

Chiral EFT for dark matter detection experiments

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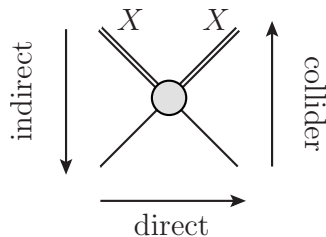
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Dark Matter in Compact Objects, Stars, and Low-Energy Experiments

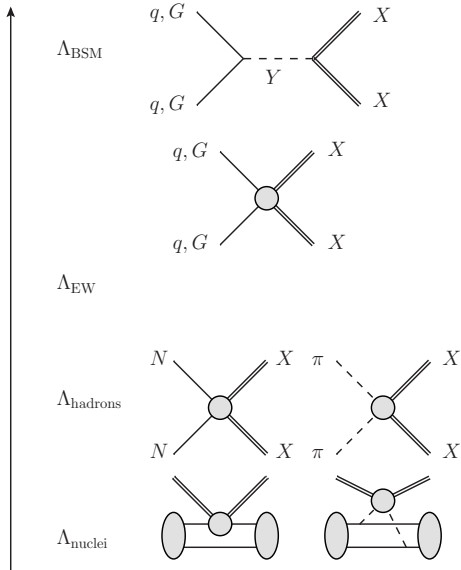
Rate for WIMP–nucleus scattering

$$\frac{dR}{dE_r} = \underbrace{\frac{\sigma_{\chi N}^{\text{SI}}}{m_\chi \mu_N^2}}_{\text{particle + hadronic physics}} \times \underbrace{|\mathcal{F}_+^M(q^2)|^2}_{\text{nuclear physics}} \times \underbrace{\rho_0 \int_{v_{\text{min}}}^{v_{\text{esc}}} \frac{f(\mathbf{v}, t)}{v} d^3v}_{\text{astrophysics}}$$

- Decomposition follows from EFT
- $\sigma_{\chi N}^{\text{SI}}$ subsumes physics from QCD to BSM scale
 \leftrightarrow use EFT to keep track
- Can also use EFT for comparison with indirect detection or collider constraints (as long as EFT applies)
- **Chiral EFT for hadronic/nuclear part**



Direct detection of dark matter: scales



1 **BSM scale** Λ_{BSM} : \mathcal{L}_{BSM}

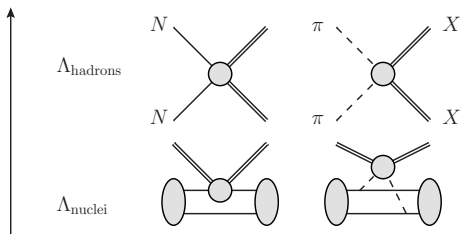
2 **Effective Operators**: $\mathcal{L}_{\text{SM}} + \sum_{i,k} \frac{1}{\Lambda_{\text{BSM}}^i} \mathcal{O}_{i,k}$
SMEFT

3 Integrate out **EW physics**

4 **Hadronic scale**: nucleons and pions
 \hookrightarrow effective interaction Hamiltonian H_I
Chiral EFT

5 **Nuclear scale**: $\langle \mathcal{N} | H_I | \mathcal{N} \rangle$
 \hookrightarrow nuclear wave function Chiral EFT, NREFT

Direct detection of dark matter: scales



- 4 **Hadronic scale:** nucleons and pions

\leftrightarrow effective interaction Hamiltonian H_I

Chiral EFT

- 5 **Nuclear scale:** $\langle \mathcal{N} | H_I | \mathcal{N} \rangle$

\leftrightarrow nuclear wave function Chiral EFT, NREFT

- Typical WIMP–nucleon **momentum transfer**

$$|\mathbf{q}_{\max}| = 2\mu_{\mathcal{N}\chi}|\mathbf{v}_{\text{rel}}| \sim 200 \text{ MeV} \quad |\mathbf{v}_{\text{rel}}| \sim 10^{-3} \quad \mu_{\mathcal{N}\chi} \sim 100 \text{ GeV}$$

- **Chiral EFT:** pions, nucleons, and WIMPs as degrees of freedom

Prézeau et al. 2003, Cirigliano et al. 2012, 2013, Menéndez et al. 2012, Klos et al. 2013, MH et al. 2015, Bishara et al. 2017

