

Probing exotic matter in neutron star cores with g -mode oscillations

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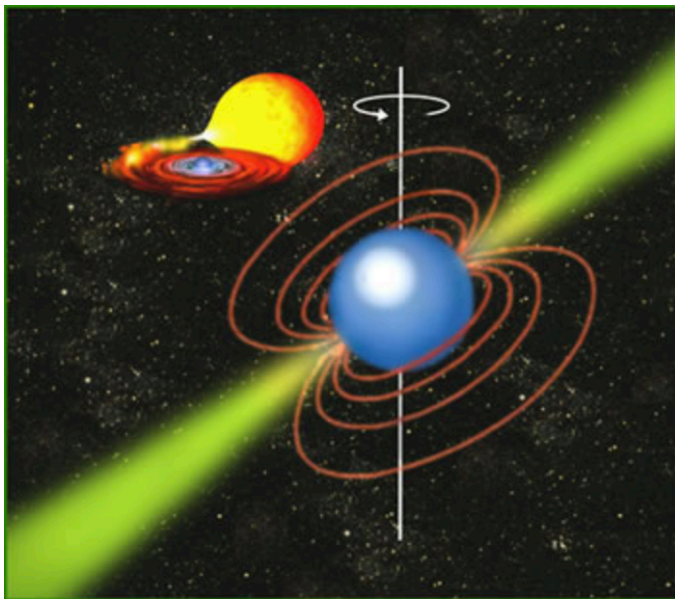


INT Workshop INT-22r-2a
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In collaboration with
C. Constantinou (INFN-TIFPA), P. Jaikumar (CSU Long Beach), T. Zhao (Ohio U) & M. Prakash (Ohio U)

Motivation

the most compact objects second only to black holes



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- still far from understanding NSs' composition since their discovery over half a century ago..
- NS mass-radius \leftrightarrow pressure vs. energy density (EoS): important, but **insufficient**

main sources for LIGO-Virgo-KAGRA:

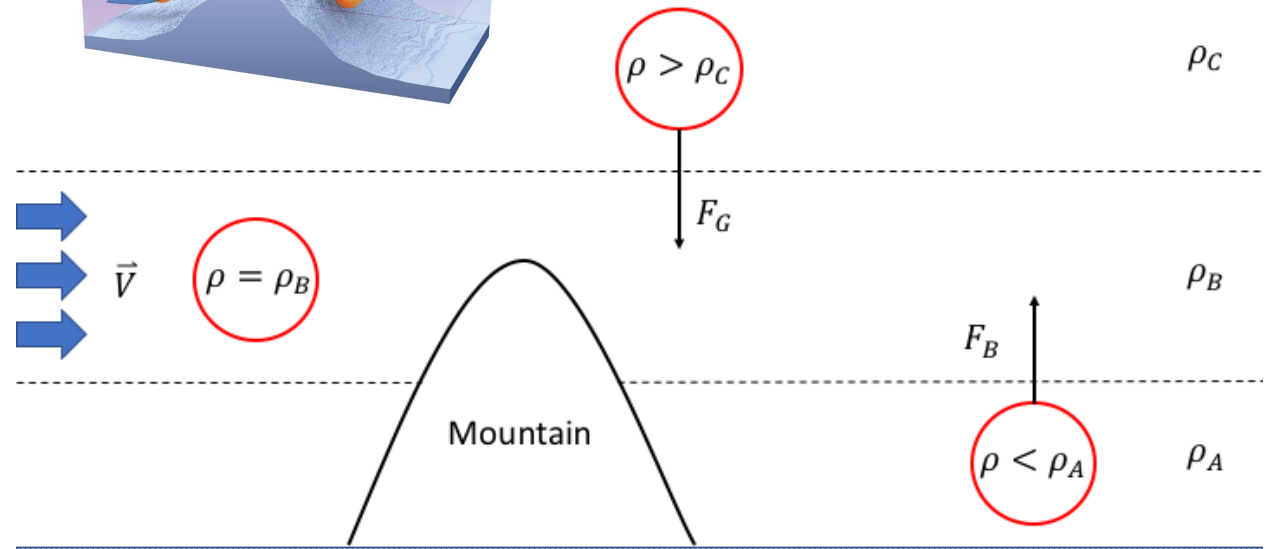
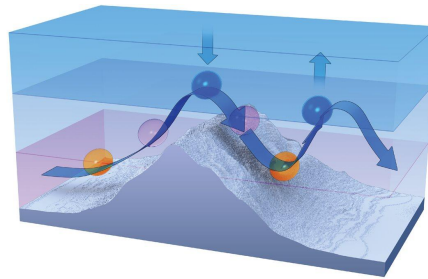
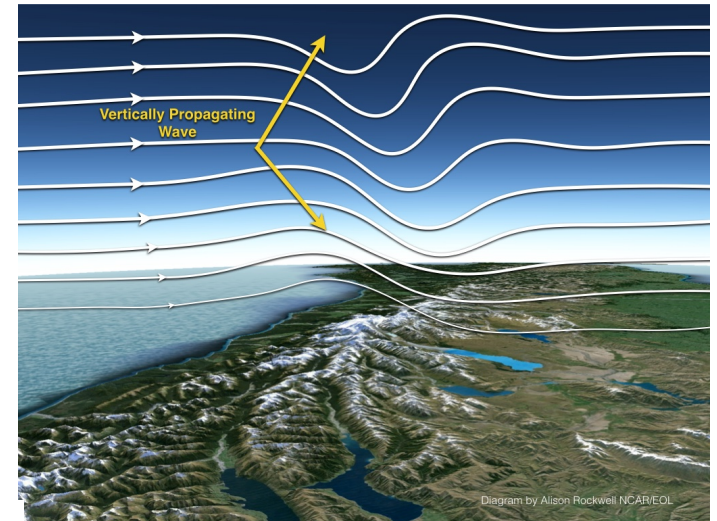
- NS, BH binary mergers WDs: $M/R \sim 10^{-4}$
- supernovae, NS/BH formation NSs: $M/R \sim 0.2$
- spinning NSs in X-ray binaries BHs: $M/R = 0.5$
- isolated NSs: instabilities, deformations

mass	radius	density	initial temp
$\sim 1.4 M_{\odot}$	$\mathcal{O}(10 \text{ km})$	$\gtrsim \rho_{\text{nuclear}}$	$\sim 30 \text{ MeV}$

g -modes (gravity modes)

restoring forces from buoyancy/gravity

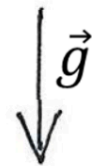
- e.g. atmospheric or ocean waves
- hydrostatic equilibrium: gravitational force balanced by pressure gradient force
- perturbed from equilibrium \rightarrow gravity or buoyancy pulls/pushes it back \rightarrow oscillation



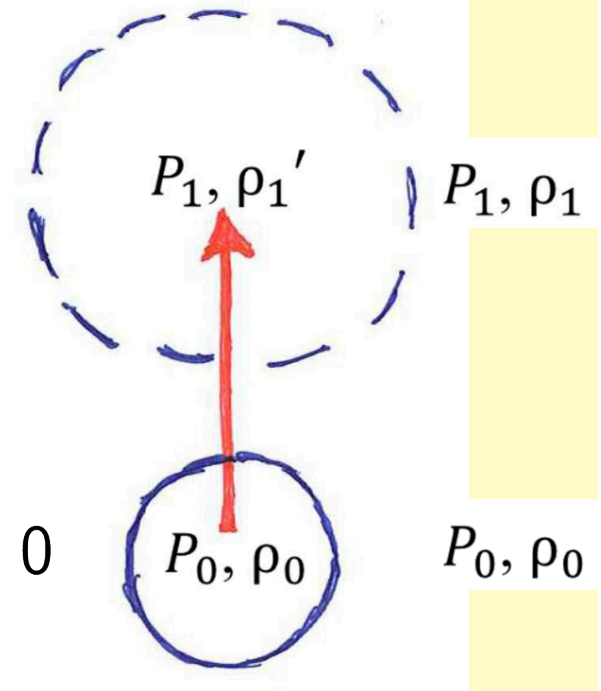
Brunt-Väisälä (buoyancy) frequency

- pressure instantaneously equilibrated, but not for composition and density
- continuity equation & the equation of motion
- “adiabatic” (composition frozen) sound speed vs. “equilibrium” sound speed

$$\Delta\rho = -\rho \frac{\partial^2 \xi}{\partial t^2}$$



$$\frac{\partial^2 \xi}{\partial t^2} + N^2 \xi = 0$$



credit: Andreas Reisenegger

$$\frac{dp}{dr} = -\rho g$$

(hydrostatic equilibrium)

$$N^2 \equiv g^2 \left(\frac{1}{c_{eq}^2} - \frac{1}{c_{ad}^2} \right) e^{\nu-\lambda}$$

← local metric coefficients

$$c_{eq}^2 = \left(\frac{dp/dr}{d\varepsilon/dr} \right)$$

$$c_{ad}^2 = \left(\frac{\partial p}{\partial \varepsilon} \right)_{y_i}$$

NS core g -modes

$$N^2 \equiv g^2 \left(\frac{1}{c_{eq}^2} - \frac{1}{c_{ad}^2} \right) e^{\nu - \lambda}$$

