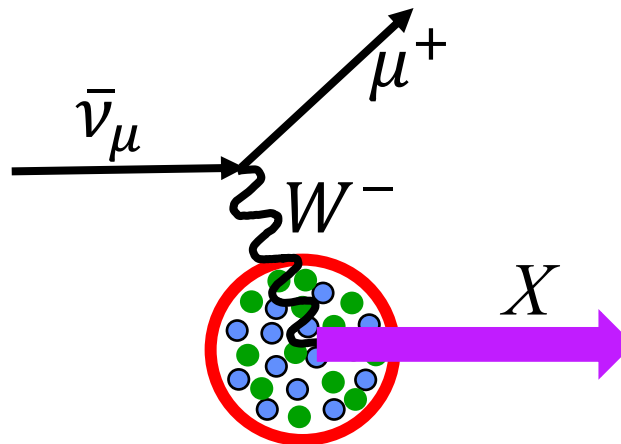
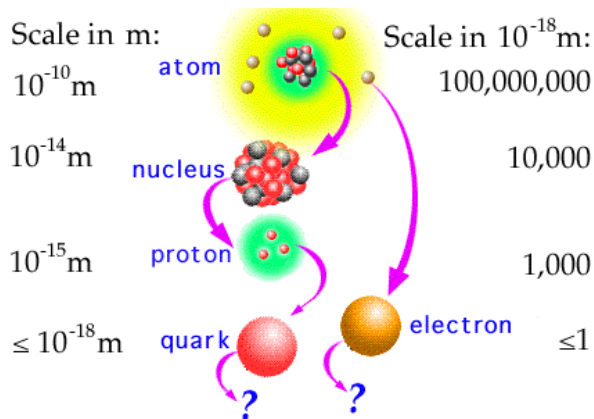


The Status of the Axial-vector Form Factor and its Uncertainties in Lattice QCD

Rajan Gupta
Theoretical Division, T-2
Los Alamos National Laboratory, USA



Elementary Particles

Quarks	u up	c charm	t top	γ photon
	d down	s strange	b bottom	
Leptons	ν_e electron neutrino	ν_μ muon neutrino	ν_τ tau neutrino	Z Z boson
	e electron	μ muon	τ tau	
	I	II	III	Force Carriers

Three Families of Matter

Theoretical Physics Uncertainties to Empower Neutrino Experiments,
 INT, University of Washington, Seattle, USA

Acknowledgements

Thanks to my collaborators (PNDME and NME collaborations)

Tanmoy Bhattacharya, Vincenzo Cirigliano, Yong-Chull Jang, Balint Joo, Huey-Wen Lin, Emanuele Mereghetti, Santanu Mondal, Sungwoo Park, Oleksandr (Sasha) Tomalak, Frank Winter, Junsik Yoo, Boram Yoon

Thanks for computer resources

OLCF (INCITE HEP133), ERCAP@NERSC (HEP, NP), USQCD@JLAB, LANL IC

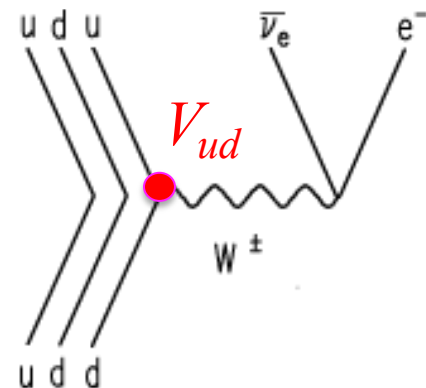
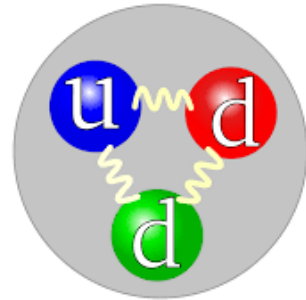
- USQCD Community white paper:
Lattice QCD and Neutrino-Nucleus Scattering, *Eur.Phys.J.A* 55 (2019) 11, 196
- Snowmass 2021 White Paper
Theoretical tools for neutrino scattering: interplay between lattice QCD, EFTs, nuclear physics, phenomenology, and neutrino event generators. e-Print: [2203.09030](https://arxiv.org/abs/2203.09030) [hep-ph]

Publications on Form Factors

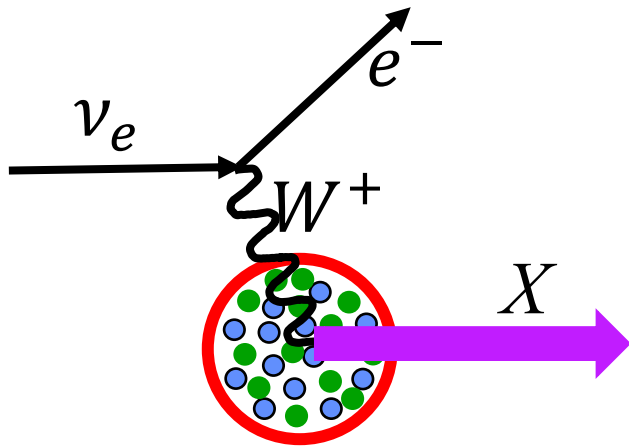
- AFF: R. Gupta et al, (PNDME) PhysRevD.96.114503 (2017)
- VFF: Y-C Jang, et al, (PNDME) PhysRevD.101.014507 (2020)
- AFF: Y-C Jang et al, (PNDME) PRL 124 (2020) 072002
- Both: S. Park, et al, (NME) PRD 105, 054505 (2022)
- AFF: Y-C Jang, et al, (PNDME) arXiv:2305:11330

Outline:

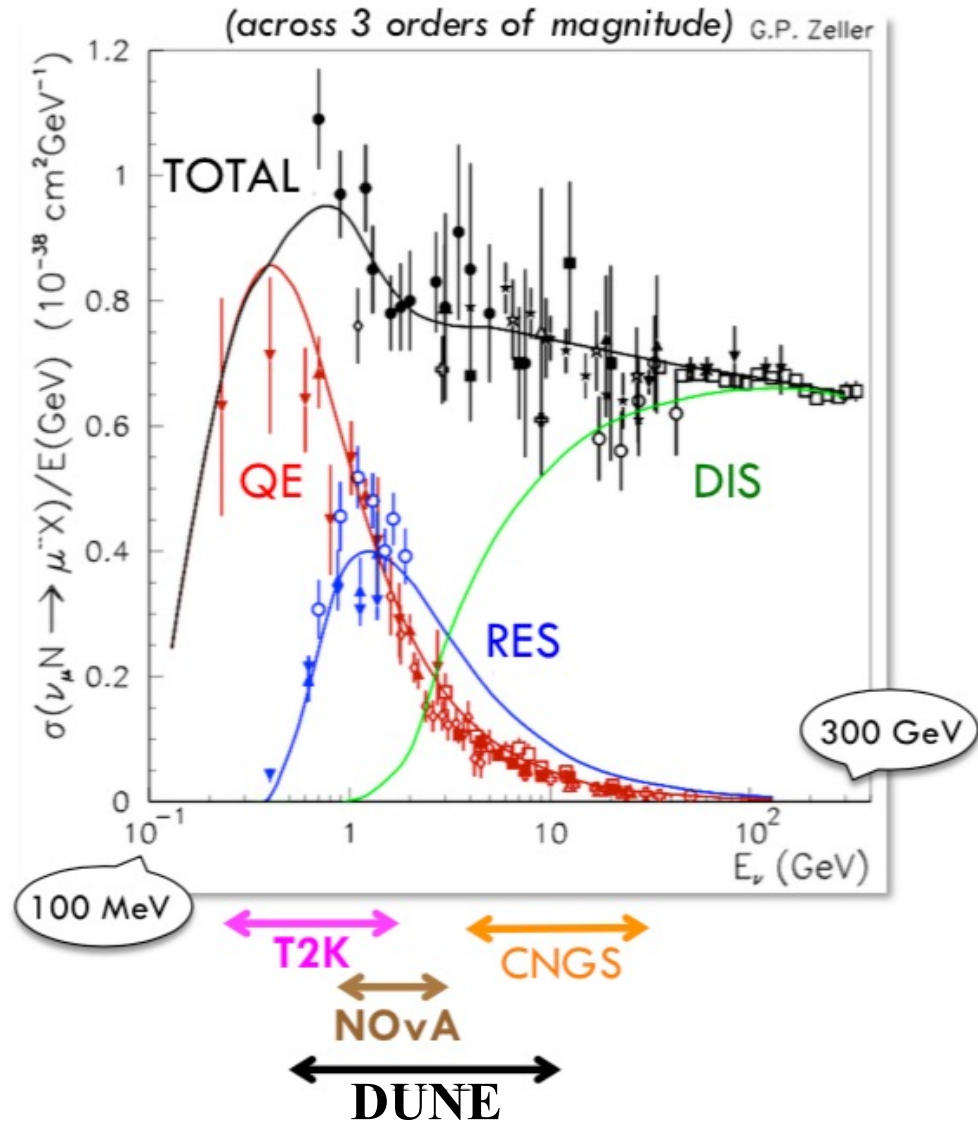
- What LQCD can provide for ν -nucleus oscillation experiments
 - Axial and vector form factors of nucleons [nuclei are much more challenging]
 - Nuclear corrections in nuclei: $(p, n) \rightarrow (C^{12}, O^{16}, Ar^{40})$
- Challenges to the calculations of nucleon matrix elements
 - Signal-to-noise falls as $e^{-(M_N - 1.5M_\pi)\tau}$
 - Excited states in nucleon correlation functions
 - Extrapolation in $\{a, M_\pi, M_\pi L\}$
- FF must satisfy PCAC
 - What we learned from $\langle N(p_f) | A_4(q) | N(p_i) \rangle$
 - Towers of $N\pi, N\pi\pi$, states contribute to axial and PS correlators
- Comparison of published results for g_A, G_A
- Comparison with MINERvA and ν -D analyses
- Summary of unpublished results for g_A, G_A
- Transition matrix elements
- Results for G_E, G_M
- Future



ν energy range covers complex physics



- Incoming neutrino energy and flux not known precisely
- Dynamics of struck Argon nucleus is too complex to simulate directly and connect to final states seen in the detectors



Goal: Inputs for DUNE

Matrix elements (form factors) for $\nu - {}^{40}\text{Ar}$ scattering

$$\langle X | A_\mu(q) | {}^{40}\text{Ar} \rangle$$

$$\langle X | V_\mu(q) | {}^{40}\text{Ar} \rangle$$

Building blocks:

Starting with nucleons and different energy regions:

$$\langle p | J_\mu^W(q) | n \rangle \quad \text{Quasi-elastic}$$

$$\langle n\pi | J_\mu^W(q) | n \rangle, \langle \Delta | J_\mu^W(q) | n \rangle \quad \text{Resonant}$$

$$\langle X | J_\mu^W(q) | n \rangle \quad \text{DIS}$$

Including nuclear effects in scattering off complex nuclear targets

Nuclear many body Hamiltonian takes as input matrix elements involving successively more multi-particles

– One nucleon $\langle p | J_{\mu}^{+}(q) | n \rangle$

– Transition $\langle n\pi | J_{\mu}^{W}(q) | n \rangle, \langle \Delta | J_{\mu}^{W}(q) | n \rangle$

– Two nucleon $\langle n p | J_{\mu}^{W+}(q) | n n \rangle$

The ν -n differential cross-section:

$$\frac{d\sigma}{dQ^2} \begin{pmatrix} \nu_l + n \rightarrow l^- + p \\ \bar{\nu}_l + p \rightarrow l^+ + n \end{pmatrix} = \frac{M^2 G_F^2 \cos^2 \theta_c}{8\pi E_\nu^2} \left\{ A(Q^2) \pm B(Q^2) \frac{(s-u)}{M^2} + C(Q^2) \frac{(s-u)^2}{M^4} \right\},$$

$$A(Q^2) = \frac{(m^2 + Q^2)}{M^2} \left[(1 + \tau) F_A^2 - (1 - \tau) F_1^2 + \tau(1 - \tau) F_2^2 + 4\tau F_1 F_2 - \frac{m^2}{4M^2} \left((F_1 + F_2)^2 + (F_A + 2F_P)^2 - 4 \left(1 + \frac{Q^2}{4M^2} \right) F_P^2 \right) \right],$$

$$B(Q^2) = \frac{Q^2}{M^2} F_A (F_1 + F_2),$$

$$C(Q^2) = \frac{1}{4} (F_A^2 + F_1^2 + \tau F_2^2).$$

$\langle N A_\mu N \rangle \rightarrow$ linear combination of F_A, \tilde{F}_P

$\langle N V_\mu N \rangle \rightarrow G_E, G_M$

F_A = axial form factor
 $G_E = F_1 - \tau F_2$ Electric
 $G_M = F_1 + F_2$ Magnetic
 $\tau = Q^2 / 4M^2$
 $M = M_p = 939$ MeV
 m = mass of the lepton

