Towards a microscopic description of cluster in-medium effects

F.Gulminelli
LPC and Normandie Université, Caen

institut universitaire de France

Transport properties in compact stars

- Compact star dynamics involves transport properties
- Some of them concern « low » density n-p-e matter ≡ clusters
  - NS cooling: B-thermal evolution
  - Relaxation after accretion & deep crust heating
  - CC, PNS cooling & mergers

} e-Z, $T \approx 10^8 K$

ν-Z $T > 10^{10} K$

Schmitt&Shternin Springer 2018

- Key micro feature: charge distribution => resistivity, opacity

- **Present situation:**
  - **Cluster abundancies from Nuclear Statistical Equilibrium (classical Beth-Uhlenbeck) with ad-hoc modifications**

$$n_{AZ} = \frac{g_{AZ}}{2\pi^2\hbar^3}\int_0^\infty dp\, exp - \beta\left(\frac{p^2}{2M_{AZ}} - \boldsymbol{B_{AZ}} - A\mu - Z\mu_p\right)$$

Nuclei=quasi-particles
B: vacuum values

Z.Lin et al,PRC **102**(2020)045801

The p-n system (deuteron)

$$n_{free} = \frac{4}{\lambda^3} e^{\frac{\mu}{T}}$$

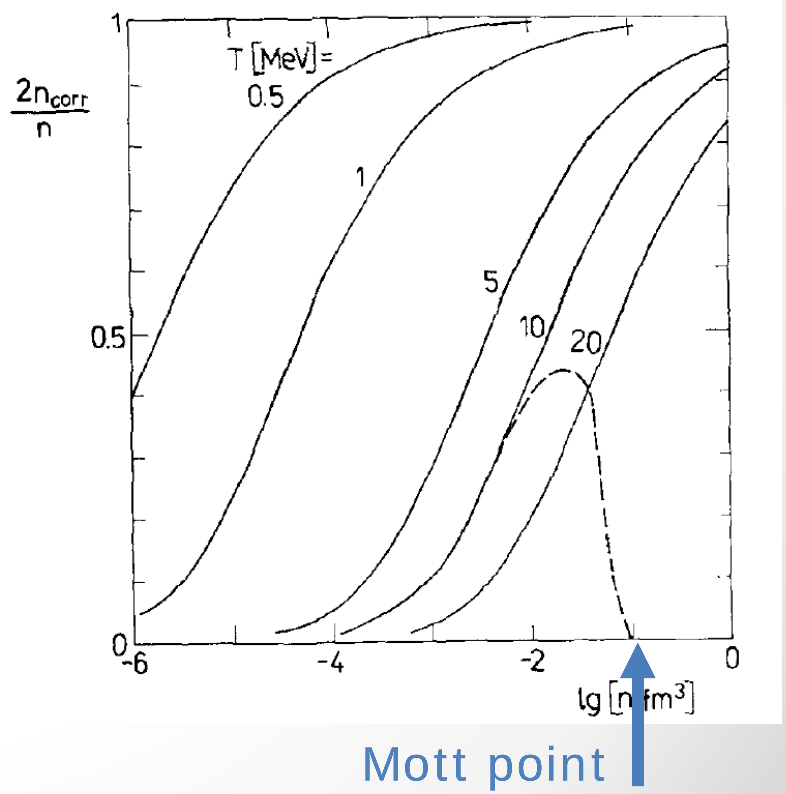
$$I = \frac{3\sqrt{2}\lambda^3}{8}\left(e^{-\frac{B}{T}} - 1\right)$$

- Only monomers and dimers, symmetric matter $n_n = n_p$
- $n = n_{free} + 2n_{cor}$

  $= n_{free} + 2n_{free}^2 I$

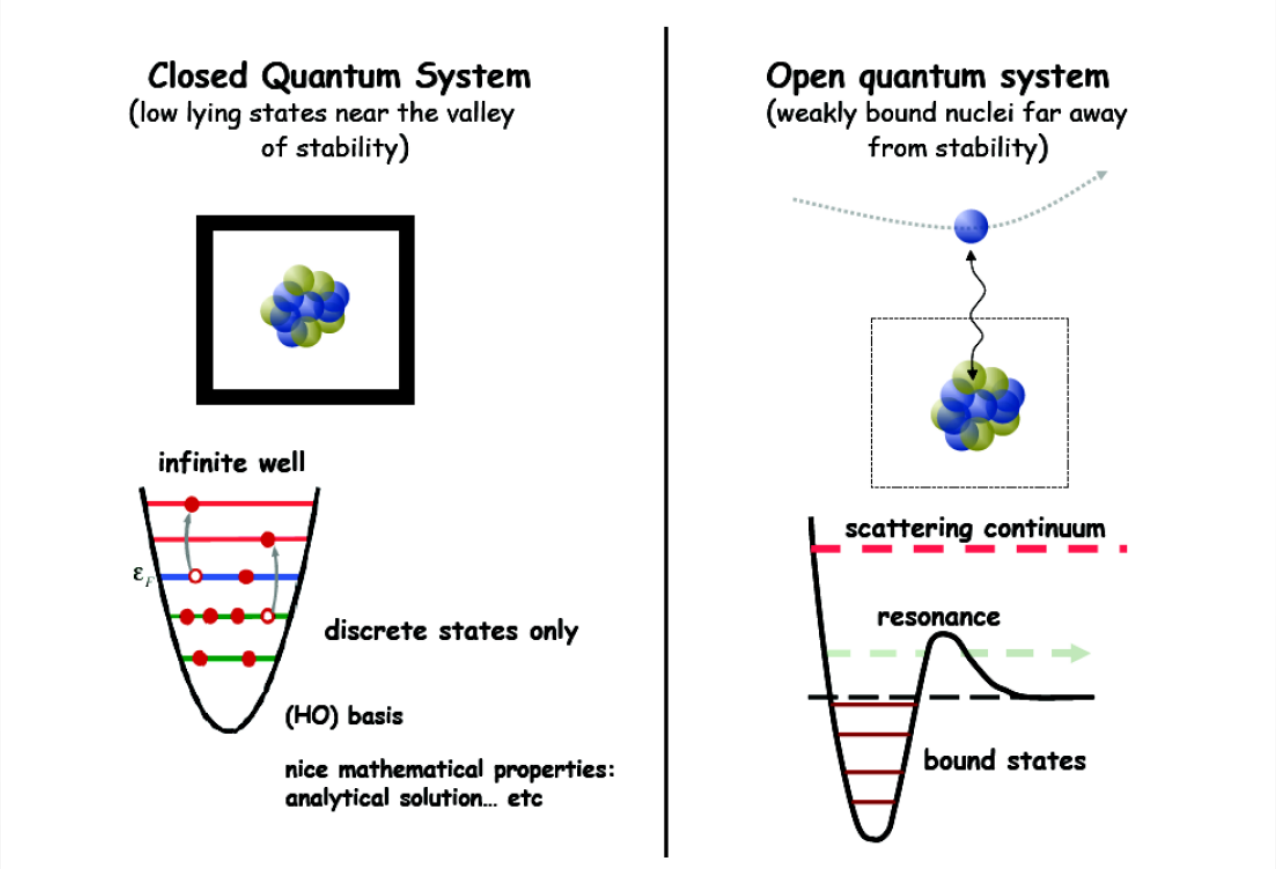
- => $\frac{n_{cor}}{n} = n_{free}\frac{I(T)}{1+2n_{free}I(T)}$
- This simple theory misses the physical dissolution of clusters in dense matter: Mott effect
- We need: quantum treatment & interactions with the medium: binding energy shift $B_{AZ} \rightarrow B_{AZ}^0 - \Delta B_{AZ}(\mathrm{n,T})$
- The Mott point is defined by $\boldsymbol{B_{AZ}(P_{Mott}, n, T) = 0}$

