

# From Low-Energy QCD to Nuclear Structure

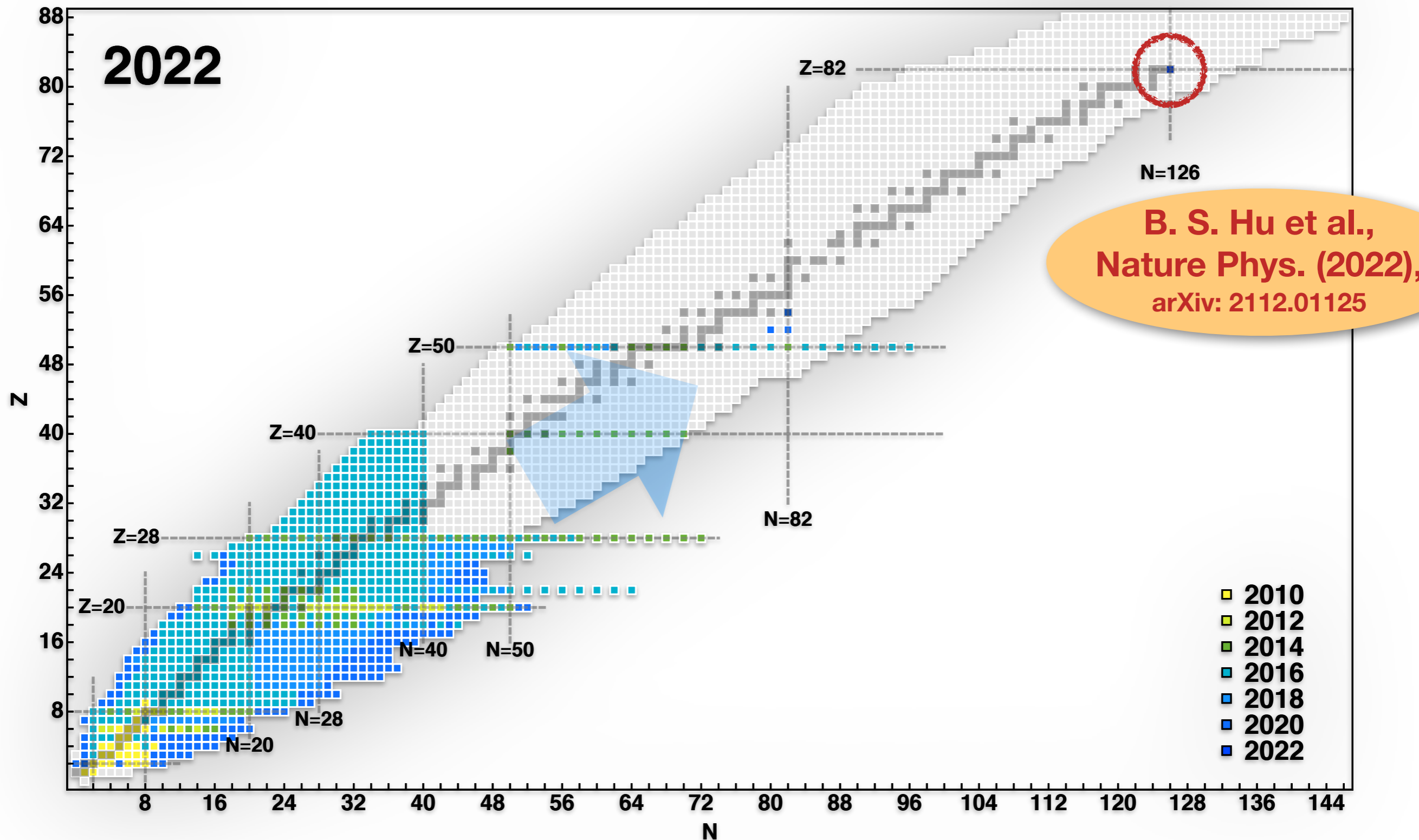
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Facility for Rare Isotope Beams  
& Department of Physics and Astronomy  
Michigan State University



# Progress in *Ab Initio* Calculations



[ cf. HH, *Front. Phys.* 8, 379 (2020) ]



**Chiral EFT**



**RG**

(similarity trasfos)



**many-body  
method**

- **Interactions (& Operators) from Chiral EFT**
  - symmetries of low-energy QCD
  - power counting
- **(Similarity) Renormalization Group**
  - systematically dial resolution scales (cutoffs) of theory
  - trade-off: enhanced convergence & accuracy of many-body methods vs. omitted induced  $4N$ , ...,  $AN$  forces
- ***Ab Initio* Many-Body Methods**
  - systematically improvable towards exact solution

# Sources of Uncertainty



**Chiral EFT**



**RG**  
(similarity trafos)



**many-body  
method**

- selection of degrees of freedom
  - regulators
  - truncation
  - low-energy constant (**LEC**) uncertainties
- 
- selection of operator basis / model space
  - truncation
- 
- symmetry restrictions
  - model-space & many-body truncation(s)
  - continuum

# Nuclear Interactions from Chiral Effective Field Theory

## **Recent(-ish) Reviews:**

E. Epelbaum, H. Krebs and P. Reinert, *Front. Phys.* **8**, 98 (2002)

M. Piarulli and I. Tews, *Front. Phys.* **7**, 245 (2020)

R. Machleidt and F. Sammarruca, *Phys. Scripta* **91**, 083007 (2016)

# Interactions from Chiral EFT



	Two-nucleon force	Three-nucleon force	Four-nucleon force
LO ( $Q^0$ )			
NLO ( $Q^2$ )			
N <sup>2</sup> LO ( $Q^3$ )			
N <sup>3</sup> LO ( $Q^4$ )			

- organization in powers  $(Q/\Lambda_\chi)^\nu$  allows **systematic improvement**
- low-energy constants **fit to NN, 3N data** (future: from Lattice QCD (?))
- **consistent NN, 3N, ... interactions & operators** (e.g. **electroweak transitions**)



# Current Interactions



PHYSICAL REVIEW C 91, 051301(R) (2015)

## Accurate nuclear radii and binding energies from a chiral interaction

A. Ekström,<sup>1,2</sup> G. R. Jansen,<sup>2,1</sup> K. A. Wendt,<sup>1,2</sup> G. Hagen,<sup>2,1</sup> T. Papenbrock,<sup>1,2</sup> B. D. Carlsson,<sup>3</sup> C. Forssén,<sup>3,1,2</sup> M. Hjorth-Jensen,<sup>4,5</sup> P. Navrátil,<sup>6</sup> and W. Nazarewicz<sup>7,2,7</sup>

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(Received 5 December 2014; revised manuscript received 10 April 2015; published 1 May 2015)

With the goal of developing precise predictions for light and medium-mass nuclei, we study the chiral two-nucleon scattering data, as well as binding energies and radii of few-nucleon systems and selected isotopes of carbon and oxygen. Coupled-cluster calculations based on interaction-regularized N<sup>3</sup>LO<sub>1</sub> yield accurate binding energies and radii of nuclei. In addition, the  $\Delta$  isobaric states in <sup>16</sup>O and <sup>16</sup>Ca are described accurately, while spectra for selected *p*- and *sd*-shell nuclei are in reasonable agreement with experiment.

DOI: 10.1103/PhysRevC.91.051301 PACS number(s): 21.30.-x, 21.10.-k, 21.45.-v, 21.60.De

PHYSICAL REVIEW C 102, 054301 (2020)

## • NN and 3N forces at through order N3LO, some NN forces available at even higher orders [cf. Epelbaum et al., Front. Phys. 8, 98 (2020) and references therein]

## • local, semilocal, nonlocal regulators

## • with and without virtual $\Delta$ isobars

## • need to account for **also see talk by M. Piarulli** between LECs as well as observables

## [see, e.g., Reinert et al., EPJA 54, 86; Wesolowski et. al, PRC 104, 064001 and refs. therein]

### Light-Nuclei Spectra from Chiral Dynamics

M. Piarulli,<sup>1</sup> A. Baroni,<sup>2</sup> L. Gandola,<sup>3,4</sup> A. Schwenk,<sup>3,4,5</sup> S. Pastore,<sup>6</sup> M. Piarulli,<sup>1</sup> Steven C. Pieper,<sup>1</sup> R. Schiavilla,<sup>1</sup> J. W. Holt,<sup>7</sup> and T. Papenbrock<sup>1</sup>

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<sup>2</sup>Department of Physics, Old Dominion University, Norfolk, Virginia 23529, USA

<sup>3</sup>Department of Mathematics and Physics, University of Salento, 73100 Lecce, Italy

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(Received 31 July 2017; published 1 February 2018)

In recent years local chiral interactions have been employed as a means to describe few- and many-body systems. In this Letter, we present Green's function Monte Carlo calculations of light nuclei based on the family of chiral interactions presented in Ref. [1], with the addition of conjunction with chiral three-nucleon interactions (N<sup>3</sup>LO<sub>1</sub>) and the inclusion of virtual  $\Delta$  isobaric states in their two-pion-exchange components. We obtain predictions for the energy level and level ordering of nuclei in the mass range  $A = 4-12$ , accurate to  $\leq 2\%$  in agreement with experimental data.

DOI: 10.1103/PhysRevLett.120.052503

PHYSICAL REVIEW LETTERS 120, 122502 (2018)

Eur. Phys. J. A (2018) 54: 86  
DOI 10.1140/epja/i2018-12516-4

THE EUROPEAN  
PHYSICAL JOURNAL

Regular Article – Theoretical Physics

## Semilocal coordinate-space regularized chiral two-nucleon interactions

**Abstract.** We introduce new semilocal two-nucleon potentials up to fifth order in the chiral expansion. We employ a simple regularization approach for the pion exchange contributions which maintains the locality of the interaction while removing the high-momentum divergences. The removal of the redundant contact terms results in a drastic simplification of the fits to scattering data and leads to interactions which are much softer (i.e., more perturbative) than our recent semilocal coordinate-space regularized potentials. Using the pion-nucleon low-energy constants from matching pion-nucleon Roy-Steiner equations to chiral perturbation theory, we find that the number of parameters required for the description of the pion-nucleon interaction is significantly reduced by the high-precision fit. The resulting interactions yield accurate binding energies and radii for a range of nuclei from  $A = 16$  to  $A = 132$ , and provide accurate equations of state for nuclear matter and realistic symmetry energies. Selected excited states are also in agreement with data.

## Probing chiral interactions in medium-mass nuclei

**Abstract.** We study the saturation properties of nuclei on novel chiral interactions up to fifth order in the chiral expansion. We employ a simple regularization approach for the pion exchange contributions which maintains the locality of the interaction while removing the high-momentum divergences. The removal of the redundant contact terms results in a drastic simplification of the fits to scattering data and leads to interactions which are much softer (i.e., more perturbative) than our recent semilocal coordinate-space regularized potentials. Using the pion-nucleon low-energy constants from matching pion-nucleon Roy-Steiner equations to chiral perturbation theory, we find that the number of parameters required for the description of the pion-nucleon interaction is significantly reduced by the high-precision fit. The resulting interactions yield accurate binding energies and radii for a range of nuclei from  $A = 16$  to  $A = 132$ , and provide accurate equations of state for nuclear matter and realistic symmetry energies. Selected excited states are also in agreement with data.

(Received 29 April 2019; published 12 August 2019)

## • **tensions** revealed in LEC fits: e.g., optimal 3NF LECs for medium-mass nuclei vs. nuclear matter

## [cf. Hoppe et al., PRC 100, 024318; Hüther et al., PLB 808, 135651, ...]

D. Lonardoni,<sup>1,2</sup> J. Carlson,<sup>2</sup> S. Gandolfi,<sup>2</sup> J. E. Lynn,<sup>3,4</sup> K. E. Schmidt,<sup>5</sup> A. Schwenk,<sup>3,4,6</sup> and X. B. Wang<sup>7</sup>

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<sup>5</sup>Department of Physics, Arizona State University, Tempe, Arizona 85287, USA

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(Received 26 September 2017; revised manuscript received 29 January 2018; published 22 March 2018)

We report accurate quantum Monte Carlo calculations of nuclei up to  $A = 16$  based on local chiral two- and three-nucleon interactions up to next-to-next-to-leading order. We examine the theoretical uncertainties associated with the chiral expansion and the cutoff in the theory, as well as the associated operator choices in the three-nucleon interactions. While in light nuclei the cutoff variation and systematic uncertainties are rather small, in <sup>16</sup>O these can be significant for large coordinate-space cutoffs. Overall, we show that chiral interactions constructed to reproduce properties of few-body systems and nucleon-nucleon scattering give an excellent description of binding energies, charge radii, and spectra for selected *p*- and *sd*-shell nuclei in open-shell systems in  $A = 6$  and  $12$ .

PHYSICAL REVIEW C 101, 014318 (2020)

## Novel chiral Hamiltonian and observables in light and medium-mass nuclei

V. Somà,<sup>1,2</sup> P. Navrátil,<sup>2,1</sup> F. Raimondi,<sup>3,4,1</sup> C. Barbieri,<sup>4,3</sup> and T. Duguet<sup>1,5,1</sup>

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(Received 23 July 2019; revised manuscript received 6 November 2019; published 22 January 2020)

**Background:** Recent advances in nuclear structure theory have led to the availability of several complementary *ab initio* many-body techniques applicable to light and medium-mass nuclei as well as nuclear matter. After successful benchmarks of different approaches, the focus is moving to the development of improved models of nuclear Hamiltonians, currently representing the largest source of uncertainty in *ab initio* calculations of nuclei. In particular, the use of the existing two- plus three-body interactions is capable of satisfactorily describing the ground state properties of nuclei. **Purpose:** A novel parametrization of a Hamiltonian based on chiral effective field theory is introduced. Specifically, three-nucleon operators at next-to-next-to-leading order are combined with an existing (and successful) two-body interaction containing terms up to next-to-next-to-next-to-leading order. The resulting potential is labeled  $NV+3N(\ln)$ . The objective of the present work is to investigate the performance of this new Hamiltonian across light and medium-mass nuclei.

PHYSICAL REVIEW C 102, 054301 (2020)

## Accurate bulk properties of nuclei from $A = 2$ to $\infty$ from potentials with $\Delta$ isobars

W. G. Jiang,<sup>1,2,3</sup> A. Ekström,<sup>3</sup> C. Forssén,<sup>3</sup> G. Hagen,<sup>2,1</sup> G. R. Jansen,<sup>4,2</sup> and T. Papenbrock<sup>1,2</sup>

<sup>1</sup>Department of Physics and Astronomy, University of Tennessee, Knoxville, Tennessee 37996, USA

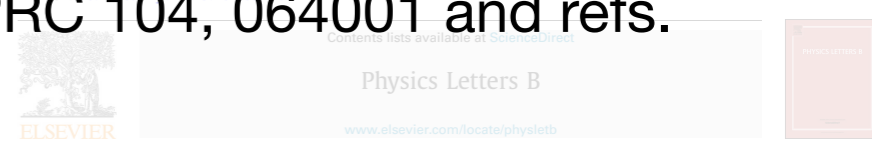
<sup>2</sup>Physics Division, Oak Ridge National Laboratory, Oak Ridge, Tennessee 37831, USA

<sup>3</sup>Department of Physics, Chalmers University of Technology, SE-412 96 Göteborg, Sweden

<sup>4</sup>National Center for Computational Sciences, Oak Ridge National Laboratory, Oak Ridge, Tennessee 37831, USA

(Received 2 July 2019; accepted 16 October 2020; published 2 November 2020)

We apply the interaction-regularized chiral two-nucleon plus three-nucleon interactions in the  $\Delta$  isobaric channel to calculate the bulk properties of nuclei from  $A = 2$  to  $A = 132$ , and provide accurate equations of state for nuclear matter and realistic symmetry energies. Selected excited states are also in agreement with data.



Family of chiral two-nucleon and three-nucleon interactions for accurate description of nuclei and nuclear matter. To this end, we perform in-medium similarity renormalization group (IM-SRG) calculations based on chiral interactions at next-to-leading order (NLO), N<sup>2</sup>LO, and N<sup>3</sup>LO, where the 3N interactions at N<sup>2</sup>LO and N<sup>3</sup>LO are fit to the empirical saturation point of nuclear matter and to the binding energy of nuclei. The resulting interactions yield accurate binding energies and radii for a range of nuclei from  $A = 16$  to  $A = 132$ , and provide accurate equations of state for nuclear matter and realistic symmetry energies. Selected excited states are also in agreement with data.

**ARTICLE INFO**  
ABSTRACT  
We present a family of nucleon-nucleon (NN) plus three-nucleon (3N) interactions up to N<sup>3</sup>LO in the chiral expansion that provides an accurate *ab initio* description of ground-state energies and charge radii up to medium-mass regime with quantified theory uncertainties. Starting from the NN interactions proposed by Entem, Machleidt and Nosyk, we construct 3N interactions with consistent chiral order, non-local regulator, and cutoff value and explore the dependence of nuclear observables over a range of mass numbers on the 3N low-energy constants. By fixing these constants using the <sup>4</sup>He and <sup>16</sup>O ground-state energies, we obtain interactions that robustly reproduce experimental energies and radii for a large range from *p*-shell nuclei to the nickel isotopic chain and resolve many of the deficiencies of previous interactions. Based on the order-by-order convergence and the cutoff dependence of nuclear observables, we assess the uncertainties due to the interaction, which yield a significant contribution to the total theory uncertainty.  
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DOI: 10.1103/PhysRevC.100.024318



# The Similarity Renormalization Group

## Review:

S. Bogner, R. Furnstahl, and A. Schwenk, Prog. Part. Nucl. Phys. **65**, 94 (2010)

E. Anderson, S. Bogner, R. Furnstahl, and R. Perry, Phys. Rev. C **82**, 054001 (2011)

E. Jurgenson, P. Navratil, and R. Furnstahl, Phys. Rev. C **83**, 034301 (2011)

R. Roth, S. Reinhardt, and H. H., Phys. Rev. C **77**, 064003 (2008)

H. H. and R. Roth, Phys. Rev. C **75**, 051001 (2007)

## Basic Idea

**continuous unitary transformation** of the Hamiltonian to band-diagonal form w.r.t. a given “uncorrelated” many-body basis

- **flow equation** for Hamiltonian  $H(\mathbf{s}) = U(\mathbf{s})HU^\dagger(\mathbf{s})$  :

$$\frac{d}{ds}H(\mathbf{s}) = [\eta(\mathbf{s}), H(\mathbf{s})], \quad \eta(\mathbf{s}) = \frac{dU(\mathbf{s})}{ds}U^\dagger(\mathbf{s}) = -\eta^\dagger(\mathbf{s})$$

- choose  $\eta(\mathbf{s})$  to achieve desired behavior, e.g.,

$$\eta(\mathbf{s}) = [H_d(\mathbf{s}), H_{od}(\mathbf{s})]$$

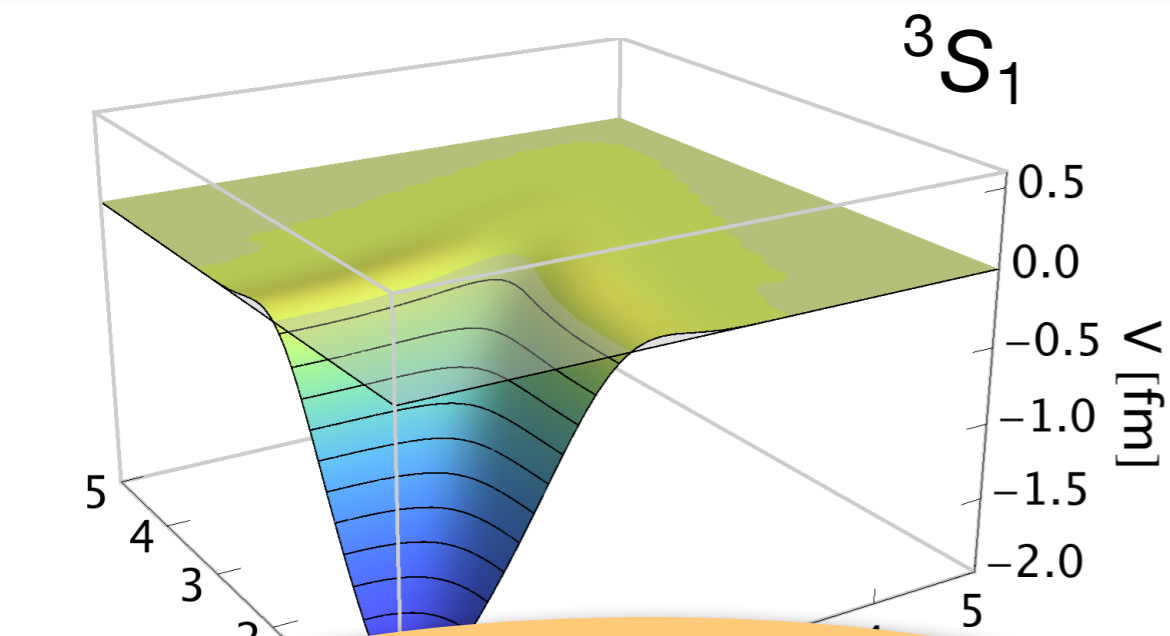
to **suppress** (suitably defined) **off-diagonal Hamiltonian**

- **consistent evolution** for all **observables** of interest

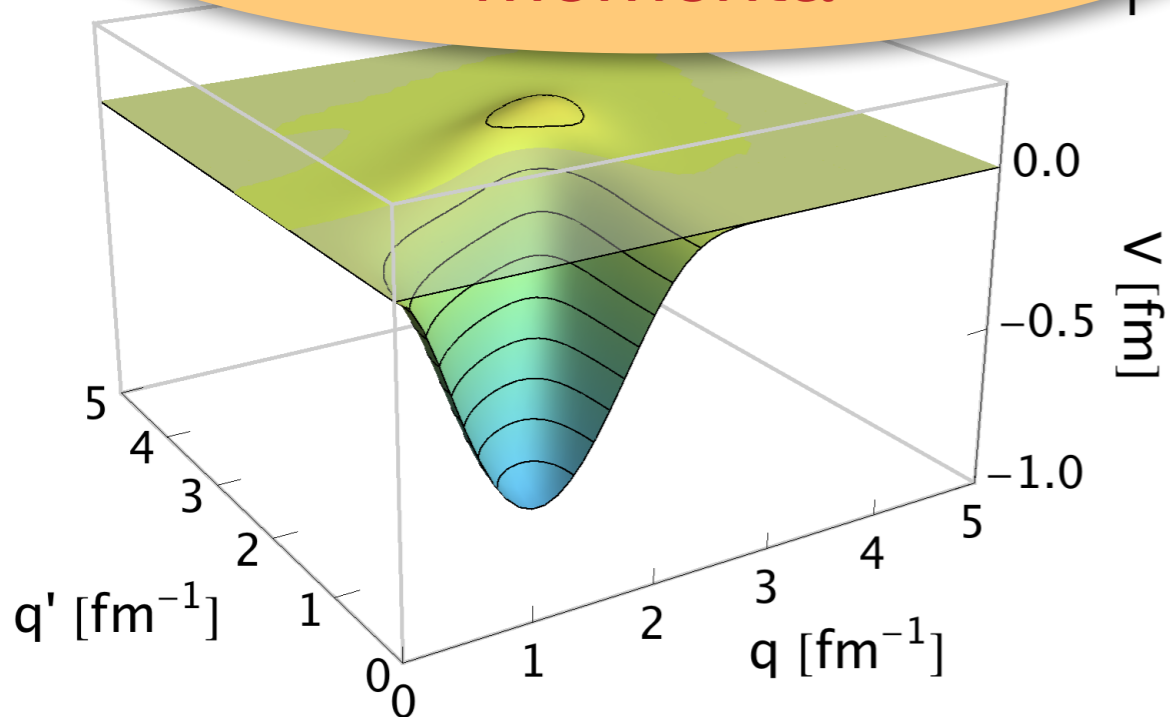
# SRG in Two-Body Space



momentum space matrix elements



lowering resolution scale  $\lambda$   
 $\Leftrightarrow$  decoupling of low and high momenta