- **NREFT:** all degrees of freedom integrated out but nucleons and WIMPs

Fan et al. 2010, Fitzpatrick et al. 2012, Anand et al. 2013

- Effective theory of QCD based on **chiral symmetry**

$$\mathcal{L}_{\text{QCD}} = \bar{q}_L i \not{D} q_L + \bar{q}_R i \not{D} q_R - \bar{q}_L M q_R - \bar{q}_R M q_L - \frac{1}{4} G_{\mu\nu}^a G_a^{\mu\nu}$$

- Expansion in momenta p/Λ_χ and quark masses $m_q \sim p^2$

↪ **scale separation**

- Breakdown scale: $\Lambda_\chi = M_\rho \dots 4\pi F_\pi \sim 1 \text{ GeV}$

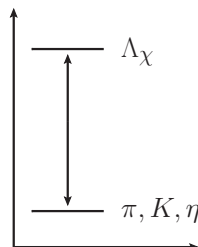
- Two variants

- **$SU(2)$** : u - and d -quark **dynamical**, m_s fixed at **physical value**

↪ expansion in M_π/Λ_χ , usually nice convergence

- **$SU(3)$** : u -, d -, and s -quark dynamical

↪ expansion in M_K/Λ_χ , sometimes tricky



Chiral EFT: a modern approach to nuclear forces

- Traditionally: meson-exchange potentials
- Chiral effective field theory
 - Based on **chiral symmetry** of QCD
 - **Power counting**
 - **Low-energy constants**
 - Hierarchy of multi-nucleon forces
 - Consistency of NN and $3N$

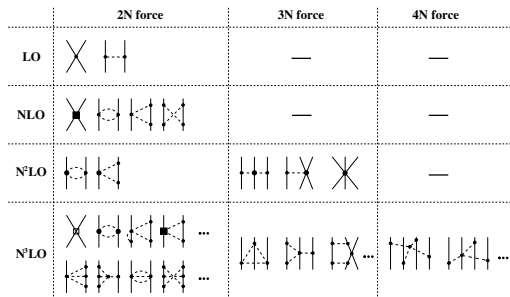
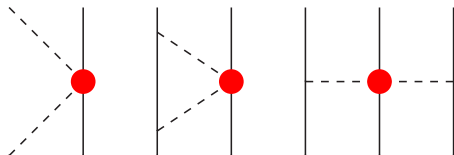


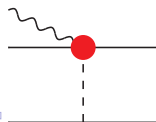
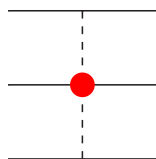
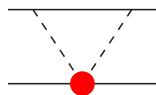
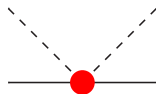
Figure taken from 1011.1343

↪ modern theory of nuclear forces

- Long-range part related to **pion–nucleon scattering**



- Coupling to **external sources** $\mathcal{L}(v_\mu, a_\mu, s, p)$
- Same LECs appear in **axial current**
 $\hookrightarrow \beta$ decay, neutrino interactions, dark matter
- Vast literature for v_μ and a_μ , up to one-loop level
 - With unitary transformations: Kölling et al. 2009, 2011, Krebs et al. 2016, 2019
 - Without unitary transformations: Park et al. 2003, Pastore et al. 2008, Baroni et al. 2015
- For **dark matter** further currents: s , p , tensor, spin-2, θ_μ^μ



- **Effective WIMP Lagrangian** for spin-1/2 SM singlet χ [Goodman et al. 2010](#)

$$\begin{aligned} \mathcal{L}_\chi = & \frac{1}{\Lambda^3} \sum_q \left[C_q^{SS} \bar{\chi} \chi m_q \bar{q} q + C_q^{PS} \bar{\chi} i \gamma_5 \chi m_q \bar{q} q + C_q^{SP} \bar{\chi} \chi m_q \bar{q} i \gamma_5 q + C_q^{PP} \bar{\chi} i \gamma_5 \chi m_q \bar{q} i \gamma_5 q \right] \\ & + \frac{1}{\Lambda^2} \sum_q \left[C_q^{VW} \bar{\chi} \gamma^\mu \chi \bar{q} \gamma_\mu q + C_q^{AV} \bar{\chi} \gamma^\mu \gamma_5 \chi \bar{q} \gamma_\mu q + C_q^{VA} \bar{\chi} \gamma^\mu \chi \bar{q} \gamma_\mu \gamma_5 q + C_q^{AA} \bar{\chi} \gamma^\mu \gamma_5 \chi \bar{q} \gamma_\mu \gamma_5 q \right] \\ & + \frac{1}{\Lambda^2} \sum_q \left[C_q^{TT} \bar{\chi} \sigma^{\mu\nu} \chi \bar{q} \sigma_{\mu\nu} q + \tilde{C}_q^{TT} \bar{\chi} \sigma^{\mu\nu} \gamma_5 \chi \bar{q} \sigma_{\mu\nu} q \right] + \frac{1}{\Lambda^3} \left[C_g^S \bar{\chi} \chi \alpha_s G_{\mu\nu}^a G_a^{\mu\nu} + \dots \right] \end{aligned}$$