- stability criterion: $N^2 > 0 \iff$ stable stratification
- assuming **cold** NS (zero temperature/entropy); no convection or turbulence

osc. amplitude $\sim e^{i\omega t}$, $\omega \propto \sqrt{N^2}$

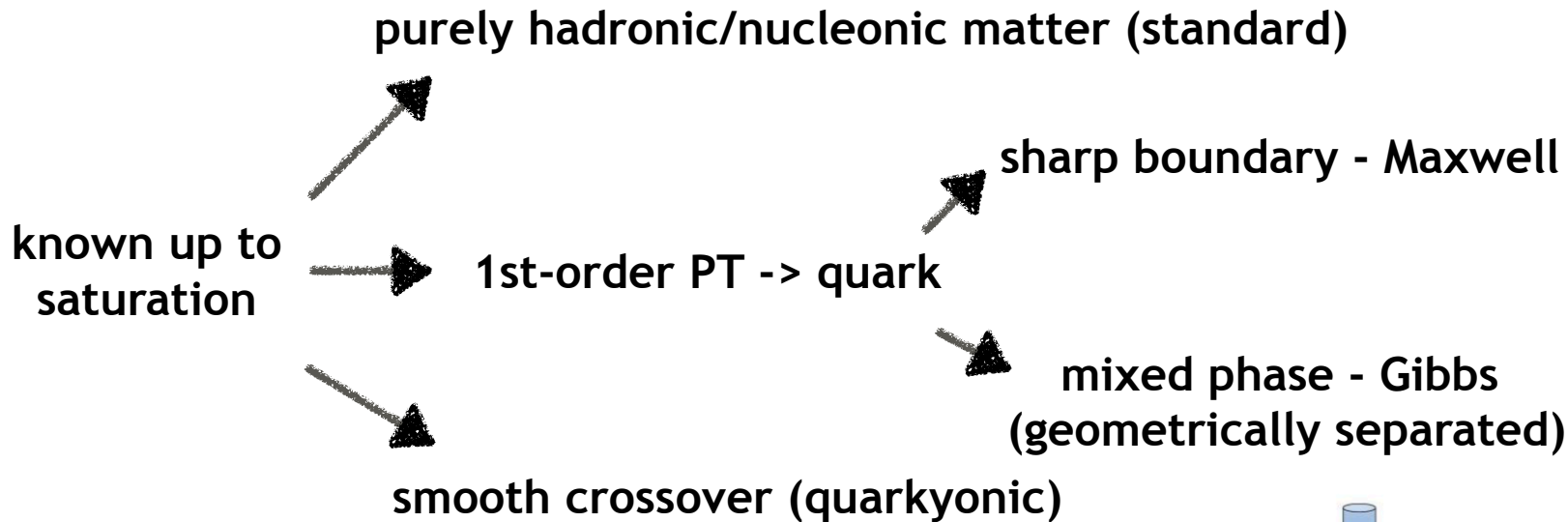
$$c_{ad}^2 \geq c_{eq}^2 \Rightarrow \text{mode stabilized}$$

- in bulk region of the NS liquid core: restored by buoyancy due to the chemical composition gradient e.g. proton fraction
- crustal modes behave differently and are expected to be quite small

$$\tilde{\mu} = \mu_e + \mu_p - \mu_n \quad c_{ad}^2 - c_{eq}^2 = - \left(\frac{\partial p}{\partial Y_e} \right)_\varepsilon \left(\frac{dY_e}{d\varepsilon} \right)$$

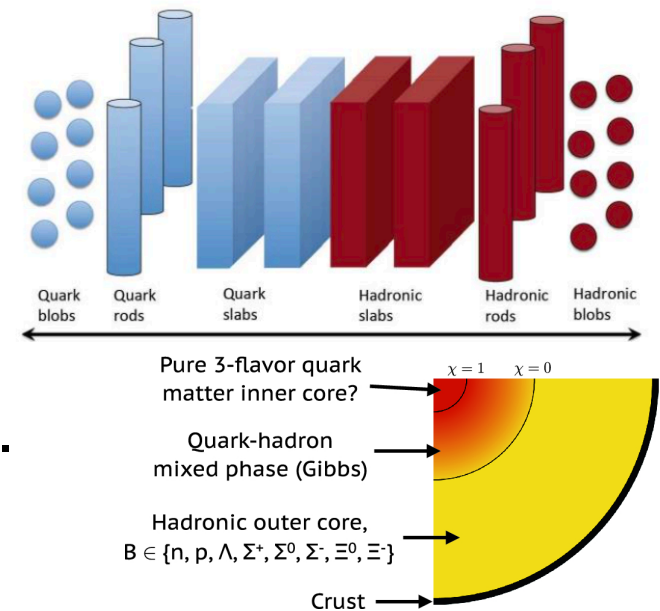
- e.g. in n - p - e matter

Hybrid stars with exotic matter



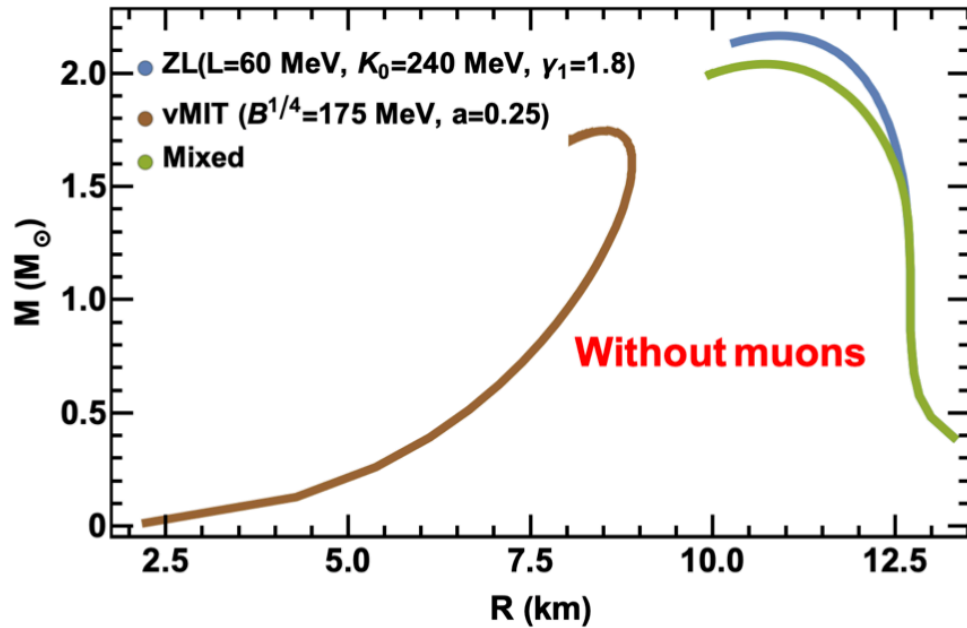
see Isaac's talk on Tuesday

- **masquerade** problem: EoSs with or without PTs may hardly be distinguishable via observations that constrain M-R only
- crossover models motivated by e.g. lattice calc.
- 1st-**OPT**: mixed phase (Gibbs) is favored if the hadron/quark surface tension is small

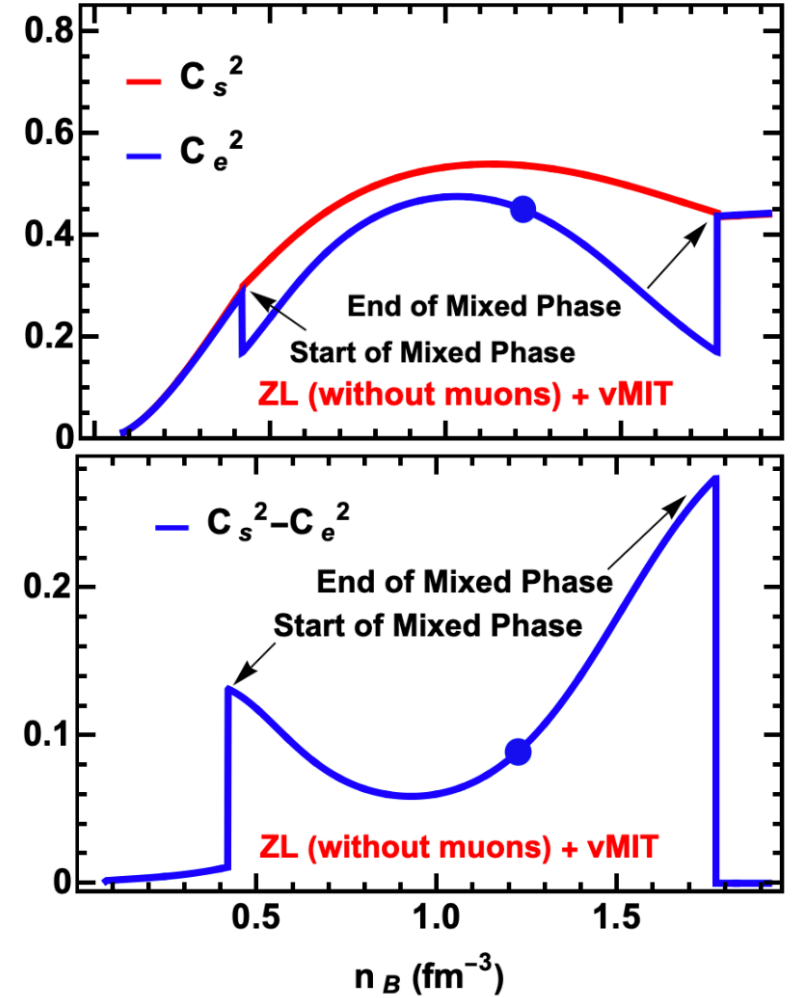


1st-OPT mixed phase (Gibbs)

$$\frac{1}{c_{ad}^{*2}} = \frac{1 - \chi}{c_{ad,H}^2} + \frac{\chi}{c_{ad,Q}^2}$$



$$\frac{1}{c_{eq}^{*2}} = \frac{1 - \chi}{c_{eq,H}^2} + \frac{\chi}{c_{eq,Q}^2} + (\varepsilon_Q - \varepsilon_H) \frac{d\chi/dn_B}{dp/dn_B}$$



