Nucleon D-term in the Sakai-Sugimoto model

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with Mitsutoshi Fujita, Shigeki Sugimoto, Takahiro Ueda, preprint tonight

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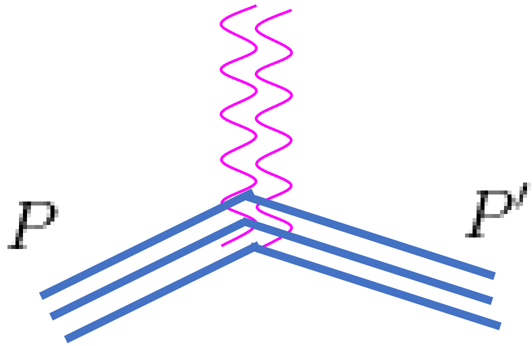
Contents

- Gravitational form factors
- D-term and its experimental probes
- Holographic calculation of the D-term

  Naïve approach
  More serious approach

Nucleon gravitational form factors



Off-forward matrix element of the QCD energy momentum tensor

$$T^{\mu\nu} = -F^{\mu\alpha}F^{\nu}{}_{\alpha} + \frac{\eta^{\mu\nu}}{4}F^{\alpha\beta}F_{\alpha\beta} + \bar{\psi}i\gamma^{(\mu}D^{\nu)}\psi$$

$$\langle P'|T^{\mu\nu}|P\rangle = \bar{u}(P')\left[A(t)\gamma^{(\mu}\bar{P}^{\nu)} + B(t)\frac{\bar{P}^{(\mu}i\sigma^{\nu)\alpha}\Delta_\alpha}{2M} + D(t)\frac{\Delta^\mu\Delta^\nu - g^{\mu\nu}\Delta^2}{4M}\right]u(P)$$

Form factors associated with scattering off a graviton

For a spin-1/2 hadron, there are 3 independent form factors.

Cannot be measured directly, but indirectly, we can!

1 graviton $\approx$ 2 photons, or 2 gluons

D-term: the last global unknown

$D(t=0)$ is a fundamental conserved charge of the proton, just like mass and spin!

The value, even the sign, is unknown at the moment. No entry in the Particle Data Group.

Spatial components of the energy momentum tensor
→ May be interpreted as internal `pressure' exerted by quarks and gluons

Polyakov (2003)

$$T^{ij}(\boldsymbol{r}) = \left(\frac{r^i r^j}{r^2} - \frac{1}{3}\delta^{ij}\right) s(r) + \delta^{ij}\, p(r) \qquad D = M \int d^3r\, r^2 p(r)$$

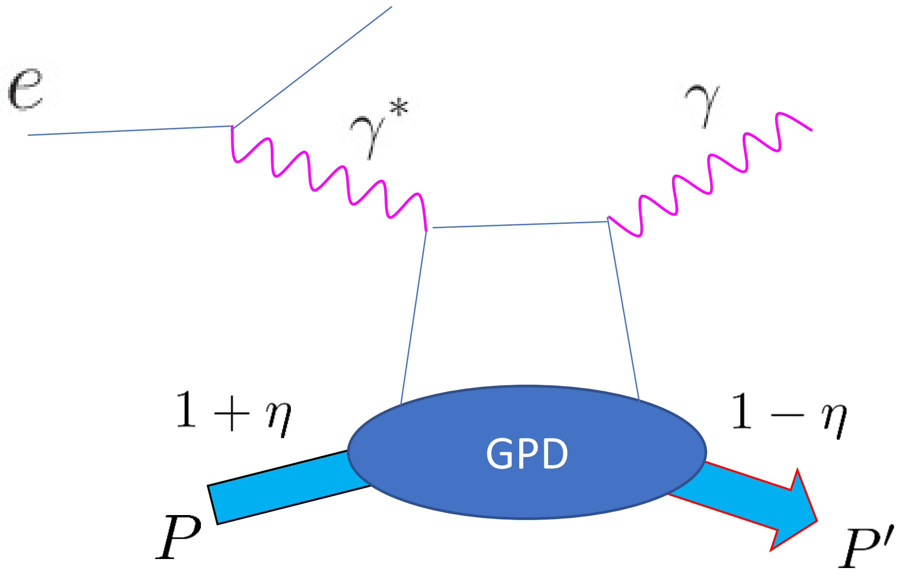
Conjecture: Stable hadrons must have a negative D-term $D(t=0) < 0$

No field theoretical proof.
The word `pressure' should not be taken literally in its usual sense in thermodynamics

Deeply Virtual Compton Scattering (DVCS)

Experimental access to GPDs
Measured at HERA, CERN, Jlab,…

1 graviton $\approx$ 2 photons



$$i\int d^4y e^{iqy}\langle P'|T\{J^\mu(y)J^\nu(0)\}|P\rangle$$

$$= -(g^{\mu+}g^{\nu-}+g^{\nu+}g^{\mu-}-g^{\mu\nu})\underbrace{\int\frac{dx}{2}\left(\frac{1}{x+\xi-i\epsilon}+\frac{1}{x-\xi+i\epsilon}\right)H_q(x,\eta,\Delta)}_{\text{Compton form factor } \mathcal{H}_q}\bar{u}(P')\gamma^+u(P)+\cdots$$