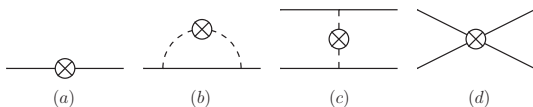
- **Chiral power counting**

$$\partial = \mathcal{O}(p), \quad m_q = \mathcal{O}(p^2) = \mathcal{O}(M_\pi^2), \quad a_\mu, v_\mu = \mathcal{O}(p), \quad \frac{\partial}{m_N} = \mathcal{O}(p^2)$$

↪ construction of effective Lagrangian for nucleon and pion fields

↪ organize in terms of **chiral order** ν , $\mathcal{M} = \mathcal{O}(p^\nu)$

Classes of contributions



- Three classes of corrections:
 - **Subleading one-body responses** (a)
 - **Radius corrections** (b)
 - **Two-body currents** (c), (d)
- (a)+(b) just **ChPT for nucleon matrix elements**, but (c)+(d) genuinely new effects
- Hierarchy predicted from chiral EFT

Example: scalar currents

- For a Majorana WIMP χ : **scalar quark current**

$$\mathcal{L} = \sum_q C_q^{SS} \bar{\chi} \chi m_q \bar{q} q$$

leading contribution to responses with coherent enhancement

↪ **spin-independent scattering**

- Need nucleon matrix elements

$$\langle N | m_q \bar{q} q | N \rangle = f_q^N m_N \quad m_N (f_U^N + f_D^N) = \sigma_{\pi N}$$

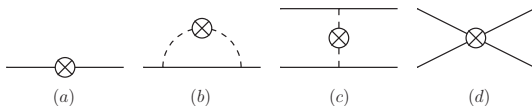
to extract BSM information from cross section

$$\sigma_{\text{SI}} = \frac{4\mu_N^2}{\pi} \left| m_N \sum_q C_q^{SS} f_q^N + \dots \right|^2 \quad \mu_N = \frac{m_N m_\chi}{m_N + m_\chi}$$

- $\sigma_{\pi N}$ critical for interpretation of direct-detection searches, especially in SUSY models [Bottino et al. 2000](#), [Ellis et al. 2008](#)

- $\sigma_{\pi N} = \mathcal{O}(p^2)$ suppressed due to absence of scalar current at leading order

Two-body currents: scalar operators



- Diagram (a) scales like $\mathcal{O}(p^2)$ in the chiral counting
- In contrast to the nucleon, the pion has a **leading-order scalar coupling**

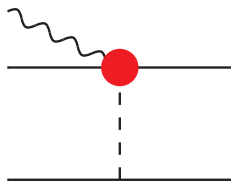
$$\mathcal{M}_{(b)} \simeq \underbrace{p \times \frac{1}{p} \times p}_{\text{nucleon line}} \times \underbrace{\frac{1}{p^2} \times p^2 \times \frac{1}{p^2}}_{\text{pion line}} \times \underbrace{p^4}_{\text{loop}} = \mathcal{O}(p^3)$$

$$\mathcal{M}_{(c)} \simeq \underbrace{p \times \frac{1}{p^2} \times p^2 \times \frac{1}{p^2} \times p}_{\text{pion line}} \times \underbrace{p^3}_{\delta\text{-function for spectator nucleon}} = \mathcal{O}(p^3)$$

↔ two-body effects as important as loop corrections

- Chiral corrections only **suppressed by a single chiral order**

Two-body currents: axial-vector operators



- Axial-vector current can couple via pion pole
↔ two-body effects proportional to c_i , including $\Delta(1232)$ enhancement
- **One-body current**

$$\mathbf{J}_{i,1b}^3 = \frac{1}{2} \tau_i^3 \left(G_A^3(\mathbf{q}^2) \boldsymbol{\sigma}_i - \frac{G_P^3(\mathbf{q}^2)}{4m_N^2} (\mathbf{q} \cdot \boldsymbol{\sigma}_i) \mathbf{q} \right)$$

