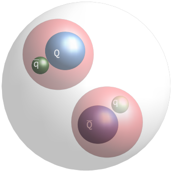
HOW TO IDENTIFY HADRONIC MOLECULES

by combining results from Lattice QCD, EFTs and Experiment

March 19, 2023 | Christoph Hanhart | IKP/IAS Forschungszentrum Jülich

HADRONIC MOLECULES



- are few-hadron states, bound by the strong force
- do exist: light nuclei.
  e.g. deuteron as *pn* & hypertriton as Λd bound state
- are located typically close to relevant continuum threshold;
  e.g., for $E_B = m_1 + m_2 - M$ and $\gamma = \sqrt{2\mu E_B}$
  - $E_B^{\rm deuteron} = 2.22$ MeV ($\gamma = 45$ MeV)
  - $E_B^{\rm hypertriton} = (0.13 \pm 0.05)$ MeV (to Λd) ($\gamma = 13$ MeV)
- can be identified in observables (Weinberg compositeness):

$$\frac{g_{\rm eff}^2}{4\pi} = \frac{4M^2\gamma}{\mu}(1-\lambda^2) \;\rightarrow\; a = -2\left(\frac{1-\lambda^2}{2-\lambda^2}\right)\frac{1}{\gamma}\;; \quad r = -\left(\frac{\lambda^2}{1-\lambda^2}\right)\frac{1}{\gamma}$$

where $(1-\lambda^2)$=probability to find molecular component in bound state wave function

Are there mesonic molecules?

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DISCLAIMERS AND OUTLINE

The method presented is 'diagnostic' — especially,

- it does not allow for conclusions on the binding force;
- it allows one only to study individual states;
- quantitative interpretation gets lost when states get bound too deeply ('uncertainty' $\sim R\gamma$)

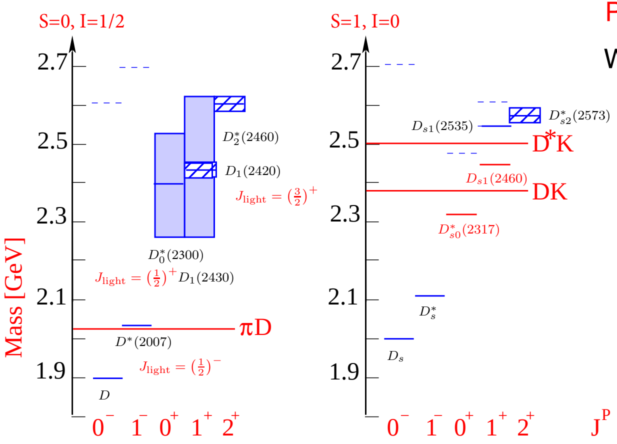
To go beyond tailor made effective field theories needed

In this talk I present how unitarized ChPT can be applied to

- Goldstoneboson D meson scattering $\Longrightarrow D^*_{s0}(2317)$, $D^*_0(2300)$ ...
- DD^* scattering $\Longrightarrow T_{cc}(3875)$

to allow for simultaneous study of experimental and lattice data to eventually reveal the nature of the states

PART 1: ONE CHARM QUARK



Quark Modell: M. Di Pierro and E. Eichten, PRD 64 (2001) 114004

Puzzles:

Why are/is

1. $M(D_{s1})\&M(D_{s0}^*)$ so light?
2. $M(D_{s1})-M(D_{s0}^*) \simeq M(D^*)-M(D)$?
3. $M(D_0^*) \simeq M(D_{s0}^*)$? $M(D_1) \simeq M(D_{s1})$?