skewness $\eta = \frac{-\Delta\cdot q}{2P\cdot q}$

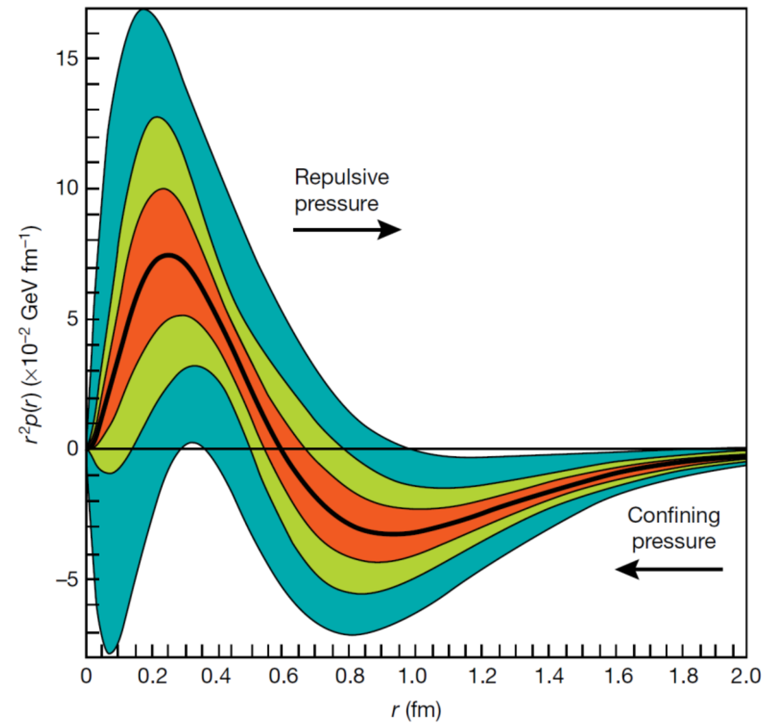
Extraction of (quark) D-term from DVCS

$$D = D_u + D_d + D_s + D_g + \cdots$$

$D_{u,d}$ related to the subtraction constant in the dispersion relation for the Compton form factor Teryaev (2005)

$$\mathrm{Re}\mathcal{H}_q(\xi,t) = \frac{1}{\pi}\int_{-1}^{1} dx \mathrm{P}\frac{\mathrm{Im}\mathcal{H}_q(x,t)}{\xi - x} + 2\int_{-1}^{1} dz \frac{D_q(z,t)}{1-z}$$

Burkert, Elouadrhiri, Girod (2018)

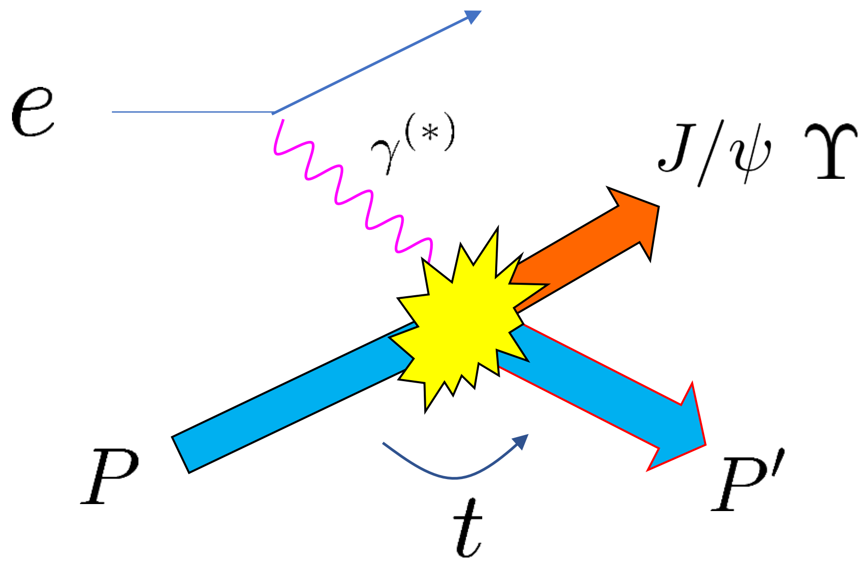


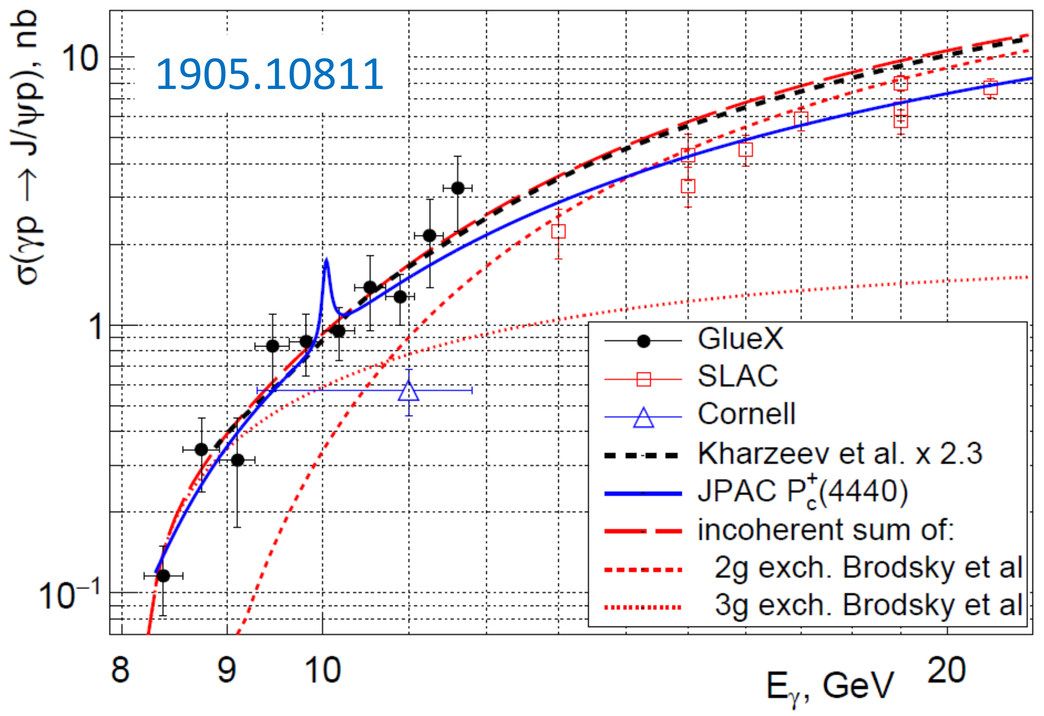
HOWEVER, this is not directly proportional to what we want $\int_{-1}^{1} dz z D_q(z,t) = D_q(t)$

The difference is due to twist-2, higher-spin ($j > 2$) operators $\bar{q}\gamma^+(\overleftrightarrow{D}^+)^{j-1}q$

Two-photon states probe not just spin-2, but infinitely many higher spin correlations.
How can we isolate the spin-2 component?