- **Two-body current**

$$\begin{aligned} \mathbf{J}_{12,2b}^3 = & -\frac{g_A}{2F_\pi^2} [\tau_1 \times \tau_2]^3 \left[c_4 \left(1 - \frac{\mathbf{q}}{q^2 + M_\pi^2} \mathbf{q} \cdot \right) (\boldsymbol{\sigma}_1 \times \mathbf{k}_2) + \frac{c_6}{4} (\boldsymbol{\sigma}_1 \times \mathbf{q}) + i \frac{\mathbf{p}_1 + \mathbf{p}'_1}{4m_N} \right] \frac{\boldsymbol{\sigma}_2 \cdot \mathbf{k}_2}{M_\pi^2 + k_2^2} \\ & - \frac{g_A}{F_\pi^2} \tau_2^3 \left[c_3 \left(1 - \frac{\mathbf{q}}{q^2 + M_\pi^2} \mathbf{q} \cdot \right) \mathbf{k}_2 + 2c_1 M_\pi^2 \frac{\mathbf{q}}{q^2 + M_\pi^2} \right] \frac{\boldsymbol{\sigma}_2 \cdot \mathbf{k}_2}{M_\pi^2 + k_2^2} + \text{contact terms} \end{aligned}$$

- **Non-relativistic EFT for dark matter** Fan et al. 2010, Fitzpatrick et al. 2012, Anand et al. 2013
↪ integrate out the pions
- Similar to **pionless EFT** for nuclear physics
↪ only remaining degrees of freedom nucleons and WIMPs
- Calculation organized in terms of

$$\mathbf{q} \quad \mathbf{v}^\perp = \mathbf{v} + \frac{\mathbf{q}}{2\mu_N} \quad \mathbf{S}_X \quad \mathbf{S}_N$$

↪ expand in \mathbf{q} , \mathbf{v}^\perp (using $\mathbf{v}^\perp \cdot \mathbf{q} = 0$)

Matching to NREFT

- Operator basis for **WIMP and nucleon fields** Fan et al. 2010, Fitzpatrick et al. 2012

$$\begin{aligned}\mathcal{O}_1 &= \mathbb{1} & \mathcal{O}_2 &= (\mathbf{v}^\perp)^2 & \mathcal{O}_3 &= i\mathbf{S}_N \cdot (\mathbf{q} \times \mathbf{v}^\perp) & \mathcal{O}_4 &= \mathbf{S}_\chi \cdot \mathbf{S}_N \\ \mathcal{O}_5 &= i\mathbf{S}_\chi \cdot (\mathbf{q} \times \mathbf{v}^\perp) & \mathcal{O}_6 &= \mathbf{S}_\chi \cdot \mathbf{q} \mathbf{S}_N \cdot \mathbf{q} & \mathcal{O}_7 &= \mathbf{S}_N \cdot \mathbf{v}^\perp & \mathcal{O}_8 &= \mathbf{S}_\chi \cdot \mathbf{v}^\perp \\ \mathcal{O}_9 &= i\mathbf{S}_\chi \cdot (\mathbf{S}_N \times \mathbf{q}) & \mathcal{O}_{10} &= i\mathbf{S}_N \cdot \mathbf{q} & \mathcal{O}_{11} &= i\mathbf{S}_\chi \cdot \mathbf{q} & & \dots\end{aligned}$$

- Matching to relativistic amplitudes

$$\begin{aligned}\mathcal{M}_{1,NR}^{SS} &= \mathcal{O}_1 f_N(t) & \mathcal{M}_{1,NR}^{SP} &= \mathcal{O}_{10} g_5^N(t) & \mathcal{M}_{1,NR}^{PP} &= \frac{1}{m_\chi} \mathcal{O}_6 h_5^N(t) \\ \mathcal{M}_{1,NR}^{VV} &= \mathcal{O}_1 \left(f_1^{V,N}(t) + \frac{t}{4m_N^2} f_2^{V,N}(t) \right) + \frac{1}{m_N} \mathcal{O}_3 f_2^{V,N}(t) + \frac{1}{m_N m_\chi} (t\mathcal{O}_4 + \mathcal{O}_6) f_2^{V,N}(t) \\ \mathcal{M}_{1,NR}^{AV} &= 2\mathcal{O}_8 f_1^{V,N}(t) + \frac{2}{m_N} \mathcal{O}_9 \left(f_1^{V,N}(t) + f_2^{V,N}(t) \right) \\ \mathcal{M}_{1,NR}^{AA} &= -4\mathcal{O}_4 g_A^N(t) + \frac{1}{m_N^2} \mathcal{O}_6 g_P^N(t) & \mathcal{M}_{1,NR}^{VA} &= \left\{ -2\mathcal{O}_7 + \frac{2}{m_\chi} \mathcal{O}_9 \right\} h_A^N(t)\end{aligned}$$