M. Schmidt et al. Ann.Phys. 202 57 (1990)



Open Quantum Systems

- Particles bound in clusters are correlated and interact with their continuum states: same phenomenology as dripline nuclei



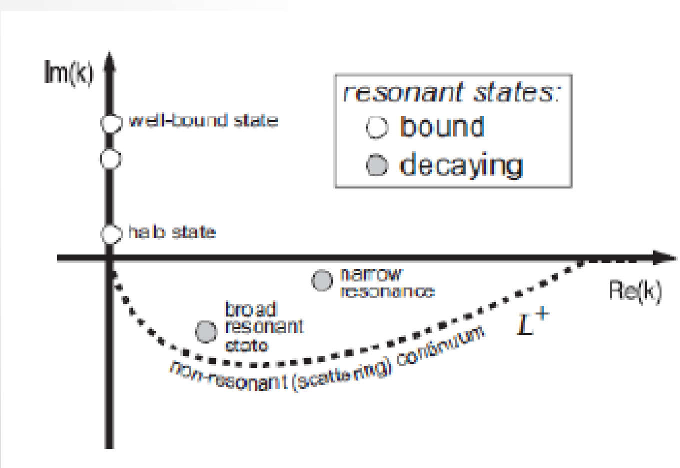
N.Michel,M.Ploszajczak, Gamow Shell Model, Lecture Notes in Physics 983, 2021

Open Quantum Systems

Dripline nuclei

- Bound, scattering states, resonances are S-matrix poles in the complex plane
- Rigged Hilbert space and Berggren basis

$$\sum |u_n\rangle\langle u_n| + \int_{L^+} dk\ |u_k\rangle\langle u_k| = 1$$

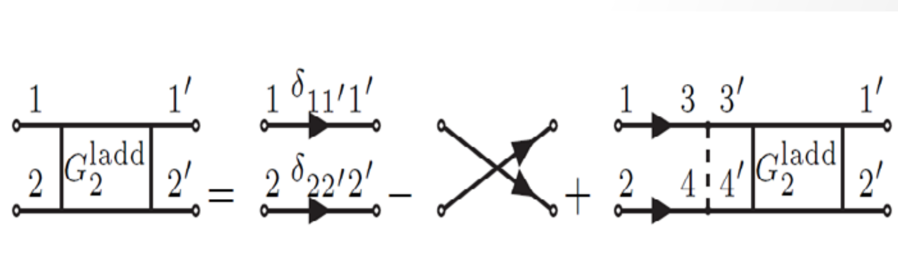


A.Bohm 1978, N.Michel 2001....

Clusters in the medium

- Solution of the Bethe-Salpeter equation in the ladder approximation with thermal GF
- In-medium modified 2-body Schroedinger equation up to the scattering threshold

$$G_2(121'2') = \sum_n \Psi_n(12) \frac{1 - f_{1'} - f_{2'}}{i\omega_m - E_n + \mu} \Psi_n^*(1'2')$$

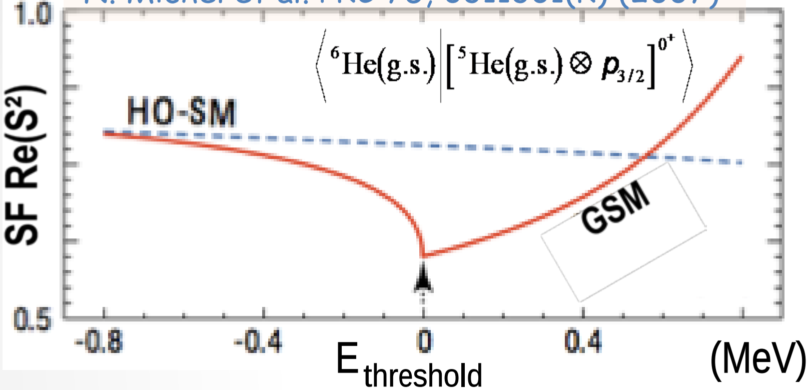


G.Roepke 1982, W.Kraeft 1986...

Open Quantum Systems

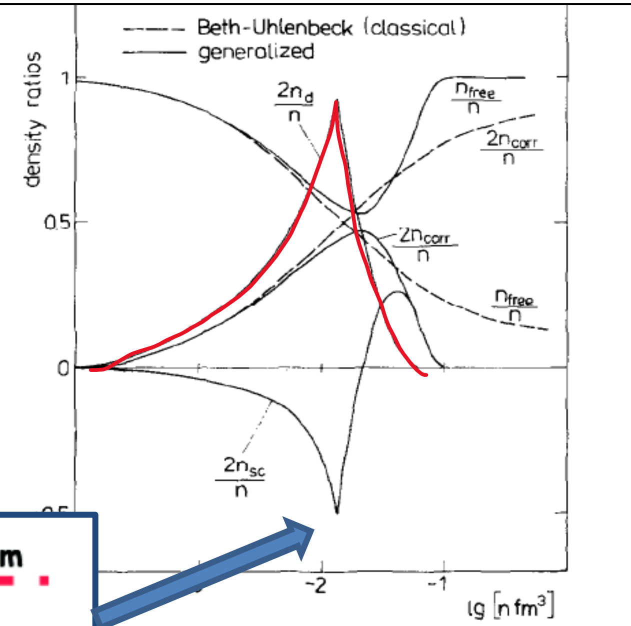
Dripline nuclei

N. Michel et al. PRC 75, 0311301(R) (2007)

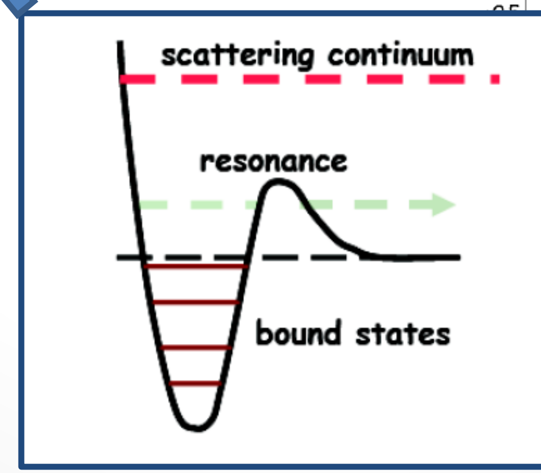


Clusters in the medium

$$n_{cor}(P) = f_{BE}\left(B_d(n, P < P_{th})\right) + \int de\, h\left(\{\delta_n(e)\}\right)$$



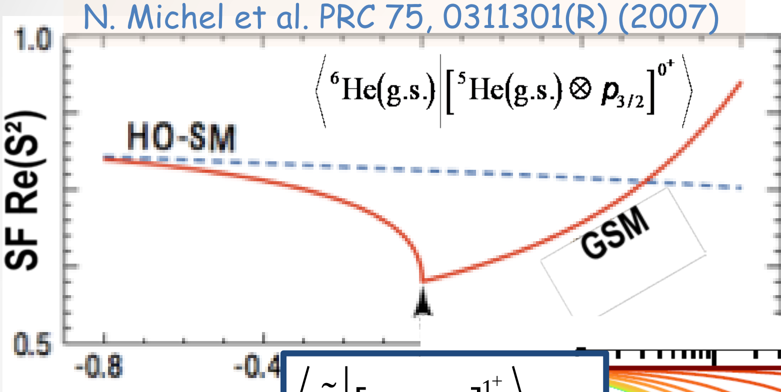
M. Schmidt et al. Ann.Phys. 202 57 (1990)