Quarkonium production near threshold



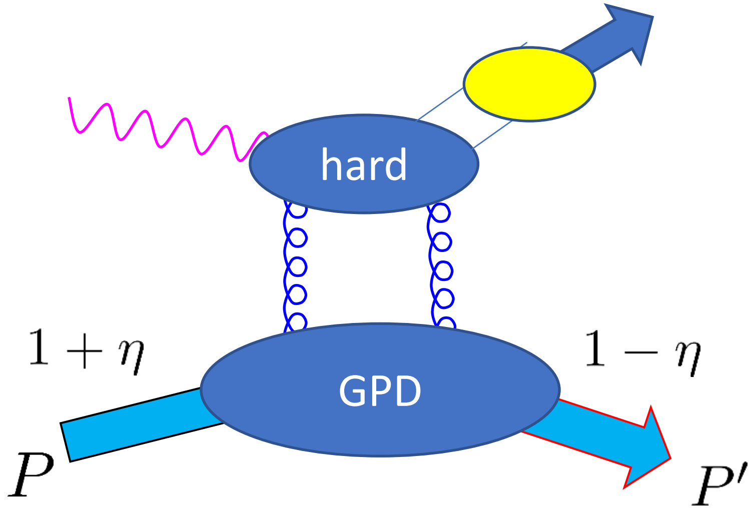


Ongoing experiments at Jlab (Glue-X collaboration), future measurement at EIC?

Heavy quarks interact with proton only via gluon exchanges.
→ Study of GFFs, in particular, the gluon D-term $D_g(t)$

1 graviton $\approx$ 2 gluons

Connection to the gluon energy momentum tensor



Assume factorization works.
Amplitude proportional to the Compton form factor

$$\int_{-1}^{1}\frac{dx}{x}\left(\frac{1}{\eta-x-i\epsilon}-\frac{1}{\eta+x-i\epsilon}\right)H_g(x,\eta,t)$$

skewness $\eta=\dfrac{P^+-P'^+}{P'^++P^+}$

gluon GPD

Near the threshold, and when $Q^2\to\infty$ or $M_q\to\infty$, $\boxed{\eta\approx 1}$

Just like in DVCS, two-gluon state couples to infinitely many operators with different spins, not just $T^{\mu\nu}$.

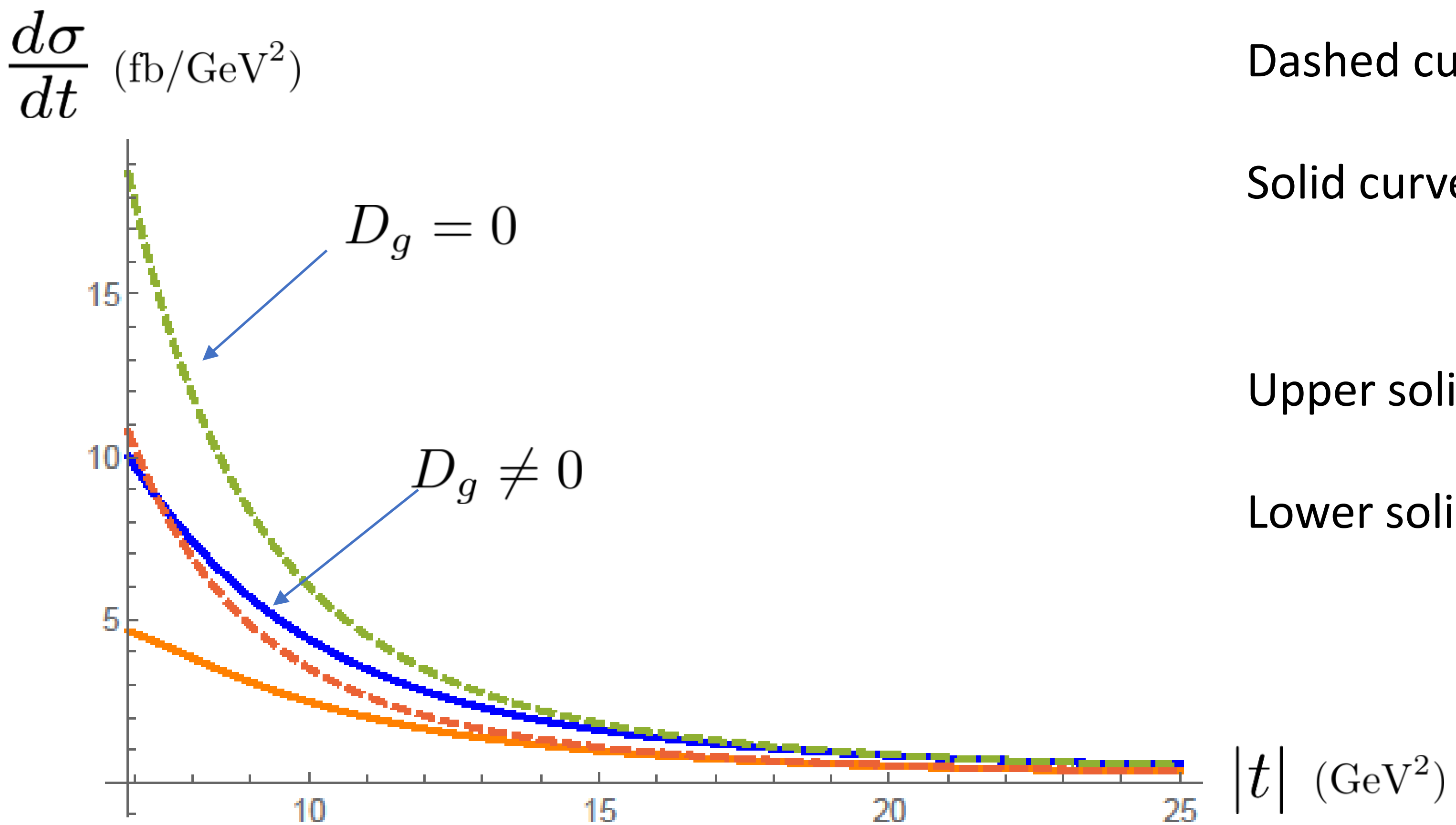
HOWEVER, unlike in DVCS, $T^{\mu\nu}$ dominates over all the other twist-2 operators combined!

YH, Strikman (2021); Guo, Ji, Liu (2021)

Prediction for the Electron-Ion Collider (EIC)

Boussarie, YH (2020)

$J/\psi \qquad Q^2 = 64\,\mathrm{GeV}^2 \quad \sqrt{S_{ep}} = 20\,\mathrm{GeV} \qquad W = 4.4\,\mathrm{GeV}$



Dashed curves: without gluon D-term

Solid curves: with gluon D-term

Upper solid $b = 1$

Lower solid $b = 0$

$$\langle P|\frac{\beta}{2g}F^2|P\rangle = 2M^2(1-b)$$

`D’ for D-brane
Calculating the D-term in holographic QCD

Fujita, YH, Sugimoto, Ueda, arXiv:2206:xxxxx

D-term has been calculated in all sorts of models.

