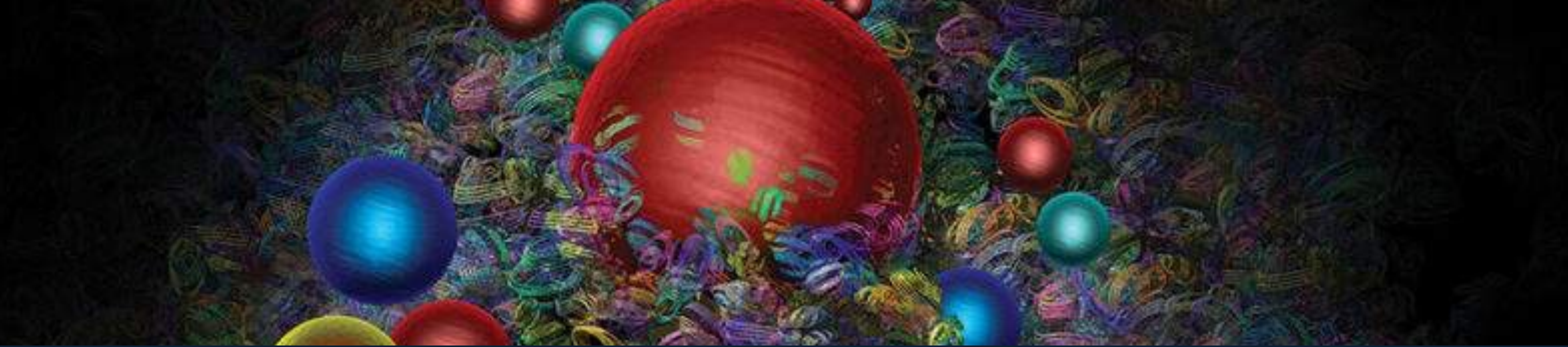

Global extraction of Generalized Parton Distributions (GPDs) with moment space parameterization.

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QCD at the Femtoscale in the Era of Big Data