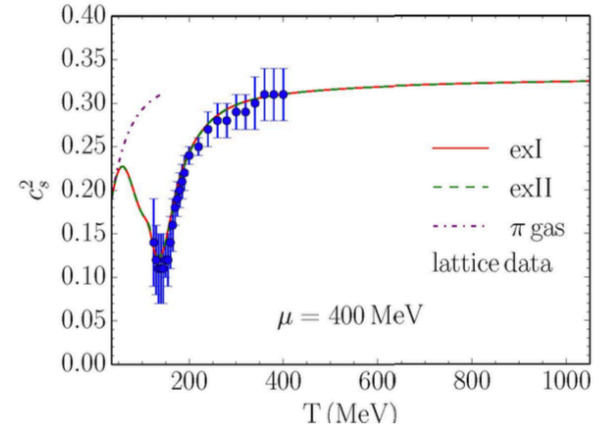
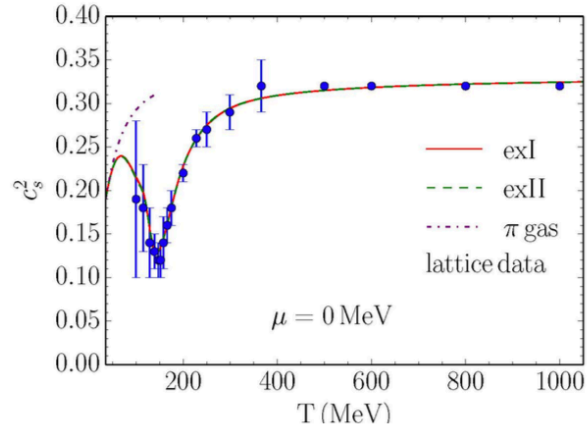
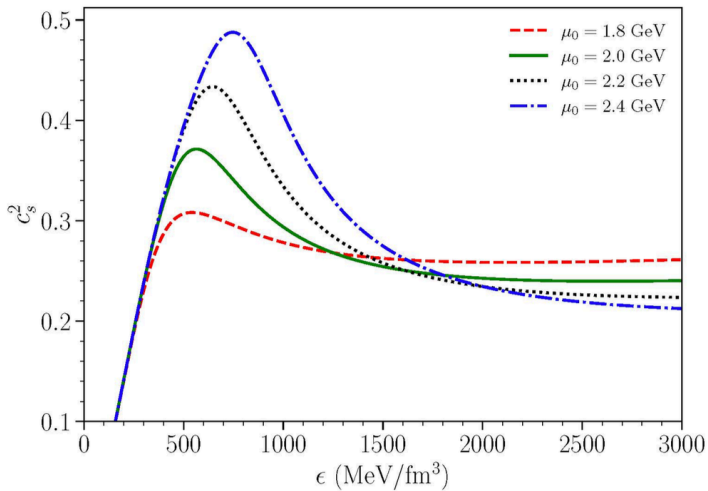
arXiv:2101.06349

Crossover matter

- Kapusta-Welle approach: switching function of baryon chemical potential (arXiv:2103.16633)

$$P_B = (1 - S)P_H + SP_Q$$

$$S = \exp \left[-(\mu_0/\mu)^4 \right]$$

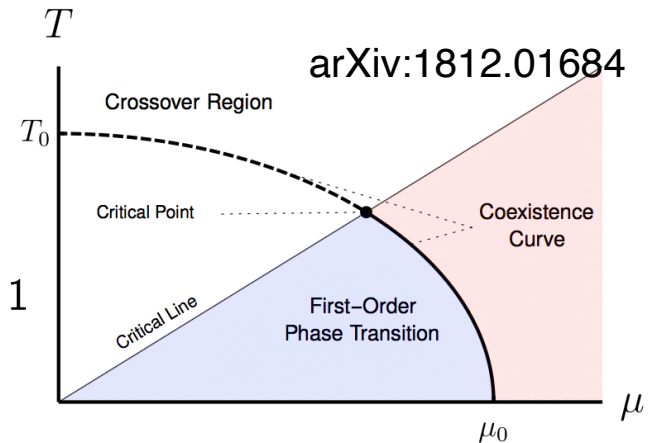


$$P(T, \mu) = SP_{\text{pQCD}} + (1 - S)P_{\text{hadron}} \quad S = \exp \left[-\frac{1}{(T/T_0)^n + (\mu/3\pi T_0)^n} \right]$$

Albright, Kapusta & Young 2015.

- analogy: lattice QCD shows a crossover at finite temperature

$$S = 1/2 \quad \left(\frac{T}{T_0} \right)^2 + \left(\frac{\mu}{\mu_0} \right)^2 = 1$$



Crossover vs. Gibbs

- Kapusta-Welle approach: switching function of baryon chemical potential
(arXiv:2103.16633)

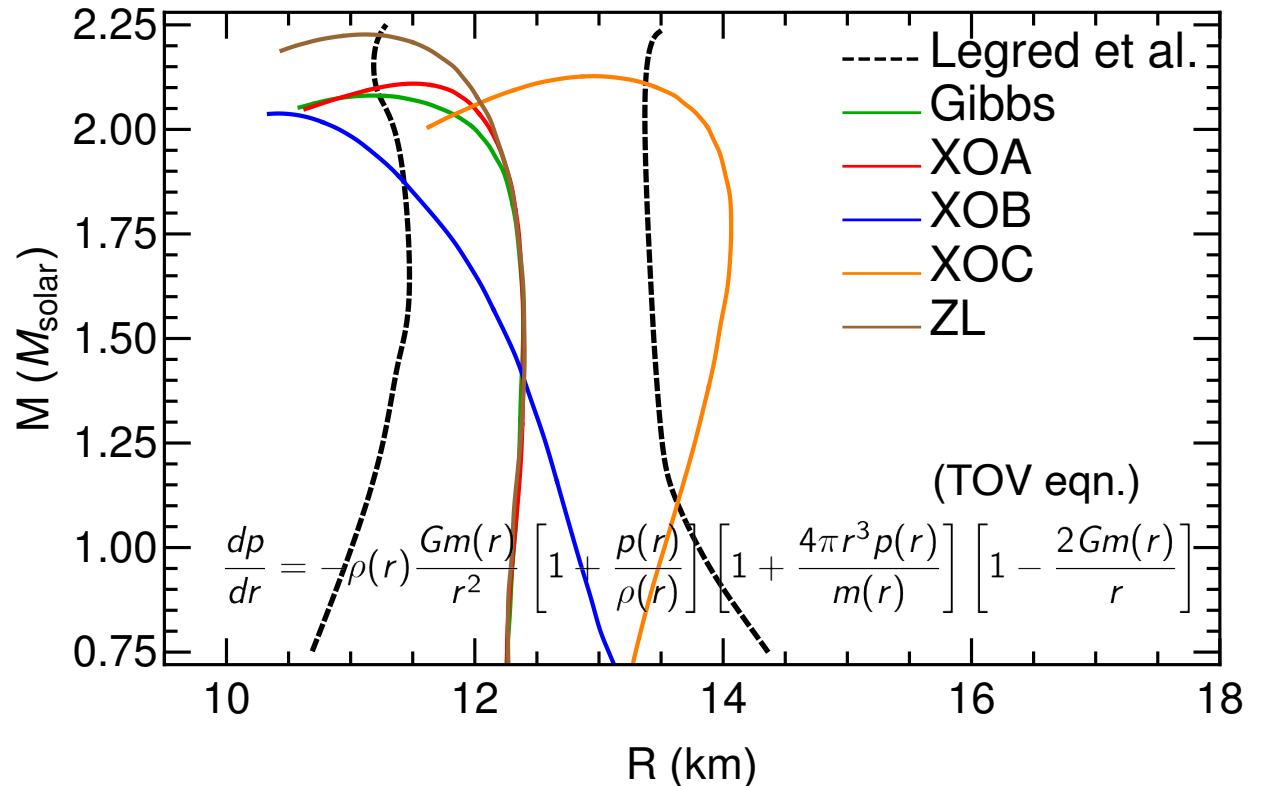
$$P_B = (1 - S)P_H + SP_Q$$

$$S = \exp \left[-(\mu_0/\mu)^4 \right]$$

$$\mu_0 \sim 2.0 \text{ GeV}$$

$$\mu = \frac{n_n \mu_n + n_p \mu_p}{n_n + n_p}$$

$$n_i = \left. \frac{\partial P}{\partial \mu_i} \right|_{\mu_j}$$



- our work: construct Gibbs and crossover XOA/XOB/XOC using ZL (nucleonic) + vMIT (quark) + KW parameters