Open Quantum Systems

Dripline nuclei

N. Michel et al. PRC 75, 0311301(R) (2007)



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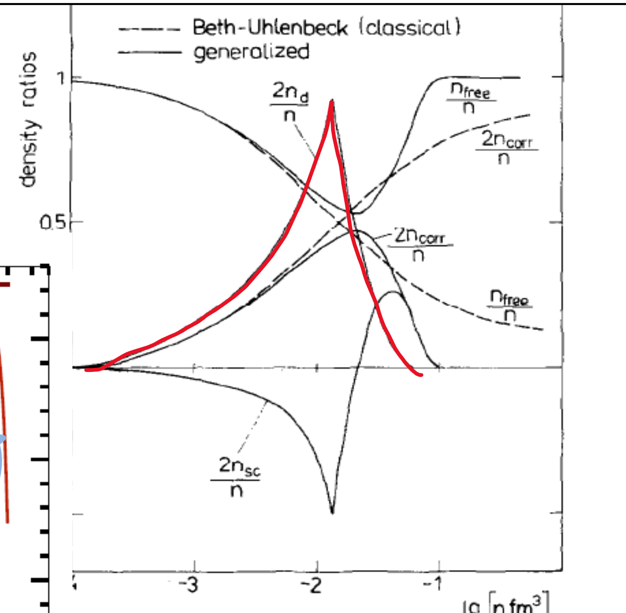
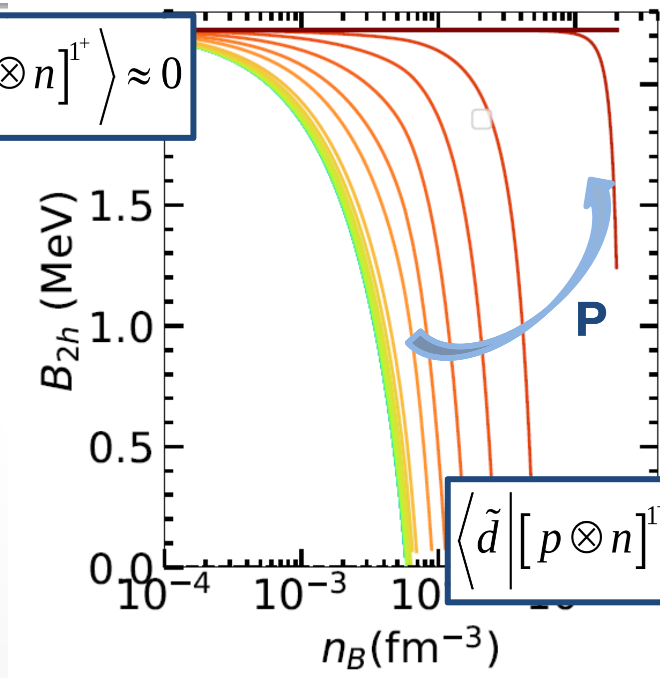


M. Schmidt et al. Ann.Phys. 202 57 (1990)

$$\left\langle \tilde{d} \middle| [p \otimes n]^{1^+} \right\rangle \approx 0$$



$$\left\langle \tilde{d} \middle| [p \otimes n]^{1^+} \right\rangle = 1$$

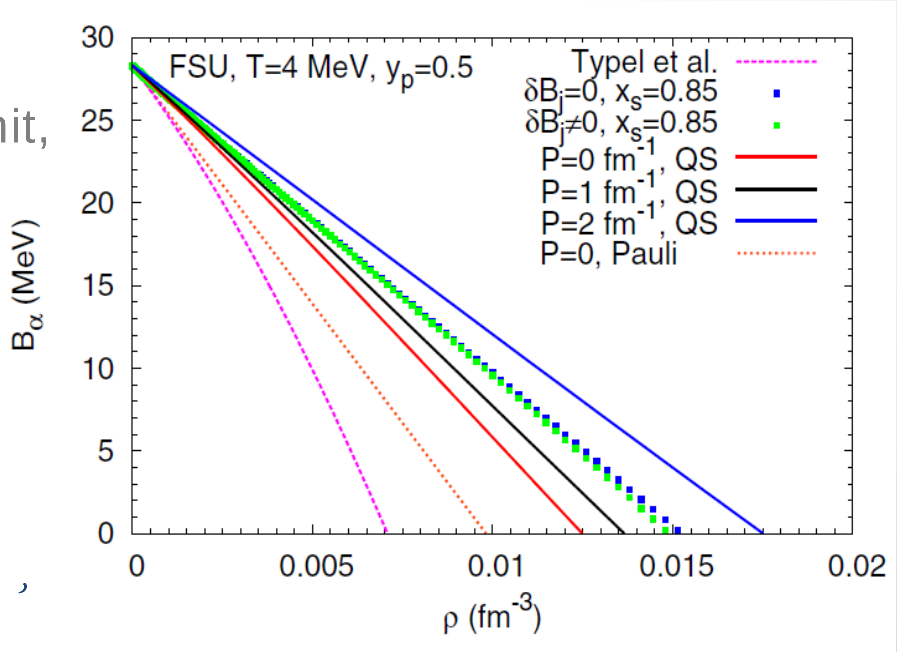
$E_{threshold} \leftrightarrow n_{Mott}$

Clusters in the medium

$$n_{AZ} = \frac{g_{AZ}}{2\pi^2\hbar^3}\int_{P_{Mott}(n,T)}^{\infty} dP\left[1 \pm exp\beta\left(\frac{P^2}{2M_{AZ}} - B_{AZ}(P,n,T) - A\mu - Z\mu_p\right)\right]^{-1}$$

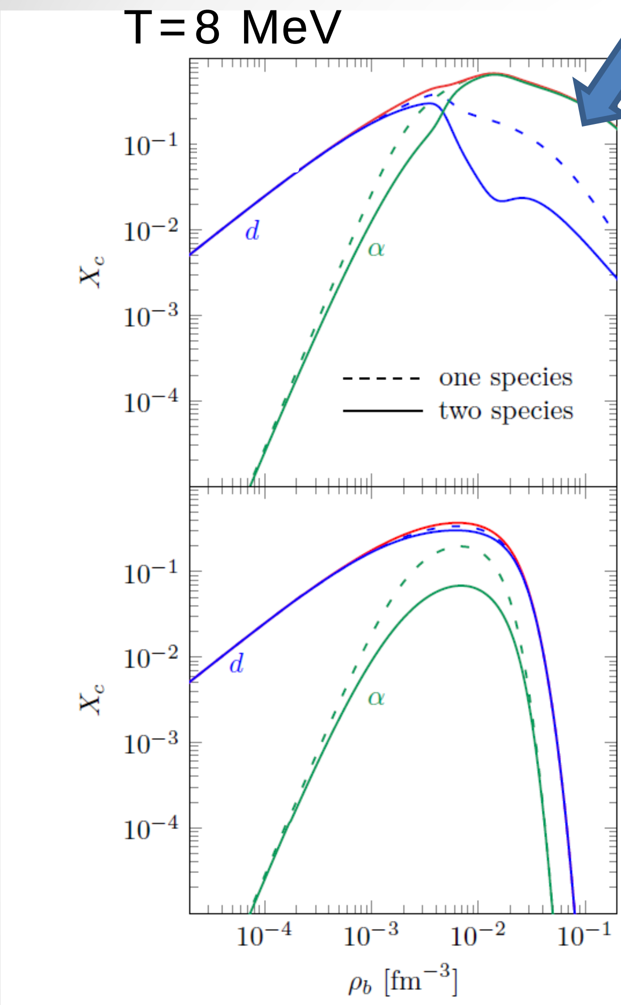
- The propagator has a q-particle structure as long as $B_{AZ} \geq 0$
- Minimum momentum for bound clusters to exist $\forall n, T$: $B_{AZ}(P_{Mott}) = 0$
- Exact solutions for B_{AZ} only exist for the deuteron

