* connected contribution, same for Aubin et al. 2022, χQCD 2022

R-ratio result from Colangelo et al. 2022

Role of isospin breaking: phenomenological estimates

	SD window		int window		LD window		full HVP	
	$\mathcal{O}(e^2)$	$\mathcal{O}(\delta)$	$\mathcal{O}(e^2)$	$\mathcal{O}(\delta)$	$\mathcal{O}(e^2)$	$\mathcal{O}(\delta)$	$\mathcal{O}(e^2)$	$\mathcal{O}(\delta)$
$\pi^0\gamma$	0.16(0)	–	1.52(2)	–	2.70(4)	–	4.38(6)	–
$\eta\gamma$	0.05(0)	–	0.34(1)	–	0.31(1)	–	0.70(2)	–
ρ–ω mixing	–	0.05(0)	–	0.83(6)	–	2.79(11)	–	3.68(17)
FSR (2π)	0.11(0)	–	1.17(1)	–	3.14(3)	–	4.42(4)	–
M_{π^0} vs. $M_{\pi^\pm}$ (2π)	0.04(1)	–	$-0.09(7)$	–	$-7.62(14)$	–	$-7.67(22)$	–
FSR (K^+K^-)	0.07(0)	–	0.39(2)	–	0.29(2)	–	0.75(4)	–
kaon mass (K^+K^-)	$-0.29(1)$	0.44(2)	$-1.71(9)$	2.63(14)	$-1.24(6)$	1.91(10)	$-3.24(17)$	4.98(26)
kaon mass $(\bar{K}^0K^0)$	0.00(0)	$-0.41(2)$	$-0.01(0)$	$-2.44(12)$	$-0.01(0)$	$-1.78(9)$	$-0.02(0)$	$-4.62(23)$
total	0.14(1)	0.08(3)	1.61(12)	1.02(20)	$-2.44(16)$	2.92(17)	$-0.68(29)$	4.04(39)
BMWc 2020	–	–	$-0.09(6)$	0.52(4)	–	–	$-1.5(6)$	1.9(1.2)
RBC/UKQCD 2018	–	–	0.0(2)	0.1(3)	–	–	$-1.0(6.6)$	10.6(8.0)
JLM 2021	–	–	–	–	–	–	–	3.32(89)

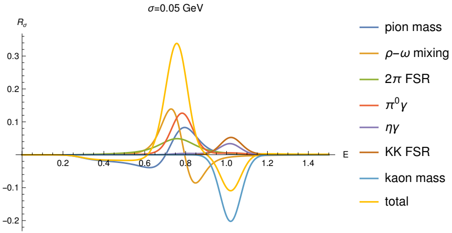
- Reasonable agreement with BMWc 2020, RBC/UKQCD 2018, and James, Lewis, Maltman 2021

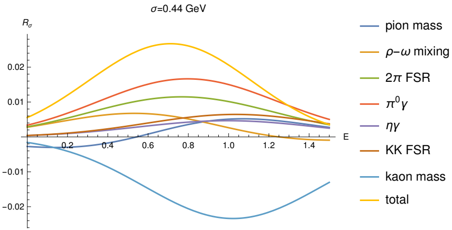
  ↪ if anything, the result would become even larger with pheno estimates

- Adding 3π (FSR and ρ–ω mixing) will remove tension in $\mathcal{O}(\delta)$

- **Cancellation of individually sizable corrections!**

Role of isospin breaking: energy dependence





- Alternative to windows: **Gaussian smearing** ETMC 2022

$$R_\sigma(s) = \int_0^\infty ds'\, G_\sigma(\sqrt{s'} - \sqrt{s})R(s') \qquad G_\sigma(\omega) = \frac{e^{-\omega^2/(2\sigma^2)}}{\sqrt{2\pi\sigma^2}}$$

- Cancellation for a_μ seems to involve a delicate balance with kernel $K(s)$
- Question: Is Gaussian smearing (expected to be) advantageous compared to linear combinations of windows? The inverse Laplace problem should persist . . .

Sixth plenary TI workshop

Muon g–2 Theory Initiative

Sixth Plenary Workshop

Bern, Switzerland, September 4–8, 2023

UNIVERSITÄT
BERN

AEC
ALBERT EINSTEIN CENTER
FOR FUNDAMENTAL PHYSICS

Local Organising Committee
Gilberto Colangelo (Chair)
Martin Hoferichter (Chair)
Bai-Long Hoid
Simon Holz
Gurtej Kanwar
Marina Marinković
Letizia Parato
Peter Stoffer
Jan-Niklas Toelstede
Urs Wenger

http://muong-2.itp.unibe.ch/

International Advisory Committee
Michel Davier (Orsay)
Aida El-Khadra (Illinois)
Christoph Lehner (Regensburg)
Laurant Lellouch (Marseille)
Tsutomu Mibe (KEK)
Lee Roberts (Boston)
Thomas Teubner (Liverpool)
Hartmut Wittig (Mainz)