- Observations

- SI: \mathcal{O}_1 , SD: combination of \mathcal{O}_4 and \mathcal{O}_6
- Not all the \mathcal{O}_i equally important, QCD implies relations among them

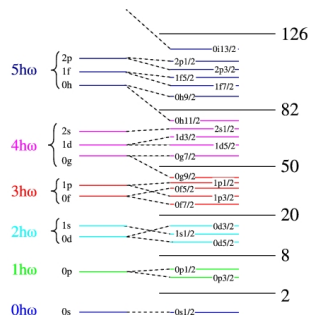
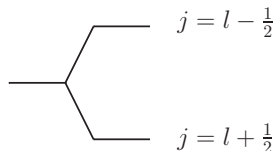
Coherence effects

- NREFT especially useful to analyze **coherence effects**

- Six distinct nuclear responses

Fitzpatrick et al. 2012, Anand et al. 2013

- $M \leftrightarrow \mathcal{O}_1 \leftrightarrow \text{SI}$
- $\Sigma', \Sigma'' \leftrightarrow \mathcal{O}_4, \mathcal{O}_6 \leftrightarrow \text{SD}$
- $\Phi'' \leftrightarrow \mathcal{O}_3 \leftrightarrow \text{quasi-coherent, spin-orbit operator}$
- $\Delta, \tilde{\Phi}'$: not coherent
- **Quasi-coherence** of Φ''
 - Spin-orbit splitting
 - Coherence until mid-shell
 - About 20 coherent nucleons in Xe
 - Interference $M-\Phi'' \leftrightarrow \mathcal{O}_1-\mathcal{O}_3$
- Further coherent M -responses from $\mathcal{O}_5, \mathcal{O}_8, \mathcal{O}_{11}$, but no interference with \mathcal{O}_1 due to sum over \mathbf{S}_X



Rate for WIMP–nucleus scattering

$$\frac{dR}{dE_r} = \underbrace{\frac{\sigma_{\chi N}^{SI}}{m_\chi \mu_N^2}}_{\text{particle + hadronic physics}} \times \underbrace{|\mathcal{F}_+^M(q^2)|^2}_{\text{nuclear physics}} \times \underbrace{\rho_0 \int_{v_{\min}}^{v_{\text{esc}}} \frac{f(\mathbf{v}, t)}{v} d^3v}_{\text{astrophysics}}$$

- **EFT constraints** on particle + hadronic part
- **Convolution with nuclear wave function** replaces traditional Helm form factor
 - ↪ nuclear shell model, ab-initio techniques
- Different options for EFT parameterizations, in terms of
 - 1 **Wilson coefficients at high scale**
 - ↪ need to specify quantum numbers of WIMP
 - 2 **effective couplings at QCD scale**, e.g., $\sigma_{\chi N}^{SI}$, $\sigma_{\chi N}^{SD}$
 - ↪ combination of Wilson coefficients and hadronic matrix elements
 - 3 **NREFT coefficients**
 - ↪ BSM constraints obtained after matching to QCD

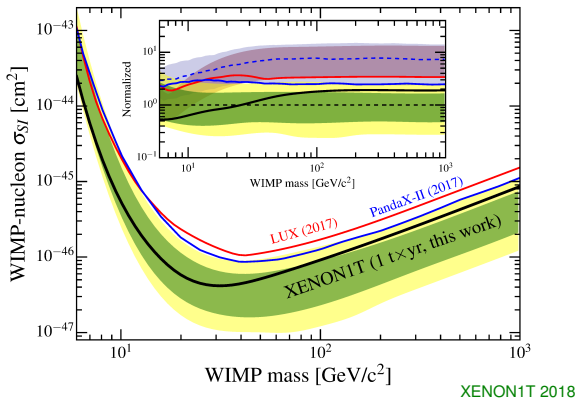
EFT decomposition of direct detection rate

