Bulk Viscosity of Two-Flavor Color Superconducting Quark Matter in Neutron Star Mergers

M. Alford, **Arus Harutyunyan**, A. Sedrakian, S. Tsiopelas

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BYURAKAN Astrophysical Observatory

YEREVAN STATE UNIVERSITY

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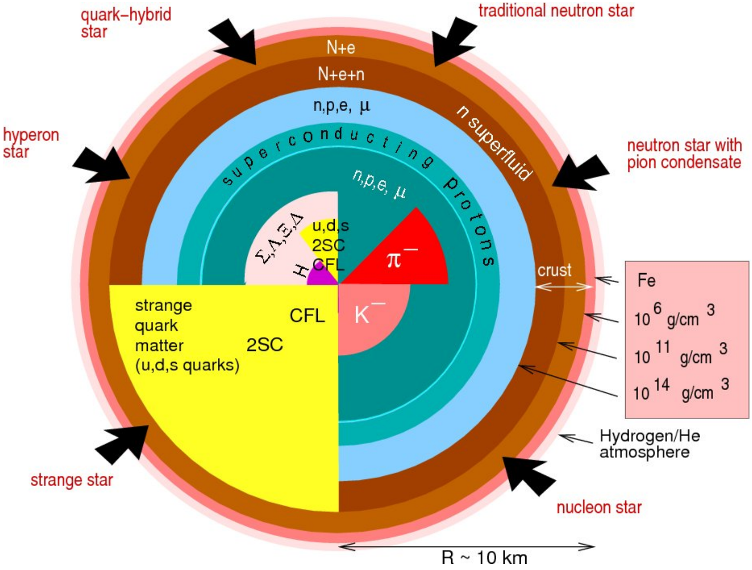
Outline

- Introduction & motivation
- Weak processes and the bulk viscosity
- Numerical results
- Conclusions

The structure of a neutron star



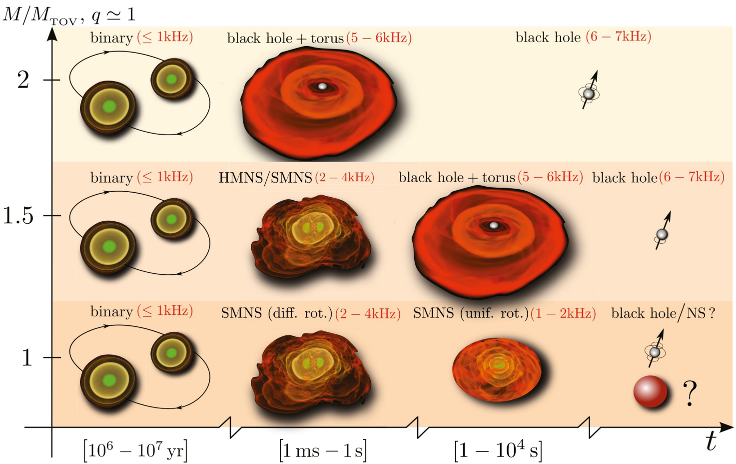
Compact star binaries

- Compact stars are natural laboratories which allow us to study the properties of nuclear matter under extreme physical conditions (strong gravity, strong magnetic fields, etc.).
- The recent detection of gravitational and electromagnetic waves originating from black hole or neutron star mergers motivates studies of compact binary systems.
- Various physical processes in the compact binary systems can be modelled in the framework of general-relativistic hydrodynamics simulations.
- Transport coefficients are key inputs in hydrodynamic modelling of compact star mergers as they measure the energy dissipation rate in hydrodynamic evolution of matter.
- The bulk viscosity might affect the hydrodynamic evolution of mergers by damping the density oscillations which can affect the form of the gravitational signal.

Aim of this work

We compute the bulk viscosity of two-flavor color-superconducting (2SC) *udse* quark matter from weak processes in the neutrino-transparent regime. We also assess the possible impact of the bulk viscous damping on the density oscillations of the post-merger object.

Binary neutron star mergers



- Characteristic timescales of the initial phase of post-merger ~ 10 ms (over which intense gravitational wave emission is expected); long-term post-merger evolution phase is ~ 1 s.
- Characteristic oscillation frequencies in BNS mergers lie in the range 1–10 kHz.