Institute for Nuclear Theory, Seattle, WA

June 13rd, 2024

BERKELEY LAB

QUARK-GLUON TOMOGRAPHY COLLABORATION

INSTITUTE for NUCLEAR THEORY

Outline

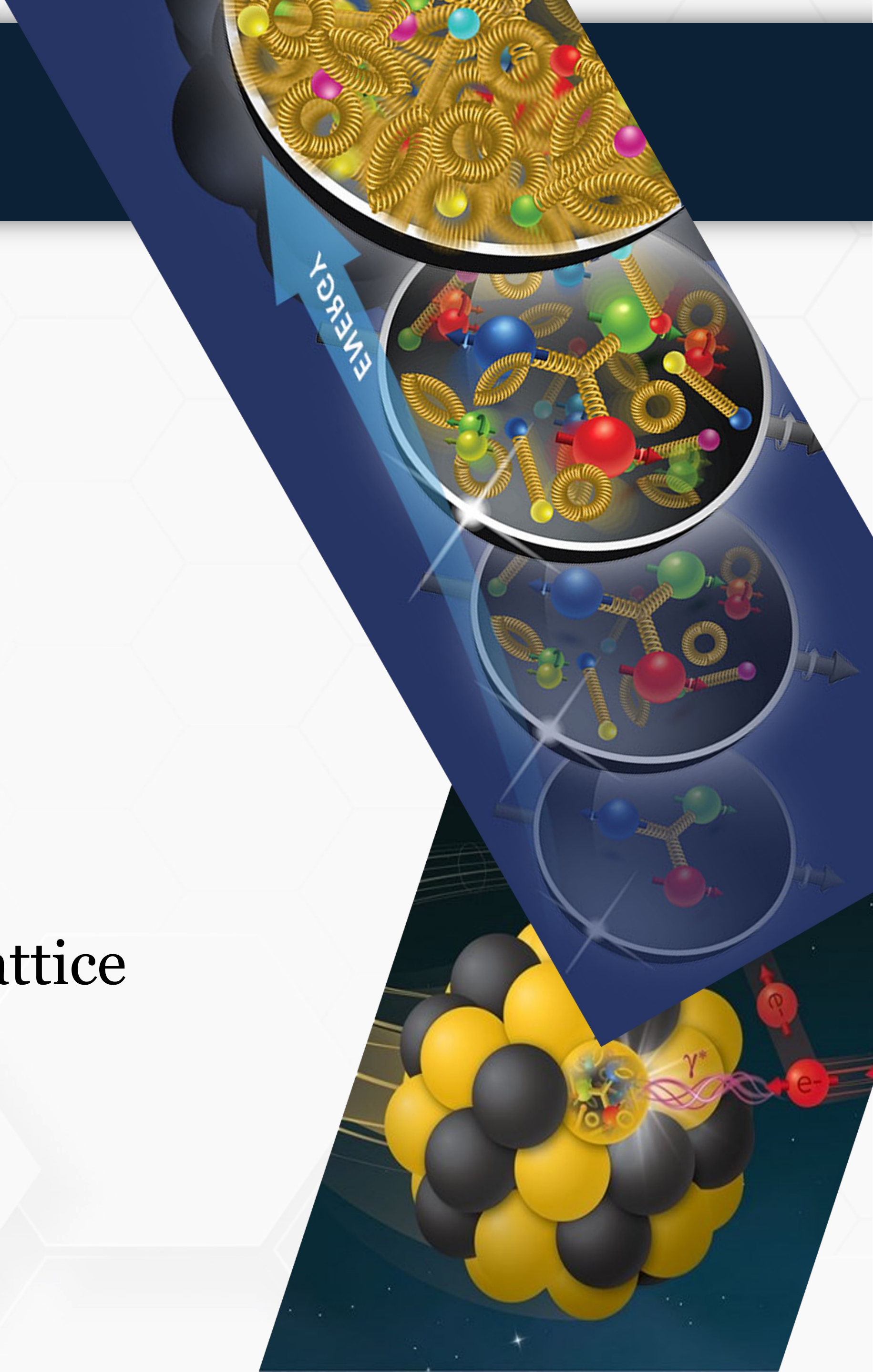
» Intro – nucleon structure and GPDs

» Inputs for GPD extraction

» Moment space parameterization of GPDs

» Global extraction of GPDs from experiments and lattice

» Summary



Nucleon spin and 3D structure

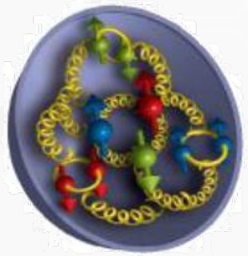
Naïve quark model — quark spin

Gluonic contribution — gluon spin

Transverse motions — quark orbital AM, gluon orbital AM

$$\frac{\hbar}{2} =$$

+



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+

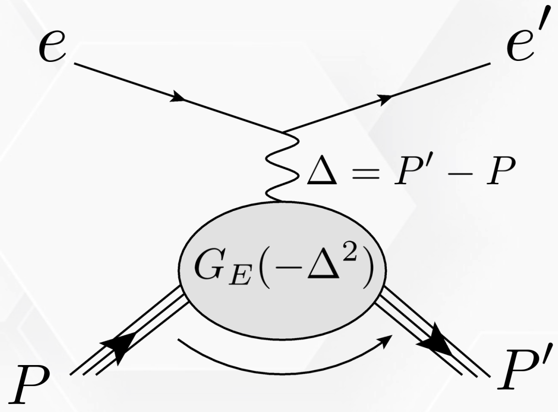
R. Jaffe and A. Manohar Nucl. Phys. B 337, 509 (1990)

Jaffe-Manohar sum rule

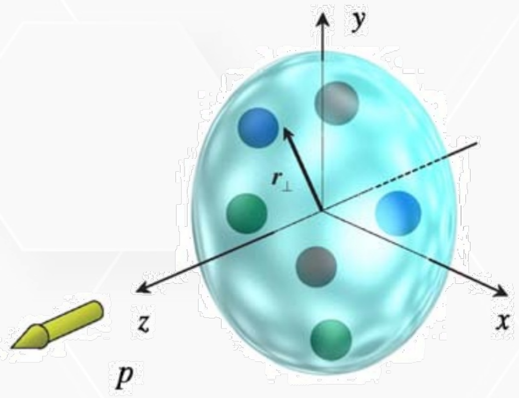
Nucleon spin can be explored with parton distributions localized in coordinate space.

Proton radius and form factors

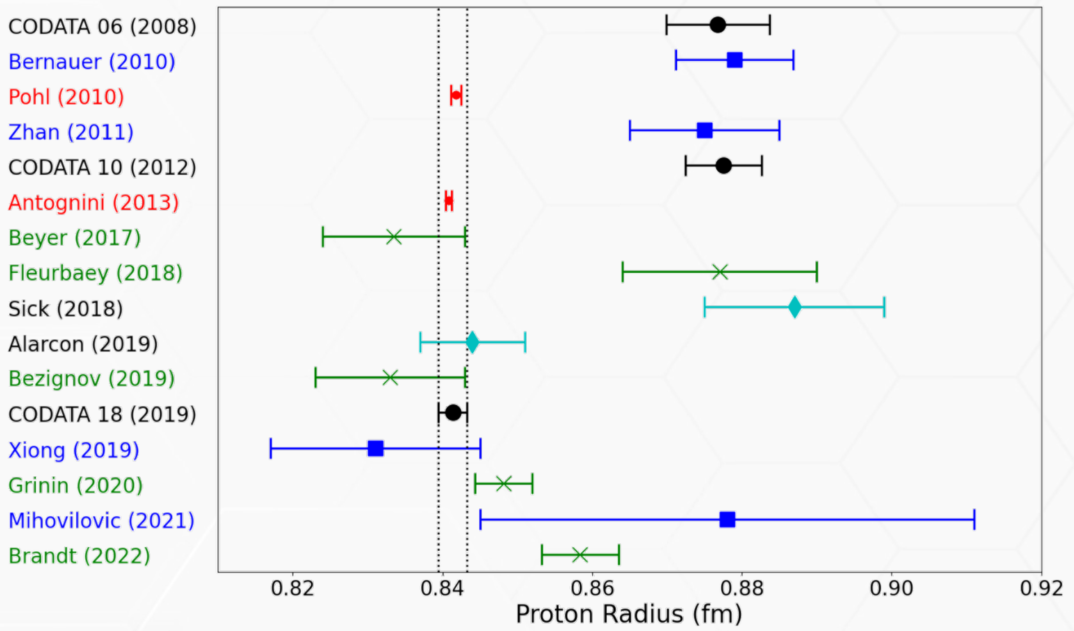
How do we know that the proton radius is around one fermi?



elastic scattering



charge distribution



Plot by E. J. Downie at SPIN 2023 at Duke

$$\rho_{\mathrm{NR}}(\boldsymbol{r}) = \int \frac{\mathrm{d}^3\boldsymbol{\Delta}}{(2\pi)^3} e^{-i\boldsymbol{\Delta}\cdot\boldsymbol{r}} G_E(-\boldsymbol{\Delta}^2)$$

Energy-momentum tensor form factors

The energy-momentum tensor (EMT) is the tool to study the mechanical properties of the nucleon. Its nucleon matrix element can be written as:

$$\langle P'|\, T^{\mu\nu}_{q,g}\, |P\rangle = \bar{u}(P') \Big[A_{q,g}(t)\gamma^{(\mu}\bar{P}^{\nu)} + B_{q,g}(t)\frac{\bar{P}^{(\mu} i\sigma^{\nu)\alpha}\Delta_\alpha}{2M_N} + C_{q,g}(t)\frac{\Delta^\mu\Delta^\nu - g^{\mu\nu}\Delta^2}{M_N} + \bar{C}_{q,g}(t) M_N g^{\mu\nu} \Big] u(P)$$