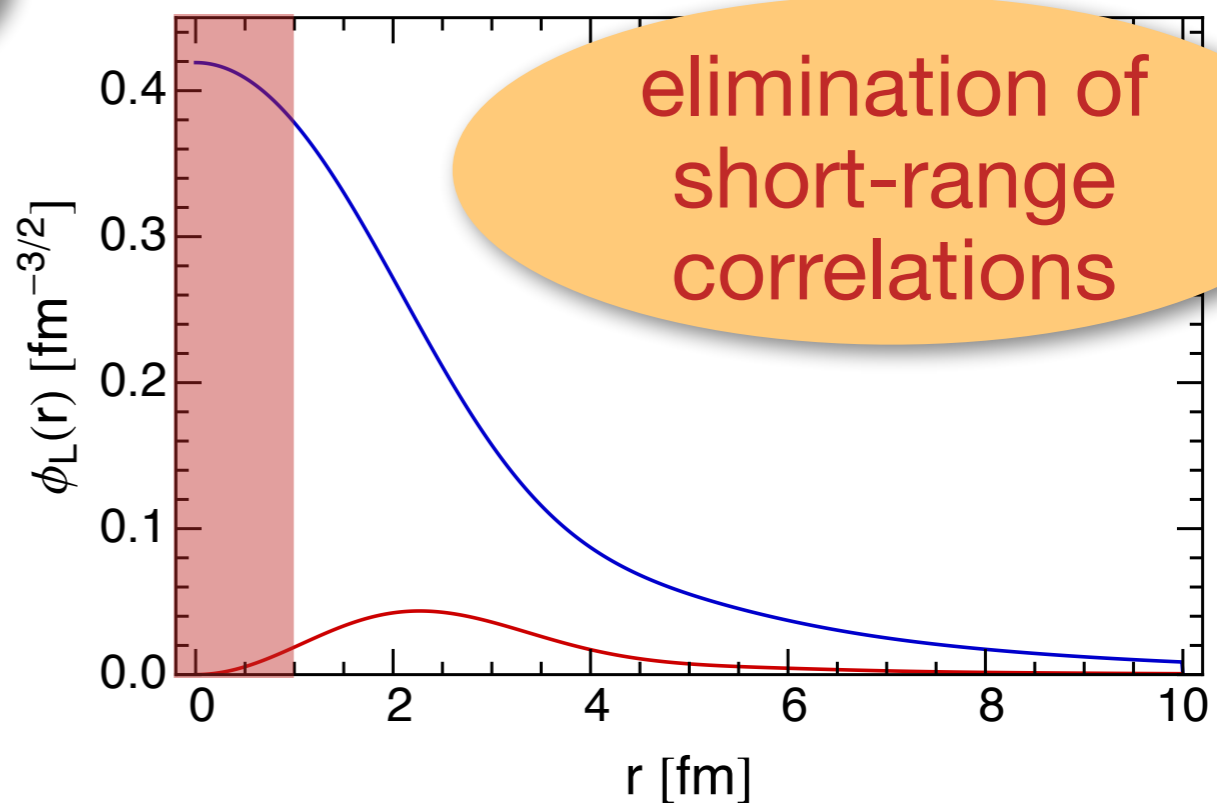


$$\lambda = 1.8 \text{ fm}^{-1}$$

$$\eta(\lambda) = 2\mu [T_{\text{rel}}, H(\lambda)]$$

$$\lambda = s^{-1/4}$$

deuteron wave function



elimination of short-range correlations

- SRG is a **unitary transformation** in **A-body space**
- up to **A-body interactions** are **induced** during the flow:

$$\frac{dH}{d\lambda} = \left[ \left[ \sum a^\dagger a, \underbrace{\sum a^\dagger a^\dagger a a}_{2\text{-body}} \right], \underbrace{\sum a^\dagger a^\dagger a a}_{2\text{-body}} \right] = \dots + \underbrace{\sum a^\dagger a^\dagger a^\dagger a a a}_{3\text{-body}} + \dots$$

- state-of-the-art: evolve in three-body space, truncate induced four- and higher many-body forces  
(Jurgenson, Furnstahl, Navratil, PRL 103, 082501; Hebeler, PRC 85, 021002; Wendt, PRC 87, 061001 )
- **$\lambda$ -dependence** of eigenvalues is a **diagnostic** for size of omitted induced interactions

# Ab Initio Many-Body Methods

## Recent(-ish) Reviews:

HH, Front. Phys. **8**, 379 (2020)

S. Gandolfi, D. Lonardoni, A. Lovato and M. Piarulli, Front. Phys. **8**, 117 (2020)

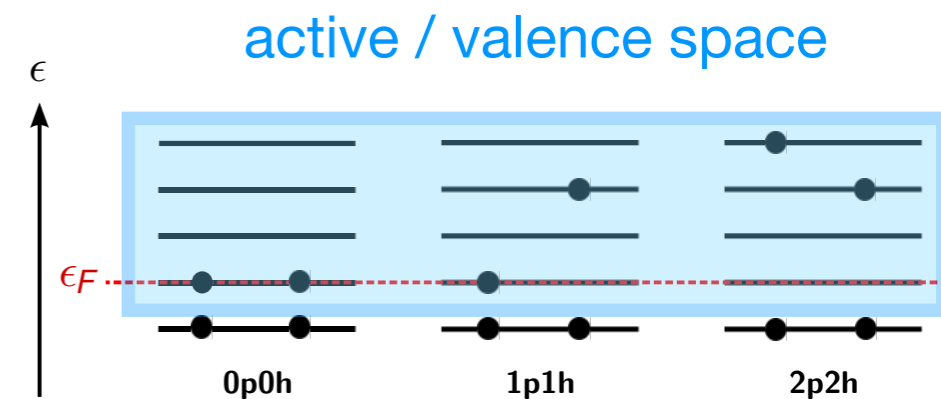
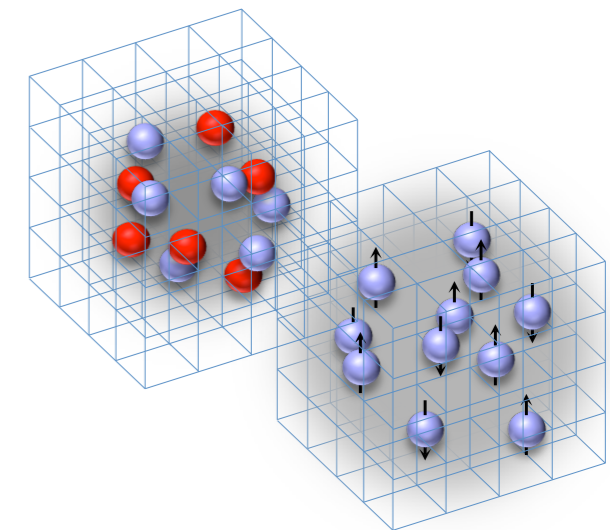
D. Lee, Front. Phys. **8**, 174 (2020)

V. Somà, Front. Phys. **8**, 340 (2020)

## also see

“What is *ab initio* in nuclear theory?”, A. Ekström, C. Forssén, G. Hagen, G. R. Jansen, W. Jiang, T. Papenbrock, arXiv:2212.11064

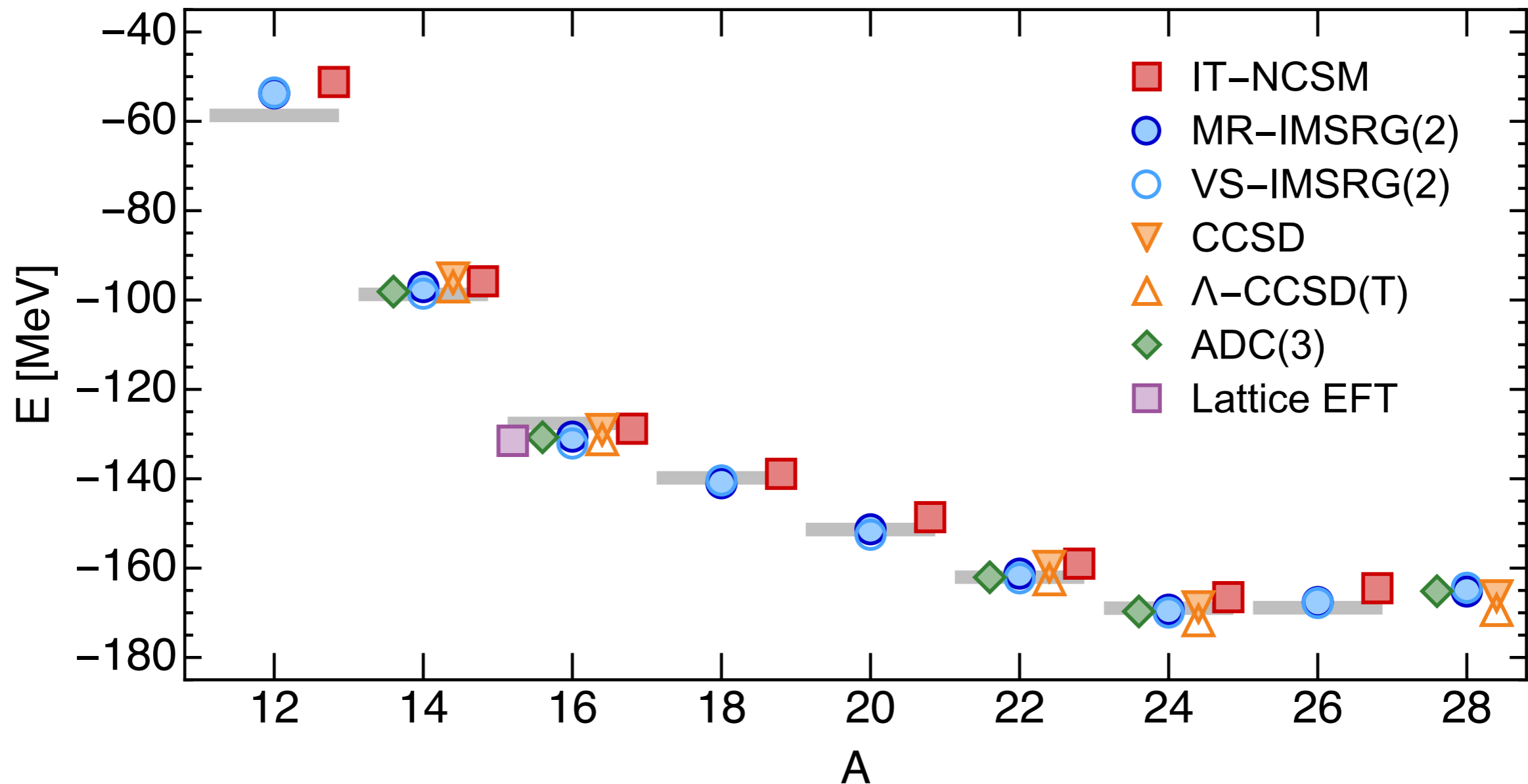
- **Coordinate Space**
  - Quantum Monte Carlo
  - Lattice EFT
- **Configuration Space: Particle-Hole Expansions**
  - Many-Body Perturbation Theory (MBPT)
  - (No-Core) Configuration Interaction (aka Shell Model, (NC)SM)
  - Coupled Cluster (CC) **talks by T. Papenbrock & G. Hagen**
  - In-Medium Similarity Renormalization Group (IMSRG)
- **Configuration Space / Coordinate Space: Geometric Expansions**
  - deformed HF(B) + projection
  - projected Generator Coordinate Method (PGCM)
  - symmetry-adapted NCSM



# Consistency: Oxygen Isotopes



*HH, Front. Phys. 8, 379 (2020)*



**consistent ground-state energies** for the **same interaction**  
(and comparable Lattice EFT action)

# (Multi-Reference) In-Medium Similarity Renormalization Group

HH, Phys. Scripta, Phys. Scripta 92, 023002 (2017)

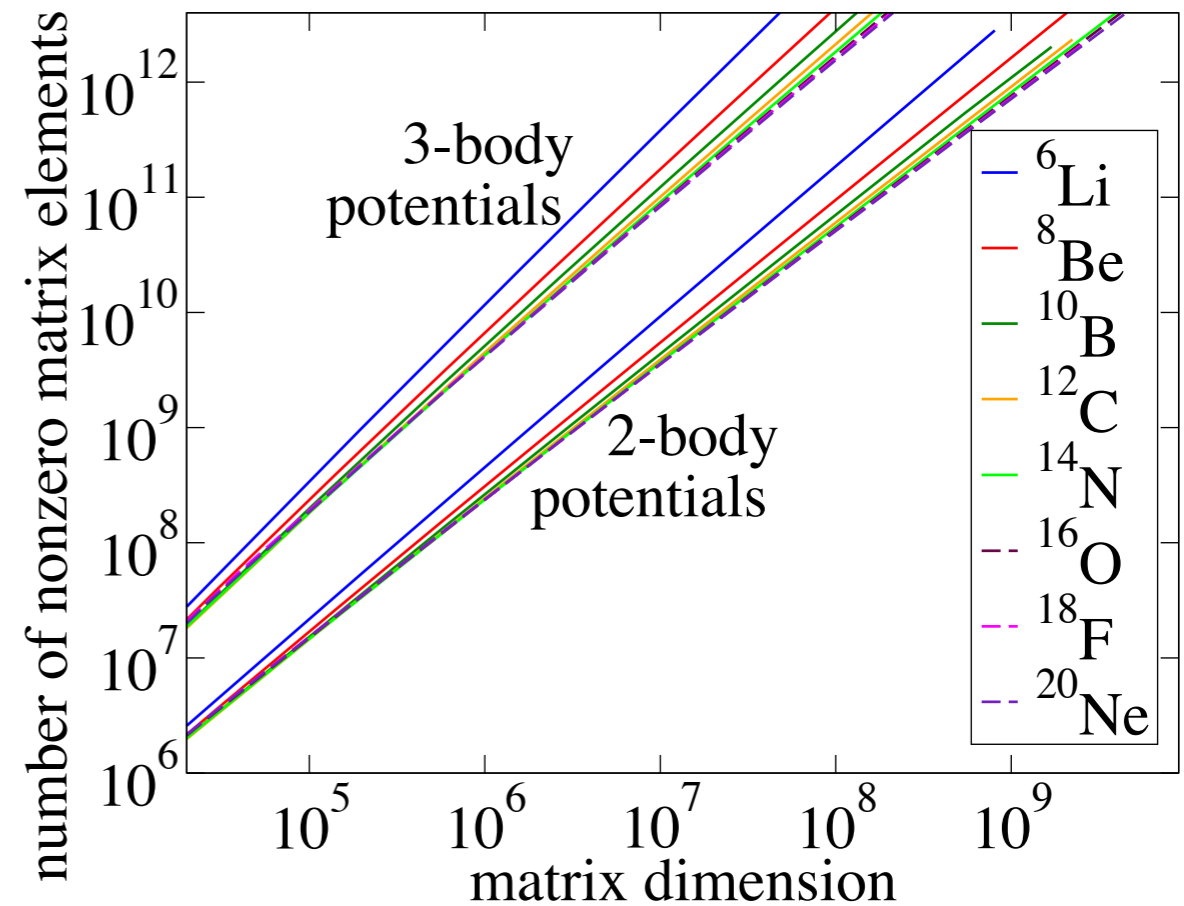
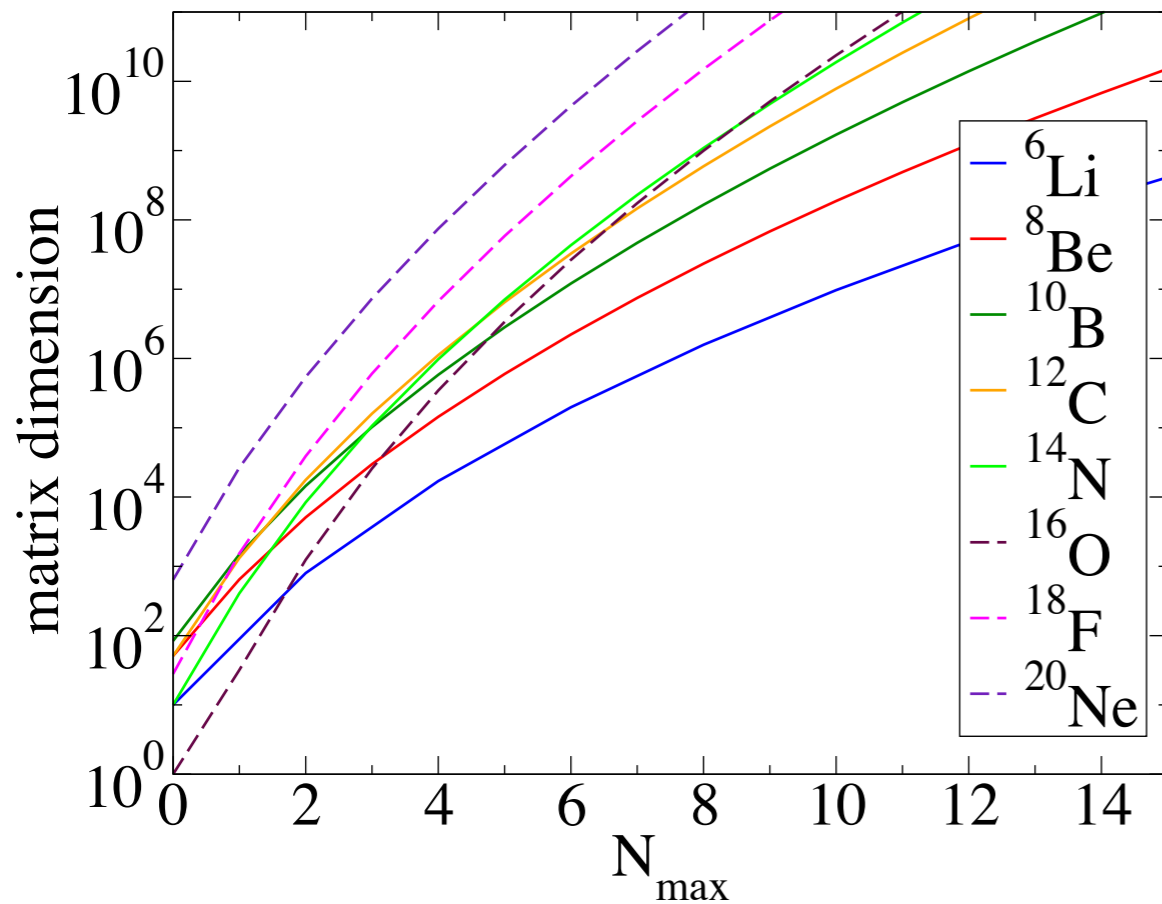
HH, S. K. Bogner, T. D. Morris, A. Schwenk, and K. Tuskiyama, Phys. Rept. **621**, 165 (2016)

HH, S. Bogner, T. Morris, S. Binder, A. Calci, J. Langhammer, R. Roth, Phys. Rev. C **90**, 041302 (2014)

HH, S. Binder, A. Calci, J. Langhammer, and R. Roth, Phys. Rev. Lett **110**, 242501 (2013)



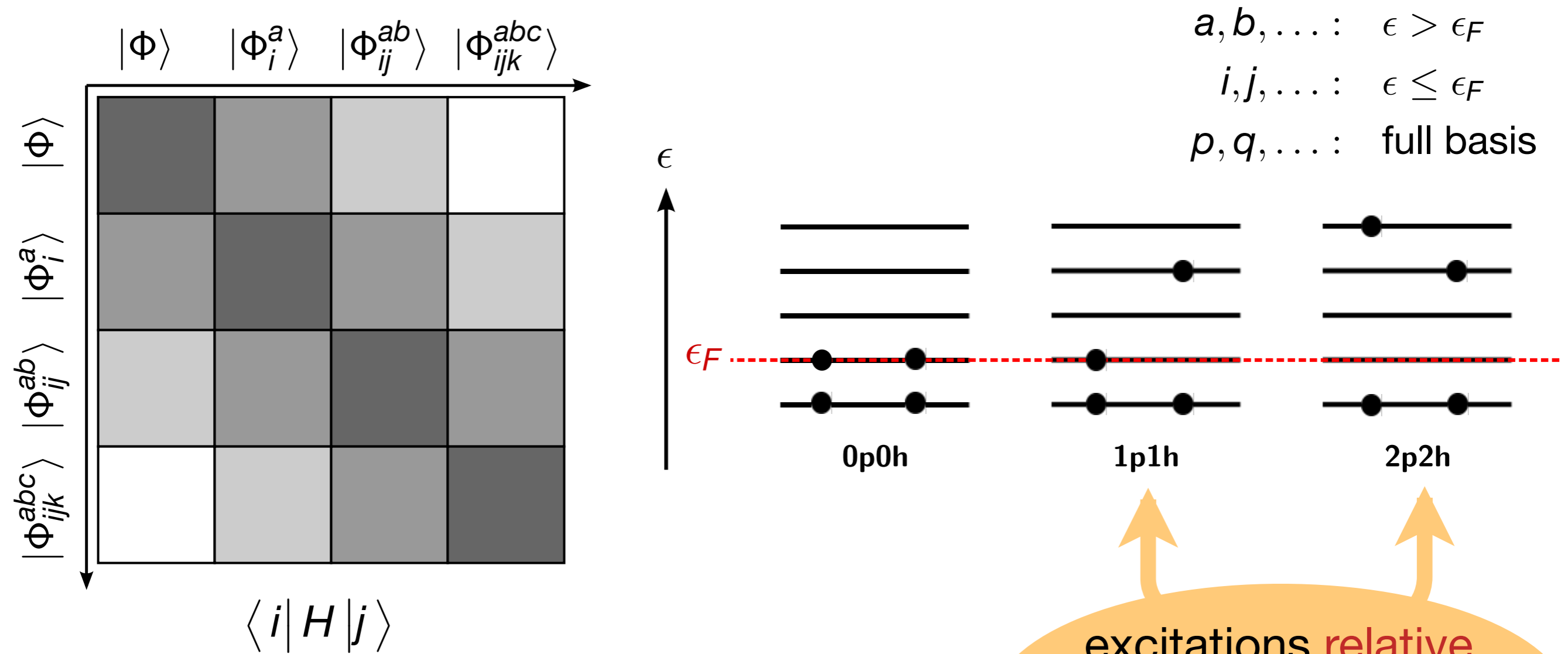
# Large-Scale Diagonalization



from: C. Yang, H. M. Aktulga, P. Maris, E. Ng, J. Vary, Proceedings of NTSE-2013

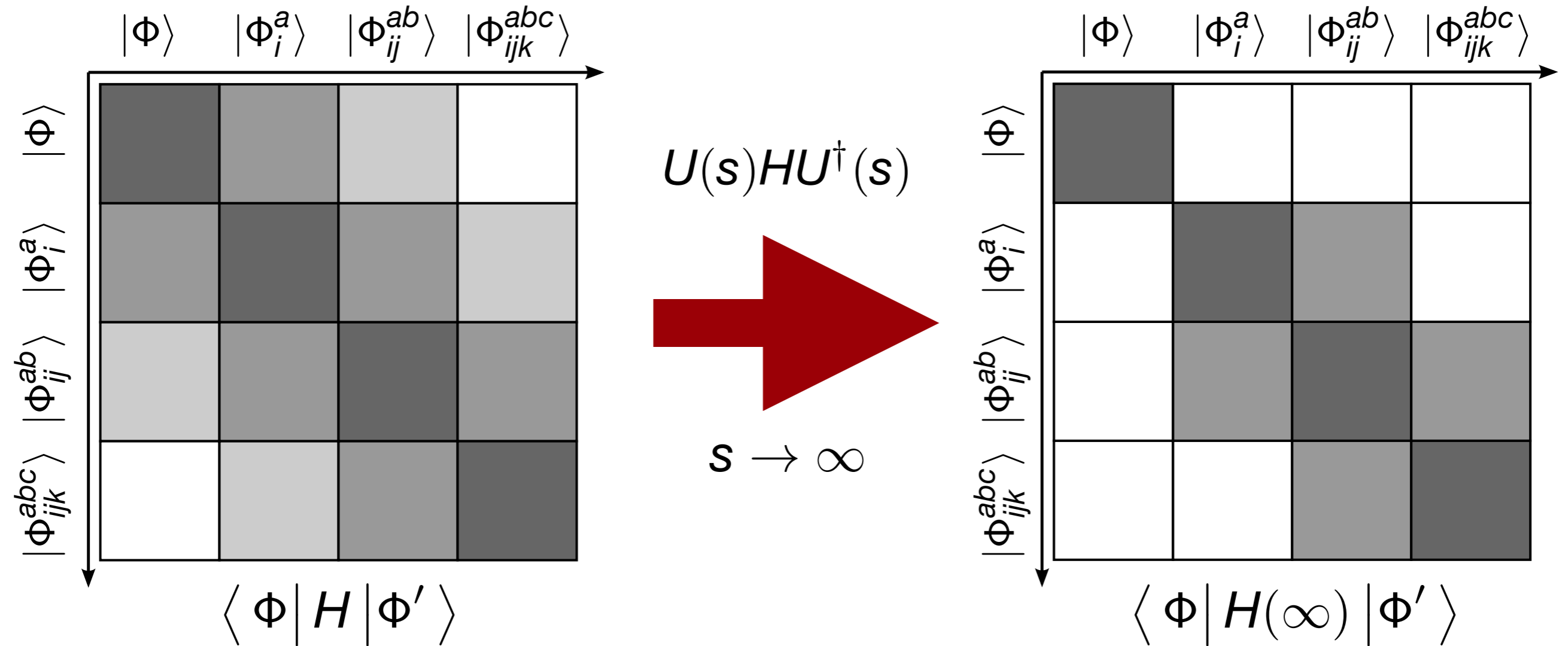
- basis-size “explosion”: **exponential growth**
- **importance truncation** etc. cannot fully compensate this growth as  $A$  increases