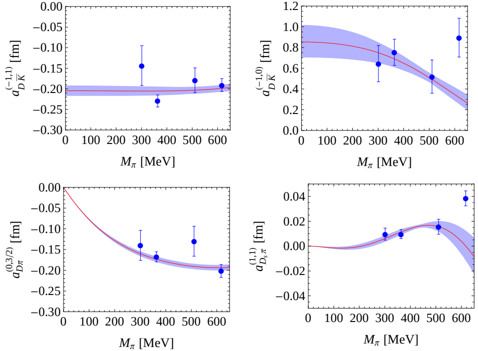
The solution provides crucial information about the nature of these states

HEAVY LIGHT SYSTEMS

Tool: Unitarized chiral perturbation theory

Truong, Dorado, Pelaez, Kaiser, Weise, Oller, Oset, Lutz, Kolomeitsev, Guo, Meißner, C.H., ...

- $\pi/K/\eta$–D/D_s scattering in ChPT to NLO unitarized (6 Parameter)
- fit LECs to lattice data for $a^{(S,I)}_{D_x\phi}$ in selected channels — Liu et al. PRD87(2013)014508



controlled quark mass dependence
Fit range up to $M_\pi = 500$ MeV

- $D^*_{s0}(2317)$ emerges as a pole with $M_{D^*_{s0}} = 2315^{+18}_{-28}$ MeV.

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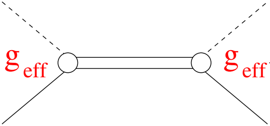
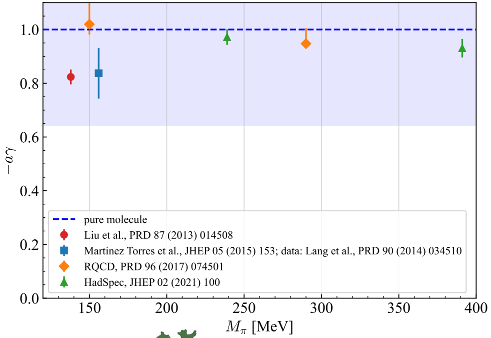
INTERPRETATION A LA WEINBERG

$D^*_{s0}(2317)$: $a=$ $g_{\rm eff}$  $g_{\rm eff} + \mathcal{O}(1/\beta) \simeq -\left(\frac{2(1-\lambda^2)}{2-\lambda^2}\right)\frac{1}{\gamma}$

$\Longrightarrow a = -(1.05 \pm 0.36)$ fm for molecule $(\lambda^2 = 0)$; smaller otherwise



Various lattice studies show under binding

study $a\gamma$ (removes E_b dep.)

All analyses consistent with purely molecular $D^*_{s0}(2317)$ (analogous for $D_{s1}(2460)$)

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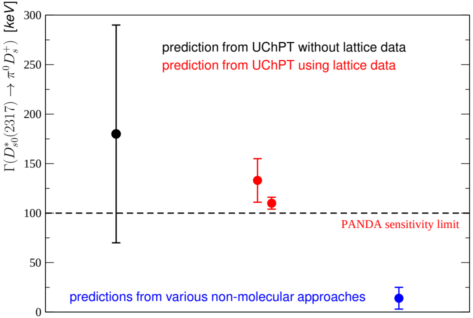
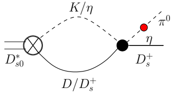
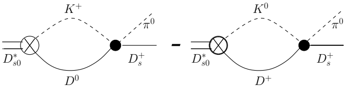
EXP. TEST: HADRONIC WIDTH



Genuine contribution:



Specific for molecules:



F.K. Guo et al., PLB666(2008)251; L. Liu et al. PRD87(2013)014508; X.Y. Guo et al., PRD98(2018)014510
and, e.g., P. Colangelo and F. De Fazio, PLB570(2003)180

Width very sensitive to D^*_{s0} molecular component

Experiment needs very high resolution $\to$ PANDA

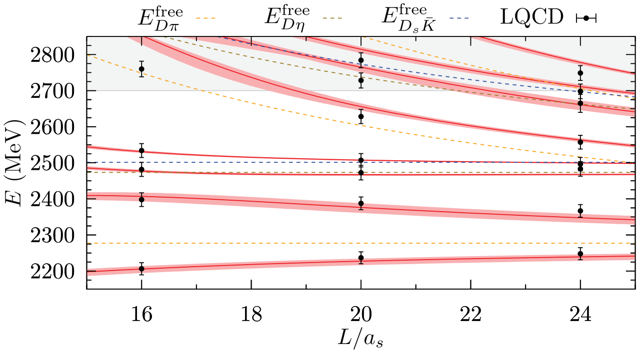
Predict $M_{B^*_{s0}} = 5722 \pm 14$ MeV and various decays