arXiv:2109.14091

Two sound speeds

- adiabatic: start with the **unconstrained** system -> compute partial derivatives -> evaluate quantities at beta-equilibrium

$$(i = n, p, u, d, s, e, \mu)$$

enforce beta-equilibrium

$$c_{ad}^2(n_B, y_i) = \left(\frac{\partial p}{\partial n_B} \right)_{y_i} \left(\frac{\partial \varepsilon}{\partial n_B} \right)_{y_i}^{-1}$$



$$y_i \rightarrow y_{i,\beta}(n_B)$$

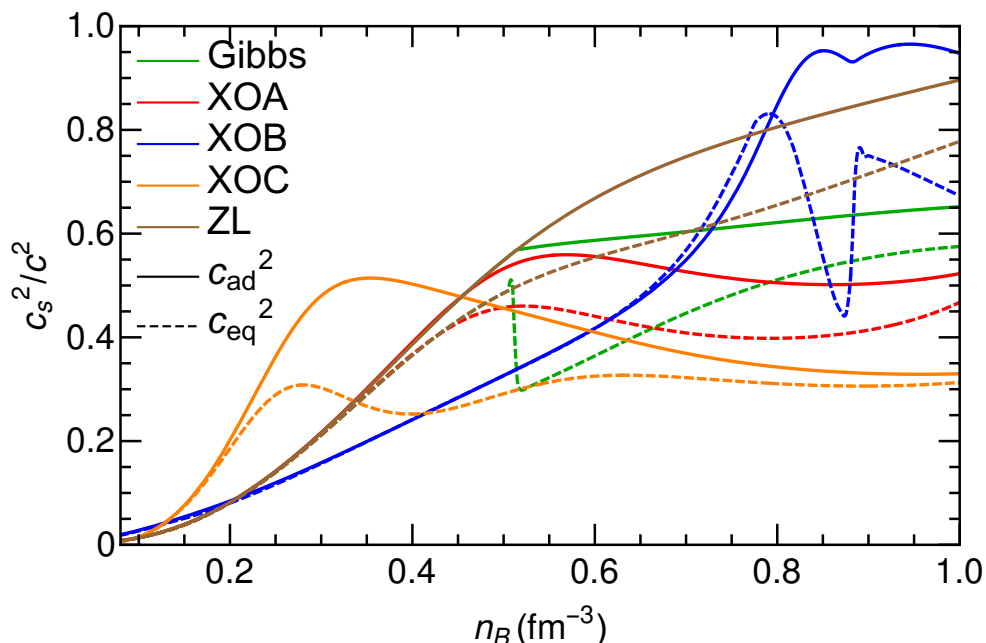
$$c_{ad,\beta}^2(n_B) = c_{ad}^2[n_B, y_{i,\beta}(n_B)]$$

$$\mu_n = \mu_p + \mu_e; \mu_e = \mu_\mu; \mu_d = \mu_s$$

$$\mu_n = \mu_u + 2\mu_d; \mu_p = 2\mu_u + \mu_d$$

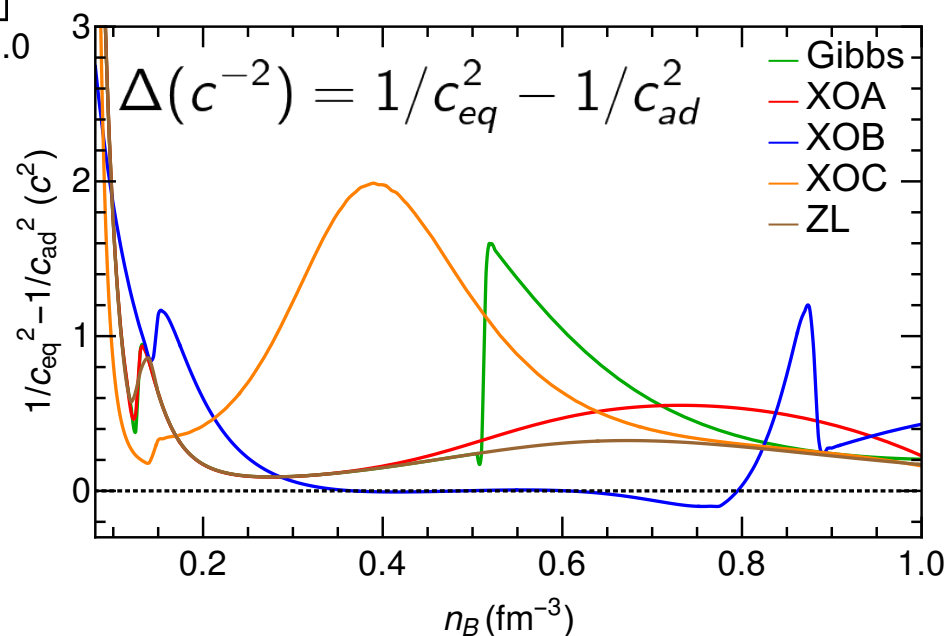
- equilibrium sound speed $c_{eq}^2(n_B) \equiv \left(\frac{dp}{d\varepsilon} \right)_\beta = \left(\frac{dp}{dn_B} \right)_\beta \left(\frac{d\varepsilon}{dn_B} \right)_\beta^{-1}$

Sound speed profiles

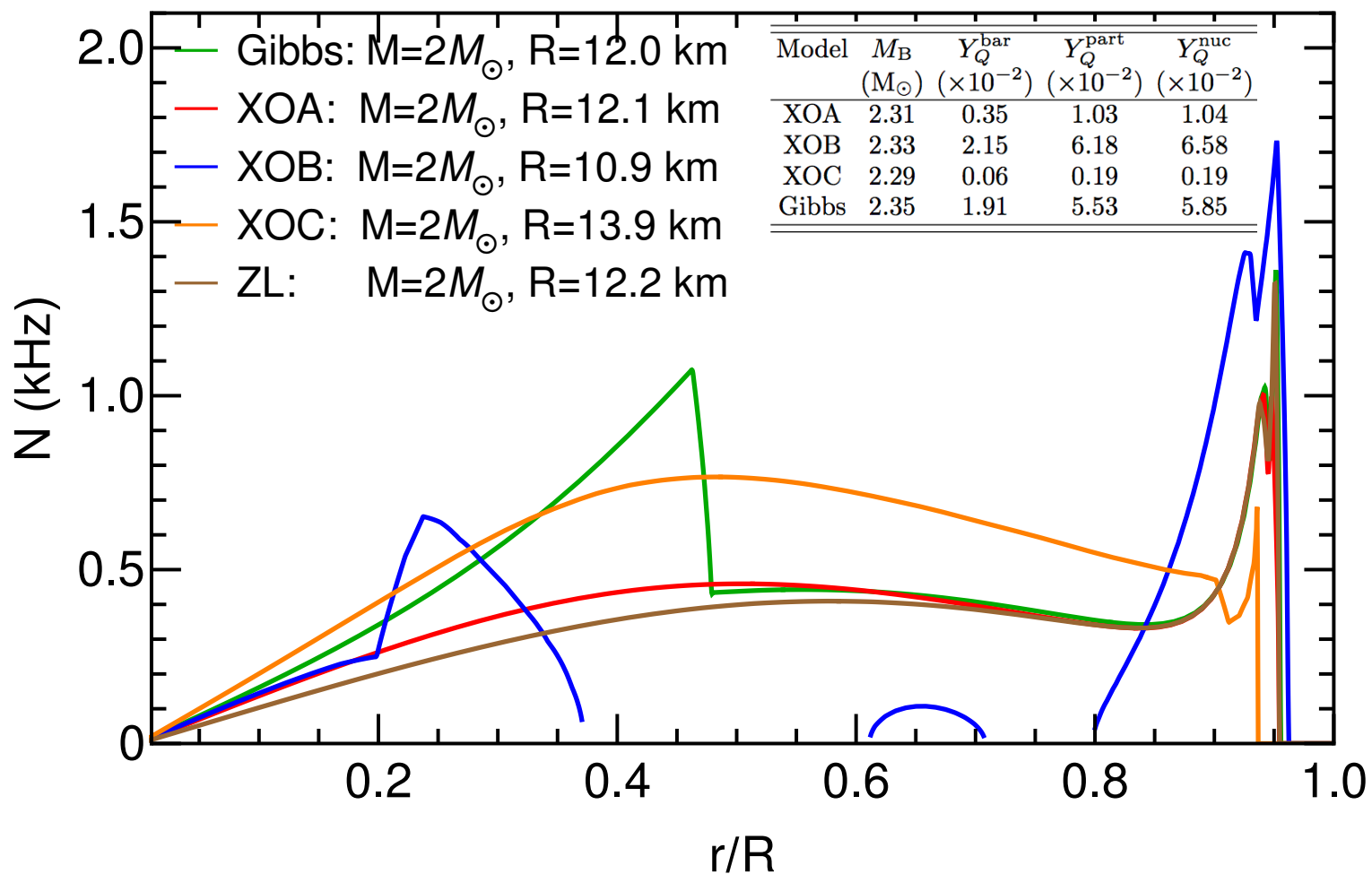


- nucleonic only (ZL) - both increase monotonically
- admixtures of nucleons and quarks (Gibbs or crossover) induce non-monotonic behavior
- $c_{ad}^2 > c_{eq}^2$ for all densities except XOB