Tensions in the V_{ud}–V_{us} plane

- Global-fit point away from unitarity line

  $(\Delta_{\text{CKM}} = |V_{ud}|^2 + |V_{us}|^2 - 1)$

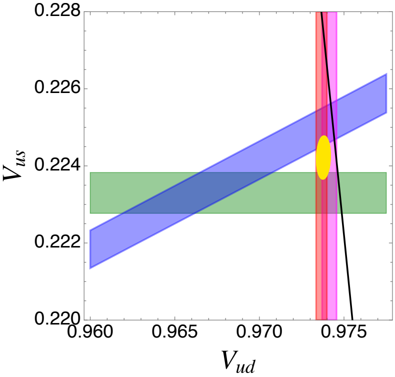
$$V_{ud} = 0.97378(26) \qquad V_{us} = 0.22422(36)$$

$$\Delta_{\text{CKM}} = -1.48(53) \times 10^{-3} \quad [2.8\sigma]$$

- Three possible measures of the CKM tension

$$\Delta^{(1)}_{\text{CKM}} = |V^{\beta}_{ud}|^2 + |V^{K_{\ell 3}}_{us}|^2 - 1$$
$$= -1.76(56) \times 10^{-3} \quad [3.1\sigma]$$
$$\Delta^{(2)}_{\text{CKM}} = |V^{\beta}_{ud}|^2 + |V^{K_{\ell 2}/\pi_{\ell 2},\, \beta}_{us}|^2 - 1$$
$$= -0.98(58) \times 10^{-3} \quad [1.7\sigma]$$
$$\Delta^{(3)}_{\text{CKM}} = |V^{K_{\ell 2}/\pi_{\ell 2},\, K_{\ell 3}}_{ud}|^2 + |V^{K_{\ell 3}}_{us}|^2 - 1$$
$$= -1.64(63) \times 10^{-2} \quad [2.6\sigma]$$

↪ already tension in kaon sector alone 2.6σ



Cirigliano, Crivellin, MH, Moulson 2022

What can we do to clarify the situation?

- Corroborating V_{ud}
  - Nuclear-structure corrections for superallowed β decays
  - Improved neutron-decay measurements (g_A, τ_n)
  - **Pion β decay** with PIONEER
- Corroborating V_{us}
  - Improved lattice calculations of F_K/F_π
  - **A new measurement of $K_{\mu 3}/K_{\mu 2}$**, possible at NA62
  - τ and hyperon decays sensitive to V_{us}, but feasible at the relevant level of accuracy?

A new measurement of $K_{\mu 3}/K_{\mu 2}$, why?

	current fit	$K_{\mu3}/K_{\mu2}$ BR at 0.5%			$K_{\mu3}/K_{\mu2}$ BR at 0.2%		
		central	$+2\sigma$	-2σ	central	$+2\sigma$	-2σ
$\frac{V_{us}}{V_{ud}}\Big\vert_{K_{\ell 2}/\pi_{\ell 2}}$	0.23108(51)	0.23108(50)	0.23085(51)	0.23133(51)	0.23108(49)	0.23071(51)	0.23147(52)
$V_{us}^{K_{\ell 3}}$	0.22330(53)	0.22337(51)	0.22360(52)	0.22309(54)	0.22342(49)	0.22386(52)	0.22287(52)
$10^2\Delta^{(3)}_{\text{CKM}}$	$-1.64(63)$	$-1.57(60)$	$-1.18(62)$	$-2.02(63)$	$-1.53(59)$	$-0.83(62)$	$-2.33(62)$
	-2.6σ	-2.6σ	-1.9σ	-3.2σ	-2.6σ	-1.4σ	-3.8σ

- Is the $K_{\ell 3}$ vs. $K_{\ell 2}$ tension real or an experimental problem?
  - $K_{\ell 2}$ data base completely dominated by KLOE 2006
  - Global fit to kaon data not great, p-value $\simeq 1\%$
- This can be clarified with **a new precision measurement of $K_{\mu 3}/K_{\mu 2}$**:
  - In case the tension were of experimental origin, there should be a positive shift compared to current fit

    $\hookrightarrow$ $\Delta^{(3)}_{\text{CKM}}$ would move from -2.6σ to -1.4σ for a $+2\sigma$ shift with a 0.2% measurement
  - In case the tension were of BSM origin, the current value would be confirmed (or move further in the other direction)

$\hookrightarrow$ **a single new precision measurement would have a huge impact!**