Fu et al., EPJA58(2022)70

Next: Study multiplet structure from GB-*D*-meson scattering

THE S = 0 SECTOR

Keeping parameters fixed one gets:



Albaladejo et al., PLB767(2017)465; Lattice: Moir et al. [Had.Spec.Coll.] JHEP10(2016)011
Fits directly to these data: Z. H. Guo et al., EPJC 79(2019)13; M. F. M. Lutz et al., PRD106(2022)114038

Poles for

- $M_\pi \simeq 391$ MeV: (2264, 0) MeV [000] & (2468, 113) MeV [110]
- $M_\pi = 139$ MeV: (2105, 102) MeV [100] & (2451, 134) MeV [110]

Questions $c\bar{q}$ nature of lowest lying 0^+ D state, $D_0^*(2300)$

NRW-FAIR

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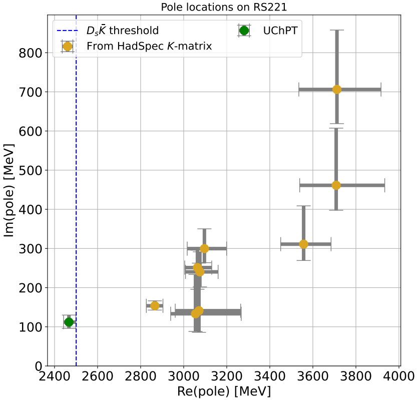
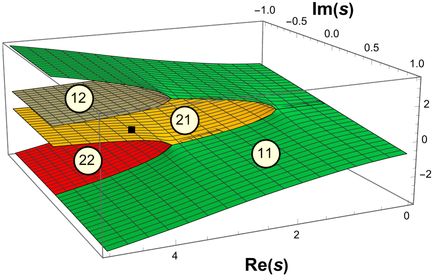
POLE STRUCTURE FROM LATTICE STUDY

Lattice study reported only bound state pole

Moir et al. [Had.Spec.Coll.] JHEP10(2016)011

Second pole was present, but location depends on amplitude model





The poles were on hidden on sheet

A. Asokan et al., [arXiv:2212.07856 [hep-ph]]

Distance to threshold balanced by size of residue

V. Baru et al.,EPJA23(2005)523

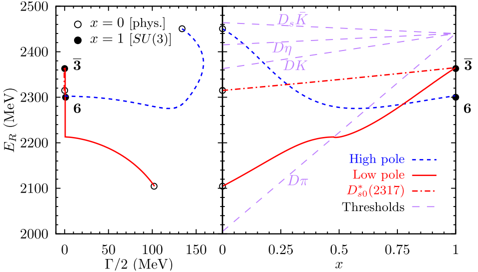
Explains correlation between Re(pole) and Im(pole)

SU(3) STRUCTURE

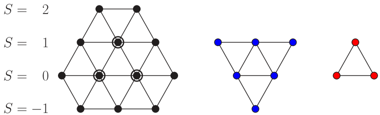
Albaladejo et al., PLB767(2017)465

$$m(x) = m^{\mathrm{phy}} + x(m - m^{\mathrm{phy}})$$

$m_\phi = 0.49$ GeV; $M_D = 1.95$ GeV



Multiplets: $[\bar{3}]\otimes[8]=[\overline{15}]\oplus[6]\oplus[\bar{3}]$



with $[\overline{15}]$ repulsive,
$[6]$ attractive,
$[\bar{3}]$ most attractive

- 3 poles give observable effect with SU(3)-breaking on
- At $SU(3)$ symmetric point $m_\phi \simeq 490$ MeV: 3 bound and 6 virtual states
- The light $D\pi$ state is the multiplet member of $D^*_{s0}(2317)$

$$\Longrightarrow \quad M_{D^*_{s0}(2317)} - M_{D^*_0(2100)} = 217\ \mathrm{MeV}$$