Analysis of (e, μ, ν) - n scattering involves

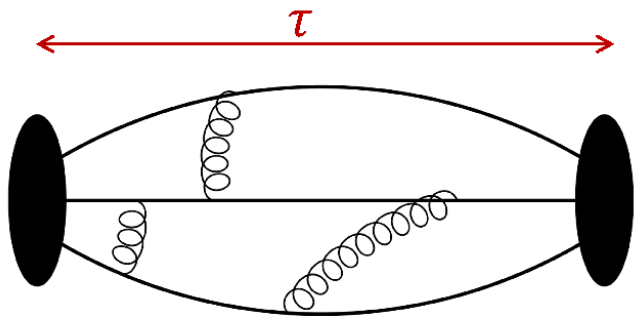
5 Form Factors & 3 charges g_A, μ, g_p^*

- $G_E(Q^2)$ Electric
- $G_M(Q^2)$ Magnetic
- $G_A(Q^2)$ Axial
- $\tilde{G}_P(Q^2)$ Induced pseudoscalar
- $G_P(Q^2)$ Pseudoscalar (extracted from $\langle NPN \rangle$)
- Lattice methodology is common: all calculated at the same time
- Precise experimental data exist for $G_E(Q^2)$ and $G_M(Q^2)$
- Axial ward identity (PCAC) relates $G_A(Q^2), \tilde{G}_P(Q^2), G_P(Q^2)$

- $G_E(Q^2 = 0) = 1$ Conserved vector charge
- $G_M(Q^2 = 0) = \mu = 4.7058$ Magnetic moment
- $G_A(Q^2 = 0) = g_A = 1.276(2)$ Axial charge
- $\tilde{G}_P(Q^2 = 0.88m_\mu^2) = g_p^* = 8.06(55)$ Induced pseudoscalar charge

Lattice QCD gives us

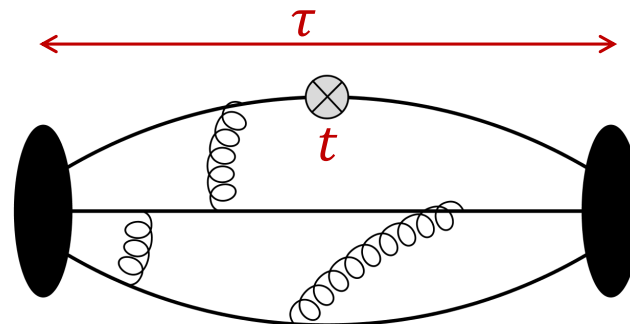
2-point function



$$\langle \Omega | \widehat{N}_\tau^\dagger \widehat{N}_0 | \Omega \rangle$$

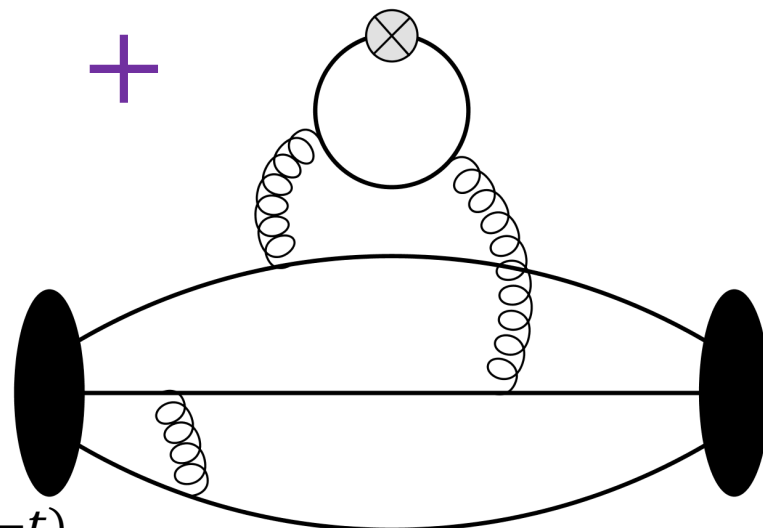
$$\Gamma^{2pt}(\tau) = \sum_i |A_i|^2 e^{-E_i \tau}$$

3-point functions



Connected

+

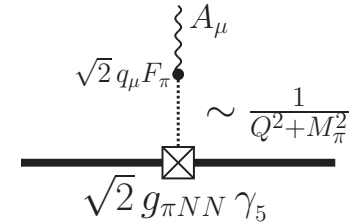
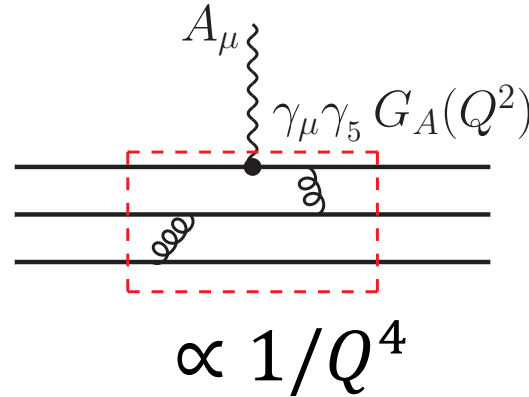
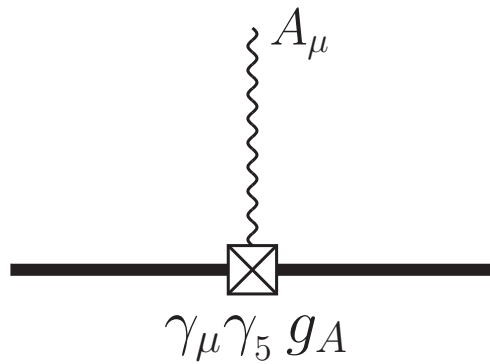


Disconnected

$$\langle \Omega | \widehat{N}_\tau^\dagger O(t) \widehat{N}_0 | \Omega \rangle$$

$$\Gamma_O^{3pt}(t, \tau) = \sum_{i,j} A_i^* A_j \langle i | O | j \rangle e^{-E_i t - E_j (\tau - t)}$$

$\Gamma^n \rightarrow ME \rightarrow$ Axial-vector Form Factors, G_A, \tilde{G}_P, G_P



On each ensemble characterized by $\{a, M_\pi, M_\pi L\}$

$$\langle N(p_f) | A^\mu(q) | N(p_i) \rangle = \bar{u}(p_f) \left[\gamma^\mu G_A(q^2) + q_\mu \frac{\tilde{G}_P(q^2)}{2M} \right] \gamma_5 u(p_i)$$

$$\langle N(p_f) | P(q) | N(p_i) \rangle = \bar{u}(p_f) G_P(q^2) \gamma_5 u(p_i)$$

PCAC [$\partial_\mu A_\mu = 2mP$] relates G_A, \tilde{G}_P, G_P

Essential steps in the analysis

- Remove ESC from correlation functions Γ^n to obtain ME within ground-state nucleon
- Decompose ME into form factors $G(Q^2)$ on each ensemble $\{a, M_\pi, M_\pi L\}$
- Parameterize this $G(Q^2)|_{a, M_\pi, M_\pi L}$
- Perform CCFV extrapolation to get $G(Q^2)|_{cont}$
- Parameterize this $G(Q^2)|_{cont}$

Model averaging should include model choices at each step that have significant effect on result

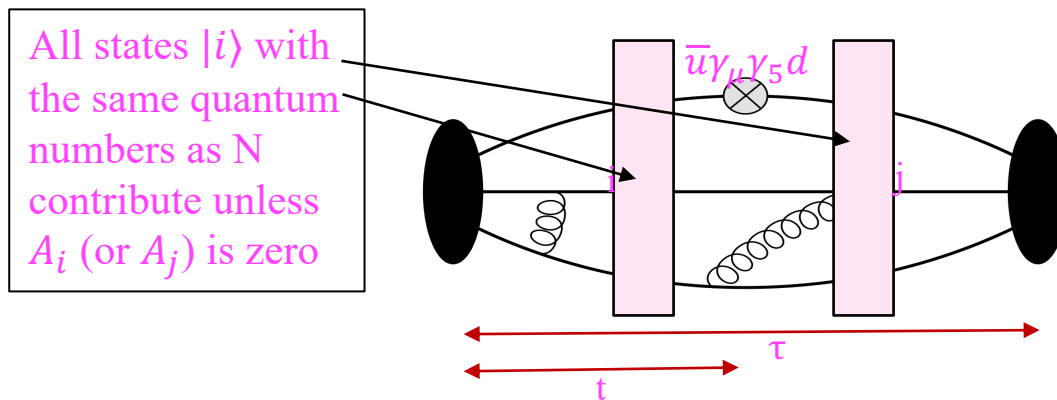
Calculations of nucleon 2,3-point functions using LQCD are mature

Spectrum (energies E_i & amplitudes A_i) and ME are extracted from fits to the spectral decomposition of 2- and 3-point functions

$$\Gamma^{2pt}(\tau) = \sum_i |A_i|^2 e^{-E_i \tau}$$

$$\Gamma_O^{3pt}(t, \tau) = \sum_{i,j} A_i^* A_j \underbrace{\langle i | O | j \rangle}_{\text{Extract } \langle 0 | O | 0 \rangle} e^{-E_i t - E_j (\tau - t)}$$

Extract $\langle 0 | O | 0 \rangle$



Radial excited States:

N(1440), N(1710)

Towers of multihadrons states

$N(\vec{k})\pi(-\vec{k}) > 1200 \text{ MeV}$

$N(0)\pi(\vec{k})\pi(-\vec{k}) > 1200 \text{ MeV}$

but removing ESC from multihadron states remains a challenge

Challenges for nucleon ME

- Need large τ to “kill” states with *small* mass gap ($\Delta M \sim 300$)

- Cannot go to large enough τ because the signal/noise degrades as $e^{-(M_N - 1.5M_\pi)\tau}$

– Signal: 2-pt: $\tau \sim 2\text{fm}$; 3-pt: $\tau \sim 1.5\text{fm}$

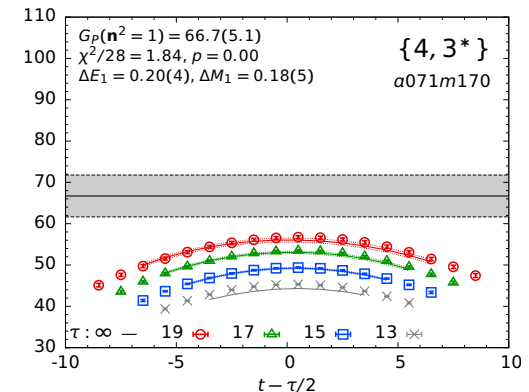
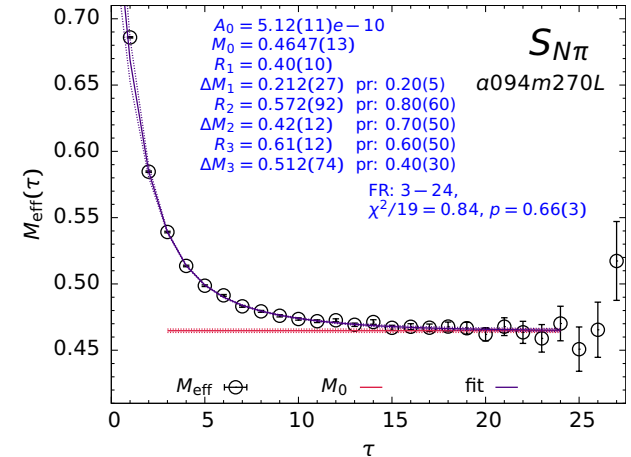
- Typical interpolating operator \hat{N} couples to the nucleon, its excitations and multi-hadron states with the same quantum numbers

- As $\vec{q} \rightarrow 0$, the towers of $N\pi$, $N\pi\pi$, states become arbitrarily dense above ~ 1230 MeV (the Δ region)

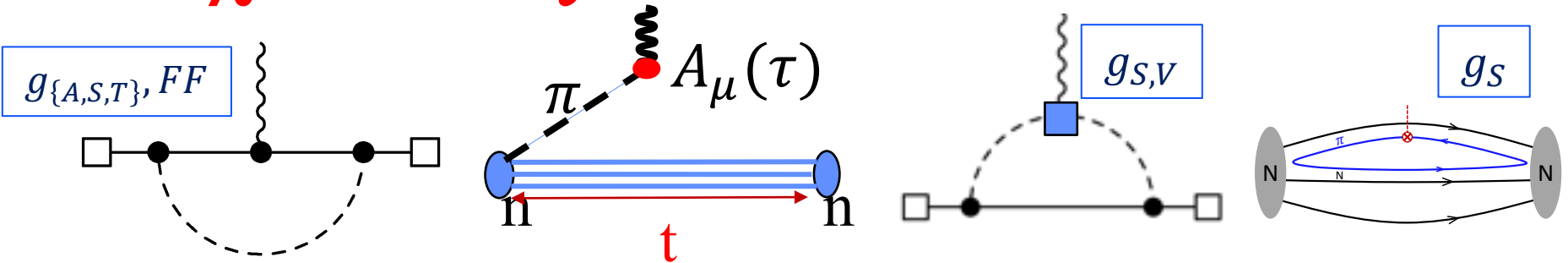
- Quantities impacted by $N\pi$, $N\pi\pi$, states should be analyzed on $M_\pi \lesssim 200$ MeV ensembles