# Transforming the Hamiltonian



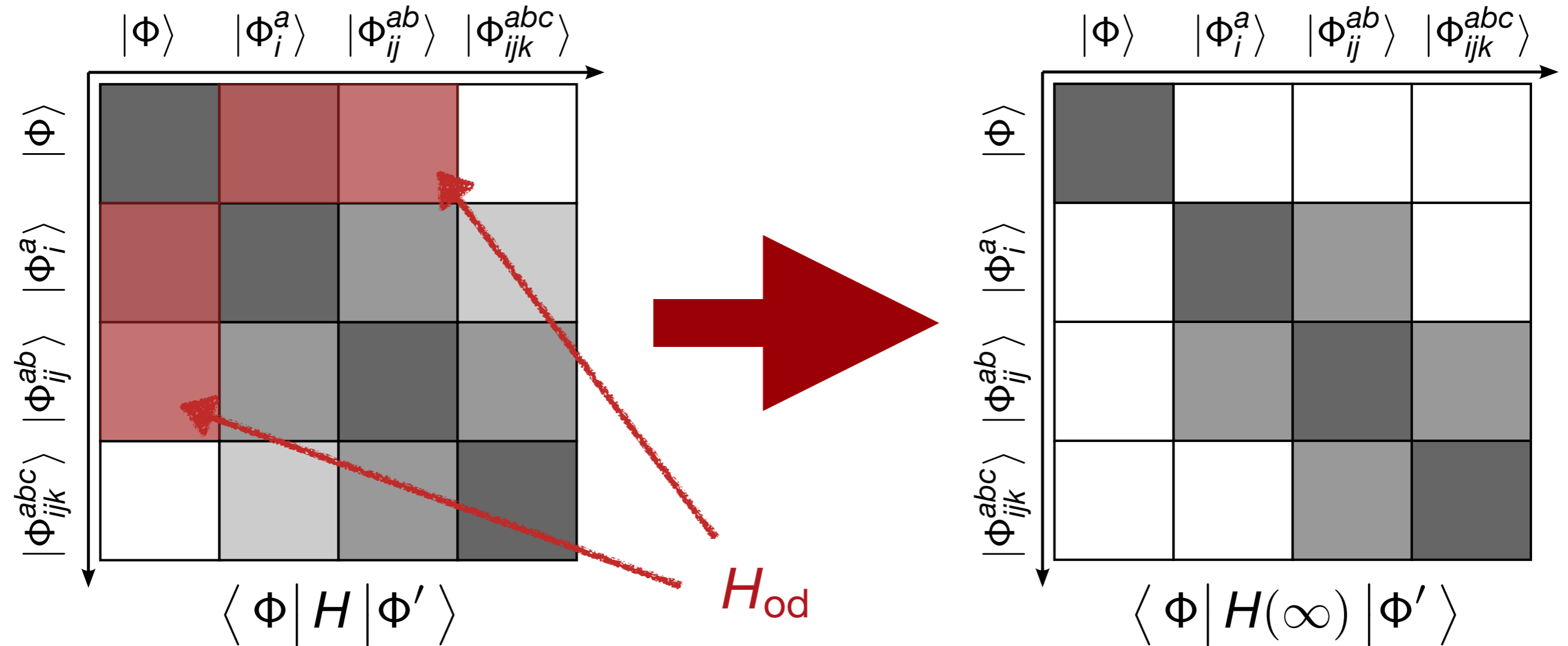
- reference state: **single Slater determinant**

# Decoupling in A-Body Space



**goal:** decouple reference state  $|\Phi\rangle$   
from excitations

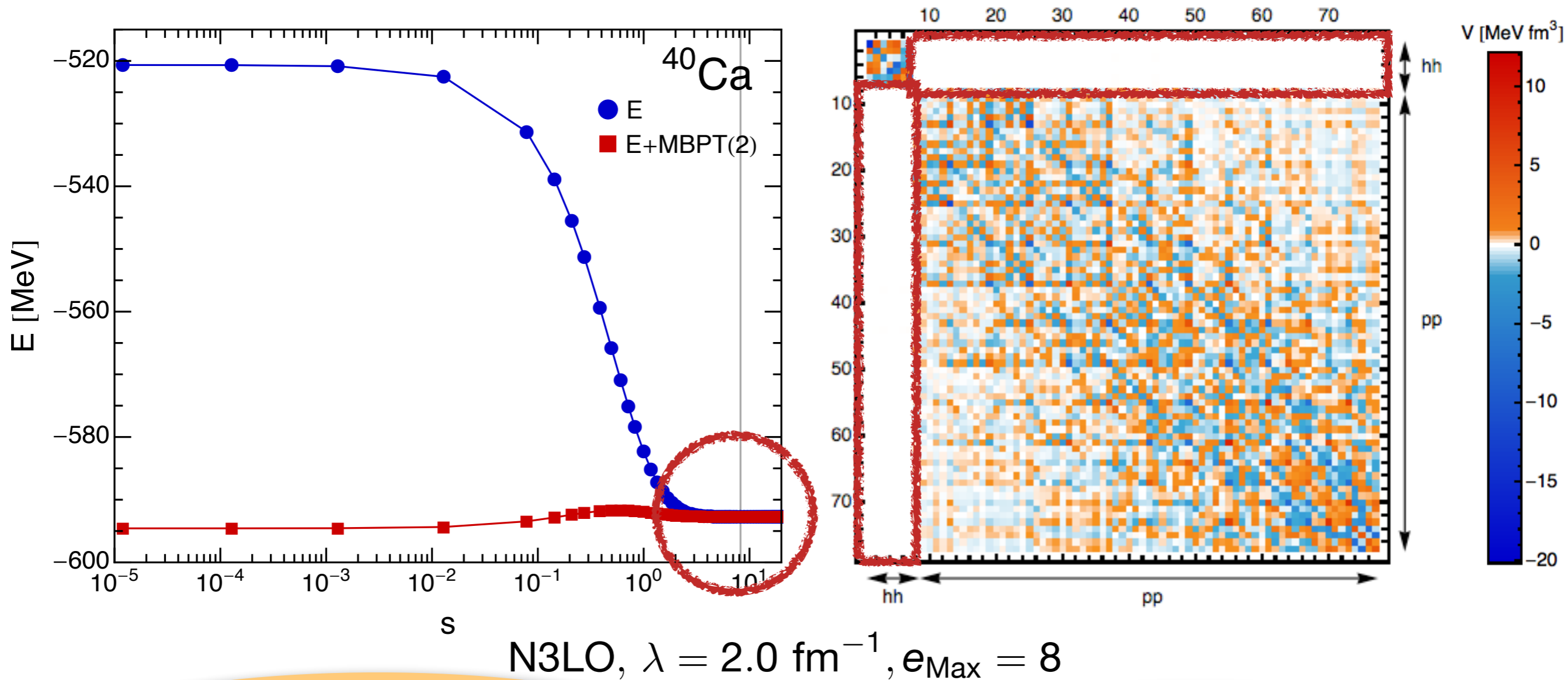
# Flow Equation



$$\frac{d}{ds} H(s) = [\eta(s), H(s)],$$

Operators truncated at **two-body level** - **matrix is never constructed explicitly!**

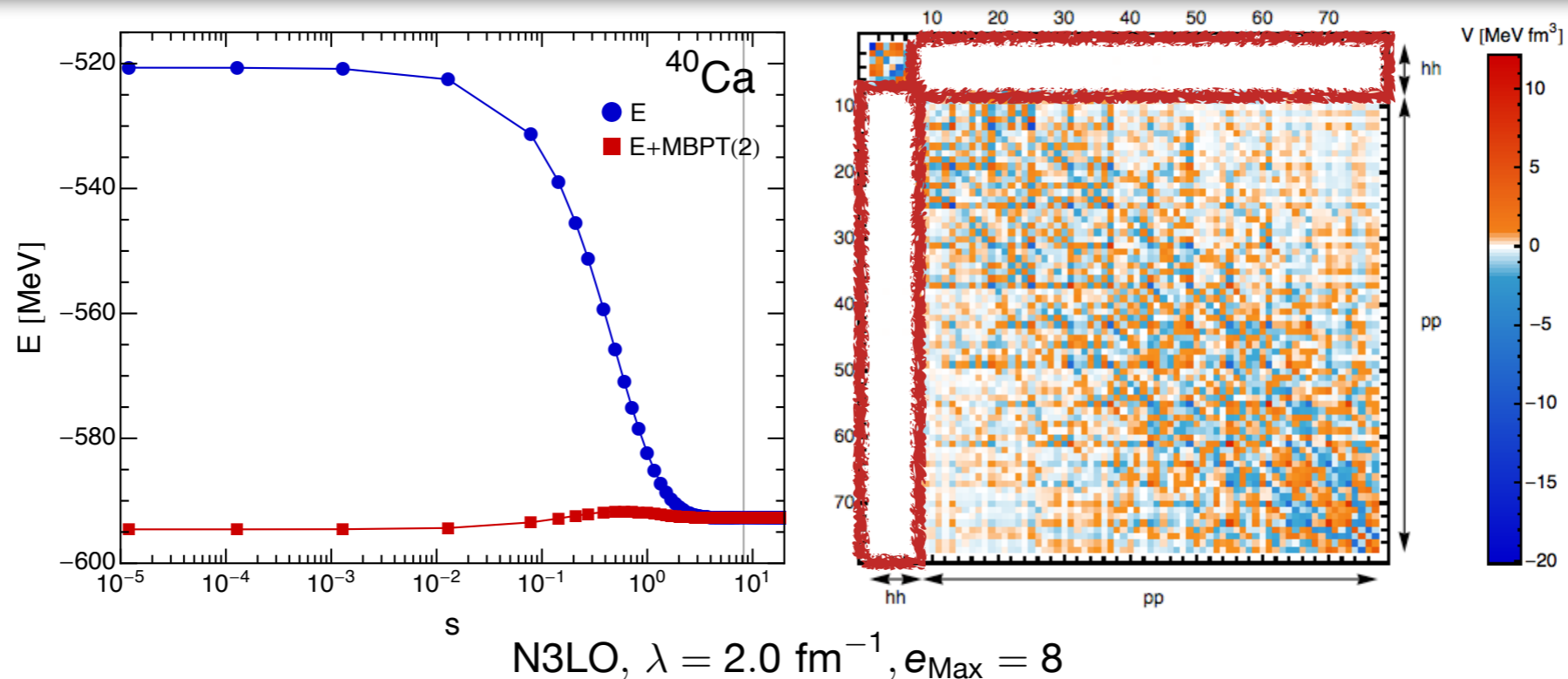
# Decoupling



non-perturbative  
resummation of MBPT series  
(**correlations**)

off-diagonal couplings  
are rapidly driven to zero

# Decoupling



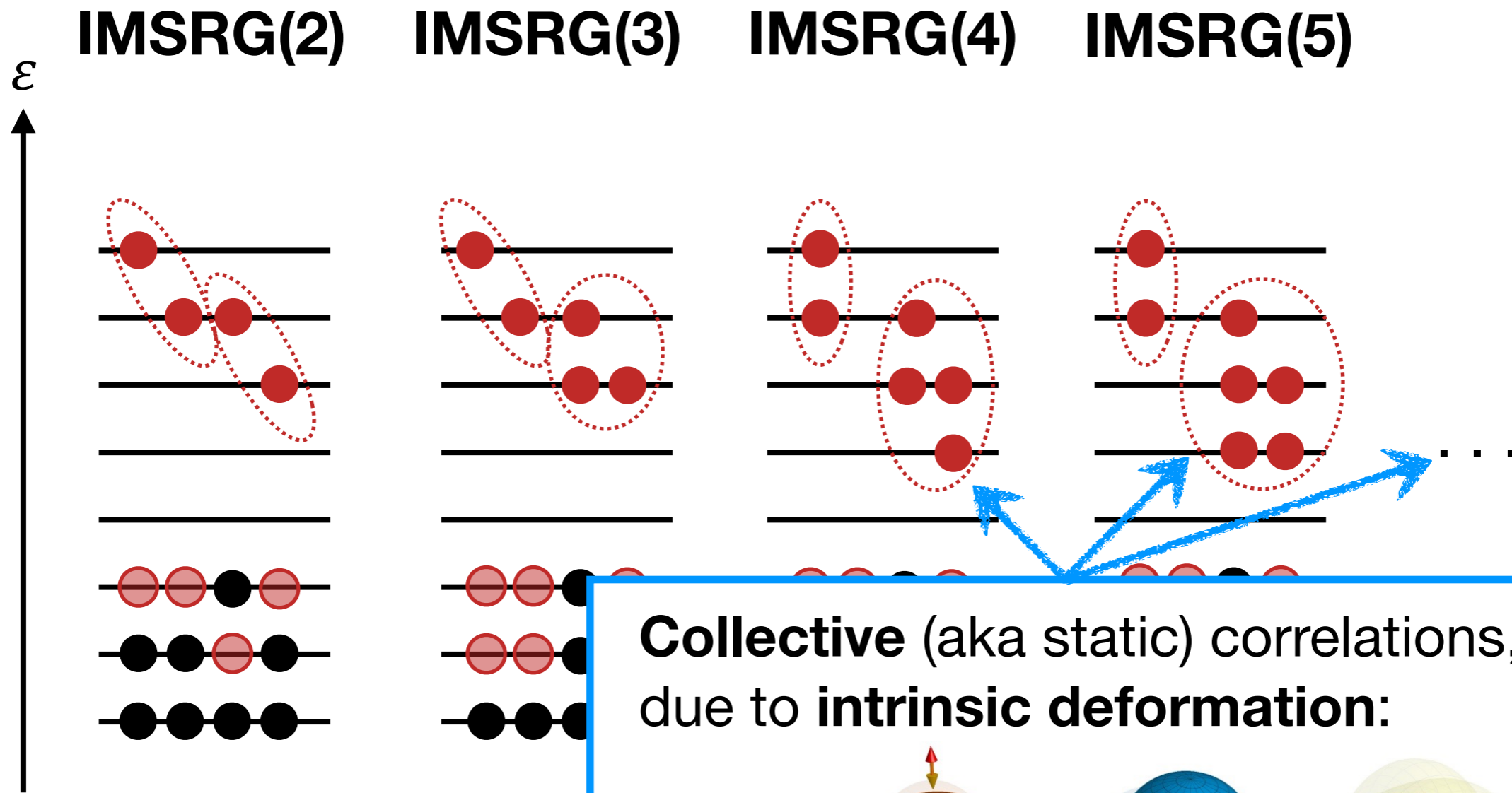
- absorb correlations into **RG-improved Hamiltonian**

$$U(s) H U^\dagger(s) U(s) |\Psi_n\rangle = E_n U(s) |\Psi_n\rangle$$

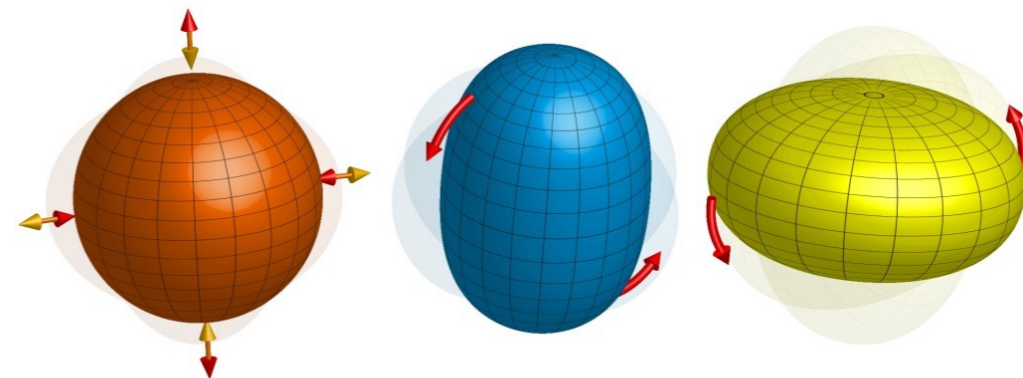
- reference state is ansatz for transformed, **less correlated** eigenstate:

$$U(s) |\Psi_n\rangle \stackrel{!}{=} |\Phi\rangle$$

# Correlated Reference States

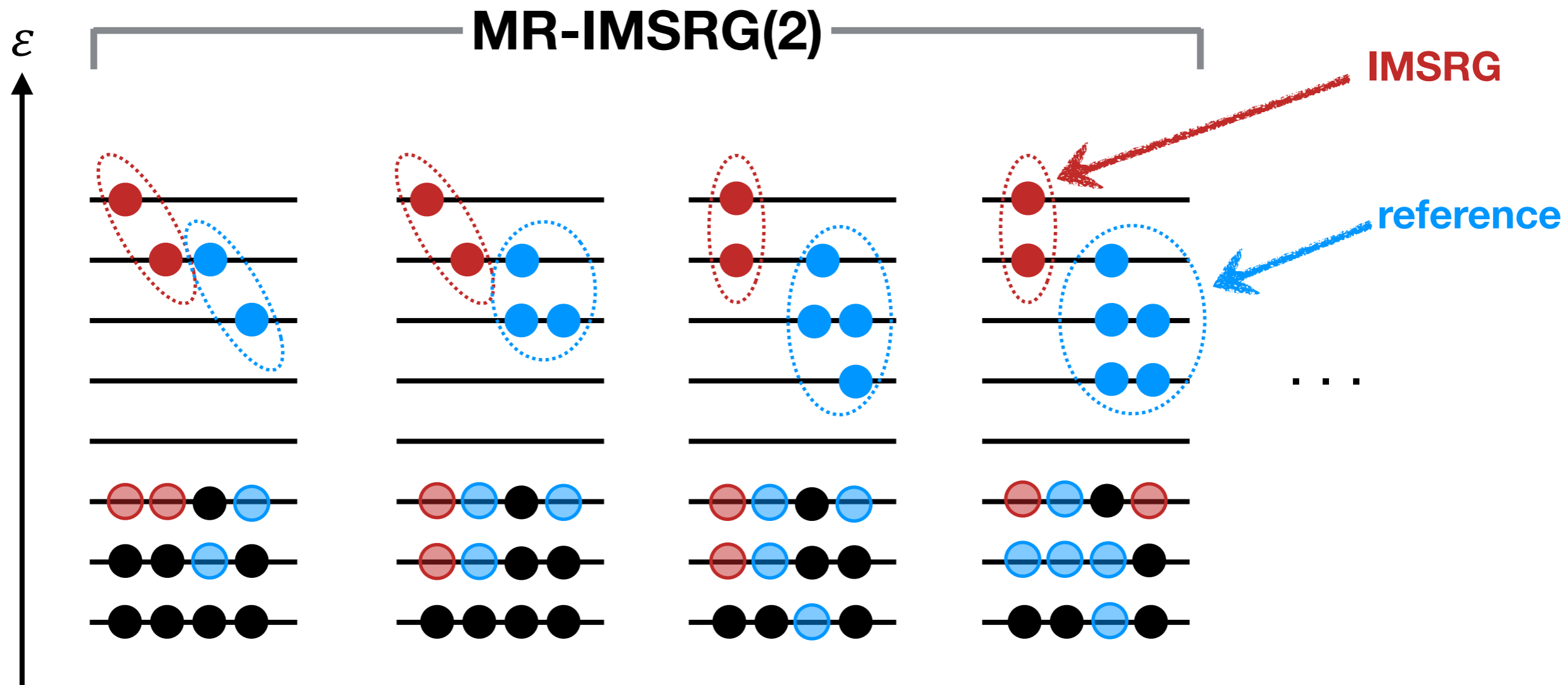


**Collective** (aka static) correlations, e.g. due to **intrinsic deformation**:



“standard” IMS  
Slater determinan

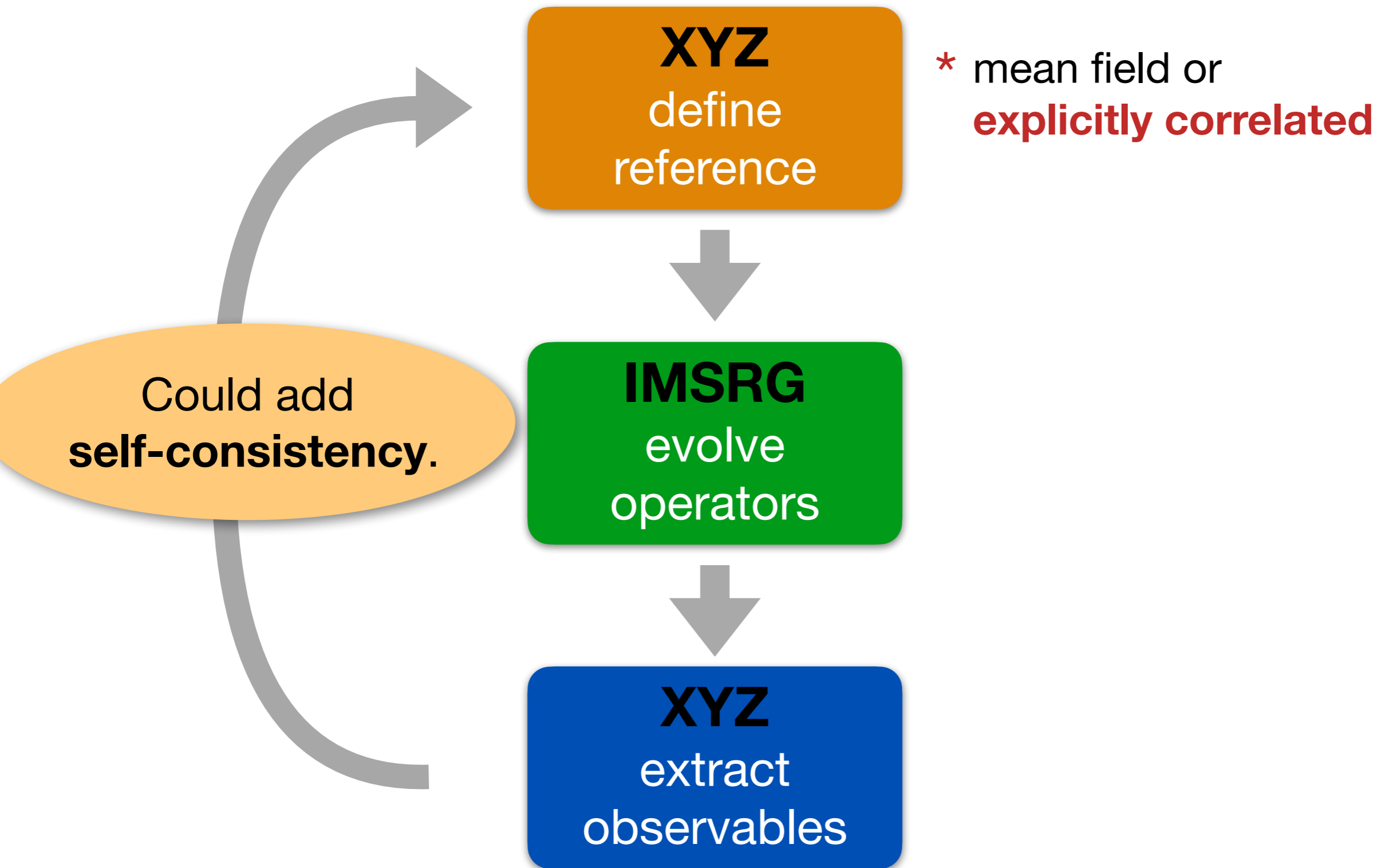
# Correlated Reference States



**MR-IMSRG:** build correlations on top of **already correlated** state (e.g., from a method that describes static correlation well)