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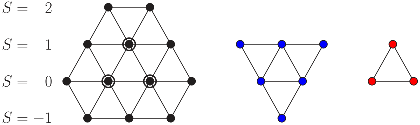
SU(3) STRUCTURE

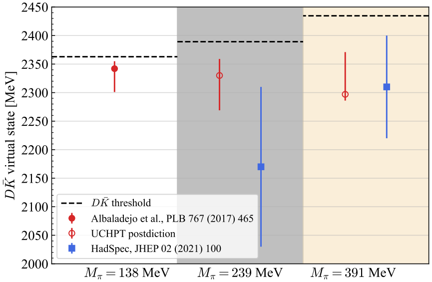


Albaladejo et al., PLB767(2017)465

- Lattice shows repulsion in $[\overline{15}]$ as predicted in UChPT
- States in [6] found in UChPT and lattice:

Hofmann and Lutz, NPA733(2004)142

  - $S = -1$



  - $S = 0$: Lattice shows evidence for pole in [6] @$M_\pi \approx 600$ MeV in line with UChPT prediction

Gregory et al., [arXiv:2106.15391 [hep-ph]]+Lüscher analysis.

- Quark Model: $[\bar{3}] \otimes [1] = [\bar{3}]$ — the [6] is absent

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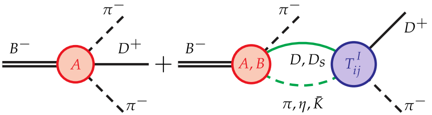
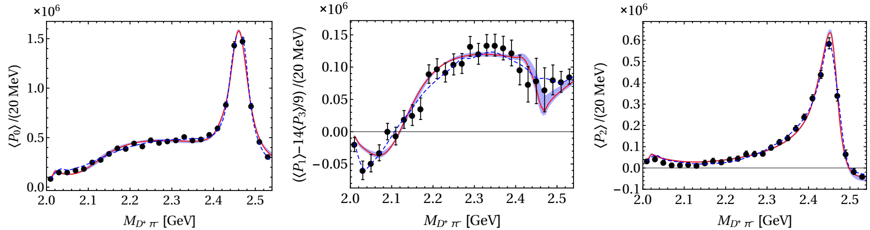
OBSERVABLE: $B^- \to D^+\pi^-\pi^-$

With the ϕD amplitude fixed we can calc. production reactions:

Du et al., PRD98(2018)094018; for more results see Du et al., PRD99(2019)114002



for the S-wave (two free para.);
other partial waves from BW-fit

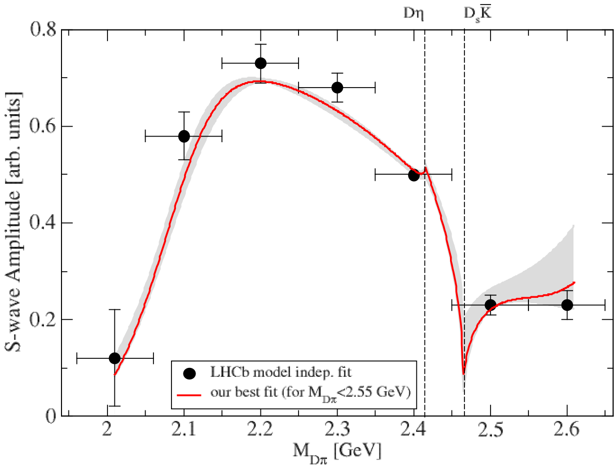


LHCb, PRD94(2016)072001

$$\langle P_0 \rangle \propto |\mathcal{A}_0|^2 + |\mathcal{A}_1|^2 + |\mathcal{A}_2|^2 , \quad \langle P_2 \rangle \propto \frac{2}{5}|\mathcal{A}_1|^2 + \frac{2}{7}|\mathcal{A}_2|^2 + \frac{2}{\sqrt{5}}|\mathcal{A}_0||\mathcal{A}_2| \cos(\delta_2 - \delta_0)$$

$$\langle P_{13} \rangle \equiv \langle P_1 \rangle - \frac{14}{9}\langle P_3 \rangle \propto \frac{2}{\sqrt{3}}|\mathcal{A}_0||\mathcal{A}_1| \cos(\delta_1 - \delta_0)$$