X. Ji Phys. Rev. Lett. 78, 610 (1997)

Momentum form factors: $A_{q,g}(t)$

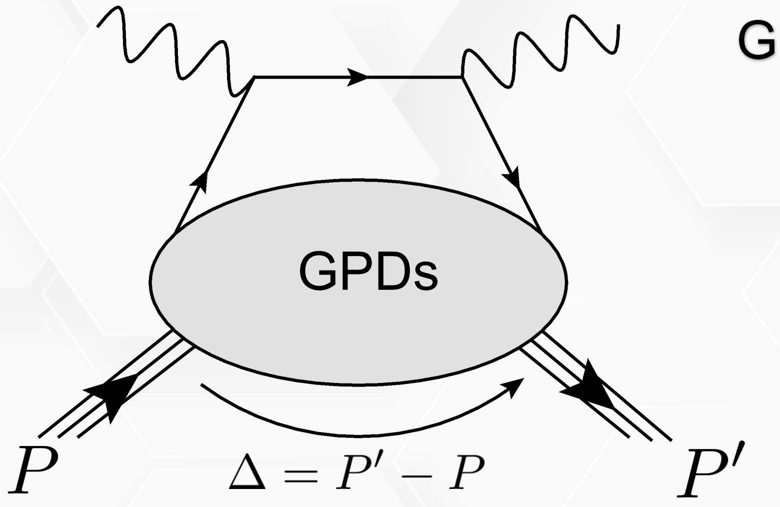
Angular momentum form factors: $J_{q,g}(t) = \frac{1}{2}\left(A_{q,g}(t) + B_{q,g}(t)\right)$

Stress tensor form factors: $C_{q,g}(t)$

Generalized parton distributions (GPDs)

To measure them, one needs to couple nucleon to spin-2 probe (graviton?).

Alternative approach: couple the parton to spin-2 probe (2 photons/2 gluons)



D. Muller et. al. Fortsch.Phys. 42 101 (1994)

X. Ji Phys. Rev. Lett. 78, 610 (1997)

GPDs are distributions unifying parton distributions and form factors

$$F(x, \Delta^\mu) = F(x, \xi, t)$$

x : average parton momentum fraction

ξ : skewness – longitudinal momentum transfer $\xi \equiv -n \cdot \Delta/2$

t : total momentum transfer squared $t \equiv \Delta^2$

GPDs are partonic form factors? — at zero skewness

3D mass & spin structures with GPDs

GPDs reduce to form factors when integrated over *x* X. Ji, J. Phys. G 24 1181-1205 (1998)

Charge FFs

$$\int \mathrm{d}x H(x,\xi,t) = F_1(t)$$

$$\int \mathrm{d}x E(x,\xi,t) = F_2(t)$$

Gravitational FFs

$$\int \mathrm{d}x\; xH(x,\xi,t) = A(t) + (2\xi)^2 C(t)$$

$$\int \mathrm{d}x\; xE(x,\xi,t) = B(t) - (2\xi)^2 C(t)$$

GPDs also provide an intuitive 3D image of nucleon:

M. Burkardt, Int. J. Mod. Phys. A 18 173-208 (2003)

$$\rho_q^{\mathrm{Unp}}(x,\boldsymbol{b}) = \int \frac{\mathrm{d}^2\boldsymbol{\Delta}}{(2\pi)^2} e^{-i\boldsymbol{\Delta}\cdot\boldsymbol{b}} H_q(x,-\boldsymbol{\Delta}^2) = \mathscr{H}_q(x,\boldsymbol{b})$$

which contains information of nucleon spin structure, e. g. transverse spin

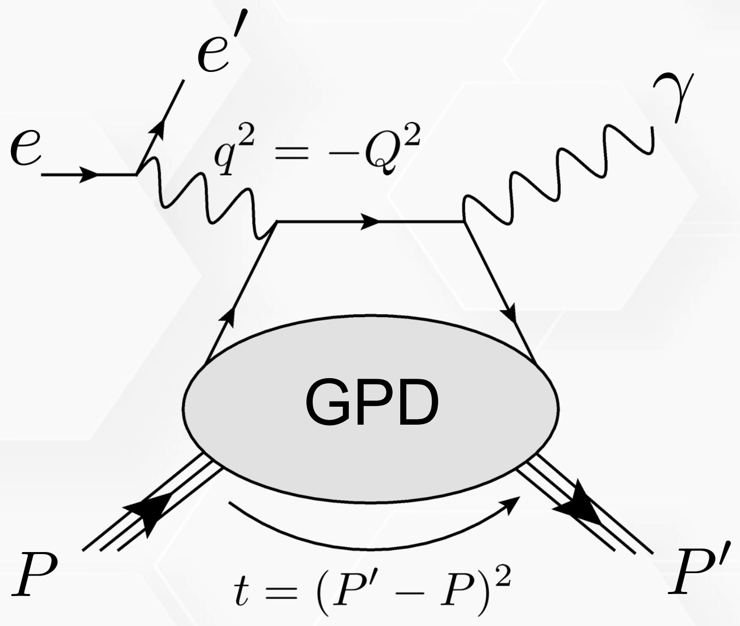
$$J_q^T(x) = \int \mathrm{d}^2\boldsymbol{b}(b^y \times xP^+)\rho_q^T(x,\boldsymbol{b})$$

Y. Guo et. al. Nucl. Phys. B 969 115440 (2021)

Inputs for GPD studies

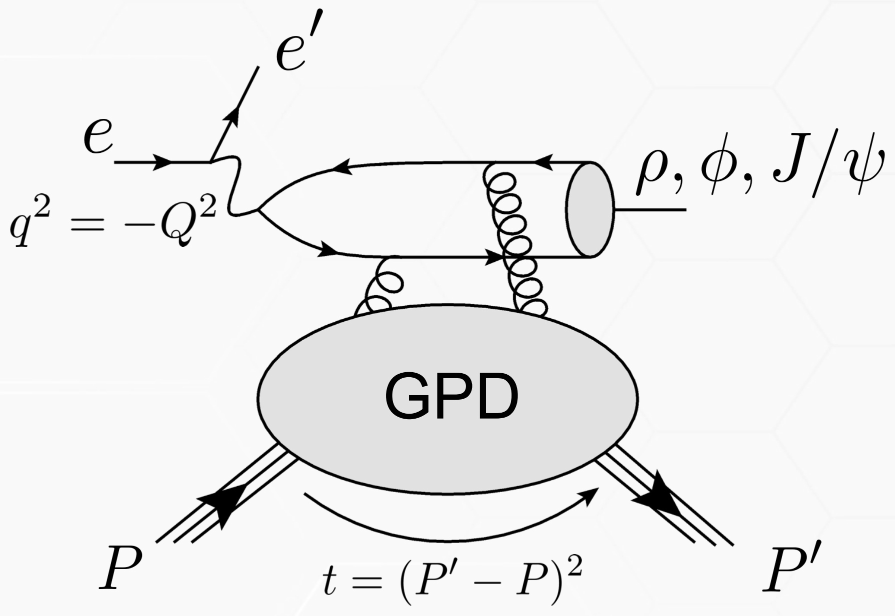
Deeply virtual processes

Diffractive processes can provide us access to the 3D structures.