- => *Approximations*: Boltzmann limit, simplified form factors, schematic interactions....
- => or *Phenomenological* expressions fitted on data



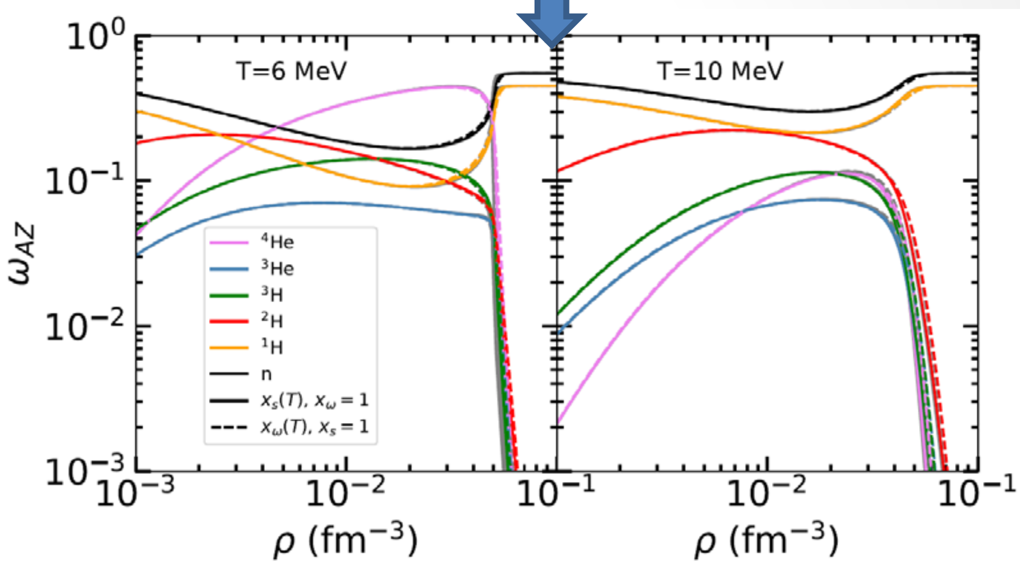
G. Röpke, Nucl. Phys. A 867, 66 (2011) , PRC 92 (2015) 054001
H.Pais et al,PRC97(2018)045805

P_{Mott} from
R.Wang et al, PRC 108(2023)L031601



S.Burrello et al, ArXiV:260302060

ΔB from
T.Custodio et al, PRL 184(2025)082304



T.Custodio et al, arXiv:2602.11996

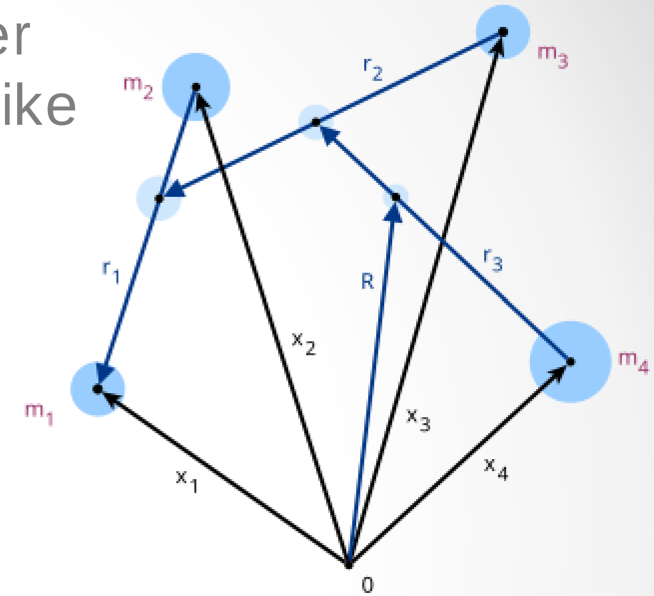
P_{Mott} from
S.Typel et al, PRC81(2010)015803

The microscopic theory of the shifts

$$\vec{q} = \{q_1, \ldots, q_{A-1}\}$$

- The solution of the BS equation in ladder approximation leads to a Schrodinger-like equation for the A-body system:

$$\left[E_{\boldsymbol{P}} - \sum_{i=1}^{A} e(\boldsymbol{p}_i)\right] \phi_A(\vec{q}) = \sum_{i<j} (1 - f_i - f_j) \int \frac{d\vec{\mathbf{p}}'}{(2\pi)^{3A}} v_{ij} \phi_A(\vec{q}'),$$



- with an effective non-hermitian Hamiltonian

$$\hat{H} = \hat{T} + \hat{V}_1 + \left(1 - \sum_{i<j} \left(f_{\tau_i} + f_{\tau_j}\right)\right) \hat{V}_2$$

M. Schmidt et al. Ann.Phys (1990)
G. Röpke, Nucl. Phys. 2009=>2014
F.Gulminelli et al, in preparation

- For separable interactions, a closed solution can be found in perturbation:

$$E_{N_n,N_p}(\boldsymbol{P}) = \frac{P^2}{2M^*} + \sum_{i=1}^{A} \mathcal{U}_{\tau_i} + E_0 - 2 \int \frac{d\vec{q}}{(2\pi)^{3(A-1)}} S(\vec{q}) \left(E_0 - \sum_{k=1}^{A-1} \frac{q_k^2}{2\bar{\mu}_k}\right) \bar{f}_{N_n,N_p}(\boldsymbol{p}_1)$$

- only depends on the ground state properties (and EDF for the self-energy shift)

$$S(\vec{q}) = |\phi_A(\vec{q})|^2$$

The microscopic theory of the shifts

- Self-energy and Pauli-blocking shifts:

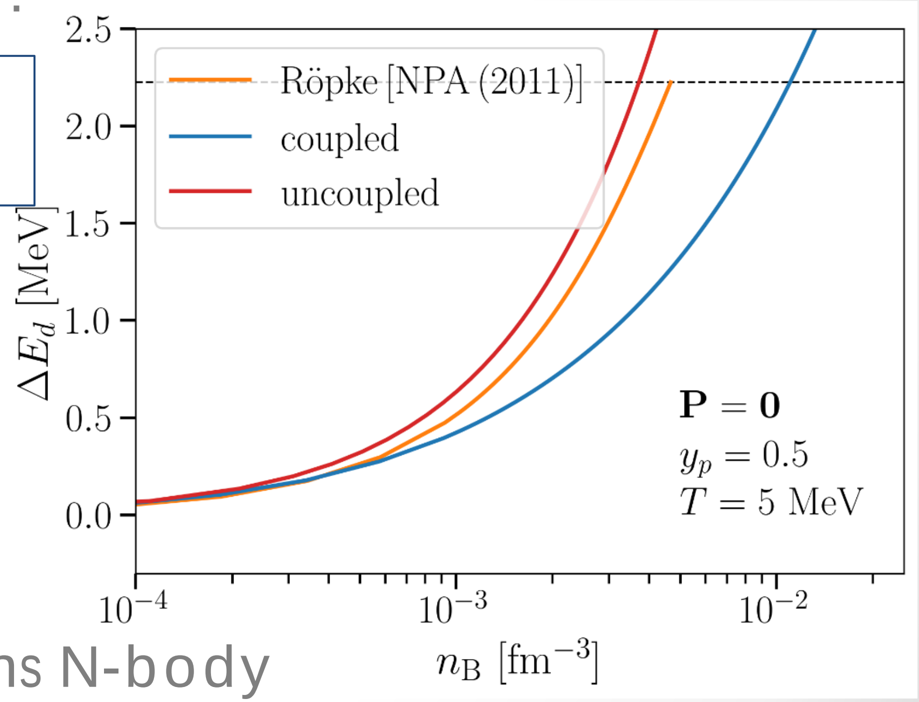
$$E_{N_n,N_p}(\mathbf{P}) = \frac{P^2}{2M^*} + \sum_{i=1}^{A} \mathcal{U}_{\tau_i} + E_0 - 2\int \frac{d\vec{\mathbf{q}}}{(2\pi)^{3(A-1)}} S(\vec{\mathbf{q}}) \left(E_0 - \sum_{k=1}^{A-1} \frac{q_k^2}{2\bar{\mu}_k} \right) \bar{f}_{N_n,N_p}(\mathbf{p}_1)$$

