Evan Rule, WH, Kevin McElvain, PRL 130 (2023) 13190
WH, Evan Rule, Kevin McElvain, Michael J. Ramsey-Musolf, Phys. Rev. C 107 (2023) 035504
WH, Tony Menzo, Ken McElvain, Evan Rule, Jure Zupan, arXiv soon

Nonrelativistic Effective Theory and $\mu \rightarrow e$ Conversion

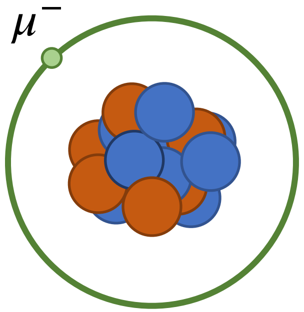
- Muonic atoms (radioactive!) & CLFV
- NRET: motivation, counting, formulation
- Connecting the NRET to higher scales

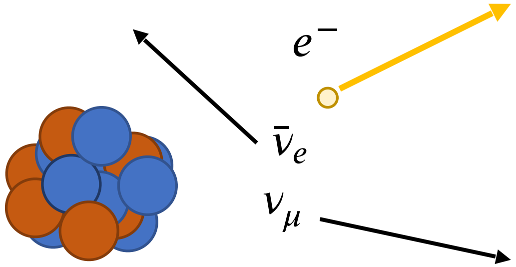
INT March 26, 2024

Radioactive Muonic Atoms

Formed when muons produced in pion decay are stopped in a metal target

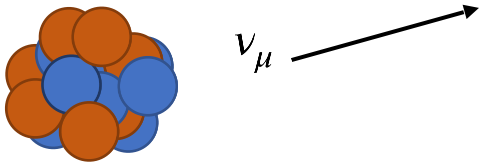
- Quickly cascade into the 1s atomic orbit around the target nucleus





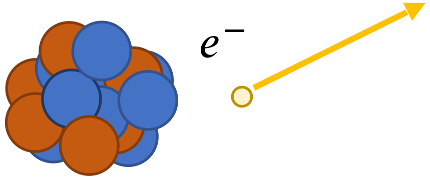
$$\mu^- \to e^- + \bar{\nu}_e + \nu_\mu$$

(free decay)



$$\mu^- + (A, Z) \to \nu_\mu + (A, Z - 1)$$

(dominant in heavier nuclei)



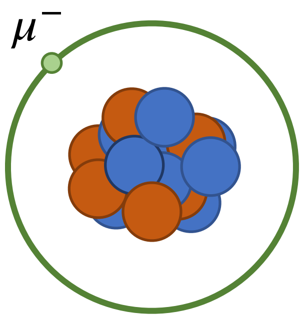
$$\mu^- + (A, Z) \to e^- + (A, Z)$$

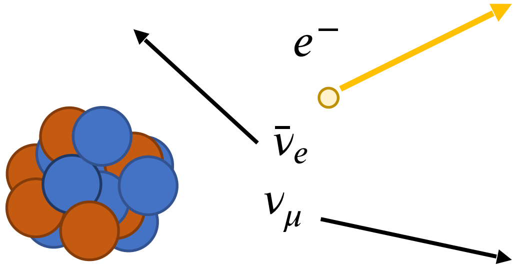
(flavor changing)

Radioactive Muonic Atoms

Formed when muons produced in pion decay are stopped in a metal target

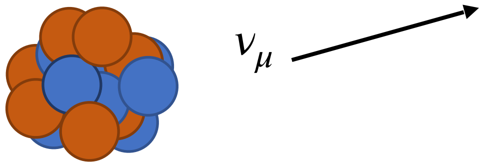
- Quickly cascade into the 1s atomic orbit around the target nucleus





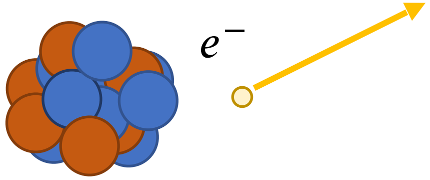
$$\mu^- \to e^- + \bar{\nu}_e + \nu_\mu$$

(free decay)



$$\mu^- + (A, Z) \to \nu_\mu + (A, Z-1)$$

(dominant in heavier nuclei)



$$\mu^- + (A, Z) \to e^- + (A, Z)$$

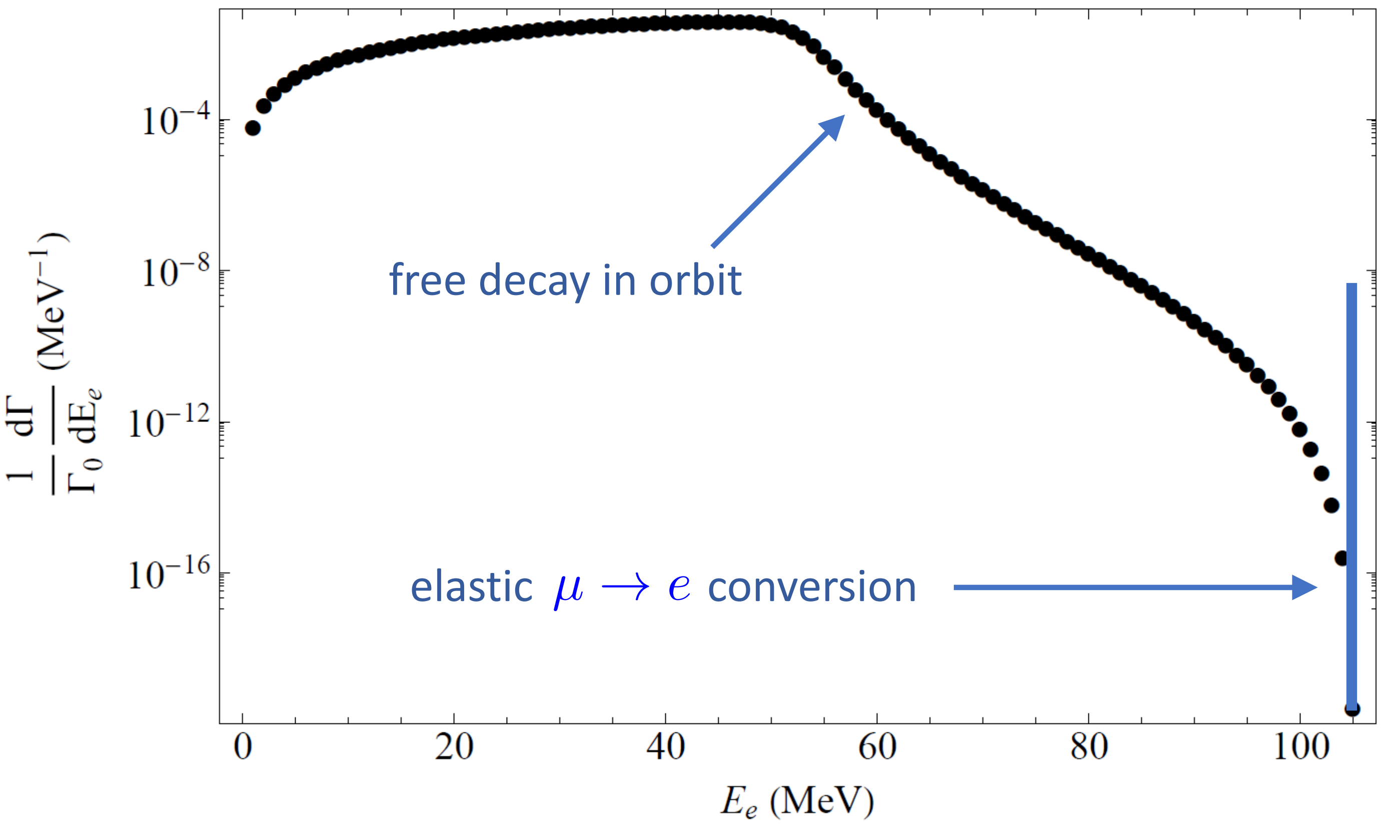
(flavor changing)

Simple kinematics:

$$E_e \sim m_\mu - E_\mu^{\text{binding}}$$

Electron Spectrum

Figure: Czarnecki, Garcia i Tormo, & Marciano, Phys. Rev. D **84**, 013006 (2011)

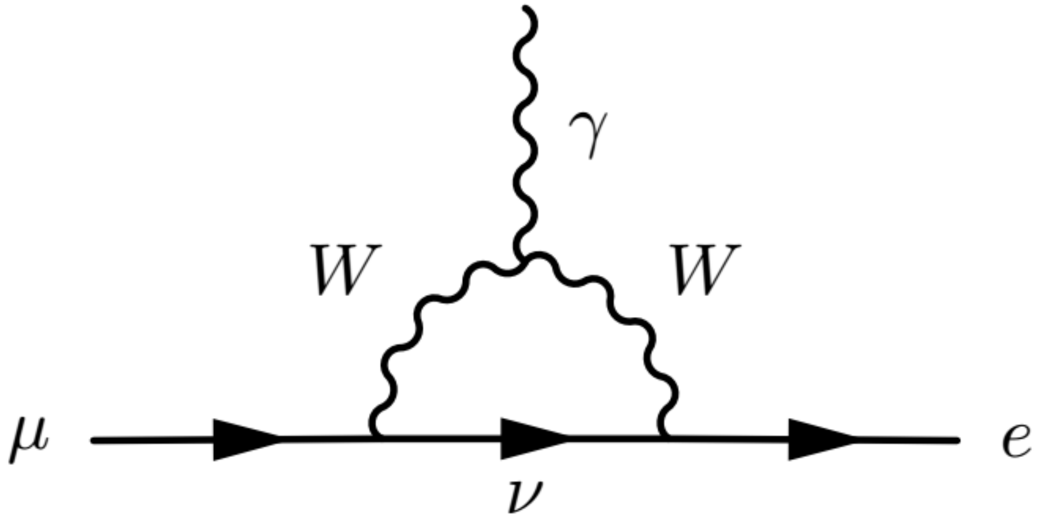


Experimental Requirements

- intense muon beams
- energy resolution

$$\mathrm{BR}\left(\mu^- + (A,Z) \to e^- + (A,Z)\right) \equiv \frac{\Gamma\left(\mu^- + (A,Z) \to e^- + (A,Z)\right)}{\Gamma\left(\mu^- + (A,Z) \to \nu_\mu + (A,Z-1)\right)}$$

single neutrino

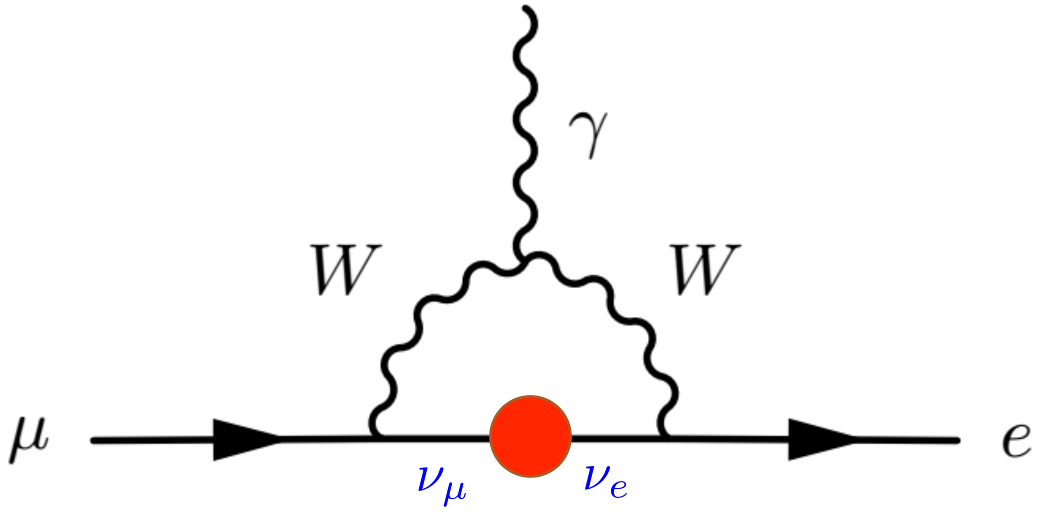


Theory Predicts: $\mathrm{BR}\left(\mu^{+} \rightarrow e^{+}\gamma\right) \approx 10^{-4}$

Feinberg, Phys. Rev. **110**, 1482 (1958)

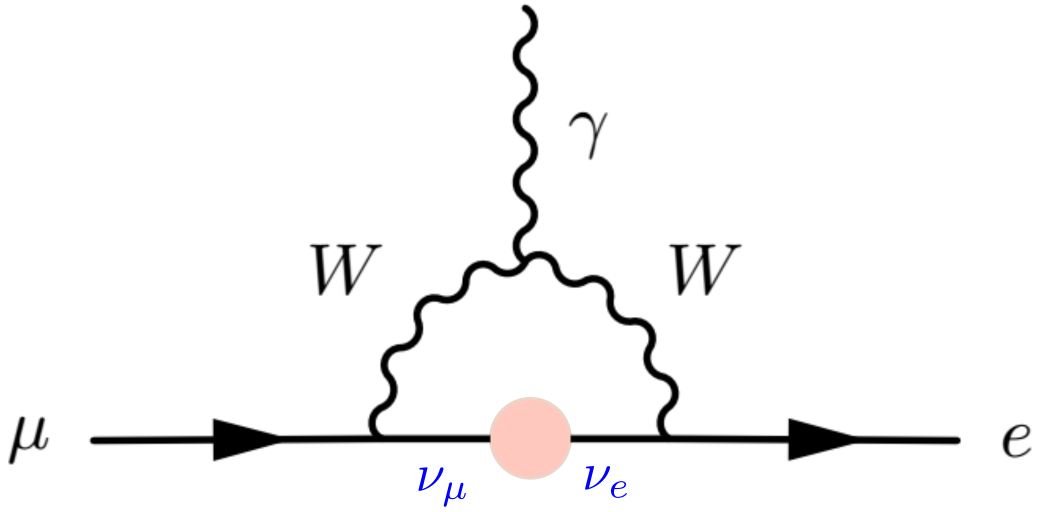
Nevis Cyclotron: $\mathrm{BR}\left(\mu^{+} \rightarrow e^{+}\gamma\right) < 2 \times 10^{-5}$

Lokanathan & Steinberger, Phys. Rev. **98**, 240 (A) (1955) two neutrinos: $\nu_\mu \neq \nu_e$



Pontecorvo, Zh. Eksp. Teor. Fiz. **37**, 1751–1757 (1959).