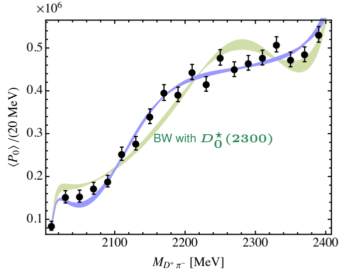
$D\pi$ S-WAVE FROM $B^- \to D^+\pi^-\pi^-$

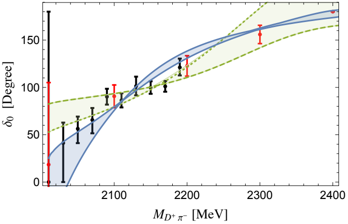


Effect of thresholds enhanced, by pole at $\sqrt{s_p} \sim (2451 - i134)$ MeV

on nearby unphysical sheet

LIGHTEST CHARMED SCALAR





Mass of lightest charmed $J^P = 0^+$ state:

- BW with $m = 2300$ MeV incompatible with data
- UChPT with $(2105 \pm 8 - i102 \pm 11)$ MeV is compatible Du et al., PRL126(2021)192001
- Low mass confirmed by Lattice QCD $(2196 \pm 64 - i210 \pm 110)$ MeV at $M_\pi = 239$ MeV HadSpec, JHEP07(2021)123

Analogous picture for $J^P = 1^+$

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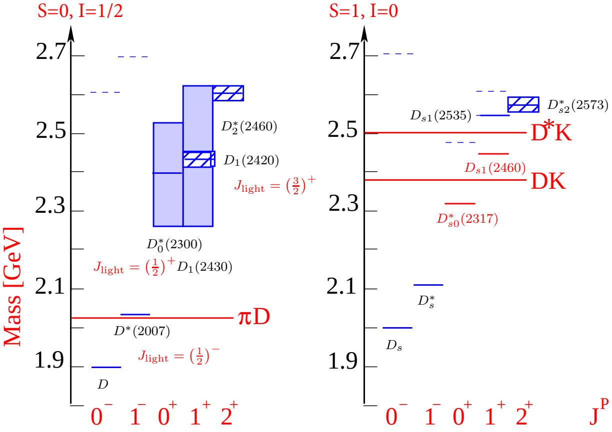
CHARMED STATES



Quark Modell: M. Di Pierro and E. Eichten, PRD 64 (2001) 114004

Puzzles solved:

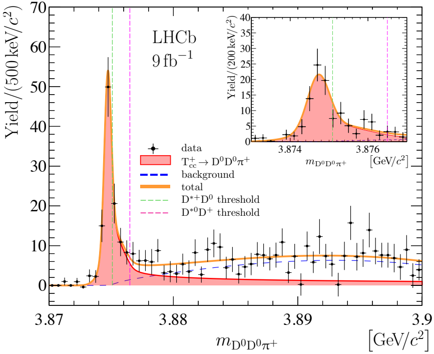
1. $M(D_{s1})\&M(D_{s0}^*)$ are DK and D^*K bound states
2. $M(D_{s1})-M(D_{s0}^*) \simeq M(D^*)-M(D)$, since spin symmetry gives equal binding
3. States with strangeness heavier
   $M(D_0^*) = 2100$ MeV
   $M(D_{s0}^*) = 2317$ MeV