- Excited states giving significant contribution to a particular ME are not known *a priori*. χ PT is a very useful guide

- *The potential of variational methods for isolating the ground state is just starting to be realized!*

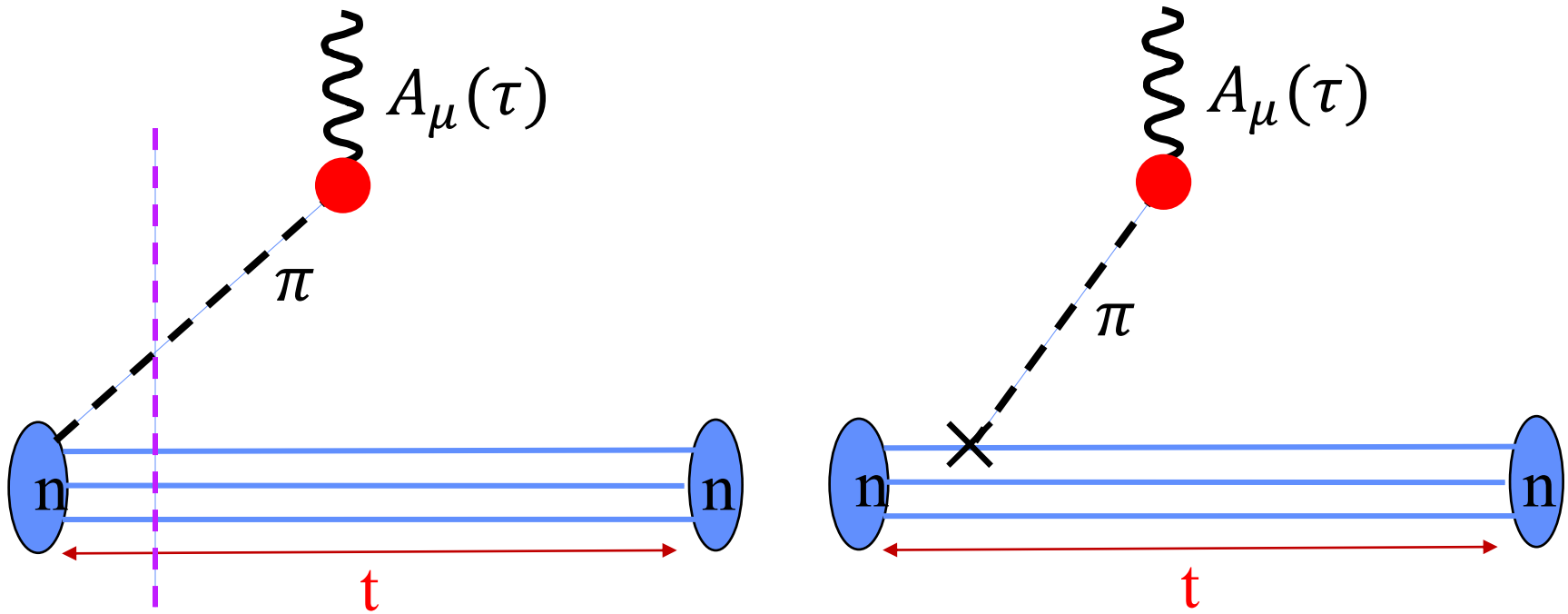


χ PT analysis of excited states



- Corrections from pion loops arise in all Γ^n
- Loops that originate or end at sources are ESC. These can be removed by a perfect nucleon source.
- Loops that originate on the nucleon line give rise to both: corrections to the physical result and excited state contributions (from pion going on-shell in Minkowski)
 - The latter are suppressed exponentially by the mass gap
 - Unless there are large cancellations, both should be considered in
 - (i) removing excited state contamination in Γ^n to get ME
 - (ii) Chiral fits to the data

χPT : $N\pi$ state coupling is large in the axial channel



Enhanced coupling to $N\pi$ state: Since the pion is light, the vertex \bullet can be anywhere in the lattice 3-volume

$$\sim V^{-1} A_i^* \langle i | A_4 | j \rangle \sim V$$

Decomposition of ground state matrix elements: $\langle N_\tau A_\mu(t) N_0 \rangle$ provides an over-determined set

Choosing “3” the direction of spin projection

$$\langle N(p_f) | A_{1,2}(q) | N(p_i) \rangle \rightarrow -\frac{q_{1,2}q_3}{2M} \tilde{G}_P$$

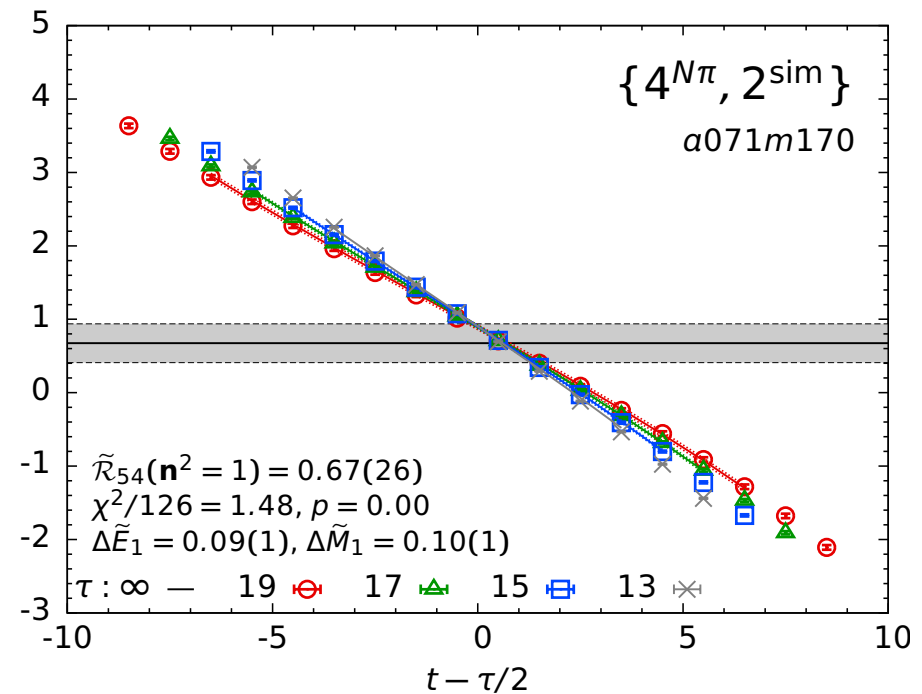
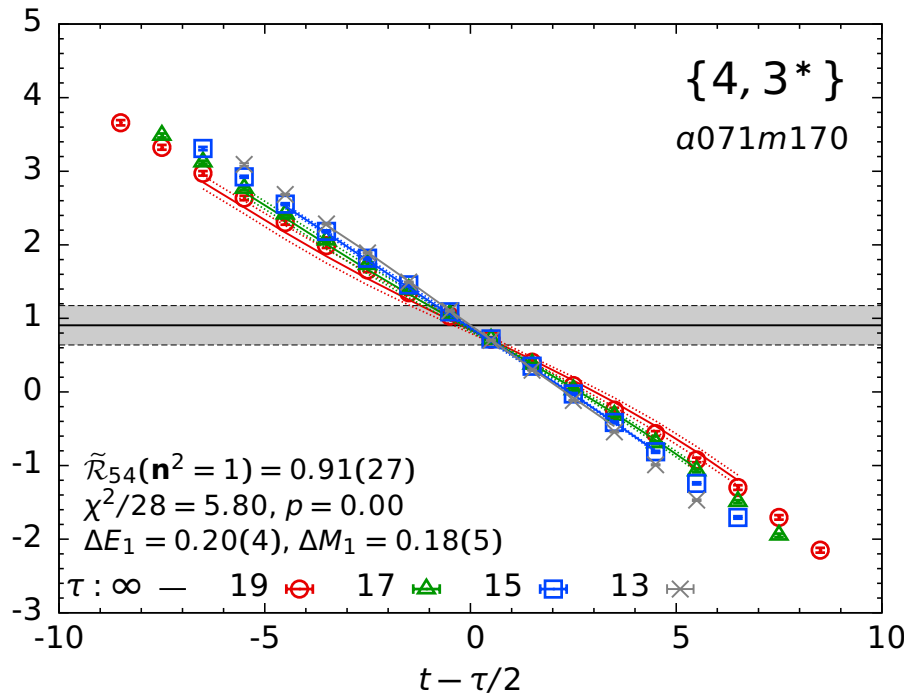
$$\langle N(p_f) | A_3(q) | N(p_i) \rangle \rightarrow -\left[\frac{q_3^2}{2M} \tilde{G}_P - (M + E)G_A \right] \left. \vphantom{\langle N(p_f) | A_3(q) | N(p_i) \rangle} \right\} \begin{array}{l} \text{Gives both} \\ G_A, \tilde{G}_P \end{array}$$

$$\langle N(p_f) | A_4(q) | N(p_i) \rangle \rightarrow -q_3 \left[\frac{E-M}{2M} \tilde{G}_P - G_A \right]$$

Redundant.
 Dominated by
 excited states

Data driven evidence for $N\pi$ excited state

- $\langle N_\tau A_4(t) N_0 \rangle$ has large ESC
- Fits with $N\pi$ as the first excited state are preferred



FF obtained including $N\pi$ state satisfy PCAC

Gupta et al, PhysRevD.96.114503 (2017) \rightarrow Jang et al, PRL 124 (2020) 072002

Constraints once FF are extracted from ground state matrix elements

1) PCAC ($\partial_u A_u = 2\hat{m}P$) requires

$$2\hat{m}G_P(Q^2) = 2M_N G_A(Q^2) - \frac{Q^2}{2M_N} \tilde{G}_P(Q^2)$$

2) In any [nucleon] ground state

$$\partial_4 A_4 = (E_q - M_0)A_4$$

3) G_A , \tilde{G}_P extracted from $\langle N(p_f) | A_i(q) | N(p_i) \rangle$
must be consistent with $\langle N(p_f) | A_4(q) | N(p_i) \rangle$

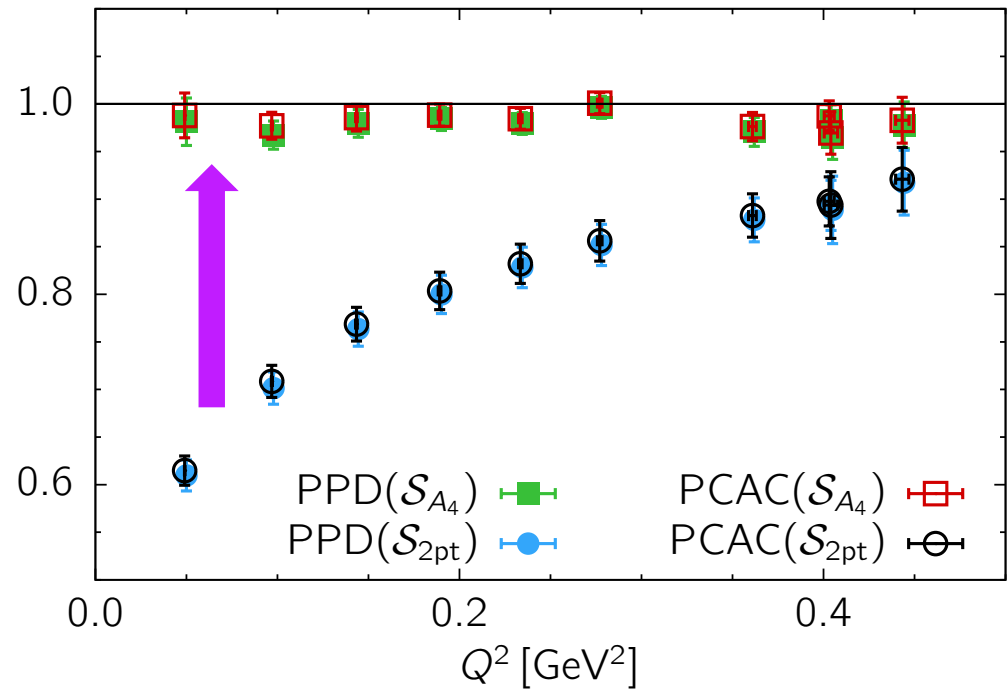
2017 → 2019: Resolution with PCAC and PPD