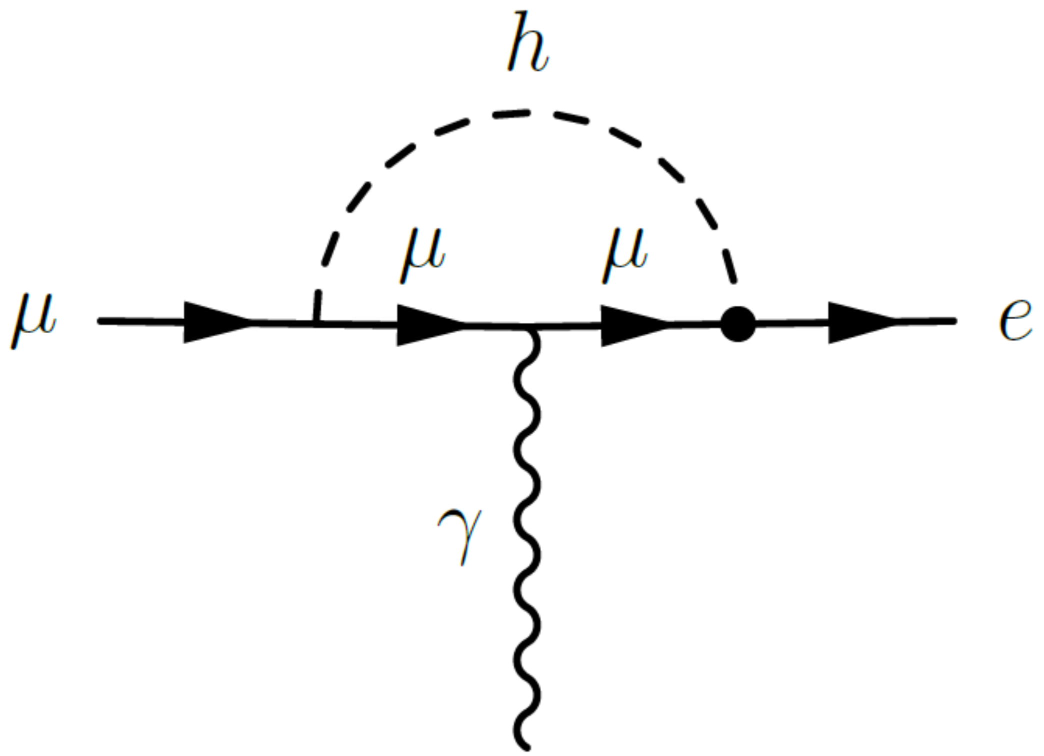
massive neutrinos & flavor mixing



$$\left(\frac{m_\nu}{M_W}\right)^2$$

branching ratio suppression $\sim \left(\frac{m_\nu}{M_W}\right)^4 \sim 10^{-50}$

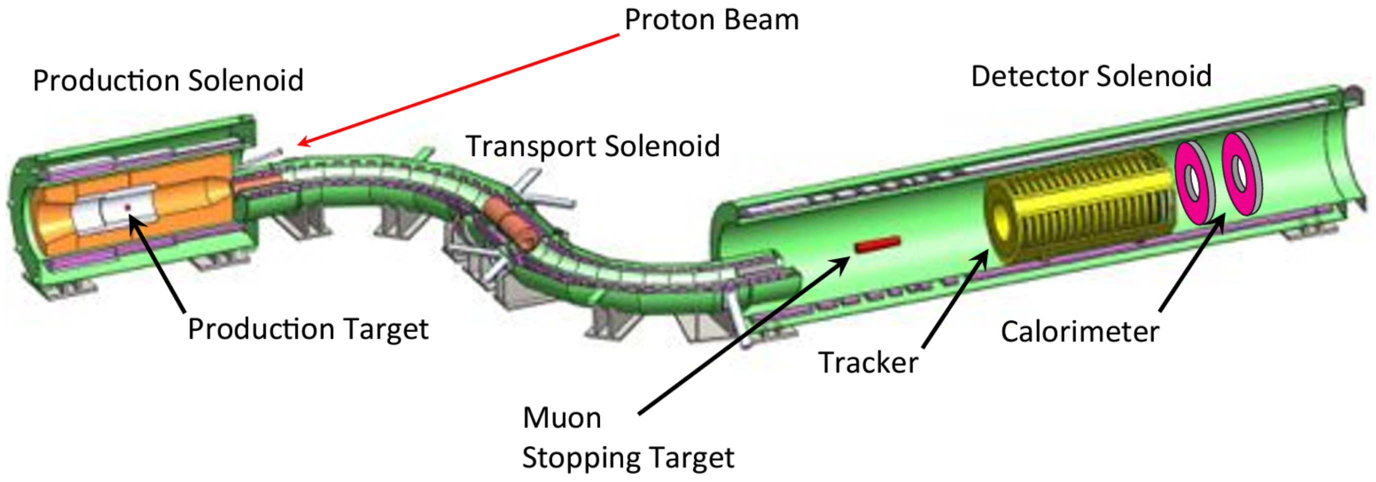
nevertheless, establishes that CLFV occurs but many other BSM possibilities other than those induced by neutrinos



CLFV a probe off new sources of flavor mixing

NRET: motivation, counting, formulation

New experiments



- Huge fluence: 10^{18} muons will be captured
- Clean CLFV signal, free of SM backgrounds
- Target can be varied to obtain complimentary constraints
- Restriction to elastic nuclear response imposes selection rule due to good nuclear g.s. P, CP

Both experiments under construction

Expected to improve the branching ratio by $\sim 10^4$ to $\sim 10^{-17}$

Theory: operator content

- Of $\sim$ two dozen papers in the literature, 90% focused on one operator, the coherent vector charge, 10% on spin-flip

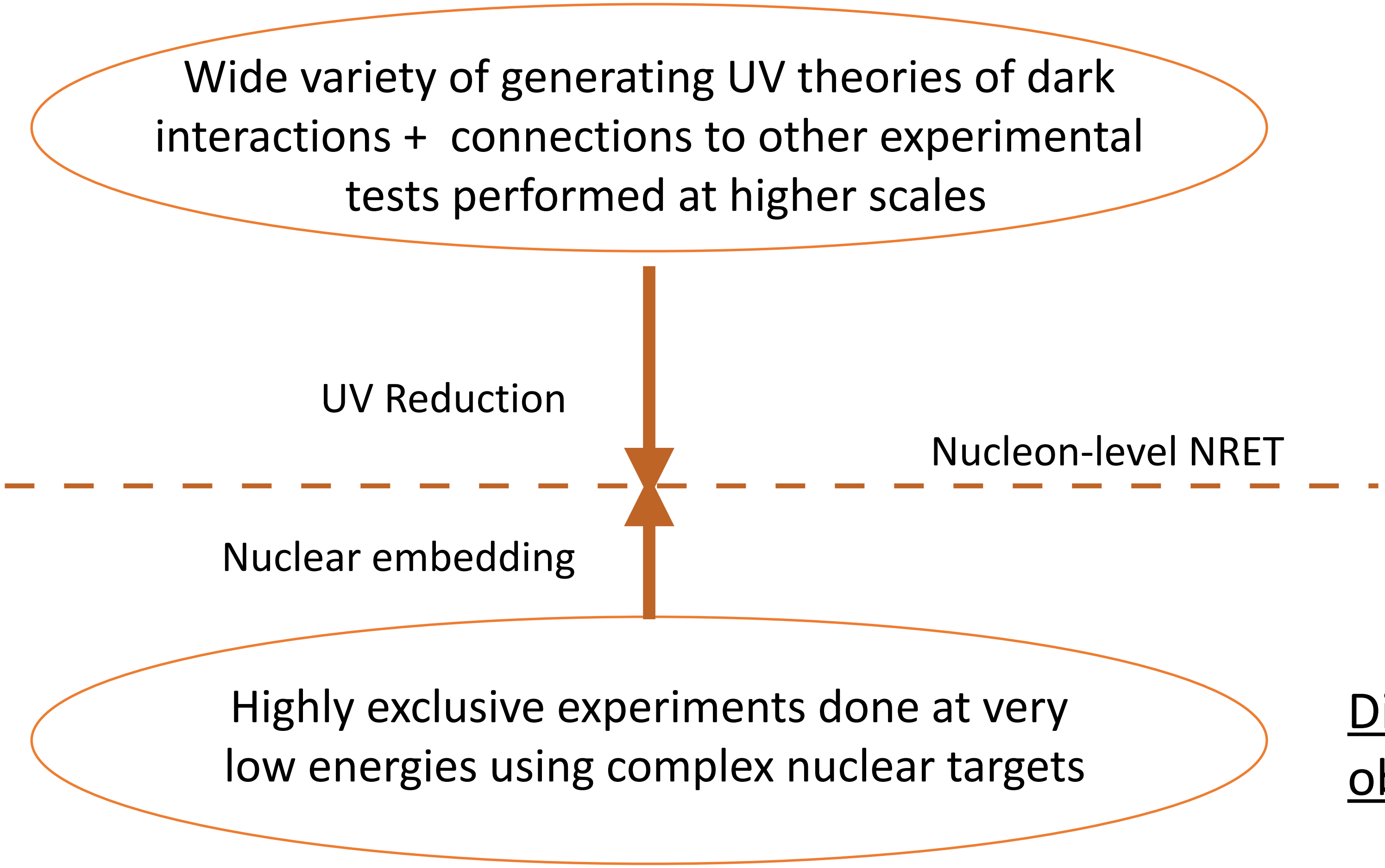
- Unclear what can or cannot be determined about BSM CLFV from experiment

- Lack of an operator organization according to the hierarchy of physical parameters relevant to the nuclear scale

$$y = \left(\frac{qb}{2}\right)^2 \sim \left(\frac{m_\mu R}{2}\right)^2 \sim 1 \quad > \quad |\vec{v}_N| \quad > \quad |\vec{v}_\mu| \quad > \quad |\vec{v}^{\,\mathrm{recoil}}_{\mathrm{nucleus}}| \sim 0$$

The problem has many similarities to WIMP direct detection, kinematically (elastic, but with q R $\sim$ 1) and in terms of the need to connect nuclear scale observations to high energy

An efficient alternative to top-down reductions was developed using NRET

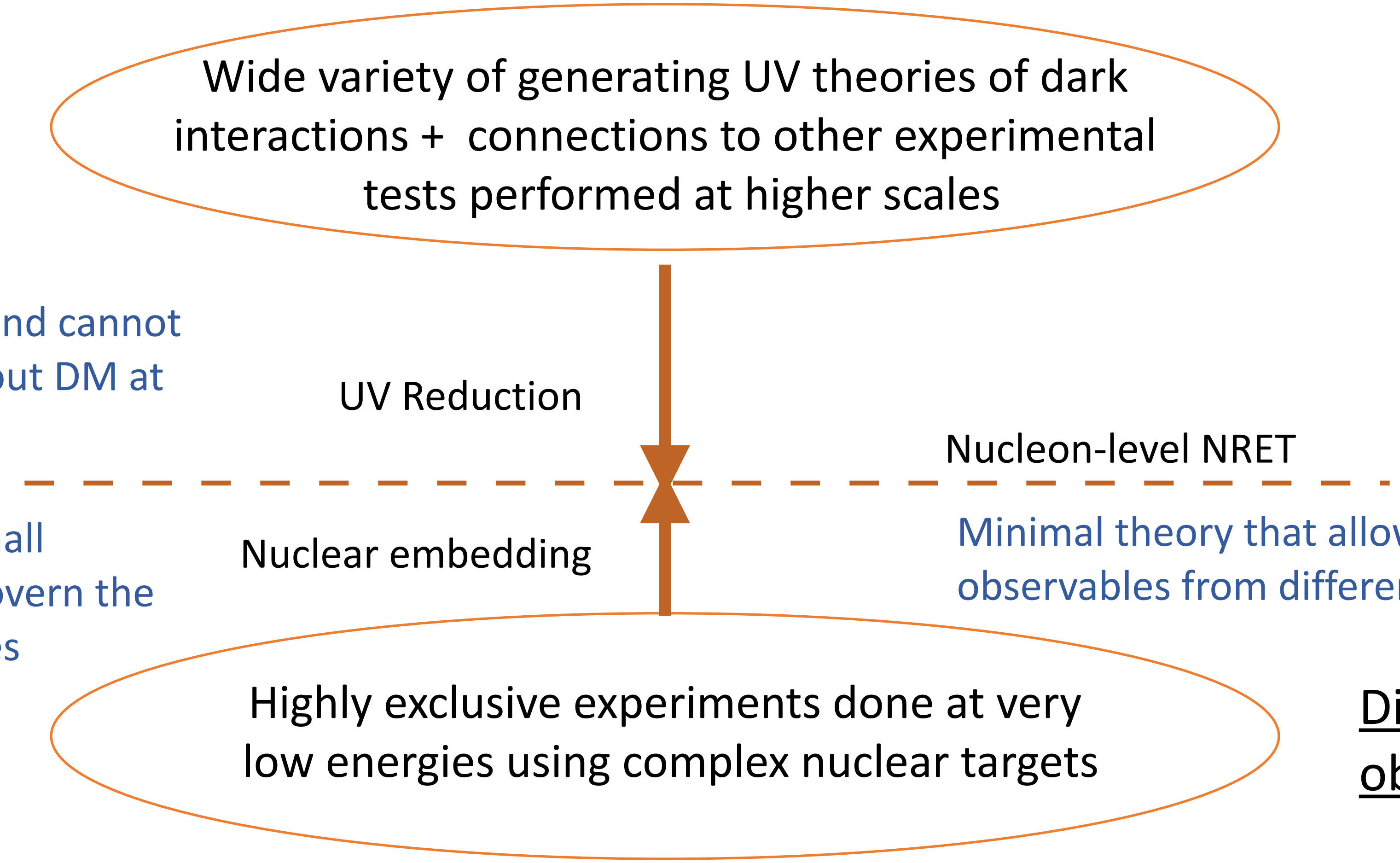


Direct detection observables

Fitzpatrick, WH, Katz, Lubbers, Xu, JCAP 02 (2013) 004
Fan, Reece, Wang, JCAP 1011 (2010) 042