Figure from *L. Rezzolla and O. Zanotti*, Relativistic Hydrodynamics (Oxford University Press, Oxford, UK, 2013).

Weak processes in *udse* matter

- The simplest (semi-leptonic) weak-interaction processes in neutrino-transparent quark matter are the direct Urca processes

$$d \to u + e^- + \bar{\nu}_e \qquad u + e^- \to d + \nu_e$$
$$s \to u + e^- + \bar{\nu}_e \qquad u + e^- \to s + \nu_e$$

- In β-equilibrium $\mu_d = \mu_s = \mu_u + \mu_e$. Out of equilibrium there are chemical imbalances

$$\mu_{\Delta_{d/s}} = \mu_{d/s} - \mu_u - \mu_e \neq 0$$

- In addition, we have the following nonleptonic processes

$$u + d \leftrightarrow u + s$$

  which equilibrate much faster than Urca processes for frequencies of interest.
- As a result, $\mu_s = \mu_d$ and $\mu_{\Delta_s} = \mu_{\Delta_d} = \mu_\Delta$, which is the relevant measure of how much the system is driven out of β-equilibrium state by compression and rarefaction.

Rates of the Urca processes

- The rates of the Urca processes are given by $[\bar{f}(k) = 1 - f(k)]$

$$\Gamma_{d/s\to ue\bar{\nu}} = \int d\Omega_k \sum |\mathscr{M}_{\rm Urca}|^2 \bar{f}(k)\bar{f}(p)f(p')(2\pi)^4\delta^{(4)}(k+p+k'-p')$$

$$\Gamma_{ue\to(d/s)\nu} = \int d\Omega_k \sum |\mathscr{M}_{\rm Urca}|^2 f(k)f(p)\bar{f}(p')(2\pi)^4\delta(k+p-k'-p')$$

- The spin-averaged squared matrix element is ($\cos\theta_c \to \sin\theta_c$ for s-quark)

$$\sum |\mathscr{M}_{\rm Urca}|^2 = 128 G_F^2 \cos^2\theta_c (k\cdot p)(k'\cdot p')$$

- For small departures from equilibrium $\Gamma_{d/s\to ue\bar{\nu}} - \Gamma_{ue\to(d/s)\nu} = \lambda_{d/s}\,\mu_\Delta$, with

$$\lambda_{d/s} = \left.\frac{\partial(\Gamma_{d/s\to ue\bar{\nu}} - \Gamma_{ue\to(d/s)\nu})}{\partial\mu_\Delta}\right|_{\mu_\Delta=0}$$

- At low temperatures the rates are given by (note that there is no threshold for direct Urca)

$$\lambda_d \simeq 0.2 G_F^2 \cos^2\theta_c p_{Fd}^2 T^5, \qquad \lambda_s \simeq 0.03 G_F^2 \sin^2\theta_c \mu_s^* m_s^{*2} T^4$$

- The rate of non-leptonic processes is given by

$$\lambda_{\rm non-lep} = \frac{64}{5\pi^3} G_F^2 \sin^2\theta_c \cos^2\theta_c \mu_d^{*5} T^2$$

Density oscillations in quark matter

- Consider now small-amplitude density oscillations in quark matter with frequency ω

$$n_j(t) = n_{j0} + \delta n_j(t), \quad \delta n_j(t) \sim e^{i\omega t}, \qquad j = \{u, d, s, e\}$$

- The shifts in the particle densities δn_j lead to chemical imbalance

$$\mu_\Delta = A_d \delta n_d + A_s \delta n_s - A_u \delta n_u - A_e \delta n_e$$

- The susceptibilities A_j are given by

$$A_d = A_{dd} - A_{ud}, \quad A_u = A_{uu} - A_{du}, \quad A_s = A_{ds} - A_{us}, \quad A_e = A_{ee}, \quad A_{ij} = \partial \mu_i / \partial n_j$$

  Off-diagonal elements $i \neq j$ do not vanish because of strong interactions between quarks.

