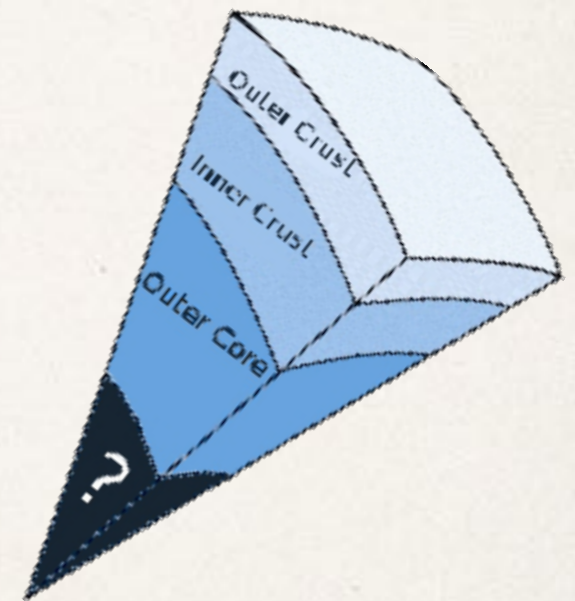
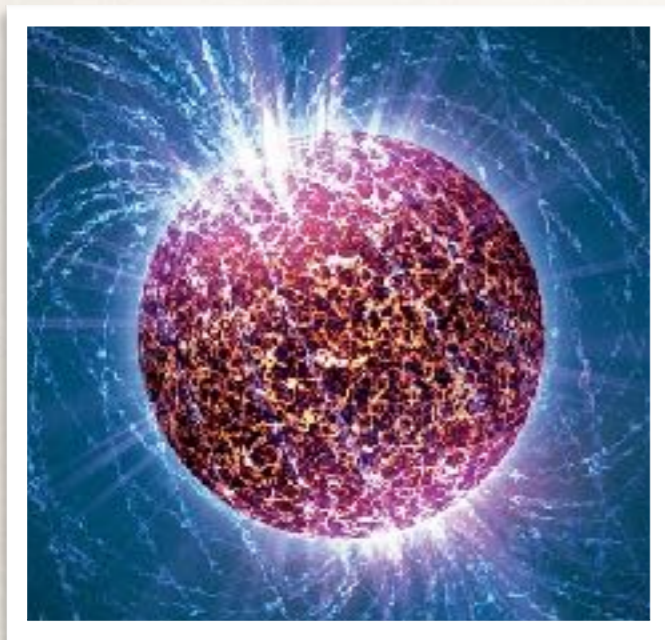
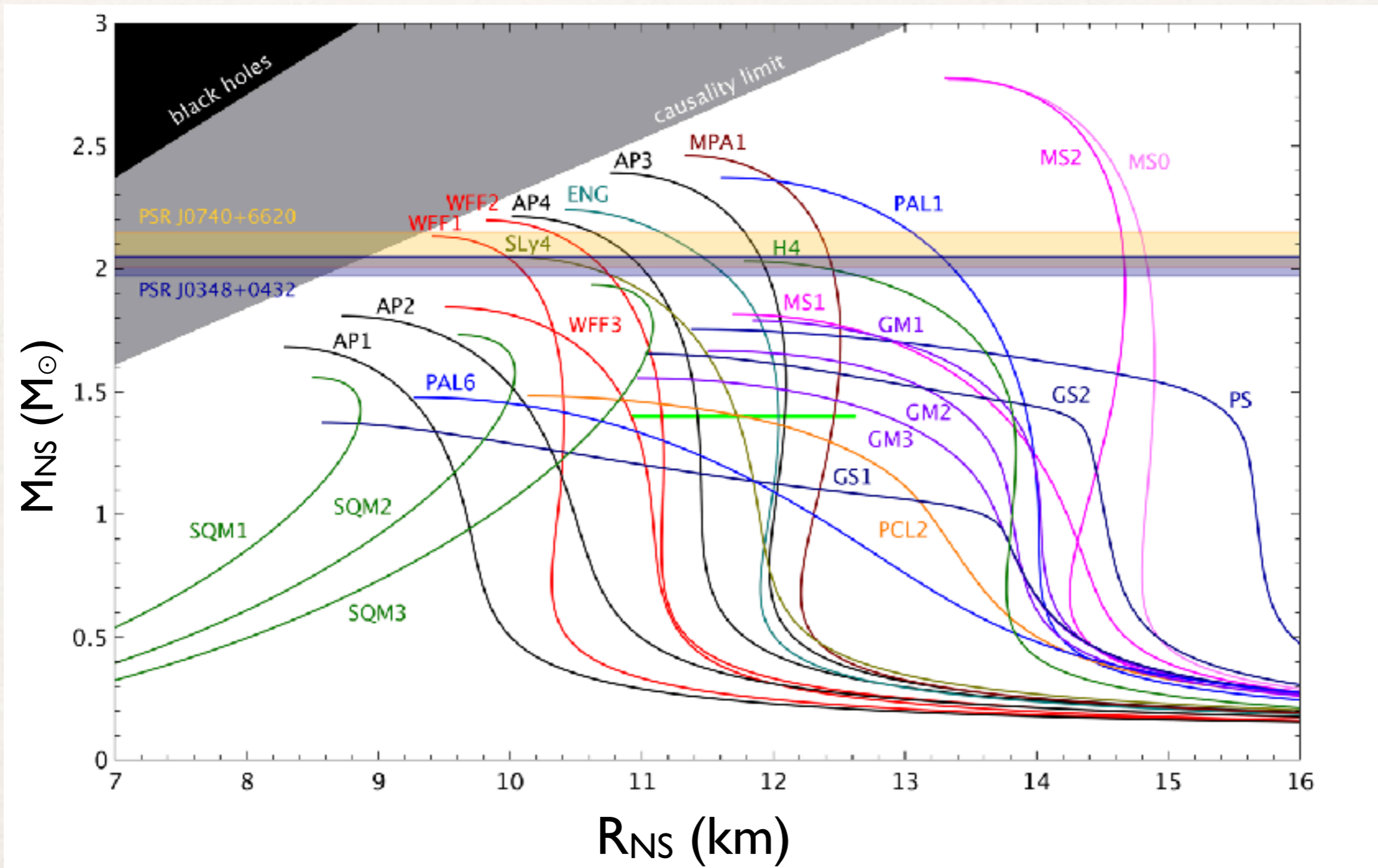


Mass–Radius constraints on the equation of state with phase–averaged X–ray observations

Sebastien Guillot

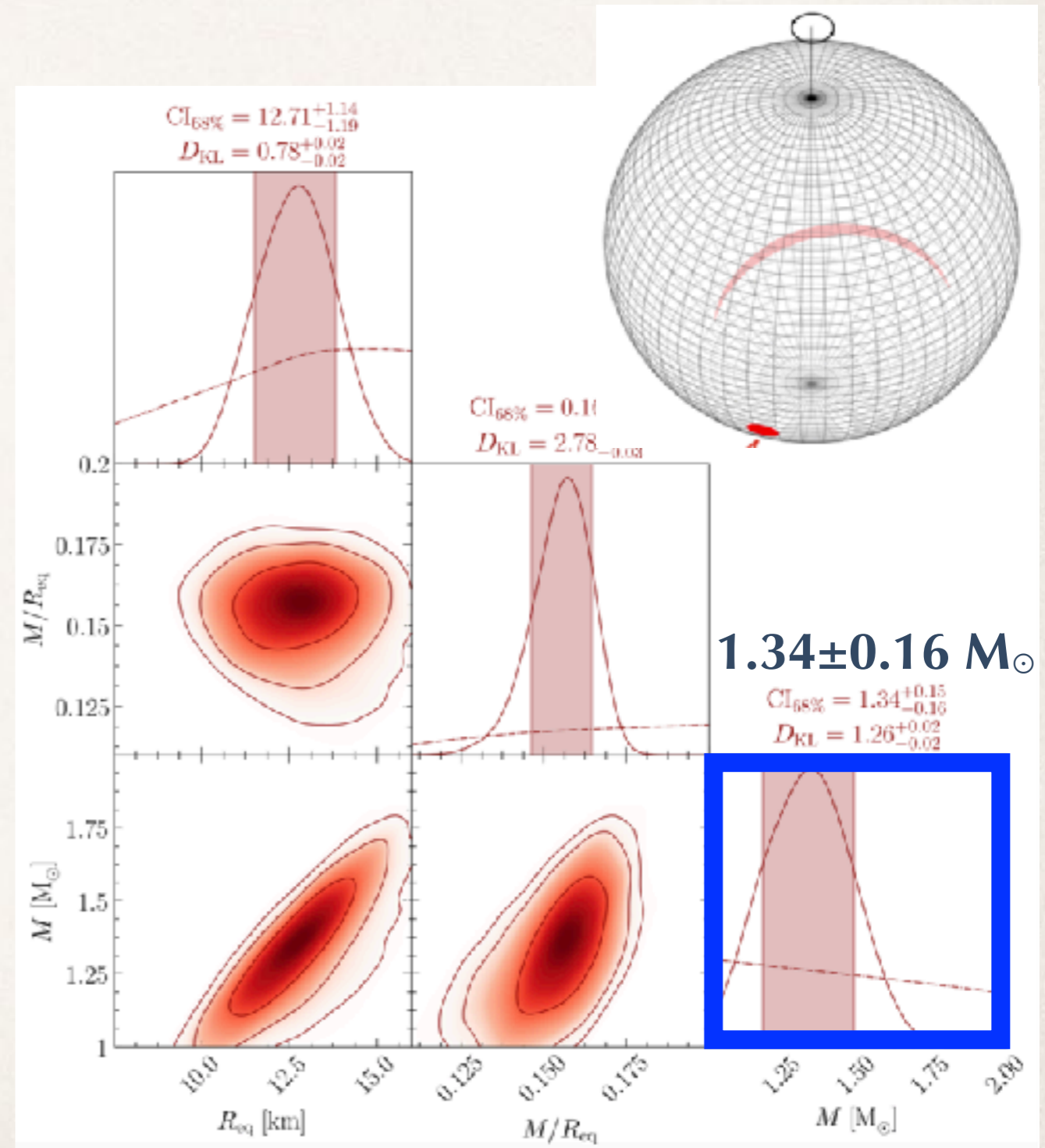
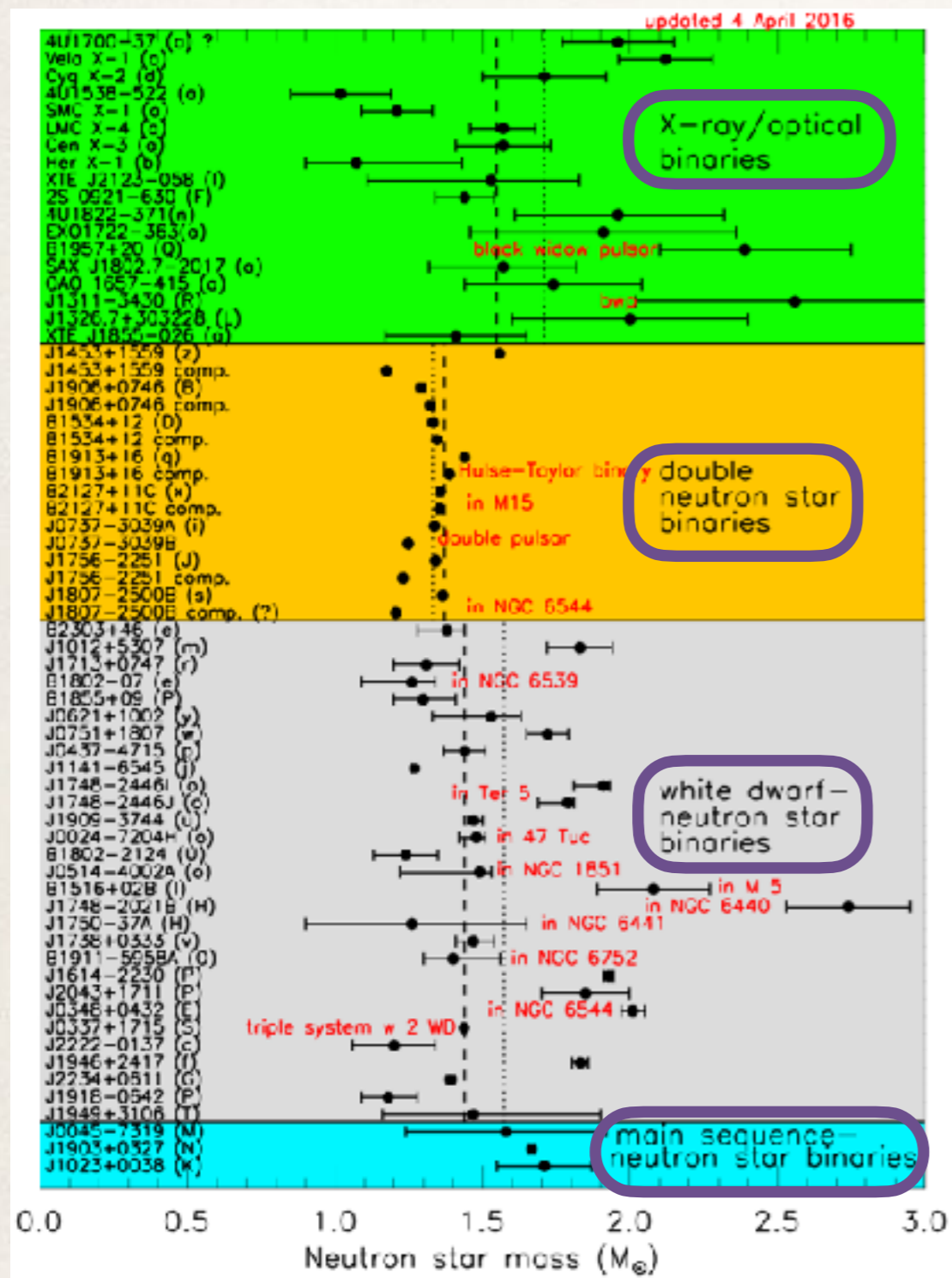


To determine the equation of state $P(\rho)$, one needs to measure M_{NS} and/or R_{NS} .



Credits: N. Wex

Measuring the masses requires neutron stars in binary systems or pulse profile modelling.

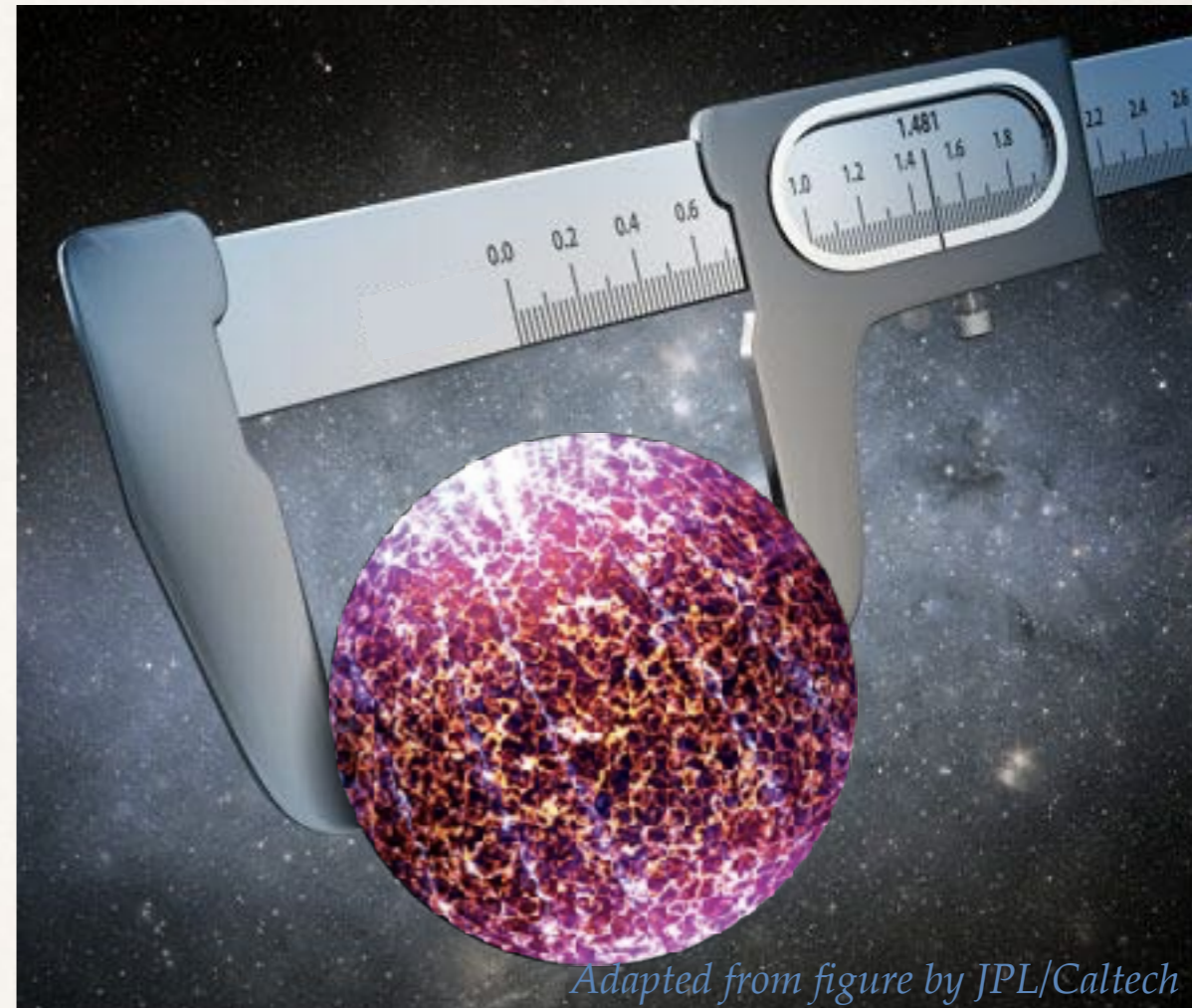


Riley et al. 2019

Measuring the radius precisely is rather difficult for neutron stars.

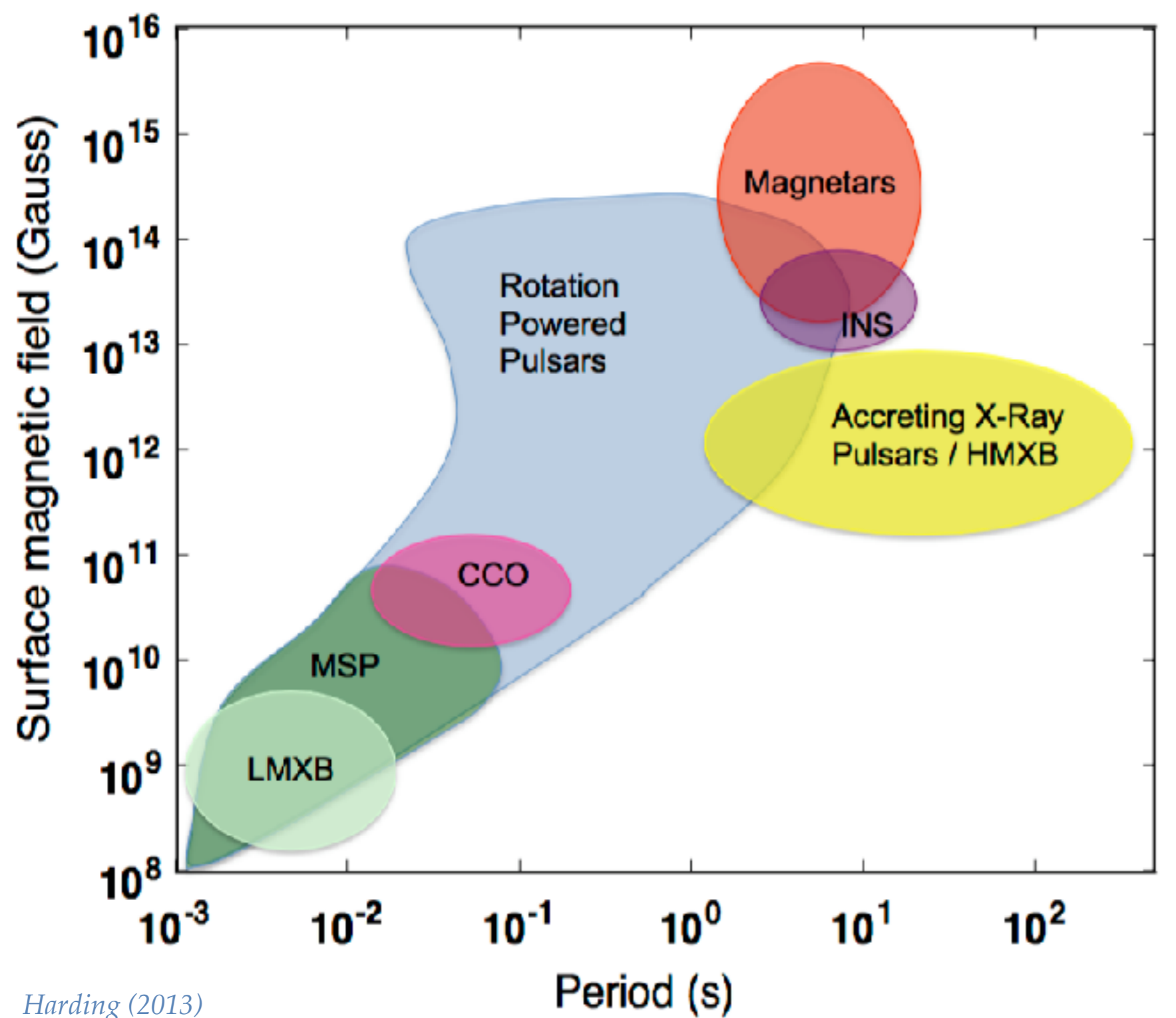
To measure the radius of a star, we need to:

1. observe the surface thermal emission
2. correctly model this emission
3. know the distance

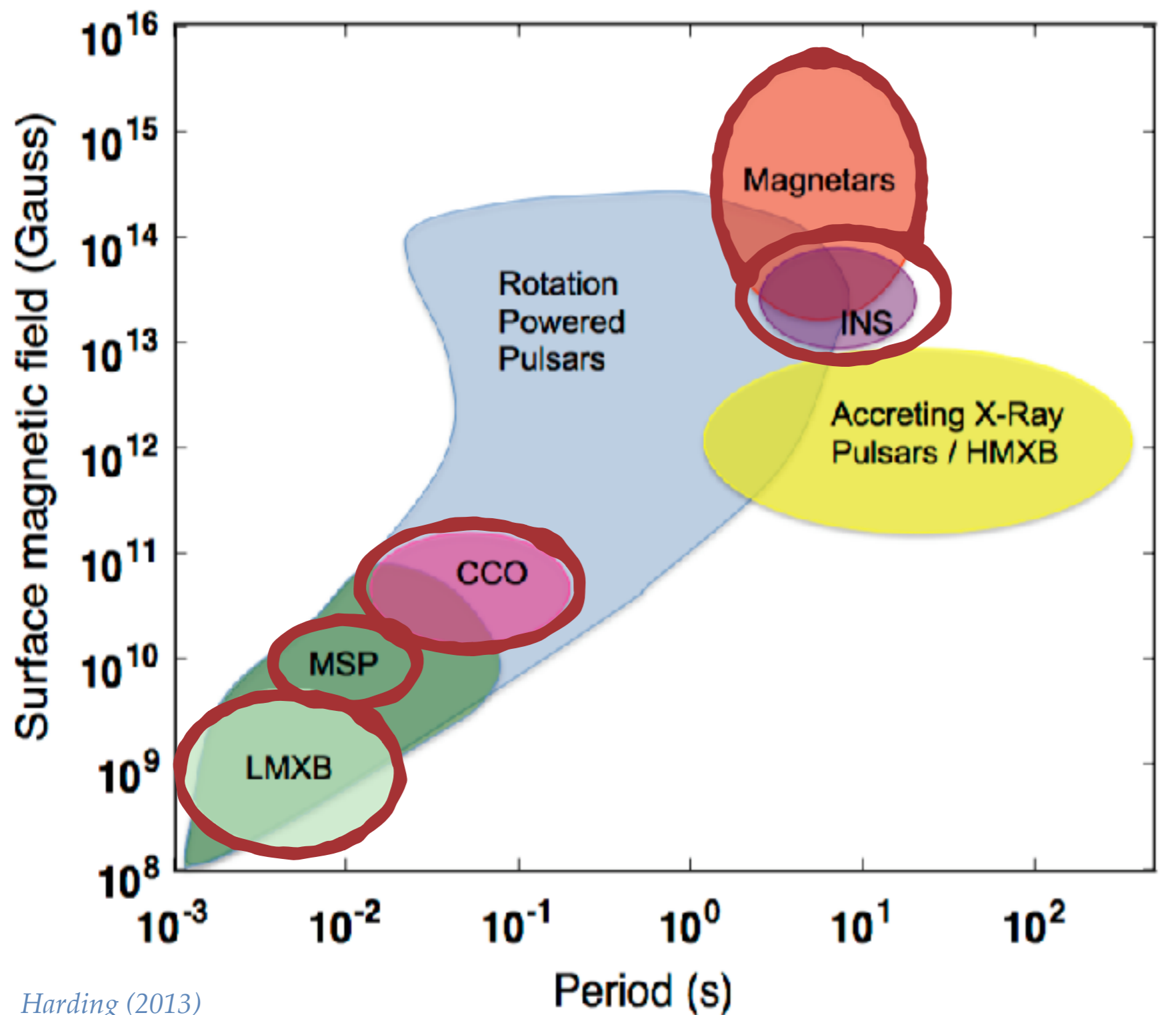


$$L = 4\pi R^2 \sigma T_{\text{eff}}^4 \longrightarrow F = \left(\frac{R}{D} \right)^2 \sigma T_{\text{eff}}^4$$

Neutron stars come in many flavours, with different properties and observational signatures.