- the **difference** $\Delta(c^{-2}) = 1/c_{eq}^2 - 1/c_{ad}^2$ drives the restoring force for g -mode oscillations
- smooth variation in composition is unlikely observable; needs radical changes in (new) particle species



Brunt-Väisälä in a hybrid star



arXiv:2109.14091

Global g -mode frequency

$$\frac{dU}{dr} = \frac{g}{c_{ad}^2} U + e^{\lambda/2} \left[\frac{l(l+1)e^\nu}{\omega^2} - \frac{r^2}{c_{ad}^2} \right] V$$

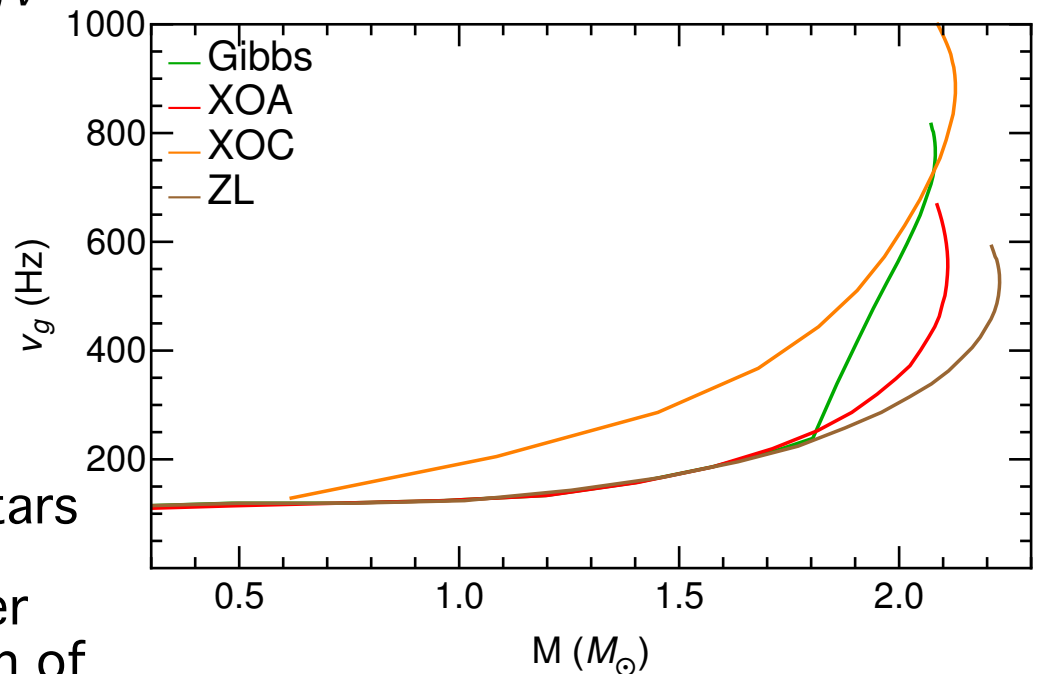
$$\frac{dV}{dr} = e^{\lambda/2-\nu} \left(\frac{\omega^2 - N^2}{r^2} \right) U + g\Delta(c^{-2})V$$

$$\Delta(c^{-2}) = 1/c_{eq}^2 - 1/c_{ad}^2$$

$$\nu_g = \omega / (2\pi)$$

- $\nu_g \gtrsim 600$ Hz only for hybrid stars
- signature of exotic phases - higher frequency indicates larger fraction of quark matter

- most distinct feature of Gibbs
- mixed phase onset \rightarrow peak in local BV frequency \rightarrow kink in global g -mode frequency

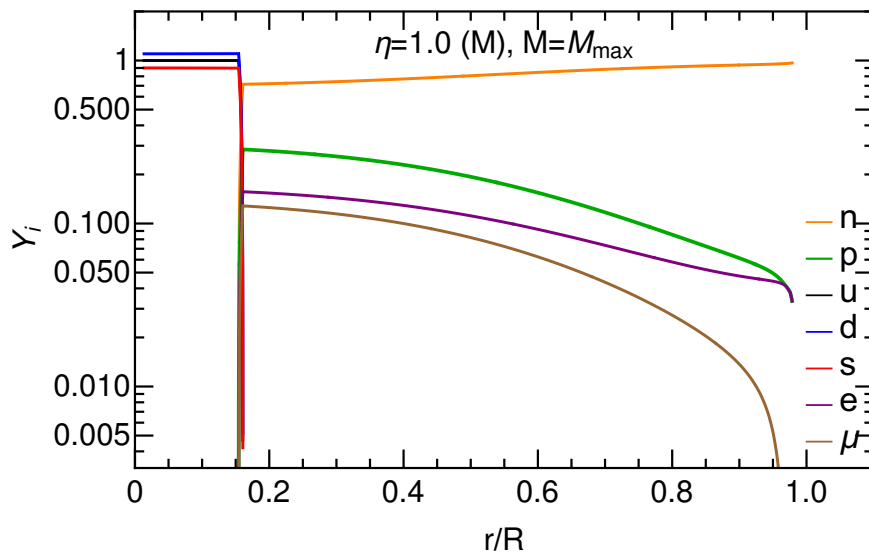


arXiv:2109.14091

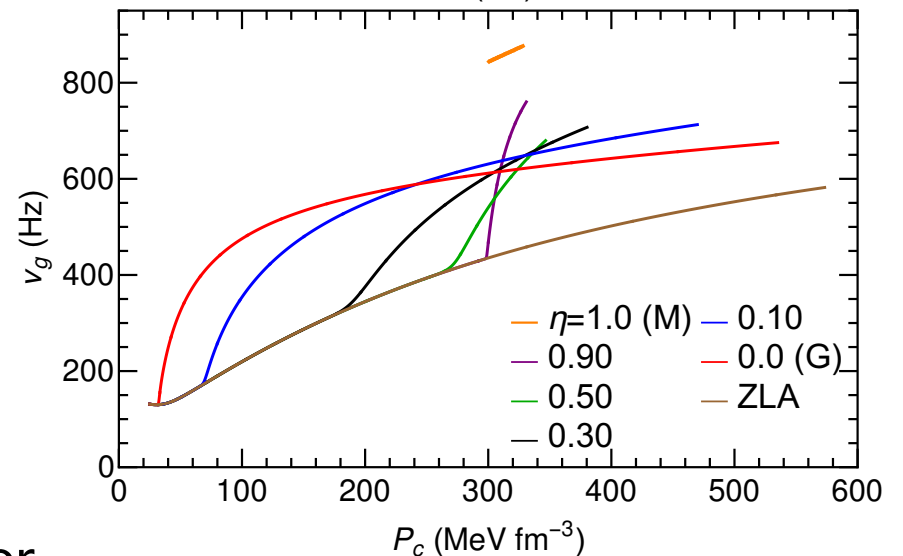
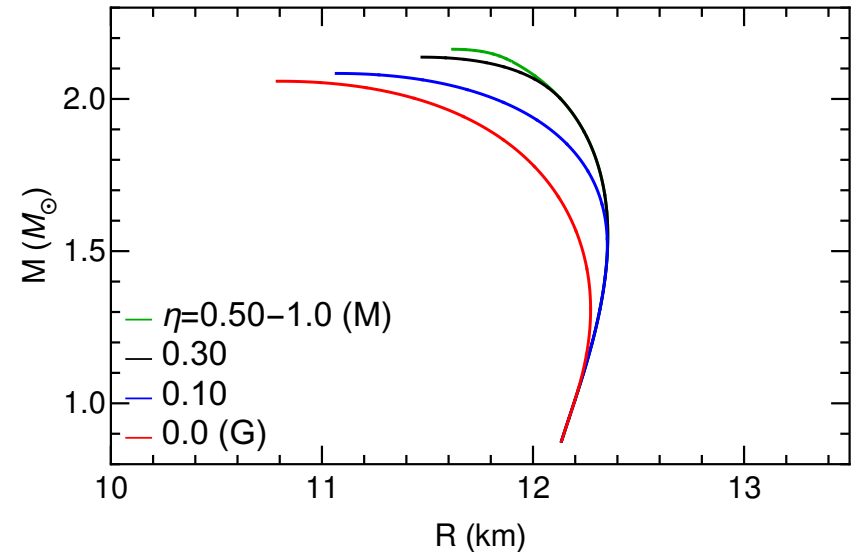
Interpolation between Gibbs and Maxwell