- If the weak processes were switched off, all particle species would be conserved

$$\frac{\partial}{\partial t} \delta n_j^0(t) + \theta n_{j0} = 0, \quad \delta n_j^0(t) = -\frac{\theta}{i\omega} n_{j0}, \qquad \theta = \partial_i v^i$$

Balance equations

- The rate equations which take into account the net production of particles read

$$
\begin{aligned}
\frac{\partial}{\partial t}\delta n_d &= -\theta n_{d0} - \lambda_d \mu_\Delta - I_{ud\to us} \\
\frac{\partial}{\partial t}\delta n_s &= -\theta n_{s0} - \lambda_s \mu_\Delta + I_{ud\to us} \\
\frac{\partial}{\partial t}\delta n_u &= -\theta n_{u0} + (\lambda_d + \lambda_s)\mu_\Delta \\
\frac{\partial}{\partial t}\delta n_e &= -\theta n_{e0} + (\lambda_d + \lambda_s)\mu_\Delta
\end{aligned}
$$

- The quantity $I_{ud\to us}$ denotes the rate of the non-leptonic reaction $d + u \to s + u$, which is driven by a nearly vanishing chemical potential difference $\delta\mu_d - \delta\mu_s \ll \mu_\Delta$, but cannot be neglected because the relevant λ-coefficient can be very large.
- The charge neutrality (both color and electric) and baryon conservation imply

$$
\begin{aligned}
\tilde{n}_u + \tilde{n}_d + \tilde{n}_s = 2(n_u + n_d + n_s) &= 2n_b \\
\frac{2}{3}(n_u + \tilde{n}_u) - \frac{1}{3}(n_d + n_s + \tilde{n}_d + \tilde{n}_s) &= n_e + n_\mu = n_u + \tilde{n}_u - n_b
\end{aligned}
$$

where n_i are the densities of blue quarks, $\tilde{n}_i$ are the summed densities of red and green quarks, and n_b is the baryon density.

Bulk viscosity

- Solving these equations we can compute the pressure out of equilibrium

$$p = p(n_j) = p(n_{j0} + \delta n_j) = p_{\rm eq} + \delta p',$$

where the non-equilibrium part of the pressure - the bulk viscous pressure, is given by

$$\Pi \equiv \delta p' = \sum_j \frac{\partial p}{\partial n_j} \delta n_j'$$

- The non-equilibrium shifts are found from $\delta n_j' = \delta n_j - \delta n_j^0$, and the bulk viscosity is then identified from $\Pi = -\zeta\theta$, where

$$\boxed{\zeta(\omega) = \frac{C^2}{A} \frac{\gamma}{\omega^2 + \gamma^2}}$$

with susceptibilities $A = \left.\frac{\partial \mu_\Delta}{\partial n_u}\right|_{n_b}$, $C = n_b \left.\frac{\partial \mu_\Delta}{\partial n_b}\right|_{x_u}$, and relaxation rate $\gamma = (\lambda_d + \lambda_s)A$.

- At the maximum of the bulk viscosity ($\omega = \gamma$), in the high-frequency ($\omega \gg \gamma$) and low-frequency ($\omega \ll \gamma$) limits we find

$$\zeta_{\rm max} = \frac{C^2}{2A\omega}, \qquad \zeta_{\rm high} = \frac{2\gamma}{\omega}\zeta_{\rm max}, \qquad \zeta_{\rm low} = \frac{2\omega}{\gamma}\zeta_{\rm max}$$

Lagrangian of the model

- To describe the properties of 2SC quark matter, we adopt 3-flavor NJL Lagrangian with vector interactions and the 't Hooft determinant term

$$\begin{aligned}\mathcal{L}_{NJL} &= \bar{\psi}(i\gamma^{\mu}\partial_{\mu} - \hat{m})\psi + G_S \sum_{a=0}^{8} \left[(\bar{\psi}\lambda_a\psi)^2 + (\bar{\psi}i\gamma_5\lambda_a\psi)^2\right] \\ &+ G_V(\bar{\psi}i\gamma^{\mu}\psi)^2 + G_D \sum_{\gamma,c} \left[\bar{\psi}^a_{\alpha} i\gamma_5 \epsilon^{\alpha\beta\gamma}\epsilon_{abc}(\psi_C)^b_{\beta}\right]\left[(\bar{\psi}_C)^r_{\rho} i\gamma_5 \epsilon^{\rho\sigma\gamma}\epsilon_{rsc}\psi^s_{\sigma}\right] \\ &- K\left\{\det_f[\bar{\psi}(1+\gamma_5)\psi] + \det_f[\bar{\psi}(1-\gamma_5)\psi]\right\}\end{aligned}$$