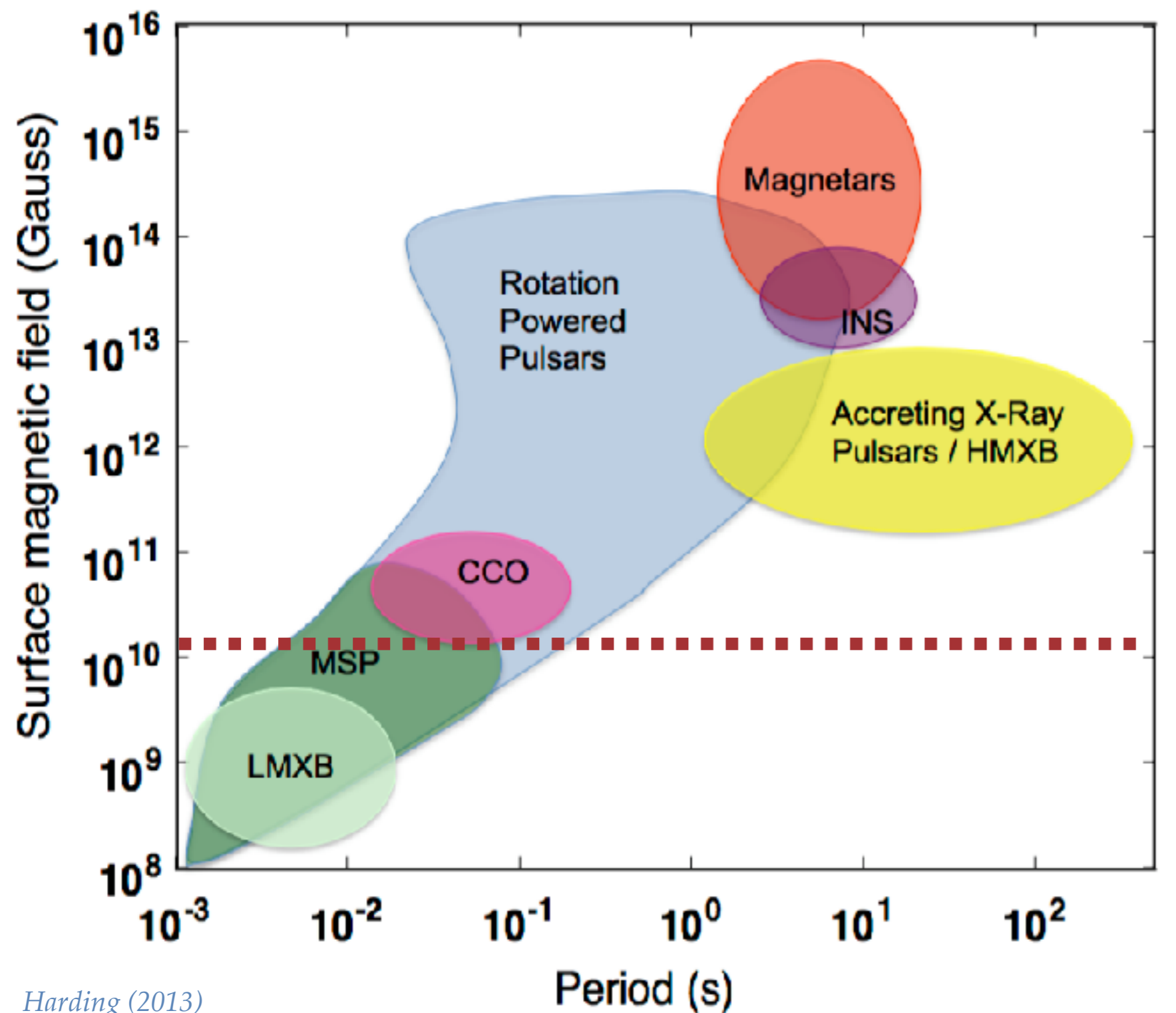


Neutron stars come in many flavours, with different properties and observational signatures.



The emission from the entire surface needs to be visible

Neutron stars come in many flavours, with different properties and observational signatures.

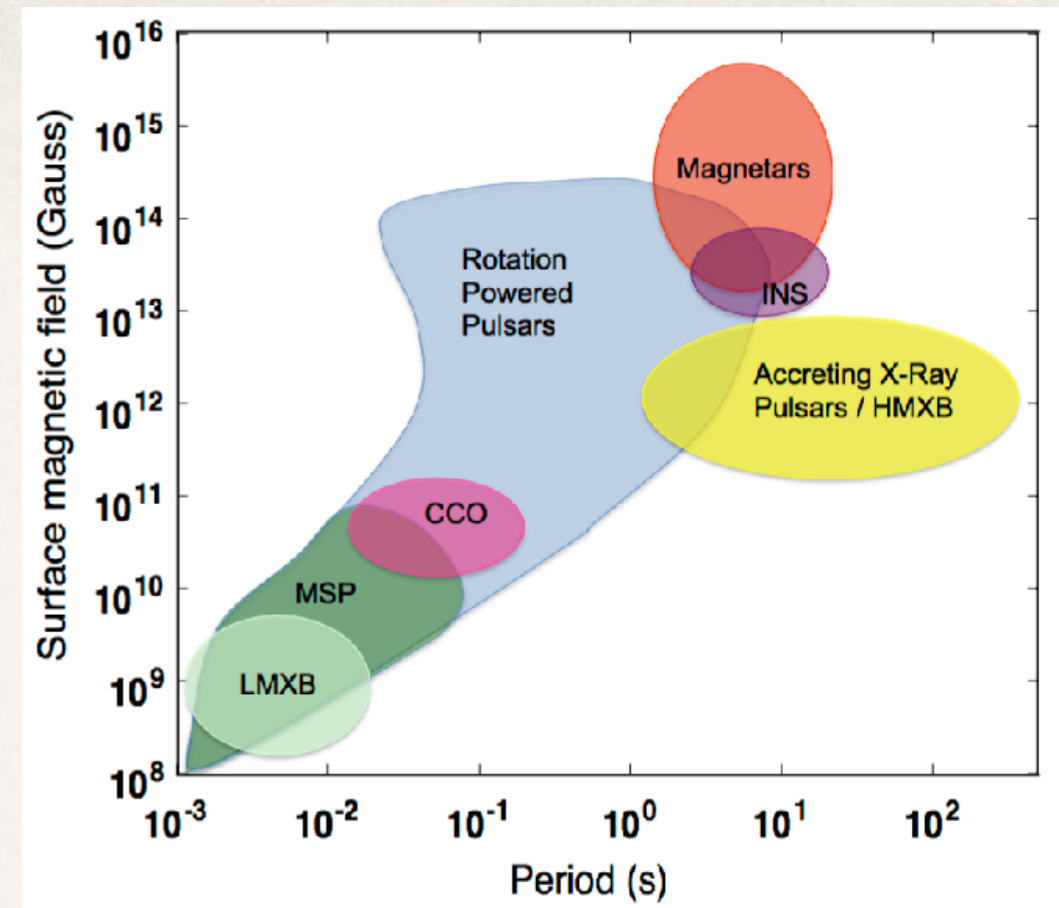


Harding (2013)

Highly magnetised atmospheres are difficult to model

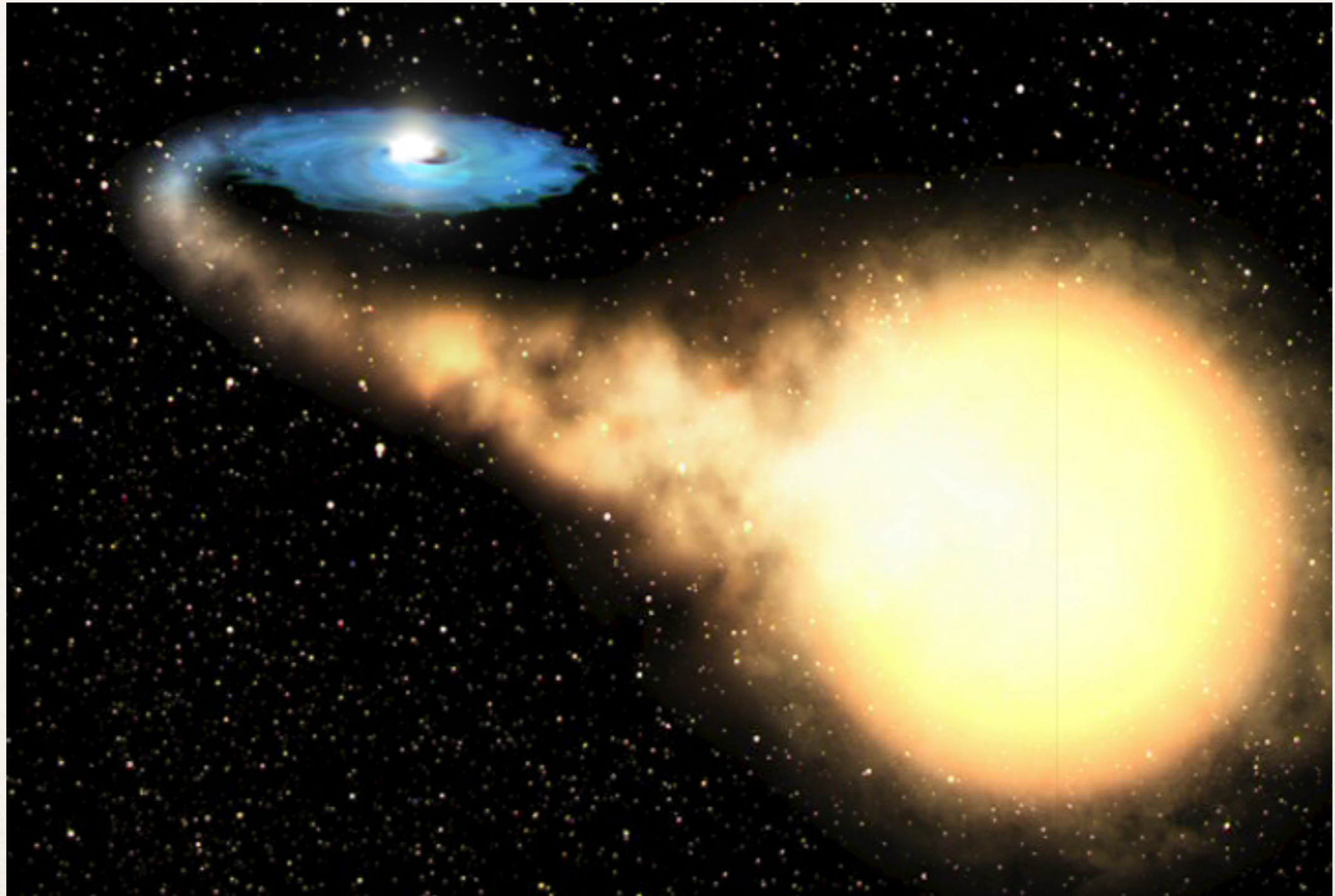
For $B \lesssim 10^{10}$ G, opacities of free-free processes in 10^6 K atmosphere are unaffected

OUTLINE



1. Low mass X-ray binaries in quiescence
2. Millisecond pulsars
3. Thermonuclear bursts in X-ray binaries

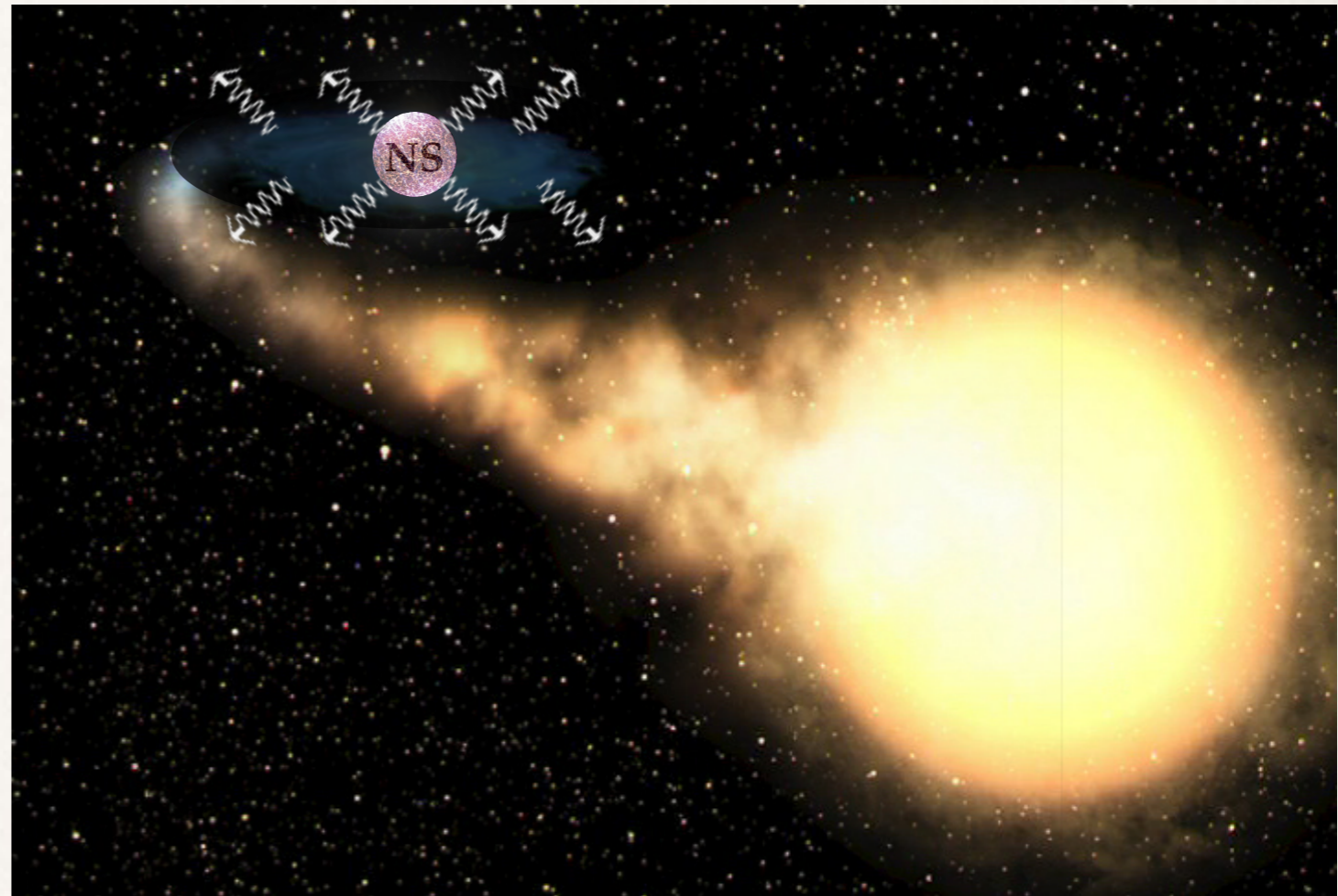
1. We will start with low-mass X-ray binaries.



Quiescent low-mass X-ray binaries are ideal systems for radius measurements.

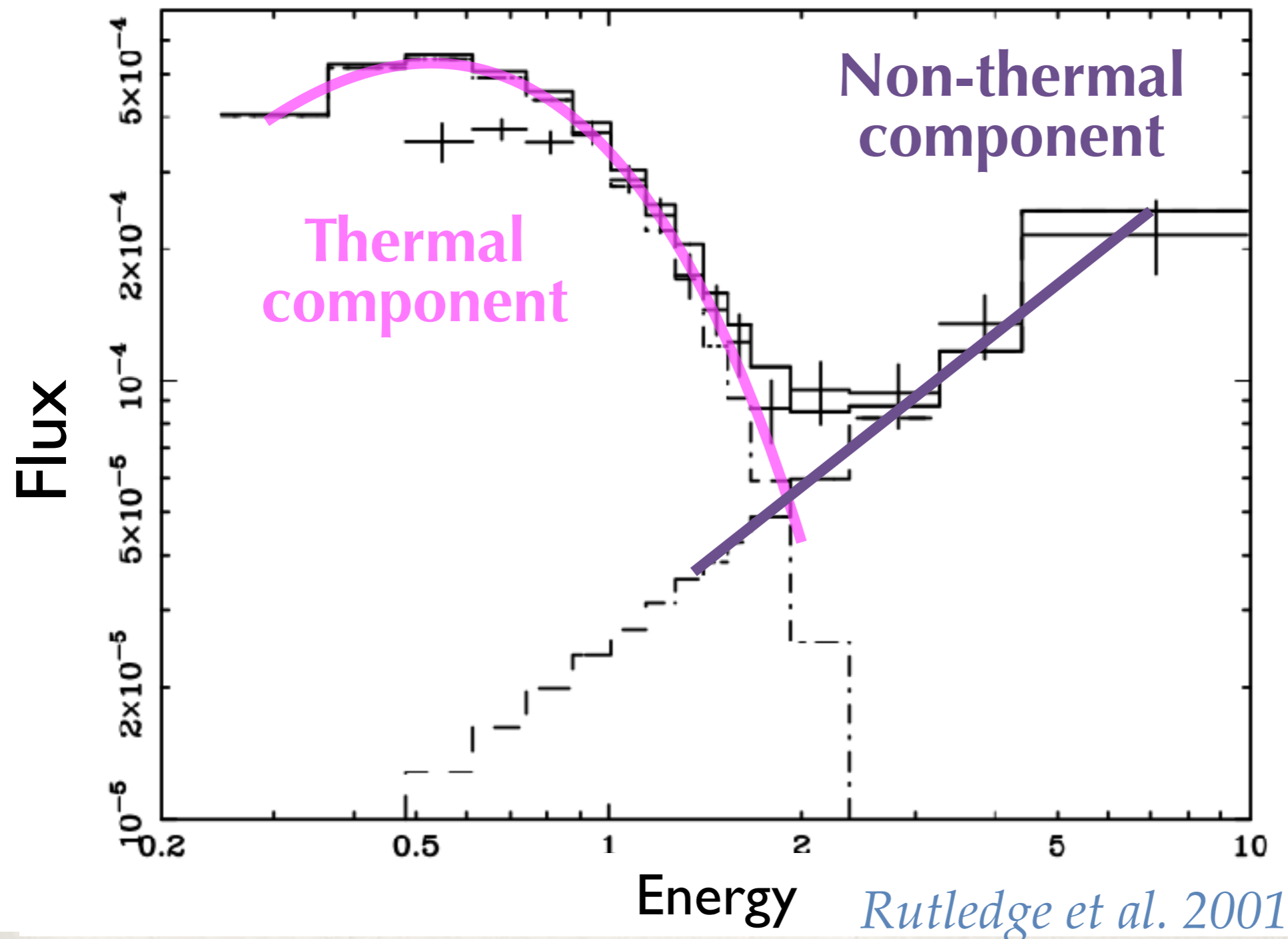
Surface thermal emission at $T_{\text{eff}} \sim 10^6$ K, powered by residual heat from the deep crust radiating outwards through the **atmosphere** with $L_X = 10^{32-33}$ erg/sec

Spectral fitting of this surface emission gives us T_{eff} and $F_X \propto (R_\infty/D)^2$



$$R_\infty = R_{\text{NS}} (1 + z) = R_{\text{NS}} \left(1 - \frac{2GM_{\text{NS}}}{R_{\text{NS}} c^2} \right)^{-1/2}$$

A radius measurement was obtained from Cen X-4 (a known field LMXB) observed during quiescence.