- Different EFT approaches basically differ by **choice of scale** at which EFT coefficients are determined
- All can be used to **generalize the standard SI/SD picture**
- Example for approach 2 (most natural from **chiral-EFT perspective**):

$$\begin{aligned} \frac{d\sigma_{\chi\mathcal{N}}}{dq^2} &= \frac{1}{4\pi v^2} \left| \sum_{I=\pm} \left(c_I^M - \frac{q^2}{m_N^2} \dot{c}_I^M \right) \mathcal{F}_I^M(q^2) + c_\pi \mathcal{F}_\pi(q^2) + c_b \mathcal{F}_b(q^2) + \frac{q^2}{2m_N^2} \sum_{I=\pm} c_I^{\phi''} \mathcal{F}_I^{\phi''}(q^2) \right|^2 \\ &+ \frac{1}{4\pi v^2} \sum_{i=5,8,11} \left| \sum_{I=\pm} \xi_i(q, v_T^\perp) c_i^{M,i} \mathcal{F}_I^M(q^2) \right|^2 \\ &+ \frac{1}{v^2(2J+1)} \left(|a_+|^2 S_{00}(q^2) + \text{Re}(a_+ a_-^*) S_{01}(q^2) + |a_-|^2 S_{11}(q^2) \right) \end{aligned}$$

green: Wilson coefficients + hadronic matrix elements, red: nuclear structure factors

Case 1: spin-independent scattering

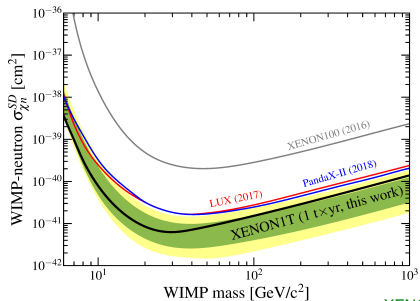


XENON1T 2018

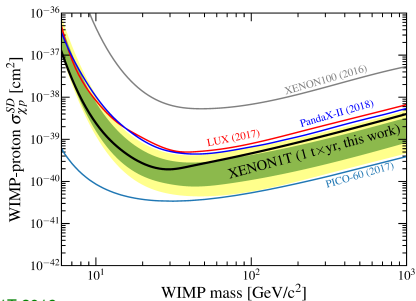
- All $c = 0$, $a = 0$ except for c_+^M : **spin-independent scattering**

$$\frac{d\sigma_{\chi N}}{dq^2} = \frac{\sigma_{\chi N}^{\text{SI}}}{4\mu_N^2 v^2} |\mathcal{F}_+^M(q^2)|^2 \quad \sigma_{\chi N}^{\text{SI}} = \frac{\mu_N^2}{\pi} |c_+^M|^2 \quad \mu_N = \frac{m_N m_\chi}{m_N + m_\chi}$$

Case 2: spin-dependent scattering

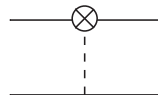


XENON1T 2019



- All $c = 0$, $a_+ = \pm a_-$: **spin-dependent scattering**

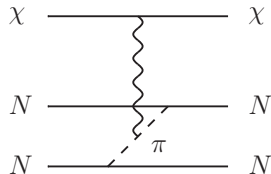
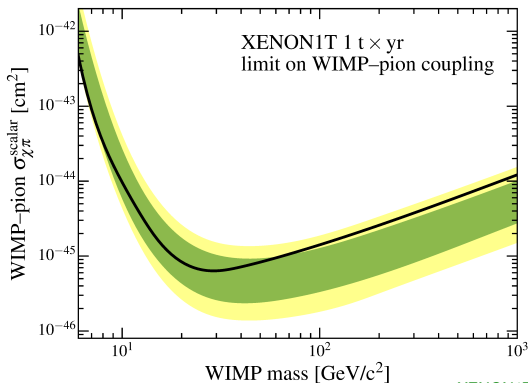
$$\frac{d\sigma_{\chi N}}{dq^2} = \frac{\sigma_{\chi N}^{SD}}{3\mu_N^2 v^2} \frac{\pi}{2J+1} S_N(q^2) \quad \sigma_{\chi N}^{SD} = \frac{3\mu_N^2}{\pi} |a_+|^2$$