Gauge/string duality may be very useful. One can literally exchange gravitons, albeit in extra dimensions.

Previous attempts in `bottom-up’ AdS/QCD models

Abidin, Carlson (2009) $D=0$
Mamo, Zahed (2019~) $D\neq 0$ only after including $1/N_c$ corrections

The Sakai-Sugimoto model—one of the most successful, `top-down’ holographic QCD models.

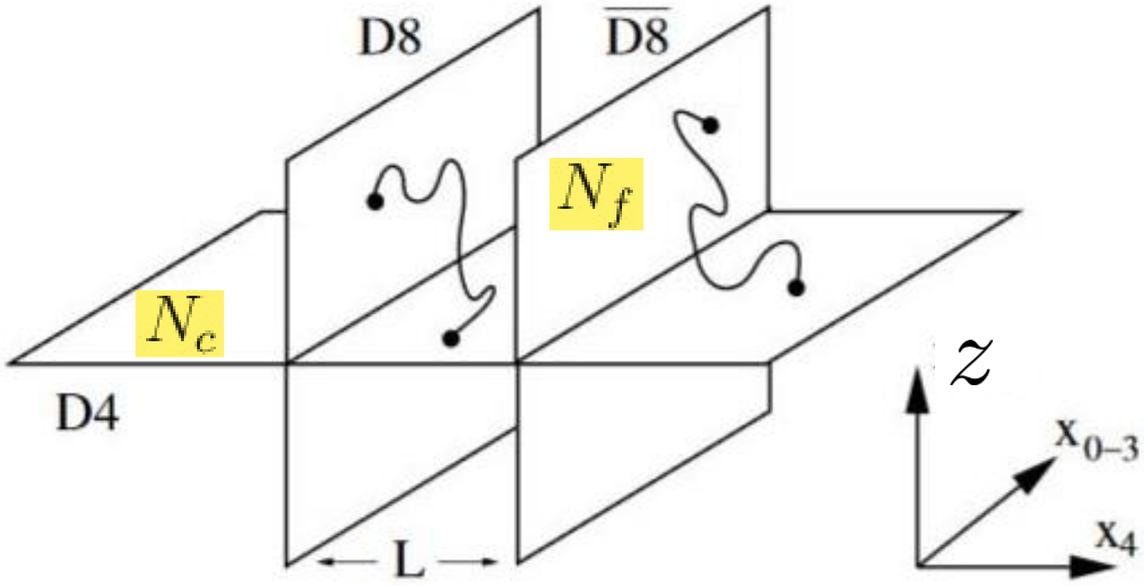
The Sakai-Sugimoto model (2004)

Dp brane: (p+1)-dim object on which strings can end.

D4/D8 brane configuration in type IIA superstring in 10 dimensions.

Low-energy effective model of QCD at large-Nc

and at strong coupling.



Only two parameters

$$\lambda = g^2 N_c = 16.63 \qquad M_{KK} = 949\,\text{MeV}$$

Features confinement, chiral symmetry breaking and its restoration at finite-T.

Successful phenomenology: meson/baryon/glueball masses, excited states, couplings and decay constants, EM form factors, U_A(1) anomaly, but no application to GFFs till now.

Hadrons in the Sakai-Sugimoto model

Two-flavor SS model→ 5D U(2) gauge theory on the `flavor' D8 branes

$$S=-\kappa\int d^4x dz \,\mathrm{tr}\left[\frac{1}{2}h(z)F_{\mu\nu}^2+k(z)F_{\mu z}^2\right]-\frac{\kappa}{2}\int d^4x dz\left(\frac{1}{2}h(z)\widehat{F}_{\mu\nu}^2+k(z)\widehat{F}_{\mu z}^2\right)+S_{CS}$$

SU(2) U(1) Chern-Simons

Warp factors $h(z)=(1+z^2)^{-1/3}, \qquad k(z)=1+z^2,$

Mesons→ eigenmodes of the EOM for gauge fields

SU(2) $\pi, \rho, a_1, \cdots$

U(1) $\omega, \eta', \cdots$

Baryons → static soliton in 4D (x^1, x^2, x^3, z)

Near horizon $z\approx 0$ → BPST instanton → collective coordinate quantization

(not done in this work)

Nucleon mass and stability

Hata, Sakai, Sugimoto, Yamato (2007)

$$M = \int d^3x T^{cl}_{00}(\vec{x}) = \kappa \int d^3x dz \mathrm{tr} \left[\frac{h(z)}{2} F_{ij}^2 + k(z) F_{iz}^2 \right] + \frac{\kappa}{2} \int d^3x dz \left[h(z)(\partial_i \widehat{A}_0)^2 + k(z)(\partial_z \widehat{A}_0)^2 \right]$$

$$= 8\pi^2\kappa \left(1 + \mathcal{O}(\rho^2) + \mathcal{O}\left(\frac{1}{\lambda^2 \rho^2} \right) \right)$$

SU(2) U(1)

$$F_{ij} = \frac{2\rho^2}{(\xi^2+\rho^2)^2} \epsilon_{ija} \tau^a$$

$$F_{iz} = -\frac{2\rho^2}{(\xi^2+\rho^2)^2} \tau_i$$

$$\xi^2 \equiv (\vec{x} - \vec{X})^2 + (z - Z)^2$$

Radius ρ (instanton size) stabilized by the attractive (isovector, SU(2)) and repulsive (isoscalar, U(1)) forces.

$$\rho = \sqrt{\frac{27\pi}{\lambda} \sqrt{\frac{6}{5}}}$$

Sounds familiar from the Skyrme model? Adkins, Nappi (1984)

One can **derive** the Skyrme model from the SS model.