An interpretation in terms of right-handed currents

- Modify **right-handed current**

  $\hookrightarrow$ vector $\sim 1+\varepsilon_R$, axial-vector $\sim 1-\varepsilon_R$

$$\Delta^{(1)}_{\text{CKM}} = 2\varepsilon_R + 2\Delta\varepsilon_R V_{us}^2 \qquad \text{(blue)}$$

$$\Delta^{(2)}_{\text{CKM}} = 2\varepsilon_R - 2\Delta\varepsilon_R V_{us}^2 \qquad \text{(red)}$$

$$\Delta^{(3)}_{\text{CKM}} = 2\varepsilon_R + 2\Delta\varepsilon_R (2 - V_{us}^2) \qquad \text{(green)}$$

  where $\Delta\varepsilon_R \equiv \varepsilon_R^{(s)} - \varepsilon_R$

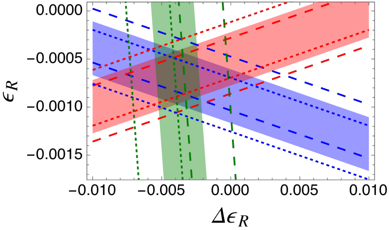
- Current fit

$$\varepsilon_R = -0.69(27)\times 10^{-3} \qquad [2.5\sigma]$$

$$\Delta\varepsilon_R = -3.9(1.6)\times 10^{-3} \qquad [2.4\sigma]$$

- Impact of new $K_{\mu 3}/K_{\mu 2}$ measurement mainly

  on $\Delta\varepsilon_R$ (dashed and dotted lines $\pm 2\sigma$ benchmark)



Cirigliano, Crivellin, MH, Moulson 2022

Modification of the Fermi constant

- **Fermi constant**
  - Best determination from muon decay MuLan 2013

$$G_F^\mu = 1.1663787(6) \times 10^{-5}\,\text{GeV}^{-2}$$

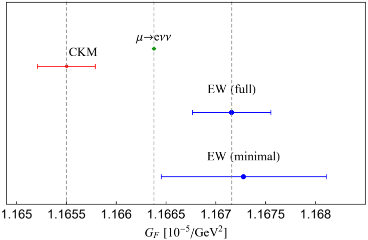
  - Electroweak fit Marciano 1999, update using HEPFit

$$G_F^{\text{EW}}\Big|_{\text{full}} = 1.16716(39) \times 10^{-5}\,\text{GeV}^{-2}$$

  - CKM deficit interpreted as modification of G_F in β decays

$$G_F^{\text{CKM}} = 1.16550(29) \times 10^{-5}\,\text{GeV}^{-2}$$

  - Does not explain tension in kaon sector



Crivellin, MH, Manzari 2021

SMEFT analysis of G_F tensions

- Possible explanations in terms of effective operators
  - A. four-fermion operators in $\mu \to e\nu\nu$: only viable for SM operator $Q_{\ell\ell}^{2112} = \bar\ell_2\gamma^\mu\ell_1\bar\ell_1\gamma_\mu\ell_2$
  - B. four-fermion operators in $u \to de\nu$: now excluded by LHC bounds
  - C. modified W–u–d couplings: possible in terms of Belfatto, Berezhiani 2021

$$Q_{\phi q}^{(3)ij} = \phi^\dagger i\overset{\leftrightarrow}{D}{}^I_\mu\phi\bar q_i\gamma^\mu\tau^I q_j \qquad Q_{\phi ud}^{ij} = \tilde\phi^\dagger iD_\mu\phi\bar u_i\gamma^\mu d_j$$

↪ generate left- and right-handed currents, respectively

  - D. modified W–ℓ–ν couplings: operator

$$Q_{\phi\ell}^{(3)ij} = \phi^\dagger i\overset{\leftrightarrow}{D}{}^I_\mu\phi\bar\ell_i\gamma^\mu\tau^I\ell_j$$

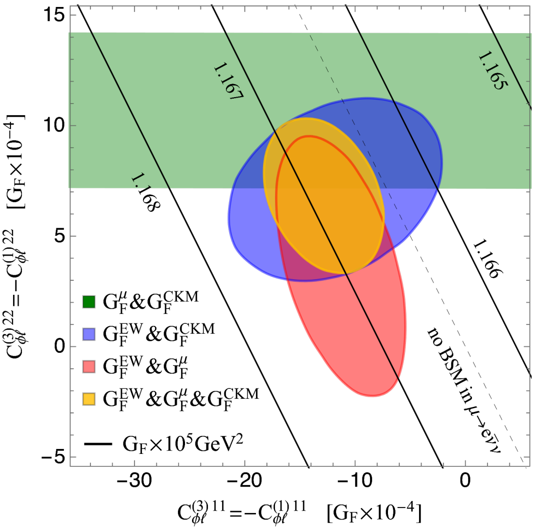
leads to interpretation in terms of LFUV Crivellin, MH 2020

  - E. other operators affecting the EW fit, $Q_{\phi\ell}^{(3)ij}$ and

$$Q_{\phi\ell}^{(1)ij} = \phi^\dagger i\overset{\leftrightarrow}{D}_\mu\phi\bar\ell_i\gamma^\mu\ell_j$$