- system under local vs. global charge neutrality in Maxwell (M) vs. Gibbs (G) construction

see Constantinos' talk on Wednesday

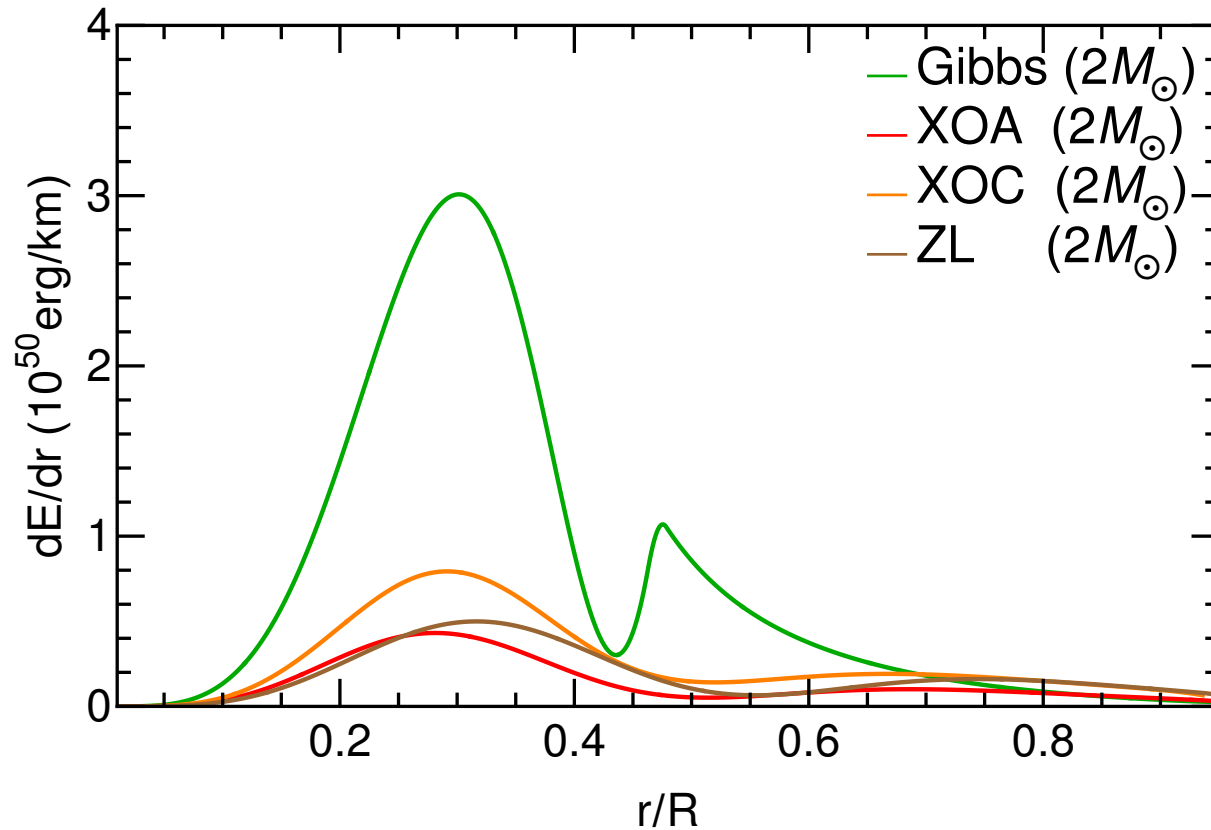


- “discontinuity” g -mode observed when there exists a sharp boundary
- new variable introduced as a proxy for the highly uncertain surface tension



arXiv:2302.04289

Mode energies and tidal forcing



$$\frac{dE}{dr} = \frac{\omega^2 r^2}{2} (\varepsilon + p) e^{(\lambda - \nu)/2} [\xi_r^2 e^\lambda + l(l+1) \xi_h^2]$$

$$U = r^2 e^{\lambda/2} \xi_r, \text{ radial}$$

$$V = \omega^2 r \xi_h, \text{ tangential}$$

- energy per unit radial distance contained in the oscillatory motion

Estimated impact on the GW waveform

Vick & Lai (2019)

$$\left| \frac{\Delta E}{E} \right| \simeq 2.3 \times 10^{-3}, \text{ NS}$$

$$\simeq 5.9 \times 10^{-3}, \text{ HS (Gibbs)}$$

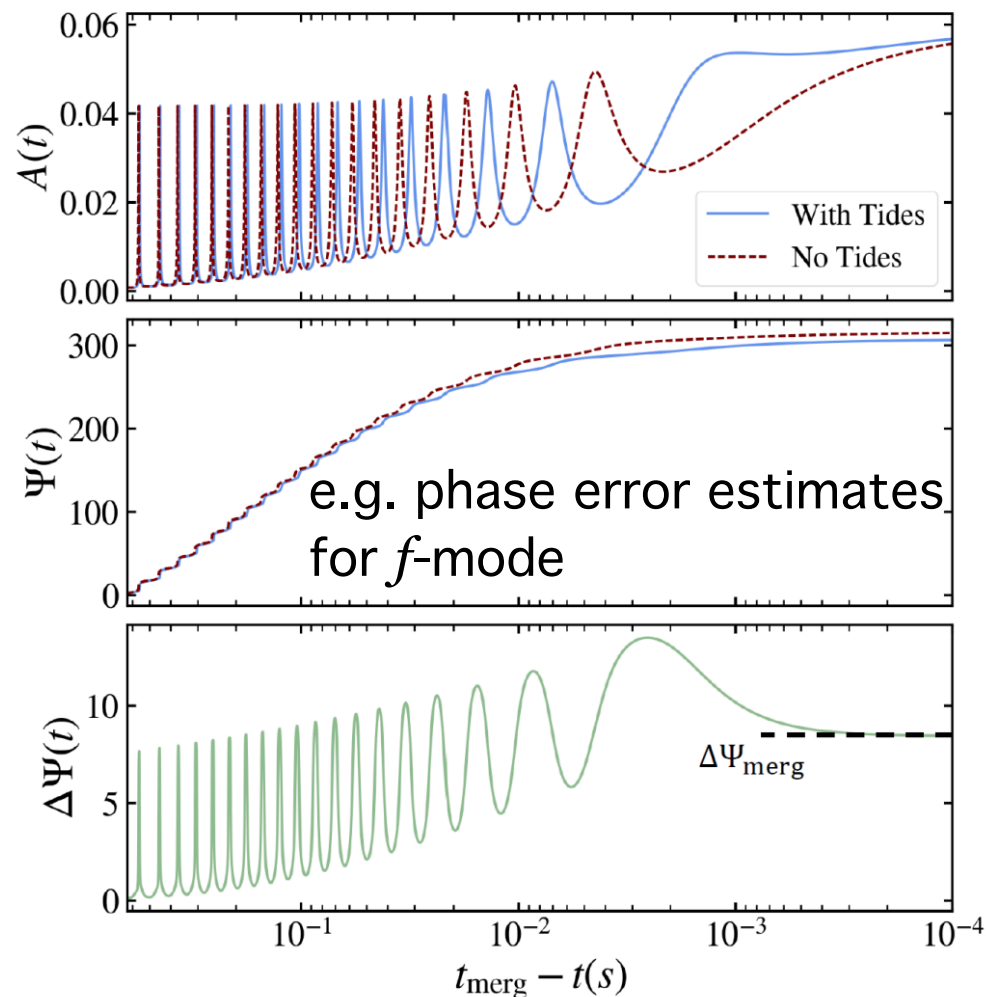
$$\simeq 2 \times 10^{-9}, \text{ SQS}$$

- resonant excitation leads to phase shift later in the inspiral (at higher frequencies); shorter duration of accumulation

$$\Delta\phi \simeq 0.8, \text{ NS}$$

$$\simeq 0.45, \text{ HS (Gibbs)}$$

$$\simeq 6 \times 10^{-4}, \text{ SQS}$$



Conclusions and outlook

- **Stellar oscillation modes carry imprints of the phase of NS interior through resonant excitation frequencies; compared to bulk properties such as masses and radii, they are more sensitive to matter composition**
- **g-modes can probe stratification: mixed phase/crossover/sharp discontinuity inside neutron stars; assuming exotic contents such as quark matter in the inner core, scenarios of crossover vs. 1st-OPT (Gibbs) can be distinguished**
- **Detection of oscillation modes is worth pursuing with improved sensitivity and more detectors as in XG network; may need continuous GW sources and/or large amplitudes**
- **Future directions: microphysics of reaction rates; thermal effects involving exotic matter; damping times, viscosity..**

THANK YOU!