# IMSRG-Improved Methods



# IMSRG-Improved Methods



- **IMSRG for closed and open-shell nuclei: IM-HF and IM-PHFB**

- HH, Phys. Scripta, Phys. Scripta 92, 023002 (2017)
- HH, S. K. Bogner, T. D. Morris, A. Schwenk, and K. Tuskuyama, Phys. Rept. 621, 165 (2016)

- **Valence-Space IMSRG (VS-IMSRG)**

- S. R. Stroberg, HH, S. K. Bogner, J. D. Holt, Ann. Rev. Nucl. Part. Sci. 69, 165

- **In-Medium No Core Shell Model (IM-NCSM)**

- E. Gebrerufael, K. Vobig, HH, R. Roth, PRL 118, 152503

- **In-Medium Generator Coordinate Method (IM-GCM)**

- J. M. Yao, J. Engel, L. J. Wang, C. F. Jiao, HH PRC 98, 054311 (2018)
- J. M. Yao et al., PRL 124, 232501 (2020)

XYZ  
define  
reference

IMSRG  
evolve  
operators

XYZ  
extract  
observables

# Merging IMSRG and CI: Valence-Space IMSRG

## **Review:**

S. R. Stroberg, HH, S. K. Bogner, and J. D. Holt, *Ann. Rev. Part. Nucl. Sci.* **69**, 165 (2019)

## **Full CI:**

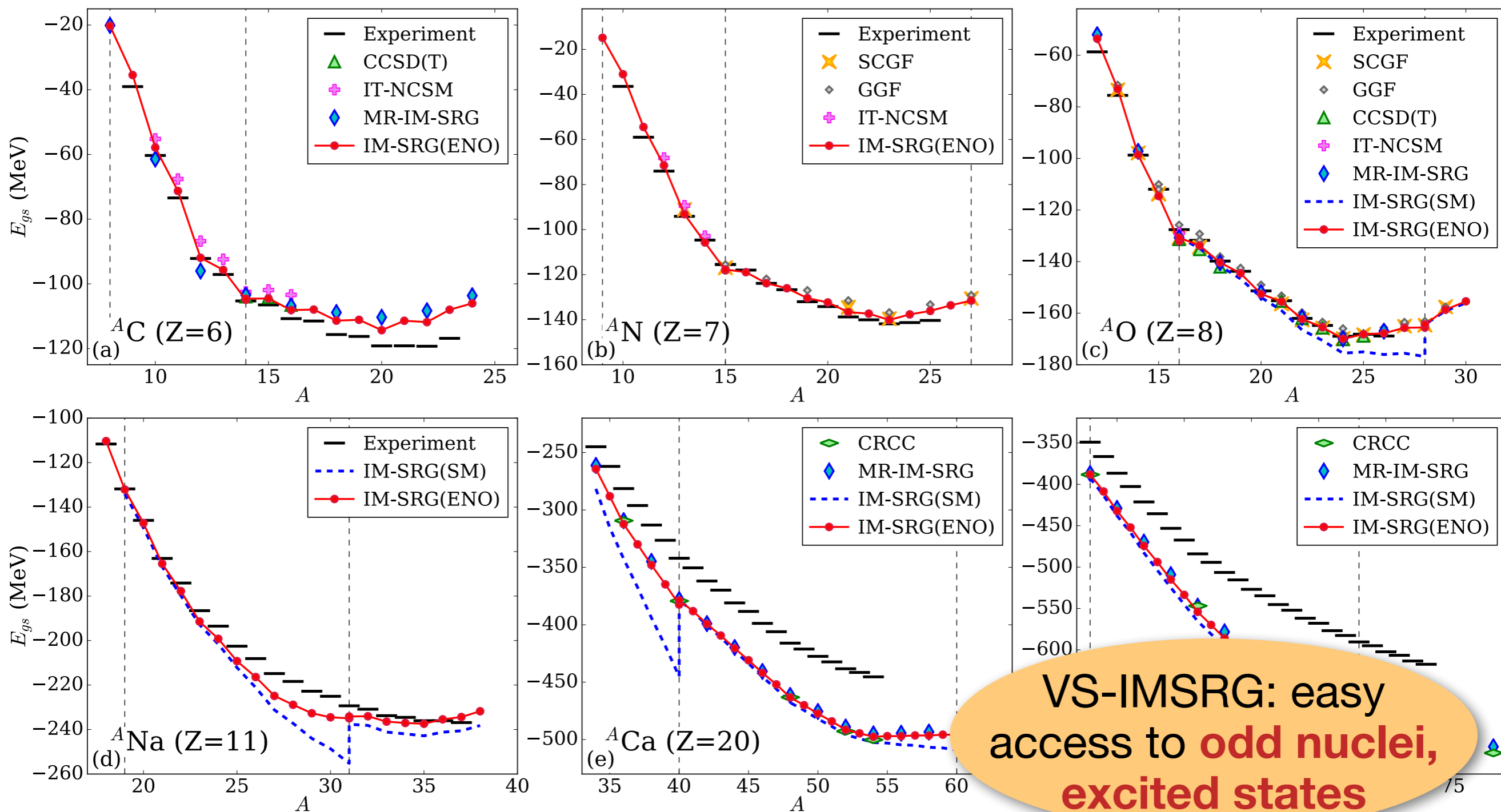
E. Gebrerufael, K. Vobig, HH, and R. Roth, *Phys. Rev. Lett.* **118**, 152503 (2017)

# Ground-State Energies



S. R. Stroberg, A. Calci, HH, J. D. Holt, S. K. Bogner, R. Roth, A. Schwenk, *PRL* **118**, 032502 (2017)

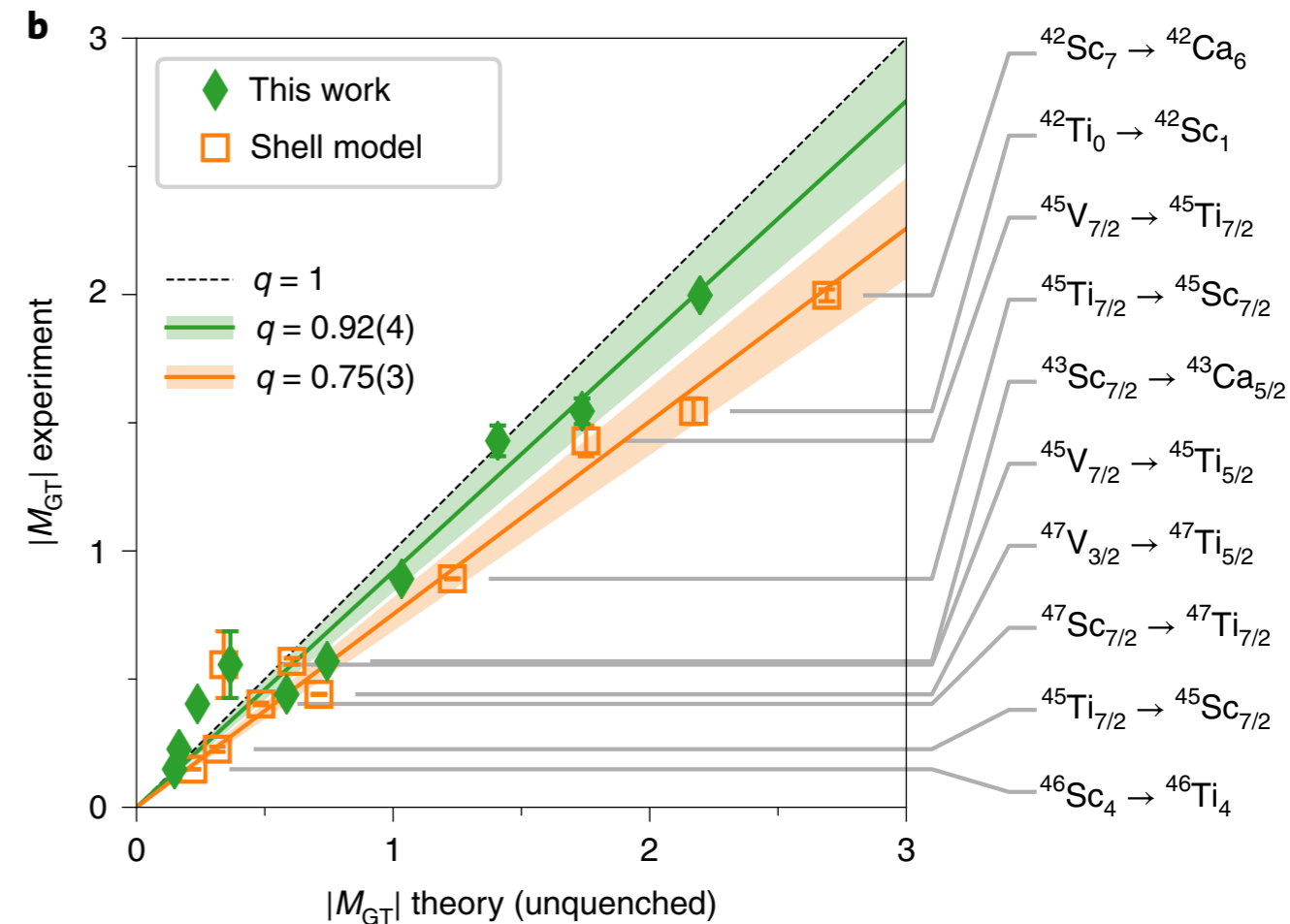
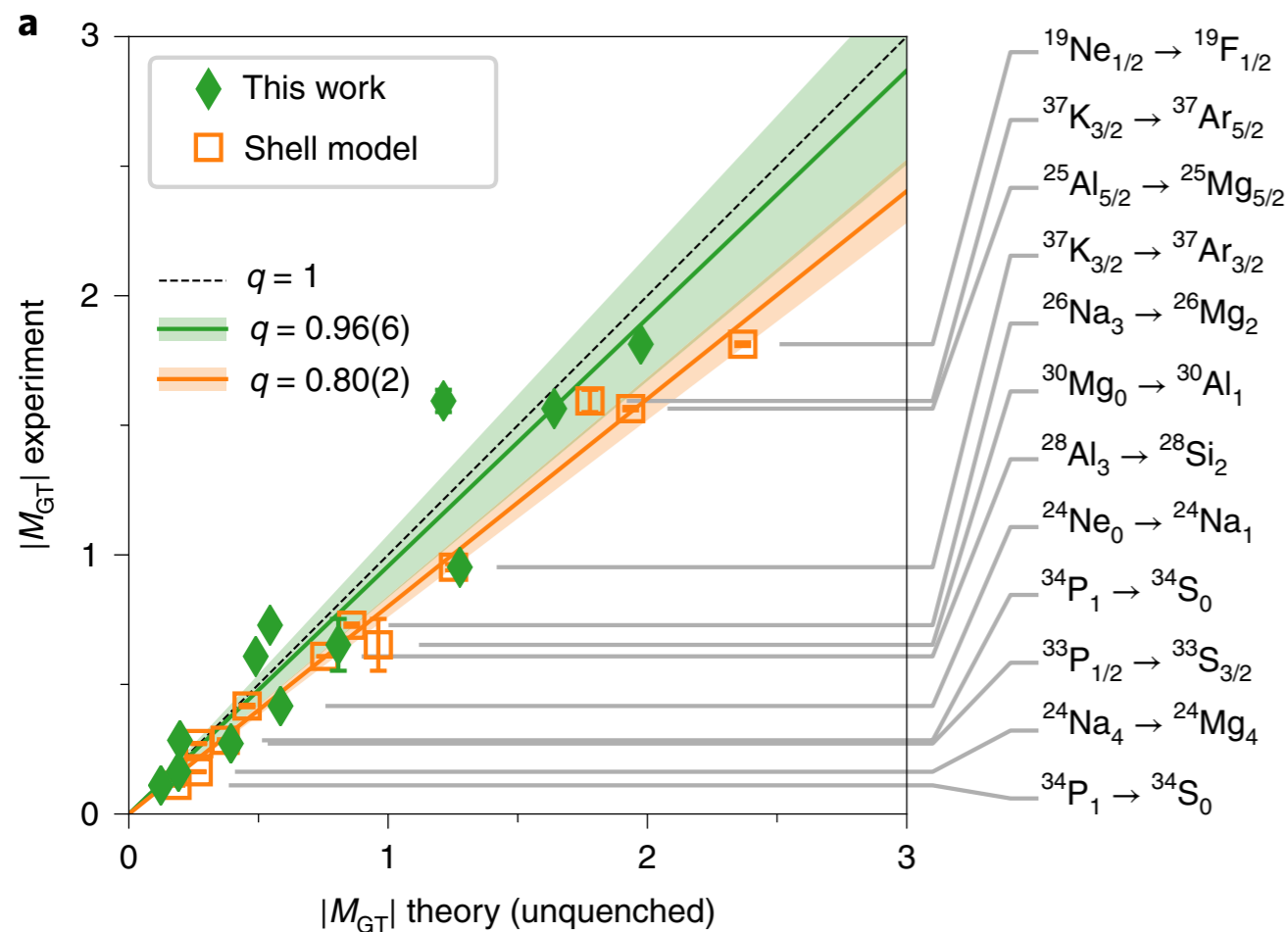
S. R. Stroberg, HH, S. K. Bogner, J. D. Holt, *Ann. Rev. Part. Nucl. Sci.* **69**, 307 (2019)



# Quenching of Gamow-Teller Decays



*P. Gysbers et al., Nature Physics 15, 428 (2019)*



- **empirical Shell model** calculations require **quenching factors** of the weak axial-vector coupling  $g_A$
- **VS-IMSRG** explains this through consistent **renormalization** of transition operator, incl. **two-body currents**

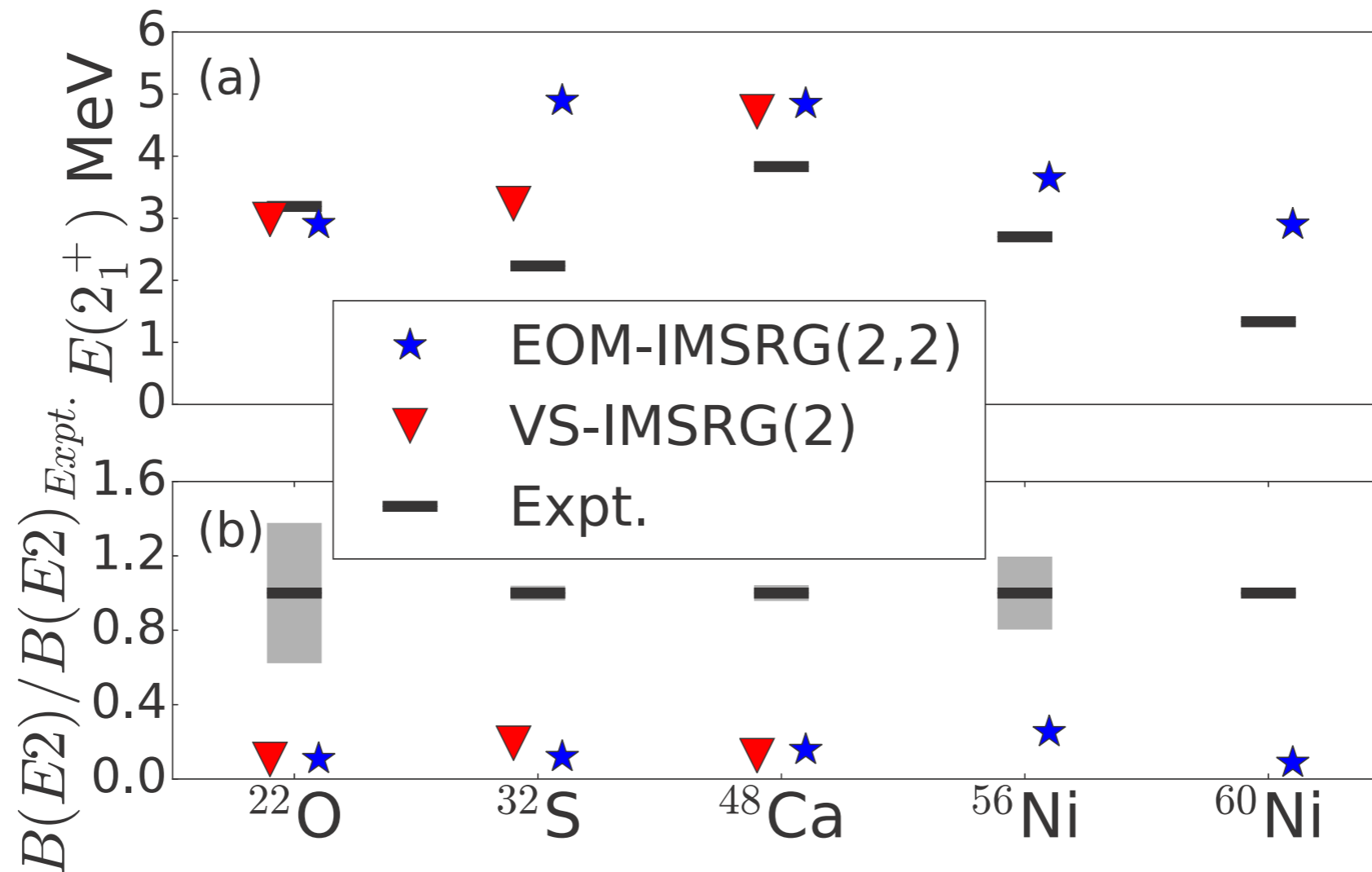
# Transitions



N. M. Parzuchowski, S. R. Stroberg et al., *PRC* **96**, 034324

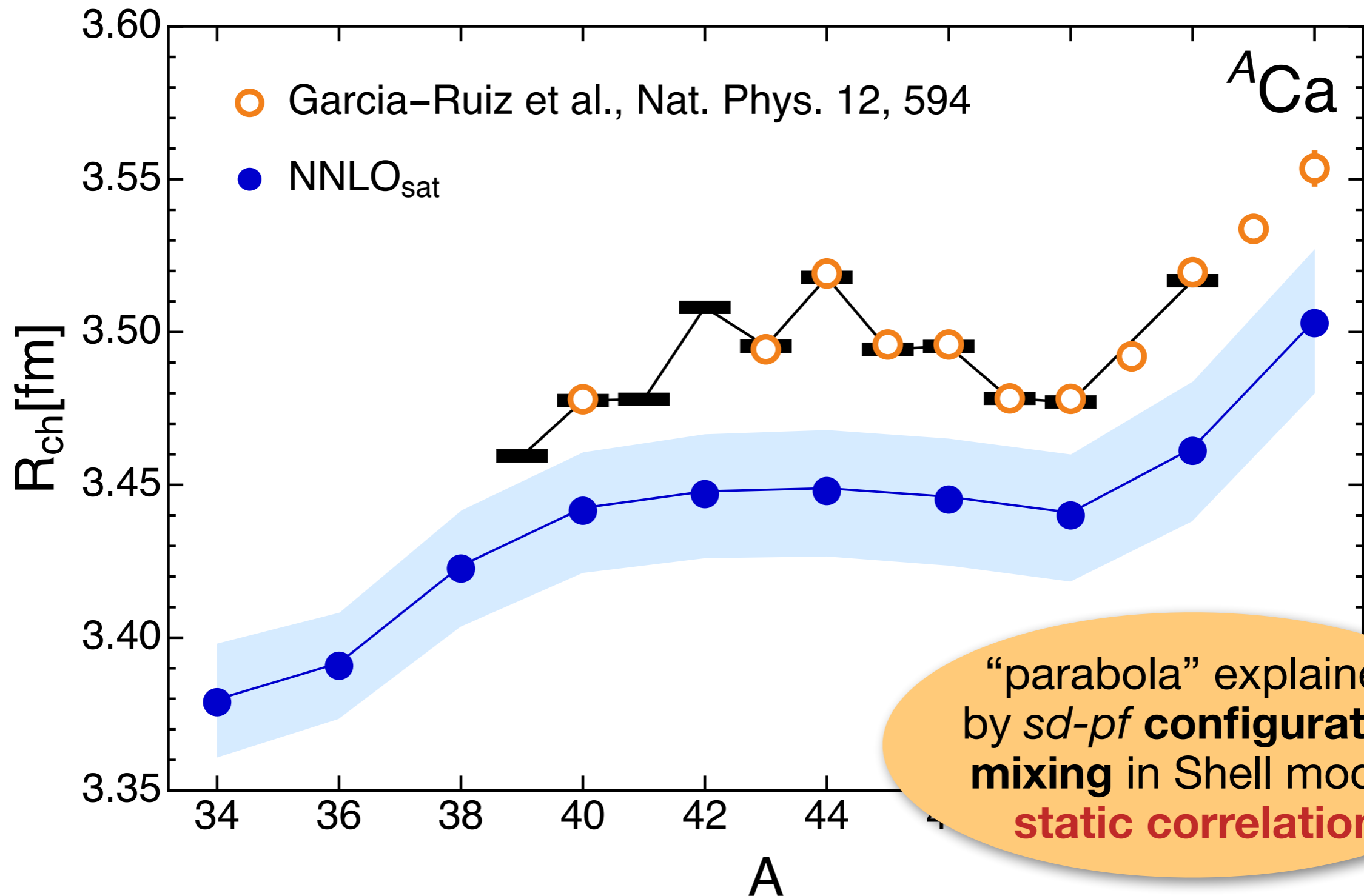
S. R. Stroberg, HH, S. K. Bogner, J. D. Holt, *Ann. Rev. Part. Nucl. Sci.* **69**, 307 (2019)

S. R. Stroberg et al. *PRC* **105**, 034333 (2022)



- **$B(E2)$  much too small:** missing collectivity due to intermediate 3p3h, ... states that are truncated in IMSRG evolution (**static correlation**)