↪ effect can be minimized by turning off $Z \to \ell\ell$ with $C_{\phi\ell}^{(1)ij} = -C_{\phi\ell}^{(3)ij}$

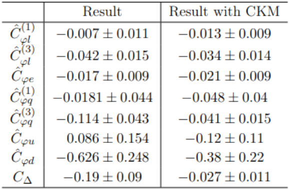
SMEFT analysis of G_F tensions



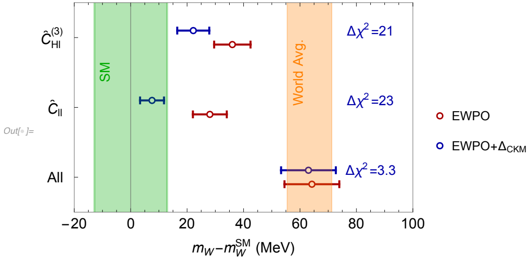
Crivellin, MH, Manzari 2021

- Common explanation in terms of $C_{\phi\ell}^{(1)11}=-C_{\phi\ell}^{(3)11}$ and $C_{\phi\ell}^{(1)22}=-C_{\phi\ell}^{(3)22}$ possible
- For BSM sensitivity the **second-most-precise determination of G_F is crucial**

Impact of CKM unitarity on explanations of W-boson mass



	Result	Result with CKM
$\hat{C}_{\varphi l}^{(1)}$	-0.007 ± 0.011	-0.013 ± 0.009
$\hat{C}_{\varphi l}^{(3)}$	-0.042 ± 0.015	-0.034 ± 0.014
$\hat{C}_{\varphi e}$	-0.017 ± 0.009	-0.021 ± 0.009
$\hat{C}_{\varphi q}^{(1)}$	-0.0181 ± 0.044	-0.048 ± 0.04
$\hat{C}_{\varphi q}^{(3)}$	-0.114 ± 0.043	-0.041 ± 0.015
$\hat{C}_{\varphi u}$	0.086 ± 0.154	-0.12 ± 0.11
$\hat{C}_{\varphi d}$	-0.626 ± 0.248	-0.38 ± 0.22
$\hat{C}_{\Delta}$	-0.19 ± 0.09	-0.027 ± 0.011



Cirigliano, Dekens, de Vries, Mereghetti, Tong 2022

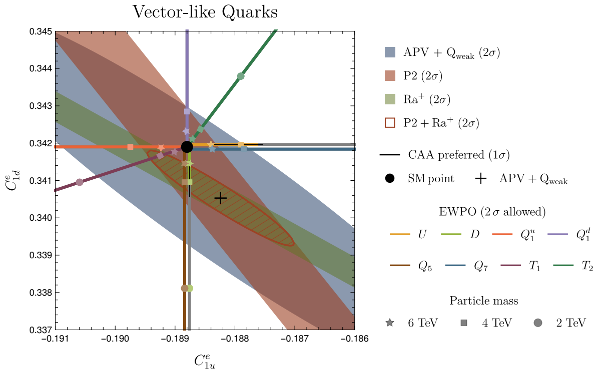
Falkowski, Gonzáles-Alonso, . . .

- Δ_{CKM} excludes certain explanations of M_W

  $\hookrightarrow$ should be included in EW fit

- Otherwise, generic explanations tend to produce a percent-level Δ_{CKM}

Correlations with parity violation in simplified models



Crivellin, MH, Kirk, Manzari, Schnell 2021

- **Low-energy parity violation** conventionally parameterized in terms of

$$\mathcal{L}_{\text{eff}}^{ee} = \frac{G_F}{\sqrt{2}} \sum_{q=u,d,s} \Big(C^e_{1q} \big[\bar{q}\gamma^\mu q\big] \big[\bar{e}\gamma_\mu\gamma_5 e\big] + C^e_{2q} \big[\bar{q}\gamma^\mu\gamma_5 q\big] \big[\bar{e}\gamma_\mu e\big] \Big)$$

- In simplified models, Cabibbo angle anomaly defines a preferred parameter range

  ↪ can be tested in parity-violating electron scattering and atomic parity violation

Lepton flavor universality violation

- Let us parameterize the ***W* couplings** as $\mathcal{L} = -i\frac{g_2}{\sqrt{2}}\bar{\ell}_i\gamma^\mu P_L\nu_j W_\mu\big(\delta_{ij} + \varepsilon_{ij}\big)$
- Modifies Fermi constant in **muon decay**

$$\frac{1}{\tau_\mu} = \frac{(G_F^{\mathcal{L}})^2 m_\mu^5}{192\pi^3}(1+\Delta q)(1+\varepsilon_{ee}+\varepsilon_{\mu\mu})^2$$