An efficient alternative to top-down reductions was developed using NRET

- Defines what can and cannot be determined about DM at low energies
- Determines the small parameters that govern the nuclear observables

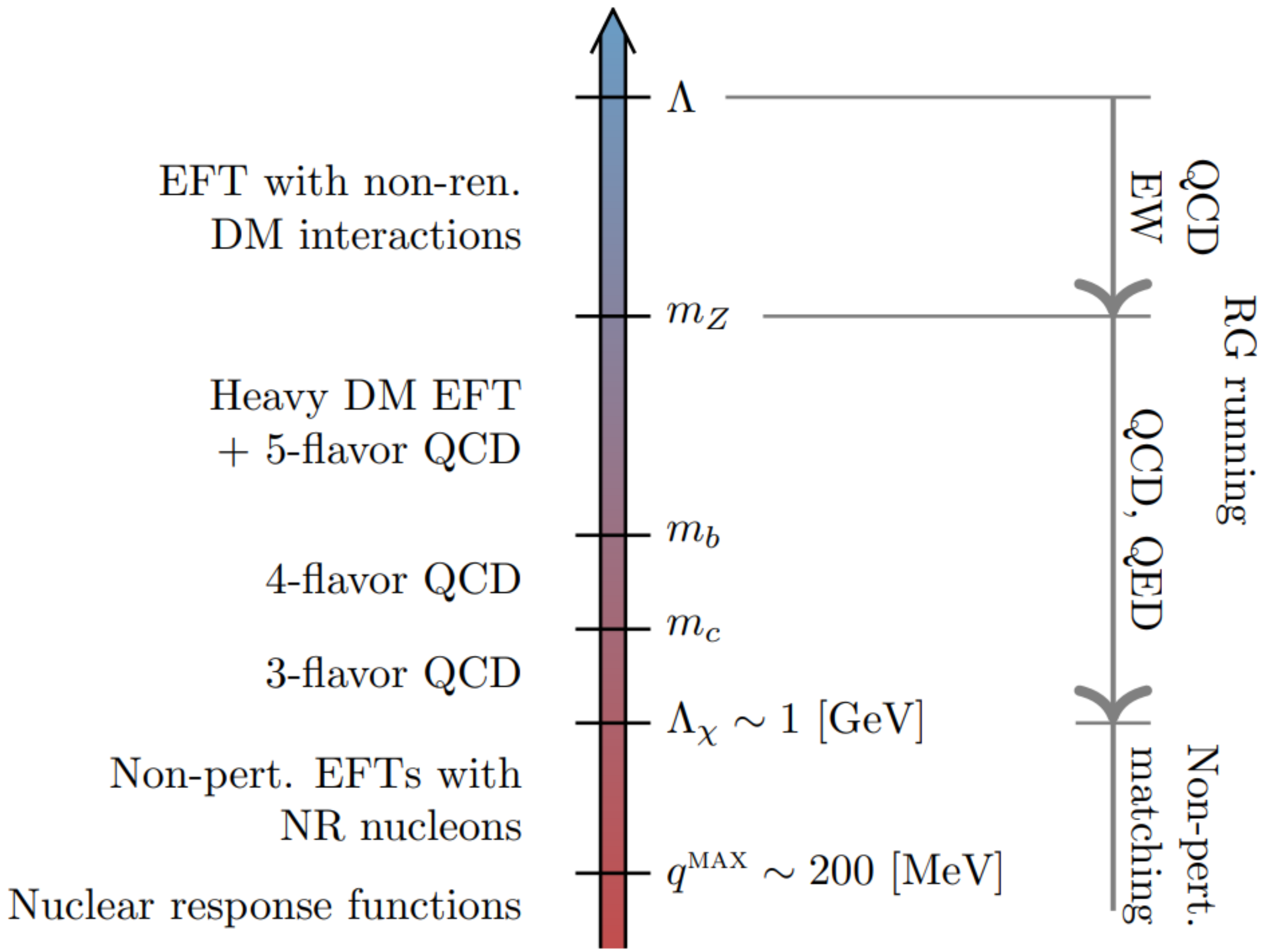


Minimal theory that allows one to relate observables from different targets

Direct detection observables

Fitzpatrick, WH, Katz, Lubbers, Xu, JCAP 02 (2013) 004
Fan, Reece, Wang, JCAP 1011 (2010) 042

Figure: Bishara, Brod, Grinstein, and Zupan, JHEP **03** 089 (2020)



Snowmass White Paper: Effective Field Theories for DM Phenomenology
M. Baumgart et al., arXiv:2203.08204

Figure: Bishara, Brod, Grinstein, and Zupan, JHEP **03** 089 (2020)

Links built

Wilson coefficients of a relativistic EFT of light quarks, gluons, photon

↓

NRET of WIMPS, neutrons, protons

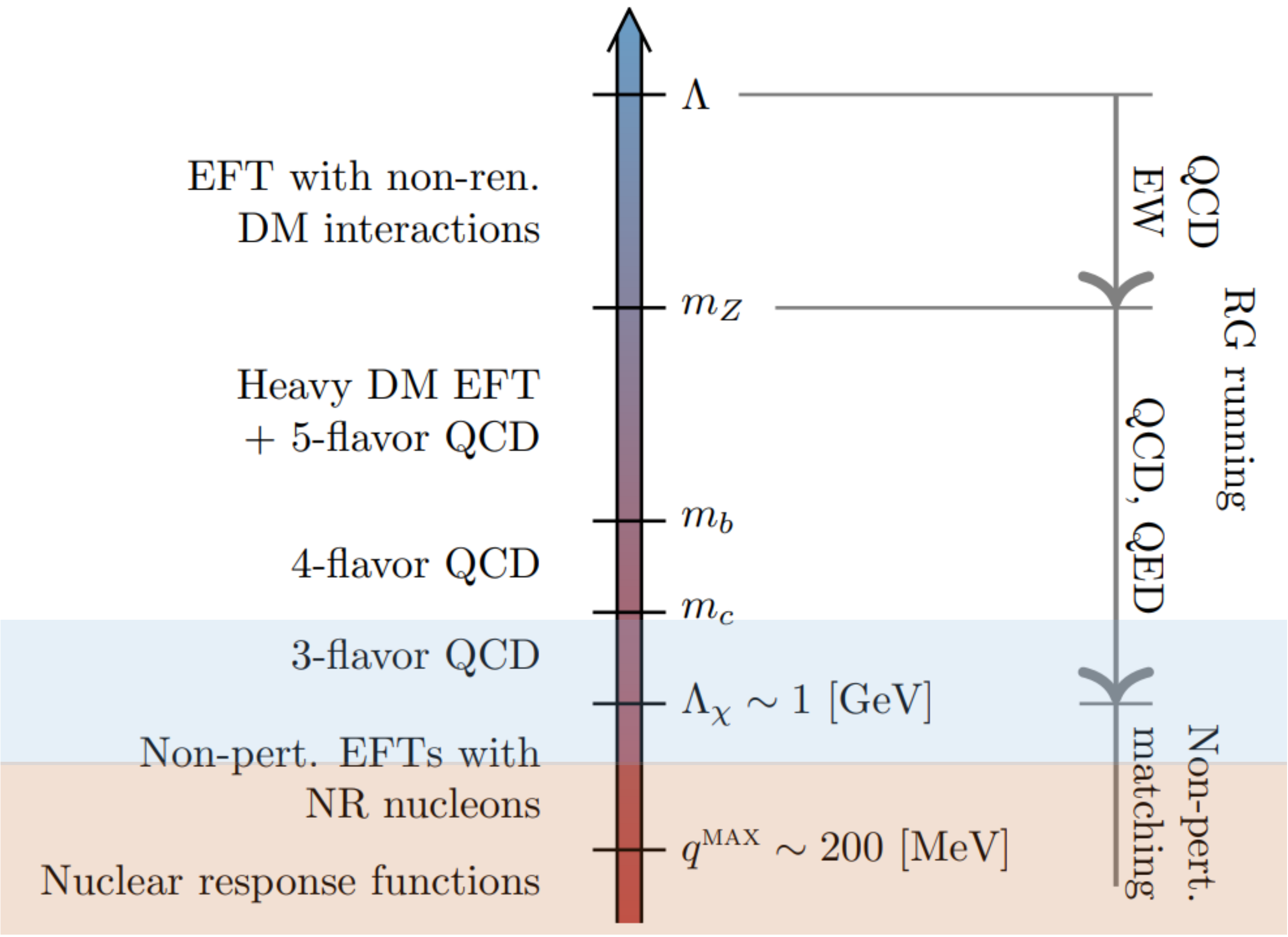
DirectDM: 1708.02678

DMFormFactor: PRC89,065501

NRET of WIMPS, neutrons protons

↑

nuclear observables



Snowmass White Paper: Effective Field Theories for DM Phenomenology
M. Baumgart et al., arXiv:2203.08204

First: Count the nuclear observables

$$\sum_i e^{i\vec{q}\cdot\vec{r}_i}$$

		multipoles even	multipoles odd	
1_N	vector	C_0	C_1	charges
$\vec{\sigma}_N \cdot \vec{v}_N$	axial	C_0^5	C_1^5	

		multipoles even	odd	even	odd	even	odd	
$\vec{\sigma}_N$	axial spin	L_0^5	L_1^5	$T_2^{5\text{el}}$	$T_1^{5\text{el}}$	$T_2^{5\text{mag}}$	$T_1^{5\text{mag}}$	
$\vec{v}_N$	vector velocity	L_0	L_1	T_2^{el}	T_1^{el}	T_2^{mag}	T_1^{mag}	currents
$\vec{\sigma}_N \times \vec{v}_N$	vector spin – velocity	L_0	L_1	T_2^{el}	T_1^{el}	T_2^{mag}	T_1^{mag}	

- We now impose the symmetries that constrain an elastic transition

Parity

		multipoles even	multipoles odd	
1_N	vector	C_0		charges
$\vec{\sigma}_N \cdot \vec{v}_N$	axial		C_1^5	

		multipoles						
		even	odd	even	odd	even	odd	
$\vec{\sigma}_N$	axial spin		L_1^5		$T_1^{5\mathrm{el}}$	$T_2^{5\mathrm{mag}}$		
$\vec{v}_N$	vector velocity	L_0		T_2^{el}			T_1^{mag}	currents
$\vec{\sigma}_N \times \vec{v}_N$	vector spin – velocity	L_0		T_2^{el}			T_1^{mag}	

- CP conservation

		multipoles	
		even	odd
1_N	vector	C_0	
$\vec{\sigma}_N \cdot \vec{v}_N$	axial		

charges

				multipoles			
		even	odd	even	odd	even	odd
$\vec{\sigma}_N$	axial spin		L_1^5		$T_1^{5\mathrm{el}}$		
$\vec{v}_N$	vector velocity						T_1^{mag}
$\vec{\sigma}_N \times \vec{v}_N$	vector spin – velocity	L_0		T_2^{el}			

currents

- There are in principle <u>six observables</u> that experimentalists could extract

Construction of the nucleon-level NRET

- Begin by enumerating the available charges and currents

charges: 1_N $\vec{\sigma}_N \cdot \vec{v}_N$ currents: $\vec{\sigma}_N$ $\vec{v}_N$ $\vec{\sigma}_N \times \vec{v}_N$

- Include the possibility of curls and gradients of these quantities

- The response of the nucleus $\sum_i e^{i\vec{q}\cdot\vec{r}_i}$

- Form all possible operators, organized according to small parameters

$$y = \left(\frac{qb}{2}\right)^2 \sim \left(\frac{m_\mu R}{2}\right)^2 \sim 1 \quad > \quad |\vec{v}_N| \quad > \quad |\vec{v}_\mu| \quad > \quad |\vec{v}^{\,\mathrm{recoil}}_{\mathrm{nucleus}}| \sim 0$$

- One can use this hierarchy to construct interactions that are successively more complete

Significant angular moment transfer
from the leptons to the nucleus:
needed multiple electron partial waves

$$y = \left(\frac{qb}{2}\right)^2 \sim \left(\frac{m_\mu R}{2}\right)^2 \sim 1 \quad > \quad |\vec{v}_N| \quad > \quad |\vec{v}_\mu| \quad > \quad |\vec{v}_{\text{nucleus}}^{\text{recoil}}| \sim 0$$