Calculation of the D form factor: A first look

Naively, the D-form factor D(k) is obtained by simply Fourier transforming the `classical' energy momentum tensor evaluated on-shell

$$\begin{aligned} T^{cl}_{ij}(\vec{x}) \;&=\; 2\kappa\int_{-\infty}^{\infty} dz\,\mathrm{tr}\left[h(z)F_{il}F_{jl} + k(z)F_{iz}F_{jz} - \frac{\delta_{ij}}{2}\left(\frac{h(z)}{2}F_{lm}^2 + k(z)F_{lz}^2\right)\right] \\ &\quad +\kappa\int_{-\infty}^{\infty} dz\left[-h(z)\partial_i\widehat{A}_0\partial_j\widehat{A}_0 + \frac{\delta_{ij}}{2}\left(h(z)(\partial_i\widehat{A}_0)^2 + k(z)(\partial_z\widehat{A}_0)^2\right)\right]. \end{aligned}$$

$$T^{cl}_{ij}(\vec{k}) = (k_ik_j - \delta_{ij}\vec{k}^2)\frac{D(|\vec{k}|)}{4M}$$

Similar to what's been done in the Skyrme and chiral soliton models
e.g., Cebulla, Goeke, Ossmann, Schweitzer (2007)

The instanton approximation insufficient (unlike in the calculation of mass).
Need to know the soliton solution everywhere $|z| < \infty$, not just near the horizon $z \approx 0$

Warning: This procedure is valid only at $k = 0$

Solution near the Minkowski boundary

When $|z| \gg 1$, SU(2) gauge fields Abelianize

$$\widehat{A}_0 \approx \frac{-108\pi^3}{\lambda} G,$$
$$A_i^g \approx 2\pi^2\rho^2\tau^a(\epsilon_{ial}\partial_l + \delta_{ia}\partial_Z)G$$
$$A_z^g \approx 2\pi^2\rho^2\tau^a\partial_a H,$$

$$G(k,z,Z) = -\kappa\sum_{n=1}^{\infty}\frac{\psi_n(z)\psi_n(Z)}{k^2+m_n^2}$$ Vector mesons

$$H(k,z,Z) = -\kappa\sum_{n=0}^{\infty}\frac{\phi_n(z)\phi_n(Z)}{k^2+m_n^2}$$ Axial vector mesons

massless pion

$$\frac{m_n}{M_{KK}} = \underset{\rho}{0.818}, \quad \underset{a_1}{1.2525}, \quad 1.695, \quad 2.132, \quad 2.567, \quad 3.001, \quad 3.435, \quad 3.868, \quad 4.300, \ldots$$

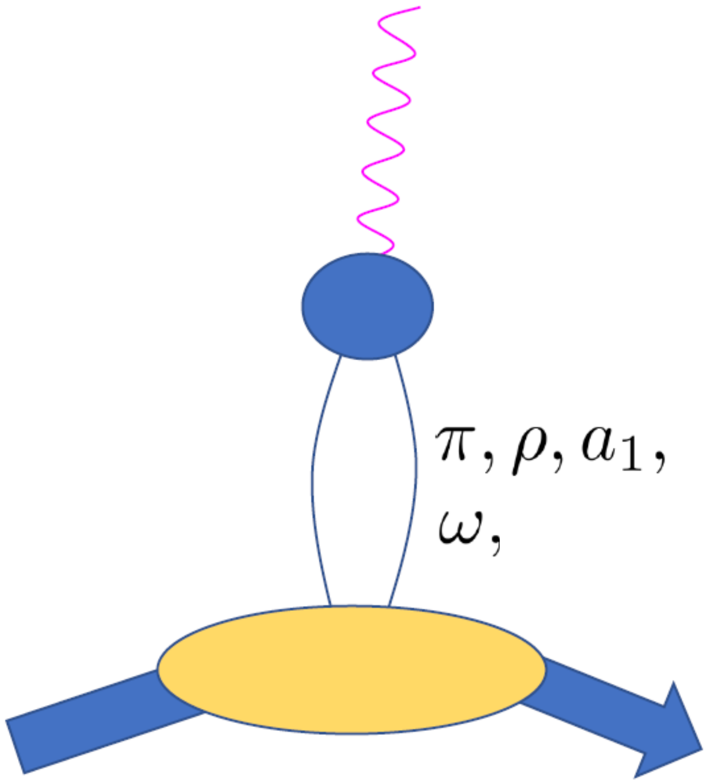
We have constructed an approximate solution that smoothly interpolates the asymptotic solution at $|z| \gg 1$ and the BPST instanton at $z \approx 0$.

Numerical result

Positive contribution from U(1) fields—isoscalar mesons ω

$$D(0) \approx 0.54 - 0.68 \approx -0.14$$

Negative contribution from SU(2) fields
isovector mesons $\pi, \rho, a_1, \cdots$, or `pion cloud'



Isoscalar mesons tend to expand the system, while isovector mesons tend to shrink it.

→ Similar to the stability argument for the nucleon mass (slide 14)

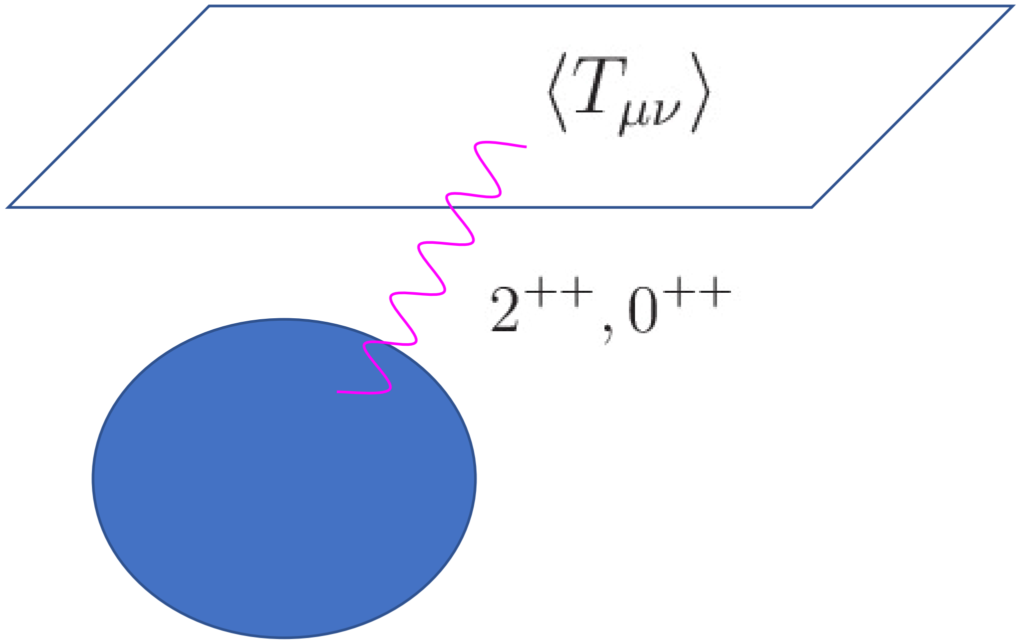
Non-abelian nature of flavor SU(2) group is crucial to get a negative sign.