   $M(D_1) = 2247$ MeV
   $M(D_{s1}) = 2460$ MeV

... role of left-hand cuts needs to be clarified

Lutz et al., PRD106(2022)114038; Korpa et al., PRD107(2023)L031505

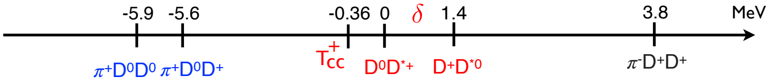
PART 2: TWO CHARM QUARKS: $[T_{cc}(3875)]$



R. Aaij et al. [LHCb], Nature Phys.18(2022)751

- no signal in D^+D^{*+}
  $\Longrightarrow$ isoscalar
- $\sim$ 100%: $D^0D^{*+} \to D^0[D^0\pi^+]$
  $\Longrightarrow$ D^{*+} building block
- Width from D^* decay
  $\Longrightarrow$ $\Gamma_{T_{cc}} < \Gamma_{D^*}$: phase sp.
- only lower bound for g^2
  $\Longrightarrow$ $|r_0| \propto 1/g^2$ small
- no other inelasticities

Excellent candidate for a molecular state!



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EFT FOR DOUBLY HEAVY STATES

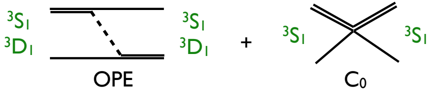
Schemes currently proposed (only initial work cited)

- Contact EFT: no pions — AlFiky et al., PLB640(2006)238
- X-EFT: pert. pions — Fleming et al., PRD76(2007)034006
- chiral EFT: non.-pert. pions — Baru et al., PRD84(2011)074029
  - largest energy range of applicability
  - Pion dynamics fully under control

$\Longrightarrow$ use this in what follows

Thus we solve LS-equation — Du et al., PRD105(2022)014024

$T = V + VGT$ with $V_{LO} =$



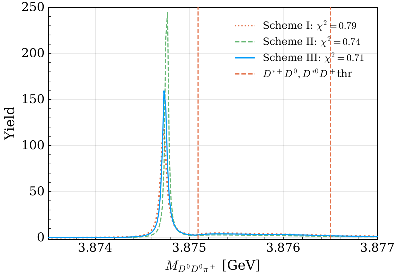
employing the expansion parameter $\chi = \sqrt{2\mu\delta}/\Lambda_\chi \approx 0.05$

@ LO : One free parameter (for each cut off) $\rightarrow$ Width fixed by mass

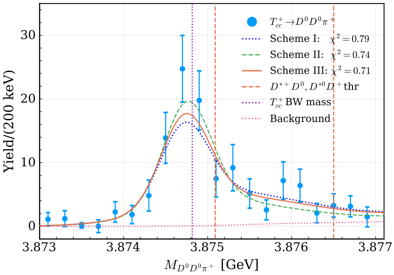
@ NLO: Energy dep. counter terms + loops

RESULTS AND DISCUSSION

w/o resolution



with resolution



Scheme	I	II	III
Description	2-body unitarity: No OPE, static D^* width	Incomplete 3-body unitarity: No OPE, dynamical D^* width	full 3-body unitarity: OPE + dynamical D^* width
Pole [keV]	$-368^{+43}_{-42} - i(37 \pm 0)$	$-333^{+41}_{-36} - i(18 \pm 1)$	$-356^{+39}_{-38} - i(28 \pm 1)$
χ^2	0.79	0.74	0.71

- Precision needs 3 body dynamics (problem: experimental resolution)
- $r = -2.4 \pm 0.01 \pm 0.85$ fm, but $r_0 = +1.38 \pm 0.01 \pm 0.85$ fm

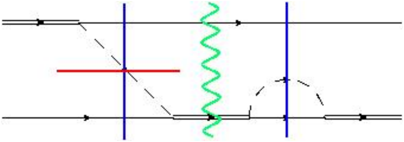
$\Longrightarrow$ T_{cc}^+ qualifies as isoscalar DD^* molecule

LATTICE STUDY

We here focus on a DD^* Lüscher analysis @ $M_\pi = 280$ MeV

Padmanath & Prelovsek, PRL129(2022)032002

Then D^* is stable ($M_\pi > \Delta M = M_{D^*} - M_D$) $\rightarrow$ cut structure:

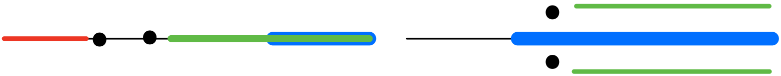


right-hand three-body cuts

right-hand two-body cut

left-hand cut

$(a) \quad m_\pi > \Delta M$ $\qquad$ $(b) \quad m_\pi < \Delta M$



Left-hand cut appears as new non-analyticity very near-by

$\Longrightarrow$ Study its impact on pole extraction

M. L. Du et al., [arXiv:2303.09441 [hep-ph]]

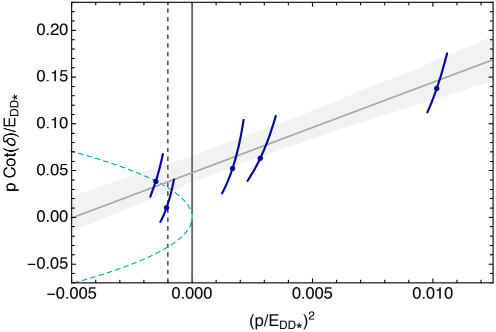
We take lattice data above lhc for granted! lattice with lhc: Raposo and Hansen, PoS LATTICE2022(2023)051

RESULTS AND DISCUSSION

From unitarity one gets $T(E) = -(2\pi/\mu)1/(p\cot(\delta(E)) - ip)$

Often employed: Taylor expansion (effective range expansion (ERE))

$$p\cot(\delta(E)) = 1/a + (r/2)p^2 \implies E_{\rm pole} = -9.9\text{ MeV } [(p/E_{DD*})^2 = -0.001]$$



green dashed: $\text{Re}(ip)/E_{DD*}$

<0: first sheet;

>0: second sheet

black dashed:

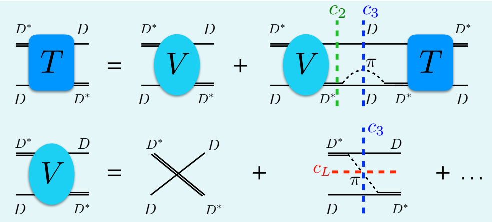
location of left-hand cut (lhc)

gray band (line): (best) ERE fit

However, lhc sets radius of convergence for ERE

IMPROVED ANALYSIS

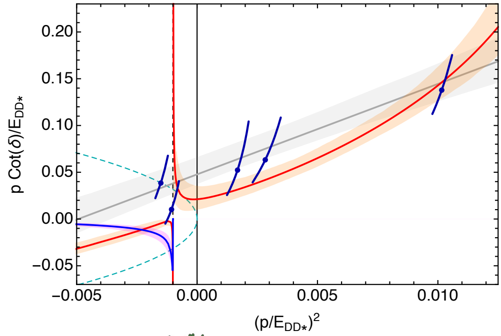
M. L. Du et al., [arXiv:2303.09441 [hep-ph]].

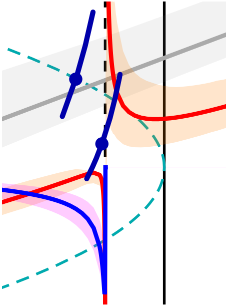


We still have

$$p \cot(\delta(E)) = -\frac{2\pi}{\mu} T(E)^{-1} + ip$$

However, it now becomes
complex below lhc





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SUMMARY AND CONCLUSION

- For near threshold states Weinberg criterion provides proper diagnostics
- View extended by studying the $SU(3)_f$ multiplet structure
  - what kinds of multiplets are there?
  - pattern of symmetry breaking important
- Cuts can be of significance
  - Two-body cuts
  - Three-body cuts (from three-body thresholds)
  - Left-hand cuts (from t-channel exchanges)