# Calcium Isotopes



# Capturing Collective Correlations: In-Medium Generator Coordinate Method

J. M. Yao, A. Belley, R. Wirth, T. Miyagi, C. G. Payne, S. R. Stroberg, HH, J. D. Holt, PRC **103**, 014315 (2021)

J. M. Yao, B. Bally, J. Engel, R. Wirth, T. R. Rodriguez, HH, PRL **124**, 232501 (2020)

J. M. Yao, J. Engel, L. J. Wang, C. F. Jiao, H. H., PRC **98**, 054311 (2018)

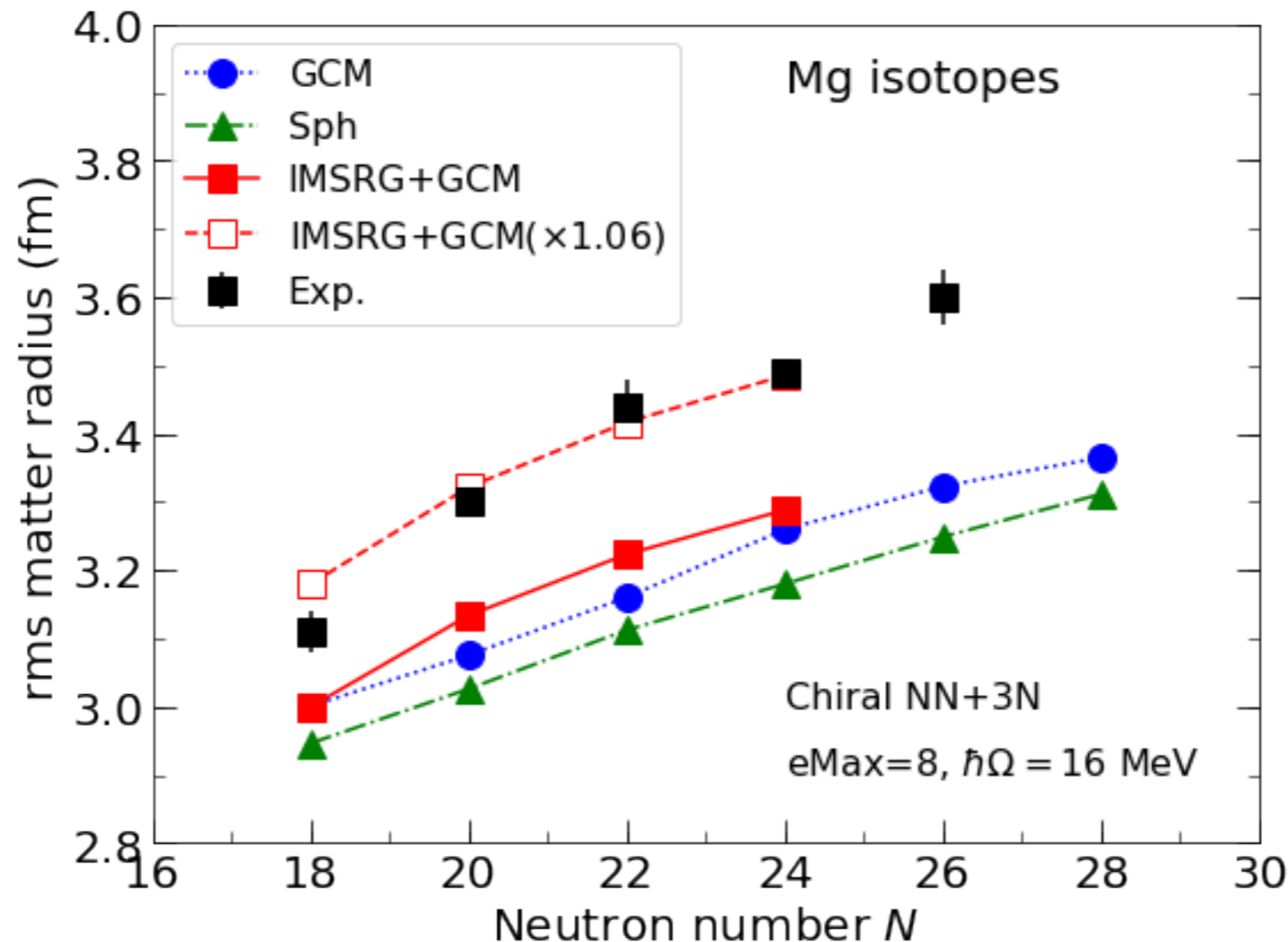
HH, J. M. Yao, T. D. Morris, N. M. Parzuchowski, S. K. Bogner and J. Engel, J. Phys. Conf. Ser. 1041, 012007 (2018)



# Magnesium Isotopes



J. M. Yao, HH, in preparation

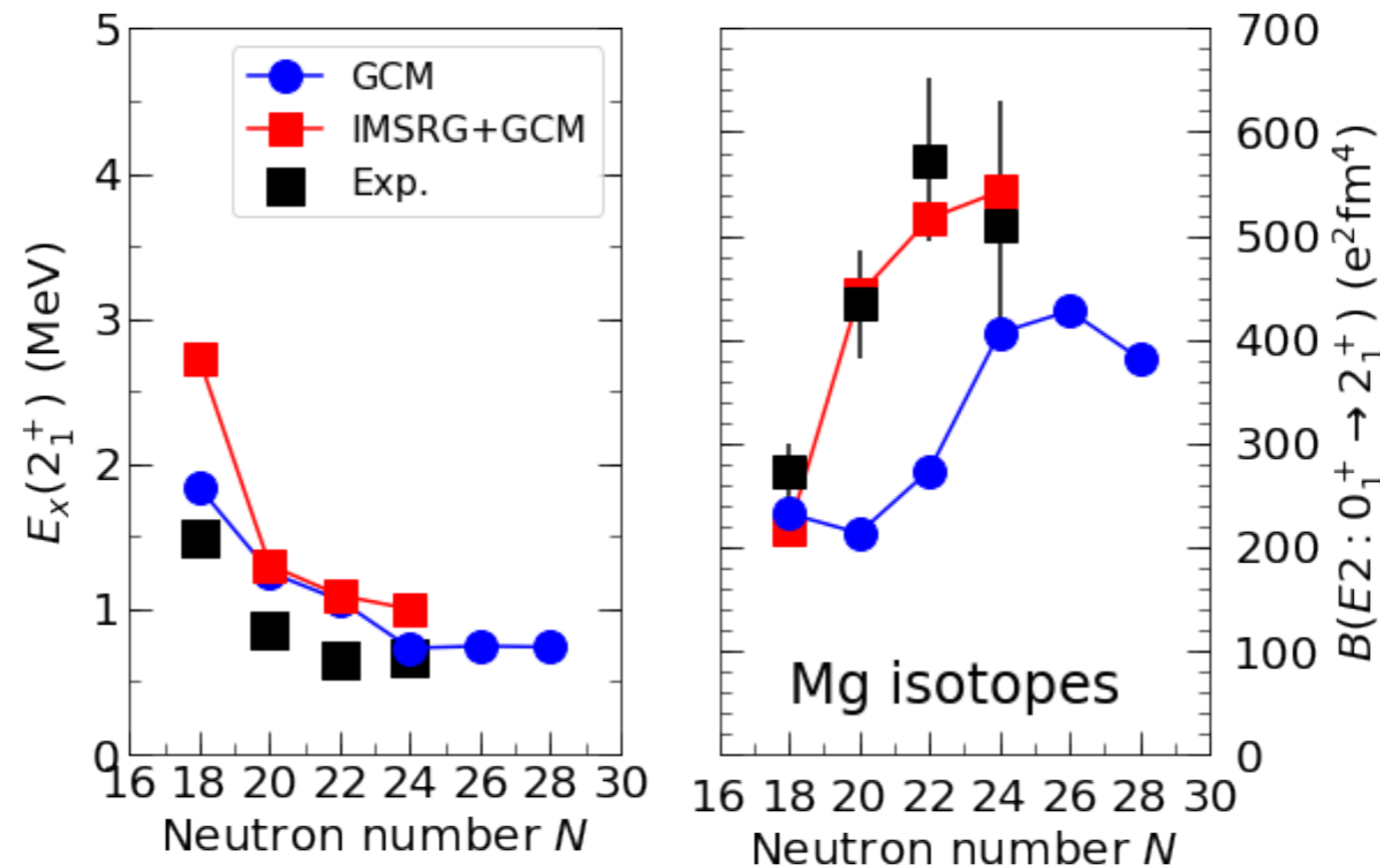


- note **improvement of rms radius trend** from IM-GCM
- global shifts (and/or rotation around “pivot”) often associated with cutoff dependence of interactions

# Magnesium Isotopes



J. M. Yao, HH, in preparation

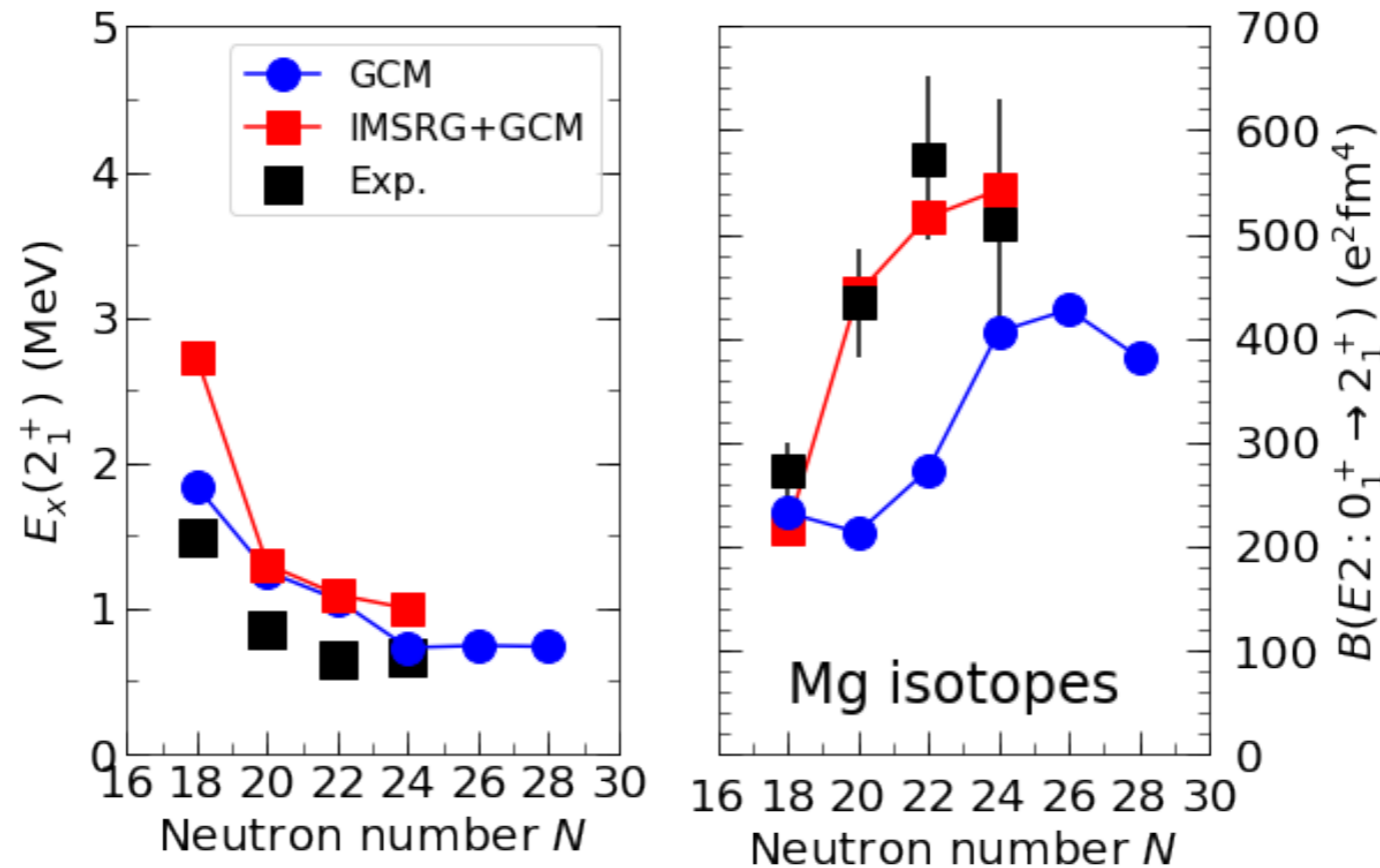


- much **improved  $B(E2)$**  values compared to standard GCM or VS-IMSRG calculations: IM-GCM captures **dynamical and static correlations!**

# Magnesium Isotopes



*J. M. Yao, HH, in preparation*



$$O = O^{(1)} \xrightarrow{s \rightarrow \infty} O(s) = O^{(1)}(s) + \underbrace{O^{(2)}(s) + \dots}_{\text{induced contributions}}$$

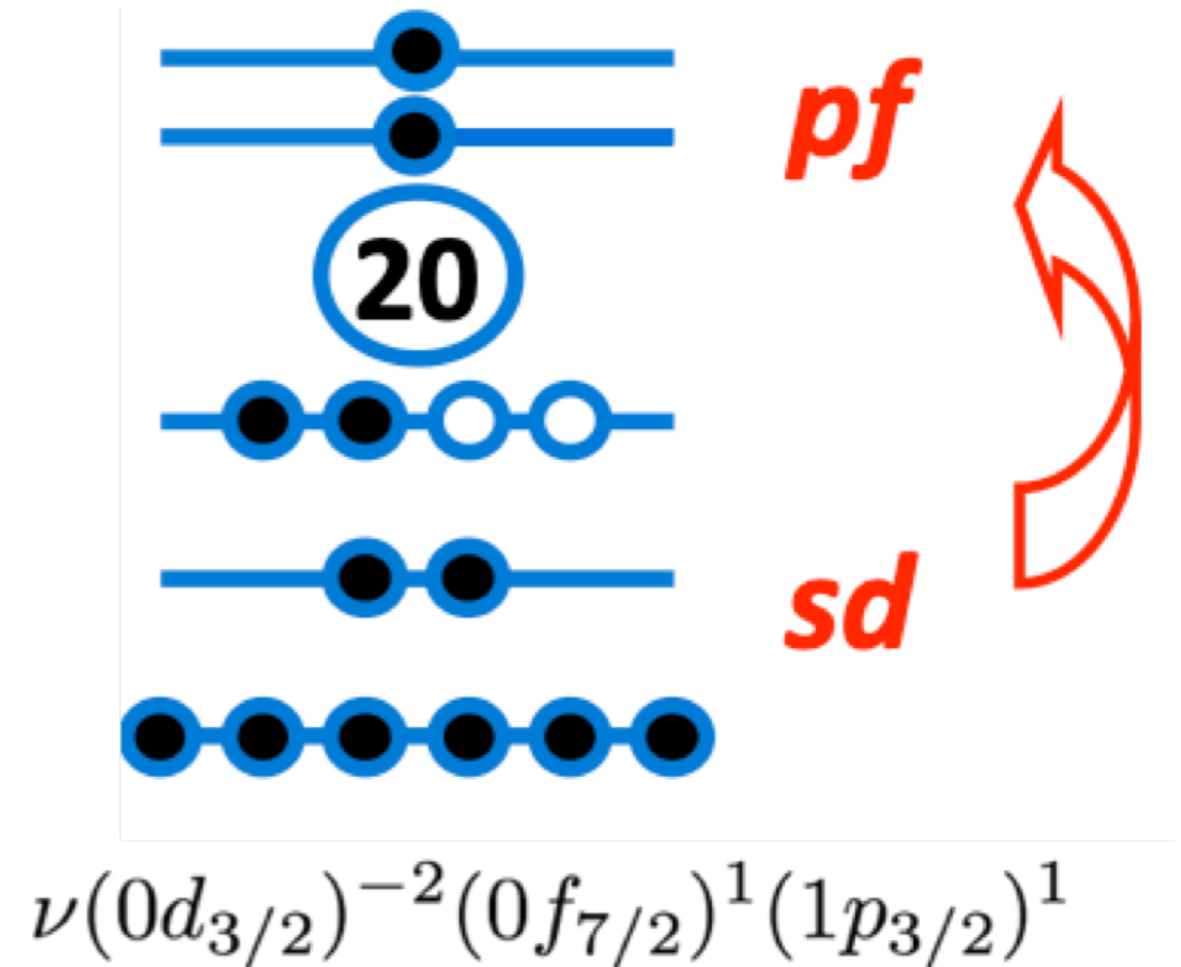
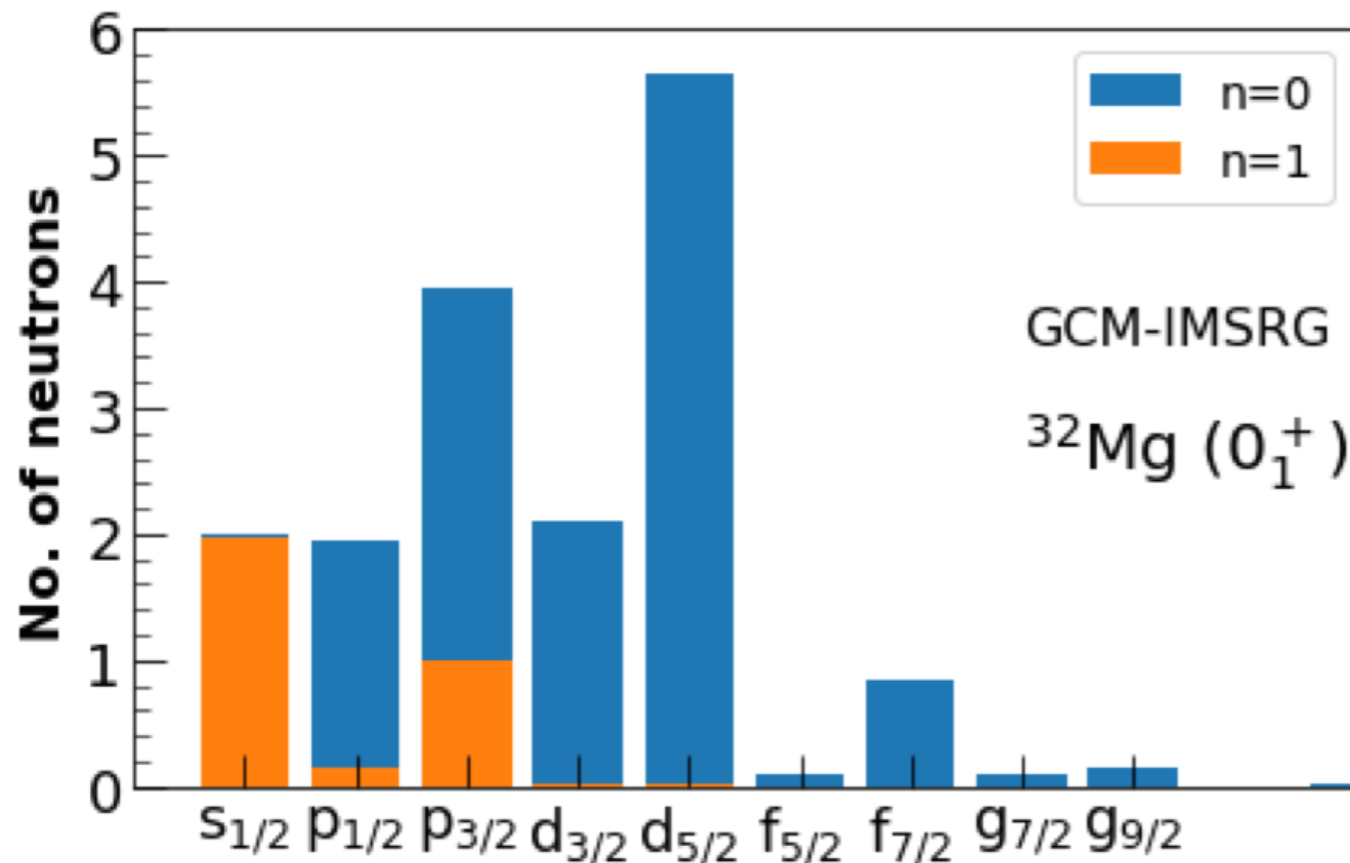
**induced contributions**

- **induced 2B quadrupole operator is small (~5%),** contrary to typical VS-IMSRG (~50%): GCM reference equips operator basis with better capability to capture collectivity

# Collectivity in Magnesium Isotopes



J. M. Yao, HH, in preparation



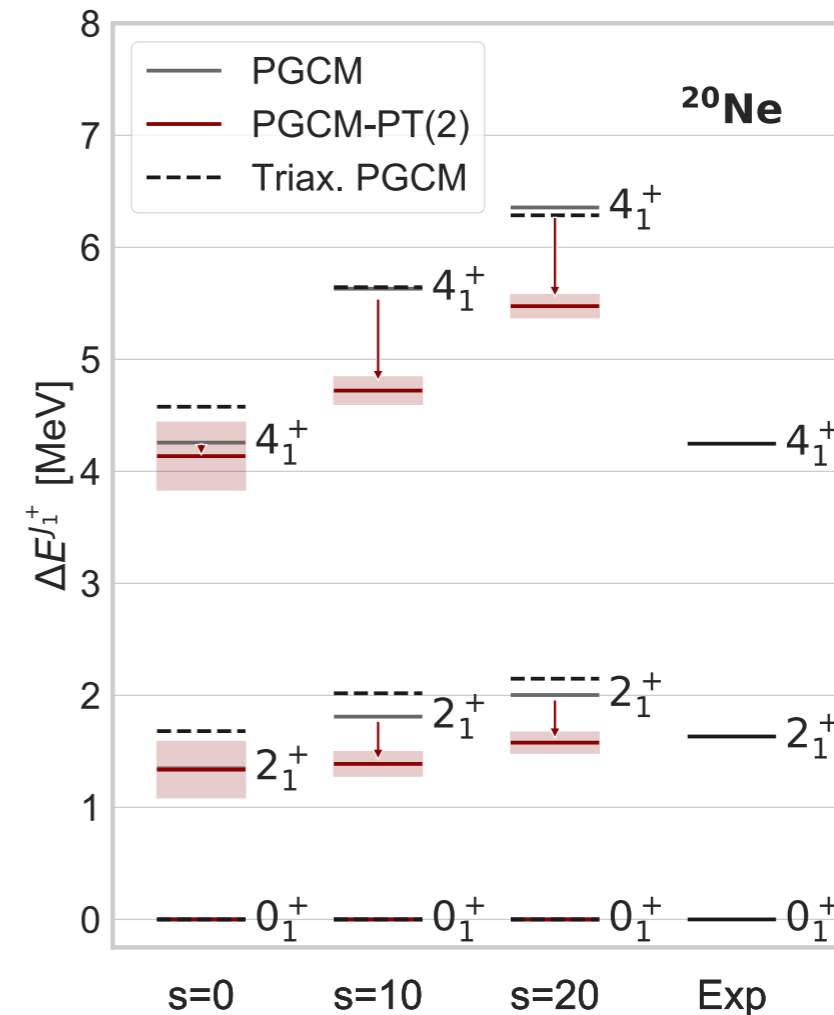
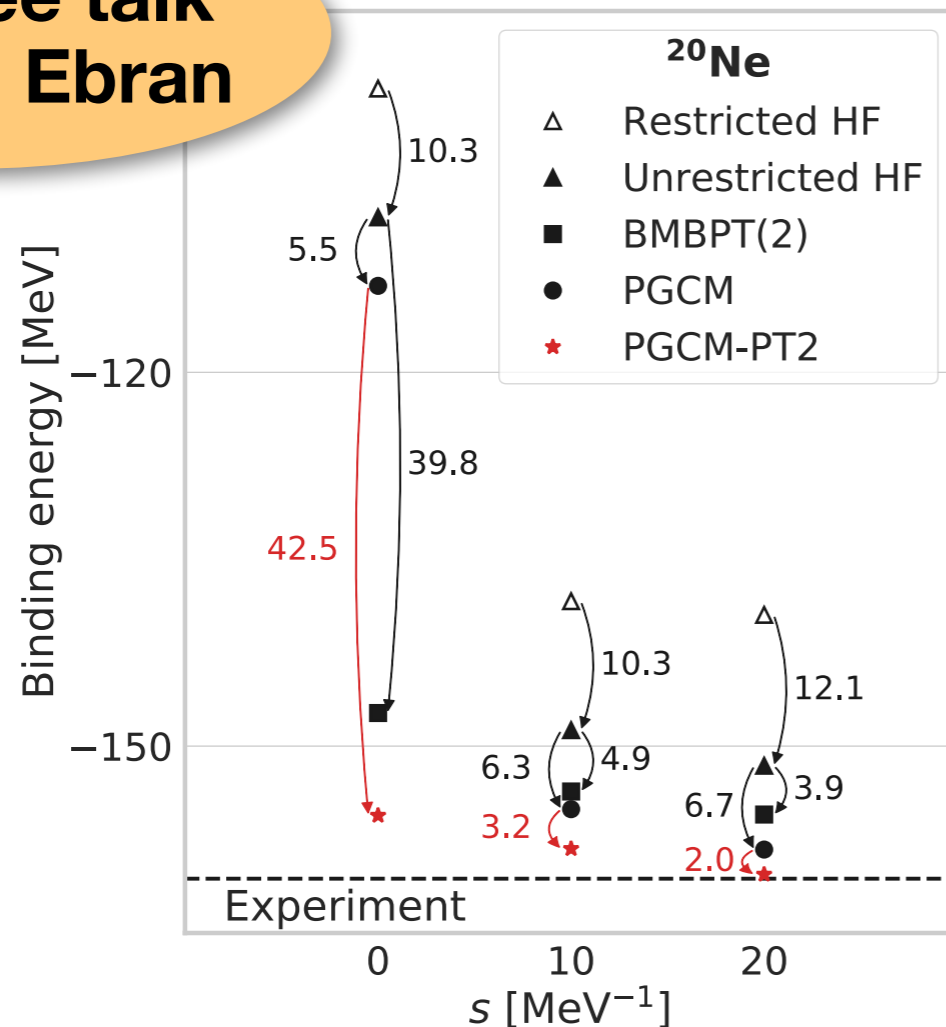
- **Caution:** occupation numbers are **not observables**, interpret with care (scale/scheme dependence)!
- For **low-resolution interactions**, the (no-core) IM-GCM and Shell Model interpretations of  $^{32}\text{Mg}$  are **qualitatively** the same: two neutrons are excited from the sd- into the pf shell.