Q & A

BACKUP

SLIDES

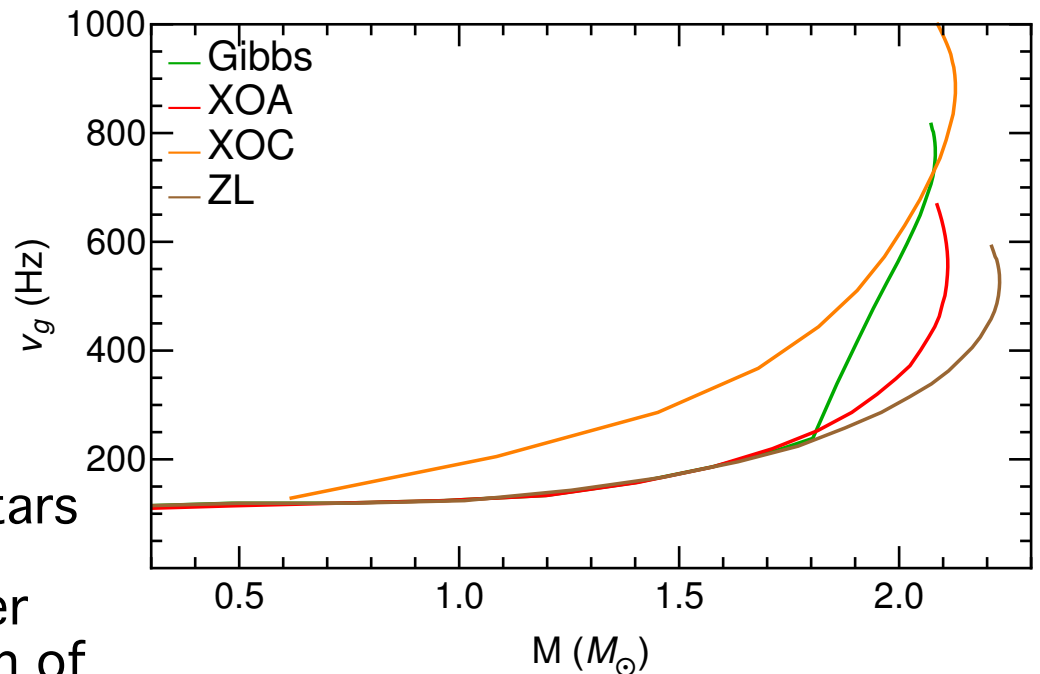
Global g -mode frequency

- comparison with other works

Authors [Ref.]	Core Composition	M [M_\odot]	$f_g = \omega_g/(2\pi)$ [kHz]
Reisenegger & Goldreich [17]	npe	1.405	0.215
Lai [20]	npe	1.4	0.073
Kantor & Gusakov [29]	npe	1.4	0.13
Kantor & Gusakov [29]	$npe\mu$	1.4	0.19
Kantor & Gusakov [29]	$npe\mu(\text{SF})$	1.4	0.46
Dommes & Gusakov [30]	$npe\mu\Lambda(\text{SF})$	1.634	0.742
Yu & Weinberg [33]	$npe\mu$	1.4	0.13
Yu & Weinberg [33]	$npe\mu(\text{SF})$	2.0	0.45
Rau & Wasserman [34]	$npe\mu(\text{SF})$	2.0	0.45

- $\nu_g \gtrsim 600$ Hz only for hybrid stars
- signature of exotic phases - higher frequency indicates larger fraction of quark matter

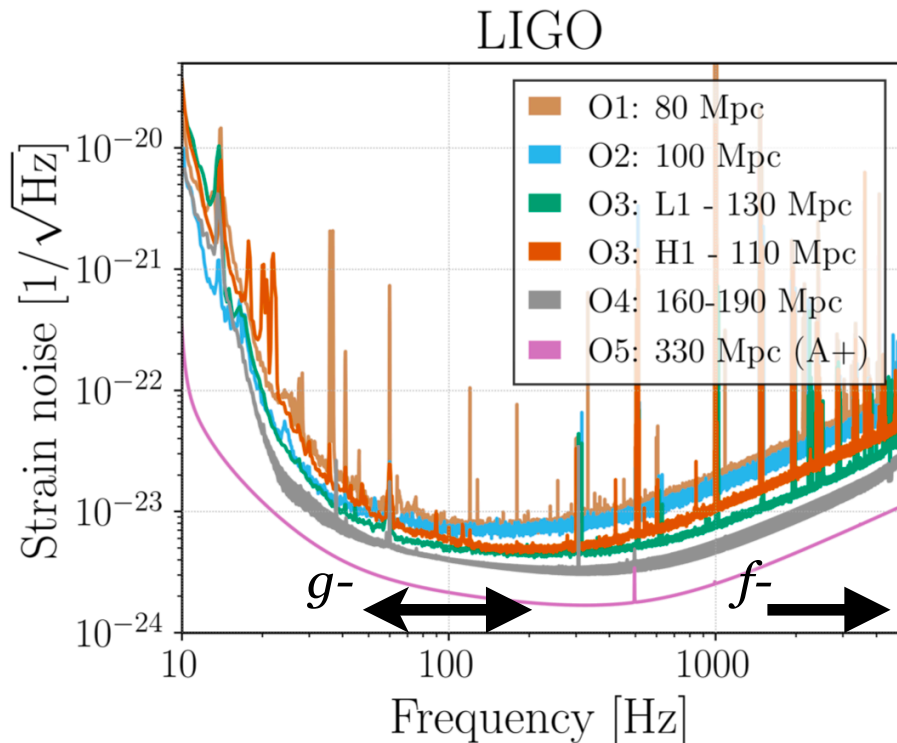
- most distinct feature of Gibbs
- mixed phase onset \rightarrow peak in local BV frequency \rightarrow kink in global g -mode frequency



arXiv:2109.14091

Restoring forces and frequency

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stable oscillation modes (“ringing”)
 -> **continuous** GWs

- p -mode/ f -mode: main restoring force is the pressure (>1.5 kHz)

$$\nu \approx \sqrt{\frac{GM}{R^3}}$$

- inertial modes (r -modes): main restoring force is the Coriolis force

$$\nu \approx \Omega$$

- w -modes: pure space-time modes i.e. only in GR (>5 kHz)

$$\nu \approx \frac{1}{R} \left(\frac{GM}{Rc^2} \right)$$

- shear-/torsional-; many other more

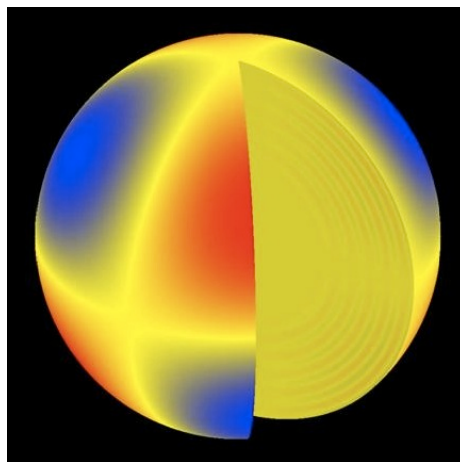
Stellar oscillations

matter imprints in **transient** GWs

- tidal effects on pre-merger (inspiral) waveform of BNS mergers
- tidal disruption in NS-BH mergers
- oscillations of merger remnant
- oscillation in supernova post-collapse phase

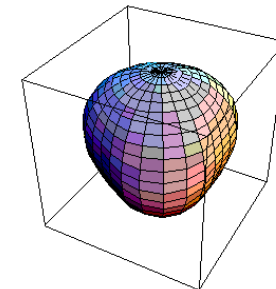
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..unless they become unstable



stable oscillation modes (“ringing”)
-> **continuous** GWs

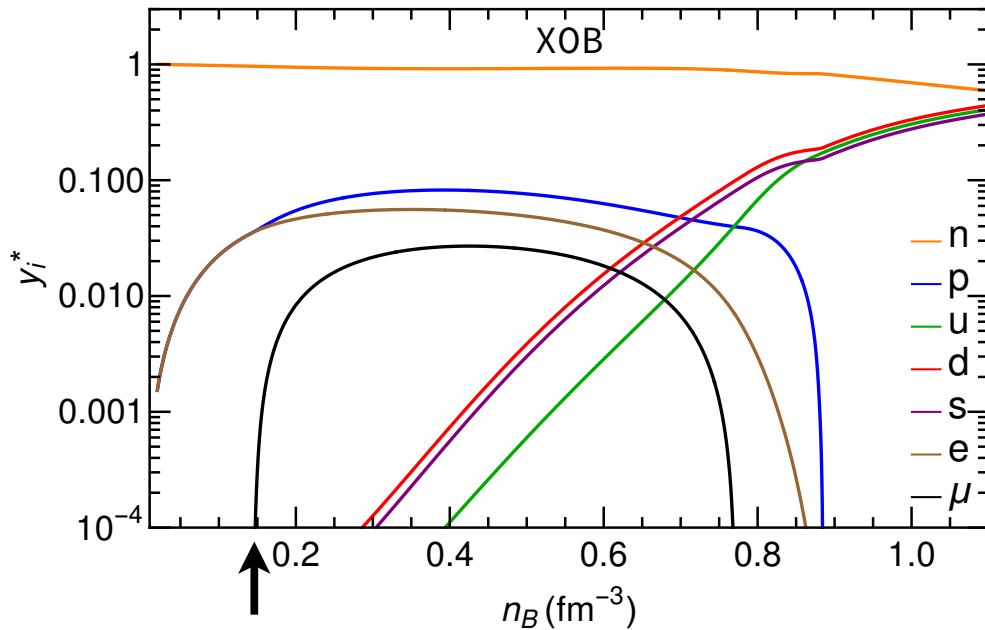
- *f*-mode (fundamental mode) scales with average density
- *p*-mode (pressure mode) probes the sound speed
- *g*-mode (gravity mode) sensitive to **composition**/thermal gradients
- *w*-mode, *s*-mode, *i*-mode/*r*-mode..