- The momentum transfer between leptons and nucleus is sufficient that a partial wave expansion is needed, accounting for the angular momentum transfer

- One can use this hierarchy to construct interactions that are successively more complete

Currents:
generates new kinds of coherence
plays a key role in selection rules

$$y = \left(\frac{qb}{2}\right)^2 \sim \left(\frac{m_\mu R}{2}\right)^2 \sim 1 \quad > \quad |\vec{v}_N| \quad > \quad |\vec{v}_\mu| \quad > \quad |\vec{v}^{\,\mathrm{recoil}}_{\mathrm{nucleus}}| \sim 0$$

- In the NRET, this is a bound-state (Jacobi) velocity — the inter-nucleon velocity

- One can use this hierarchy to construct interactions that are successively more complete

Generates the muon's lower component:
plays no role in selection rules:
amounts to a nuclear form factor
of about 5% for Al

$$y = \left(\frac{qb}{2}\right)^2 \sim \left(\frac{m_\mu R}{2}\right)^2 \sim 1 \quad > \quad |\vec{v}_N| \quad > \quad |\vec{v}_\mu| \quad > \quad |\vec{v}^{\,\mathrm{recoil}}_{\mathrm{nucleus}}| \sim 0$$

- In NRET also a bound-state velocity, the muon velocity w.r.t. the nuclear center of mass

- One can use this hierarchy to construct interactions that are successively more complete

contributes in the amplitude to order

$$\frac{m_\mu}{M_T} < 1\%$$

$$y = \left(\frac{qb}{2}\right)^2 \sim \left(\frac{m_\mu R}{2}\right)^2 \sim 1 \quad > \quad |\vec{v}_N| \quad > \quad |\vec{v}_\mu| \quad > \quad |\vec{v}^{\text{recoil}}_{\text{nucleus}}| \sim 0$$

- Neglected

Effective Theory Variations

Coherent charge operator

Coherent Response
$1_N, 1_L, \hat{q}, \vec{\sigma}_L$

2 Nucleon-level Operators

1 Response function

M_J

- Even multipoles of the charge operator: A single response

$\mathcal{O}_1 = 1_L 1_N$

$\mathcal{O}_{11} = i\hat{q} \cdot \vec{\sigma}_L 1_N$

Effective Theory Variations

"Allowed" operators

Coherent Response	Allowed Response
$1_N, 1_L, \hat{q}, \vec{\sigma}_L$	$1_N, 1_L, \hat{q}, \vec{\sigma}_L, \vec{\sigma}_N$
2 Nucleon-level Operators	6 Nucleon-level operators
1 Response function	3 Response functions
M_J	$M_J, \Sigma'_J, \Sigma''_J$

- Most general CLFV response of point-like nucleus

Distinct transverse and longitudinal spin responses

$\mathcal{O}_1 = 1_L 1_N$

$\mathcal{O}_4 = \vec{\sigma}_L \cdot \vec{\sigma}_N$

$\mathcal{O}_6 = i\hat{q} \cdot \vec{\sigma}_L \, i\hat{q} \cdot \vec{\sigma}_N$

$\mathcal{O}_9 = \vec{\sigma}_L \cdot \left(i\hat{q} \times \vec{\sigma}_N \right)$

$\mathcal{O}_{10} = 1_L \, i\hat{q} \cdot \vec{\sigma}_N$

$\mathcal{O}_{11} = i\hat{q} \cdot \vec{\sigma}_L \, 1_N$

Effective Theory Variations

Nucleon velocity: yields the general form

Coherent Response	Allowed Response	General Response
$1_N, 1_L, \hat{q}, \vec{\sigma}_L$	$1_N, 1_L, \hat{q}, \vec{\sigma}_L, \vec{\sigma}_N$	$1_N, 1_L, \hat{q}, \vec{\sigma}_L, \vec{\sigma}_N, \vec{v}_N$
2 Nucleon-level Operators	6 Nucleon-level operators	16 Nucleon-level operators
1 Response function	3 Response functions	6 Response functions
	$M_J, \Sigma'_J, \Sigma''_J$	$M_J, \Sigma'_J, \Sigma''_J, \Delta_J, \tilde{\Phi}'_J, \Phi'_J$

- All nuclear responses allowed by P and T symmetries

$$\mathcal{O}_1 = 1_L 1_N$$
$$\mathcal{O}'_2 = 1_L\, i\hat{q} \cdot \vec{v}_N$$
$$\mathcal{O}_3 = 1_L\, i\hat{q} \cdot (\vec{v}_N \times \vec{\sigma}_N)$$
$$\mathcal{O}_4 = \vec{\sigma}_L \cdot \vec{\sigma}_N$$
$$\mathcal{O}_5 = \vec{\sigma}_L \cdot (i\hat{q} \times \vec{v}_N)$$
$$\mathcal{O}_6 = i\hat{q} \cdot \vec{\sigma}_L\, i\hat{q} \cdot \vec{\sigma}_N$$
$$\mathcal{O}_7 = 1_L\, \vec{v}_N \cdot \vec{\sigma}_N$$
$$\mathcal{O}_8 = \vec{\sigma}_L \cdot \vec{v}_N$$
$$\mathcal{O}_9 = \vec{\sigma}_L \cdot (i\hat{q} \times \vec{\sigma}_N)$$
$$\mathcal{O}_{10} = 1_L\, i\hat{q} \cdot \vec{\sigma}_N$$
$$\mathcal{O}_{11} = i\hat{q} \cdot \vec{\sigma}_L\, 1_N$$
$$\mathcal{O}_{12} = \vec{\sigma}_L \cdot (\vec{v}_N \times \vec{\sigma}_N)$$
$$\mathcal{O}'_{13} = \vec{\sigma}_L \cdot [i\hat{q} \times (\vec{v}_N \times \vec{\sigma}_N)]$$
$$\mathcal{O}_{14} = i\hat{q} \cdot \vec{\sigma}_L\, \vec{v}_N \cdot \vec{\sigma}_N$$
$$\mathcal{O}_{15} = i\hat{q} \cdot \vec{\sigma}_L\, i\hat{q} \cdot (\vec{v}_N \times \vec{\sigma}_N)$$
$$\mathcal{O}'_{16} = i\hat{q} \cdot \vec{\sigma}_L\, i\hat{q} \cdot \vec{v}_N$$

Effective Theory Variations

Coherent Response	Allowed Response	General Response
$1_N, 1_L, \hat{q}, \vec{\sigma}_L$	$1_N, 1_L, \hat{q}, \vec{\sigma}_L, \vec{\sigma}_N$	$1_N, 1_L, \hat{q}, \vec{\sigma}_L, \vec{\sigma}_N, \vec{v}_N$
2 Nucleon-level Operators	6 Nucleon-level operators	16 Nucleon-level operators
1 Response function	3 Response functions	6 Response functions
M_0	$M_J, \Sigma'_J, \Sigma''_J$	$M_J, \Sigma'_J, \Sigma''_J, \Delta_J, \tilde{\Phi}'_J, \Phi''_J$

- All nuclear responses allowed by P and T symmetries

This theory is complete for nuclear analysis

$$\mathcal{O}_1 = 1_L 1_N$$
$$\mathcal{O}'_2 = 1_L\, i\hat{q} \cdot \vec{v}_N$$
$$\mathcal{O}_3 = 1_L\, i\hat{q} \cdot \left(\vec{v}_N \times \vec{\sigma}_N\right)$$
$$\mathcal{O}_4 = \vec{\sigma}_L \cdot \vec{\sigma}_N$$
$$\mathcal{O}_5 = \vec{\sigma}_L \cdot \left(i\hat{q} \times \vec{v}_N\right)$$
$$\mathcal{O}_6 = i\hat{q} \cdot \vec{\sigma}_L\, i\hat{q} \cdot \vec{\sigma}_N$$
$$\mathcal{O}_7 = 1_L\, \vec{v}_N \cdot \vec{\sigma}_N$$
$$\mathcal{O}_8 = \vec{\sigma}_L \cdot \vec{v}_N$$
$$\mathcal{O}_9 = \vec{\sigma}_L \cdot \left(i\hat{q} \times \vec{\sigma}_N\right)$$
$$\mathcal{O}_{10} = 1_L\, i\hat{q} \cdot \vec{\sigma}_N$$
$$\mathcal{O}_{11} = i\hat{q} \cdot \vec{\sigma}_L\, 1_N$$
$$\mathcal{O}_{12} = \vec{\sigma}_L \cdot \left(\vec{v}_N \times \vec{\sigma}_N\right)$$
$$\mathcal{O}'_{13} = \vec{\sigma}_L \cdot [i\hat{q} \times \left(\vec{v}_N \times \vec{\sigma}_N\right)]$$
$$\mathcal{O}_{14} = i\hat{q} \cdot \vec{\sigma}_L\, \vec{v}_N \cdot \vec{\sigma}_N$$
$$\mathcal{O}_{15} = i\hat{q} \cdot \vec{\sigma}_L\, i\hat{q} \cdot \left(\vec{v}_N \times \vec{\sigma}_N\right)$$
$$\mathcal{O}'_{16} = i\hat{q} \cdot \vec{\sigma}_L\, i\hat{q} \cdot \vec{v}_N$$

$$\mathcal{L}_{\text{eff}} \sim \sqrt{2} G_F \sum_{\tau=0,1} \sum_{i=1}^{16} \tilde{c}_i^\tau \mathcal{O}_i t^\tau$$

Nucleon-level Operator Basis

- All scalars that can be constructed from the input elementary operators, linear in $\vec{v}_N$

$$1_L,\ 1_N,\ \vec{\sigma}_L,\ \vec{\sigma}_N,\ \hat{q},\ \vec{v}_N,\ \cdots$$

- Contact form but general as the momentum transfer is fixed, $q \sim m_\mu$: if the mediator is a photon, that can be absorbed into the LECs

- The LECs c_i can be complex

- Given a complete set of leptonic and single nucleon observables — isolating all the longitudinal and transverse projections — all c_is could be determined

$$\mathcal{O}_1 = 1_L 1_N$$

$$\mathcal{O}_2' = 1_L\, i\hat{q}\cdot\vec{v}_N$$

$$\mathcal{O}_3 = 1_L\, i\hat{q}\cdot\left(\vec{v}_N\times\vec{\sigma}_N\right)$$

$$\mathcal{O}_4 = \vec{\sigma}_L\cdot\vec{\sigma}_N$$

$$\mathcal{O}_5 = \vec{\sigma}_L\cdot\left(i\hat{q}\times\vec{v}_N\right)$$

$$\mathcal{O}_6 = i\hat{q}\cdot\vec{\sigma}_L\, i\hat{q}\cdot\vec{\sigma}_N$$

$$\mathcal{O}_7 = 1_L\,\vec{v}_N\cdot\vec{\sigma}_N$$

$$\mathcal{O}_8 = \vec{\sigma}_L\cdot\vec{v}_N$$

$$\mathcal{O}_9 = \vec{\sigma}_L\cdot\left(i\hat{q}\times\vec{\sigma}_N\right)$$

$$\mathcal{O}_{10} = 1_L\, i\hat{q}\cdot\vec{\sigma}_N$$

$$\mathcal{O}_{11} = i\hat{q}\cdot\vec{\sigma}_L\, 1_N$$

$$\mathcal{O}_{12} = \vec{\sigma}_L\cdot\left(\vec{v}_N\times\vec{\sigma}_N\right)$$

$$\mathcal{O}_{13}' = \vec{\sigma}_L\cdot[i\hat{q}\times\left(\vec{v}_N\times\vec{\sigma}_N\right)]$$

$$\mathcal{O}_{14} = i\hat{q}\cdot\vec{\sigma}_L\,\vec{v}_N\cdot\vec{\sigma}_N$$

$$\mathcal{O}_{15} = i\hat{q}\cdot\vec{\sigma}_L\, i\hat{q}\cdot\left(\vec{v}_N\times\vec{\sigma}_N\right)$$

$$\mathcal{O}_{16}' = i\hat{q}\cdot\vec{\sigma}_L\, i\hat{q}\cdot\vec{v}_N$$

- Any UV Lagrangian will reduce at low energy to the NRET form

$$\mathcal{L}_{\text{eff}} = \sqrt{2} G_F \sum_{\tau=0,1} \sum_{i=1}^{16} \tilde{c}_i \mathcal{O}_i t^\tau \qquad t^0 = 1,\ t^1 = \tau_3$$

and thus can be matched to this form, if a consistent counting is employed

<u>What happens when we embed this in a nucleus?</u>

1. An additional operator $\sim \sum_{i=1}^{A} e^{i\vec{q}\cdot\vec{r}_i}$ that transfer linear and angular momentum to the nucleus

2. Selection rules imposed by the restriction to elastic scattering, that reduce how much we can learn about the CLFV Lagrangian

How do we evaluate the rate?