Deeply virtual Compton scattering

X. Ji, Phys. Rev. D 55, 7114 (1997)

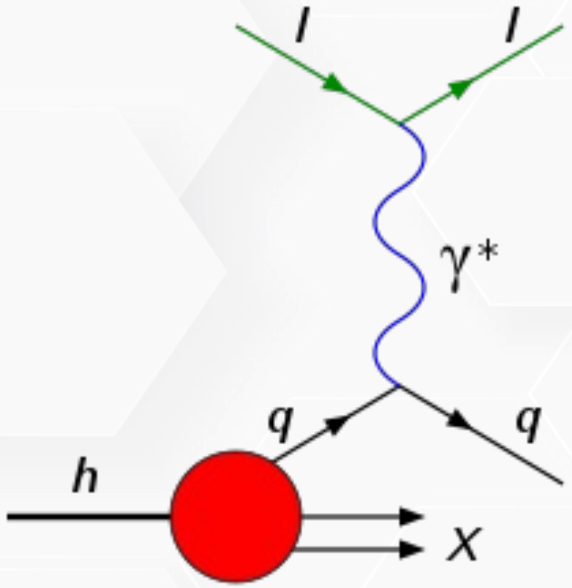


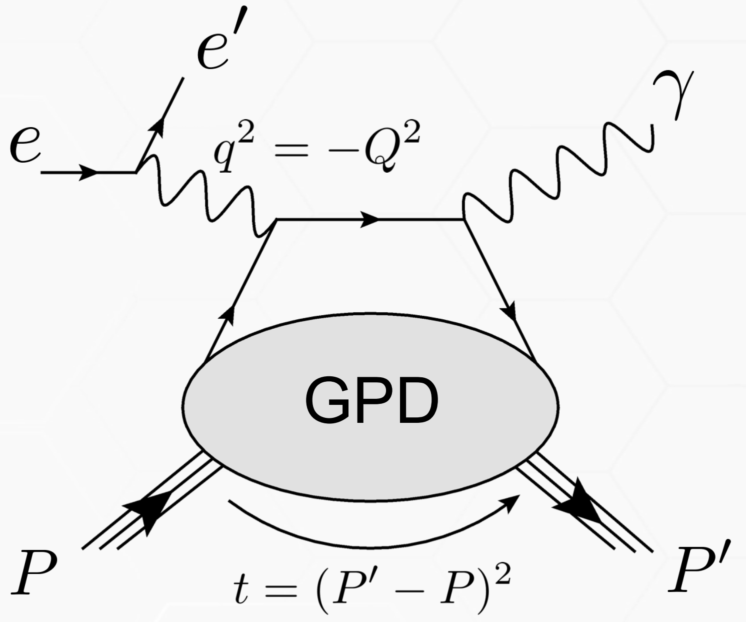
Deeply virtual meson production

A.V. Radyushkin Phys. Lett. B 385 333-342 (1996)
J. C. Collins et. al. Phys. Rev. D 56 2982-3006 (1997)

A huge difference in exclusive productions

The problem in exclusive productions: partons are not directly measure.





DIS: parton almost on-shell,

it will hadronize into final state

DVCS: parton off-shell, it will return

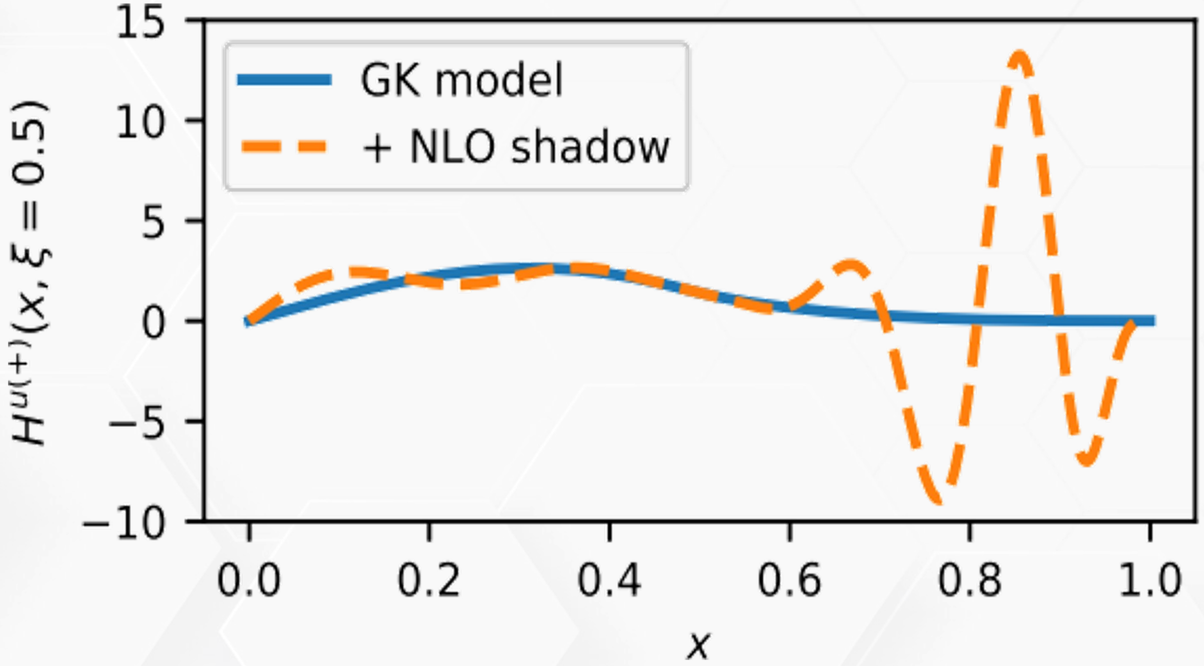
to the hadron it came out from

Inverse problem and shadow GPDs

The problem in exclusive productions: partons are not directly measure.