$\hookrightarrow$ measured Fermi constant $G_F = G_F^{\mathcal{L}}(1+\varepsilon_{ee}+\varepsilon_{\mu\mu})$

- All β-decay observables affected according to

$$V_{ud} \to V_{ud}^\beta = V_{ud}^{\mathcal{L}}(1-\varepsilon_{\mu\mu})$$

where $V_{ij}^{\mathcal{L}}$ fulfill CKM unitarity

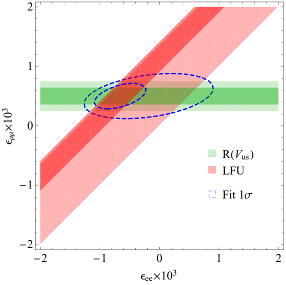
- Construct ratio Crivellin, MH 2020

$$R(V_{us}) \equiv \frac{V_{us}^{K_{\mu 2}}}{V_{us}^\beta} \equiv \frac{V_{us}^{K_{\mu 2}}}{\sqrt{1-\big(V_{ud}^\beta\big)^2 - |V_{ub}|^2}} = 1 - \bigg(\frac{V_{ud}}{V_{us}}\bigg)^2\varepsilon_{\mu\mu} + \mathcal{O}(\varepsilon^2)$$

$\hookrightarrow$ LFUV effect enhanced by $(V_{ud}/V_{us})^2 \sim 20$!

Lepton flavor universality violation

Observable	Measurement	Constraint $\times 10^3$
$\frac{K\to\pi\mu\bar\nu}{K\to\pi e\bar\nu} \simeq 1+\varepsilon_{\mu\mu}-\varepsilon_{ee}$	1.0010(25)	1.0(2.5)
$\frac{K\to\mu\nu}{K\to e\nu} \simeq 1+\varepsilon_{\mu\mu}-\varepsilon_{ee}$	0.9978(18)	−2.2(1.8)
$\frac{\pi\to\mu\nu}{\pi\to e\nu} \simeq 1+\varepsilon_{\mu\mu}-\varepsilon_{ee}$	1.0010(9)	1.0(9)
$\frac{\tau\to\mu\nu\bar\nu}{\tau\to e\nu\bar\nu} \simeq 1+\varepsilon_{\mu\mu}-\varepsilon_{ee}$	1.0018(14)	1.8(1.4)
$\frac{W\to\mu\bar\nu}{W\to e\bar\nu} \simeq 1+\varepsilon_{\mu\mu}-\varepsilon_{ee}$	0.9960(100)	−4(10)
$\frac{B\to D^{(*)}\mu\nu}{B\to D^{(*)}e\nu} \simeq 1+\varepsilon_{\mu\mu}-\varepsilon_{ee}$	0.9890(120)	−11(12)
$R(V_{us}) \simeq 1-\left(\frac{V_{ud}}{V_{us}}\right)^2\varepsilon_{\mu\mu}$	0.9891(35)	0.58(19)



Crivellin, MH 2020

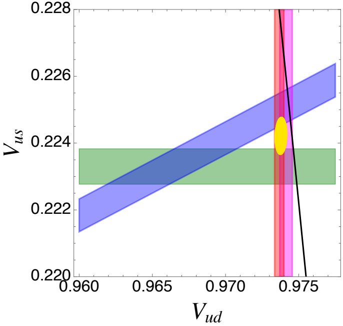
- Most stringent constraint on $\varepsilon_{\mu\mu}$ thanks to **CKM enhancement**
- Also does not explain tension in kaon sector
- Best constraint on $\varepsilon_{\mu\mu}-\varepsilon_{ee}$ from

$$R^{\pi}_{e/\mu}=\frac{\Gamma(\pi\to e\nu_e(\gamma))}{\Gamma(\pi\to\mu\nu_\mu(\gamma))}$$

- Factor 3 (10) from PEN/PiENu (PIONEER), factor 3 for τ decays from Belle II

Conclusions

- **Muon $g-2$**
  - New puzzling measurement of $e^+e^- \to \pi^+\pi^-$ from CMD-3: 5σ away from previous average
  - Tension between e^+e^- and BMWc confirmed in intermediate window at around 4σ
- **Cabibbo angle anomaly**
  - Tensions among β decays and kaon decays point to the **apparent violation of CKM unitarity**
  - **New precision measurement of $K_{\mu 3}/K_{\mu 2}$** to clarify situation in kaon sector
  - Interesting interplay with **electroweak fit** and tests of **lepton flavor universality**



ρ–ω mixing in $e^+e^- \to 3\pi$

- A coupled-channel system for $\{2\pi, \ell^+\ell^-, 3\pi\}$
  Holz, Hanhart, MH, Kubis 2022
- Developed for consistent description of $\eta' \to \pi\pi\gamma, \ell^+\ell^-\gamma$
  $\hookrightarrow$ η' transition form factor and HLbL
- $\varepsilon_{\rho\omega}$ now consistent