$$S(\vec{\mathbf{q}}) = |\phi_A(\vec{\mathbf{q}})|^2$$

- a fully self-consistent problem :

$$\bar{f}_{N_n,N_p} = \sum_{\tau=n,p} \frac{N_\tau}{A} f_\tau \; ; \; f_\tau(\mathbf{p}) = \sum_{\ell} N_{\tau,\ell} f_\ell(A_\ell \mathbf{p}),$$

$$f_\ell(\mathbf{P}) = \left[\exp\beta \left(E_\ell(P) - \sum_{\tau=n,p} N_{\tau,\ell}\mu_\tau \right) - (-1)^{A_\ell} \right]^{-1}.$$



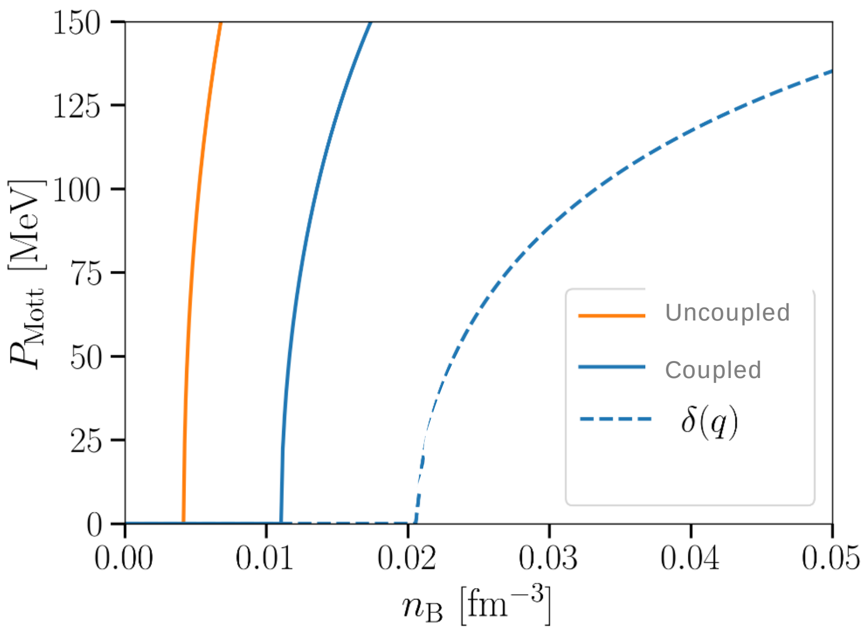
- The 1-body distribution contains N-body correlations => the shifts cannot be given in simple parametrized closed form

The Mott momentum

$S(\vec{q}) = |\phi_A(\vec{q})|^2$

- Mott momentum definition:

$$B_\ell(\boldsymbol{P}_\ell^{Mott}) = 0$$



- with the approx. $S(\vec{q}) = (2\pi)^3\delta(\vec{q})$

$$\bar{f}_\ell\left(\frac{\boldsymbol{P}_\ell^{Mott}}{A_\ell}\right) \approx \frac{1}{2},$$

- Recall $\hat{V}_2^{eff} = P_{12}\hat{V}_2$ with
  - o $\hat{V}_2 = \hat{V}_2^+$
  - o $P_{12} = (1 - f_1 - f_2)$ is quasi-hermitian for $P_{12} > 0 \rightarrow P_\ell > P_\ell^{Mott}(n_B)$

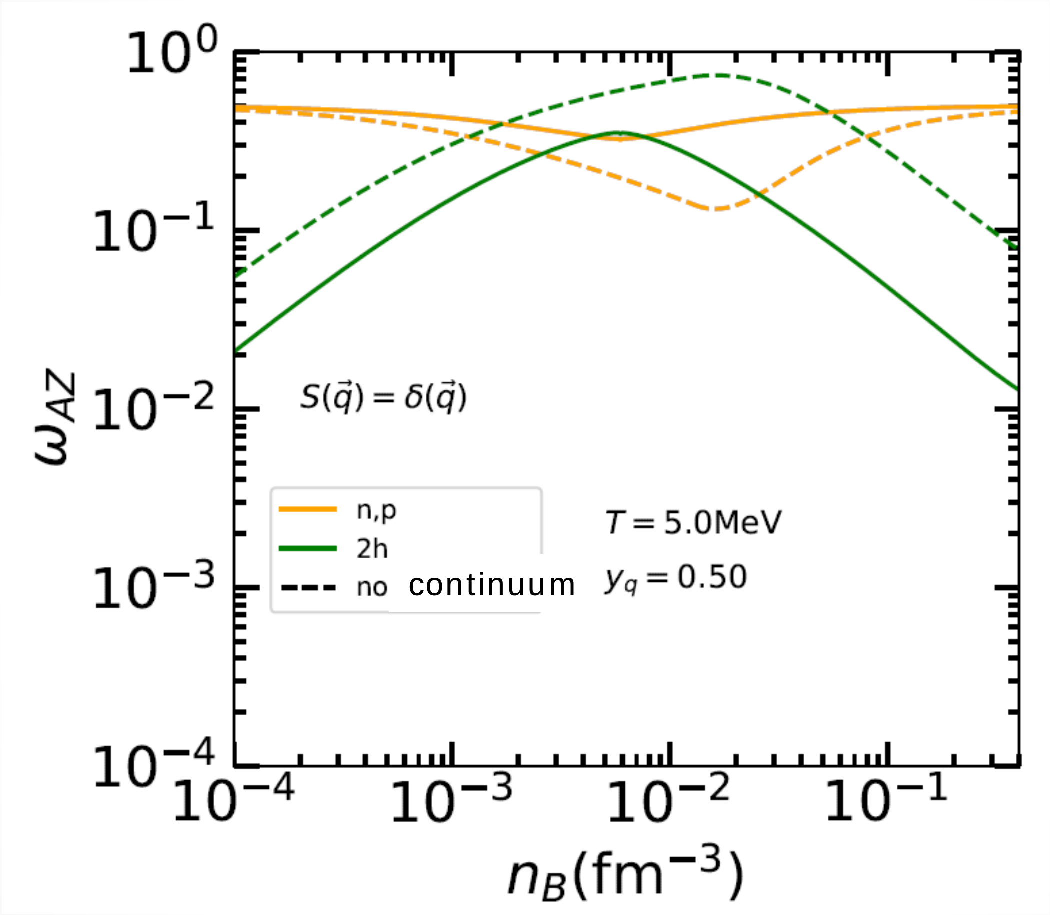
$\Rightarrow$ real eigenvalues up to P_ℓ^{Mott}