- Xe sensitive to proton spin due to **two-body currents**

Klos, Menéndez, Gazit, Schwenk 2013

Case 3: WIMP–pion scattering



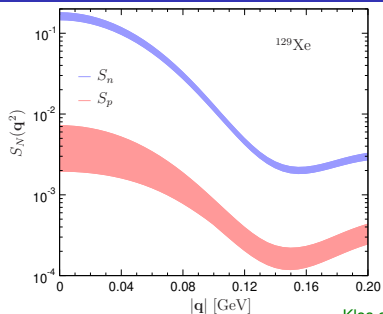
XENON1T + MH, Klos, Menéndez, Schwenk 2019

- Only c_π nonzero: **WIMP–pion scattering**

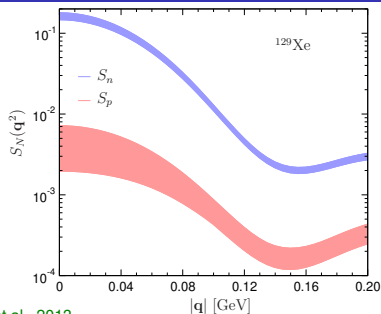
$$\frac{d\sigma_{\chi\mathcal{N}}}{dq^2} = \frac{\sigma_{\chi\pi}^{\text{scalar}}}{\mu_\pi^2 v^2} |\mathcal{F}_\pi(q^2)|^2 \quad \sigma_{\chi\pi}^{\text{scalar}} = \frac{\mu_\pi^2}{4\pi} |c_\pi|^2 \quad \mu_\pi = \frac{m_\chi M_\pi}{m_\chi + M_\pi}$$

- Expression in terms of cross section depends on underlying operator, here for a scalar $\bar{\chi}\chi\bar{q}q$

Calculation of spin-dependent responses



Klos et al., 2013

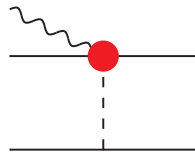


- Main challenge: **two-body currents**

↪ dominant contribution for even-numbered species

- Developments since 2013:

- New information on **low-energy constants** C_i
- **Nuclear axial-vector current** at 1-loop
Baroni et al. 2016, Krebs et al. 2016
- Improved understanding of **g_A quenching** in β decays

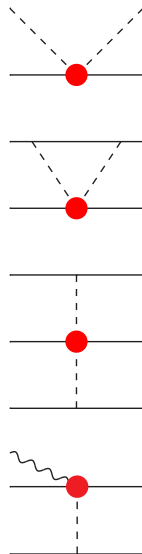


- c_i first contribute to πN scattering
- At given scheme and order, uncertainties negligible when matching in the **subthreshold region** [MH et al. 2015](#), [Siemens et al. 2016](#)
- Large shifts when including **loop effects** [Bernard et al. 2008](#)
 \hookrightarrow can be partially captured by

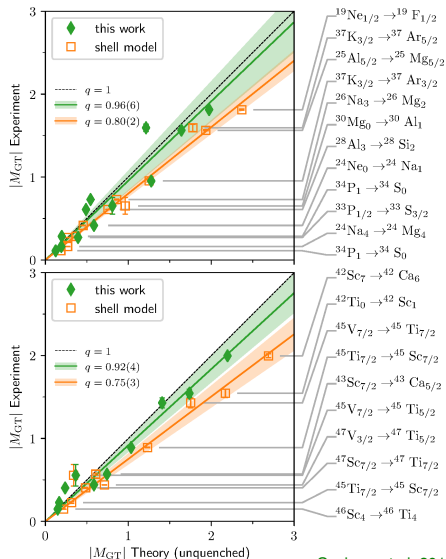
$$\delta c_1 = -\frac{g_A^2 M_\pi}{64\pi F_\pi^2} \quad \delta c_3 = -\delta c_4 = \frac{g_A^4 M_\pi}{16\pi F_\pi^2} \quad \delta c_6 = -\frac{g_A^2 M_\pi}{4\pi F_\pi^2}$$

- Absorbing relativistic corrections, we use

c_1 [GeV $^{-1}$]	c_3 [GeV $^{-1}$]	c_4 [GeV $^{-1}$]	c_6 [GeV $^{-1}$]
-1.20(17)	-4.45(86)	2.96(70)	5.01(1.06)