# Perturbative Enhancement of IM-GCM



M. Frosini et al., EPJA 58, 64 (2022)

also see talk  
by J.-P. Ebran



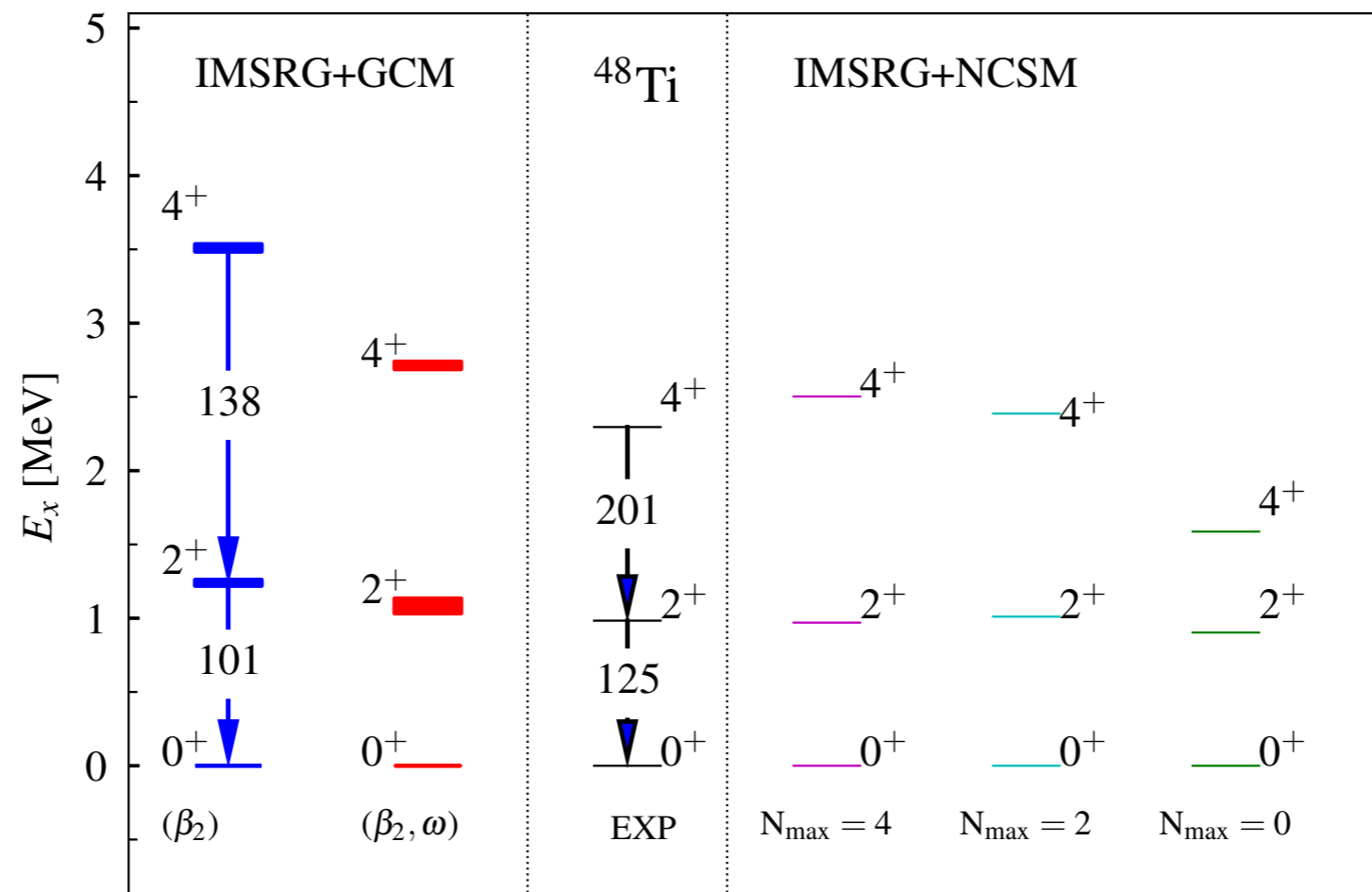
- $s$ -dependence is a **built-in diagnostic tool** for IM-GCM (**not available in phenomenological GCM**)
- if operator and wave function offer sufficient degrees of freedom, evolution of observables is unitary
- need **richer references and/or IMSRG(3)** for certain observables

# IM-GCM: $0\nu\beta\beta$ Decay of $^{48}\text{Ca}$



*J. M. Yao et al., PRL* **124**, 232501 (2020); *HH, Front. Phys.* **8**, 379 (2020)

EM1.8/2.0,  $\hbar\Omega = 16$  MeV

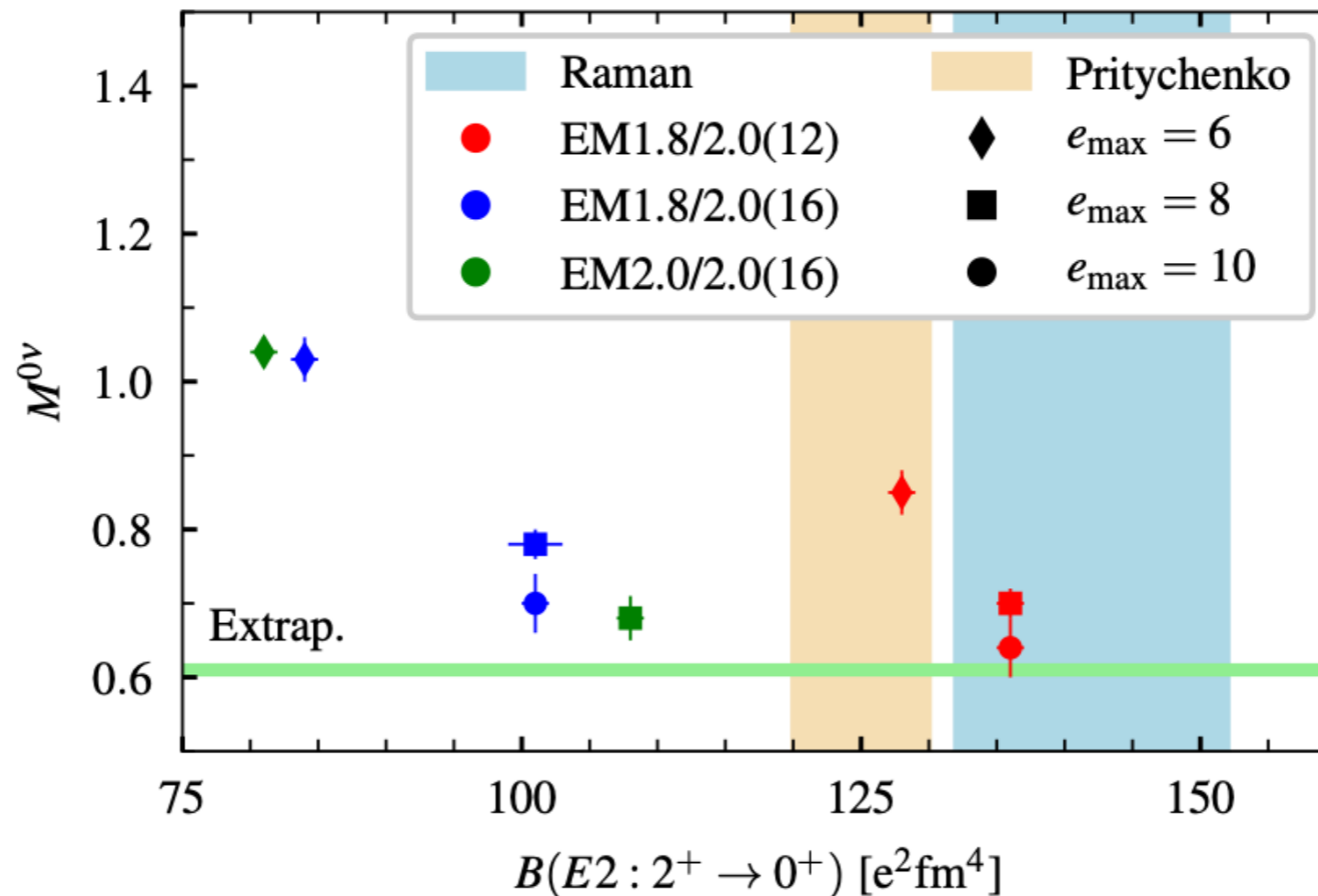


- richer GCM state through **cranking**
- **consistency** between IM-GCM and IM-NCSM

# $0\nu\beta\beta$ Decay of $^{48}\text{Ca}$



*J. M. Yao et al., PRL 124, 232501 (2020); PRC 103, 014315 (2021)*



- NME from different methods **consistent** for consistent interactions & transition operators

(A. Belley et al., PRL 126, 042502, S. Novario et al., PRC 103, 014315 (2021))

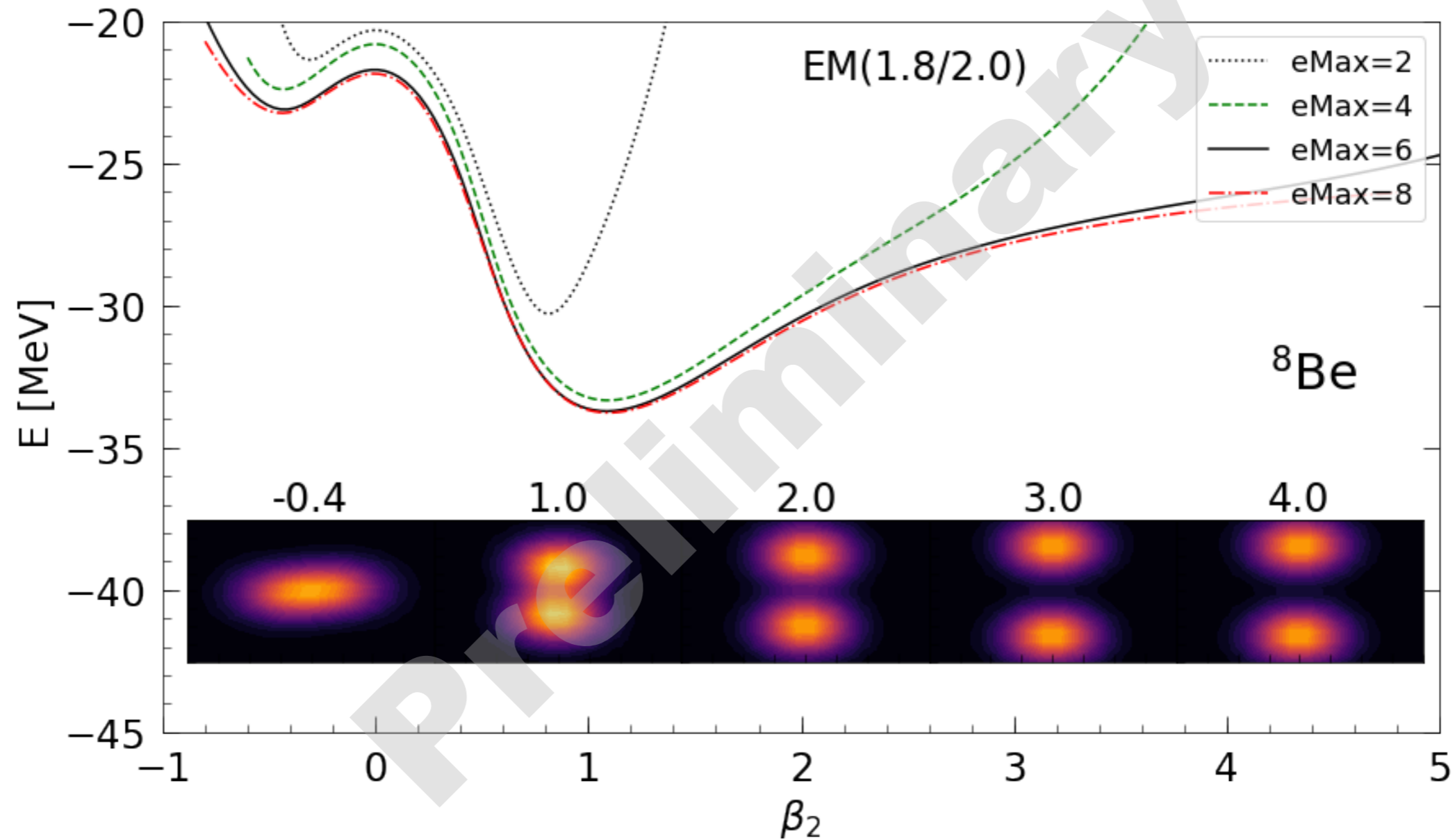
- interpretation and features differ from e.g.  $^{48}\text{Ca}$  only **weak correlation** between NME and  $B(E2)$

**not the full story yet:** improve IMSRG truncations, additional GCM correlations, include currents, ...

# Cluster Structures: $^8\text{Be}$



*J. M. Yao, R. Wirth, HH, in progress*



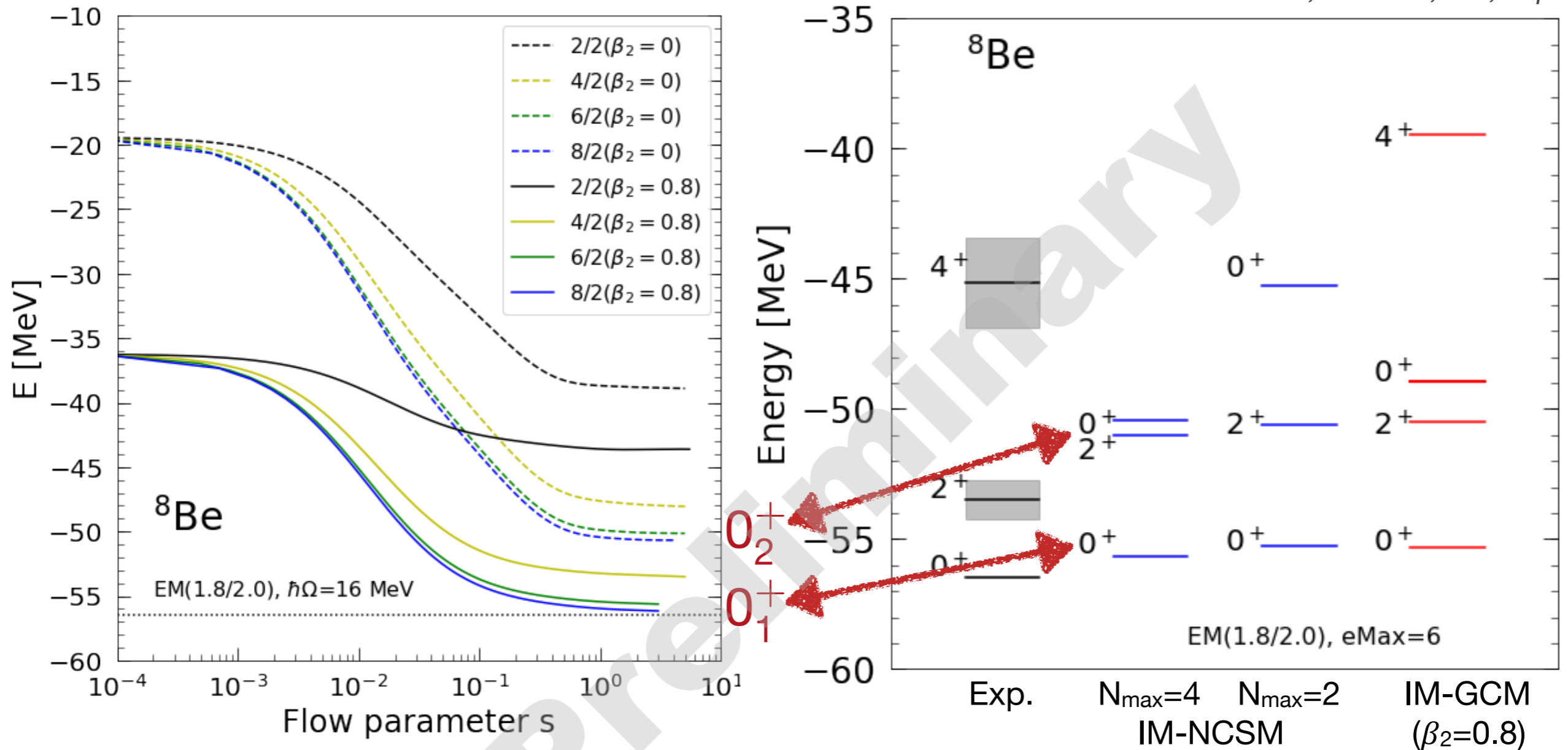
HFB potential energy surface



# Cluster Structures: $^8\text{Be}$



J. M. Yao, R. Wirth, HH, in progress

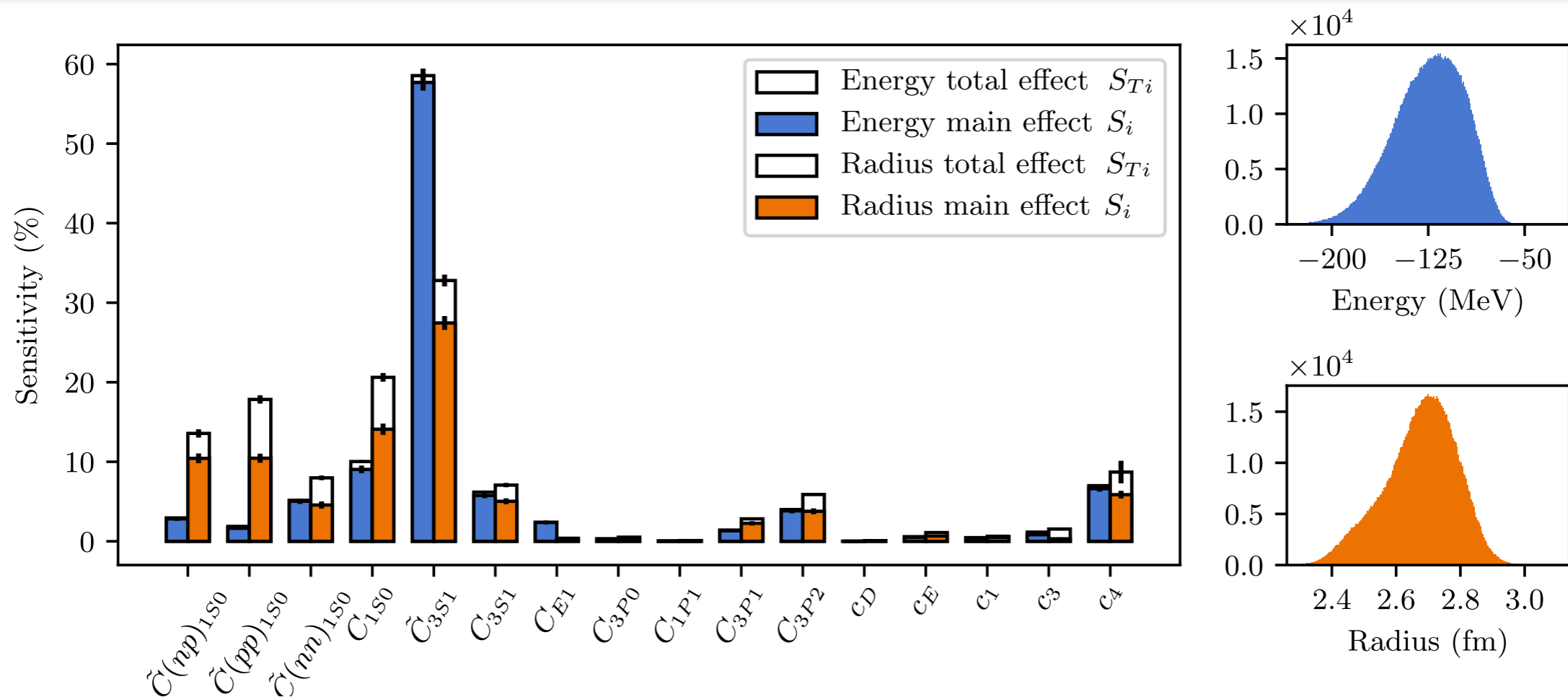


- Spherical and prolate references flow towards **different  $0^+$  states**.
- Consistent with IM-NCSM:
  - **prolate reference:** ground state and excited  $2^+$  state
  - **spherical reference:** first excited  $0^+$

Looking Ahead

- **precision** nuclear physics
  - structure of (exotic and stable) nuclei with **complex deformations**: shape coexistence, clustering, halos, ...
  - inputs for fundamental symmetry programs: **neutrinoless double beta decay, EDMs / Schiff moments**, beta decay for unitarity tests...
- **explore opportunities with heavy ion collisions**
  - **caveat**: needs careful assessment of scale and scheme dependences
- **Uncertainty Quantification / Sensitivity Analysis**
  - identify strongly constraining nuclear observables (usually difficult to compute)
  - need surrogate models/emulators (**model reduction**)

# Surrogate Models



Sensitivity of  $^{16}\text{O}$  ground-state energy to variations of chiral LECs (through NNLO) [Ekström & Hagen, PRL 123, 252501]

**Emulators from eigenvector continuation:**

D. Frame et al., PRL 121, 032501 (2017); S. König et al., PLB 810, 135814 (2018); ...

**“Reviews” of model reduction for nuclear physicists:**

E. Bonilla et al., PRC 106, 054322 (2022); J. Melendez et al., JPG 49, 102001 (2022)

- **predictive *ab initio* theory** with **systematic uncertainties** & convergence to exact result
- expanding capabilities: spectra, radii, transitions, **clustering**, bridge to **dynamics /reactions...**
- **scalable** techniques and codes: from day-to-day data analysis to leadership calculations

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# Postdoctoral Position @ FRIB



- **focus:** extensions of IMSRG Framework and applications (incl. fundamental symmetries)
- broad portfolio of nuclear theory research @ FRIB, great opportunities for collaboration
- 2 years (+ possible renewal)
- Contact me: [hergert@frib.msu.edu](mailto:hergert@frib.msu.edu) ...
- ... or apply directly at <https://careers.msu.edu/en-us/job/513047/research-associatefixed-term>
- **review of applications has started on Jan 30th, but will continue until position is filled**
- **Please encourage suitable candidates to apply!**