$$\mathcal{H}_{CFF}(\xi,t) = -\sum_q Q_q^2 \int_{-1}^{1} \mathrm{d}x \left(\frac{1}{x-\xi+i0} + \frac{1}{x+\xi-i0} \right) H_q(x,\xi,t) \ ,$$

One cannot obtain a unique solution of the GPD from the CFF measurements alone.



V. Bertone et. al. SciPost Phys.Proc. 8 (2022) 107

The Gaps

We would want:

- **Benchmark;**
- **Uncertainty quantification;**
- **Higher-order corrections;**
- **…**

However, the reality is:

- **Degeneracy in flavor space;**
- **Degeneracy in species space;**
- **Degeneracy in x-dependence; (Shadow GPD)**
- **Degeneracy in xi-dependence;**
- **Degeneracy in t-dependence; (Similar to PDF from DIS)**
- **…**

Moment space parameterization of GPD

Scheme for GPD global analysis

Parameterization of GPDs

Compute GPD observables

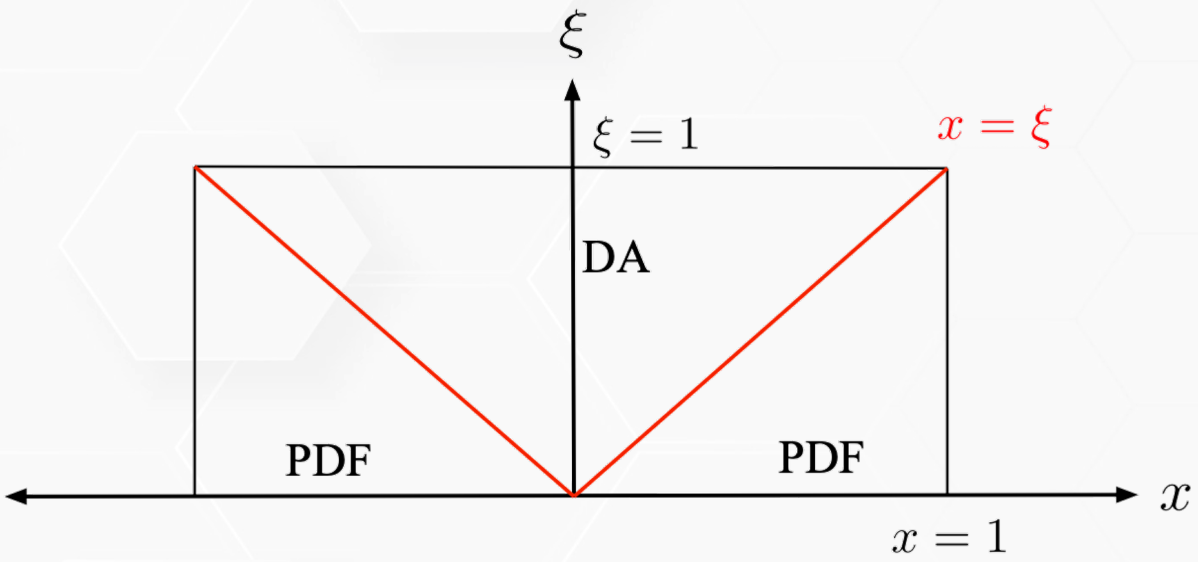
Inputs (Constraints) on GPDs

Compare and iterate

- Various GPD species and flavors
- Both x-space and moments
- QCD evolution
- Constraints in x- and moment space
- Compton form factors (with convolution)
- Computation efficiency required!

Some challenges in modeling GPDs

GPDs are not analytical function on the whole domain of definition



GPDs are continuous but not analytical on the cross-over line.

$$\mathcal{H}_{CFF}(\xi,t) = -\sum_q Q_q^2 \int_{-1}^{1} \mathrm{d}x \left(\frac{1}{x-\xi+i0} + \frac{1}{x+\xi-i0} \right) H_q(x,\xi,t) \ ,$$

Physical observables are most sensitive to the non-analytical region.

Some challenges in modeling GPDs

Like collinear PDFs, the evolution of GPDs is also given by an integro-differential equation

$$\frac{\mathrm{d}}{\mathrm{d}\ln Q^2}F\left(x,\xi,t,Q^2\right)=\frac{\alpha_s(Q)}{2\pi}\int_{-1}^{1}\frac{\mathrm{d}x'}{|\xi|}\left[V\left(\frac{x}{\xi},\frac{x'}{\xi}\right)\right]_+F\left(x',\xi,t,Q^2\right)$$

Hard constraints on the moments of GPDs:

$$\int_{-1}^{+1}dx x^{n-1}H\left(x,\xi,t\right)=\sum_{i=0,\mathrm{even}}^{n-1}\left(-2\xi\right)^iA_{ni}\left(t\right)+\left(-2\xi\right)^nC_{n0}\left(t\right)\Big|_{n\ \mathrm{even}}\quad ,$$

$$\int_{-1}^{+1}dx x^{n-1}E\left(x,\xi,t\right)=\sum_{i=0,\mathrm{even}}^{n-1}\left(-2\xi\right)^iB_{ni}\left(t\right)-\left(-2\xi\right)^nC_{n0}\left(t\right)\Big|_{n\ \mathrm{even}}\quad ,$$

GPDs in terms of Moments

GPDs can be formally expanded in the conformal moment space:

D. Mueller and A. Schafer 2006

$$F(x,\xi,t)=\sum_{j=0}^{\infty}(-1)^j p_j(x,\xi)\mathcal{F}_j(\xi,t)$$

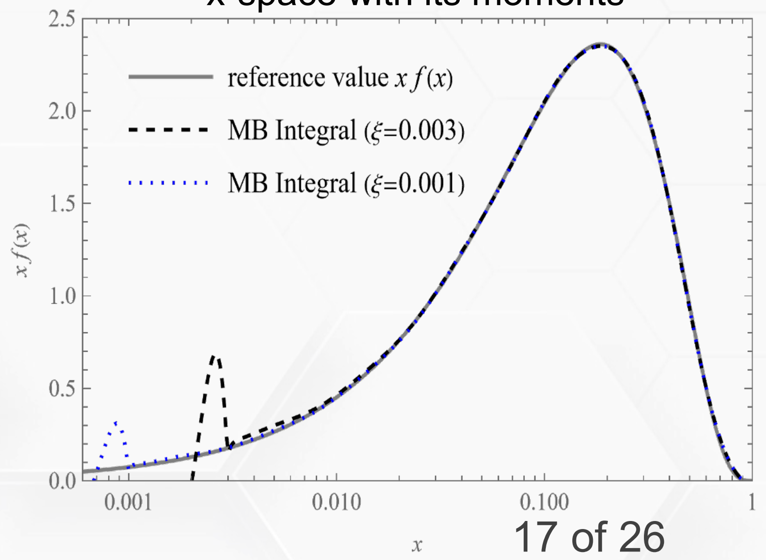
$p_j(x,\xi)$: Orthogonal basis in terms of Gegenbauer polynomials

$\mathcal{F}_j(\xi,t)$: Moments of GPDs to be parameterized

Whereas GPDs in x-space can be reconstructed by resumming all the moments through a complex integral in the moment space.

$$F(x,\xi,t)=\frac{1}{2i}\int_{c-i\infty}^{c+i\infty} dj\,\frac{p_j(x,\xi)}{\sin(\pi[j+1])}\mathcal{F}_j(\xi,t)\ ,$$

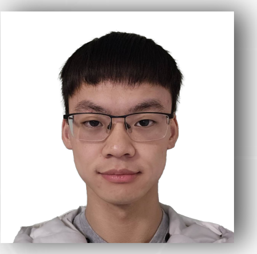
Example of the reconstruction of GPD in x-space with its moments



GPDs through Universal Moment Parameterization (GUMP)

Goal: To obtain the state-of-the-art phenomenological Generalized Parton Distributions (GPDs) through global analysis of both experimental data and lattice QCD simulations, utilizing a *universal moment parameterization* method.

Collaborators:



Yuxun Guo (Postdoc)
Lawrence Berkeley Lab.

Xiangdong Ji (PI)
University of Maryland

M. Gabriel Santiago (Postdoc)
Center for Nuclear Femtography

Kyle Shiells (PI)
University of Manitoba

Pros and cons

Advantages:

- Polynomiality condition get naturally imposed
- Avoid the non-analyticity at the cross-over line.
- LO evolution is simple and fast (order of seconds)
- NLO evolution is harder but still practical (order of minutes)

Disadvantages:

- Calculation of x-space observables takes extra time
- Not suitable for numerical x-space convolution.

  Precalculated x-space convolution / Precalculated x-space GPDs?
- Could be constraints easier to impose in x space – positivity bound of GPDs

Evolution in moment space

The amplitude can be schematically written as

$$\mathcal{A} \sim \sum_{i=q,\bar{q},g} \sum_{j=0}^{\infty} \xi^{-j-1} C^i_j F^i_j(\xi, t, Q) \ ,$$

The major cost is due to the evolution, which reads in moment space as,

$$F^i_j(\xi, t, \mu) = \sum_{i'=q,\bar{q},g} \sum_{k=0}^{j} E^{ii'}_{jk}(\xi, \mu, \mu_0) F^{i'}_k(\xi, t, \mu_0) \ .$$

The amplitude can then be written as,

$$\mathcal{A} \sim \sum_{i,i'=q,\bar{q},g} \sum_{j=0}^{\infty} \sum_{k=0}^{j} \xi^{-j-1} C^i_j E^{ii'}_{jk}(\xi, Q, \mu_0) F^i_k(\xi, t, \mu_0) \ ,$$

Efficient double integration appears to be the bottleneck

Universal Parameterization of GPD Moments

Moments of GPDs are expandable in ξ due to the polynomiality condition. For small $\xi \lesssim 0.3$ which covers most of the current data, we consider the expansion of moments

$$\mathcal{F}_j(\xi,t)=\mathcal{F}_{j,0}(t)+\xi^2\mathcal{F}_{j,2}(t)+\xi^4\mathcal{F}_{j,4}(t)+\cdots$$

The first term describes GPDs at $\xi=0$, and is parameterized with a 5-parameter ansatz (N,α,β,α',b):

$$\mathcal{F}_{j,0}(t)=NB(j+1-\alpha,1+\beta)\frac{j+1-\alpha}{j+1-\alpha(t)}\beta(t) \qquad \beta(t)=e^{-b|t|}$$

Euler Beta Function

Regge trajectory $\alpha(t)=\alpha+\alpha' t$

- Beta function $B(j+1-\alpha,1+\beta)$: corresponds to the PDF ansatz $x^{-\alpha}(1-x)^{\beta}$ in forward limit
- Regge trajectory: modify the small-x behavior at different t in the form of $x^{-\alpha(t)}$
- The residual term $\beta(t)$: motivated by the measured t-dependence in elastic scattering.

The ξ-dependence of GPD can in principle be independently parameterized. Here we instead parameterize them with simple ratios: $\mathcal{F}_{j,2}(t)=R_{\xi^2}\mathcal{F}_{j,0}(t)$ $\mathcal{F}_{j,4}(t)=R_{\xi^4}\mathcal{F}_{j,0}(t)$

to avoid unconstrained parameters due to the lack of input.