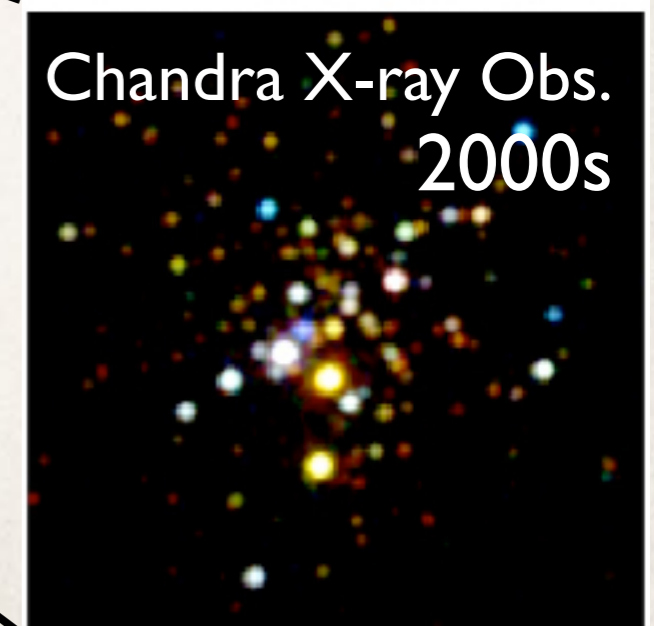
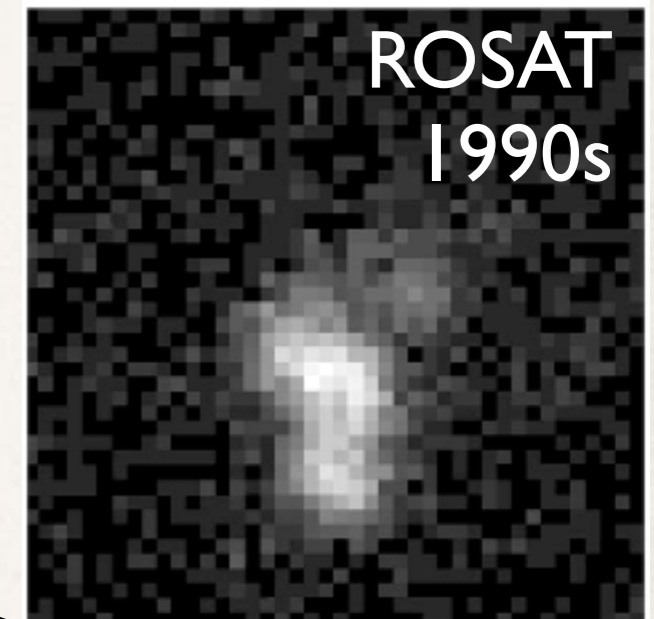
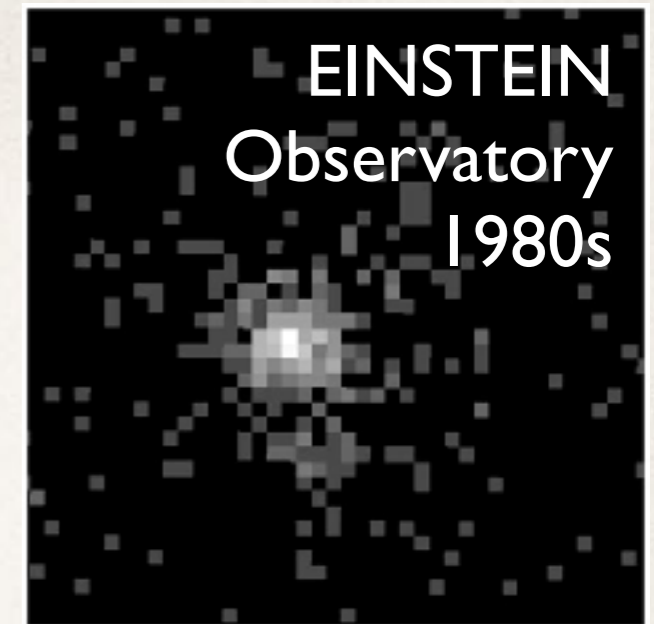
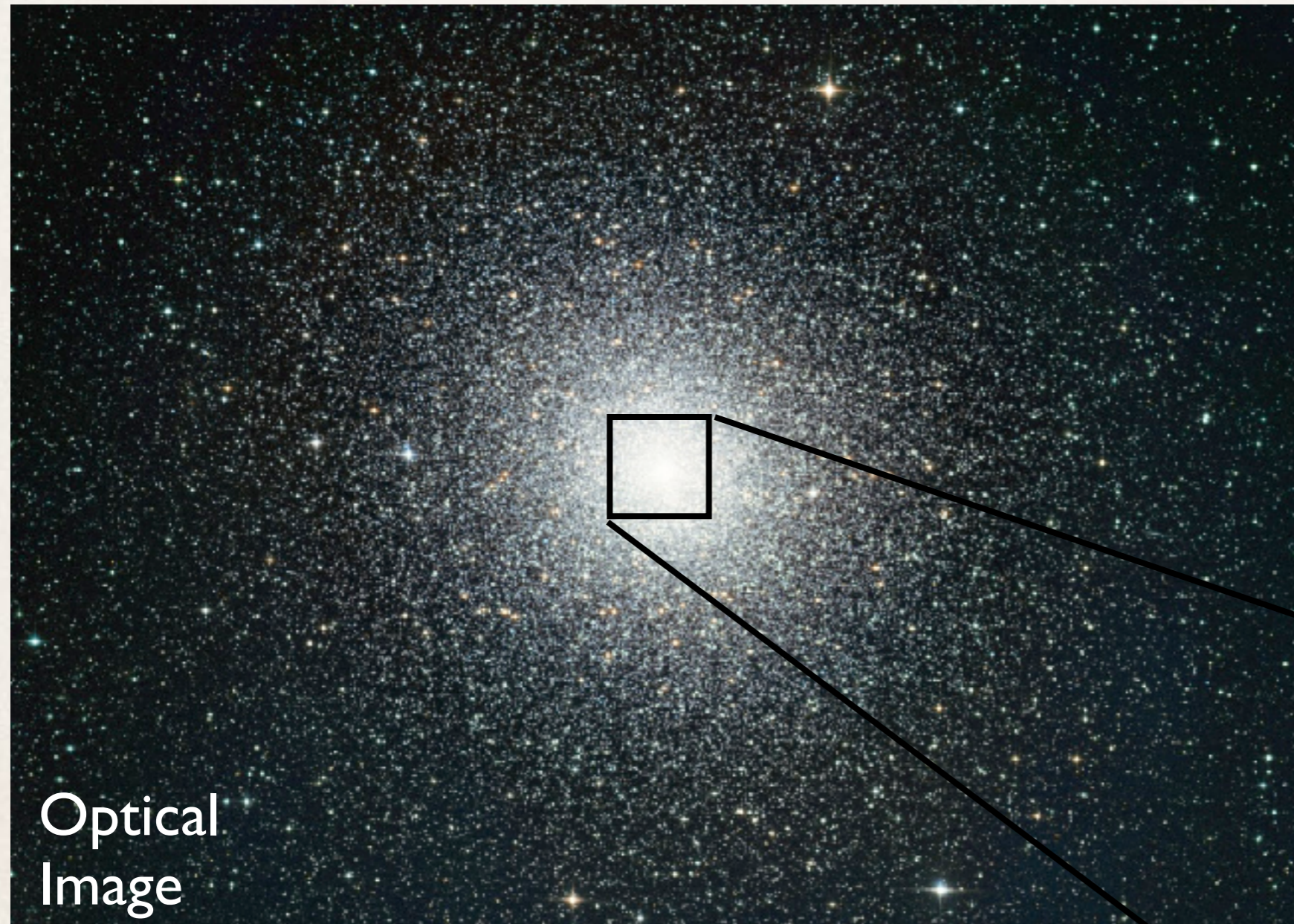


BUT

Possible evidence of residual accretion

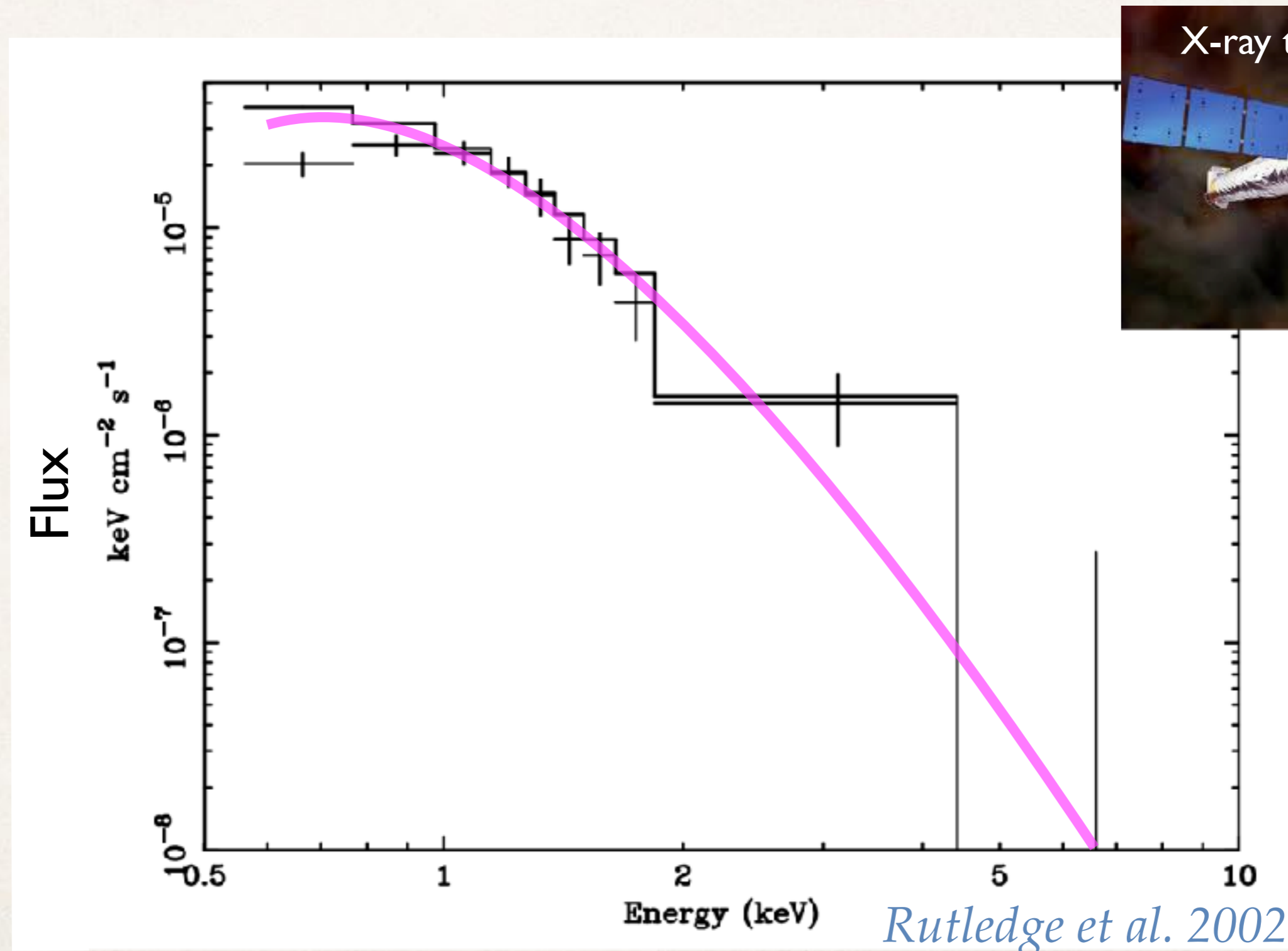
Distance poorly constrained

Globular clusters host an overabundance of LMXB systems...

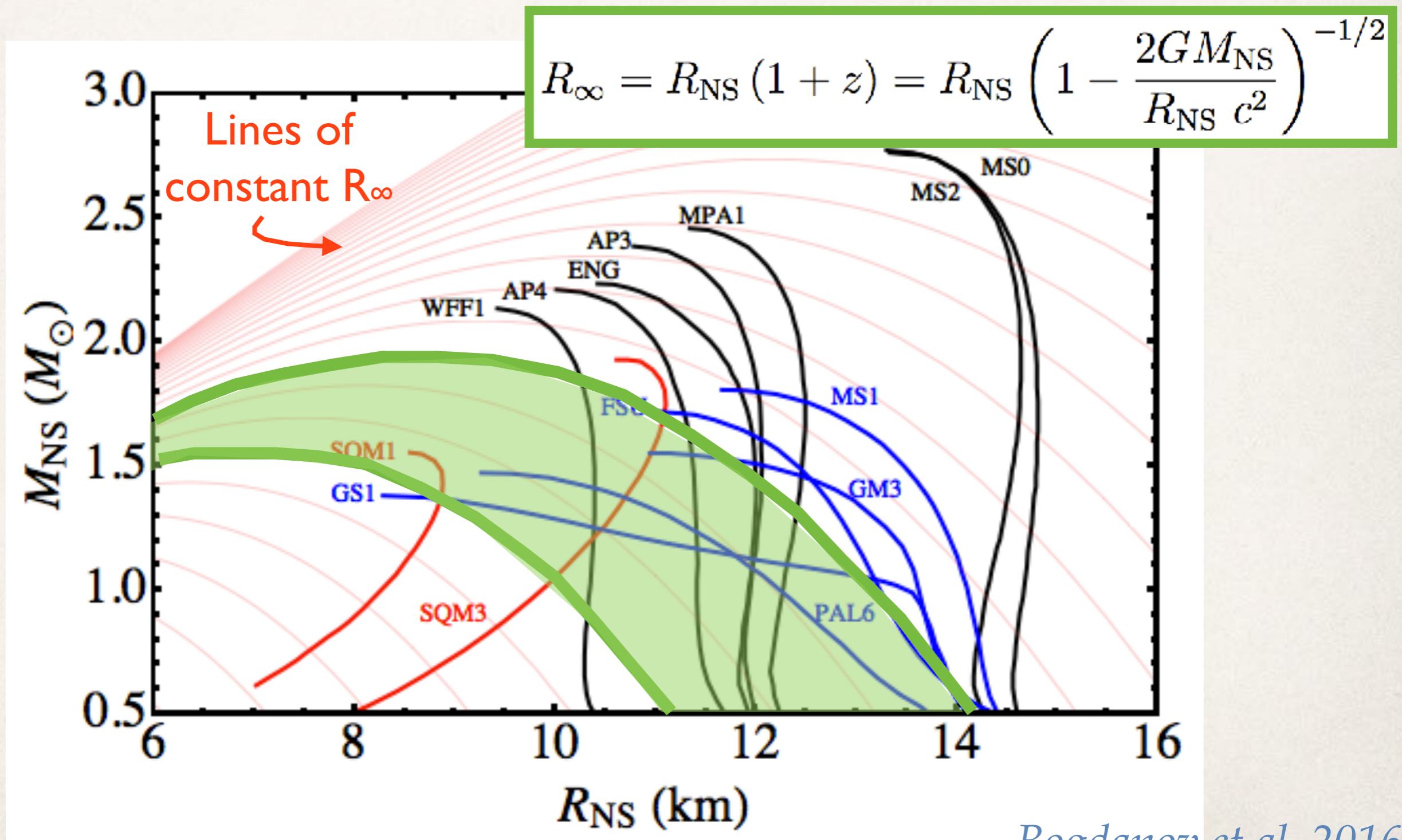


...and they have independently measured distances.

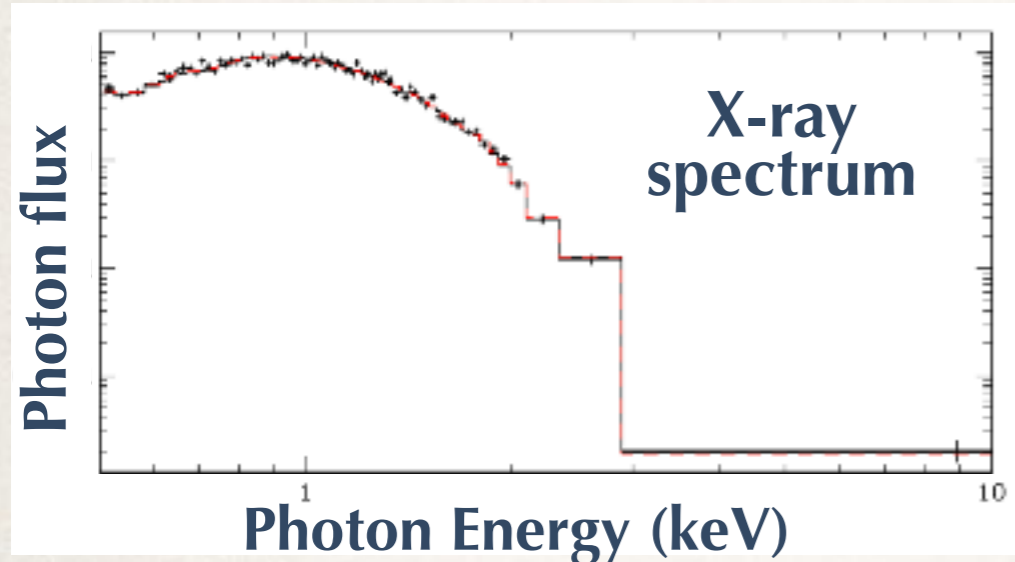
The first globular cluster qLMXB was discovered in Omega Centauri.



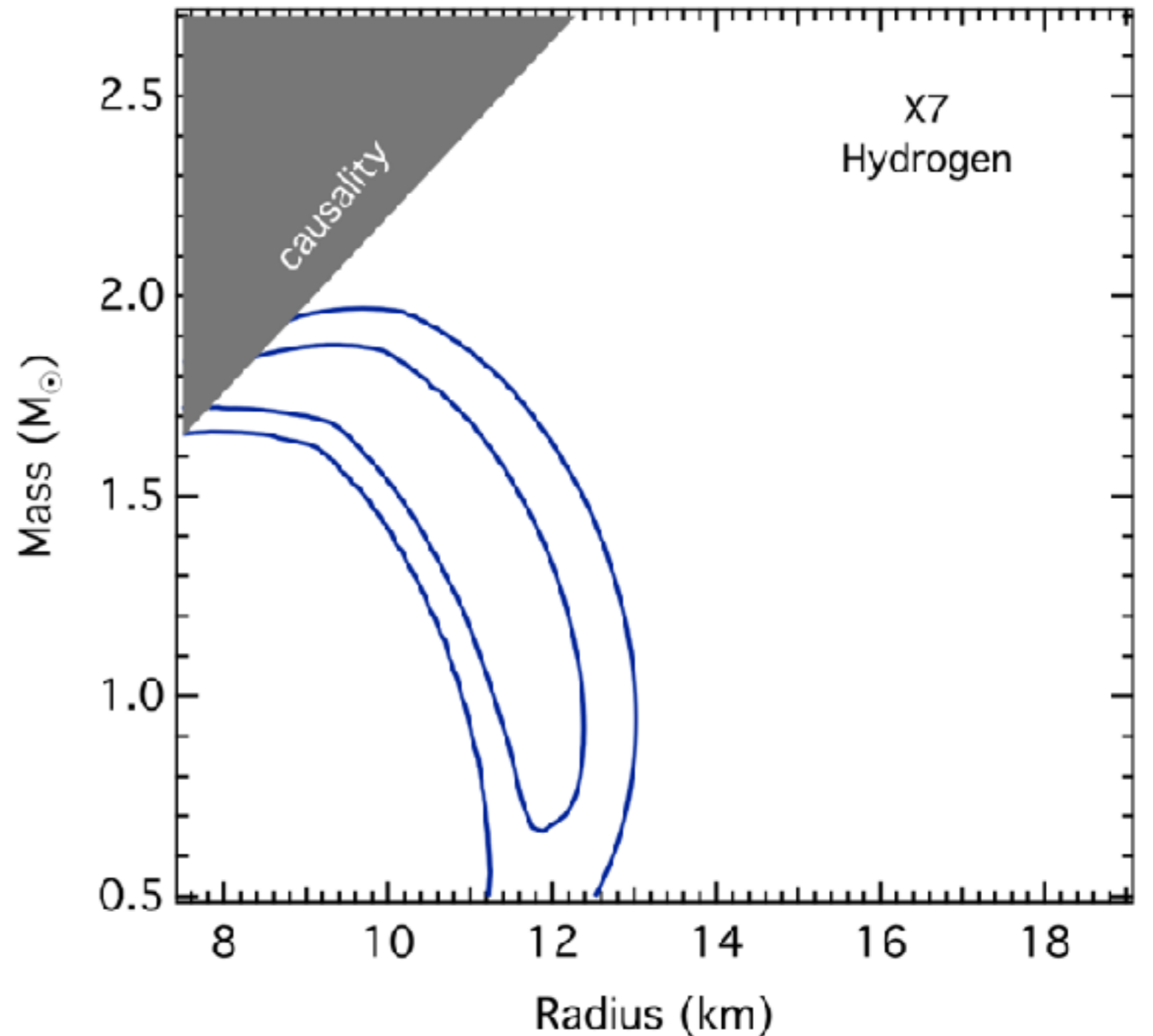
Because of gravitational redshift, the radius is degenerate with the unknown mass.



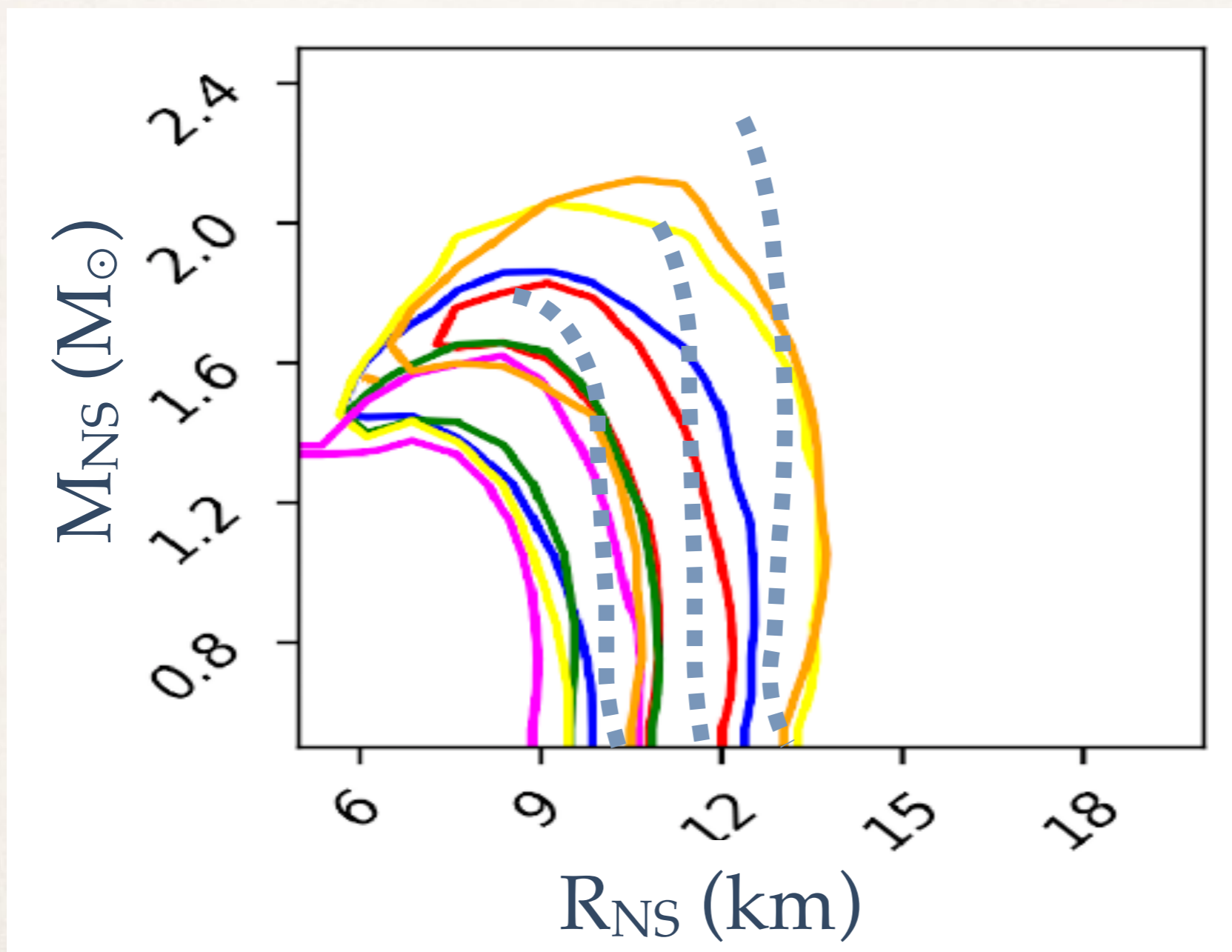
Because of gravitational redshift, the radius is degenerate with the unknown mass.