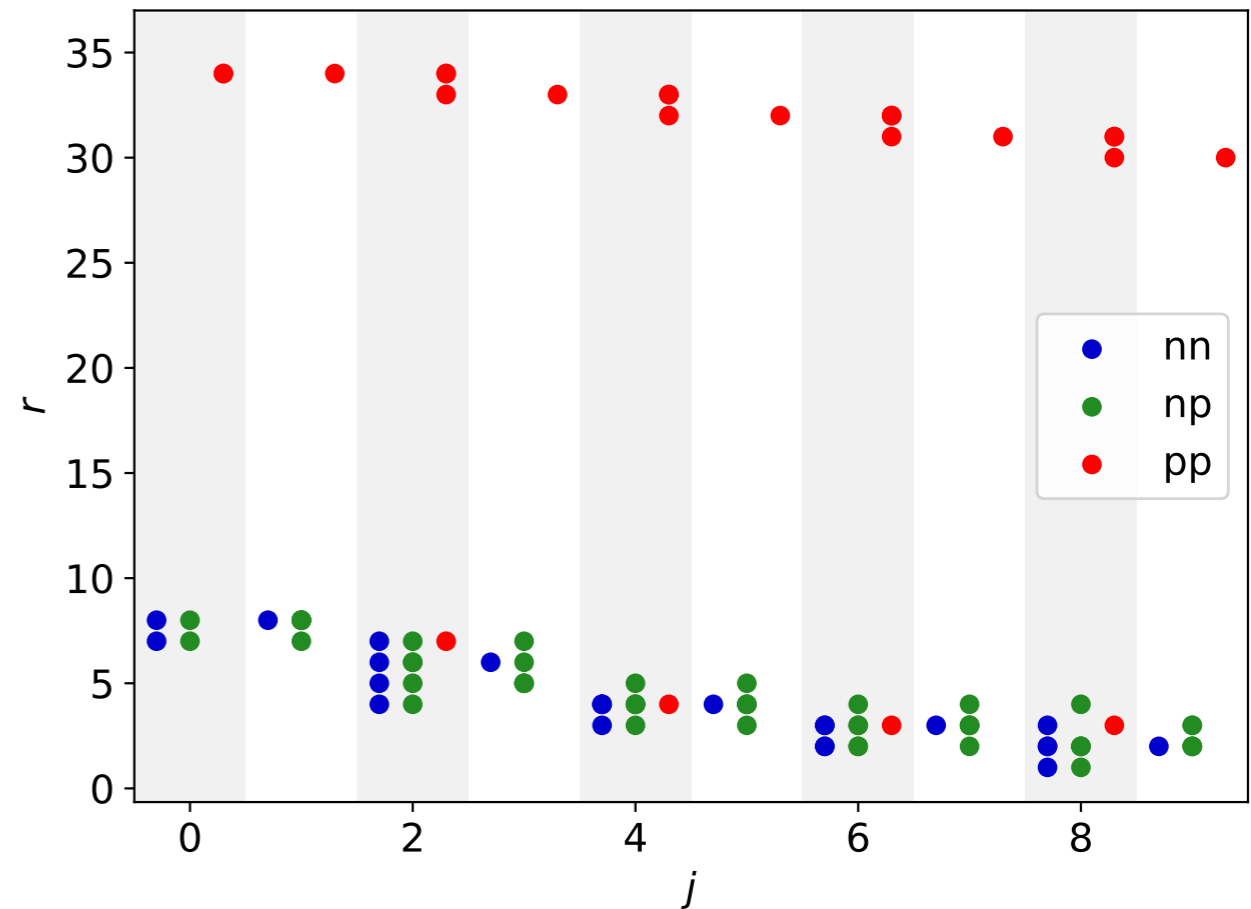
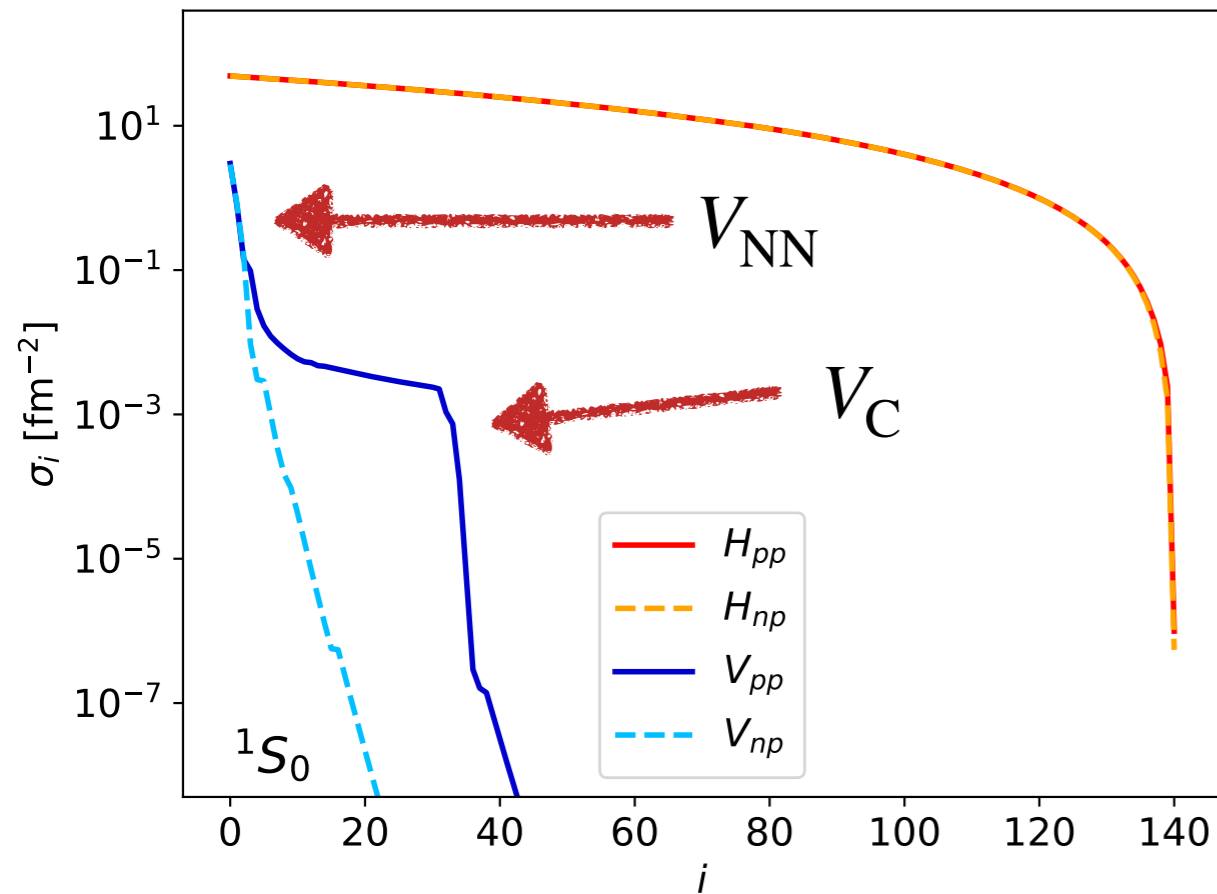
# Supplements



# Factorized Interactions



B. Zhu, R. Wirth, HH, PRC **104**, 044002 (2021)

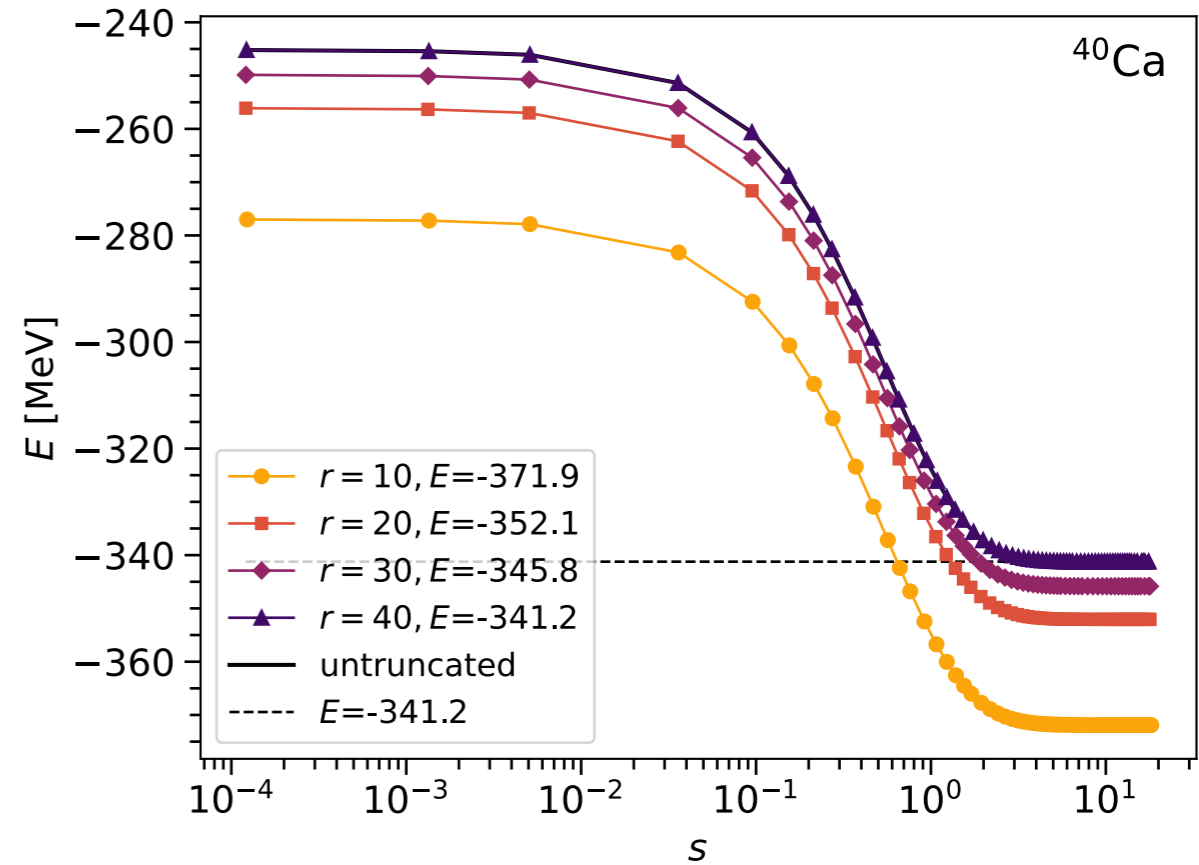
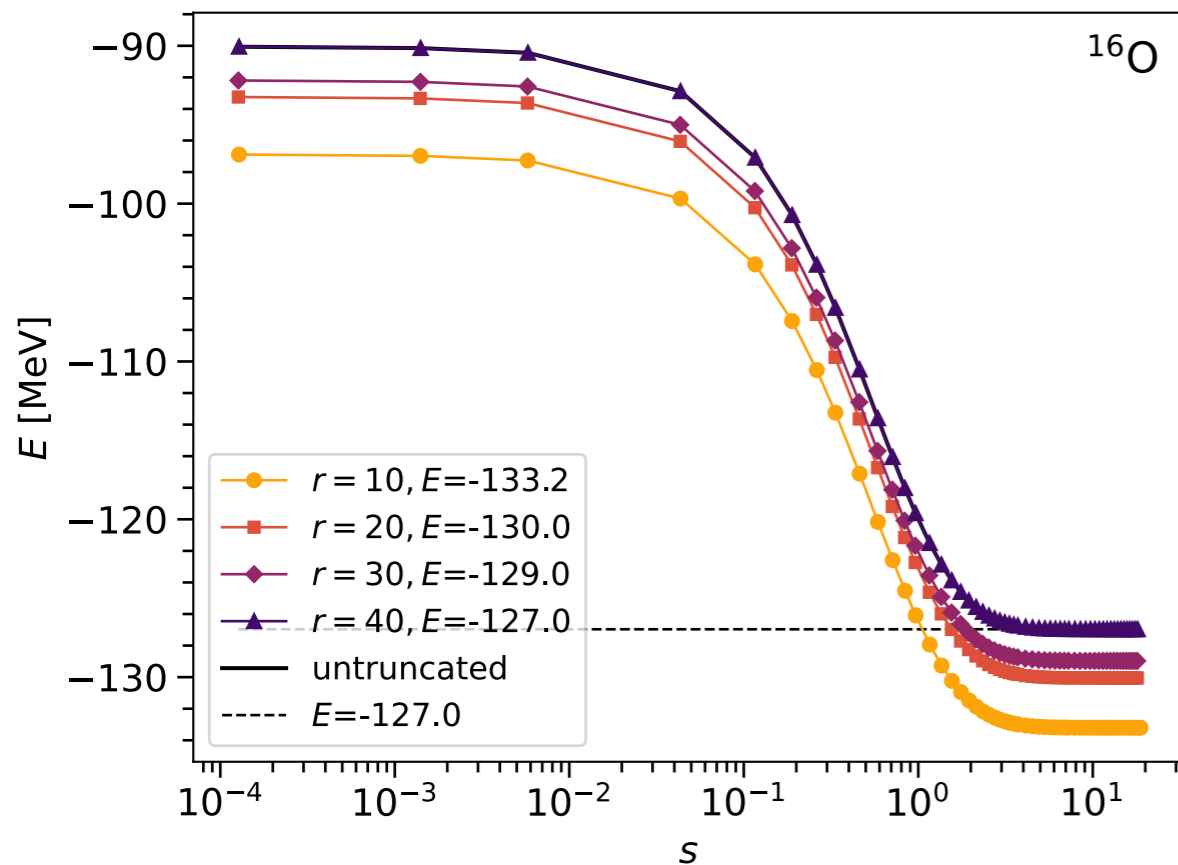


- $O(10)$  operators,  $O(100)$  particles, but  $O(10^8-10^{12})$  flow equations, basis dimension... there must be **redundancy**
- **NN interaction:** 5-10 SVD components (**short range**)
- **Coulomb interaction:** less well-behaved, but  $\sim 25-30$  components sufficient (**long range, no explicit scale**)

# Factorized Interactions



B. Zhu, R. Wirth, HH, PRC 104, 044002 (2021)

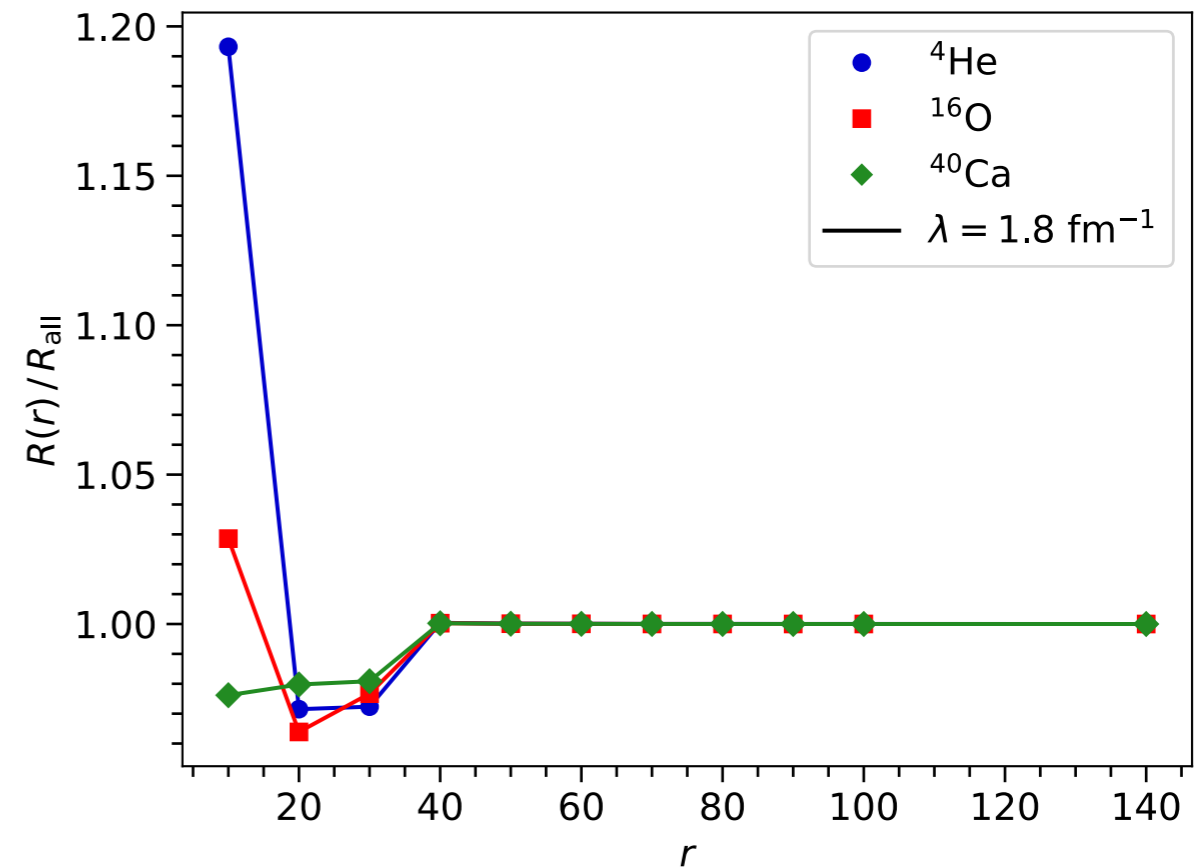
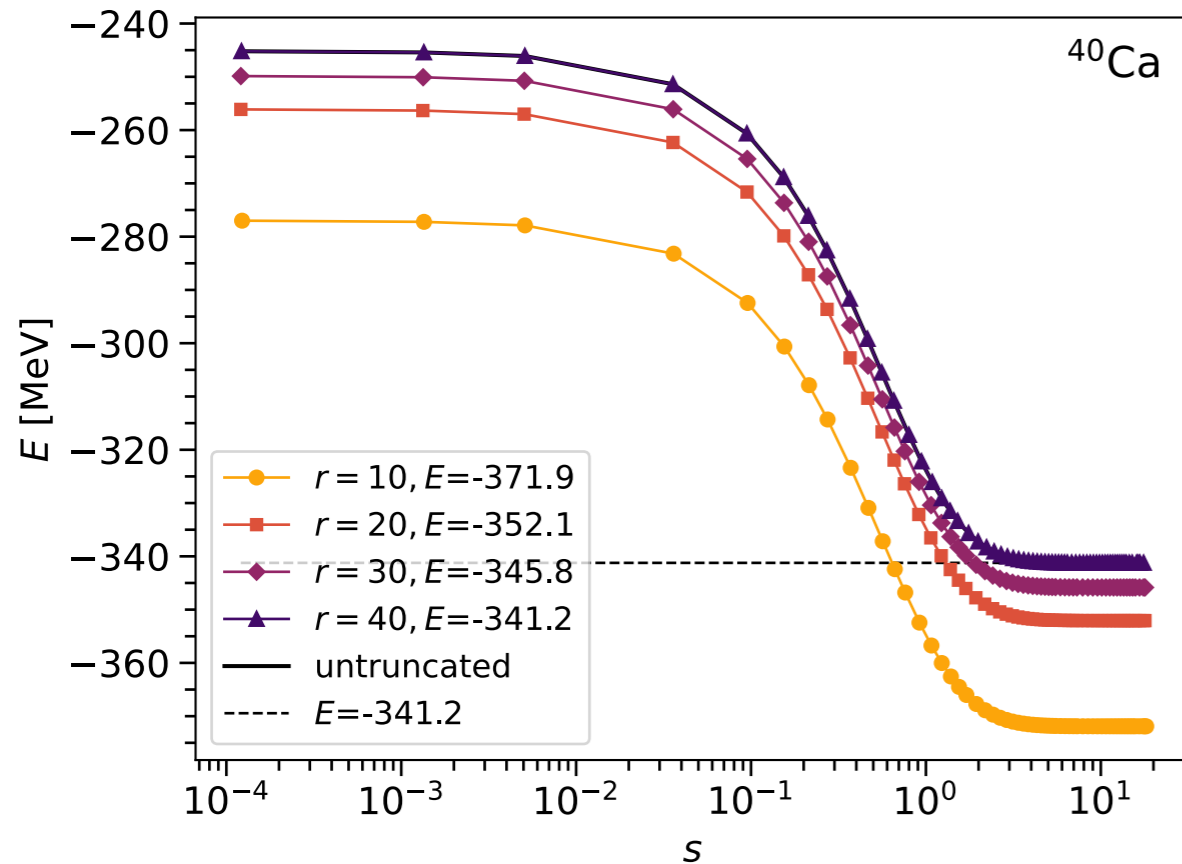


- NN interaction: free-space SRG evolution in component form (**IMSRG not yet**)
  - (3N interaction added to produce realistic binding / radii)
- free-space SRG effort and storage **reduced by several orders of magnitude**

# Factorized Interactions

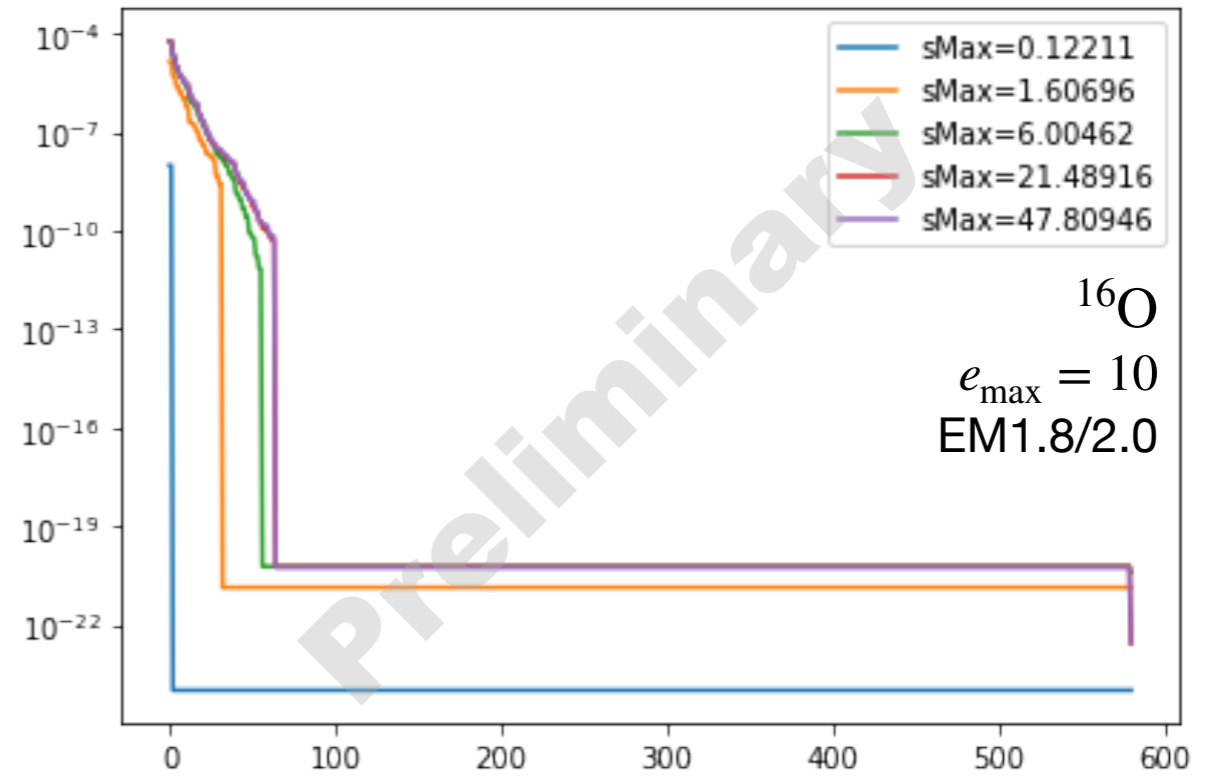
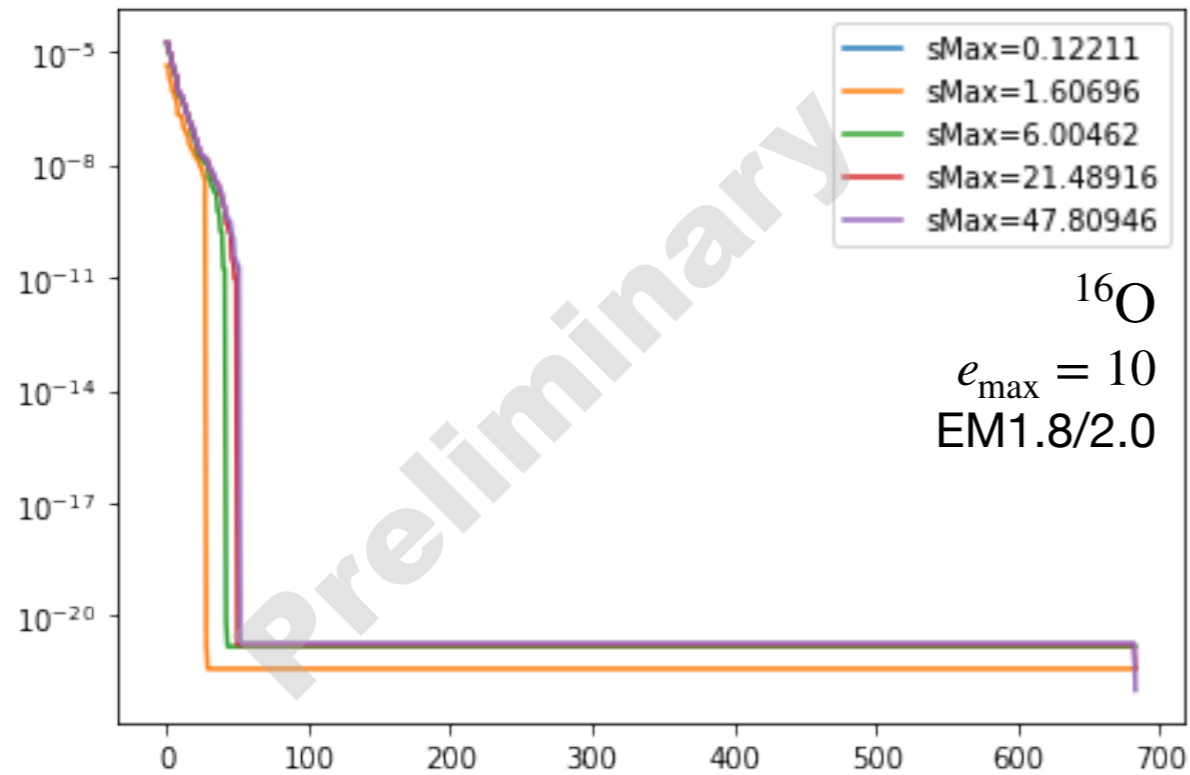