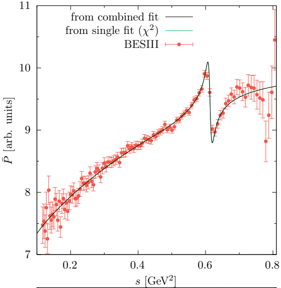
$$\text{Re}\,\varepsilon_{\rho\omega}\big|_{e^+e^-\to 2\pi} = 1.97(3)\times 10^{-3}$$

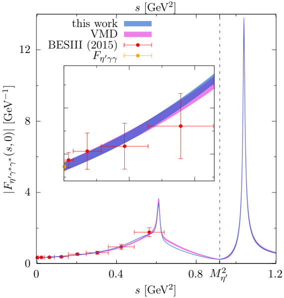
$$\varepsilon_{\rho\omega}\big|_{\eta'\to\pi\pi\gamma} = 2.00(7)\times 10^{-3}$$

- By-product: ρ–ω mixing in $e^+e^- \to 3\pi$ should enter as

$$1+\varepsilon_{\rho\omega}\, g_{\omega\gamma}^2\frac{s}{48\pi^2}\int_{4M_\pi^2}^{\infty} ds'\,\frac{\Big(1-\frac{4M_\pi^2}{s'}\Big)^{3/2}|F_\pi^V(s')|^2}{s'(s'-s-i\varepsilon)}$$

- Preliminary results:
  - BaBar fit improves significantly
  - $\varepsilon_{\rho\omega}$ (largely) consistent with $e^+e^- \to 2\pi$
  - $a_\mu^{3\pi}[\rho$–$\omega]$ sizable (and negative)





CKM unitarity

Benchmarks numbers for CKM tests from PDG “12. CKM Quark-Mixing Matrix”

first row: $|V_{ud}|^2+|V_{us}|^2+|V_{ub}|^2=0.9985(5)$

second row: $|V_{cd}|^2+|V_{cs}|^2+|V_{cb}|^2=1.025(22)$

first column: $|V_{ud}|^2+|V_{cd}|^2+|V_{td}|^2=0.9970(18)$

second column: $|V_{us}|^2+|V_{cs}|^2+|V_{ts}|^2=1.026(22)$

- **First-row unitarity test**
  - Testing consistency of V_{ud} and V_{us} at precision of a few times 10^{-4}
  - $|V_{ub}|^2 \simeq 1.5\times 10^{-5}$
  - Deficit of (2–3)σ (also deficit in first-column test, but less sensitive)

    $\hookrightarrow$ “**Cabibbo angle anomaly**”
  - Second row/column more than an order of magnitude away; third row/column $\mathcal{O}(\lambda^4)$

Determination of V_{ud} from superallowed β decays

- Master formula Hardy, Towner 2018

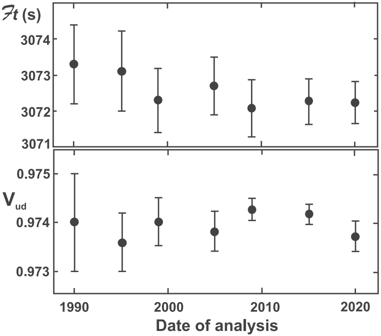
$$|V_{ud}|^2 = \frac{2984.432(3)\,\text{s}}{\mathcal{F}t(1+\Delta_R^V)}$$

with (universal) radiative corrections Δ_R^V

- Value of V_{ud} crucially depends on Δ_R^V:

Ref.	Δ_R^V
Marciano, Sirlin 2006	0.02361(38)
Seng, Gorchtein, Patel, Ramsey-Musolf 2018	0.02467(22)
Czarnecki, Marciano, Sirlin 2019	0.02426(32)
Seng, Feng, Gorchtein, Jin 2020	0.02477(24)
Hayen 2020	0.02474(31)
Shiells, Blunden, Melnitchouk 2021	0.02472(18)
Cirigliano, Crivellin, MH, Moulson 2022	0.02467(27)

↪ main uncertainty from Regge region, lattice QCD to improve?