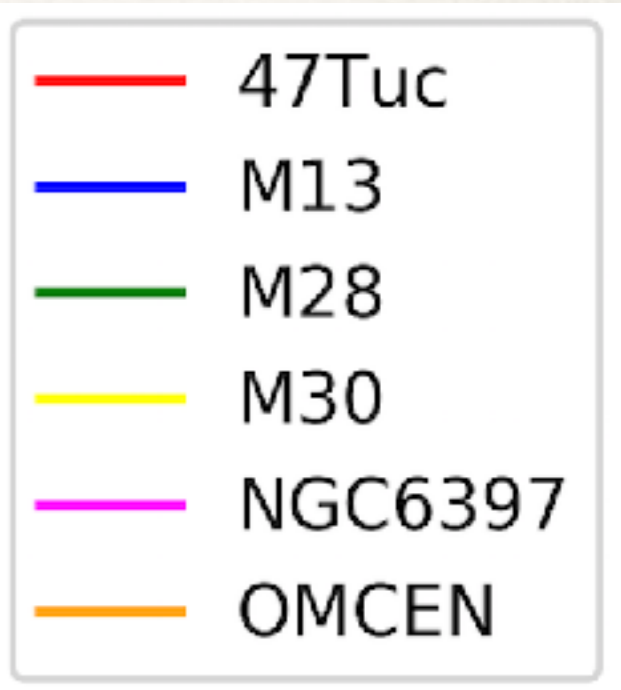
$$L \sim 4\pi \left(\frac{R_\infty}{d} \right)^2 \sigma T_{\text{eff},\infty}^4$$



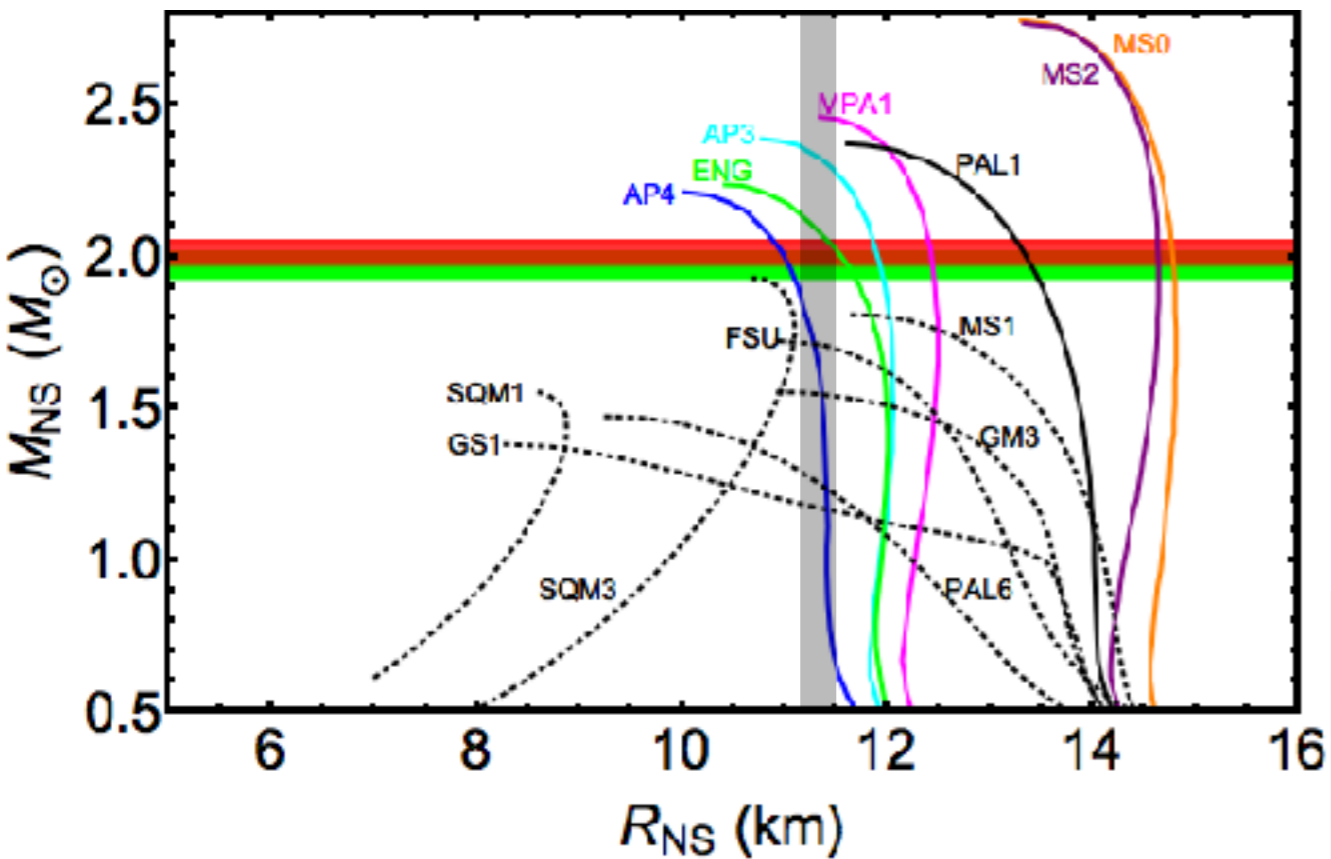
We want to find which equation of state is common to all these M-R measurements.



qLMXBs in
globular
clusters



A solution consists in combining these observations in a statistical analysis.



Constant R_{NS}

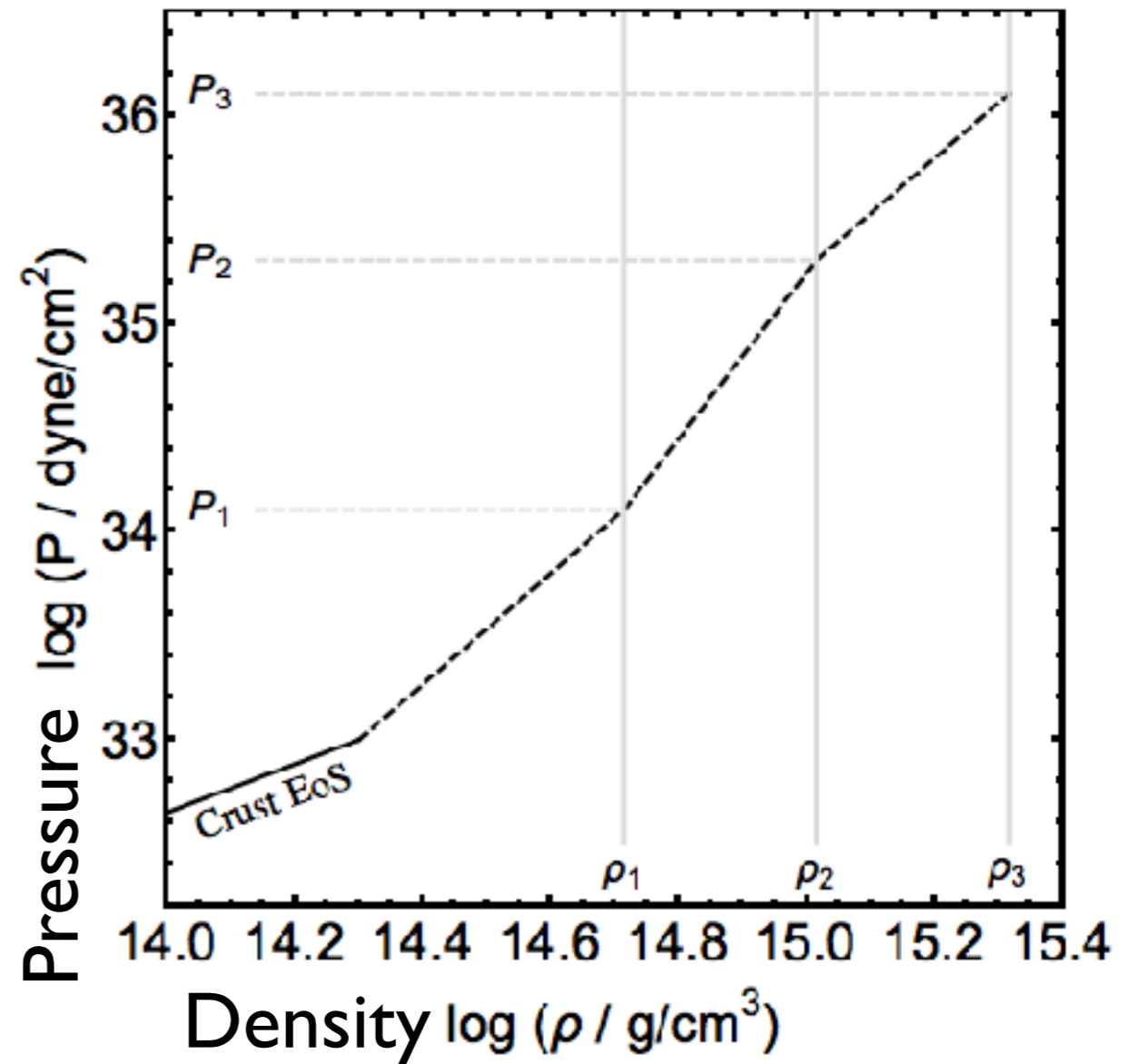
i.e., the radius is the same for all neutron stars

Guillot et al. 2013

Guillot & Rutledge 2014

Guillot 2016

Analytical
parameterizations



Piecewise polytropes

Read et al. 2009

Özel et al. 2016

We used a physically-driven, parameterisation of the equation of state is preferable.

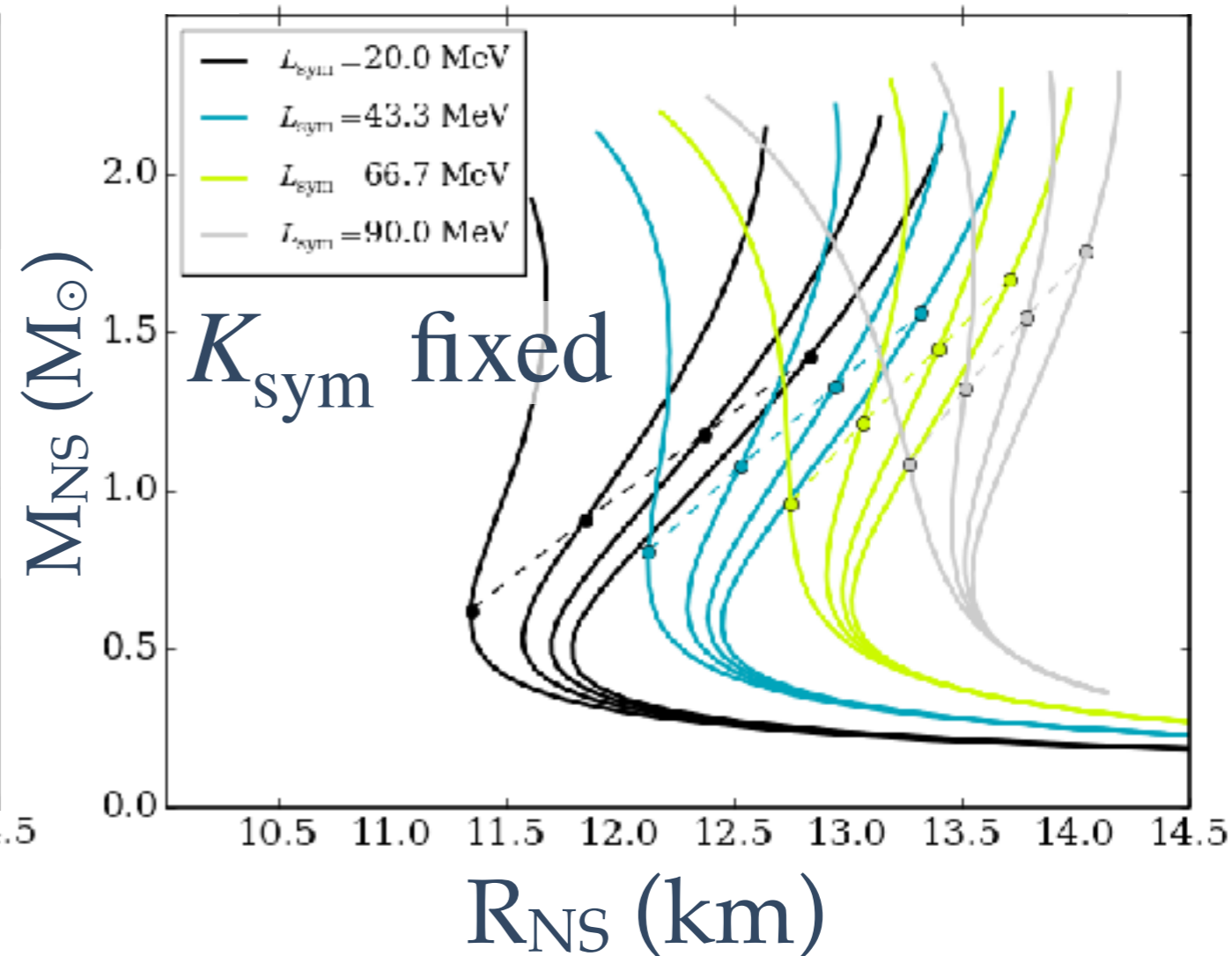
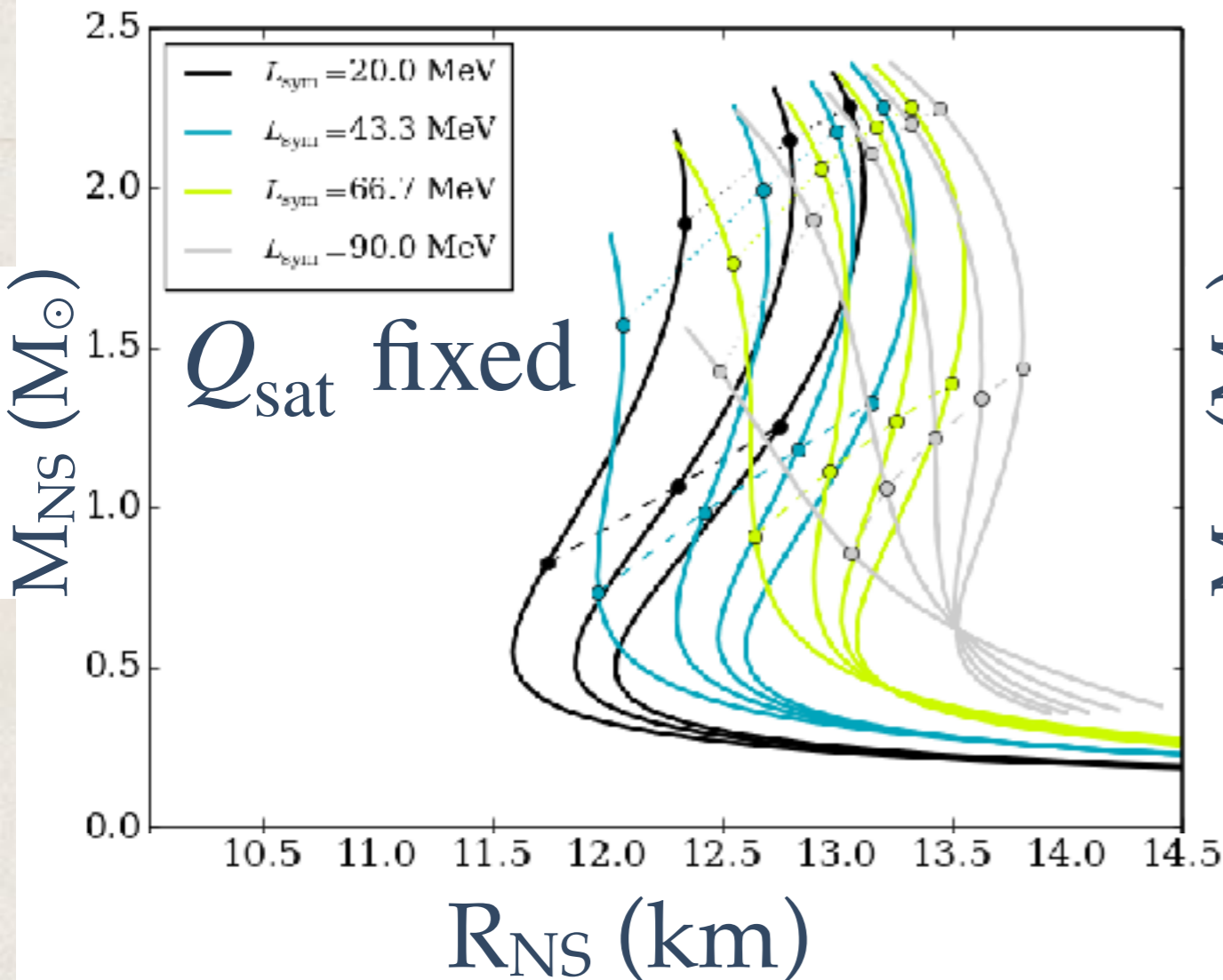
Meta-model of
J. Margueron et al.

$$e_{\text{sat}} = E_{\text{sat}} + \frac{1}{2} K_{\text{sat}} x^2 + \frac{1}{3!} Q_{\text{sat}} x^3 + \frac{1}{4!} Z_{\text{sat}} x^4 + \dots$$

$$e_{\text{sym}} = E_{\text{sym}} + L_{\text{sym}} x + \frac{1}{2} K_{\text{sym}} x^2 + \frac{1}{3!} Q_{\text{sym}} x^3 + \frac{1}{4!} Z_{\text{sym}} x^4 + \dots$$

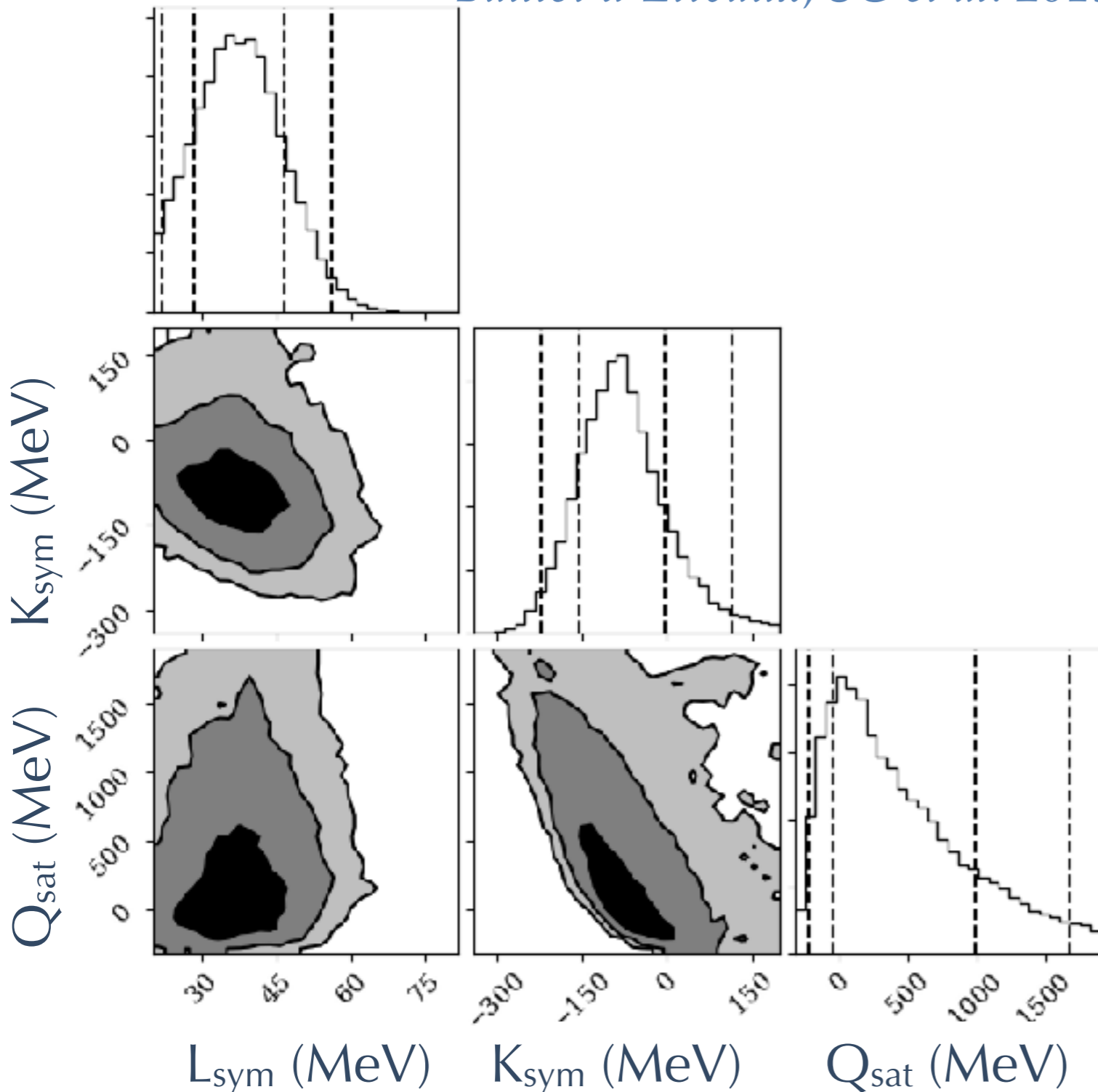
with

$$x = \frac{n - n_{\text{sat}}}{3n_{\text{sat}}}$$



The measurements of these three parameters improve over previous estimates.

Baillet d'Etivaux, SG et al. 2019



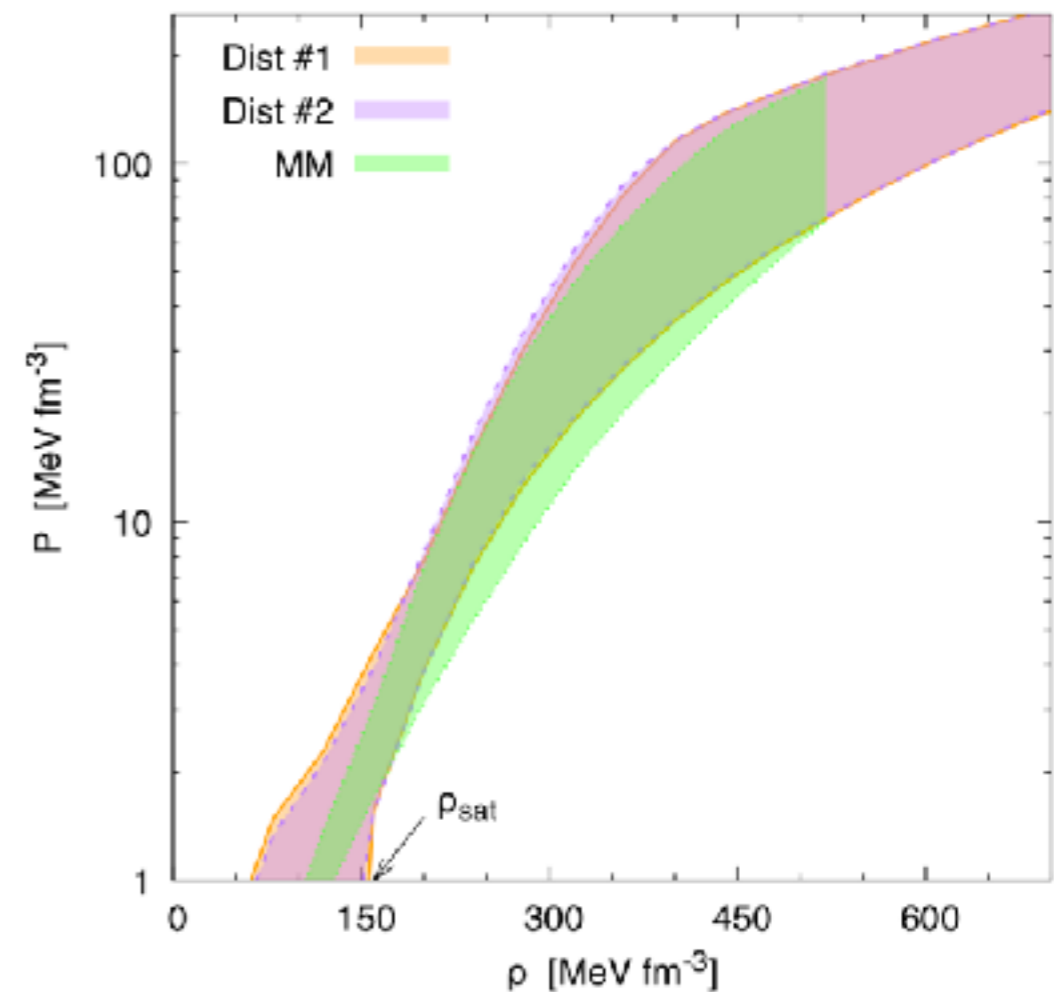
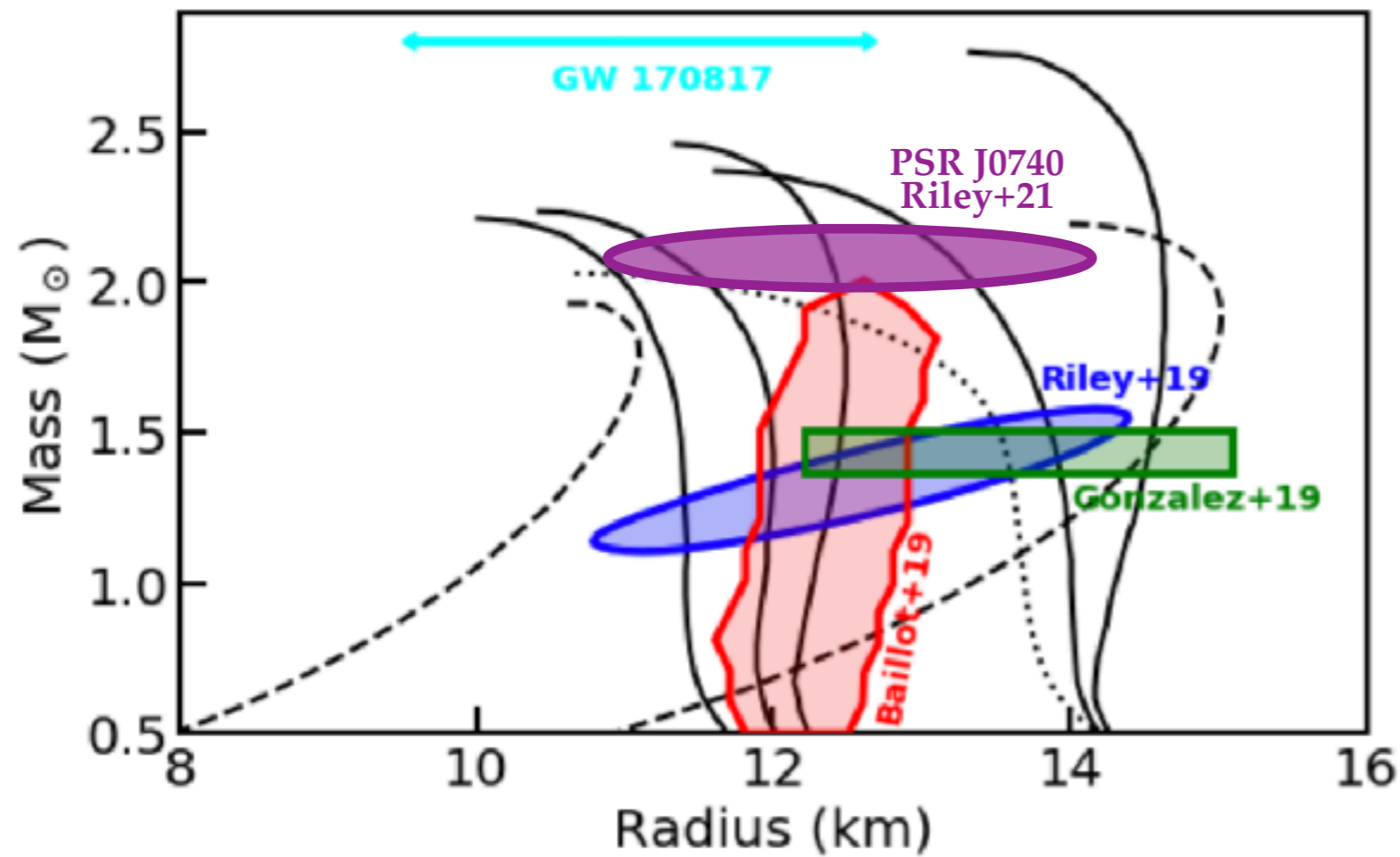
Our 2σ measurements:

- ◆ $L_{\text{sym}} \sim 25 - 60 \text{ MeV}$
- ◆ $K_{\text{sym}} \sim -250 - 130 \text{ MeV}$
- ◆ $Q_{\text{sat}} \sim -200 - 1900 \text{ MeV}$

Ranges of value from experimental and theoretical estimates:

- ◆ $L_{\text{sym}} \sim 20 - 90 \text{ MeV}$
- ◆ $K_{\text{sym}} \sim -400 - 200 \text{ MeV}$
- ◆ $Q_{\text{sat}} \sim -1300 - 1900 \text{ MeV}$

Our analysis results in $M_{NS}-R_{NS}$ or in $P-\rho$ space are consistent with other measurements.