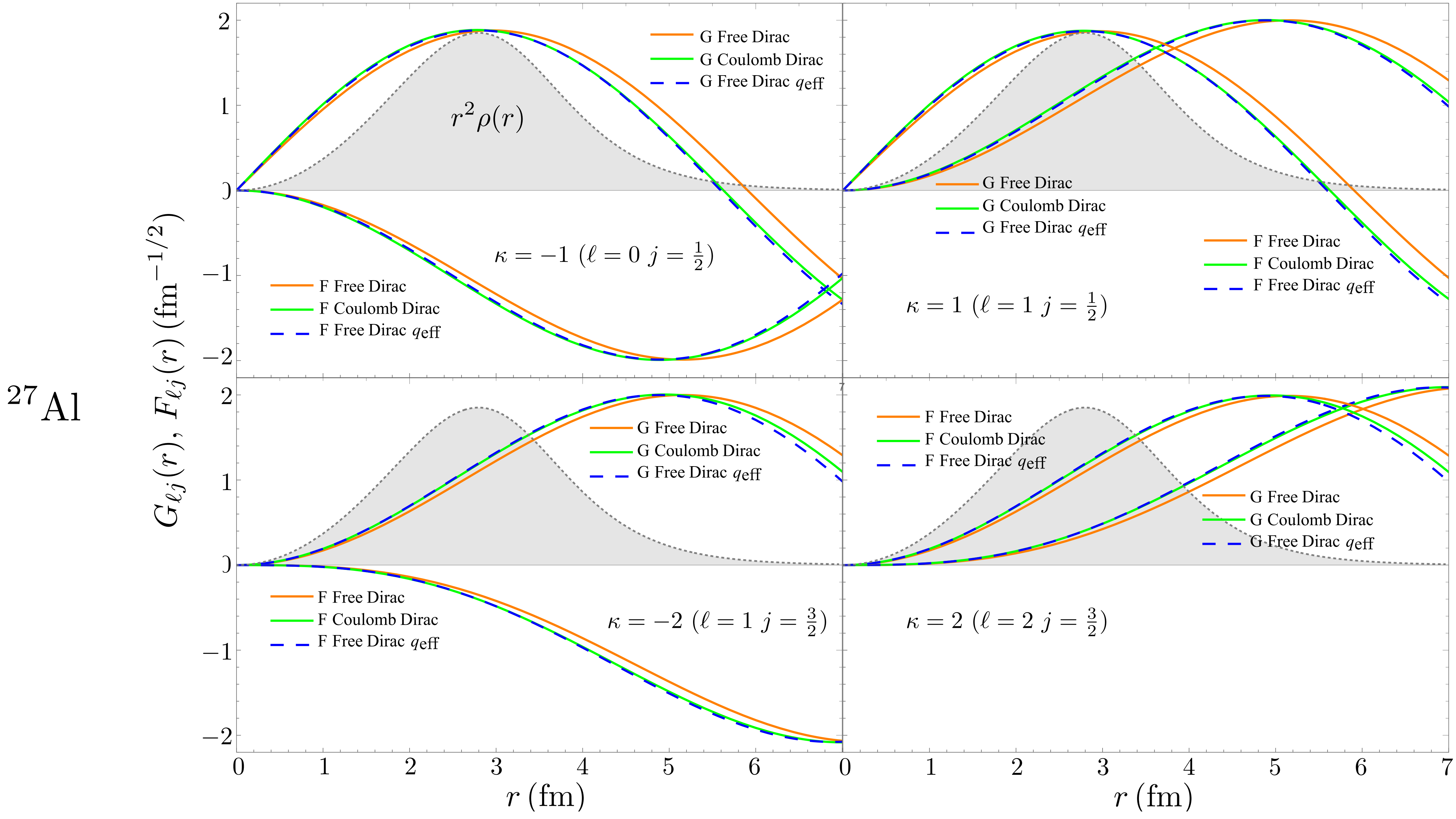
- The plane wave is actually distorted by the nuclear Coulomb field: important for heavier nuclei. Past investigators using simple CLFV operators have made a mess of this physics … retaining only the Dirac waves $\kappa = \pm 1/2$

- Now we have 16 operators, not two: what to do??

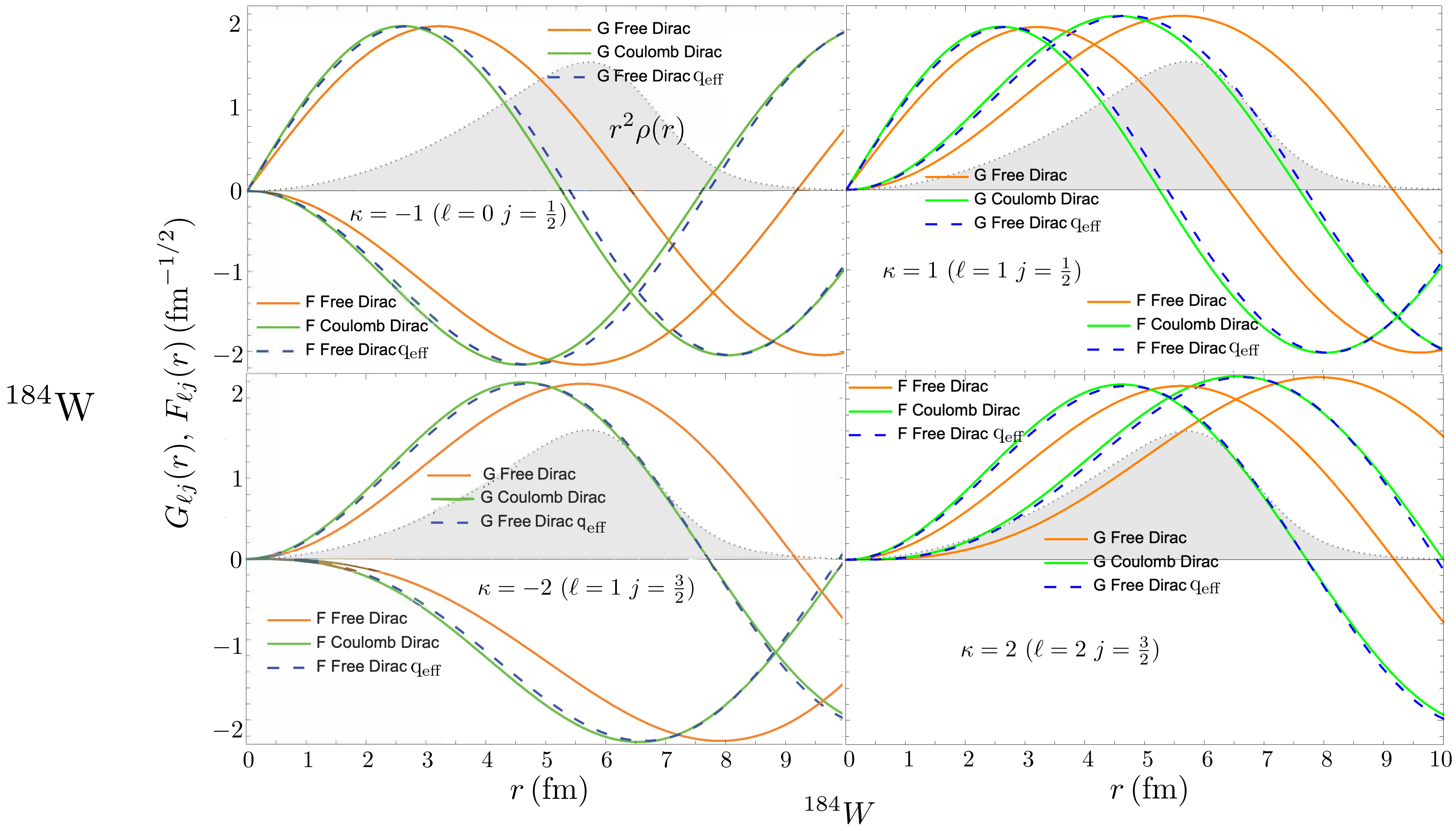
- There is a simple trick,

$$U(q,s)e^{i\vec{q}\cdot\vec{r}} \rightarrow \frac{q_{\text{eff}}}{q}\sqrt{\frac{E_e}{2m_e}} \begin{pmatrix} \xi_s \\ \vec{\sigma}_L \cdot \hat{q}\ \xi_s \end{pmatrix} e^{i\vec{q}_{\text{eff}}\cdot\vec{r}}$$

using an effective local electron momentum obtained from averaging the Coulomb potential over a nuclear volume: yields an effective plane wave, to which all the technology of spherical Bessel vector harmonics can be applied



^{27}Al
$G_{\ell j}(r),\ F_{\ell j}(r)$ (fm$^{-1/2}$)
$r^2\rho(r)$
G Free Dirac
G Coulomb Dirac
G Free Dirac $q_{\rm eff}$
F Free Dirac
F Coulomb Dirac
F Free Dirac $q_{\rm eff}$
$\kappa = -1$ ($\ell = 0$ $j = \frac{1}{2}$)
$\kappa = 1$ ($\ell = 1$ $j = \frac{1}{2}$)
$\kappa = -2$ ($\ell = 1$ $j = \frac{3}{2}$)
$\kappa = 2$ ($\ell = 2$ $j = \frac{3}{2}$)
r (fm)
r (fm)



^{184}W
$G_{\ell j}(r),\ F_{\ell j}(r)\ (\mathrm{fm}^{-1/2})$
G Free Dirac
G Coulomb Dirac
G Free Dirac $q_{\rm eff}$
$r^2\rho(r)$
$\kappa = -1\ (\ell = 0\ j = \frac{1}{2})$
F Free Dirac
F Coulomb Dirac
F Free Dirac $q_{\rm eff}$
$\kappa = 1\ (\ell = 1\ j = \frac{1}{2})$
$\kappa = -2\ (\ell = 1\ j = \frac{3}{2})$
$\kappa = 2\ (\ell = 2\ j = \frac{3}{2})$
r (fm)
^{184}W

CLFV Decay Rate

$$\omega = \frac{G_F^2}{\pi}\frac{q_{\rm eff}^2}{1+\frac{q}{M_T}}|\phi_{1s}^{Z_{\rm eff}}(\vec{0})|^2 \sum_{\tau=0,1}\sum_{\tau'=0,1}\Big\{ \Big[\tilde{R}_M^{\tau\tau'}\ W_M^{\tau\tau'}(q_{\rm eff}) + \tilde{R}_{\Sigma''}^{\tau\tau'}\ W_{\Sigma''}^{\tau\tau'}(q_{\rm eff}) + \tilde{R}_{\Sigma'}^{\tau\tau'}\ W_{\Sigma'}^{\tau\tau'}(q_{\rm eff})\Big]$$
$$+\frac{q_{\rm eff}^2}{m_N^2}\Big[\tilde{R}_{\Phi''}^{\tau\tau'}\ W_{\Phi''}^{\tau\tau'}(q_{\rm eff}) + \tilde{R}_{\tilde{\Phi}'}^{\tau\tau'}\ W_{\tilde{\Phi}'}^{\tau\tau'}(q_{\rm eff}) + \tilde{R}_{\Delta}^{\tau\tau'}\ W_{\Delta}^{\tau\tau'}(q_{\rm eff})\Big]$$
$$-\frac{2q_{\rm eff}}{m_N}\Big[\tilde{R}_{\Phi''M}^{\tau\tau'}\ W_{\Phi''M}^{\tau\tau'}(q_{\rm eff}) + \tilde{R}_{\Delta\Sigma'}^{\tau\tau'}\ W_{\Delta\Sigma'}^{\tau\tau'}(q_{\rm eff})\Big]\Big\}$$

$R_i^{\tau\tau'}(q_{\rm eff}) \leftrightarrow$ "CLFV particle physics"

$$\tilde{R}_M^{\tau\tau'} = \tilde{c}_1^\tau \tilde{c}_1^{\tau'*} + \tilde{c}_{11}^\tau \tilde{c}_{11}^{\tau'*}$$
$$\tilde{R}_{\Phi''}^{\tau\tau'} = \tilde{c}_3^\tau \tilde{c}_3^{\tau'*} + (\tilde{c}_{12}^\tau - \tilde{c}_{15}^\tau)\left(\tilde{c}_{12}^{\tau'*} - \tilde{c}_{15}^{\tau'*}\right)$$
$$\tilde{R}_{\Phi''M}^{\tau\tau'} = {\rm Re}\left[\tilde{c}_3^\tau \tilde{c}_1^{\tau'*} - (\tilde{c}_{12}^\tau - \tilde{c}_{15}^\tau)\,\tilde{c}_{11}^{\tau'*}\right]$$
$$\tilde{R}_{\tilde{\Phi}'}^{\tau\tau'} = \tilde{c}_{12}^\tau \tilde{c}_{12}^{\tau'*} + \tilde{c}_{13}^\tau \tilde{c}_{13}^{\tau'*}$$
$$\tilde{R}_{\Sigma''}^{\tau\tau'} = (\tilde{c}_4^\tau - \tilde{c}_6^\tau)(\tilde{c}_4^{\tau'*} - \tilde{c}_6^{\tau'*}) + \tilde{c}_{10}^\tau \tilde{c}_{10}^{\tau'*}$$
$$\tilde{R}_{\Sigma'}^{\tau\tau'} = \tilde{c}_4^\tau \tilde{c}_4^{\tau'*} + \tilde{c}_9^\tau \tilde{c}_9^{\tau'*}$$
$$\tilde{R}_{\Delta}^{\tau\tau'} = \tilde{c}_5^\tau \tilde{c}_5^{\tau'*} + \tilde{c}_8^\tau \tilde{c}_8^{\tau'*}$$
$$\tilde{R}_{\Delta\Sigma'}^{\tau\tau'} = {\rm Re}\left[\tilde{c}_5^\tau \tilde{c}_4^{\tau'*} + \tilde{c}_8^\tau \tilde{c}_9^{\tau'*}\right]$$