Calculation of the D form factor, more seriously

The `classical' calculation in the previous slides is not satisfactory because…

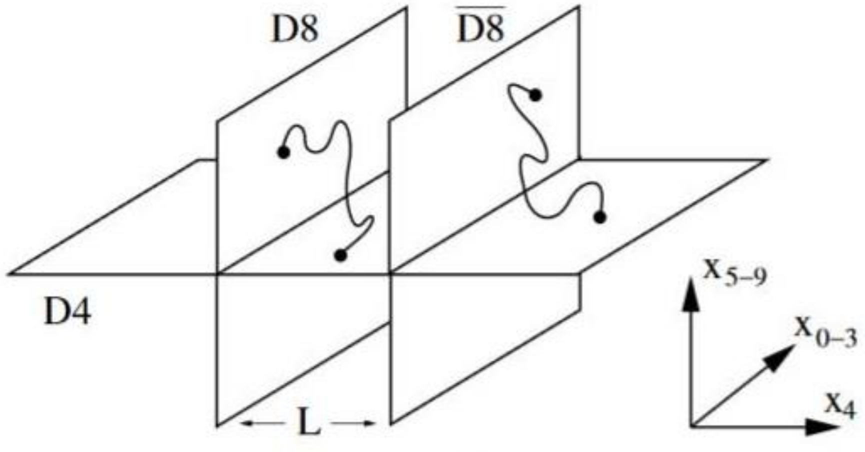
Boundary energy momentum tensor in AdS/CFT → Holographic renormalization

GFFs must be related to glueballs dual to the graviton



Mesons and baryons live in 5D (9D), but the gravitons live in 10D curved space in type IIA supergravity

There is also the dilaton. Theory is nonconformal.



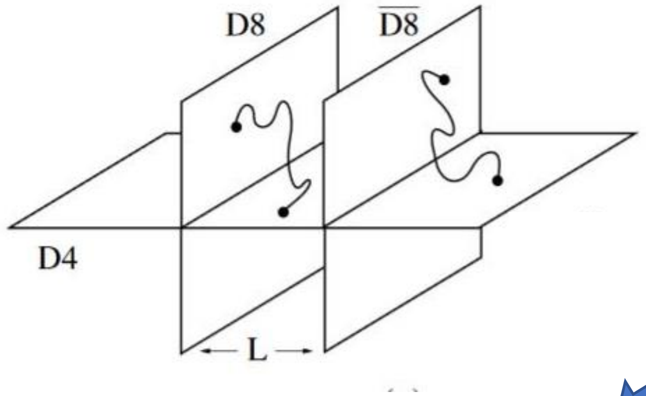
11-dimensional setup

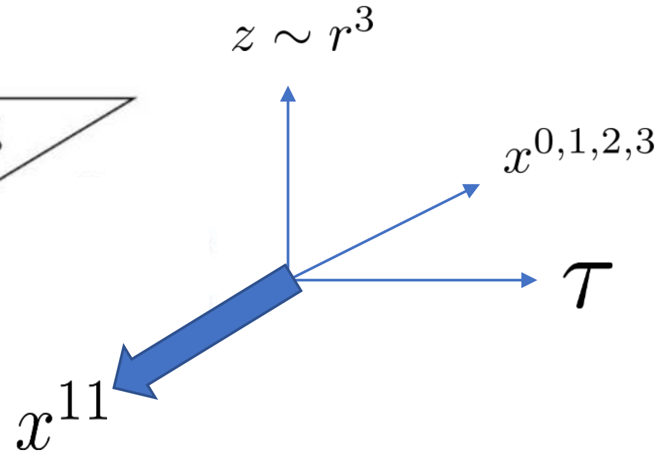
The 10D geometry in type IIA is obtained by dimensionally reducing the M-theory on $AdS_7 \times S^4$ Witten (1998)

$$ds^2 = \underbrace{\frac{r^2}{L^2}[f(r)d\tau^2 - dx_0^2 + dx_1^2 + dx_2^2 + dx_3^2 + dx_{11}^2] + \frac{L^2}{r^2}\frac{dr^2}{f(r)}}_{\mathrm{AdS}_7 \text{ black hole}} + \underbrace{\frac{L^2}{4}d\Omega_4^2}_{S^4}$$

$$f(r) = 1 - \frac{R^6}{r^6}$$

$$z = \pm\sqrt{\frac{r^6}{R^6} - 1}$$





No dilaton. Rather, the dilaton is the fluctuation of x^{11}.
Boundary theory= 6D CFT.
Compactify x^{11}, τ directions to get QCD

Glueballs

Transverse-traceless modes on AdS_7 black hole → glueballs in QCD

$$\delta g_a^a = \nabla_a \delta g^{ab} = 0$$

There are 14 TT modes, eigenfunctions of the linearized Einstein equation.

$$-\frac{d}{dr}(r^7 - r)\frac{d}{dr}T_4(r) - (m^2 r^3)T_4(r) = 0$$

Equation: J^{PC}:	T_4 $2^{++}/1^{++}/0^{++}$	V_4 $1^{-+}/0^{-+}$	S_4 0^{++}	N_4 $1^{+-}/0^{+-}$	M_4 $1^{--}/1^{--}$	L_4 0^{++}
n = 0	22.097	31.985	7.308	53.376	83.046	115.002
n = 1	55.584	72.489	46.986	109.446	143.582	189.631
n = 2	102.456	126.174	94.485	177.231	217.399	277.282
n = 3	162.722	193.287	154.981	257.958	304.536	378.099
n = 4	236.400	273.575	228.777	351.895	405.018	492.169
n = 5	323.541	368.087	315.976	459.131	518.059	619.547
n = 6	424.195	474.268	416.666	579.706	646.088	760.252
n = 7	538.487	594.231	530.950	713.638	786.559	914.307
n = 8	666.479	729.102	658.996	860.939	939.557	1081.732
n = 9	808.398	875.315	800.860	1021.613	1108.010	1262.518