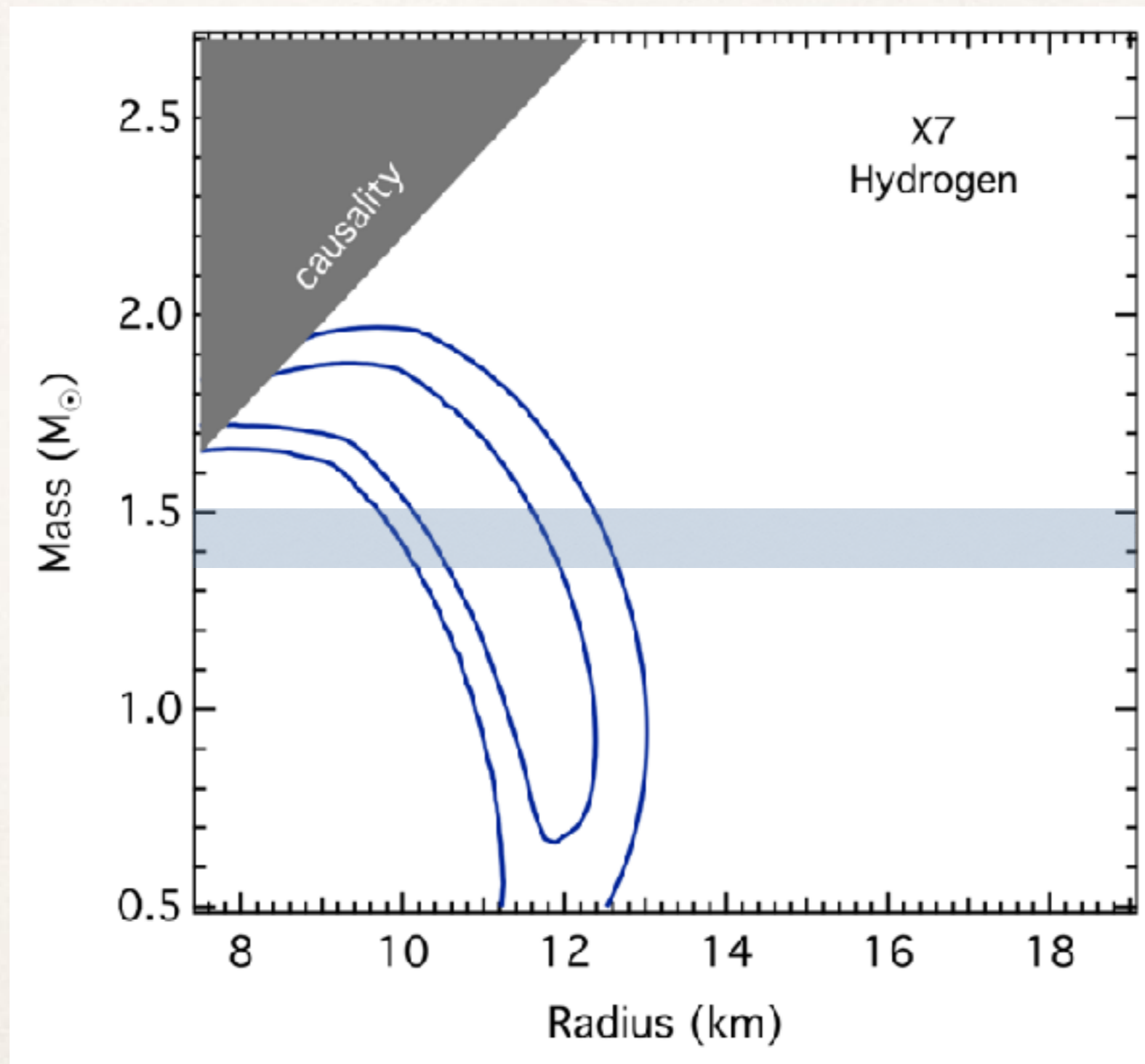


There remains some discussion points and possible caveats!



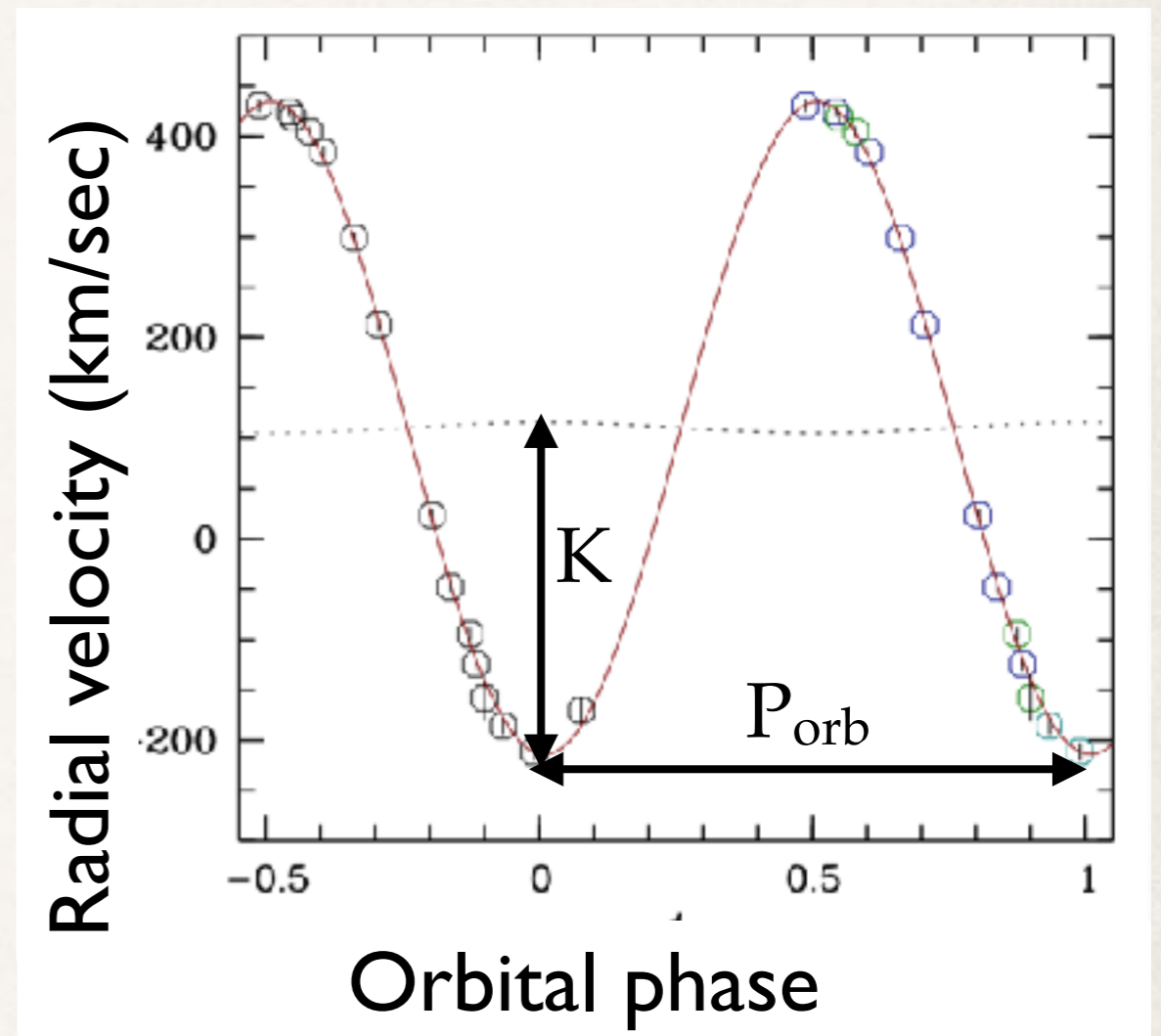
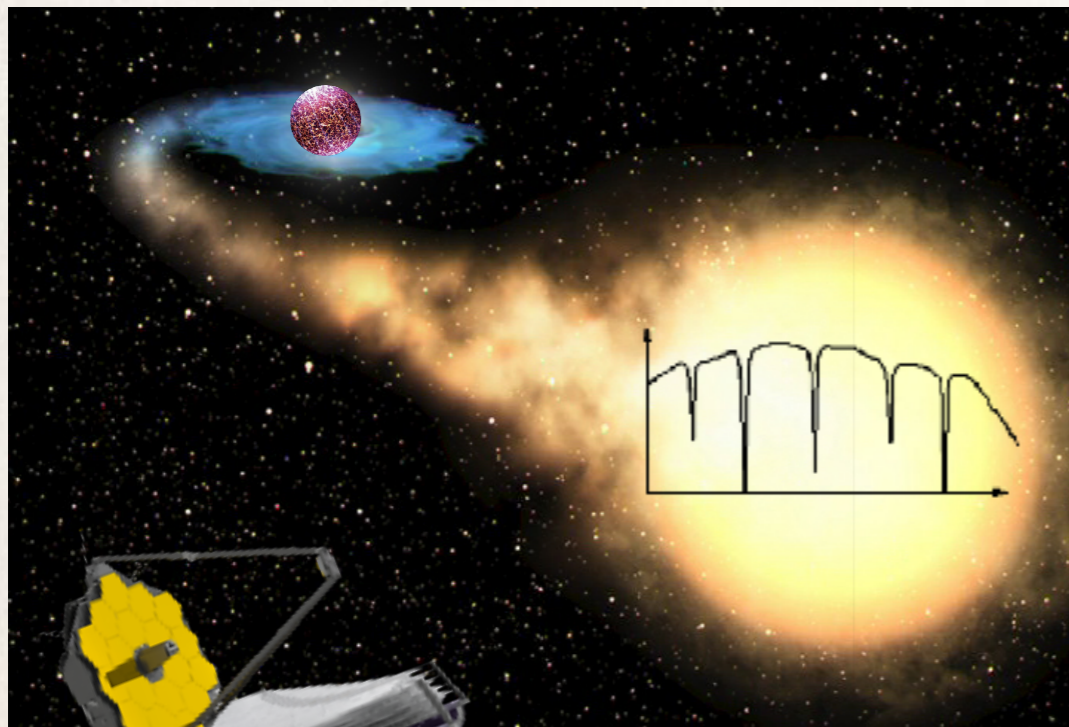
- ◆ Why only use qLMXBs in globular clusters ? → Field LMXB may not return to full quiescence
- ◆ What is the composition of the neutron star atmosphere ? → Hydrogen, Helium or something else
- ◆ Is the surface magnetic field really negligible ? → No measurement, but expected for LXMBs
- ◆ Is the emission really from the entire surface ? → No constraints exist, but ...
- ◆ What are the effects of assuming slowly rotating neutron stars? → Fast rotation may bias the R measurement

Another question: Could we measure the mass to break the M-R degeneracy?



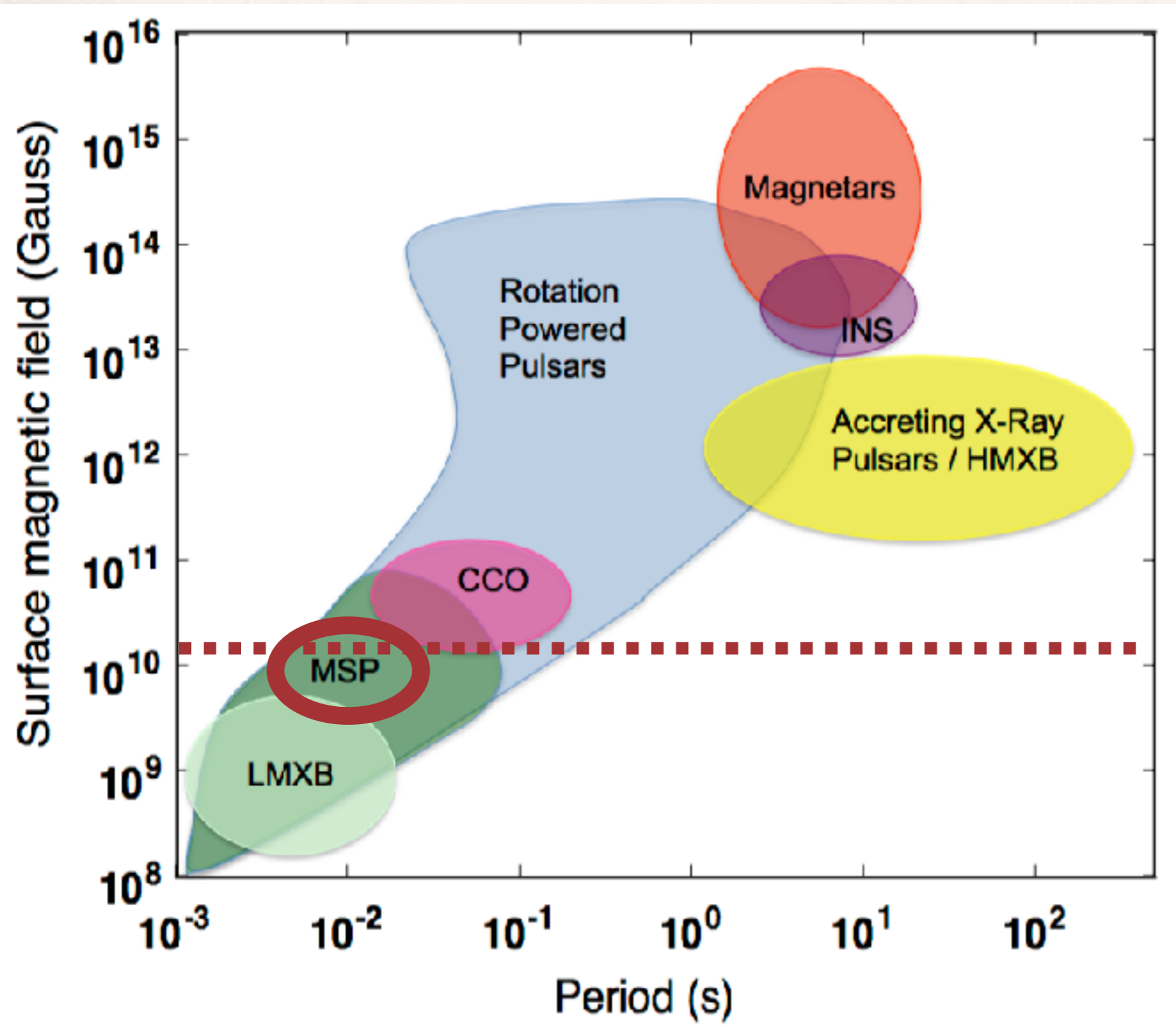
This requires identifying the companion star and determining the orbital parameters

Observing the binary companion to the NS



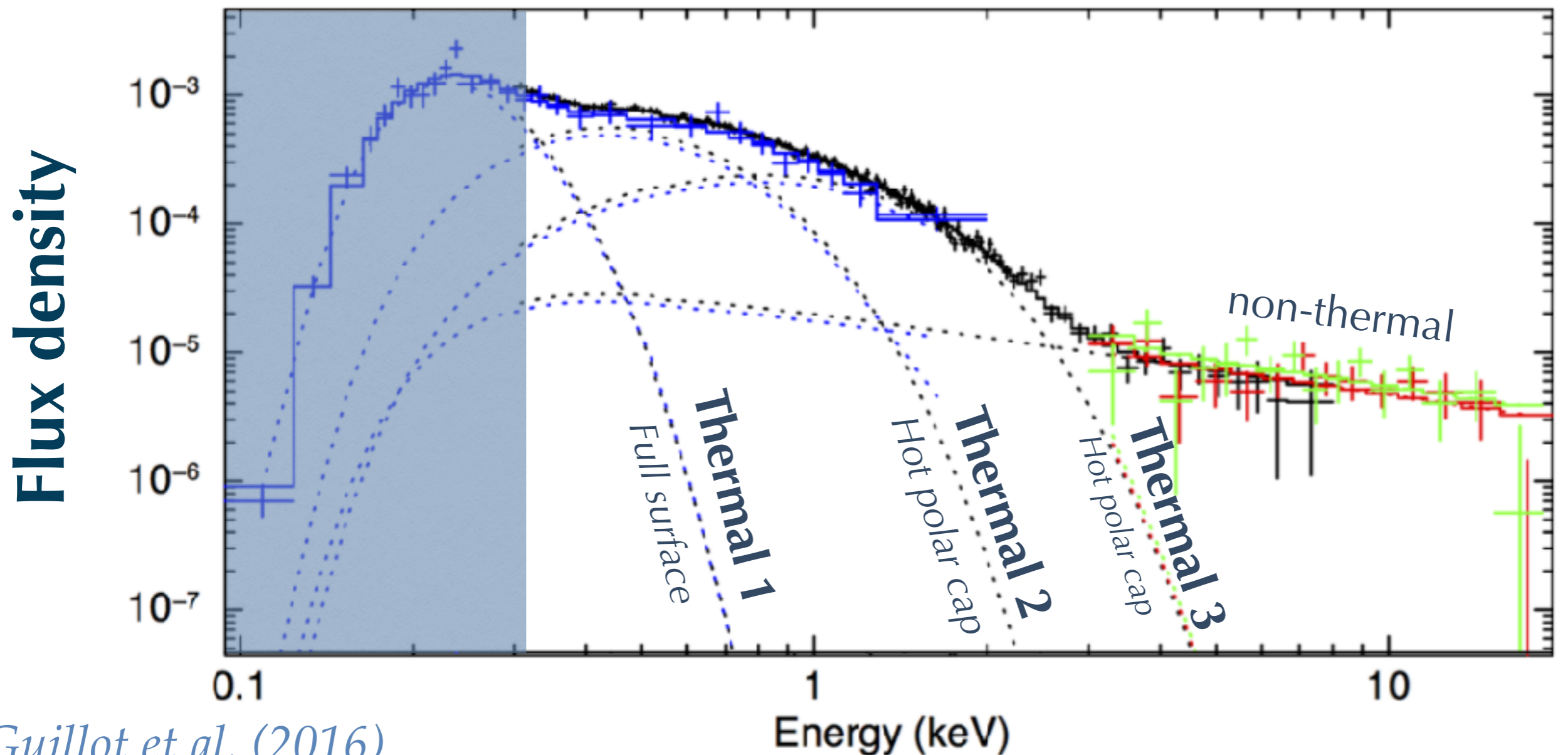
$$\frac{M_{\text{NS}}^3 \sin^3 i}{(M_{\text{NS}} + M_{\text{comp}})^2} = \frac{P_{\text{orb}} K^3}{2\pi G}$$

2. Rotation powered millisecond pulsars also have thermal surface emission.

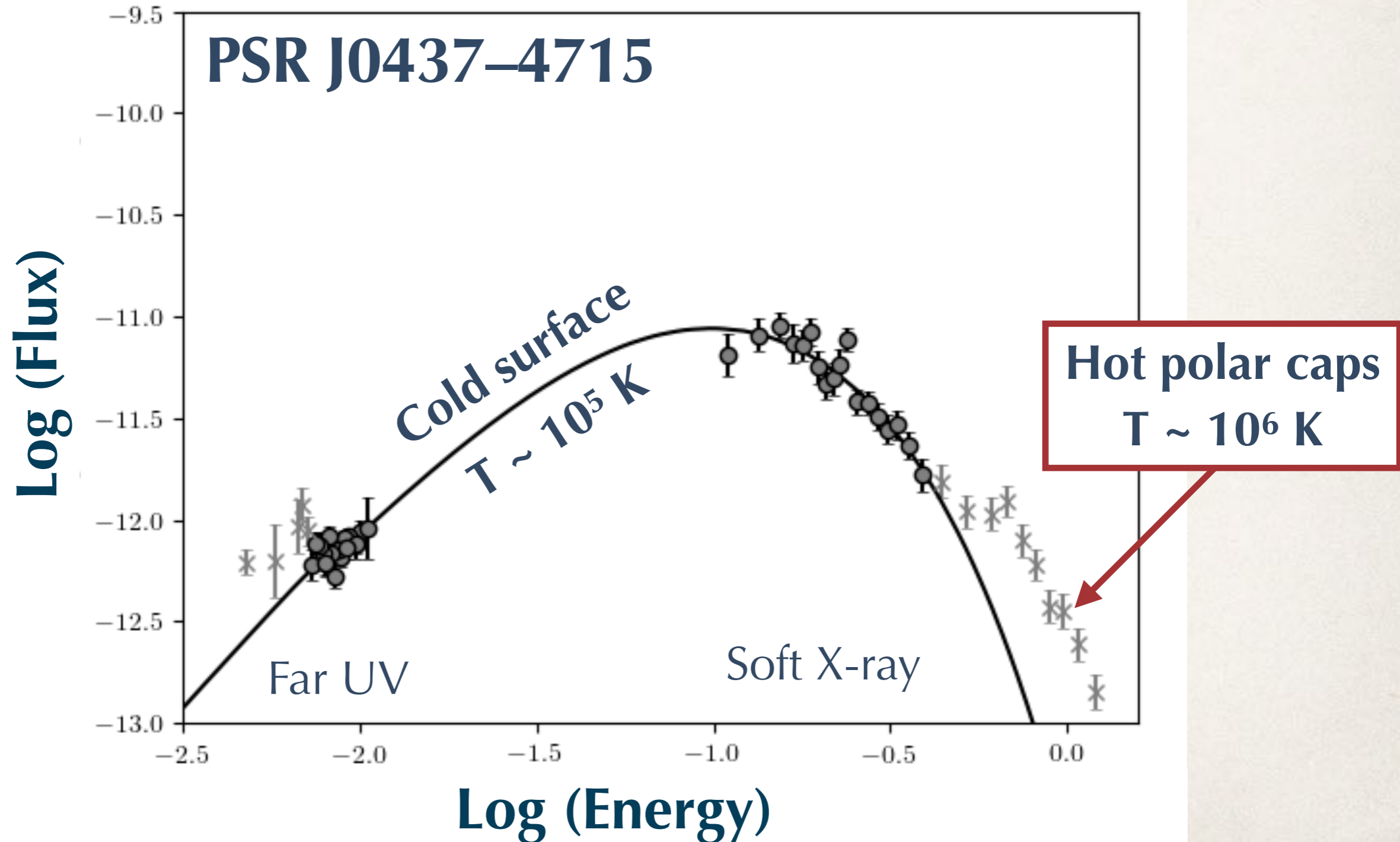


The surface emission from rotation-powered MSPs with known masses can also be used to extract the radius with phase-average spectroscopy.