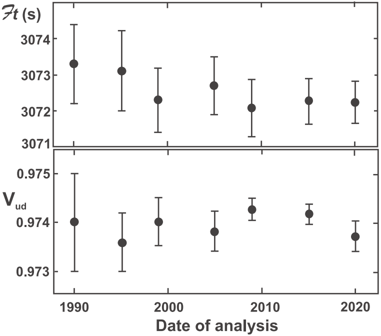
Hardy, Towner 2020

Determination of V_{ud} from superallowed β decays

- Further corrections
    - Isospin breaking Miller, Schwenk 2008, 2009, Condren, Miller 2022, Seng, Gorchtein 2022, Crawford, Miller 2022
    - Nuclear corrections Seng, Gorchtein, Ramsey-Musolf 2018, Gorchtein 2018, Seng, Gorchtein 2022
- Estimate from Gorchtein 2018 becomes dominant source of uncertainty

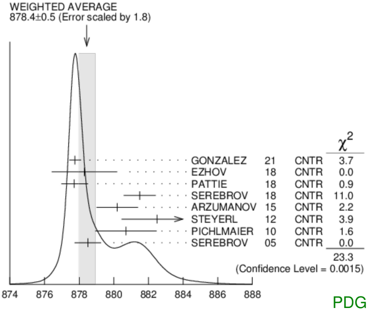
$$V_{ud}^{0^+\to 0^+} = 0.97367(11)_{\text{exp}}(13)_{\Delta_V^R}(27)_{\text{NS}}[32]_{\text{total}}$$

- Improvements from ab-initio nuclear structure? Martin, Stroberg, Holt, Leach 2021



Hardy, Towner 2020

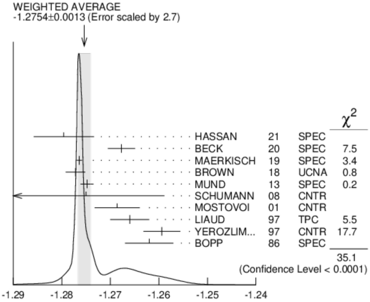
Determination of V_{ud} from neutron decay



WEIGHTED AVERAGE
878.4±0.5 (Error scaled by 1.8)

			χ^2
GONZALEZ	21	CNTR	3.7
EZHOV	18	CNTR	0.0
PATTIE	18	CNTR	0.9
SEREBROV	18	CNTR	11.0
ARZUMANOV	15	CNTR	2.2
STEYERL	12	CNTR	3.9
PICHLMAIER	10	CNTR	1.6
SEREBROV	05	CNTR	0.0
			23.3

(Confidence Level = 0.0015)



WEIGHTED AVERAGE
-1.2754±0.0013 (Error scaled by 2.7)

			χ^2
HASSAN	21	SPEC	
BECK	20	SPEC	7.5
MAERKISCH	19	SPEC	3.4
BROWN	18	UCNA	0.8
MUND	13	SPEC	0.2
SCHUMANN	08	CNTR	
MOSTOVOI	01	CNTR	
LIAUD	97	TPC	5.5
YEROZLIM...	97	CNTR	17.7
BOPP	86	SPEC	
			35.1

(Confidence Level < 0.0001)

PDG 2022

- Master formula Czarnecki, Marciano, Sirlin 2018

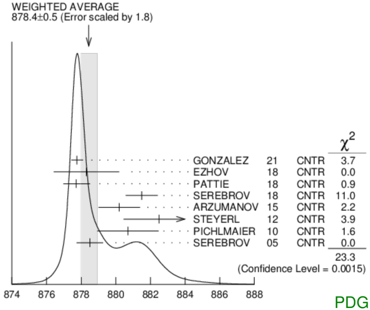
$$|V_{ud}|^2\tau_n(1+3g_A^2)(1+\Delta_{\text{RC}}) = 5099.3(3)\,\text{s}$$

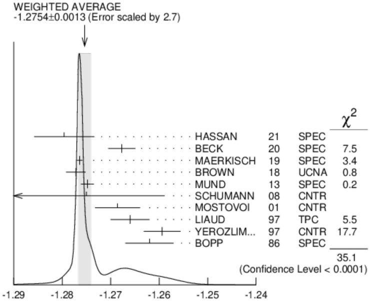
  with radiative corrections Δ_{RC}

  $\hookrightarrow$ need lifetime τ_n and asymmetry $\lambda = g_A/g_V$

- PDG average especially for g_A includes large scale factors

Determination of V_{ud} from neutron decay





PDG 2022

- Results for V_{ud}

$$V_{ud}^{\text{n, PDG}} = 0.97441(3)_f(13)_{\Delta_R}(82)_\lambda(28)_{\tau_n}[88]_{\text{total}}$$

$$V_{ud}^{\text{n, best}} = 0.97413(3)_f(13)_{\Delta_R}(35)_\lambda(20)_{\tau_n}[43]_{\text{total}}$$

$\hookrightarrow$ average of $V_{ud}^{0^+\to0^+}$ with $V_{ud}^{\text{n, best}}$ gives $V_{ud}^{\beta} = 0.97384(26)$

- Need improved measurements especially for g_A to make progress

Determination of V_{ud} from pion β decay

- Master formula Cirigliano, Knecht, Neufeld, Pichl 2003, Czarnecki, Marciano, Sirlin 2020, Feng et al. 2020

$$\Gamma(\pi^+ \to \pi^0 e^+ \nu_e(\gamma)) = \frac{G_F^2 |V_{ud}|^2 M_{\pi^\pm}^5 |f_+^\pi(0)|^2}{64\pi^3}(1 + \Delta_{\text{RC}}^{\pi\ell}) I_{\pi\ell}$$