Gupta et al, PhysRevD.96.114503 → Jang et al, PRL 124 (2020) 072002

On including low mass $N_{p=0}\pi_p$ and $N_p\pi_{-p}$ excited states neglected in previous works, FF satisfy PCAC and PPD at $\sim 5\%$

$$\frac{\hat{m}G_P}{M_N G_A} + \frac{Q^2 \tilde{G}_P}{4M_N^2 G_A} = 1$$

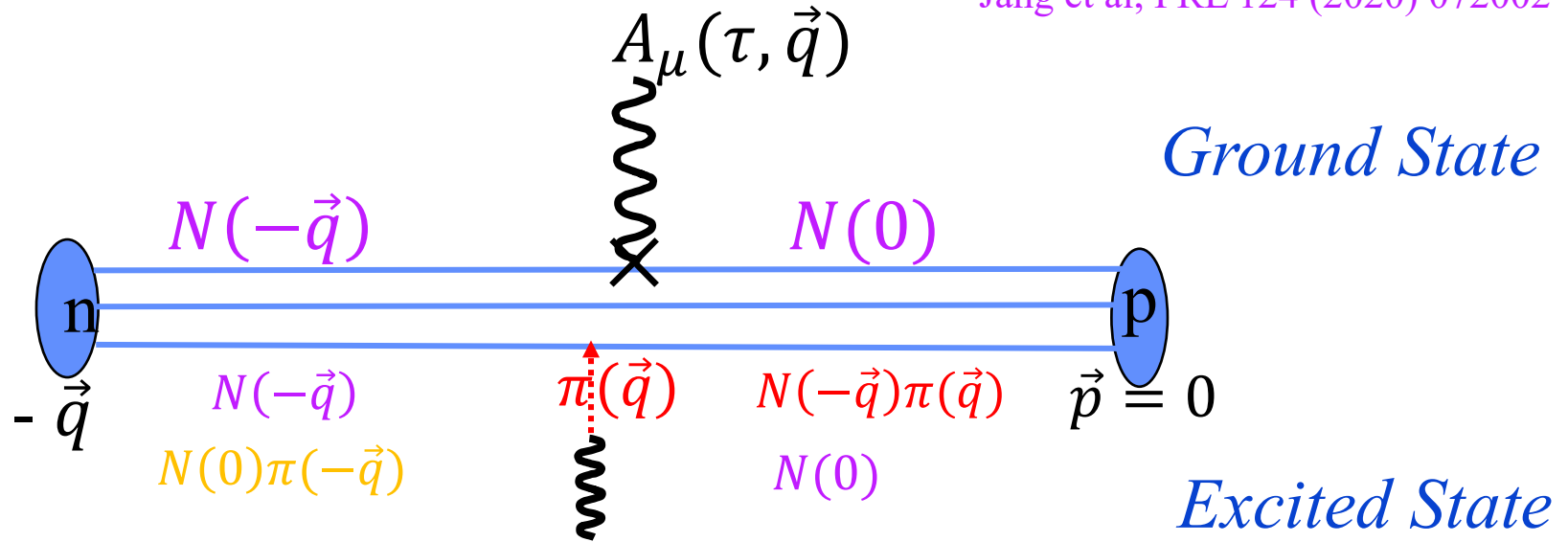
$$\frac{Q^2 + M_\pi^2}{4M_N^2} \frac{\tilde{G}_P(Q^2)}{G_A(Q^2)} = 1$$



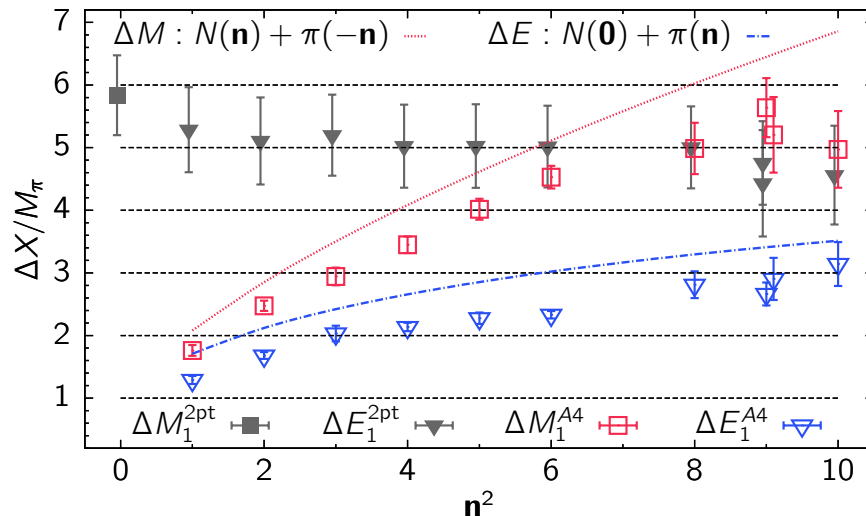
•Also see RQCD Collaboration: *JHEP* 05 (2020) 126, [1911.13150](https://arxiv.org/abs/1911.13150)

$N\pi$ state in the axial channel

Jang et al, PRL 124 (2020) 072002



Mass gaps extracted from fits match the above picture

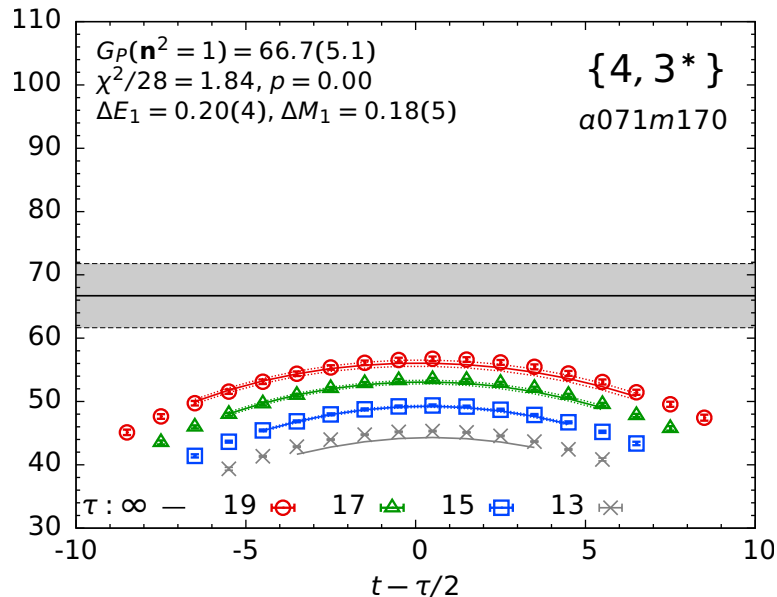


ΔM_1^{A4} and ΔE_1^{A4} are outputs of 2-state fits and not driven by priors

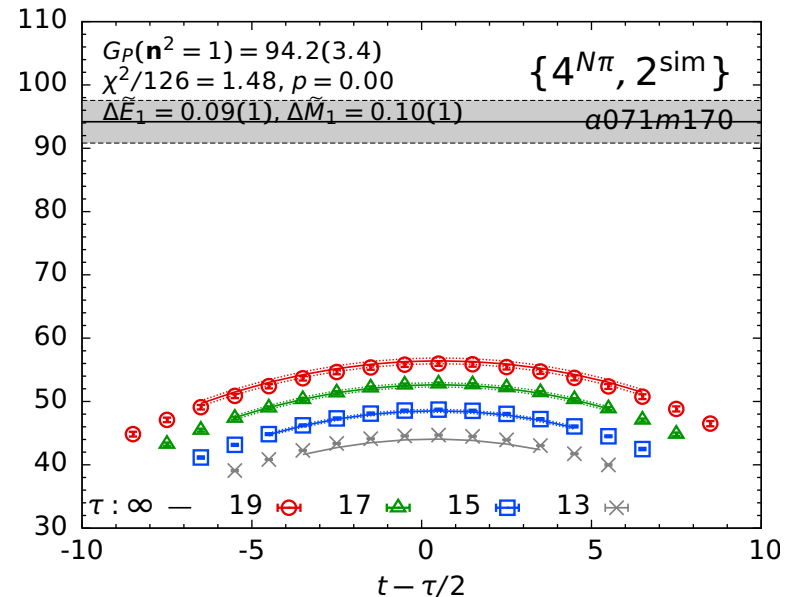
How large is the “ $N\pi$ ” effect?

Output of a simultaneous fit to $\langle A_i \rangle, \langle A_4 \rangle, \langle P \rangle$ (called $\{4^{N\pi}, 2^{sim}\}$ fit) increases the form factors by:

$$\left[\begin{array}{l} G_A \sim 5 \% \\ \tilde{G}_P \sim 45 \% \\ G_P \sim 45 \% \end{array} \right.$$



Standard 3-state fit to $\langle P \rangle$



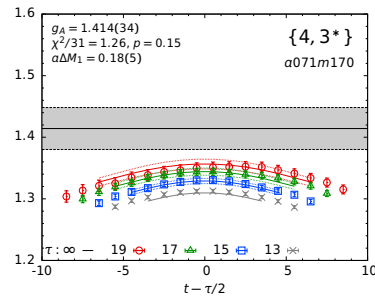
Simultaneous 2-state to $\langle A_i \rangle, \langle A_4 \rangle, \langle P \rangle$ correlators

Consistency in the extraction of g_A

- g_A from forward ME versus $g_A = G_A(Q^2 \rightarrow 0)$
- With / without including $N\pi$ state in the analysis
- PCAC

Spectrum from Γ^2

g_A (Forward ME)



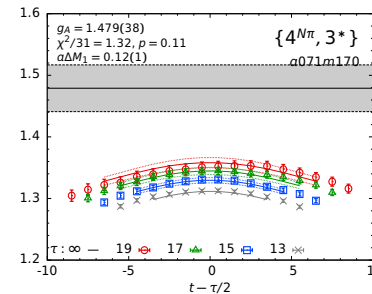
1.22(5)

$g_A = G_A(Q^2 \rightarrow 0)$

1.19(5)

G_A, \tilde{G}_P, G_P do **not**
satisfy PCAC

$N\pi$ included in fits
(via A_4 or priors)



1.28(5)

1.32(6)

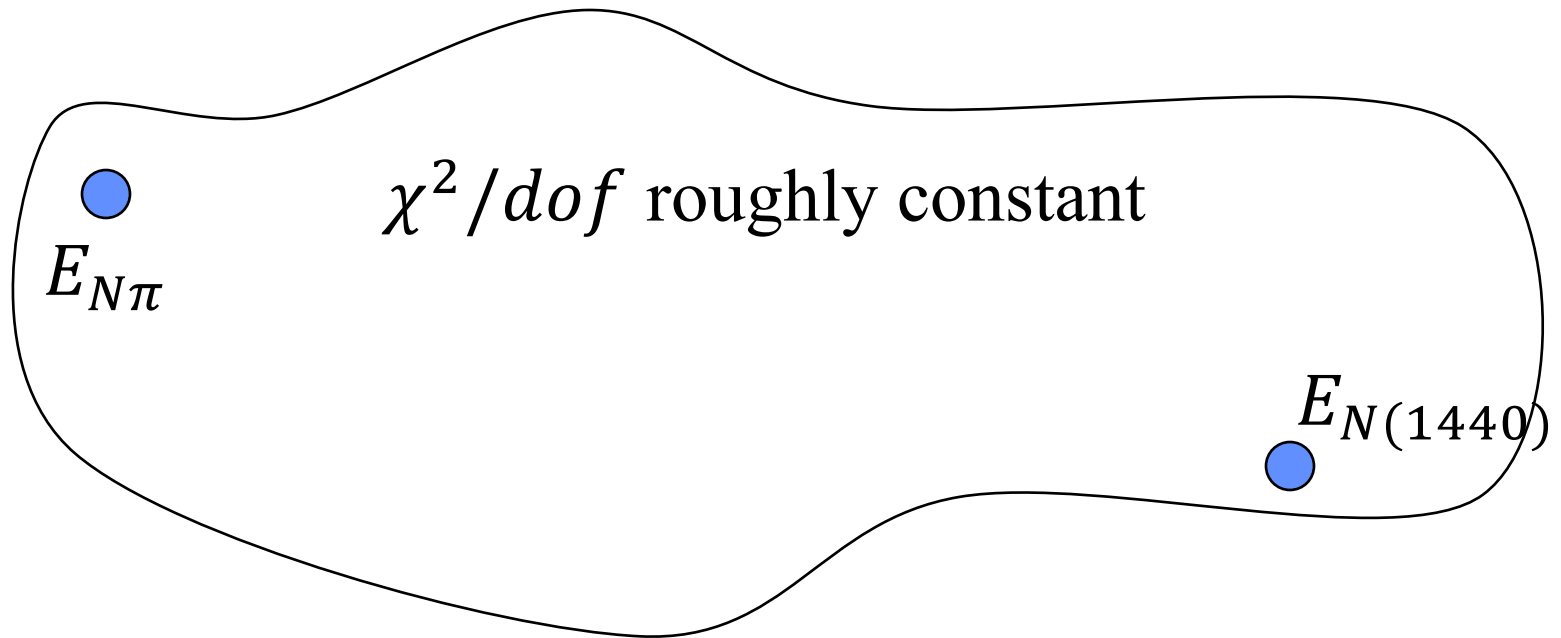
G_A, \tilde{G}_P, G_P with $N\pi$
satisfy PCAC