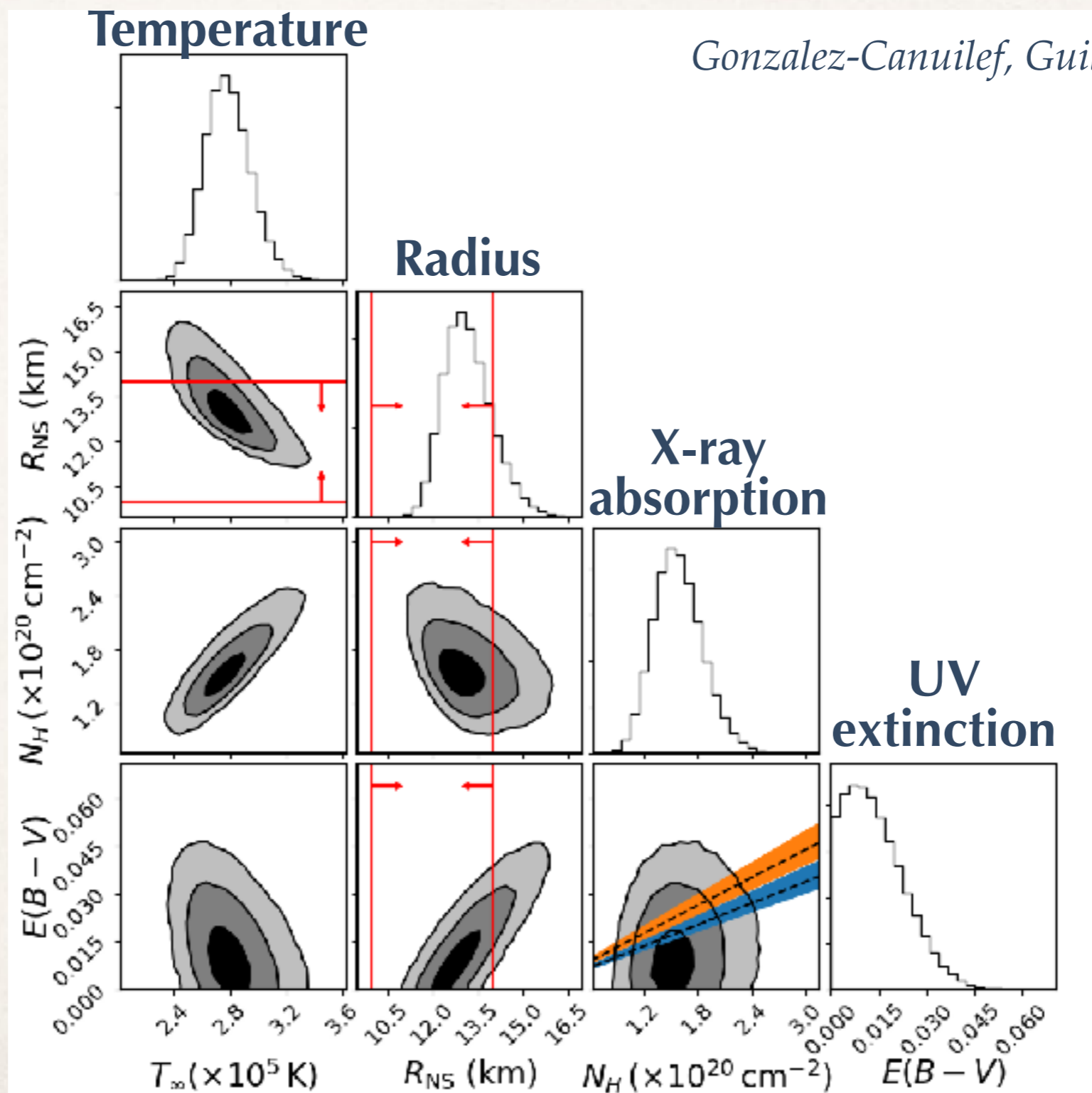
The nearest MSP PSR J0437-4715



In the far UV, the Rayleigh–Jeans tail of the surface thermal emission gives the handle to constrain the neutron star size.

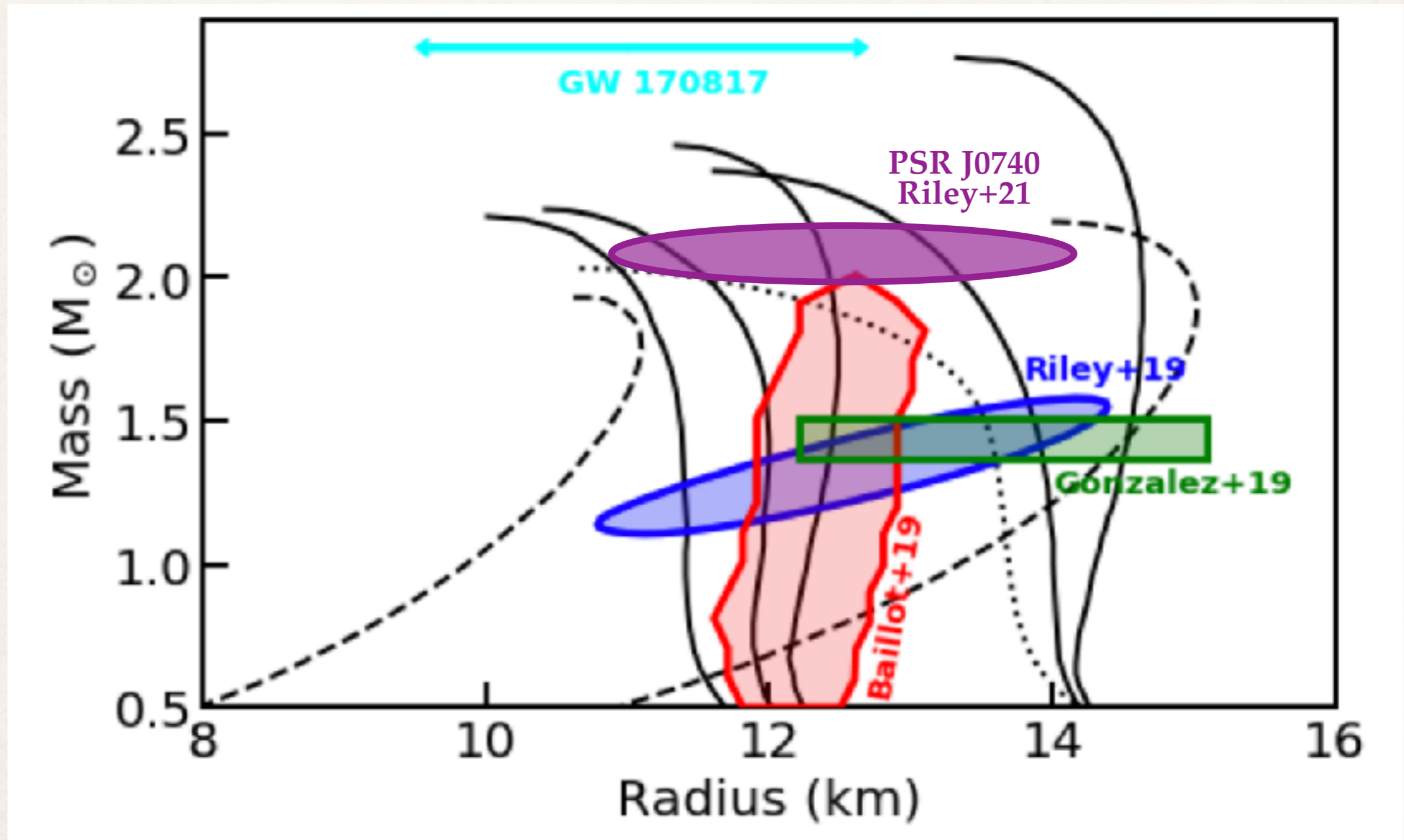


In the far UV, the Rayleigh–Jeans tail of the surface thermal emission gives the handle to constrain the neutron star size.



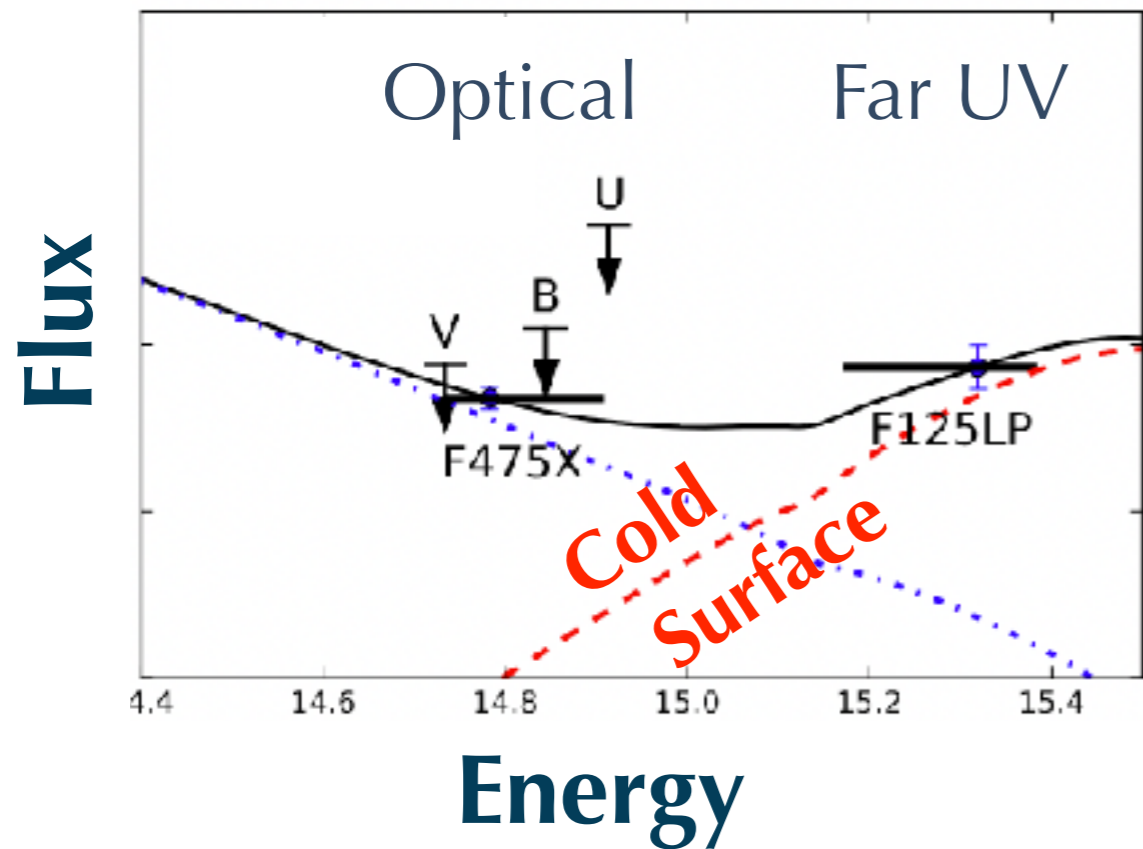
Gonzalez-Canuilef, Guillot & Reisenegger, 2019

We find a radius for **PSR J0437-4715** consistent with other measurements.

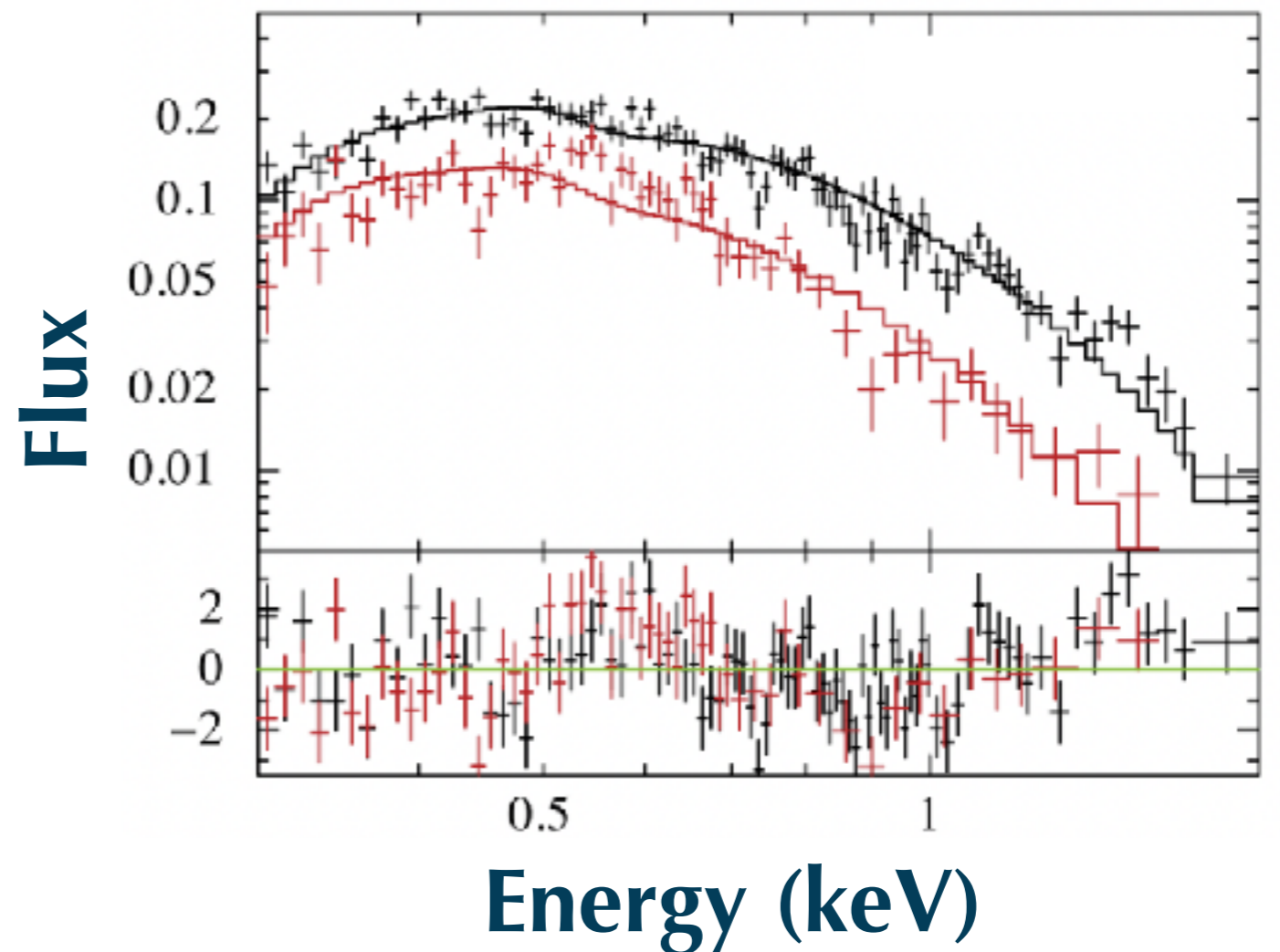


This method may be applicable to another MSP with observed far UV emission.

PSR J2124–3358

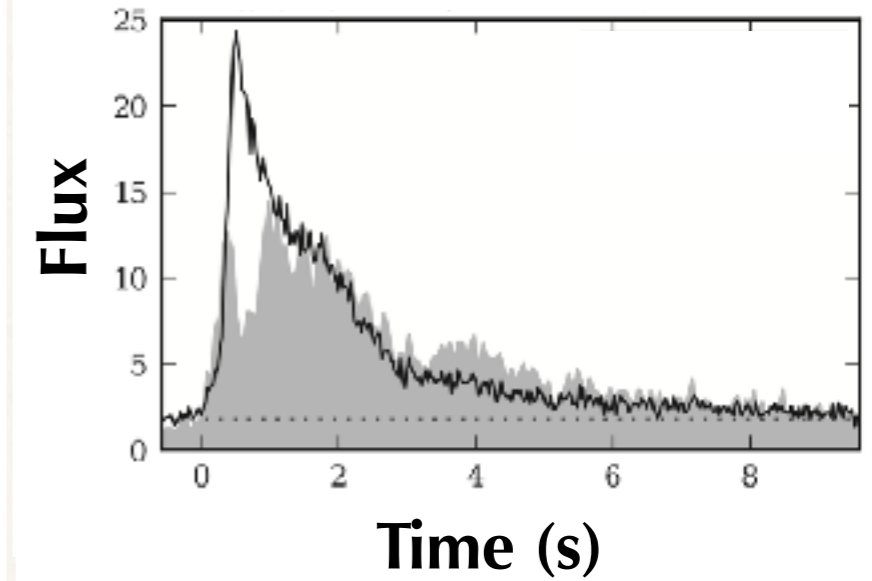
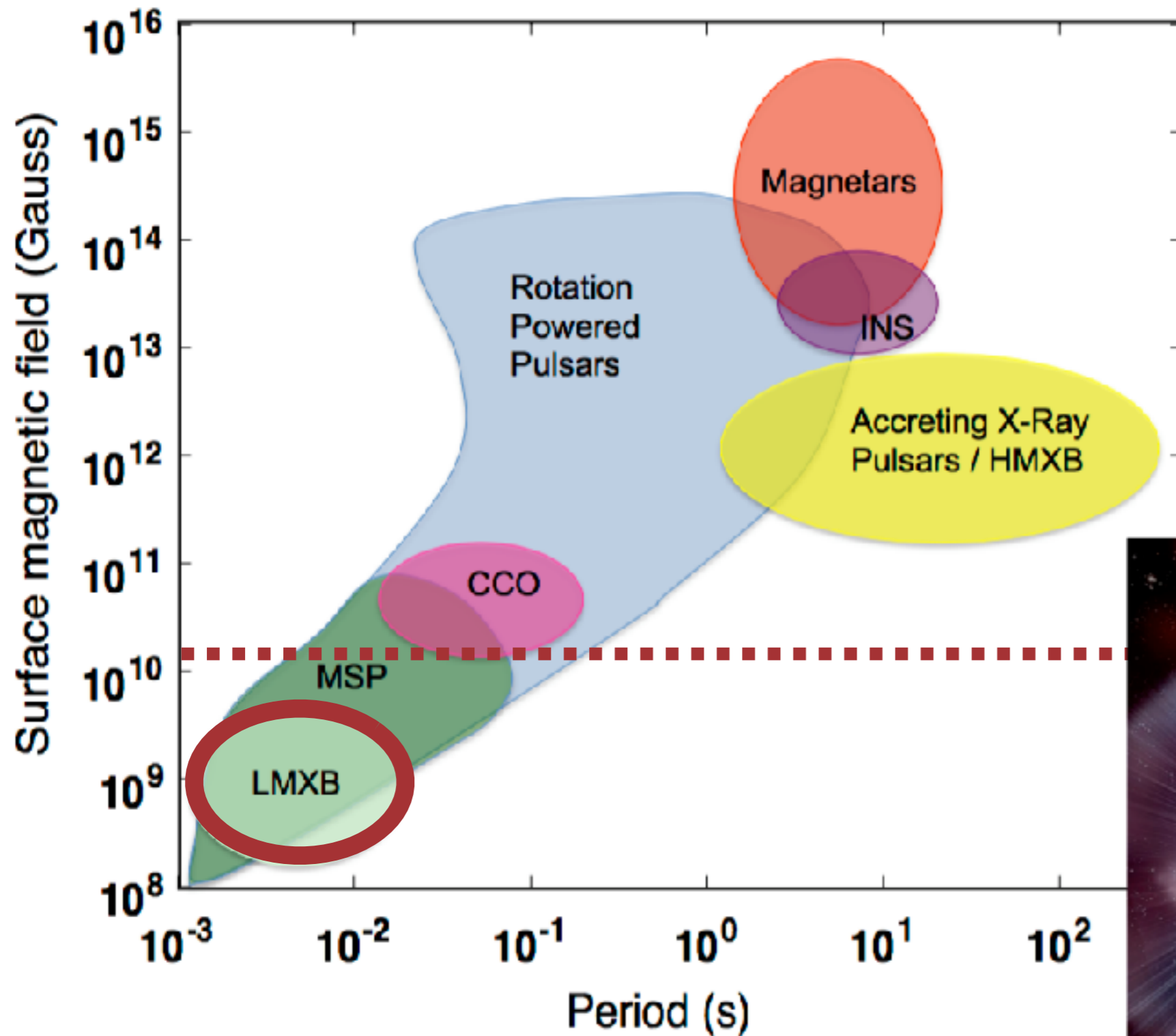