↪ need branching fraction and pion life time from experiment

- (Theory) inputs
  - Phase space $I_{\pi\ell} = 7.3766(43) \times 10^{-8}$
  - Form factor $f_+^\pi(0) = 1 - 7 \times 10^{-6}$

    ↪ protected by $SU(2)$ Ademollo–Gatto theorem (Behrends–Sirlin)
  - Radiative corrections $\Delta_{\text{RC}}^{\pi\ell} = 0.0334(10)$ ChPT, Cirigliano et al., $\Delta_{\text{RC}}^{\pi\ell} = 0.0332(3)$ lattice QCD, Feng et al.
- Resulting V_{ud} extracted from PIBETA 2004

$$V_{ud}^{\pi,\text{ChPT}} = 0.97376(281)_{\text{BR}}(9)_{\tau_\pi}(47)_{\Delta_{\text{RC}}^{\pi\ell}}(28)_{I_{\pi\ell}}[287]_{\text{total}}$$

$$V_{ud}^{\pi,\text{lattice}} = 0.97386(281)_{\text{BR}}(9)_{\tau_\pi}(14)_{\Delta_{\text{RC}}^{\pi\ell}}(28)_{I_{\pi\ell}}[283]_{\text{total}}$$

↪ factor 10 possible before other errors creep in, aim for **PIONEER experiment**

Determination of V_{us}/V_{ud} from kaon decays: $K_{\ell 2}/\pi_{\ell 2}$

- ***$K_{\ell 2}$* decays**: $K \to \ell\nu_\ell$

$$\frac{V_{us}}{V_{ud}}\frac{F_K}{F_\pi} = \left(\frac{\Gamma(K^+ \to \mu^+\nu_\mu(\gamma)M_\pi}{\Gamma(\pi^+ \to \mu^+\nu_\mu(\gamma)M_K}\right)^{1/2}\frac{1-\frac{m_\mu^2}{M_\pi^2}}{1-\frac{m_\mu^2}{M_K^2}}\left(1-\underbrace{\frac{\Delta_{\text{RC}}^K-\Delta_{\text{RC}}^\pi}{2}}_{\Delta_{\text{RC}}^{K\pi}/2}\right)$$

- Consider the ratio over $\pi_{\mu 2}$ because
  - Only need ratio of decay constant
  - Certain structure-dependent radiative corrections cancel
- Need theory input for:
  - **Decay constants** in isospin limit: $F_K/F_\pi = 1.1978(22)$ HPQCD 2013, Fermilab/MILC 2017, CalLat 2020, ETMC 2021
  - **Isospin-breaking corrections**: $\Delta_{\text{RC}}^{K\pi} = -0.0112(21)$ ChPT, Cirigliano, Neufeld 2011, $\Delta_{\text{RC}}^{K\pi} = -0.0126(14)$ lattice, Di Carlo et al. 2019
- Result:

$$\frac{V_{us}}{V_{ud}}\bigg|_{K_{\ell 2}/\pi_{\ell 2}} = 0.23108(23)_{\text{exp}}(42)_{F_K/F_\pi}(16)_{\text{IB}}[51]_{\text{total}}$$

Determination of V_{us} from kaon decays: $K_{\ell 3}$

- **$K_{\ell 3}$ decays**: $K \to \pi \ell \nu_\ell$

$$\Gamma(K \to \pi \ell \nu_\ell(\gamma)) = \frac{C_K^2 G_F^2 |V_{us}|^2 M_K^5 |f_+^{K\pi}(0)|^2}{192\pi^3} \Big(1 + \underbrace{\Delta_{\text{RC}}^{K\ell}}_{\Delta_{\text{EM}}^{K\ell} + \Delta_{SU(2)}} \Big) I_{K\ell}$$

$\hookrightarrow \ell = \mu, e$ and two charge channels

- Need theory input for:
  - **Form factor**: $f_+^{K\pi}(0) = 0.9698(17)$ ETMC 2016, Fermilab/MILC 2019
  - **Radiative corrections**: $\Delta_{SU(2)} = 0.0252(11)$ Cirigliano et al. 2002, $\Delta_{\text{EM}}^{K^0 e} = 0.0116(3)$, $\Delta_{\text{EM}}^{K^+ e} = 0.0021(5)$, $\Delta_{\text{EM}}^{K^0 \mu} = 0.0154(4)$, $\Delta_{\text{EM}}^{K^+ \mu} = 0.0005(5)$ Seng et al. 2022
- Result:

$$V_{us}^{K_{\ell 3}} = 0.22330(35)_{\text{exp}}(39)_{f_+}(8)_{\text{IB}}[53]_{\text{total}}$$