Bias in the setup

However, the reality is:

- **Degeneracy in flavor space;**
- **Degeneracy in species space;**
- **Degeneracy in x-dependence; (Shadow GPD)**
- **Degeneracy in xi-dependence;**
- **Degeneracy in t-dependence; (Similar to PDF from DIS)**
- **…**

GPDs species and flavors	Fully parameterized	GPDs linked to	Proportional constants
H_{u_V} and $\widetilde{H}_{u_V}$	✔	-	-
E_{u_V} and $\widetilde{E}_{u_V}$	✔	-	-
H_{d_V} and $\widetilde{H}_{d_V}$	✔	-	-
E_{d_V} and $\widetilde{E}_{d_V}$	✘	E_{u_V} and $\widetilde{E}_{u_V}$	$R^{E/\widetilde{E}}_{d_V}$
$H_{\bar{u}}$ and $\widetilde{H}_{\bar{u}}$	✔	-	-
$E_{\bar{u}}$ and $\widetilde{E}_{\bar{u}}$	✘	$H_{\bar{u}}$ and $\widetilde{H}_{\bar{u}}$	$R^{E/\widetilde{E}}_{\text{sea}}$
$H_{\bar{d}}$ and $\widetilde{H}_{\bar{d}}$	✔	-	-
$E_{\bar{d}}$ and $\widetilde{E}_{\bar{d}}$	✘	$H_{\bar{d}}$ and $\widetilde{H}_{\bar{d}}$	$R^{E/\widetilde{E}}_{\text{sea}}$
H_g and $\widetilde{H}_g$	✔	-	-
E_g and $\widetilde{E}_g$	✘	H_g and $\widetilde{H}_g$	$R^{E/\widetilde{E}}_{\text{sea}}$

Strategy for the GPD global analysis

Experimental data and constraints

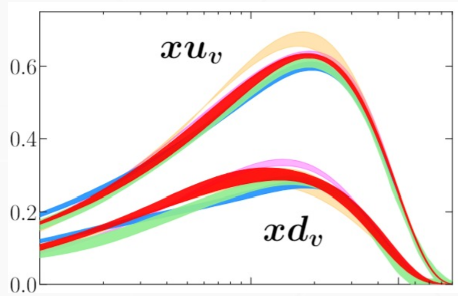
- ❑ Polarized and unpolarized PDFs from global analysis
  - ▪ Alternatively, one can fit to (polarized) DIS directly
- ❑ Neutron/ Proton charge form factors from global analysis
- ❑ Deeply virtual Compton scattering data at JLab/HERA
- ❑ Deeply virtual meson productions data at HERA

Lattice QCD simulations

- ❑ Lattice simulations of nucleon generalized form factors
- ❑ Lattice simulations of unpolarized and helicity GPDs at zero and non-zero ξ (skewness)

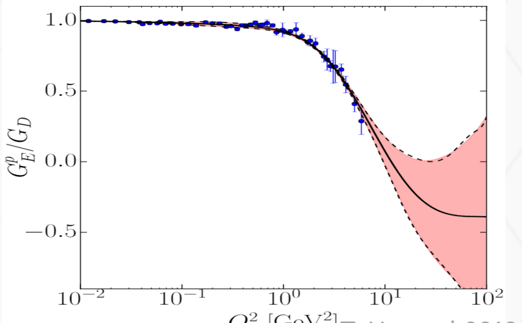
Sequential fit as first step to accelerate the convergence

Example of global PDF fit



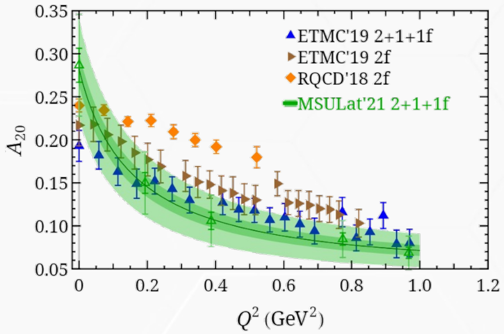
JAM 2022

Example of charge form factor fit



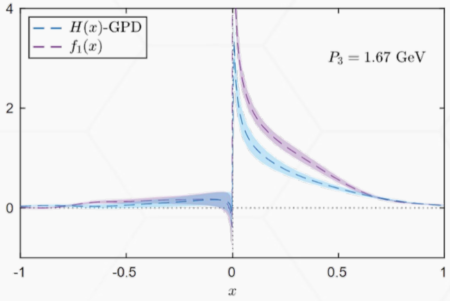
Z. Ye et al 2018

Example of lattice form factors

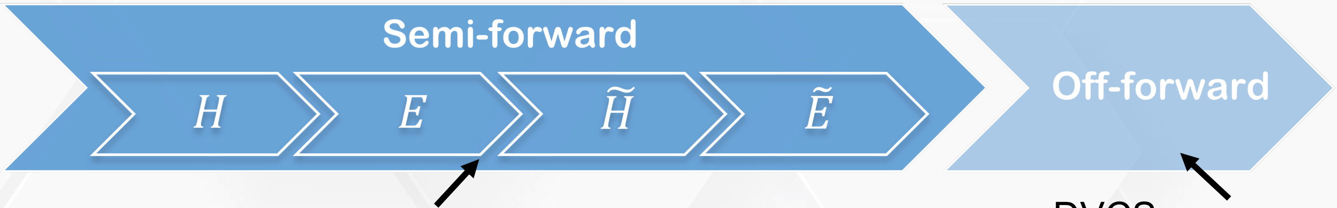


Huey-Wen Lin 2022

Example of lattice GPD



C. Alexandrou et. al. 2020



- JAM (2022) PDF global analysis results
- Globally extracted electromagnetic form factors (*Z. Ye et al* 2018)
- Lattice GPDs (*Alexandrou et al* 2020) and form factors (*Alexandrou et al* 2022)

DVCS measurements from JLab (*CLAS* 2019 & 2021, *Hall A* 2018 & 2022) and HERA (*H1* 2010)

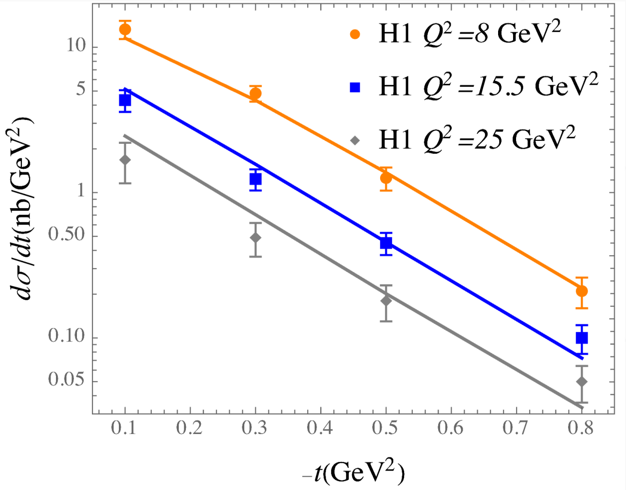
Examples of GUMP fits

The setup is rather basic:

- Around 30 parameters
- Minimization of chi^2 with *iminuit*
- Total run time with an hour for 8 core CPU at LO, no replica.