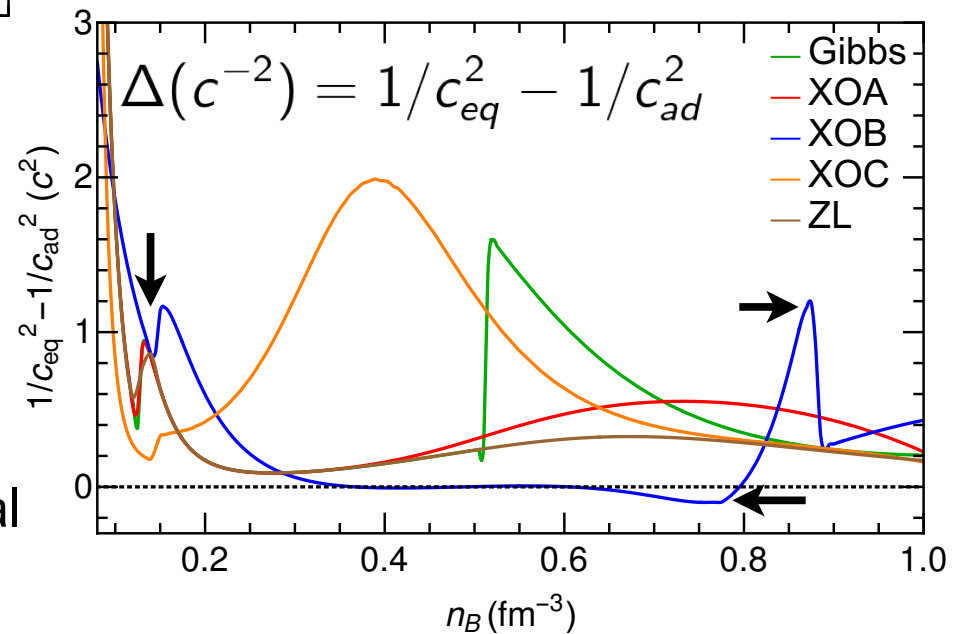
$$l = 3, m = 3$$

small amplitude oscillations ->
weak (continuous) emission of GWs

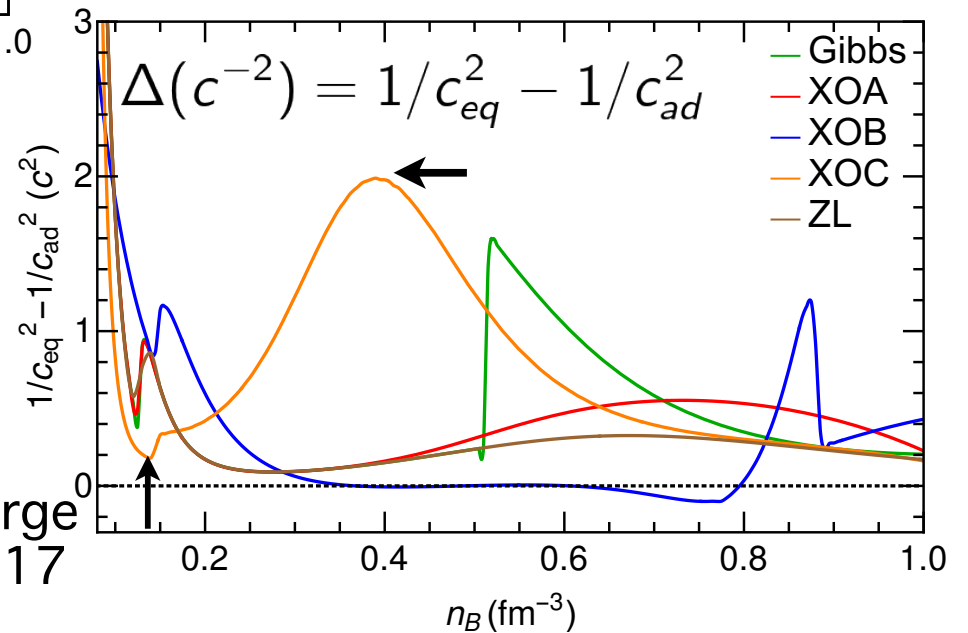
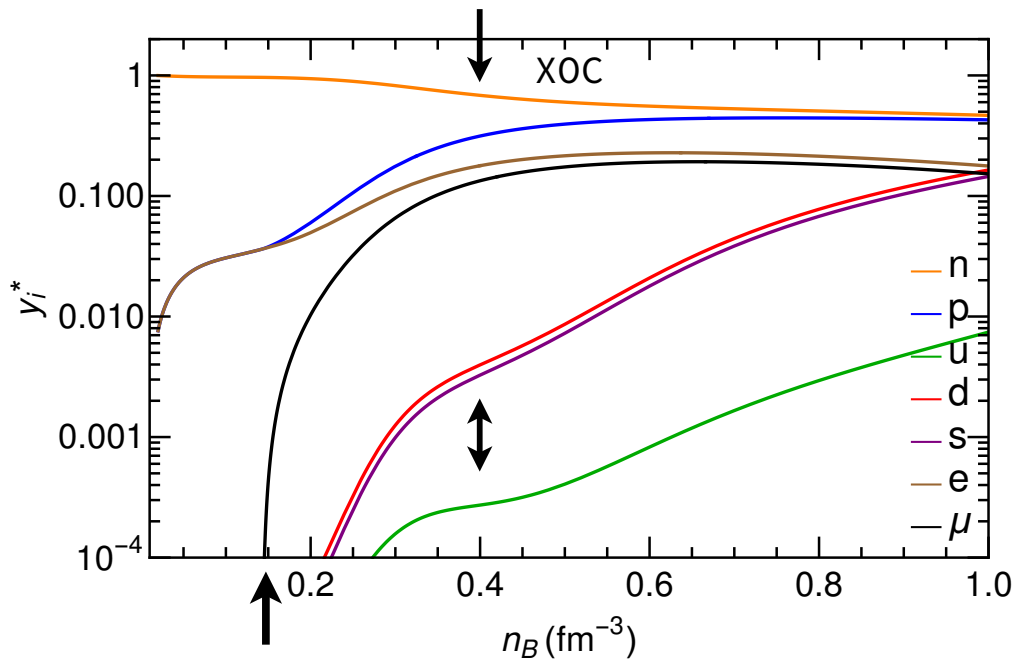
Sound speed and composition



- muons set in $\sim n_{\text{sat}}$
- peak $\sim 5.5 n_{\text{sat}}$: muons and protons **disappear**
- instability to convection - unphysical region

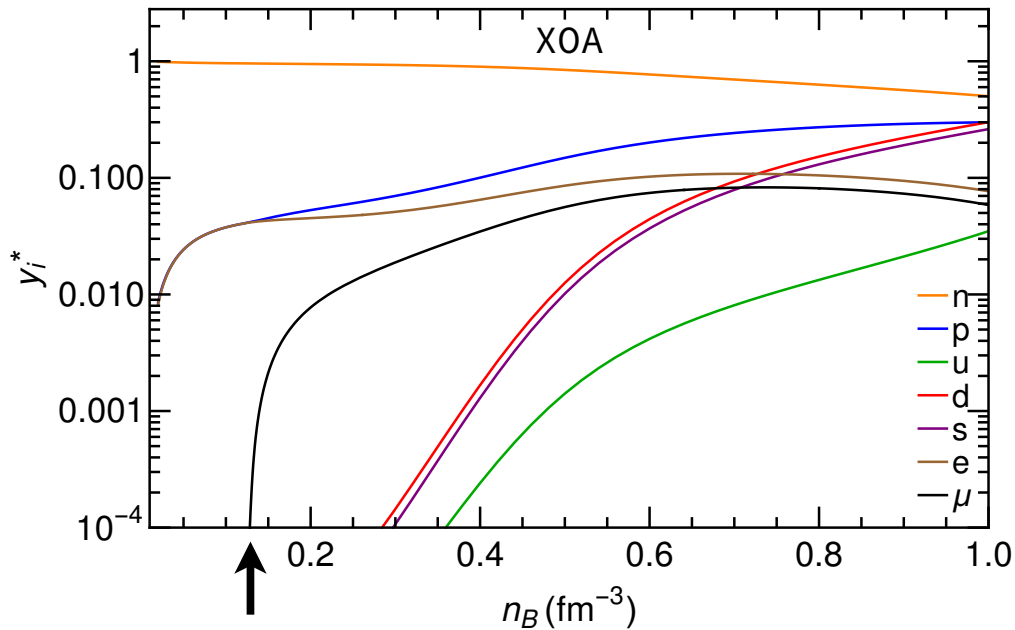


Sound speed and composition



- muons set in $\sim n_{\text{sat}}$
- peak $\sim 2.5 n_{\text{sat}}$: inflection points in quark and neutron fractions
- **too stiff** at low densities; radii too large to be compatible with e.g. GW170817

Sound speed and composition



- muons set in \sim nsat
- XOA - “good” crossover model, very similar to ZL
- Gibbs peak \sim 2.5 nsat: onset of the mixed phase