$W_i^{\tau\tau'}(q_{\rm eff}) \leftrightarrow$ "Nuclear dials"

vary these by picking the right nuclear targets

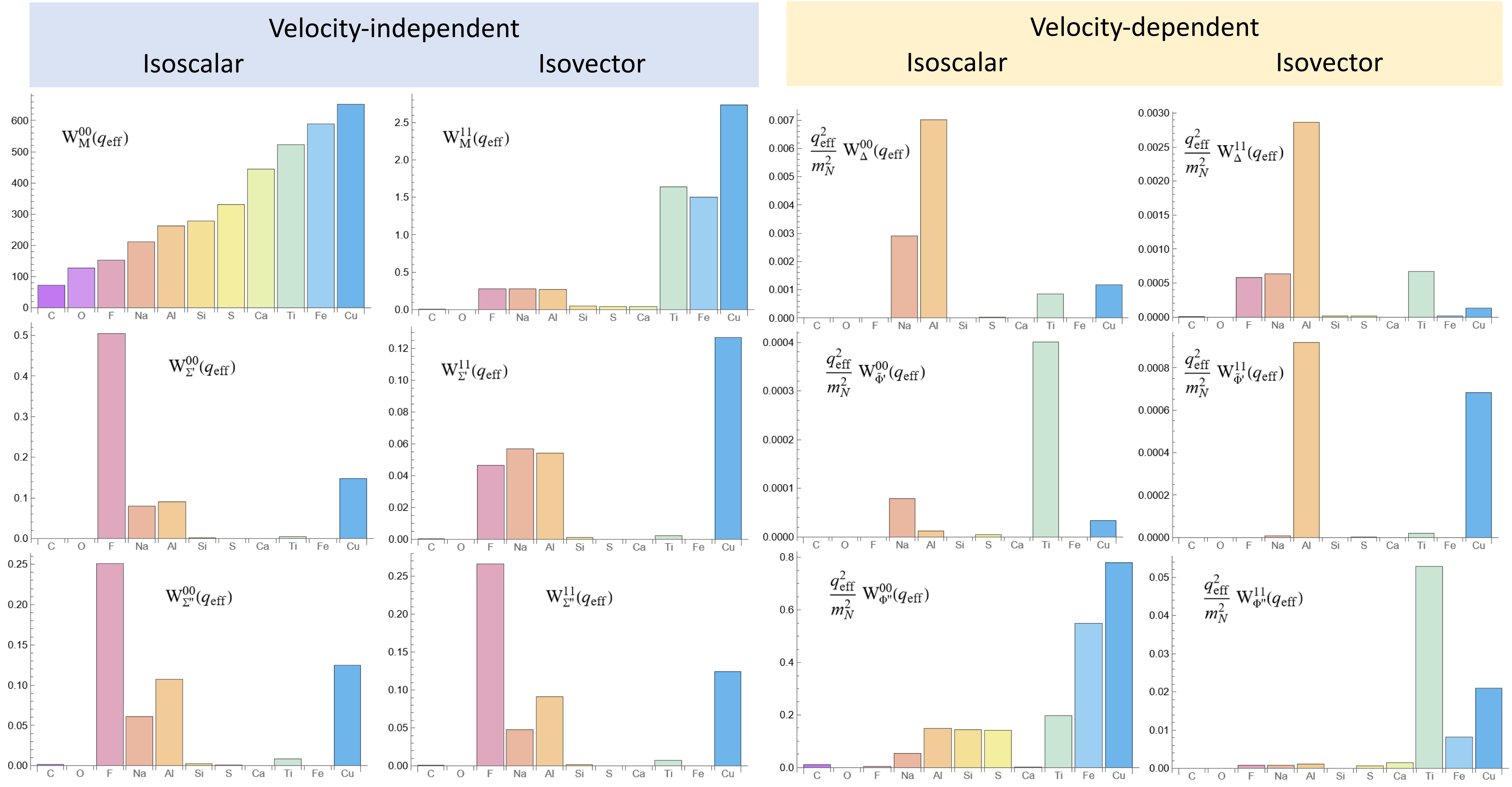
to determine these

- We find a form with six response functions: the result we deduced from symmetry arguments alone emerges from a detailed treatment of the NRET

- The experimental goal is thus most efficiently expressed at the nuclear scale: the extraction of the six $\tilde{R}_i^{\tau\tau'}$

- This in principle can be done by carefully selecting targets with the requisite properties

- Once extracted, these “nuclear LECs” become universal constraints on all higher level theories of CLFV


Velocity-independent
Isoscalar
Isovector
Velocity-dependent
Isoscalar
Isovector
$W_M^{00}(q_{eff})$
$W_M^{11}(q_{eff})$
$\frac{q_{eff}^2}{m_N^2} W_\Delta^{00}(q_{eff})$
$\frac{q_{eff}^2}{m_N^2} W_\Delta^{11}(q_{eff})$
$W_{\Sigma'}^{00}(q_{eff})$
$W_{\Sigma'}^{11}(q_{eff})$
$\frac{q_{eff}^2}{m_N^2} W_{\tilde{\Phi}'}^{00}(q_{eff})$
$\frac{q_{eff}^2}{m_N^2} W_{\tilde{\Phi}'}^{11}(q_{eff})$
$W_{\Sigma''}^{00}(q_{eff})$
$W_{\Sigma''}^{11}(q_{eff})$
$\frac{q_{eff}^2}{m_N^2} W_{\Phi''}^{00}(q_{eff})$
$\frac{q_{eff}^2}{m_N^2} W_{\Phi''}^{11}(q_{eff})$
C O F Na Al Si S Ca Ti Fe Cu

Nucleon-level NRET

Allowed

c_1^1, c_{11}^1

W_M^{11}

c_4, c_6, c_{10}

$W_{\Sigma''}$

c_4, c_9

$W_{\Sigma'}$

$v_{\rm N}$-associated

$c_3^1, c_{12}^1, c_{15}^1$

$\frac{q^2}{m_N^2} W_{\Phi''}^{11}$

c_{12}, c_{13}

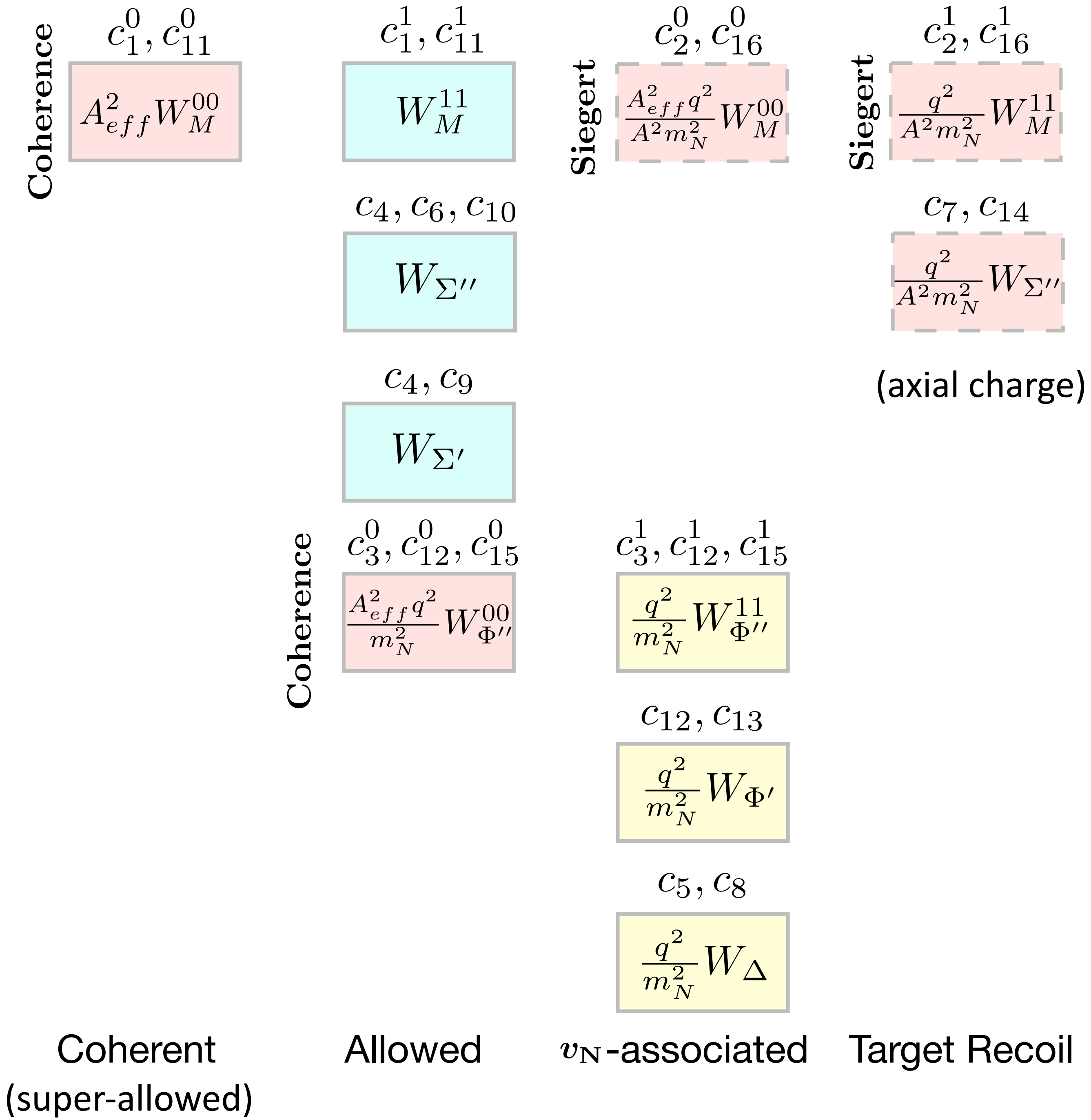
$\frac{q^2}{m_N^2} W_{\Phi'}$

c_5, c_8

$\frac{q^2}{m_N^2} W_{\Delta}$

Coherent (super-allowed) | Allowed | $v_{\rm N}$-associated | Target Recoil

altered by
nuclear embedding



Coherent vector charge operator

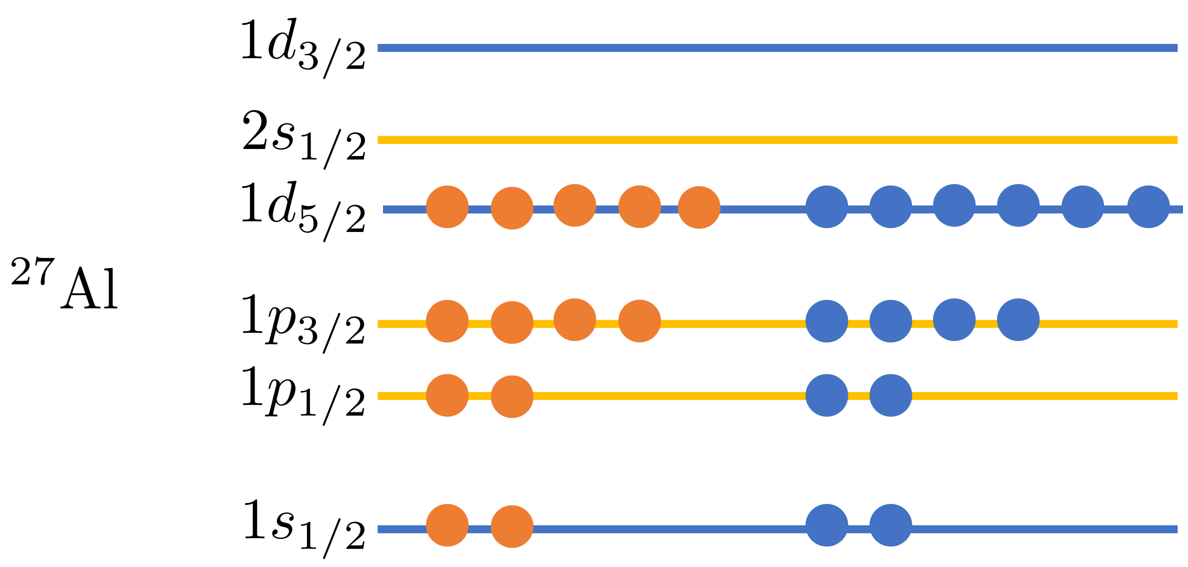
$$M_{00}(0) = \frac{1}{\sqrt{4\pi}} \sum_{i=1}^{A} 1_N(i) \sim \frac{1}{\sqrt{4\pi}} A$$

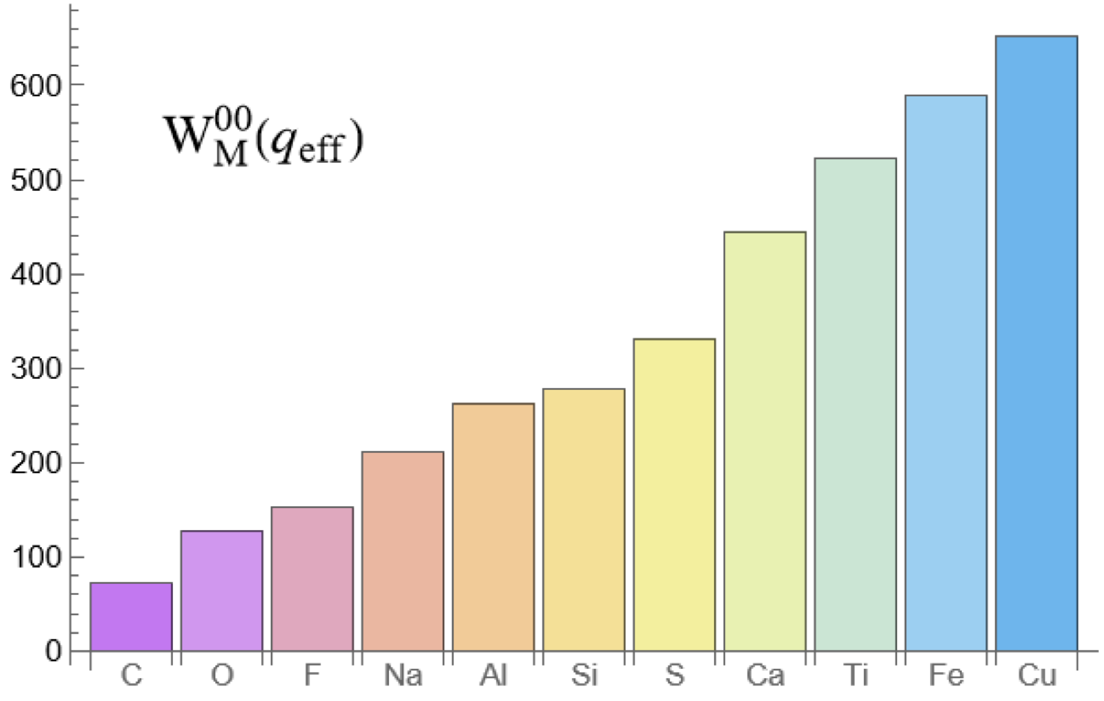
Isoscalar charge operator in the q=0 limit
sums coherently over the nucleus