Essential steps in the analysis

- Remove ESC from correlation functions Γ^n to obtain ME within ground-state nucleon
- Decompose ME into form factors $G(Q^2)$ on each ensemble $\{a, M_\pi, M_\pi L\}$
- Parameterize this $G(Q^2)|_{a, M_\pi, M_\pi L}$
- Perform CCFV extrapolation to get $G(Q^2)|_{cont}$
- Parameterize this $G(Q^2)|_{cont}$

Model averaging should include model choices at each step that have significant effect on result

If ESC is the largest systematic and fits do not select between $\{A_i, E_i\}$



- 2-state fit: Model average different E_1
- 3-state fit: Model average over $\{E_1, E_2\}$

Calculations reviewed in 2305.11330

Collab.	Ens	Lowest M_π (MeV)	Excited State	Q^2	Continuum-chiral-finite-volume extrap	g_A
PNDME 23	13	2 physical	With $N\pi$	$z^2 + z^2$	CCFV	1.292(53)(24)
Mainz 22	14	2 physical	Simultaneous ESC and Q^2	z^2	CCFV	1.225(39)(25)
NME 21*	7 (13)	2 @ 170 → 1 Phy	With $N\pi$	z^2	Ignore { $a, M_\pi^2, M_\pi^2 L$ } dependence	1.32(6)(5)
ETMC 20*	1 (3)	1 → 3 physical	Without $N\pi$	Only data for G_A	{ a }	1.283(22)
RQCD* 19/23	36 (47)	2 Phy 2 Phy	With $N\pi$ only for \tilde{G}_P, G_P	Only data for G_A		1.229—1.302 [1.284 ₂₇ ²⁸]

PNDME: arXiv:2305.11330,

NME: PRD 105, 054505 (2022),

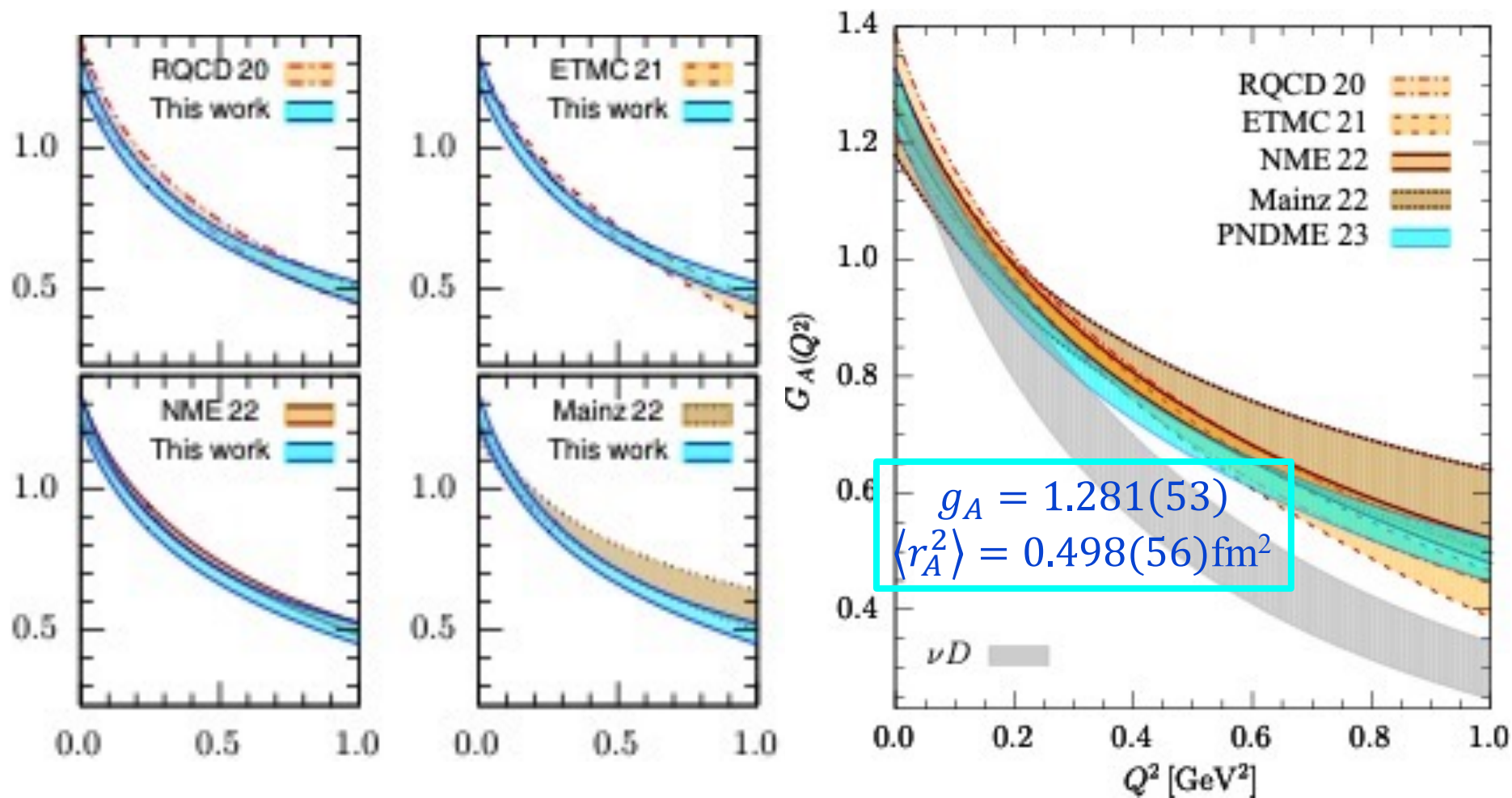
RQCD: JHEP 05, 126 (2020), PRD 107, 051505 (2023)

Mainz: PRD 106, 074503 (2022)

ETMC: PRD 103, 034509 (2021)

*New data in the pipeline

Comparing axial form factor from LQCD



A consensus is emerging

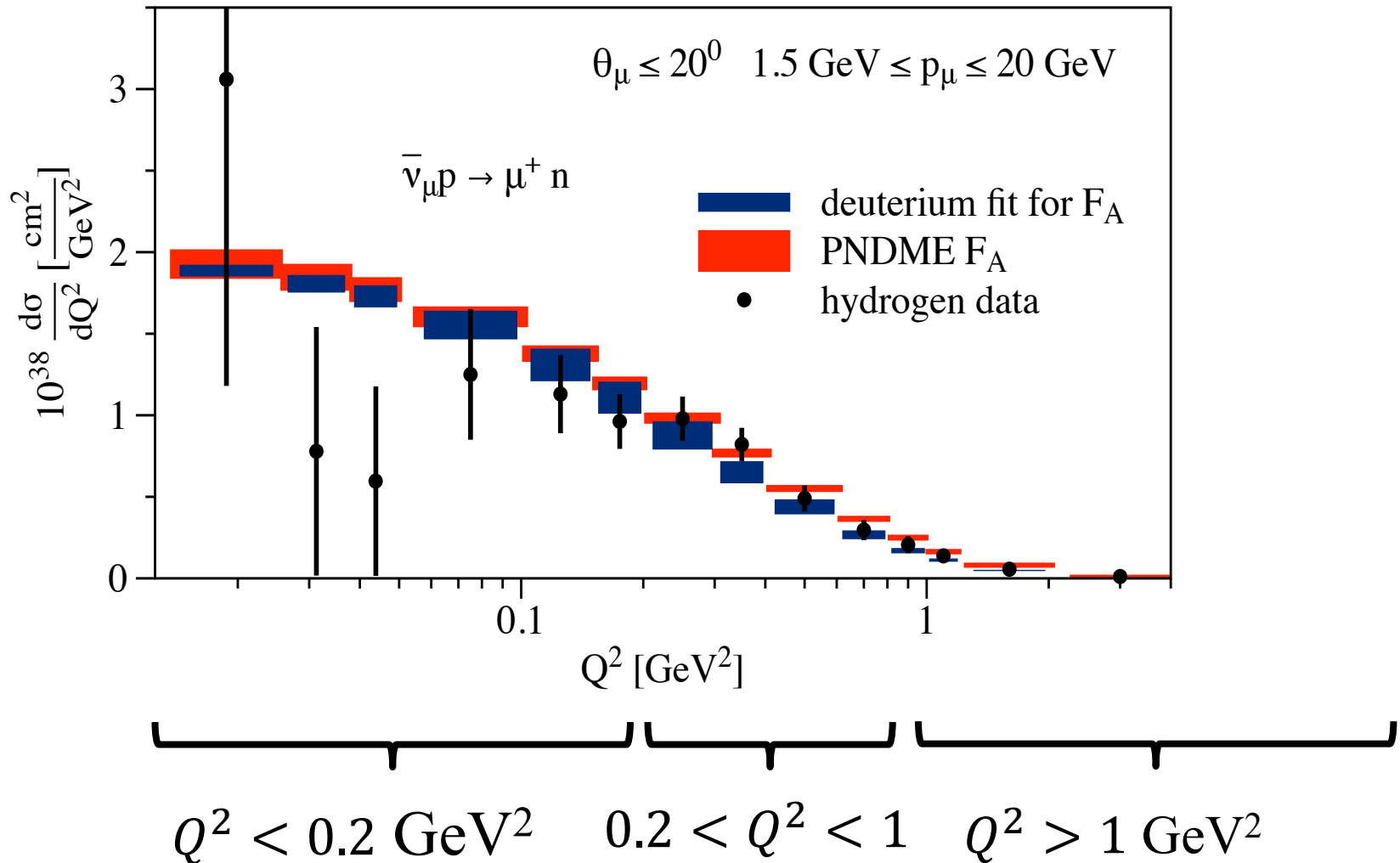
Expected improvements in lattice calculations

$$M_\pi \rightarrow 135; \quad a \rightarrow 0; \quad L \rightarrow \infty$$

- $Q^2 = p^2 - (E(p) - M)^2$
- $p = \frac{2\pi}{La} n = \frac{2\pi}{La} (i, j, k)$
- Fixed $\beta = 6/g^2$ (fixed a)
 - $M_\pi \rightarrow 135$ MeV keeping $M_\pi L$ fixed $\Rightarrow Q^2$ decreases
- Fixed M_π , take $a \rightarrow 0$ keeping L in fermi fixed
 - La fixed $\Rightarrow Q^2$ stays constant
- Fixed M_π and a : take $L \rightarrow \infty$
 - p decreases $\Rightarrow Q^2$ decreases

Q_{max}^2 in lattice data will decrease
but DUNE requires larger Q_{max}^2

Comparing prediction of x-section using AFF from $\nu - D$ and PNDME with MINERvA data



T. Cai, et al., (MINERvA) Nature volume 614, pages 48–53 (2023); Phys. Rev. Lett. 130, 161801 (2023)

Oleksandr Tomalak, Rajan Gupta, Tanmoy Bhattacharya, arXiv:2307.14920

Mapping the AFF

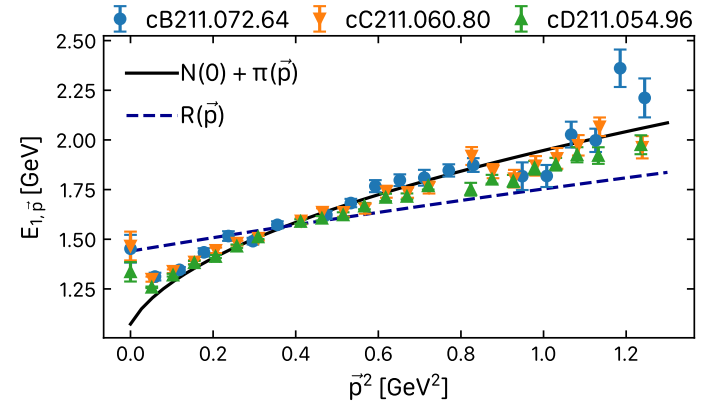
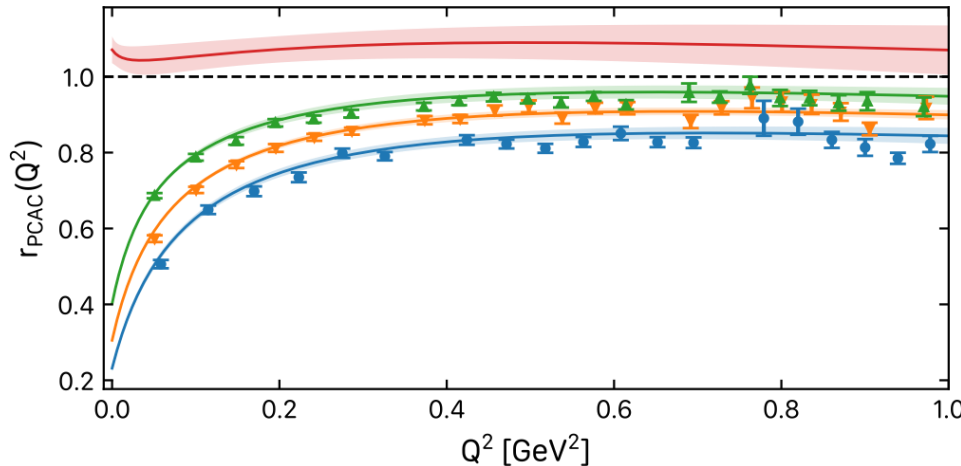
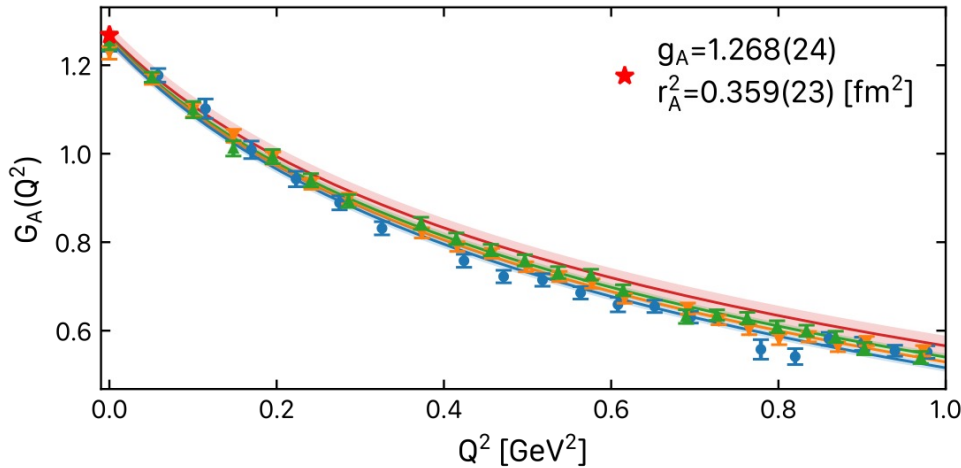
- $0 < Q^2 < 0.2 \text{ GeV}^2$
 - This region will get populated by simulations with $M_\pi \approx 135 \text{ MeV}$, $a \rightarrow 0$, $M_\pi L > 4$
 - MINER ν A data has large errors
 - Characterized by g_A and $\langle r_A^2 \rangle$ and $G_A(Q^2)$ parameterized by a z-expansion with a few terms
- $0.2 < Q^2 < 1 \text{ GeV}^2$
 - Lattice data mostly from $M_\pi > 200 \text{ MeV}$ simulations
 - Competitive with MINER ν A data. Cross check of each other
- $Q^2 > 1 \text{ GeV}^2$
 - Lattice needs new ideas
 - MINER ν A and future experiments