- This Lagrangian contains three additional terms: (i) the repulsive vector interaction with coupling G_V, (ii) the pairing interaction with coupling G_D, and (iii) the 't Hooft interaction with coupling K, which breaks the $U_A(1)$ symmetry.
- In the 2SC phase, pairing occurs in a color- and flavor-antisymmetric manner between u and d quarks, while s quarks remain unpaired. The pairing gap is given by

$$\Delta_c = G_D \left\langle (\bar{\psi}_C)^a_{\alpha} i\gamma_5 \epsilon^{\alpha\beta c}\epsilon_{abc}\psi^b_{\beta}\right\rangle$$

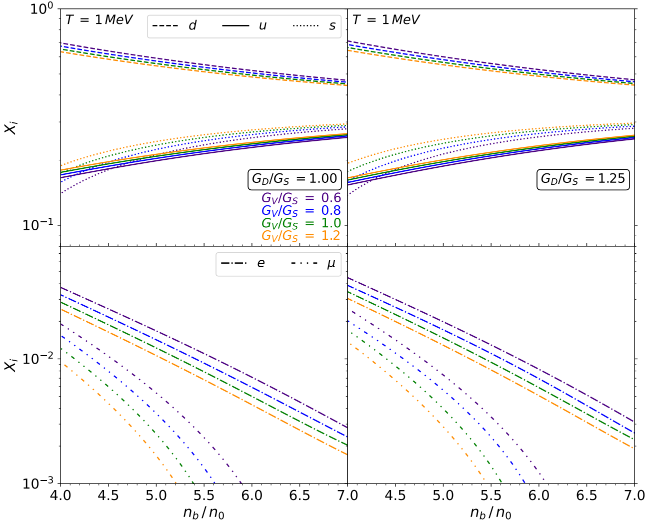
- The constituent masses and effective quark chemical potentials are given by

$$M_{\alpha} = m_{\alpha} - 4G_S\sigma_{\alpha} + 2K\sigma_{\beta}\sigma_{\gamma} \qquad \mu^* = \mathrm{diag}_f\,(\mu_u - \omega_0, \mu_d - \omega_0, \mu_s - \phi_0)$$

- The quark-antiquark condensates in the the mean field approximation are given by

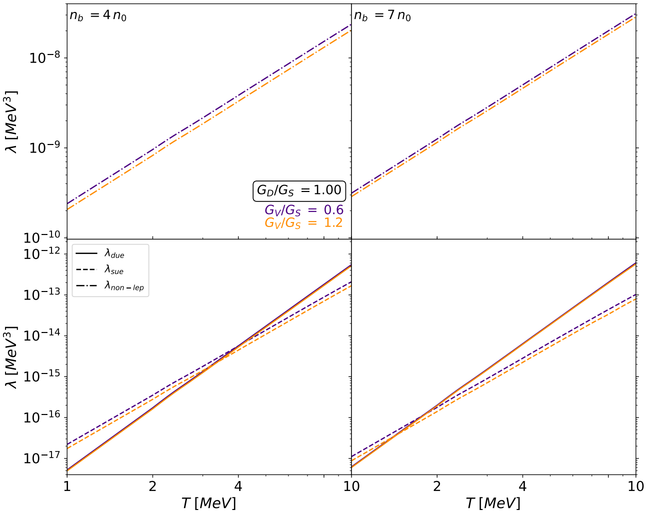
$$\sigma_{\alpha} = G_S\,\langle\bar{\psi}_{\alpha}\psi_{\alpha}\rangle \qquad \omega_0 = G_V\langle\psi^{\dagger}_u\psi_u + \psi^{\dagger}_d\psi_d\rangle \qquad \phi_0 = 2G_V\langle\psi^{\dagger}_s\psi_s\rangle$$