Easily isolated: J=0 target

vector mediator
$\bar{\chi}_e \gamma_\mu \chi_\mu \;\; \bar{N} \gamma_\mu N$

distinctive nuclear pattern

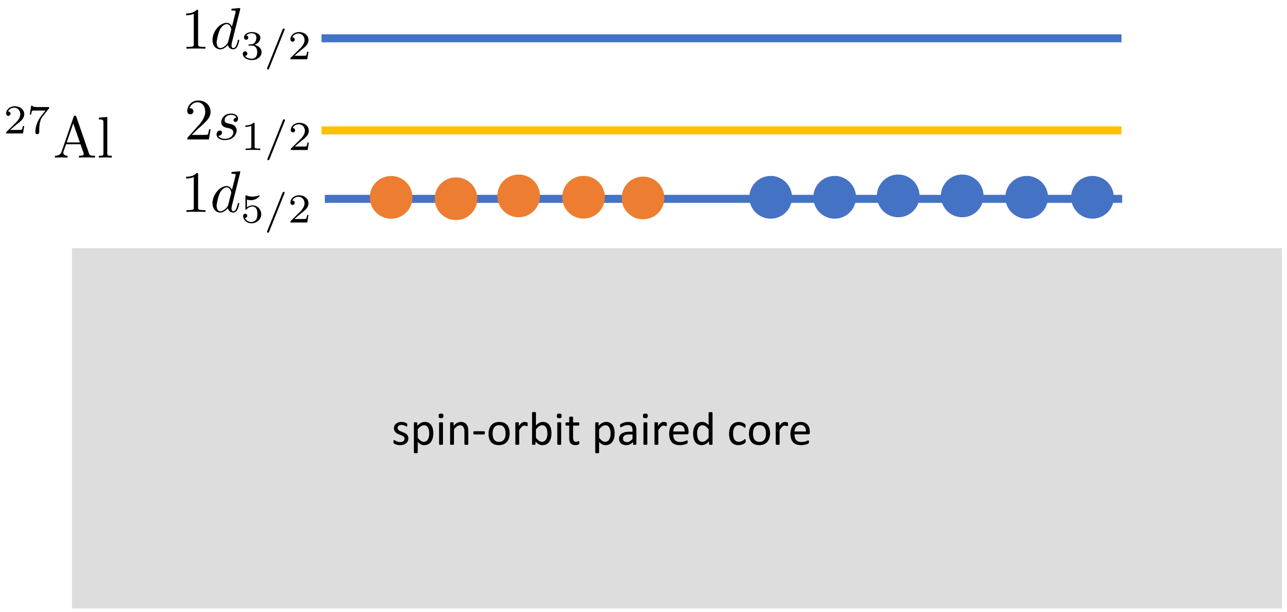




Second scalar coherent operator

$$\Phi''_{00}(0) = -\frac{1}{6\sqrt{\pi}} \sum_{i=1}^{A} \vec{\sigma}(i) \cdot \vec{\ell}(i)$$

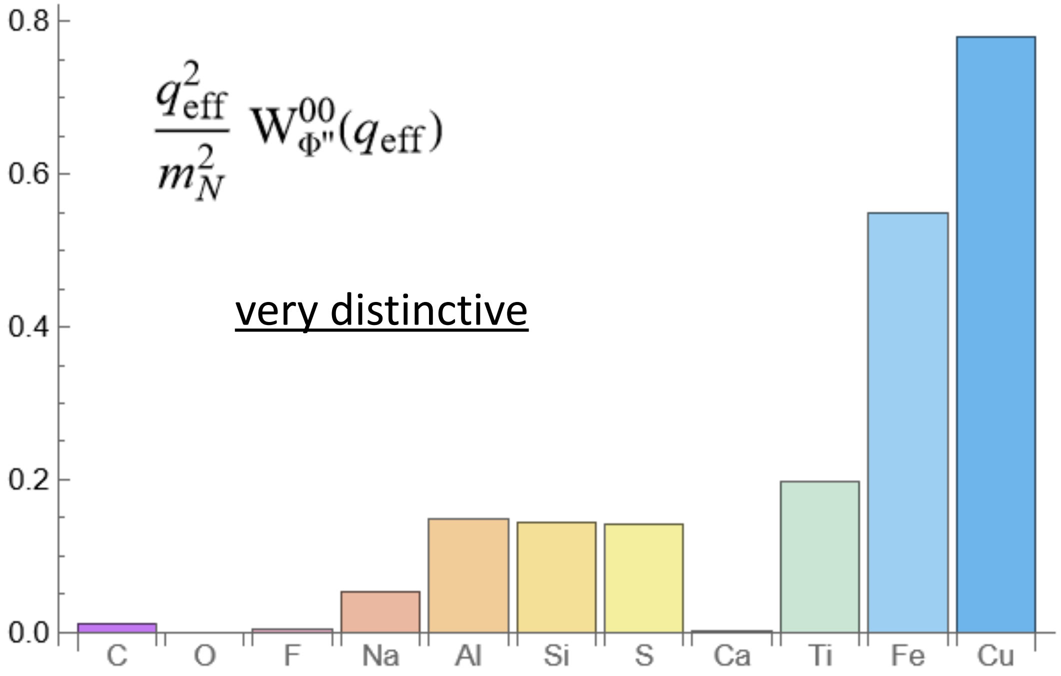
Sums coherently over $j = \ell + \frac{1}{2}$ and $j = \ell - \frac{1}{2}$ subshells but vanishes when both subshells are filled



$$\bar{\chi}_e i\sigma^{\mu\nu}\gamma^5 \chi_\mu \bar{N} i\sigma_{\mu\nu}\gamma^5 N$$

$$\omega \propto \left\langle \frac{q}{2m_N} M_{00} - \frac{q_{\rm eff}}{m_N}\Phi''_{00} \right\rangle^2$$

- Velocity-dependent contribution same order as charge monopole
- Generated by tensor-mediated interactions



Provides a nice example of experimental forensics

- Suppose a $\mu \to e$ conversion signal were found in a J=0 nucleus like ^{28}Si
- One would do a second measurement in, say, ^{40}Ca
  - If a somewhat stronger signal is seen, the vector charge operator is the source of the CLFV
  - If no signal is seen, one can deduce that the second scalar NRET operator is responsible, and consequently that the operator is tensor mediated

Limits on LECs

- Using our expression for the decay rate, we can constrain LECs using existing and future branching ratio limits for various nuclei
- We can also estimate the energy scale probed by each operator
- Assume only one operator is responsible for CLFV
- Mathematica + Python scripts to compute $B(\mu \to e)$ in terms of $\tilde{c}_i^{\tau}$ for a selection of nuclear targets

	Al		Ti	
Coupling	LEC Limit	~ Scale Probed	LEC Limit	~ Scale Probed
i=1,11 ; 0	4.0E-10	12,000 TeV	7.4E-08	910 TeV
i=1,11; 1	1.2E-08	2,200 TeV	1.3E-06	210 TeV
i=3,15; 0	1.6E-08	1,900 TeV	3.8E-06	130 TeV
i=3,15;1	1.9E-07	570 TeV	7.3E-06	91 TeV
i=4; 0	1.4E-08	2,100 TeV	1.5E-05	63 TeV
i=4; 1	1.7E-08	1,900 TeV	1.7E-05	59 TeV
i=5,8; 0	7.8E-08	880 TeV	5.8E-05	32 TeV
i=5,8; 1	1.2E-07	720 TeV	6.5E-05	30 TeV
i=6,10; 0	2.0E-08	1,800 TeV	1.8E-05	59 TeV
i=6,10; 1	2.2E-08	1,700 TeV	2.0E-05	55 TeV
i=9; 0	2.1E-08	1,700 TeV	2.8E-05	47 TeV
i=9; 1	2.8E-08	1,500 TeV	3.4E-05	42 TeV
i=12; 0	1.6E-08	1,900 TeV	3.8E-06	130 TeV
i=12; 1	1.4E-07	660 TeV	7.3E-06	91 TeV
i=13; 0	1.8E-06	180 TeV	8.4E-05	27 TeV
i=13; 1	2.1E-07	540 TeV	3.7E-04	13 TeV

†: P. Wintz, Proc. 1st Int. Symp. on Lepton and Baryon Number Violation

NRET Status

- The NRET defines in a crisp way the low-energy CLFV information available in elastic $\mu \to e$ experiments; six responses that in principle can be extracted from a program of measurements on suitably chosen targets

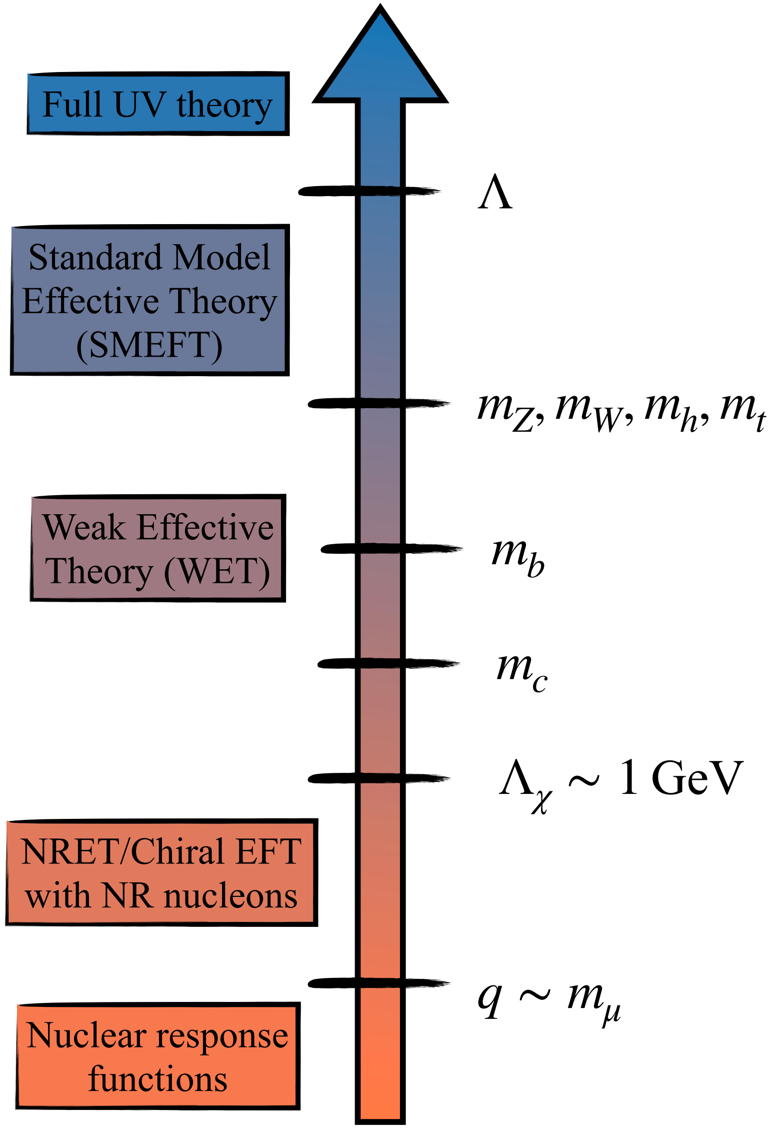
- Publicly-available Python & Mathematica codes created for $\mu \to e$ conversion, using the NRET and the best available NP; extended to include the effects of the muon's lower component ($o(v_\mu)$) — additional NRET operators

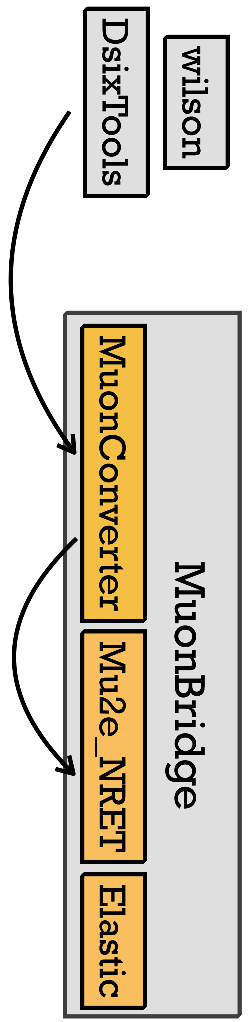
- State-of-the-art nuclear form factors via a cloud library, for almost all targets of interest