Unpublished data and looking ahead

Update from ETMC ($3 M_\pi \approx 135$ MeV ensembles)

2+1+1-flavor twisted mass ensembles

Ens. ID	latt. Vol.	a [fm]	Lm_π
cB211.072.64 (cB64)	$64^3 \times 128$	0.080	3.62
cC211.060.80 (cC80)	$80^3 \times 160$	0.069	3.78
cD211.054.96 (cD96)	$96^3 \times 192$	0.057	3.90



Excited state fits

- 2-state checked against 3-state
- $N\pi$ state not included
- 1st excited state mass $\approx xx$ MeV

PCAC test of form factors

$$r_{\text{PCAC}} = \frac{\frac{m_q}{m_N} G_5(Q^2) + \frac{Q^2}{4m_N^2} G_P(Q^2)}{G_A(Q^2)}$$

Large cut-off effects in twisted mass involving pions

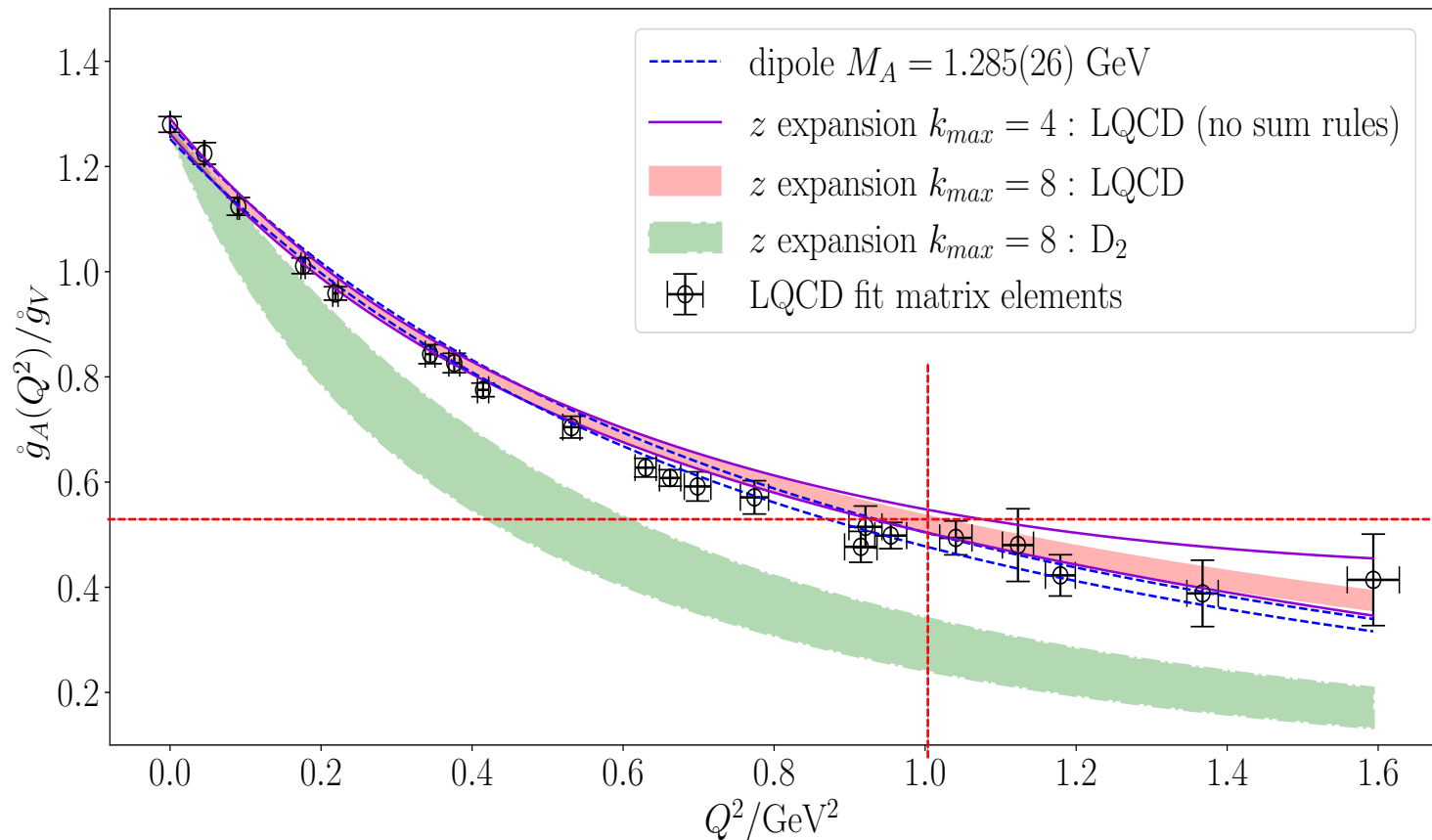
Update from CaLAT Collaboration

(A. Meyer, A. Walker-Loud)

domain-wall on HISQ calculation using sequential prop through sink
48³ × 64 ensemble (a12m130): $a^{-1} = 1.66$ GeV; $M_\pi = 132$ MeV

Gaussian sources for quark propagators

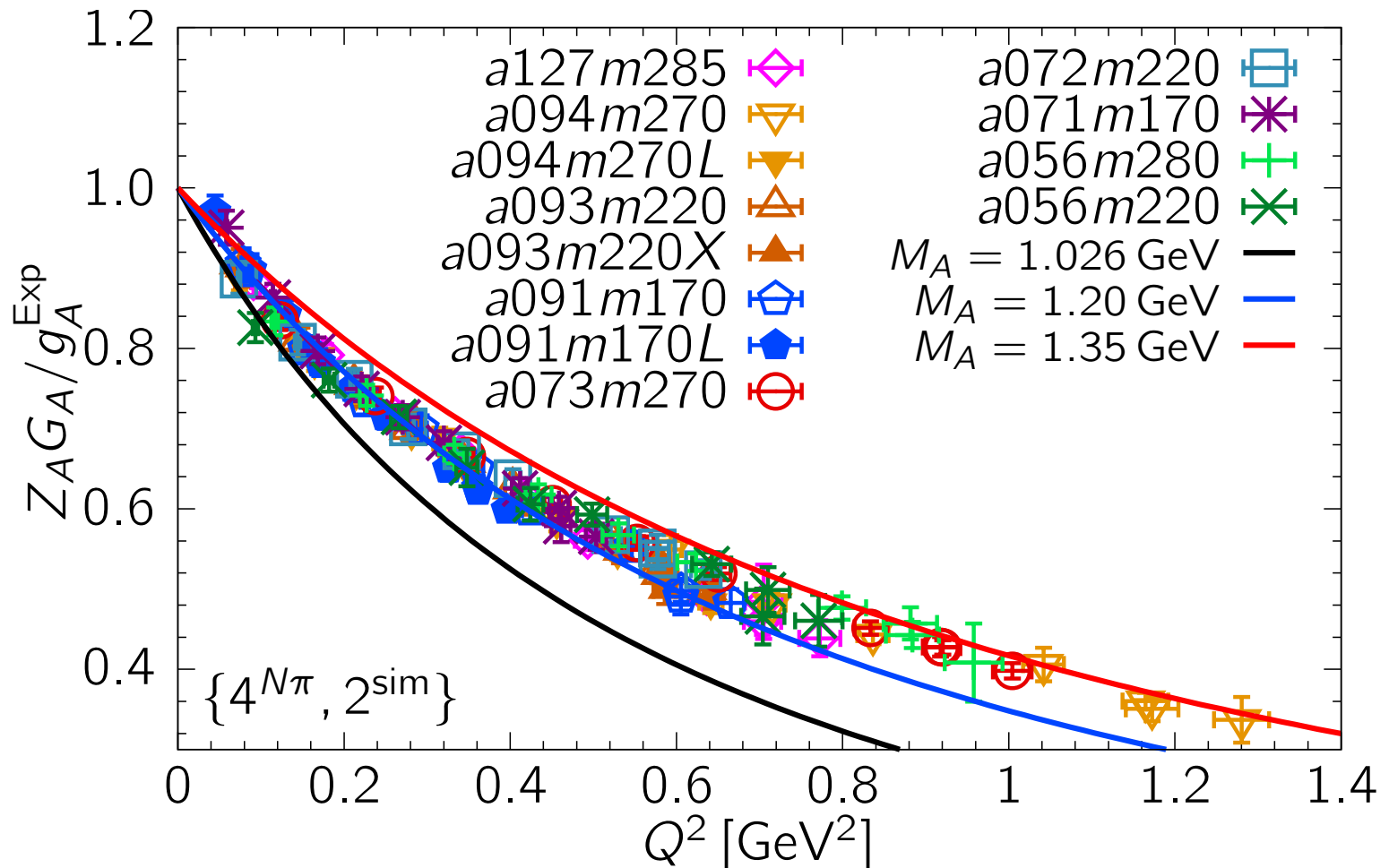
1000 X 32 (configurations X measurements)



Update from NME Collaboration

(Sungwoo Park, R.G., ...)