Brower, Mathur, Tan (2000)

TT modes in AdS_7, origin of the D-term

$$\delta g^{\mathrm{T}_4}_{\mu\nu}(2^{++}) \sim \begin{pmatrix} 2 & \ell_j \\ \ell_i & \delta_{ij} - \frac{k_i k_j}{\vec{k}^2} \end{pmatrix}$$

Purely 4-dimensional

$$\delta g^{\mathrm{T}_4}_{ab}(0^{++}) \sim \begin{pmatrix} \delta g_{\mu\nu} & \delta g_{\mu,11} \\ \delta g_{11,\mu} & \delta g_{11,11} \end{pmatrix} \sim \begin{pmatrix} \eta_{\mu\nu} - \frac{k_\mu k_\nu}{k^2} & 0 \\ 0 & -3 \end{pmatrix}$$

Analog of dilaton
in type IIA in 10D

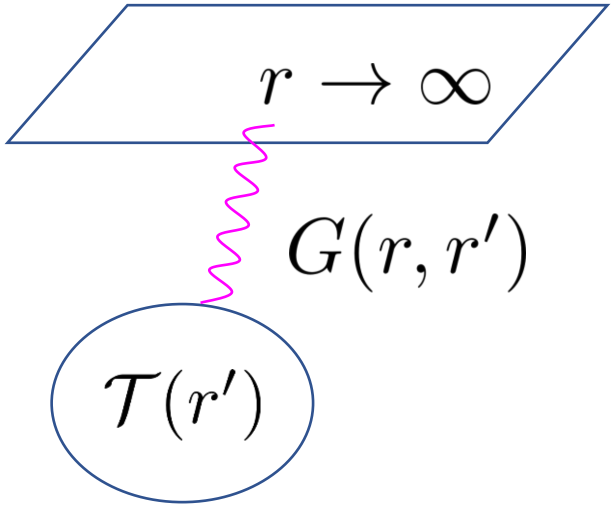
$$\delta g^{\mathrm{S}_4}_{ab} \sim \begin{pmatrix} \delta g_{\mu\nu} & & \\ & \delta g_{11,11} & \\ & & \delta g_{\tau\tau} \end{pmatrix} \sim \begin{pmatrix} \eta_{\mu\nu} - \frac{k_\mu k_\nu}{k^2} & & \\ & 1 & \\ & & -4 \end{pmatrix}$$

`Exotic' 0^{++} solution
Constable, Myers (1999)

Decouple from physical spectrum?

Boundary energy momentum tensor

Bulk baryon solution $\mathcal{T}_{AB} = \mathcal{T}_{AB}^{\mathrm{T}(2)} + \mathcal{T}_{AB}^{\mathrm{T}(0)} + \mathcal{T}_{AB}^{\mathrm{S}} + \mathcal{T}_{AB}^{\mathrm{other}}$



Metric excitation

$$\delta g_{\mu\nu} \sim G_{\mu\nu AB}^{\mathrm{T}(2)} \mathcal{T}_{\mathrm{T}(2)}^{AB} + G_{\mu\nu AB}^{\mathrm{T}(0)} \mathcal{T}_{\mathrm{T}(0)}^{AB} + G_{\mu\nu AB}^{\mathrm{S}} \mathcal{T}_{\mathrm{S}}^{AB}$$

$$\sim \frac{\langle T_{\mu\nu} \rangle}{r^4}$$

Induced energy momentum tensor on the boundary

$$\langle T_{ab} \rangle = \begin{pmatrix} (2t_2 - t_0 - s_0)\vec{k}^2 & \vec{k}^2 \ell_j & & \\ \vec{k}^2 \ell_i & \underline{(t_2 + t_0 + s_0)}(\vec{k}^2 \delta_{ij} - k_i k_j) & & \\ & & (-3t_0 + s_0)\vec{k}^2 & \\ & & & -4s_0 \vec{k}^2 \end{pmatrix}$$

Traceless in 6D,
→ trace anomaly in 4D.

$$-\langle T_\mu^\mu \rangle = \langle T_{11}^{11} \rangle + \langle T_\mathcal{T}^\mathcal{T} \rangle = M\left(A - \frac{B\vec{k}^2}{4M^2} + 3\frac{D\vec{k}^2}{4M^2}\right)$$

Solving the Einstein equation on black hole AdS_7

$$\nabla^2 \delta g_{AB} + \nabla_A \nabla_B \delta g^C_C - \nabla^C(\nabla_A \delta g_{BC} + \nabla_B \delta g_{AC}) - \frac{12}{L^2}\delta g_A \;=\; -2\kappa_7^2\left(\mathcal{T}_{AB} - \frac{g_{AB}}{5}\mathcal{T}^C_C\right)$$

$\mathrm{T}_4(2^{++})$ and $\mathrm{T}_4(0^{++})$ modes obey the same equation.

Propagation diagonal in Lorentz indices

$$G^{\mathrm{T}(2)}_{\mu\nu AB} = G^{\mathrm{T}(0)}_{\mu\nu AB} \equiv \frac{r^2 r'^2}{L^4}\tilde{G}^{\mathrm{T}}(\eta_{\mu\rho}\eta_{\nu\lambda} + \eta_{\mu\lambda}\eta_{\nu\rho})\delta^\rho_A \delta^\lambda_B$$

There is no need to distinguish them.

$$\left(T^{(\mathrm{m})}_{ab}\right) \;=\; \begin{array}{cc} \begin{array}{cccc} 0 & 1\;2\;3\;11 & r & \tau \end{array} & \\ \left(\begin{array}{c|c|c|c} T^L & V & C & 0 \\ \hline V & T^T + T^S & W & 0 \\ \hline C & W & T_{rr} & 0 \\ \hline 0 & 0 & 0 & 0 \end{array}\right) & \begin{array}{c} 0 \\ 1 \\ \vdots \\ 11 \\ r \\ \tau \end{array} \end{array}$$

$\mathcal{T}_{\tau\tau} = 0$ for the soliton solution. Naively S_4 decouples

However, the components in blue induce $\mathcal{T}_{\tau\tau}$ which must be canceled

$$\mathcal{T}_{\tau\tau} = \mathcal{T}^S_{\tau\tau} + \mathcal{T}^{\mathrm{other}}_{\tau\tau} = 0 \quad \Longrightarrow \quad \mathrm{S}_4 \text{ glueballs contribute!}$$