We are on a good path to identify the hadronic molecules in the spectrum

... thanks a lot for your attention

BACK-UP SLIDES

REMARKS OF EFFECTIVE RANGE

General parametrisation of line shapes (for stable constituents):

$$\frac{d\sigma}{dE} \sim \frac{\text{function of E}}{\left|R(E) + \frac{i}{2}[g_1^2\sqrt{2\mu_1 E} + g_2^2\sqrt{2\mu_2(E-\delta)} + \Gamma_{\text{inel.}}(E)]\right|^2}$$

where $g_1 = g_2 = g$ encodes isoscalar nature of the state, and

- $R(E)$ analytic in E: effective range expansion:

$$2R(E)/g^2 = -1/a - (r_0/2)k^2 + \mathcal{O}(k^4) \implies r_0 = -\frac{2}{\mu_1 g^2}\left(\frac{dR}{dE}\Big|_{E=0}\right)$$

modify for unstable constituents

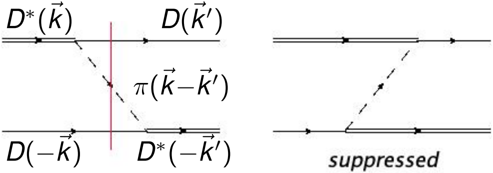
- for $|E| \approx E_b \ll \delta$:

$$i\sqrt{2\mu_2\left(\frac{k^2}{2\mu_1} - \delta\right)} = -\sqrt{2\mu_2\delta} + \underbrace{\frac{1}{2}\sqrt{\frac{\mu_2}{2\mu_1^2\delta}}}_{-(r_\delta/2)} k^2 + \mathcal{O}(k^4)$$

$\implies r_\delta \to \infty$ in the isospin limit

$\implies$ Only $r_0 \propto 1/g^2$ contains structure information!

CUTS FROM OPE FOR $\Delta M < M_\pi$



$$G_\pi(E,\vec{k}',\vec{k})^{-1} = E - 2M_D - \frac{k^2+k'^2}{2M_D} - \omega_\pi\left((\vec{k}-\vec{k}')^2\right)$$

$$\approx \Delta M + \frac{2p^2-k^2-k'^2}{2M_D} - \sqrt{(\vec{k}-\vec{k}')^2+M_\pi^2}$$

where $E = M_{D^*} + M_D + p^2/(2\mu) \approx M_{D^*} + M_D + p^2/M_D$

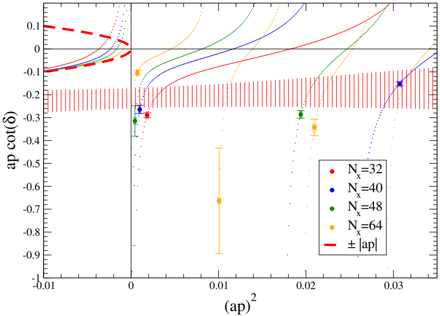
Two-body branch point: At D^*D threshold $\Longrightarrow p^2 = 0$

Three-body branch point: Smallest $p^2 > 0$ s.t. $G_\pi^{-1} = 0$
$\Longrightarrow p^2 = 2\mu(\Delta M - M_\pi)$

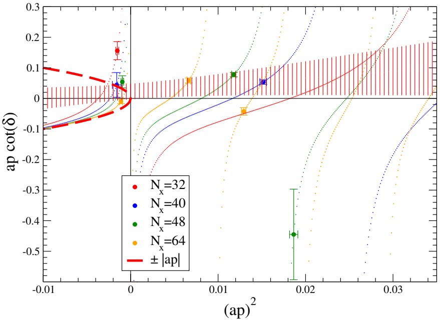
Left-hand branch point: largest $p^2 = k^2 = k'^2 < 0$ s.t. $G_\pi^{-1} = 0$
$\Longrightarrow p^2 = (\Delta M^2 - M_\pi^2)/4$

PRELIMINARY LÜSCHER RESULTS FOR πD SCATTERING @$M_\pi \approx 600$ MEV

[15]



[6]



We see

- repulsion in [15]
- attraction, leading to a virtual state in [6]