Particle fractions in equilibrium



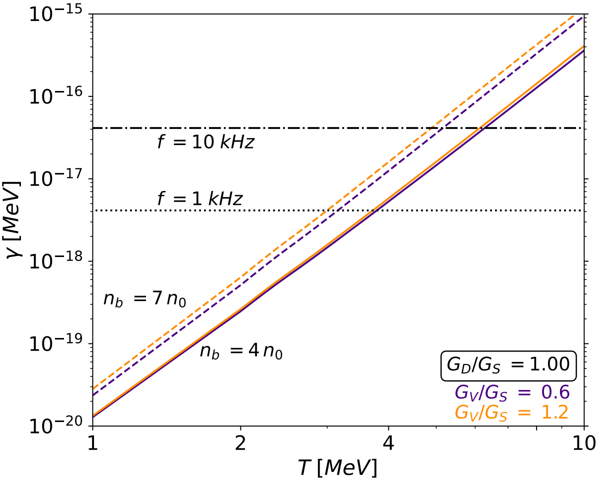
- Strengthening the repulsive vector interaction increases the u and s-quark populations while reducing the d-blue quark as well as lepton populations.
- The increase of the attractive pairing strength G_D/G_S affects the particle populations in the opposite manner.

Equilibration coefficients of Urca and non-leptonic processes



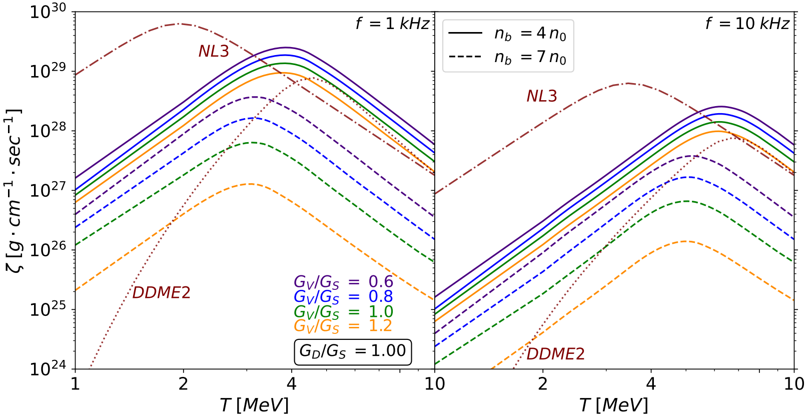
- Nonleptonic processes occur at much higher rates than the *d*-Urca and *s*-Urca processes.
- The direct *d*-Urca process is only thermally allowed with $\lambda_d \propto T^5$, whereas the *s*-Urca channel is kinematically open with $\lambda_s \propto T^4$ due to the large mass of the strange quark.

Urca relaxation rate



- The maximum of bulk viscosity is located at the temperature where $\gamma = \omega = 2\pi f$.
- Resonance occurs near $T \simeq 3 - 4$ MeV at 1 kHz, and near $T \simeq 5 - 6$ MeV at 10 kHz.
- Larger vector coupling enhances the rate γ slightly, especially at higher densities.

Bulk viscosity of *udse* matter



- The maximum of the bulk viscosity arises at several MeVs and slowly moves to lower temperatures with increasing density.
- The vector coupling G_V affects significantly the magnitude of the bulk viscosity, especially at high densities, but the location of maximum remains almost unchanged.
- In this temperature range, the bulk viscosity of 2SC quark matter is similar to that of nucleonic matter.

Estimation of oscillation damping timescale

- The energy density of baryonic oscilations with amplitude δn_B is

$$\epsilon = \frac{K}{18}\frac{(\delta n_B)^2}{n_B}.$$

- Coefficient K is the incompressibility of nuclear matter

$$K = 9\frac{\partial P}{\partial n_B}.$$

- The energy dissipation rate per volume by bulk viscosity is

$$\frac{d\epsilon}{dt} = \frac{\omega^2\zeta}{2}\left(\frac{\delta n_B}{n_B}\right)^2.$$