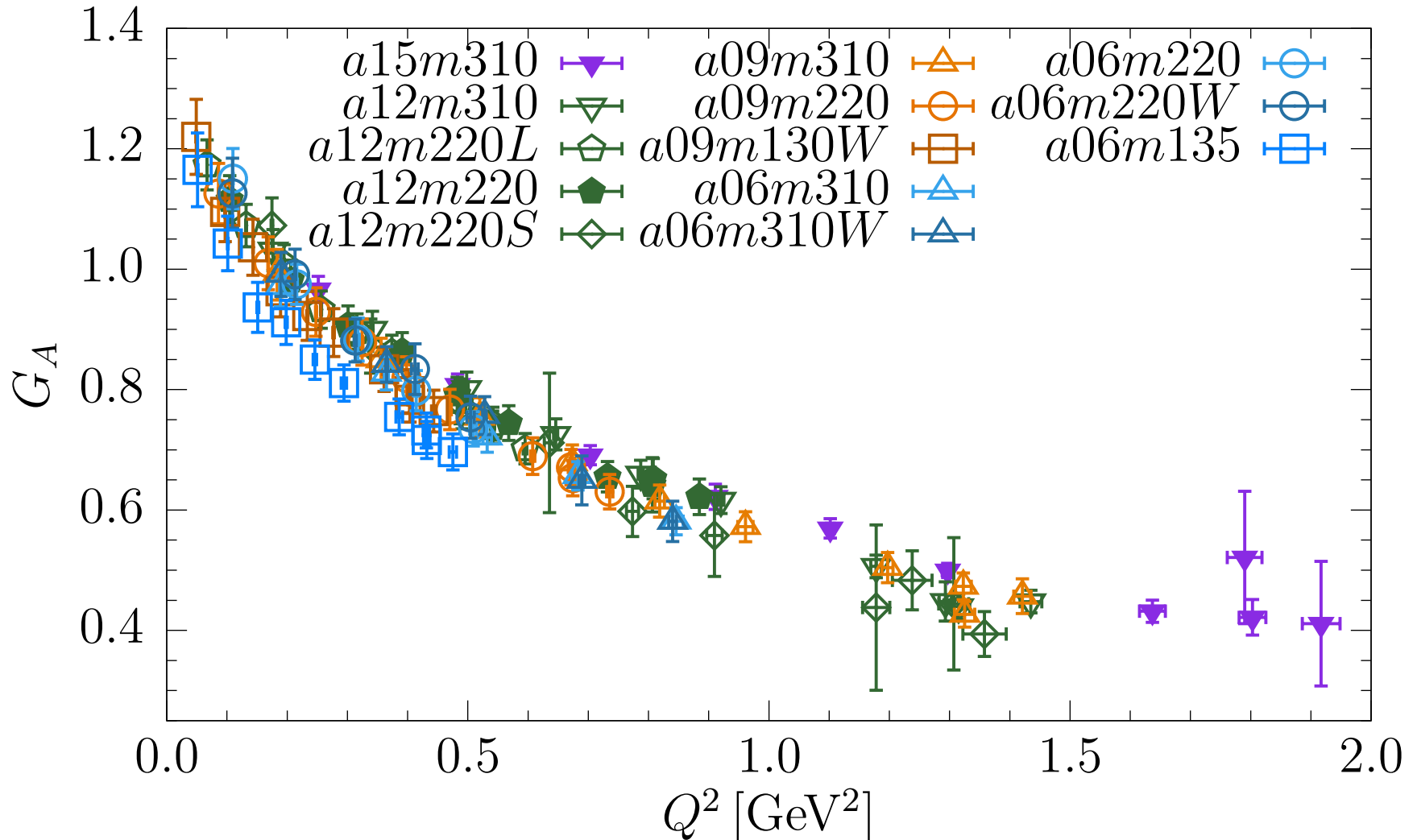
2+1-flavor clover fermions using sequential prop through sink
12 Ensembles; Gaussian sources for quark propagators



Update from PNDME Collaboration

(Y-C Jang, R.G., ...arXiv:2305:11330)

Clover-on-HISQ calculation. Thirteen 2+1+1-flavor HISQ ensembles. Sequential propagators through nucleon sink; Wuppertal sources for quark propagators

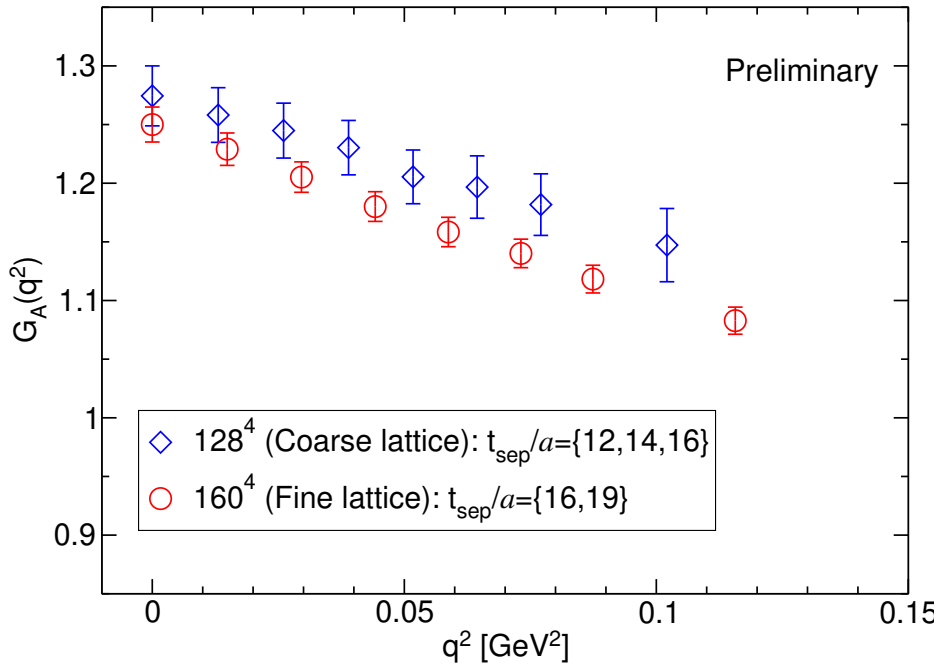


Updates from PACS

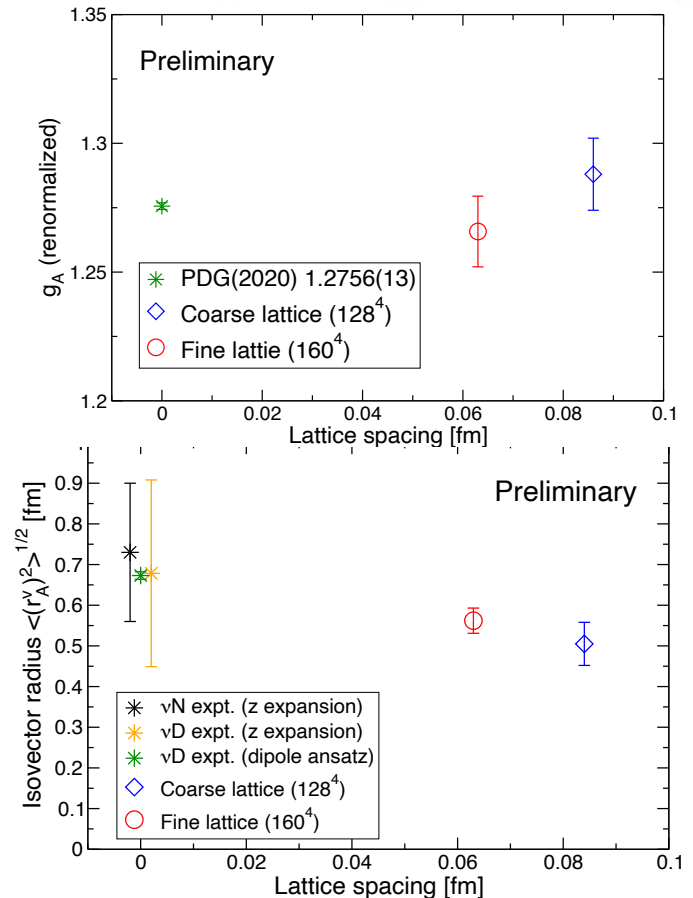
Talk by Ryutaro Tsuji

Stout smeared $O(a)$ improved
Wilson quark and Iwasaki
gauge actions. 2+1 flavors

Lattice size	128^4	160^4
Spatial volume	$(10.9 \text{ fm})^3$	$(10.1 \text{ fm})^3$
Pion mass	135 MeV	135 MeV
Nucleon mass	0.935(11) GeV	0.946(3) GeV
Lattice spacing	0.086 fm	0.063 fm
$ t_{\text{sink}} - t_{\text{src}} /a$	10, 12, 14, 16	13, 16, 19
Renormalization	SF, RI-MOM/SMOM	SF



Tuned exponential sources show very little excited-state effects in axial



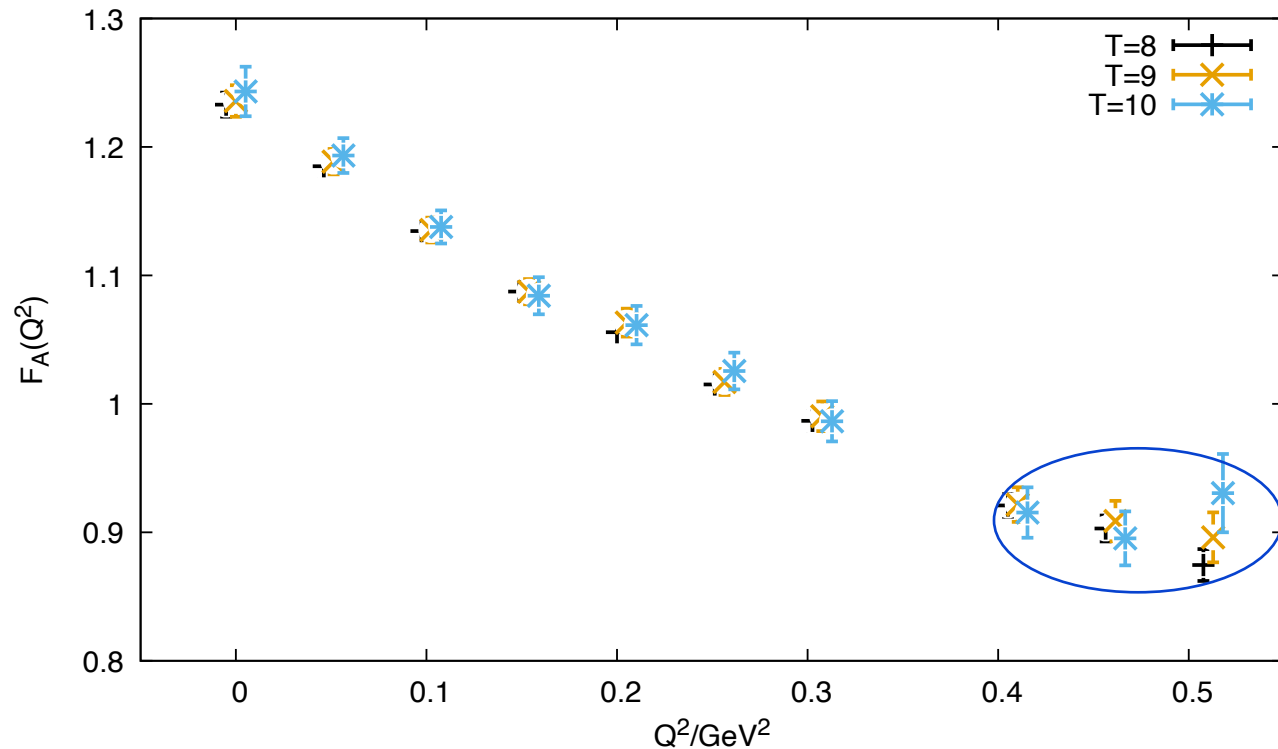
Update from LHP/RBC/UKQCD Collaboration

(S. Ohta arXiv:2211.16018)