B. Zhu, R. Wirth, HH, PRC **104**, 044002 (2021)

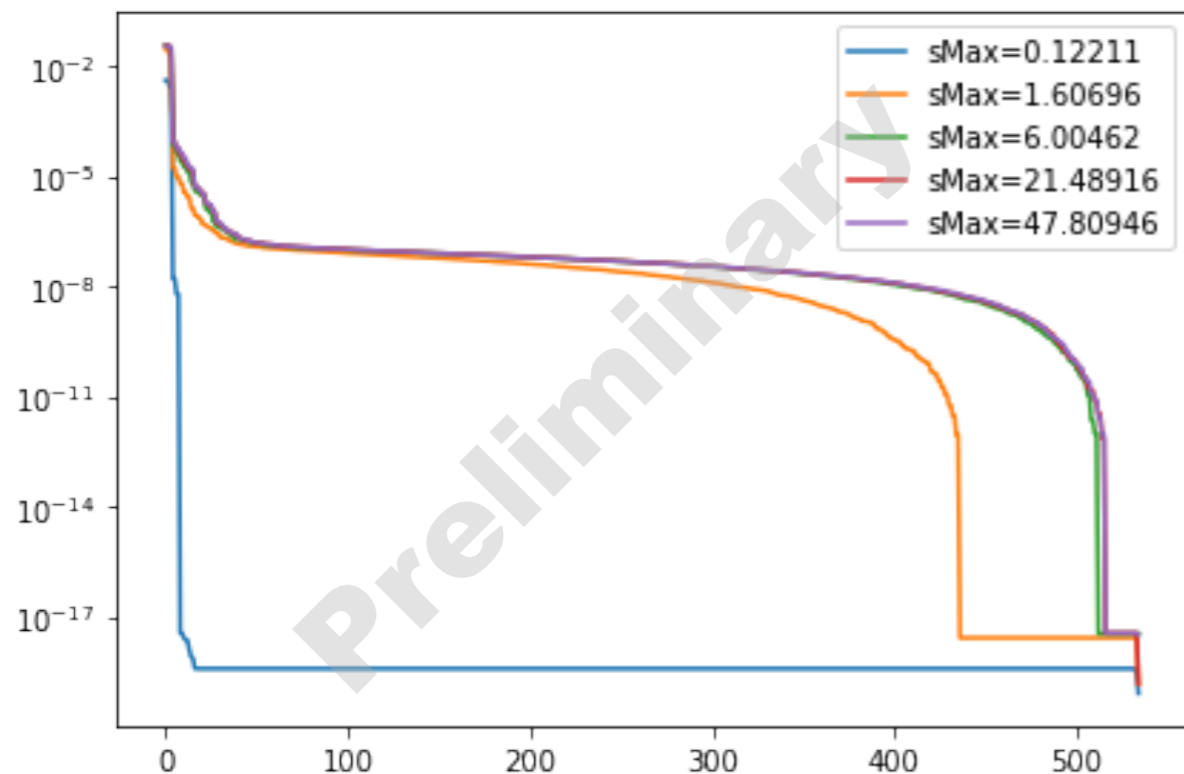


- implementing factorized SRG flow has **no adverse affect** on other observables / expectation values

# SVD for Many-Body Calculation



$^{16}\text{O}, e_{\text{max}} = 10, \text{EM1.8/2.0}$



- **Magnus-IMSRG:**

$$U(s) = e^{\Omega(s)}$$

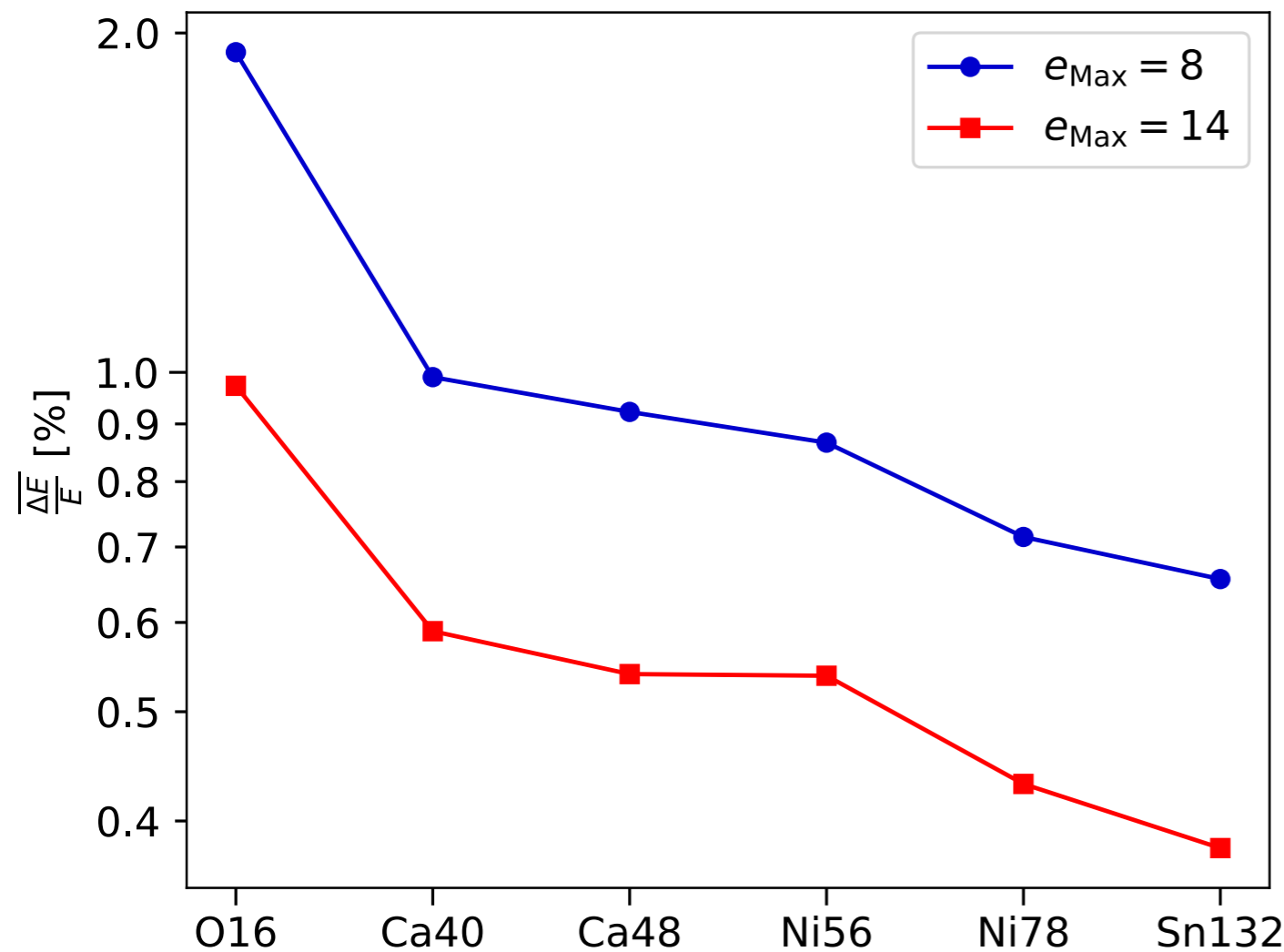
- SVD reveals that  $\Omega(s)$  **has a low rank**

# Compression with Random Projections



A. Zare, R. Wirth, C. Haselby, HH, M. Iwen, arXiv:2211.01315

EM1.8/2.0 NN+3N, MBPT(2),  $c_{\text{tot}} < 10^{-3}$



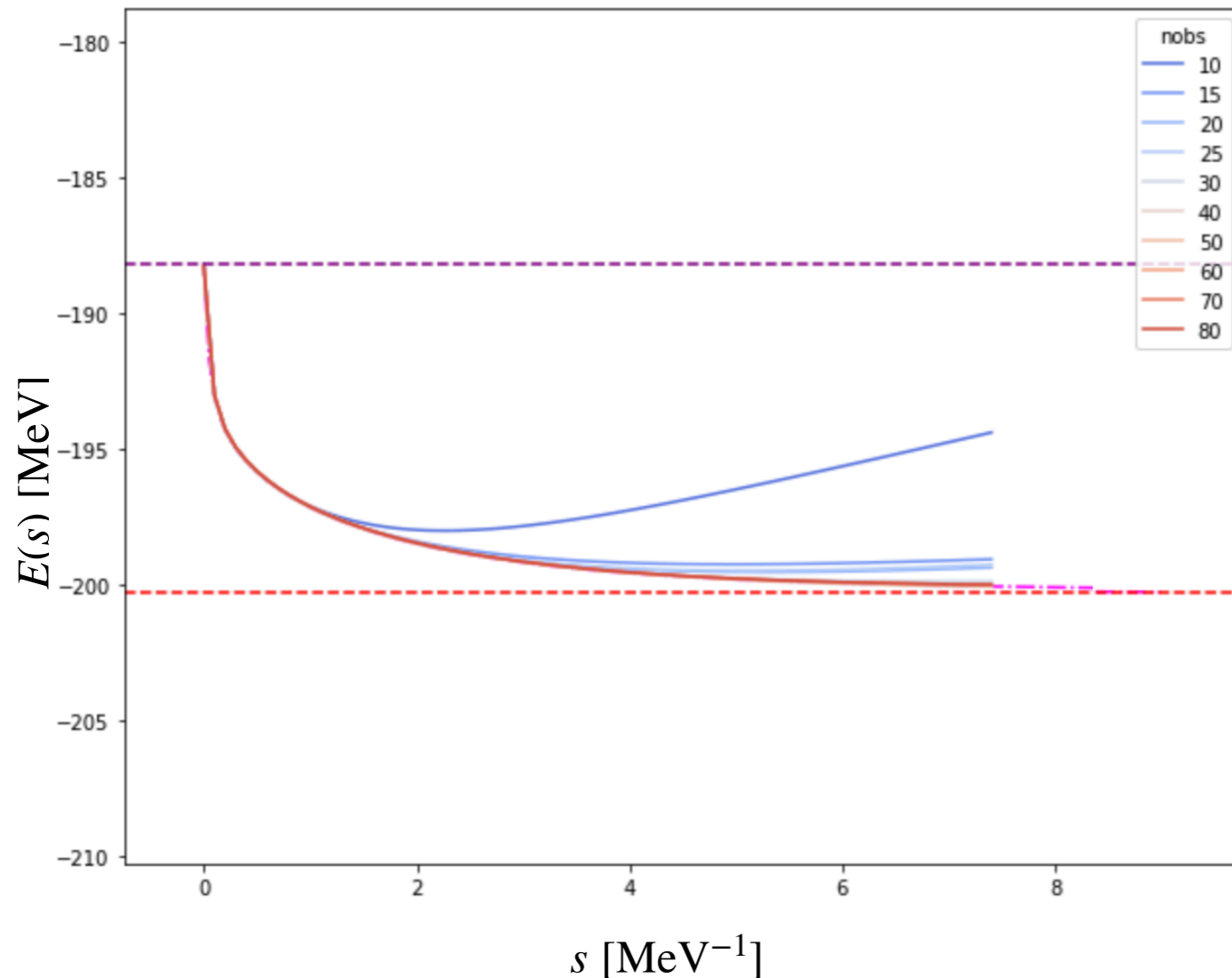
- tensorial (= modewise) **Johnson-Lindenstrauss embeddings**
- purely based on **features of (sparse) big data sets** - integrate with physics-based ideas?
- suitable for **streaming** transforms: compress on the fly while reading from disk

# Emulating IMSRG Flows



*J. Davison, J. Crawford, S. Bogner, HH, in preparation*

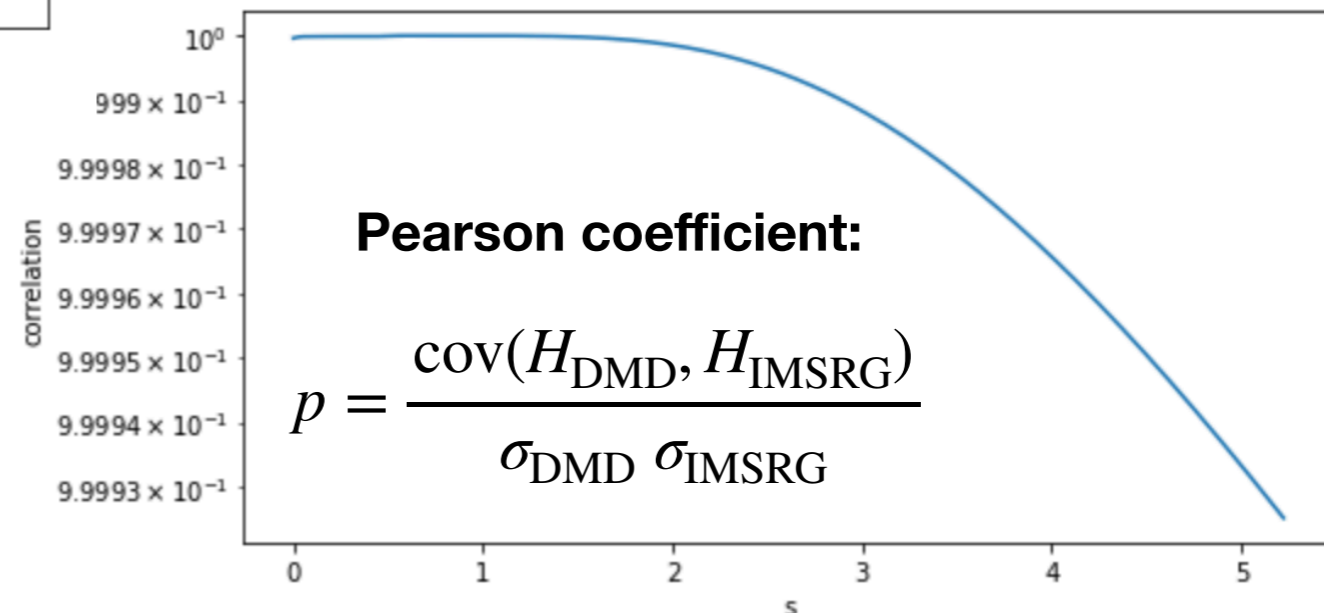
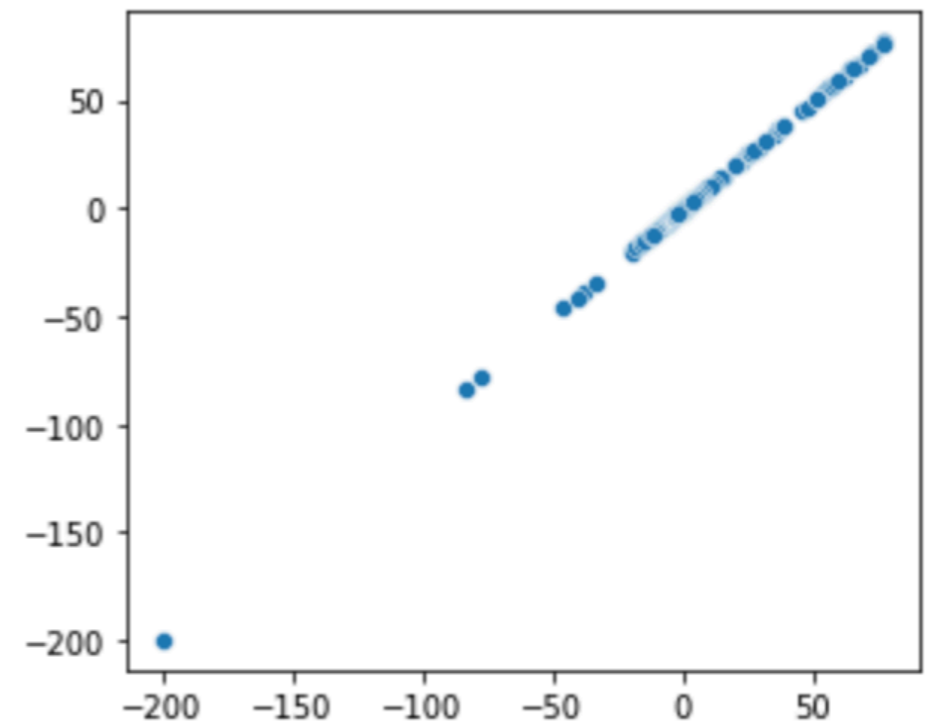
EM(500) N<sup>3</sup>LO,  $\lambda = 2.0 \text{ fm}^{-1}$



Dynamic Mode Decomposition emulator “learns” **all flowing operator coefficients** from snapshots!

$H_{\text{DMD}}(s)$  vs.  $H_{\text{IMSRG}}(s)$

$s = 5.25$

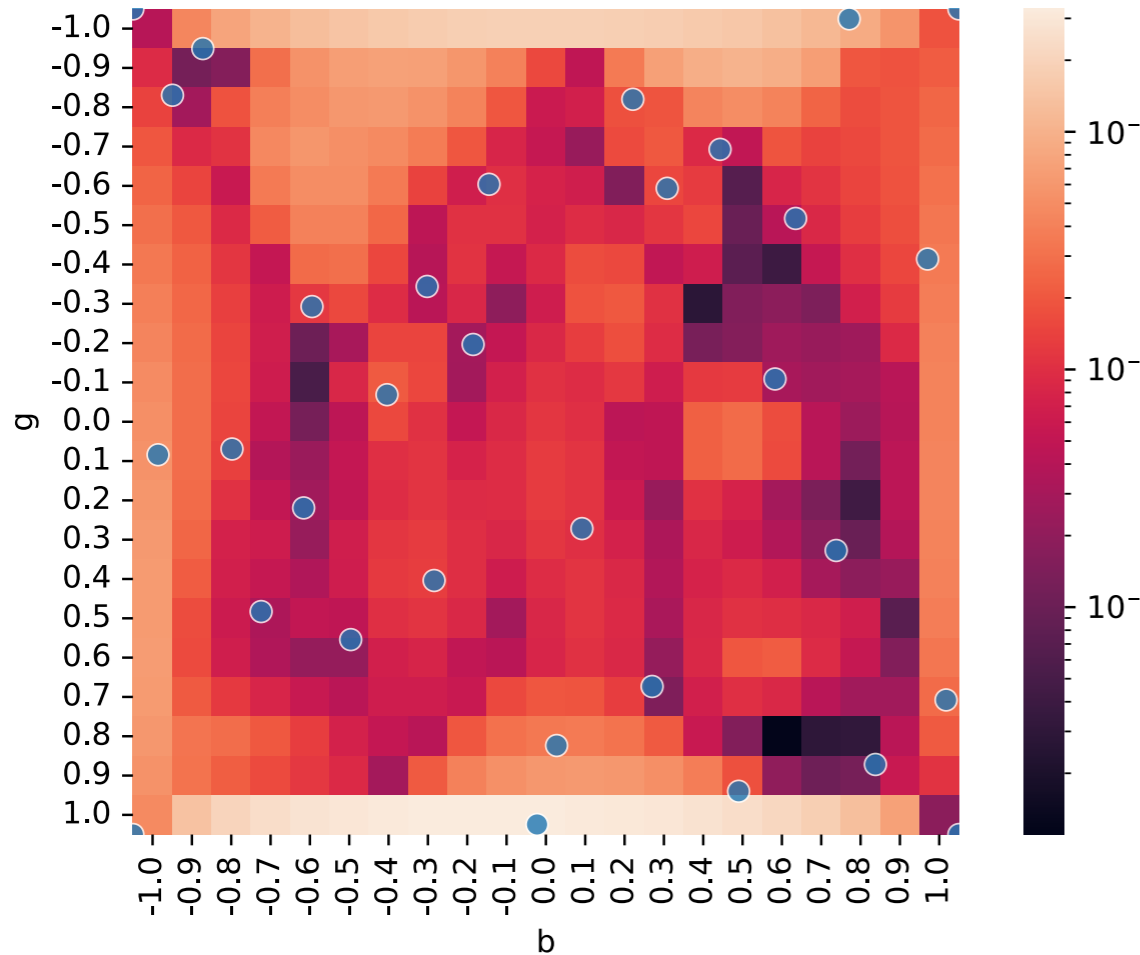


# Parametric DMD

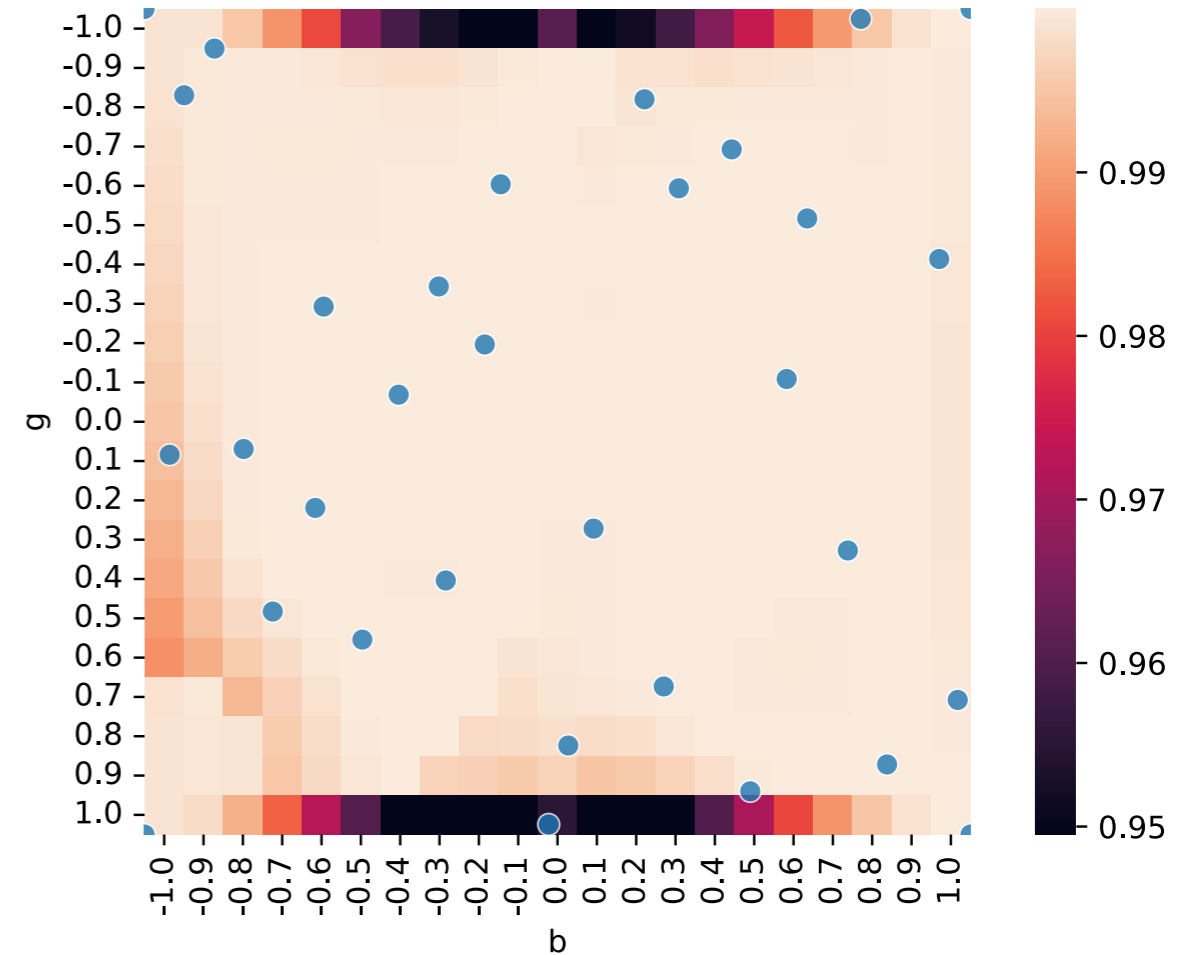


*J. Davison, J. Crawford, S. Bogner, HH, in preparation*

Absolute relative error (parametric result relative to IMSRG result)



Pearson correlation coefficient (compared to IMSRG(2) flow)



- pairing plus particle-hole model - 3 parameters + flow
- “naive” framework built for chiral LECs, but needs more optimization (more model reduction before DMD, etc.)
- (still) ambitious by trying to predict full operators, could focus on observables (zero-body part of evolving operators) only