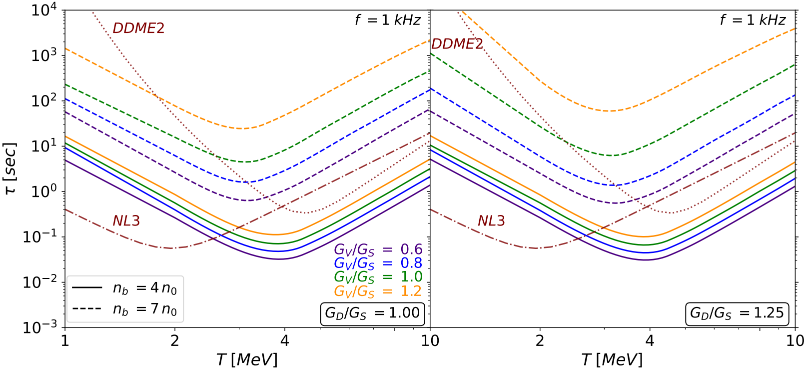
- The characteristic timescale required for dissipation is $\tau = \epsilon/(d\epsilon/dt)$

$$\tau = \frac{K n_B}{9\omega^2\zeta}.$$

- At the maximum of the bulk viscosity ($\omega = \gamma$), in the high-frequency limit ($\omega \gg \gamma$) and in the low-frequency limit ($\omega \ll \gamma$) we find

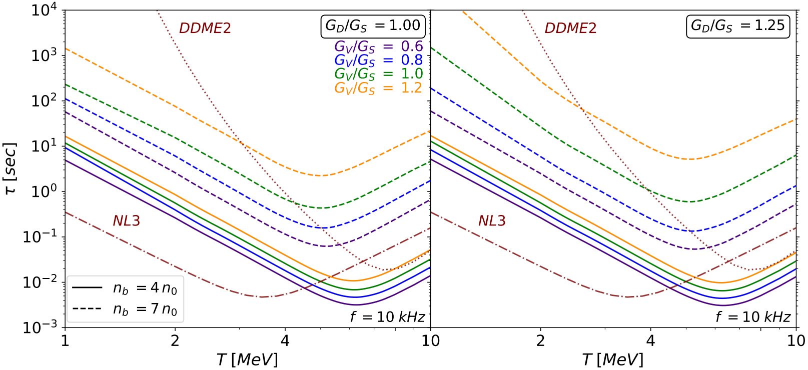
$$\tau_{\mathrm{min}} = \frac{2}{9\omega}\frac{K n_B}{C^2/A}, \qquad \tau_{\mathrm{high}} = \frac{1}{9\gamma}\frac{K n_B}{C^2/A}, \qquad \tau_{\mathrm{low}} = \frac{\gamma}{9\omega^2}\frac{K n_B}{C^2/A}.$$

Oscillation damping timescales ($f = 1$ kHz)



- Damping timescales close to the minimum are comparable to the characteristic timescales of the short-term evolution of the post-merger remnant.
- Damping timescale decreases with the increase of the vector coupling and this decrease is more pronounced at higher densities.
- Damping timescales in 2SC quark matter are similar to those of nucleonic matter.
- Thus, within the NJL model, it would be challenging to distinguish 2SC quark matter from nuclear matter based solely on their bulk viscous dissipation properties.

Oscillation damping timescales ($f = 10$ kHz)



- Damping timescales close to the minimum are comparable to the characteristic timescales of the short-term evolution of the post-merger remnant.
- Damping timescale decreases with the increase of the vector coupling and this decrease is more pronounced at higher densities.
- Damping timescales in 2SC quark matter are similar to those of nucleonic matter.
- Thus, within the NJL model, it would be challenging to distinguish 2SC quark matter from nuclear matter based solely on their bulk viscous dissipation properties.

Summary

- We studied the weak interaction rates and the bulk viscosity of the 2SC phase of finite-temperature quark matter under conditions relevant to binary neutron star mergers.
- Under these conditions the bulk viscosity mainly arises from direct Urca processes.
- We employed the vector-enhanced NJL model for three-flavor quark matter for various values of vector and diquark couplings.
- We find that varying the vector coupling by a factor of 2 changes the bulk viscosity and corresponding damping timescale by a factor of 3–20 at densities from $4n_0$ to $7n_0$.
- Damping timescales range from a few up to hundreds of milliseconds, which are comparable to the typical timescales of the evolution of the post-merger remnant.
- Bulk viscosity and damping time for 2SC quark matter are close to those of nucleonic matter, making it difficult to distinguish these states via their bulk viscous behavior.

THANK YOU FOR ATTENTION!