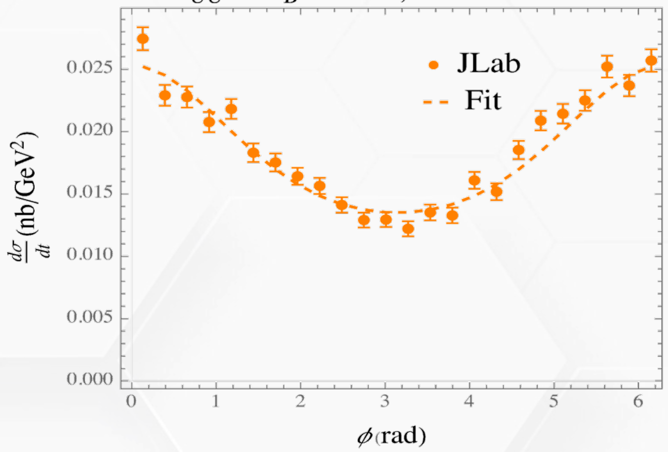
Sub-fits	χ^2	N_{data}	$\chi^2_\nu \equiv \chi^2/\nu$
Semi-forward			
tPDF H	281.7	217	1.41
tPDF E	59.7	50	1.36
tPDF $\widetilde{H}$	159.3	206	0.84
tPDF $\widetilde{E}$	63.8	58	1.23
Off-forward			
JLab DVCS	1413.7	926	~ 1.53
H1 DVCS	19.7	24	~ 0.82
Off-forward total	1433	950	**1.53**
Total	2042	1481	**1.40**

Example of fit to DVCS at H1



Example of fit to DVCS at JLab

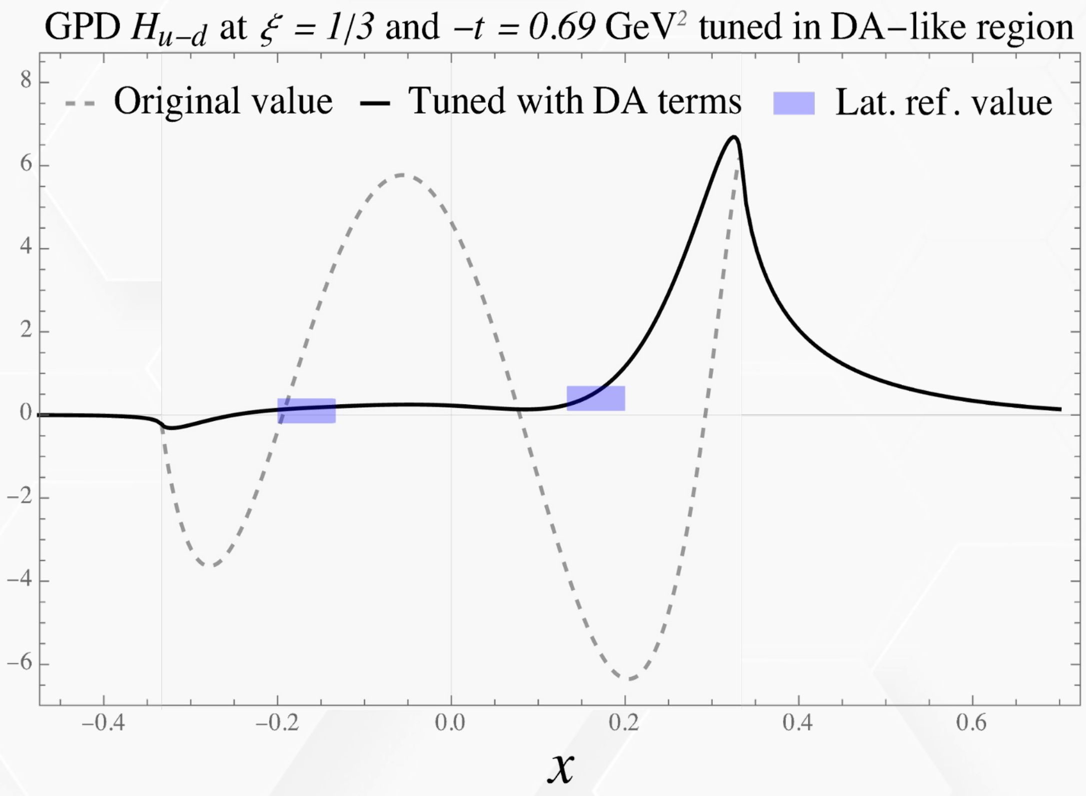
$d\sigma_{UU}$ at x_B=0.367, t=−0.197 GeV^2



Overcoming the shadow GPDs with lattice

The extracted GPDs has wiggling in the DA region, could be caused by the shadow GPDs

Example of GPDs tuned to lattice input



The wiggling can be constrained by the lattice input which gives information on the x-dependence

Summary

Summary

- Introduce the GPD and some of its relevant properties.
- Report our efforts in trying to extract them from global analysis at LO/NLO
  - DVCS at LO for the quark GPD
  - DVJ/psi P at NLO for the gluon GPD (on-going)
- It appears that the most stringent limitation is from the input (lattice/exp.) side.
- On the computation side, the bottleneck is from solving the NLO evolution.

Thank you!