$\Rightarrow$ the bound state at the continuum edge continuously evolves towards a resonant state

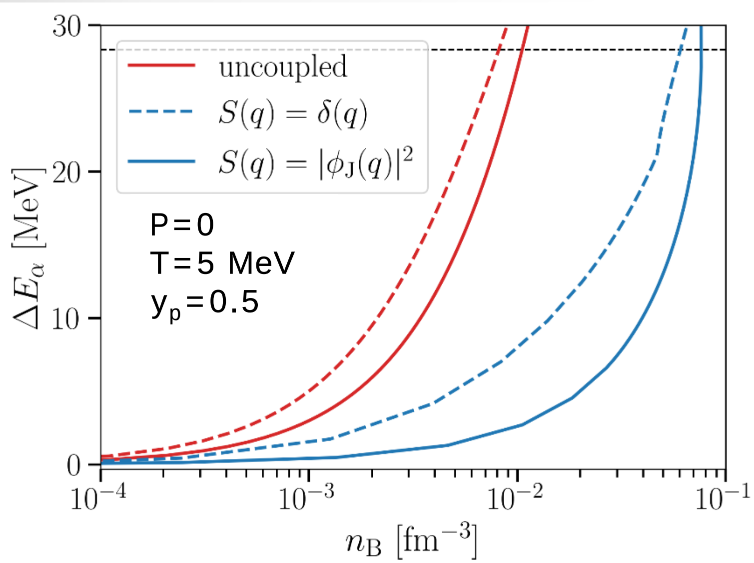
$$n_\tau^{tot} = n_\tau - \sum_{\ell, A_\ell > 1} N_{\tau,\ell} g_\ell \int_{P_\ell^{Mott}} \frac{d\boldsymbol{p}}{(2\pi)^3} f_\ell(E_\ell^{cont}(\boldsymbol{p})),$$

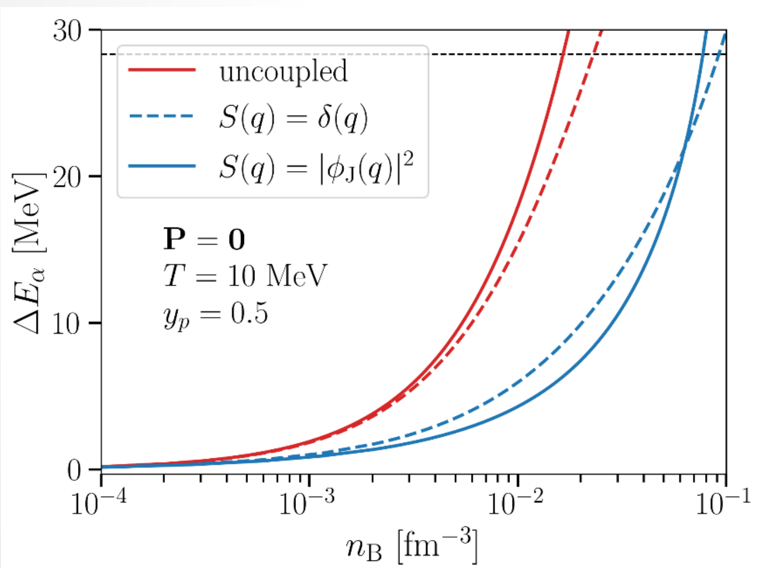
M. Schmidt et al. Ann.Phys (1990)