g_A quenching in β decays

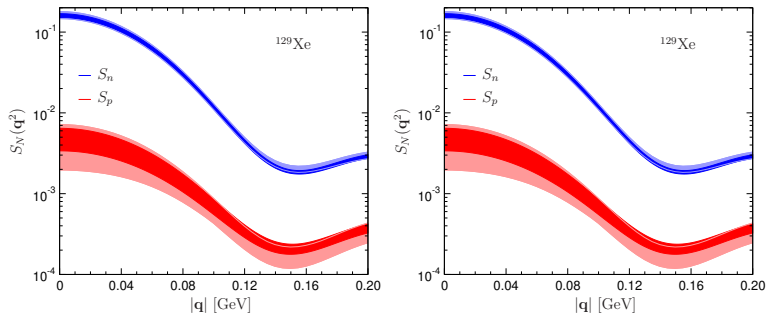


Gysbers et al. 2019

- **Ab initio calculation of β decays** explains origin of quenching:
 - Two-body currents
 - Limitations of shell model
- We adjust the normalization accordingly, use chiral prediction for q^2 dependence
- In practice done in terms of density of normal-ordering ρ and value of C_D :

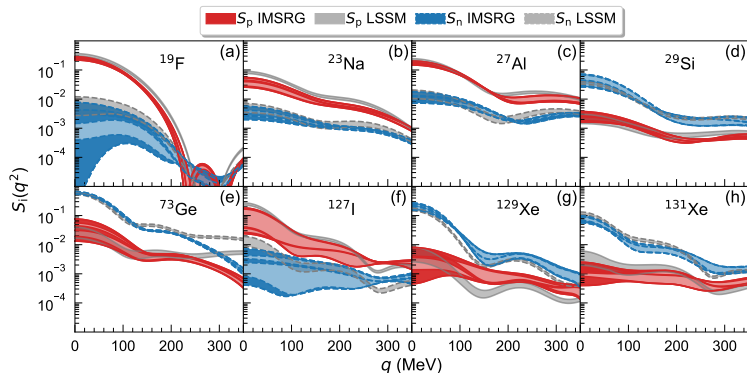
$$\{C_D, \rho\} = \{-6.08, 0.09 \text{ fm}^{-3}\} \\ \dots \{0.30, 0.11 \text{ fm}^{-3}\} \\ \rightarrow -30\% \dots -20\%$$

Improved spin-dependent responses for dark matter



- New results consistent with 2013 bands
- Uncertainties reduced especially of the suppressed (even-numbered) species

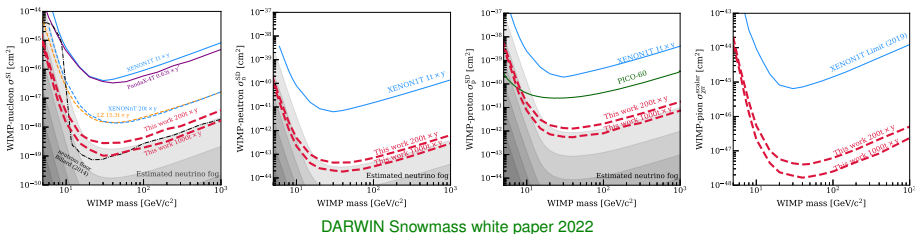
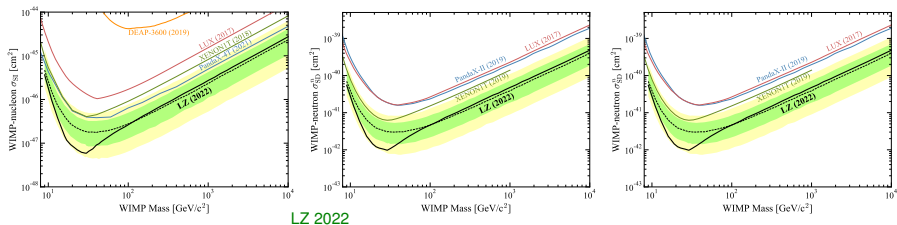
Comparison to ab-initio methods



Hu et al. 2021

- Results shown so far use nuclear shell model for nuclear wave functions
- First results from **ab-initio methods** (in-medium SRG Hu et al. 2021)
 - ↪ NN and $3N$ interactions fully consistent with currents (eventually)

New experimental results and prospects



- Impressive progress in experimental sensitivity

↪ should be analyzed with adequate theory

- EFT allows one to move systematically **from BSM to nuclear scale**
- EFT-motivated decomposition of direct-detection rate depends on **choice of scale**
↔ in principle equivalent, but more or less efficient depending on application
- **Chiral EFT:**
 - Directly generalizes standard SD/SI picture
 - Predicts hierarchy among NREFT coefficients
 - Allows one to include two-body corrections
 - Constrains Hamiltonian in ab-initio approaches