Rangelov et al. 2017



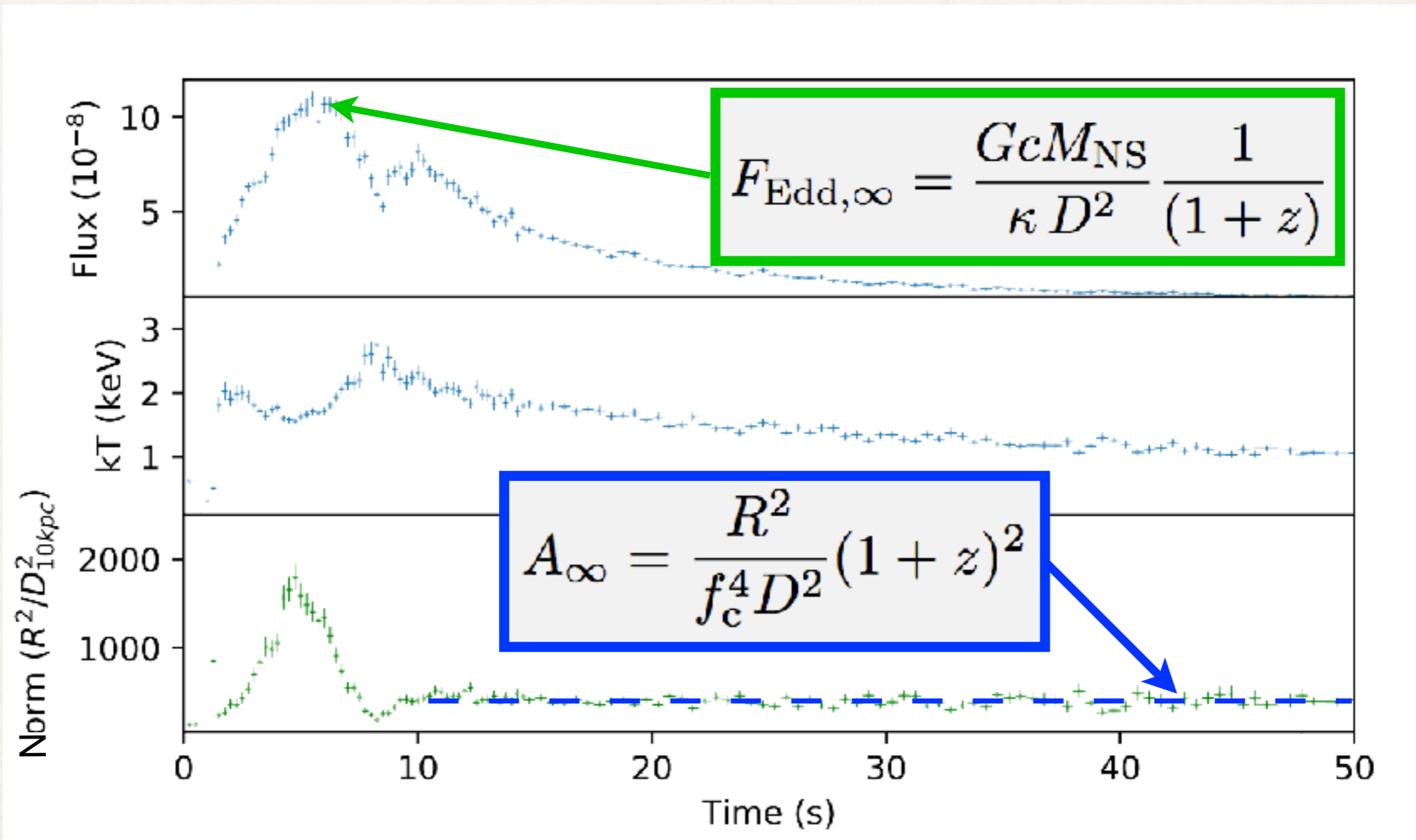
Bogdanov, SG et al. 2019

3. During accretion outbursts, LMXBs experience thermonuclear explosion from the neutron star surface.



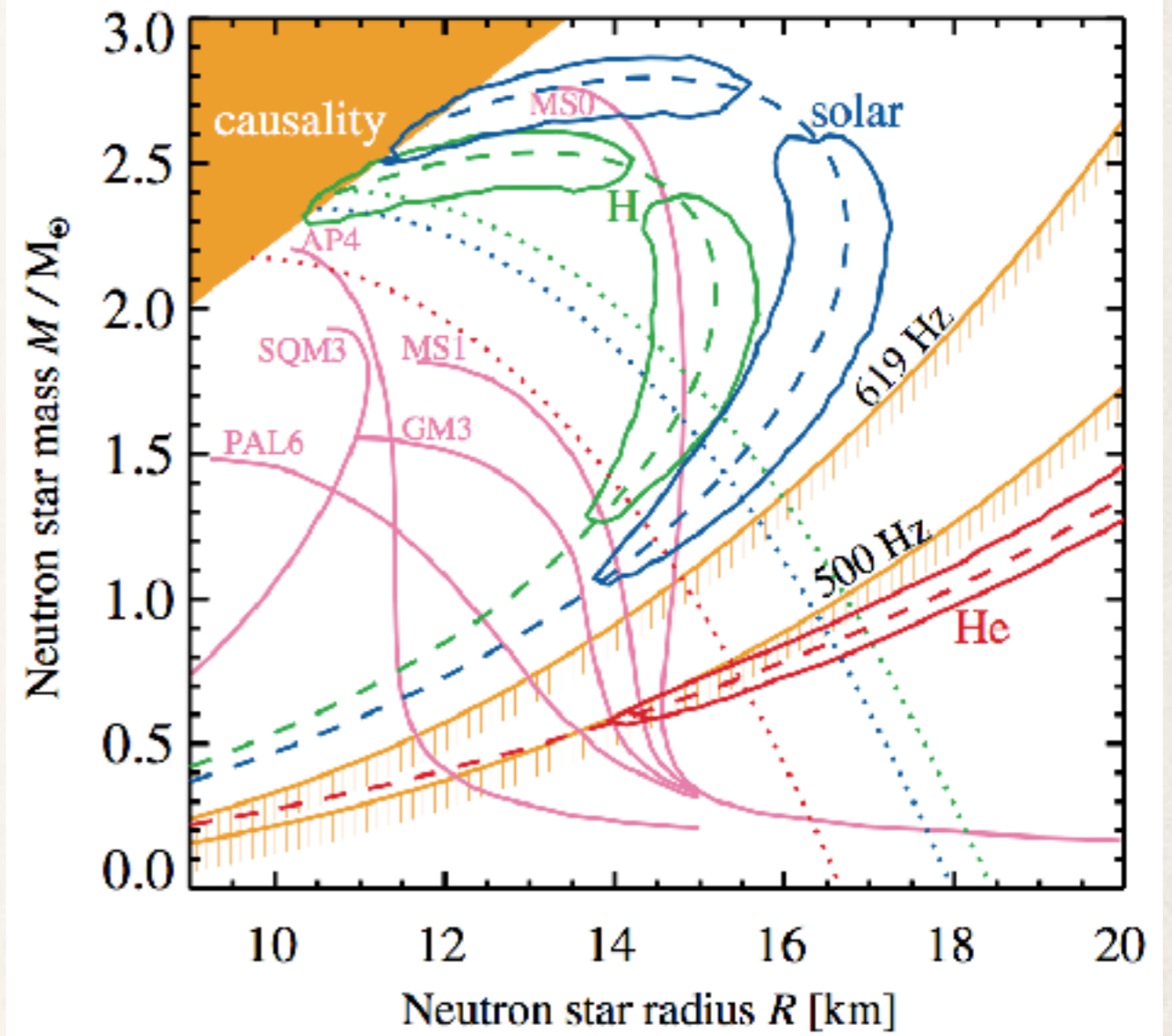
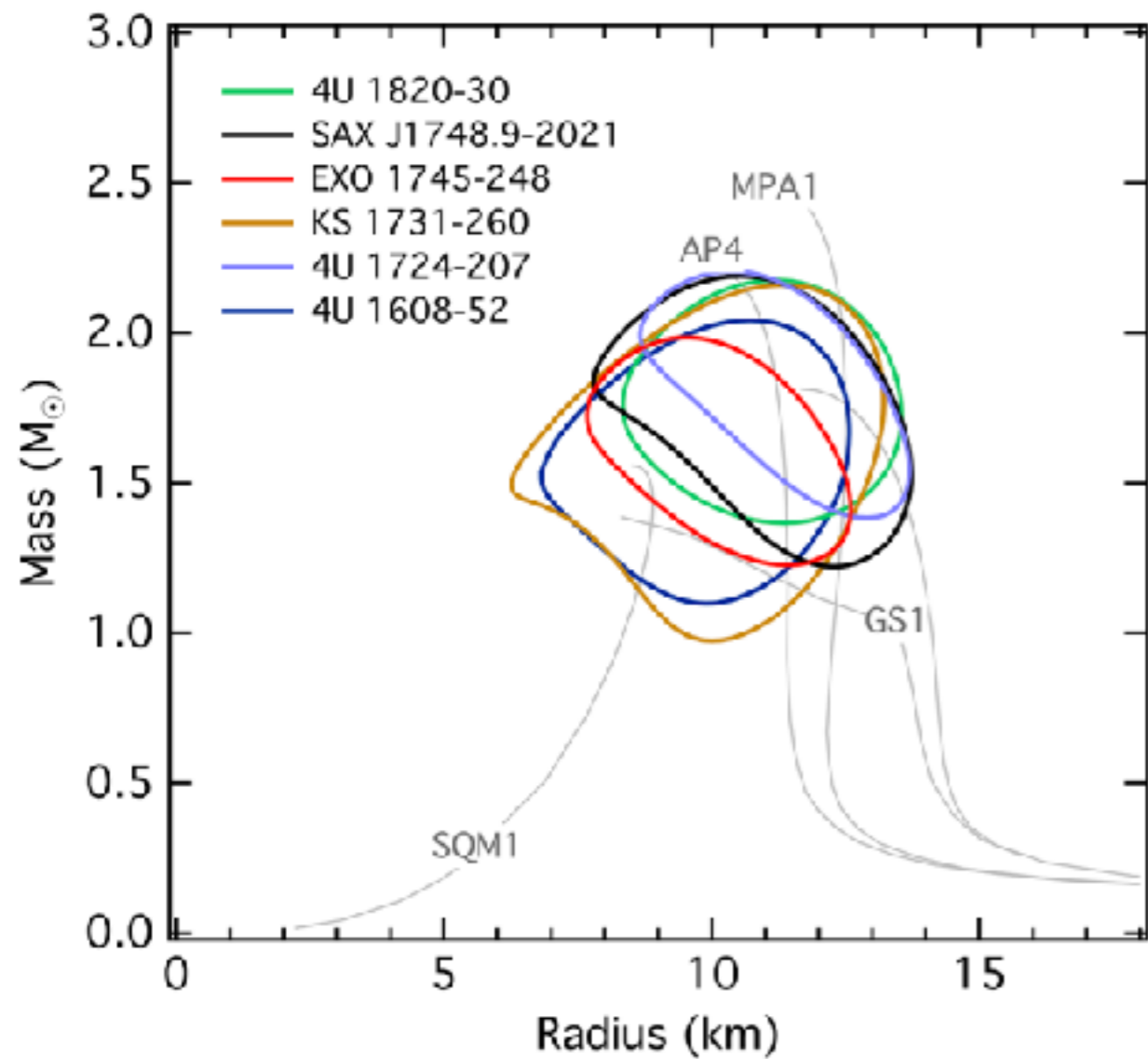
Some of these thermonuclear bursts reach a critical luminosity and push out the photosphere.

$$L_{\text{Edd}} = \frac{4\pi GcM_{\text{NS}}}{\kappa}$$



with $(1+z) = \left(1 + \frac{2GM_{\text{NS}}}{c^2 R_{\text{NS}}}\right)^{-1/2}$

Different analysis method and LMXB spectral states result in different constraints.



Data-driven selection of (high-soft state) bursts

Theory-driven selection of low-hard state bursts

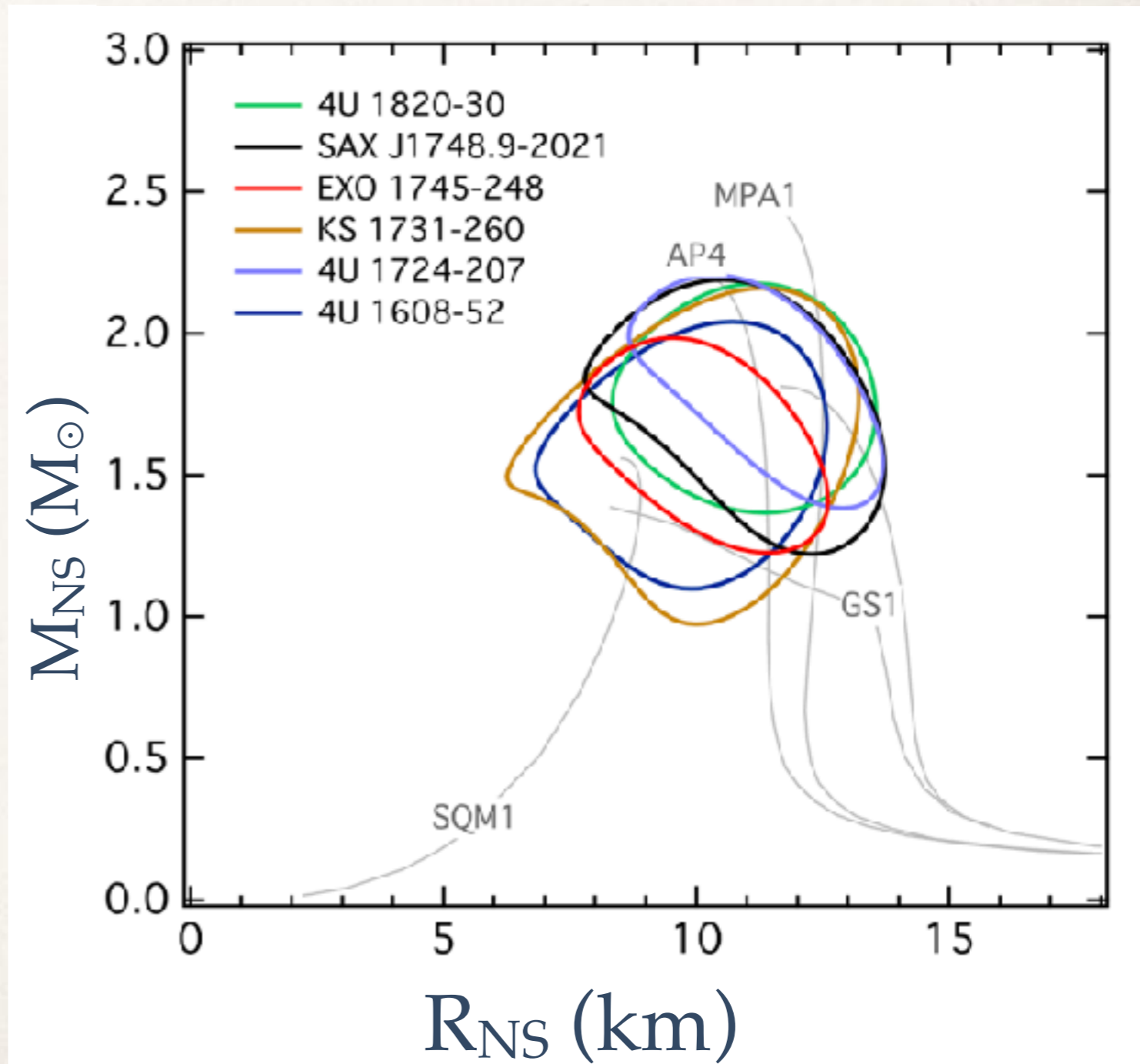
A lot of uncertainties remain and make the measurements poorly constrained.

$$F_{\text{Edd},\infty} = \frac{GcM_{\text{NS}}}{\kappa D^2} \frac{1}{(1+z)}$$

$$A_{\infty} = \frac{R^2}{f_c^4 D^2} (1+z)^2$$

Sources of uncertainty include:

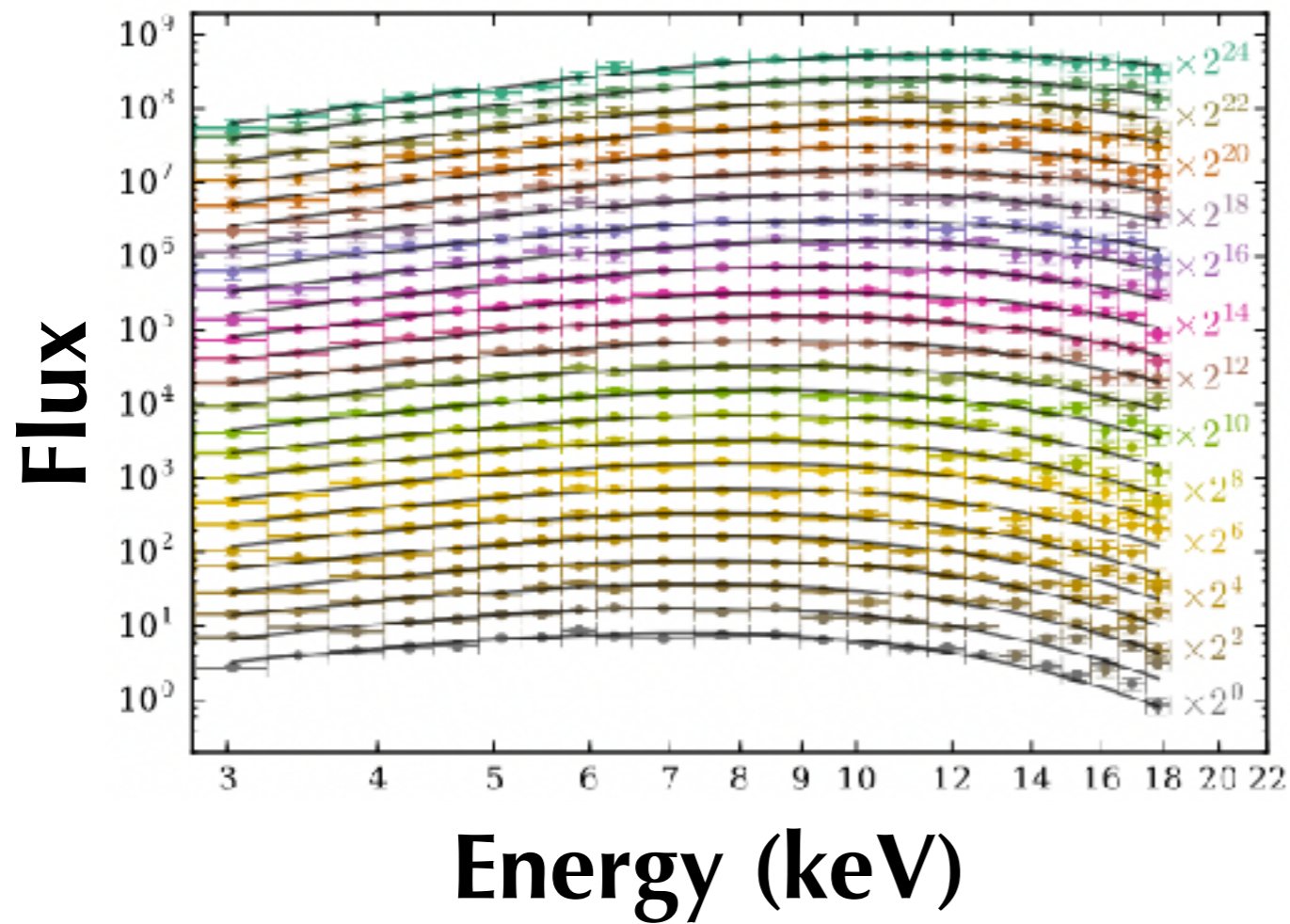
- ✦ Distance
- ✦ Atmospheric composition (via κ)
- ✦ Atmospheric modelling (via f_c)



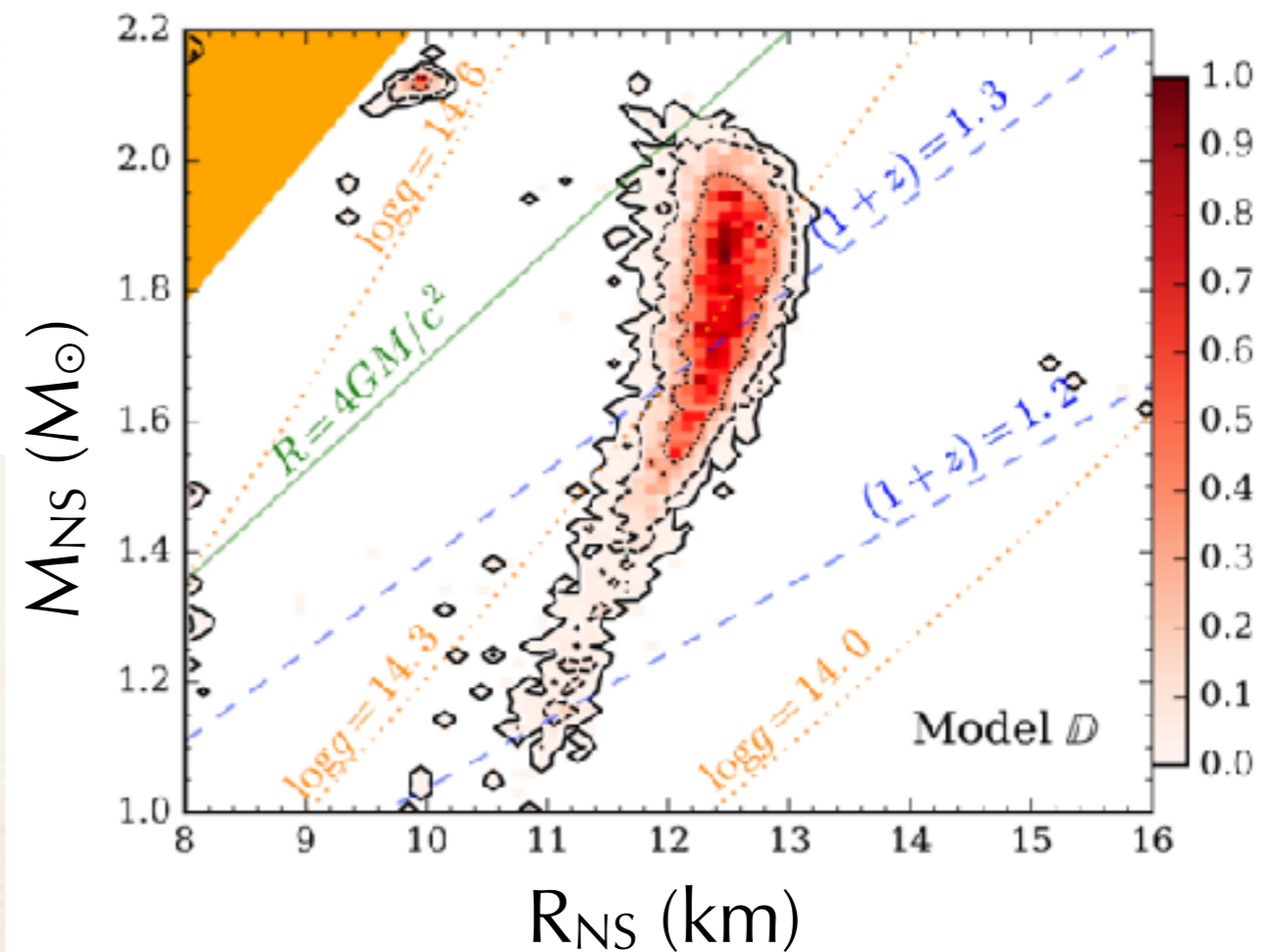
Recent developments in the field of Type I X-ray bursts ?

1. A new method

The direct spectral fits with realistic models during the burst evolution avoids using color-correction factors.