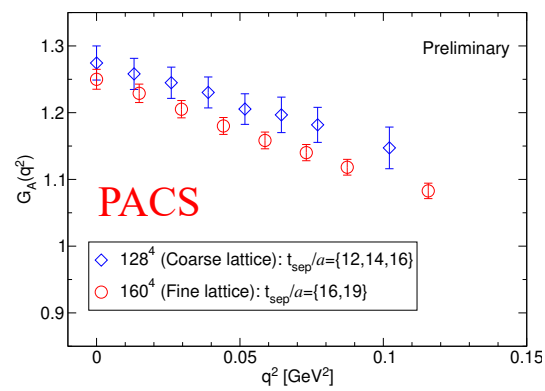
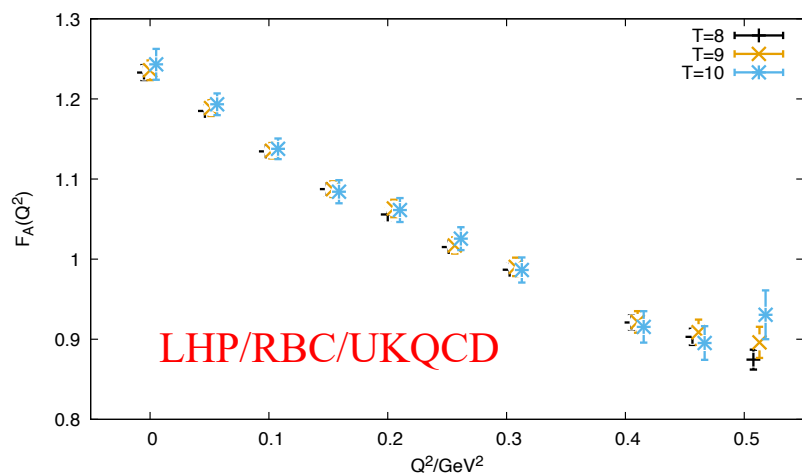
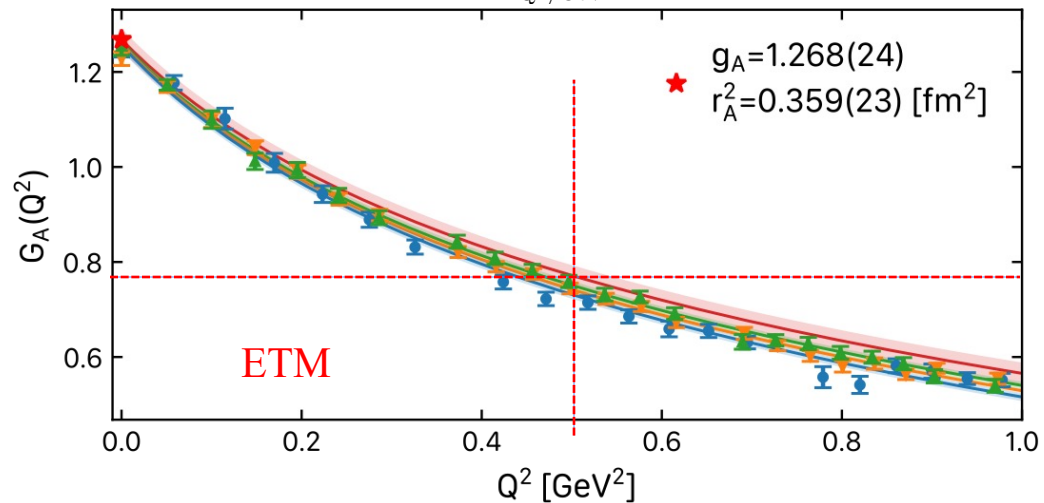
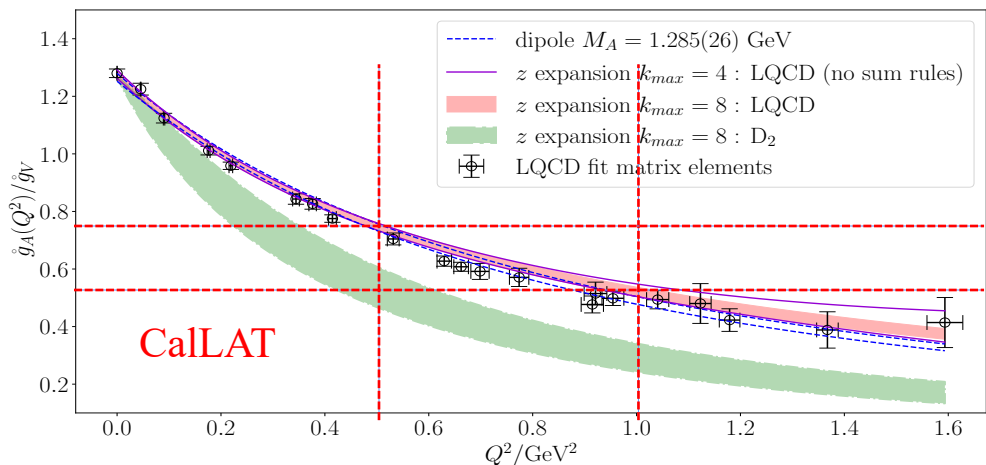
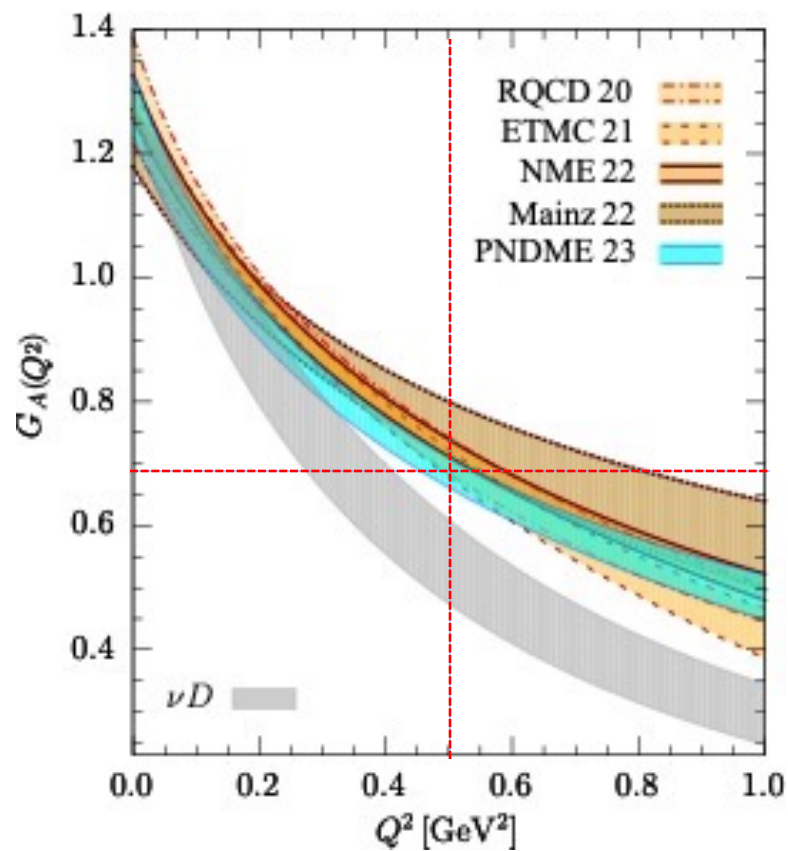
2+1-flavor domain-wall-fermions
48³×96 ensemble: $a^{-1} = 1.730(4)$ GeV
Gaussian sources for quark propagators
120 configurations, ## measurements

Data at $\tau = 8, 9, 10$ do not show significant change indicating small excited state effect

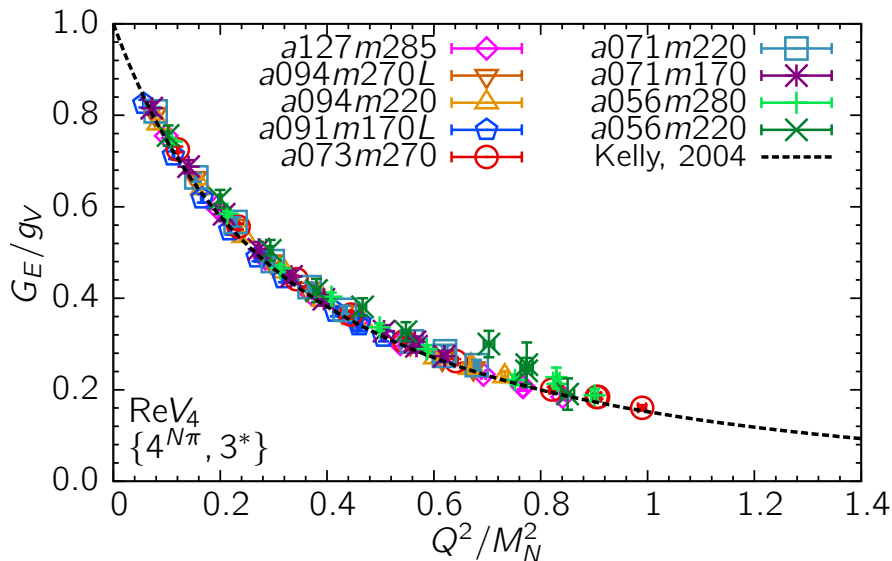
Slower fall-off than PNDME 23 data



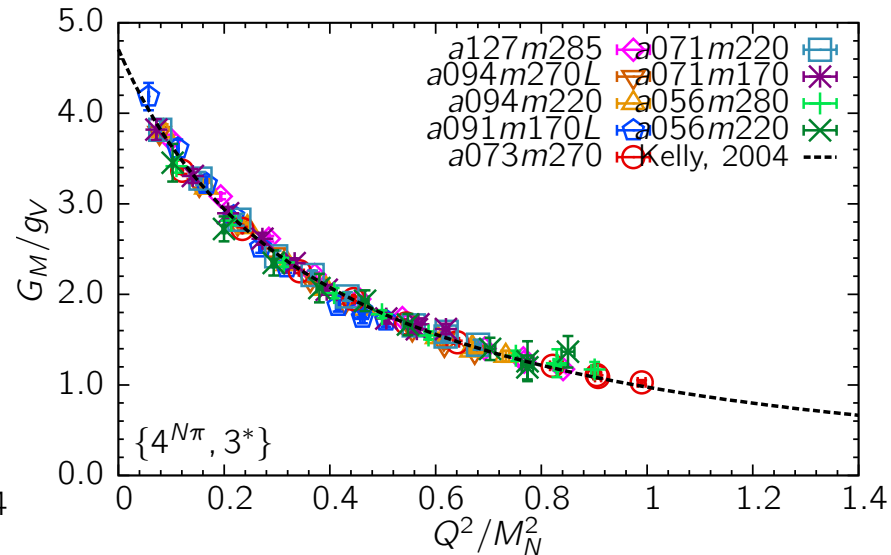
Comparison with unpublished data



Electric & Magnetic FF



Electric



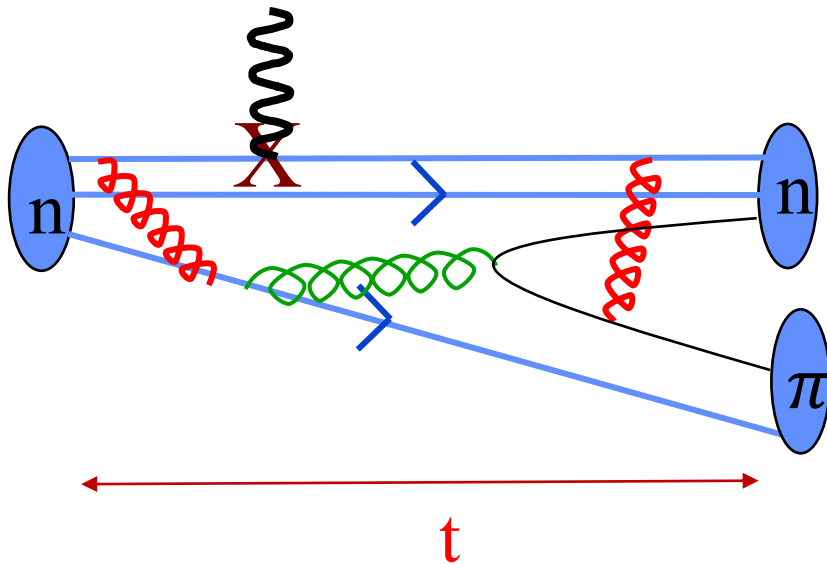
Magnetic

- The extraction of electric and magnetic form factors is insensitive to the details of the excited states
- Vector meson dominance $\rightarrow N\pi\pi$ state should contribute (some evidence)
- The form factors do not show significant dependence on the lattice spacing or the quark mass
- Good agreement with the Kelly curve. Validates the lattice methodology
- Improve precision and get data over larger range of parameter values

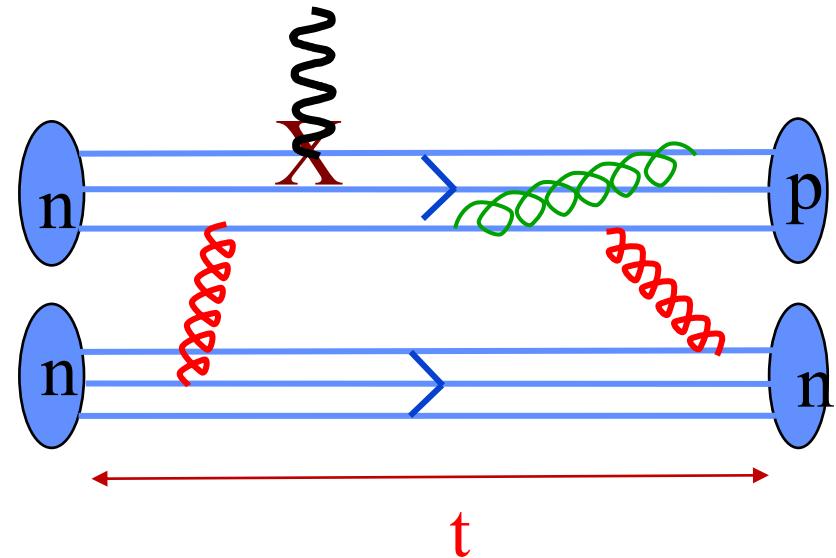
Variational with Multi-hadron states

NPB205 [FS5] (1982) 188

$$\hat{N} \rightarrow \hat{N} + c \hat{N}\pi$$



$$\langle n \pi^+ | J_\mu^+(q) | n \rangle$$



$$\langle n p | J_\mu^+(q) | n n \rangle$$

See

- Barca et al, [2211.12278](#), [2110.11908](#)
- NPLQCD Collaboration, *Phys.Rev.Lett.* 120 (2018) 15, 152002
- Nuclear matrix elements from lattice QCD for electroweak and beyond-Standard-Model processes, 2008.11160 [hep-lat]

Summary

- Challenges in lattice calculations of nucleon matrix elements:
 - Signal to noise degrades as $e^{-(M_N - 1.5M_\pi)t}$
 - removing multi-hadrons excited states to get ground state ME
 - including multi-hadrons in initial and/or final state for transition ME
- Continue to develop a robust analysis strategy for removing dominant excited states in various nucleon matrix elements
- Improve chiral and continuum extrapolation. Simulate at more $\{a, M_\pi\}$
- Current $0.04 < Q^2 < 1 \text{ GeV}^2$. Extend to larger Q^2 for DUNE
- Transition matrix elements
- Goal: Perform a comprehensive analysis of scattering data with input of lattice results for $g_A, G_E(Q^2), G_M(Q^2), G_A(Q^2), \tilde{G}_P(Q^2)$

Improvements in algorithms and computing power
are needed to reach few percent precision