\

- Nucleon-level input can be either NR or covariant
  - 4 scalar- and 16 vector-mediated interactions $\to$ 12 NRET operators
  - 12 tensor-mediated interactions needed to match the light-quark CLFV operators of dimension $\leq$ 7 to the NRET basis (the remain 4 NRET operators then needed)

Connecting to higher scales





interfaces built to existing community tools

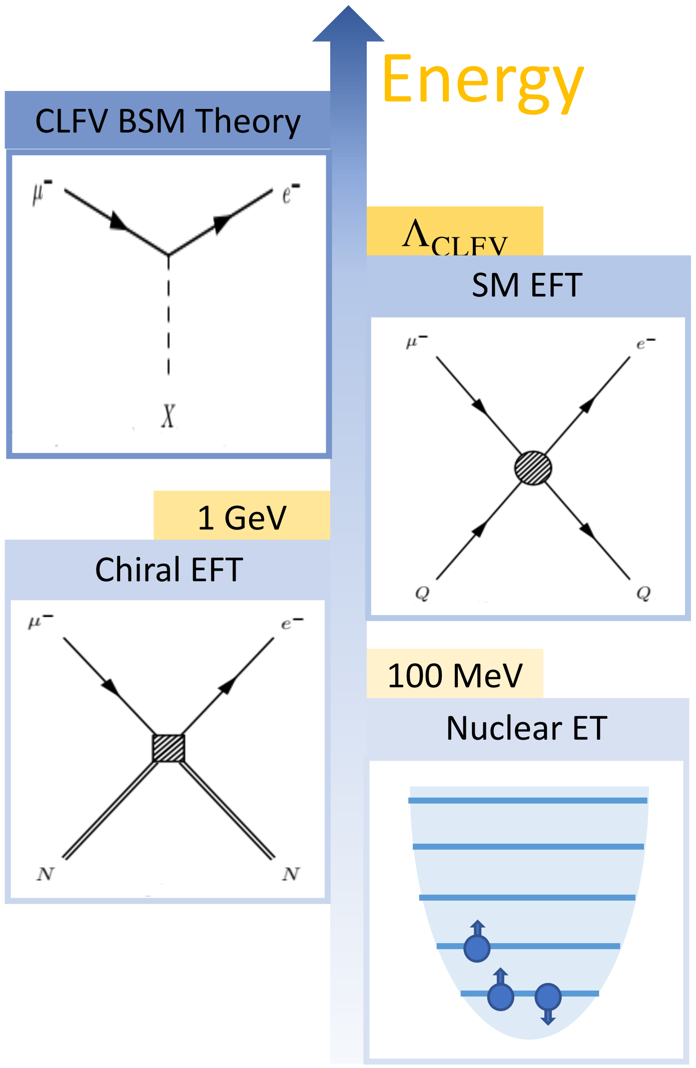
WET

$\mu = 2$ GeV

$u,\ d\ ,s$

gluons, γ s

shared with DM



Zupan, Rule, Menzo, McElvain, WH

$1 - \bar{e}_L\sigma^{\mu\nu}\mu_R F_{\mu\nu}$ $\quad 2 - (\bar{e}_L\gamma_\alpha\mu_L)(\bar{u}_L\gamma^\alpha u_L)$

$3 - (\bar{e}_L\gamma_\alpha\mu_L)(\bar{d}_L\gamma^\alpha d_L)$ $\quad 4 - (\bar{e}_L\gamma_\alpha\mu_L)(\bar{s}_L\gamma^\alpha s_L)$

$5 - (\bar{e}_R\gamma_\alpha\mu_R)(\bar{u}_R\gamma^\alpha u_R)$ $\quad 6 - (\bar{e}_R\gamma_\alpha\mu_R)(\bar{d}_R\gamma^\alpha d_R)$

$7 - (\bar{e}_R\gamma_\alpha\mu_R)(\bar{s}_R\gamma^\alpha s_R)$ $\quad 8 - (\bar{e}_L\gamma_\alpha\mu_L)(\bar{u}_R\gamma^\alpha u_R)$

$9 - (\bar{e}_L\gamma_\alpha\mu_L)(\bar{d}_R\gamma^\alpha d_R)$ $\quad 10 - (\bar{e}_L\gamma_\alpha\mu_L)(\bar{s}_R\gamma^\alpha s_R)$

$11 - (\bar{e}_R\gamma_\alpha\mu_R)(\bar{u}_L\gamma^\alpha u_L)$ $\quad 12 - (\bar{e}_R\gamma_\alpha\mu_R)(\bar{d}_L\gamma^\alpha d_L)$

$13 - (\bar{e}_R\gamma_\alpha\mu_R)(\bar{s}_L\gamma^\alpha s_L)$ $\quad 14 - (\bar{e}_L\mu_R)(\bar{u}_R u_L)$

$15 - (\bar{e}_L\mu_R)(\bar{d}_R d_L)$ $\quad 16 - (\bar{e}_L\mu_R)(\bar{s}_R s_L)$

$17 - (\bar{e}_L\mu_R)(\bar{u}_L u_R)$ $\quad 18 - (\bar{e}_L\mu_R)(\bar{d}_L d_R)$

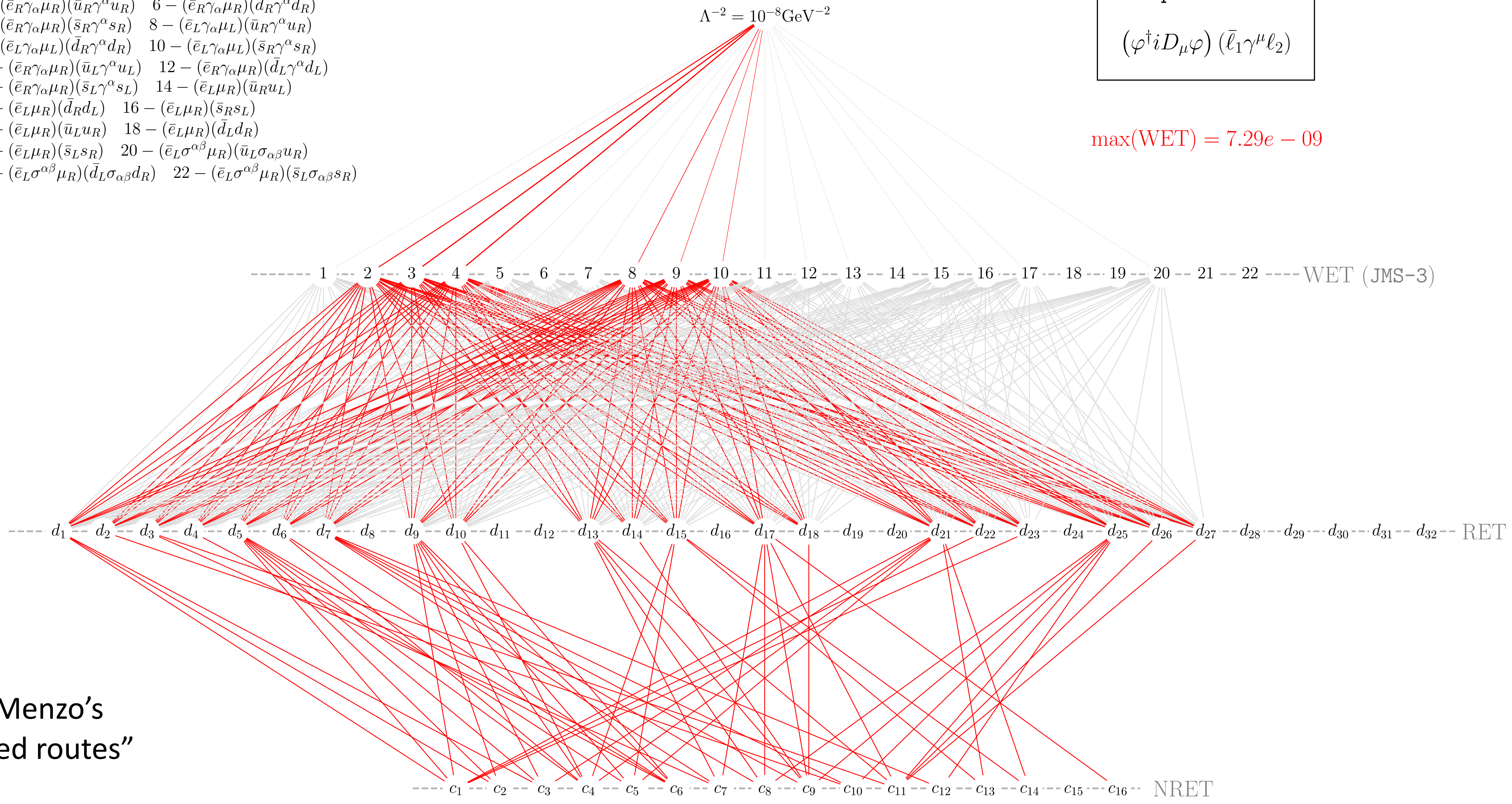
$19 - (\bar{e}_L\mu_R)(\bar{s}_L s_R)$ $\quad 20 - (\bar{e}_L\sigma^{\alpha\beta}\mu_R)(\bar{u}_L\sigma_{\alpha\beta}u_R)$

$21 - (\bar{e}_L\sigma^{\alpha\beta}\mu_R)(\bar{d}_L\sigma_{\alpha\beta}d_R)$ $\quad 22 - (\bar{e}_L\sigma^{\alpha\beta}\mu_R)(\bar{s}_L\sigma_{\alpha\beta}s_R)$

“phil1_12”

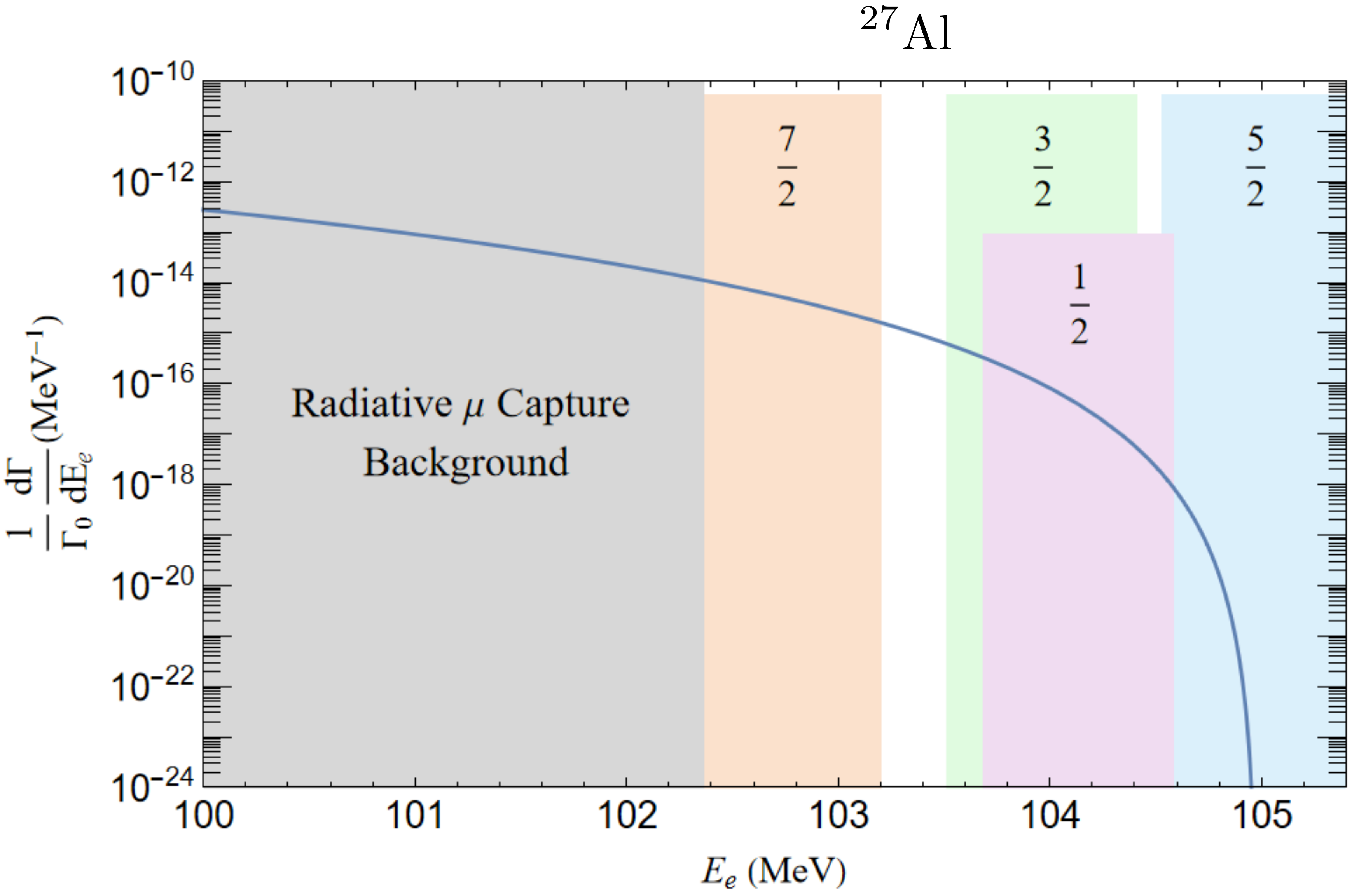
$(\varphi^\dagger i D_\mu \varphi)\,(\bar{\ell}_1\gamma^\mu\ell_2)$

$\max(\mathrm{WET}) = 7.29e - 09$



Tony Menzo’s “United routes” plot

A task remaining: inelastic



Thanks!