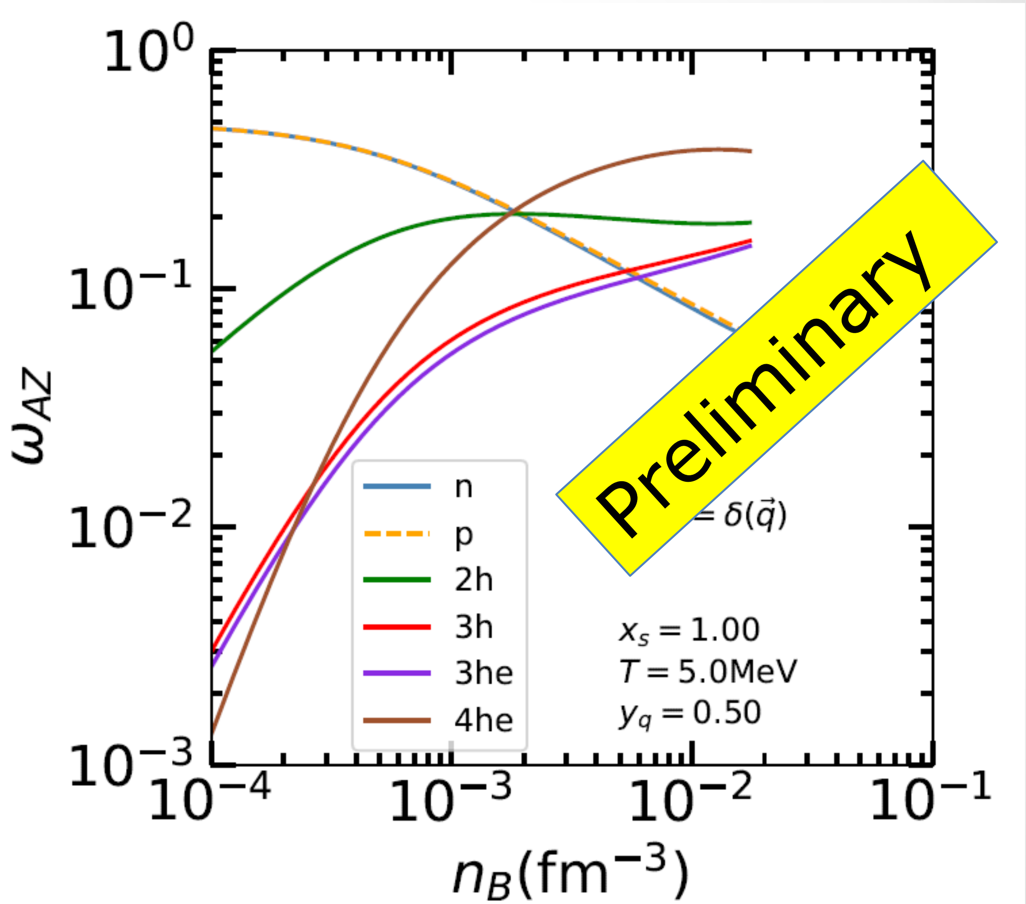
The effect of the resonant continuum



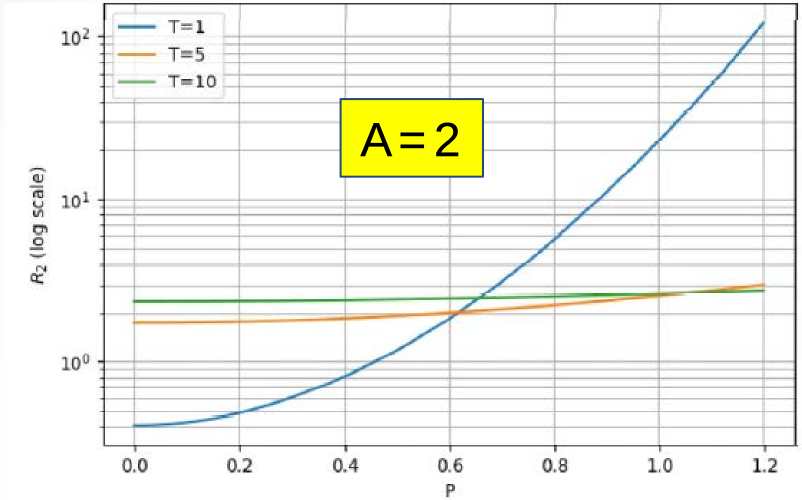
The inclusion of heavier clusters



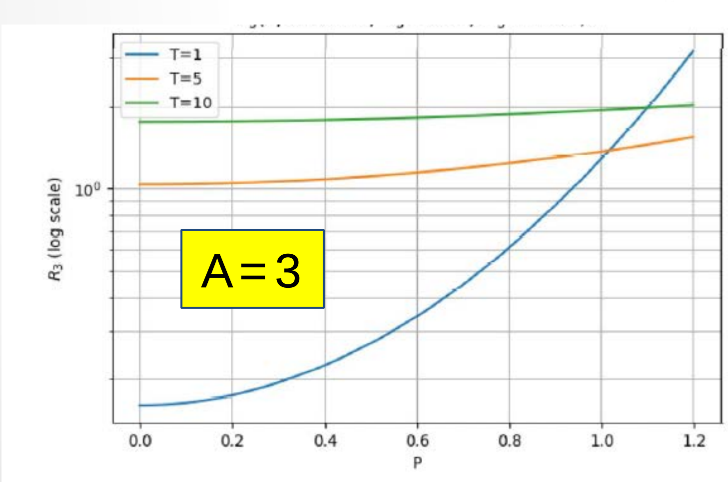


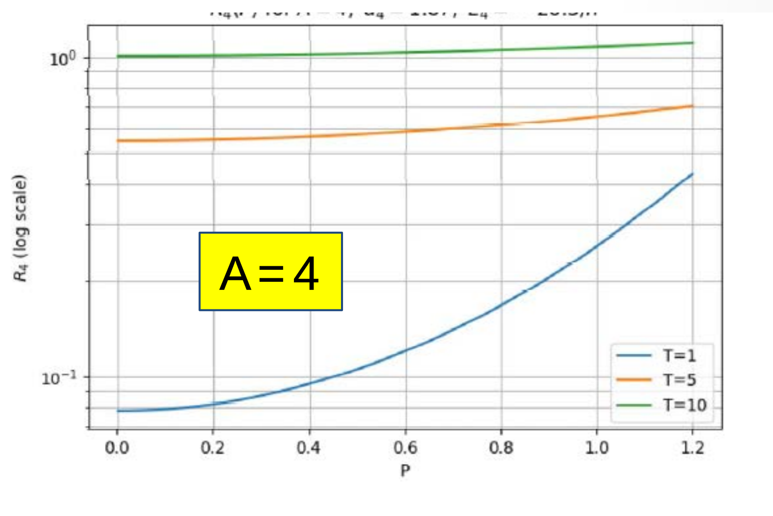


The effect of the internal cluster structure



$$R_\ell(\boldsymbol{P}) = \frac{\Delta E_\ell^{Pauli}(S = S_0)}{\Delta E_\ell^{Pauli}(S = \delta(\vec{q}))},$$

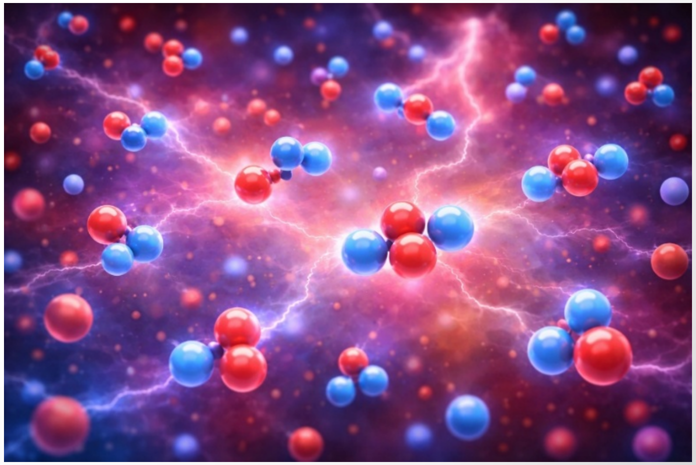




Symmetric matter, Boltzmann approximation

Conclusions

- Clusters in the astrophysical medium = Open Quantum System: bound and scattering states are coupled
- This leads to an **effective medium dependence of the clusters internal energy**
- Microscopic calculation from the Rostock GF method in perturbation
- Inclusion of bound and resonant states
- Important retro-action effects of the correlated distribution on the shifts
- Full calculation in progress