`Glueball dominance' of GFFs

$$\langle T_{\mu\nu}(\vec{k})\rangle = \langle T^{\rm T}_{\mu\nu}(\vec{k})\rangle + \langle T^{\rm S}_{\mu\nu}(\vec{k})\rangle$$

2++/0++ 0++

$$\langle T^{\rm T/S}_{\mu\nu}(\vec{k})\rangle = \frac{6}{L^{10}}\sum_{n=1}^{\infty}\alpha_n^{\rm T/S}\int_R^\infty dr'\,\frac{r'^3\Psi_n^{\rm T/S}(r')}{\vec{k}^2+(m_n^{\rm T/S})^2}\mathcal{T}^{\rm T/S}_{\mu\nu}(r',\vec{k})$$

GFFs dominated by the exchanges of infinitely many 0++ and 2++ glueballs, in perfect analogy to vector meson dominance of EM form factors.

$$D(|\vec{k}|) \sim \sum_{n=1}^{\infty}\frac{c_n^{\rm T}(|\vec{k}|)}{\vec{k}^2+(m_n^{\rm T})^2} + \sum_{n=1}^{\infty}\frac{c_n^{\rm S}(|\vec{k}|)}{\vec{k}^2+(m_n^{\rm S})^2}$$

Compare with the pQCD argument at large-k. Tanaka (2018); Tong, Ma, Yuan (2021)

$$D(|\vec{k}|) \sim \frac{1}{(\vec{k}^2+\Lambda^2)^3}$$

Seemingly different, but an infinite sum can change the analytic behavior.

Recovering the D-term at k=0

At k=0, the infinite sum can be evaluated exactly.

$$\lim_{r\to\infty} \tilde{G}^{\mathrm{T/S}}(r,r',\vec{k}=0) = \frac{1}{r^6}\sum_n \frac{\alpha_n^{\mathrm{T/S}}\Psi_n^{\mathrm{T/S}}(r')}{(m_n^{\mathrm{T/S}})^2} = \frac{L^7}{6r^6}$$

r'-dependence disappears. No convolution with the graviton propagator!

The D-term can be calculated classically (slide 17) without any reference to glueballs!

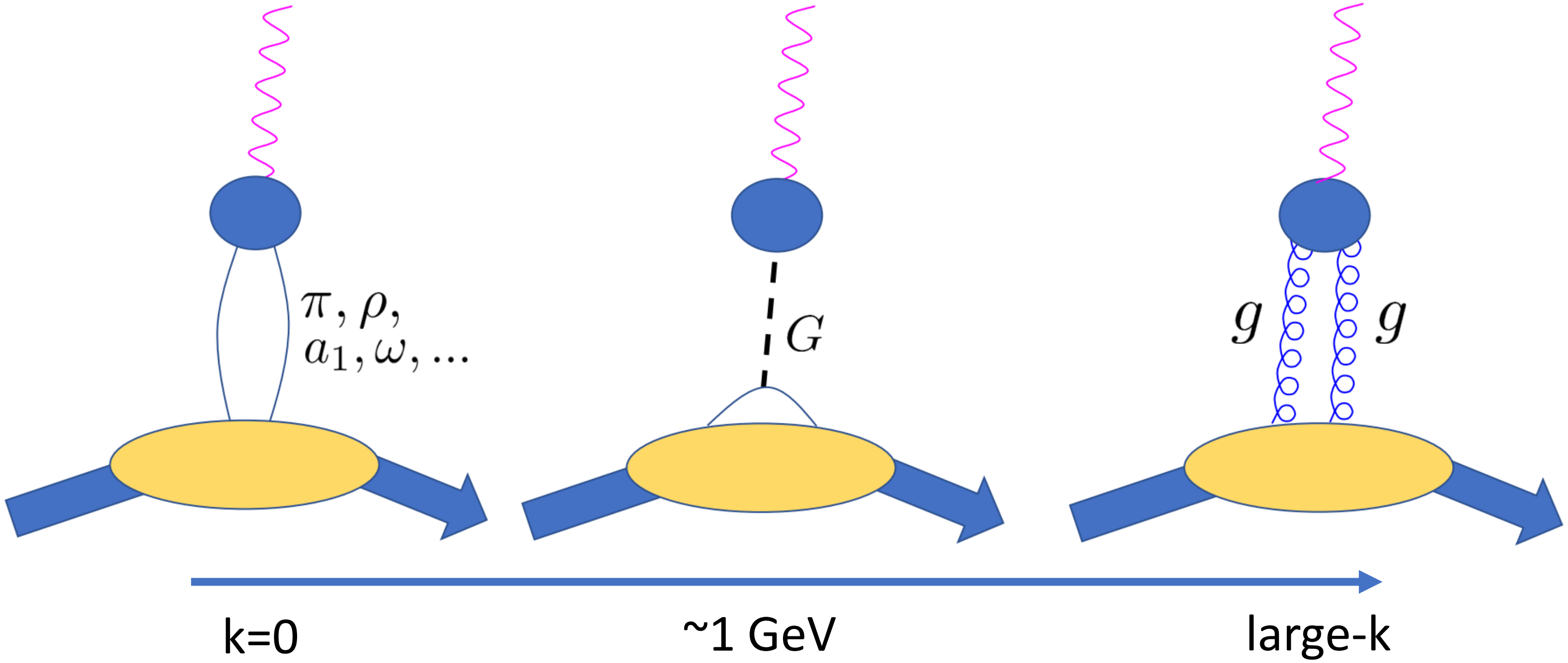
$$D \sim \kappa M \sim O(N_c^2)$$

Consistent with the known large-Nc counting,
but numerically small due to a cancellation between the U(1) and SU(2) contributions.

Summary

First study of the D-term in a top-down holographic QCD model.

Vivid physical interpretation in terms of meson and glueball exchanges.



Tong, Ma, Yuan (2021)

Mass/mechanical radius

$$\langle r^2 \rangle \equiv \frac{6D(0)}{\int_0^\infty dk^2 D(k^2)}$$

→ Need to include glueballs!

Future work

- Quantitative gravity calculation at nonzero k
- Soliton quantization
- Effect of the pion mass
- Other hadrons, nuclei