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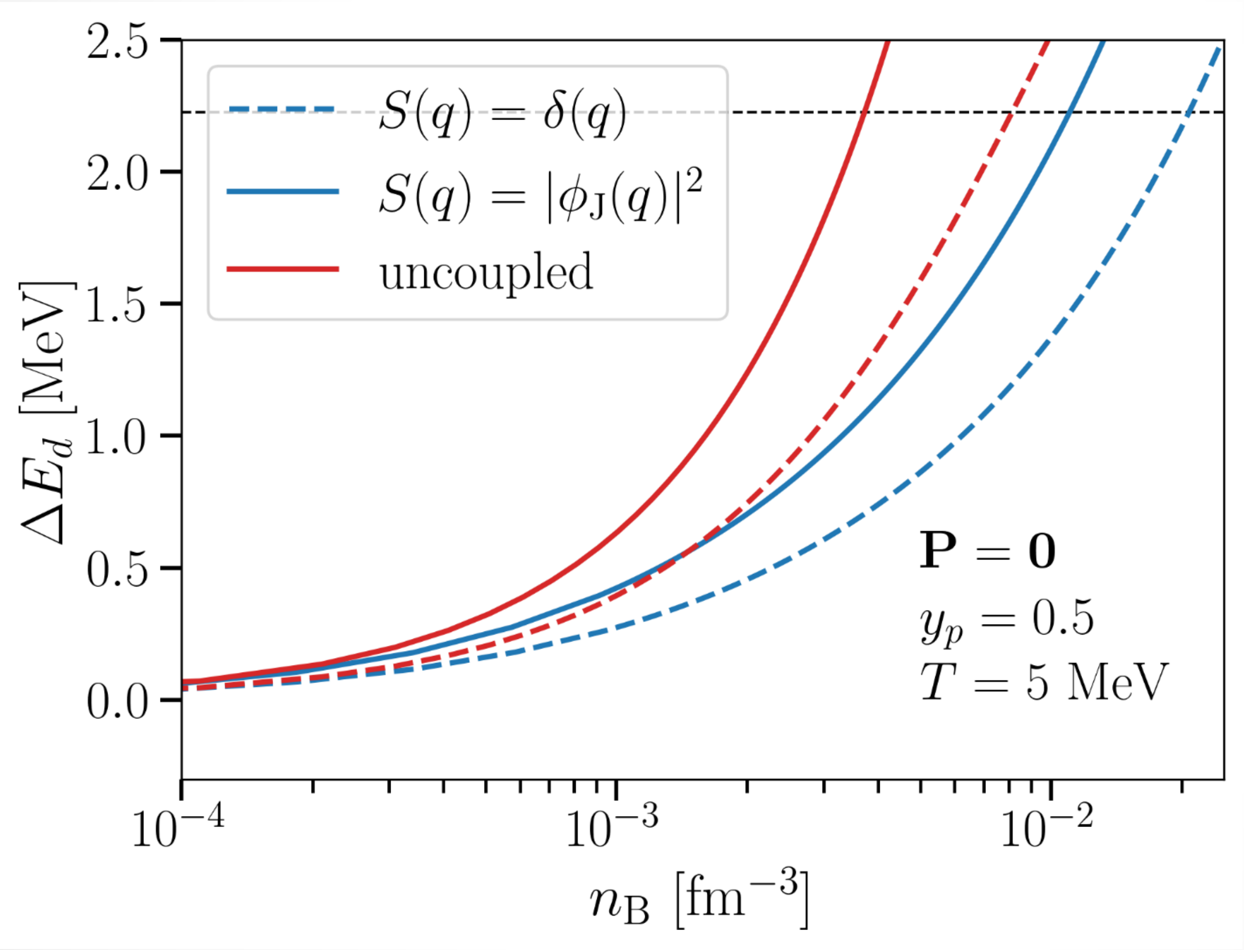
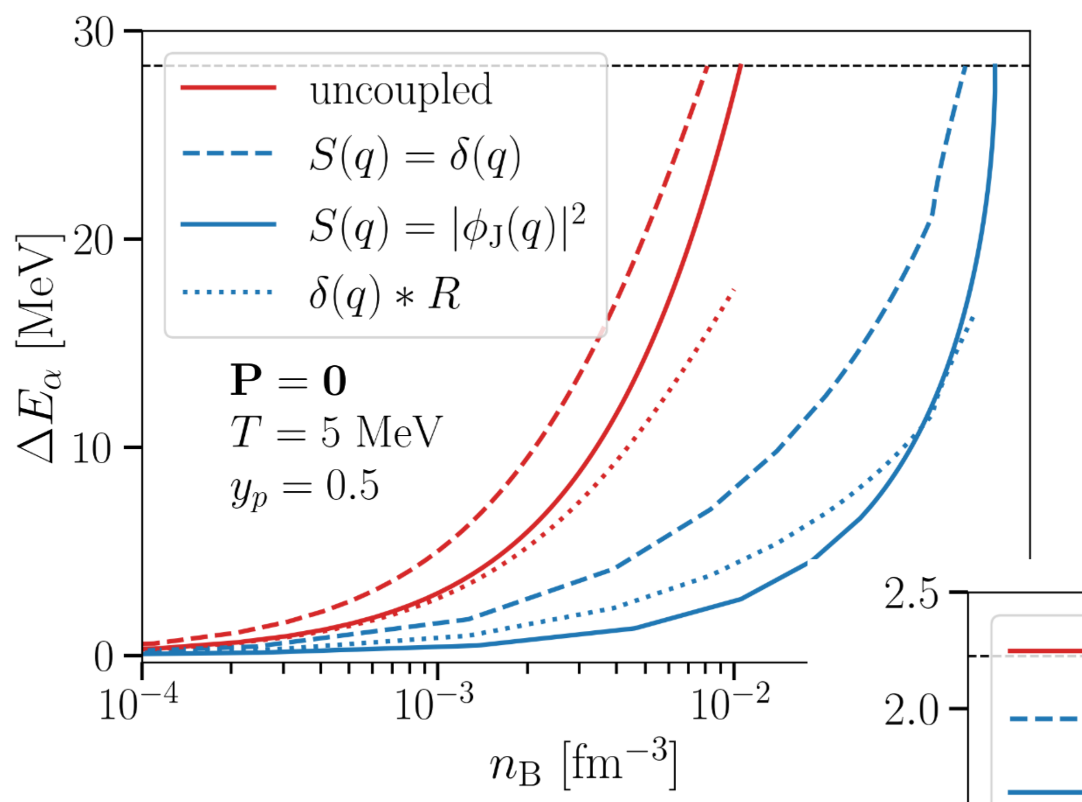
Collaboration

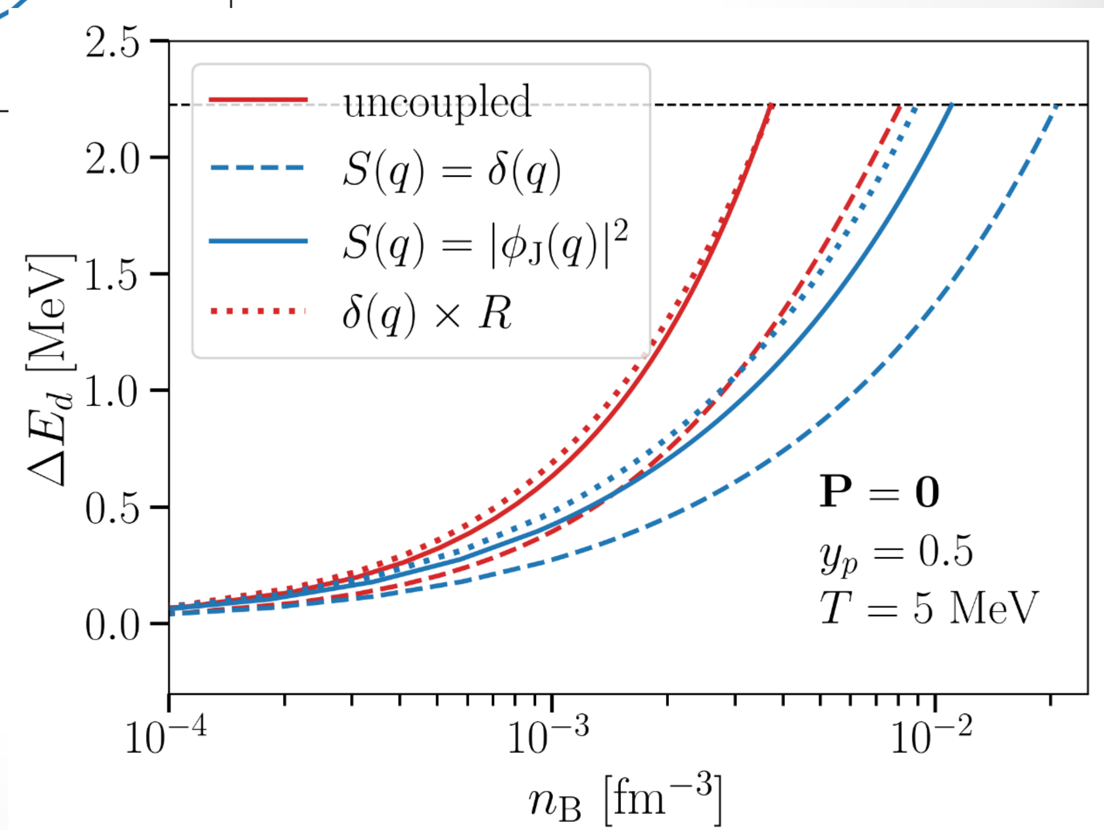
- T.Custodio (Coimbra&LPC)
- S.Burrello, M.Colonna (LNS, Catania)



$S(q) = \delta(q)$
$S(q) = |\phi_J(q)|^2$
uncoupled
$\mathbf{P} = \mathbf{0}$
$y_p = 0.5$
$T = 5$ MeV
ΔE_d [MeV]
n_B [fm^{-3}]
2.5
2.0
1.5
1.0
0.5
0.0
10^{-4}
10^{-3}
10^{-2}



30
20
10
0
ΔE_α [MeV]
uncoupled
$S(q) = \delta(q)$
$S(q) = |\phi_J(q)|^2$
$\delta(q) * R$
$\mathbf{P} = \mathbf{0}$
$T = 5$ MeV
$y_p = 0.5$
10^{-4}
10^{-3}
10^{-2}
n_B [fm^{-3}]



2.5
2.0
1.5
1.0
0.5
0.0
ΔE_d [MeV]
uncoupled
$S(q) = \delta(q)$
$S(q) = |\phi_J(q)|^2$
$\delta(q) \times R$
$\mathbf{P} = \mathbf{0}$
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10^{-4}
10^{-3}
10^{-2}
n_B [fm^{-3}]