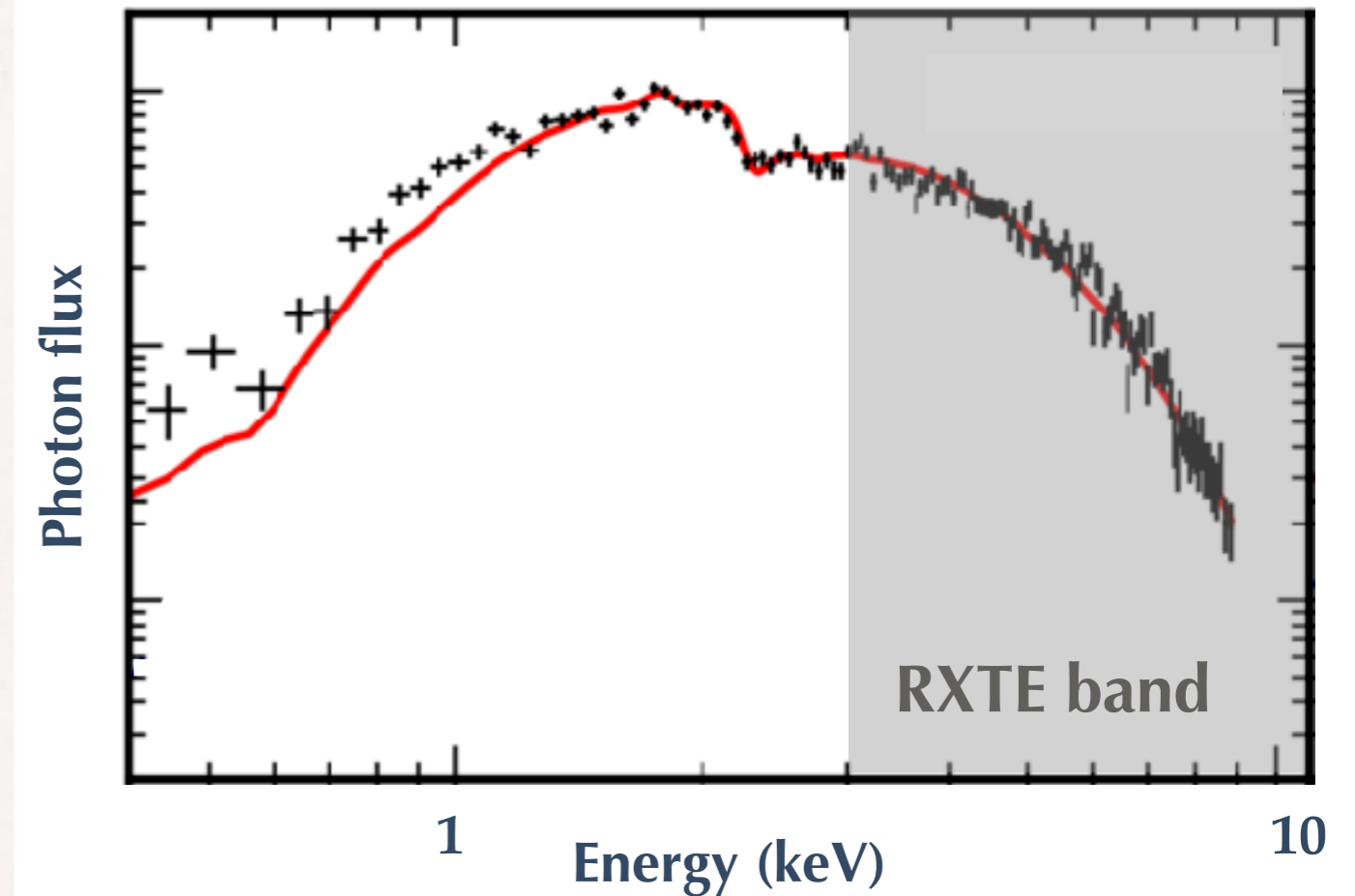
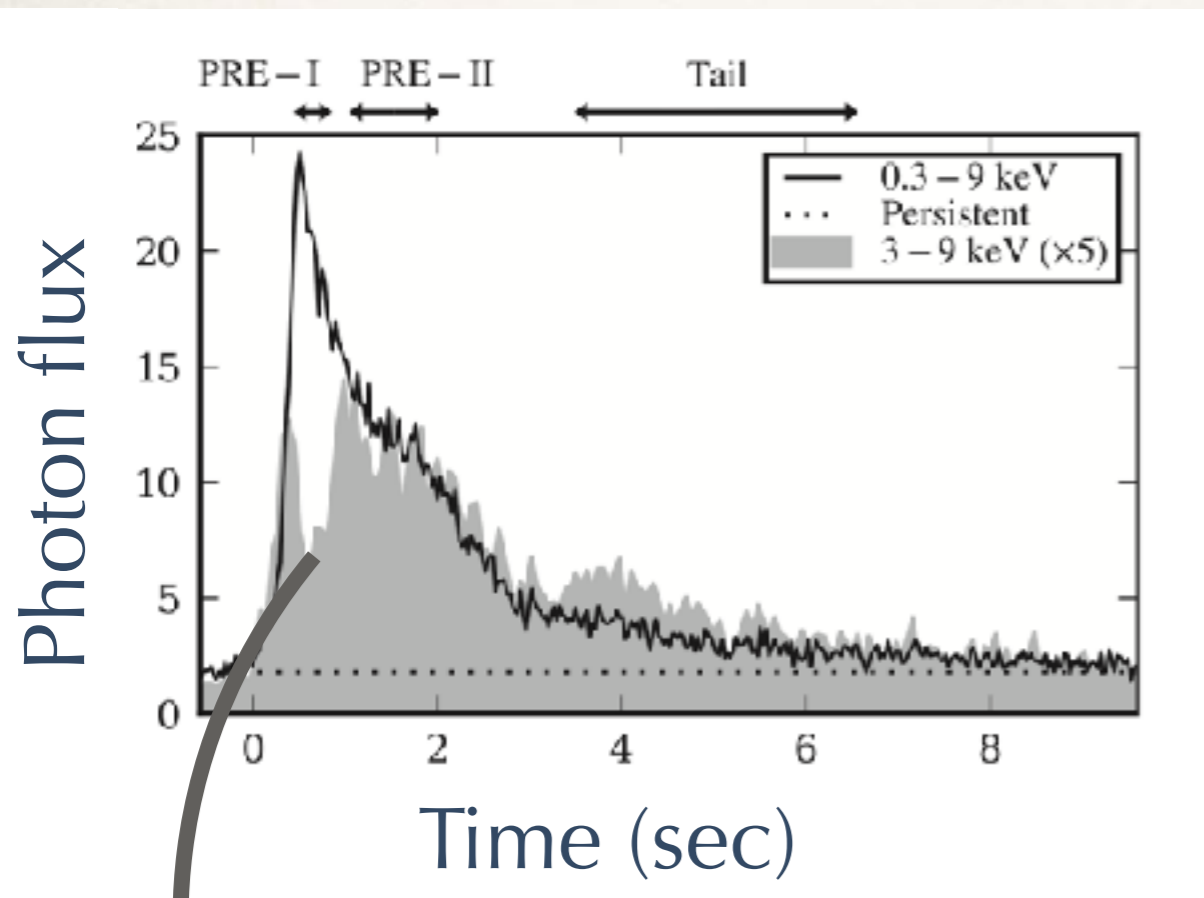
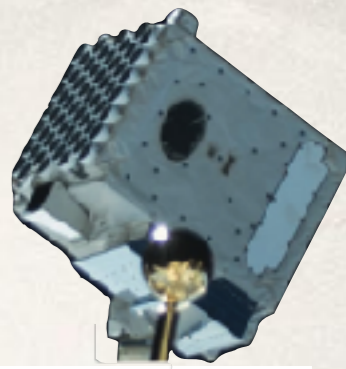
Burst from
4U 1702-429



Recent developments in the field of Type I X-ray bursts ?

1. A new method
2. A new instrument

The observation of type I X-ray bursts with NICER shows the whole burst evolution in the soft X-ray band.



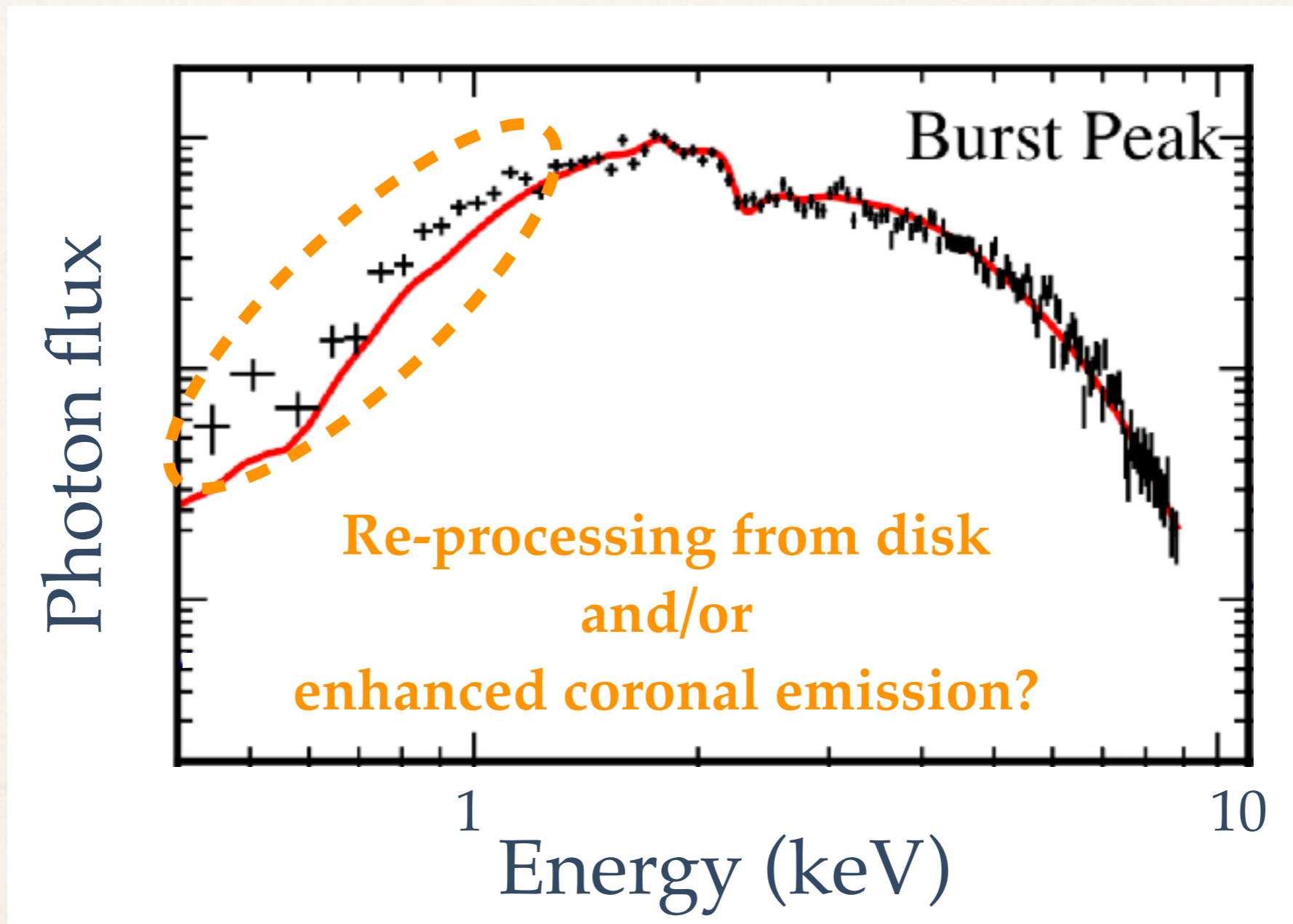
Keek et al. (2018)

In the RXTE band, the drop in flux comes from the temperature drop as the photosphere expands. With its 0.3–10 keV range, NICER sees the full evolution

Recent developments in the field of Type I X-ray bursts ?

1. **A new method**
2. **A new instrument**
3. **A new problem**

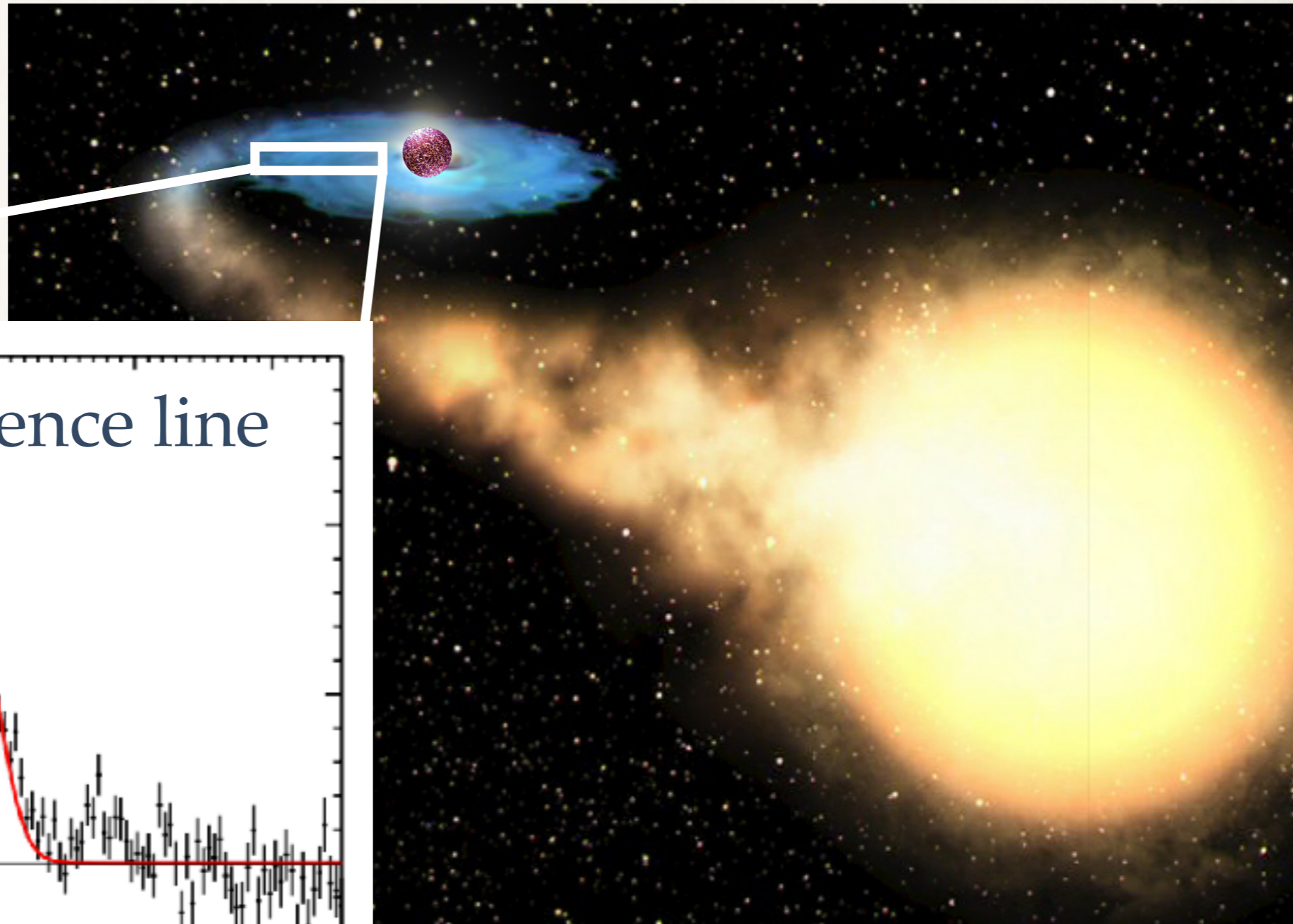
But NICER observations of type I X-ray burst also showed the presence of a un-modelled excess at low energies.



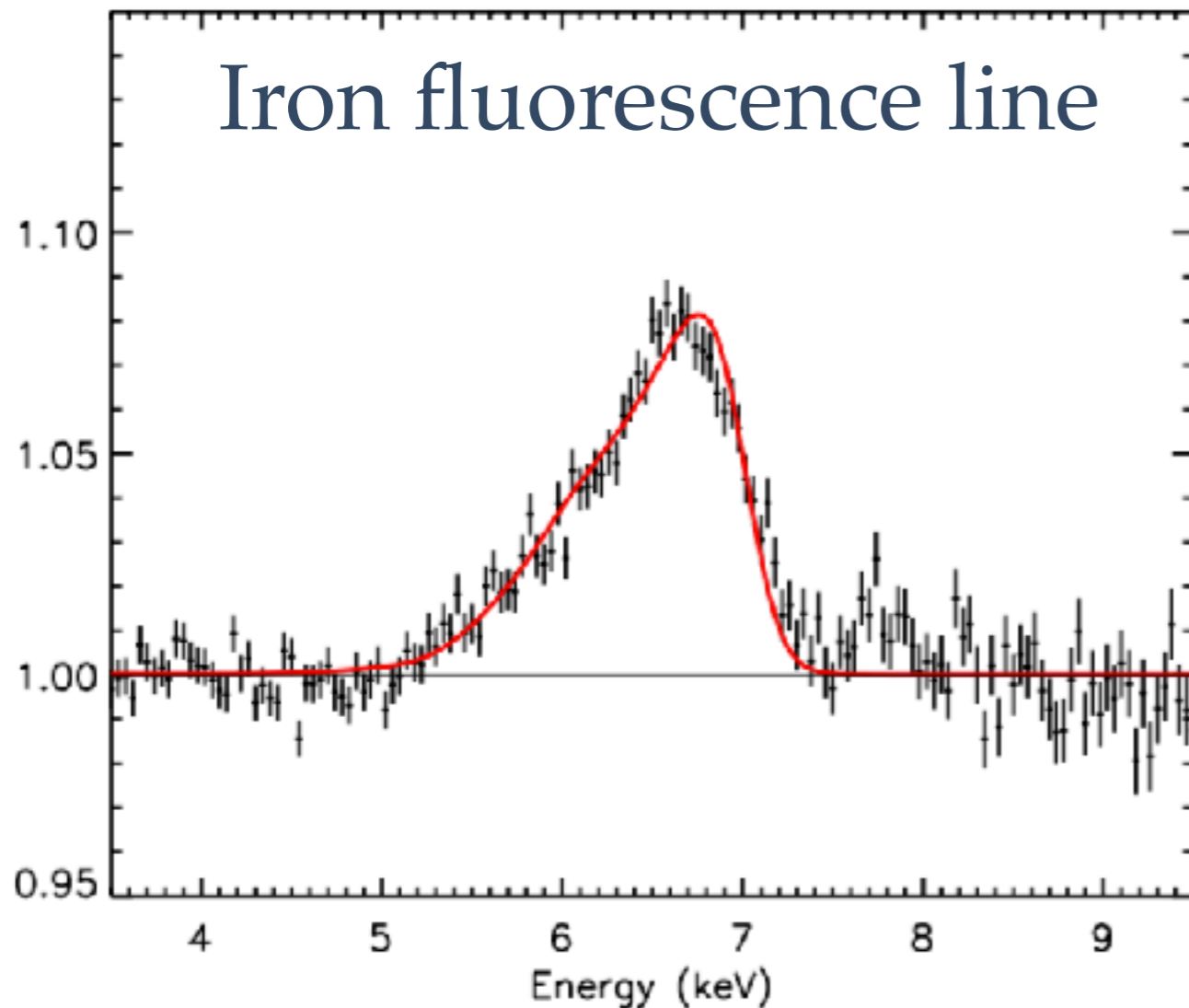
Keek et al. 2018

Güver et al. 2021, 2022

Bonus slide: The inner extent of an accretion disk gives an upper limit on the neutron star size



Iron fluorescence line

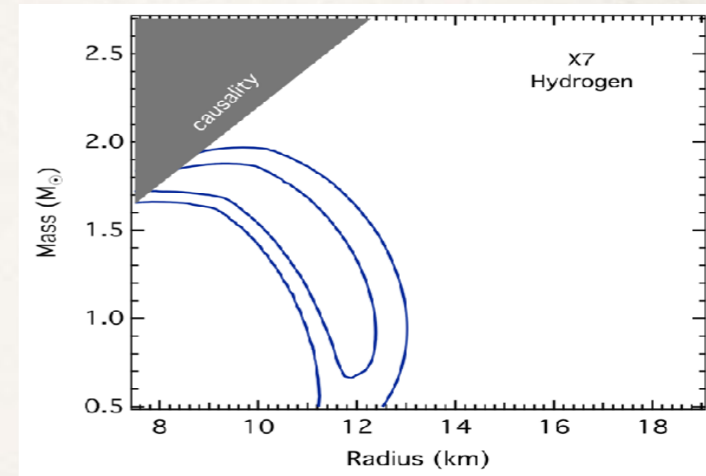


e.g., Ludlam et al. 2018, 2020

CONCLUSIONS

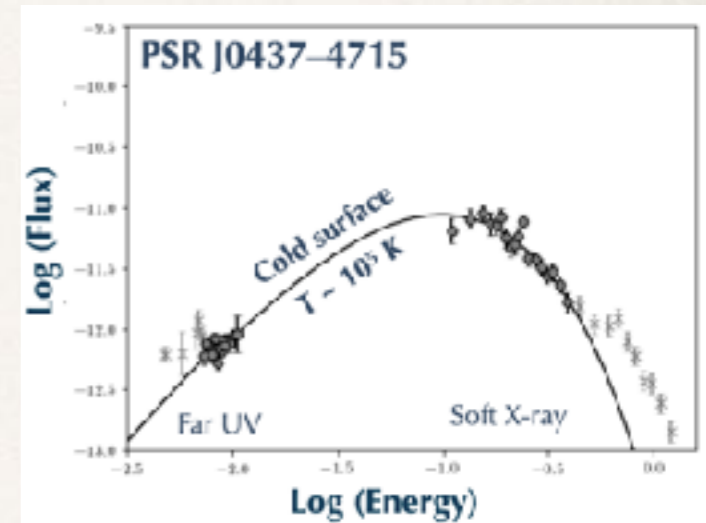
1. Quiescent Low mass X-ray binaries

Several qLMXBs can be combined, but assumptions may bias the results



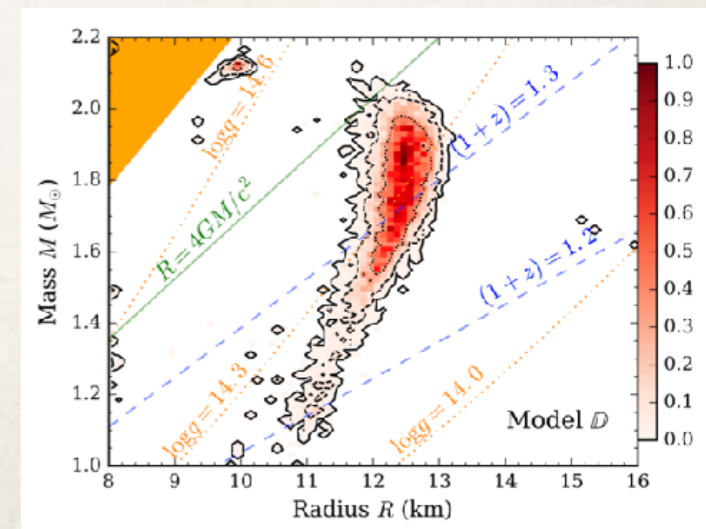
2. Cold surface of millisecond pulsars

Only 1 MSP so far, but great potential if combined with pulse profile modelling



3. Thermonuclear bursts in X-ray binaries

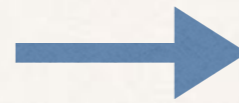
Very promising "direct spectral fit" method, but still some physical processes to clarify



There remains some discussion points and possible caveats!



- ◆ Why only use qLMXBs in globular clusters ?

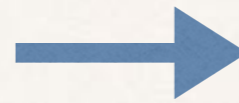


Field LMXB may not return to full quiescence

There remains some discussion points and possible caveats!

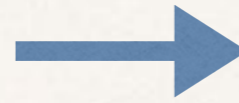


♦ Why only use qLMXBs in globular clusters ?



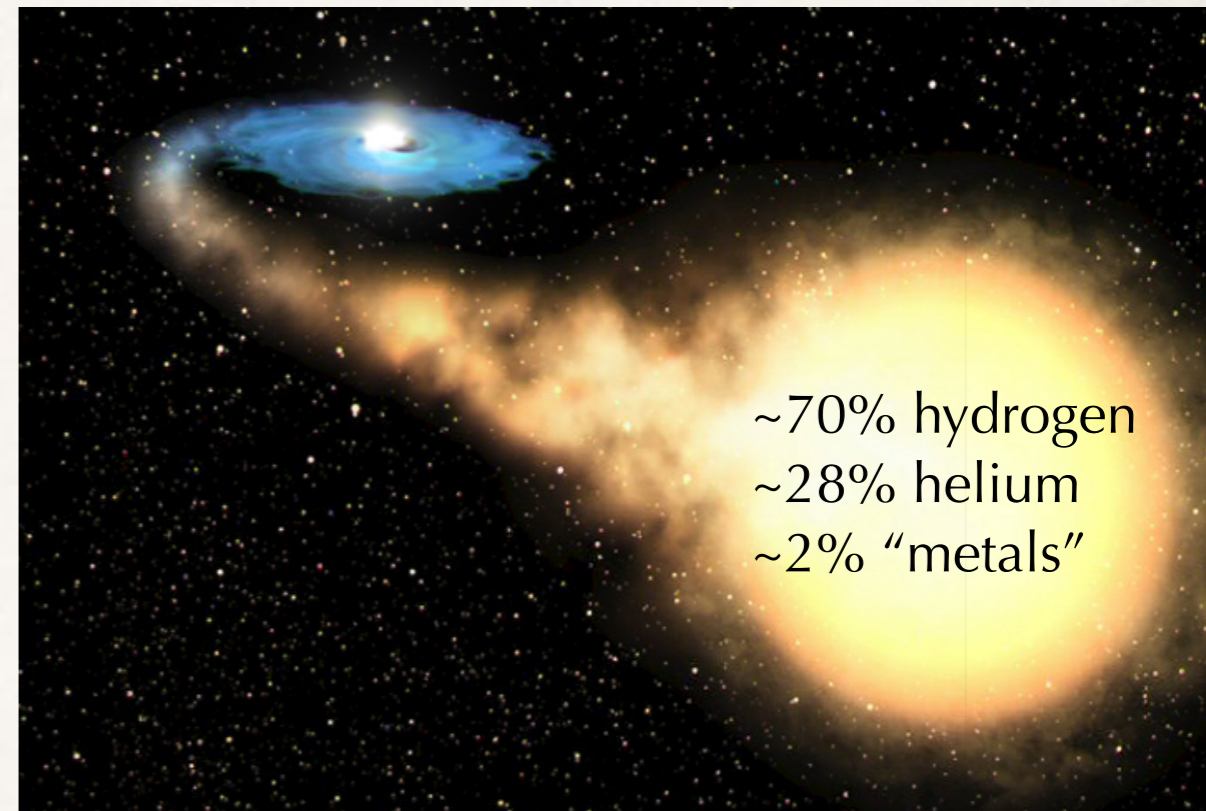
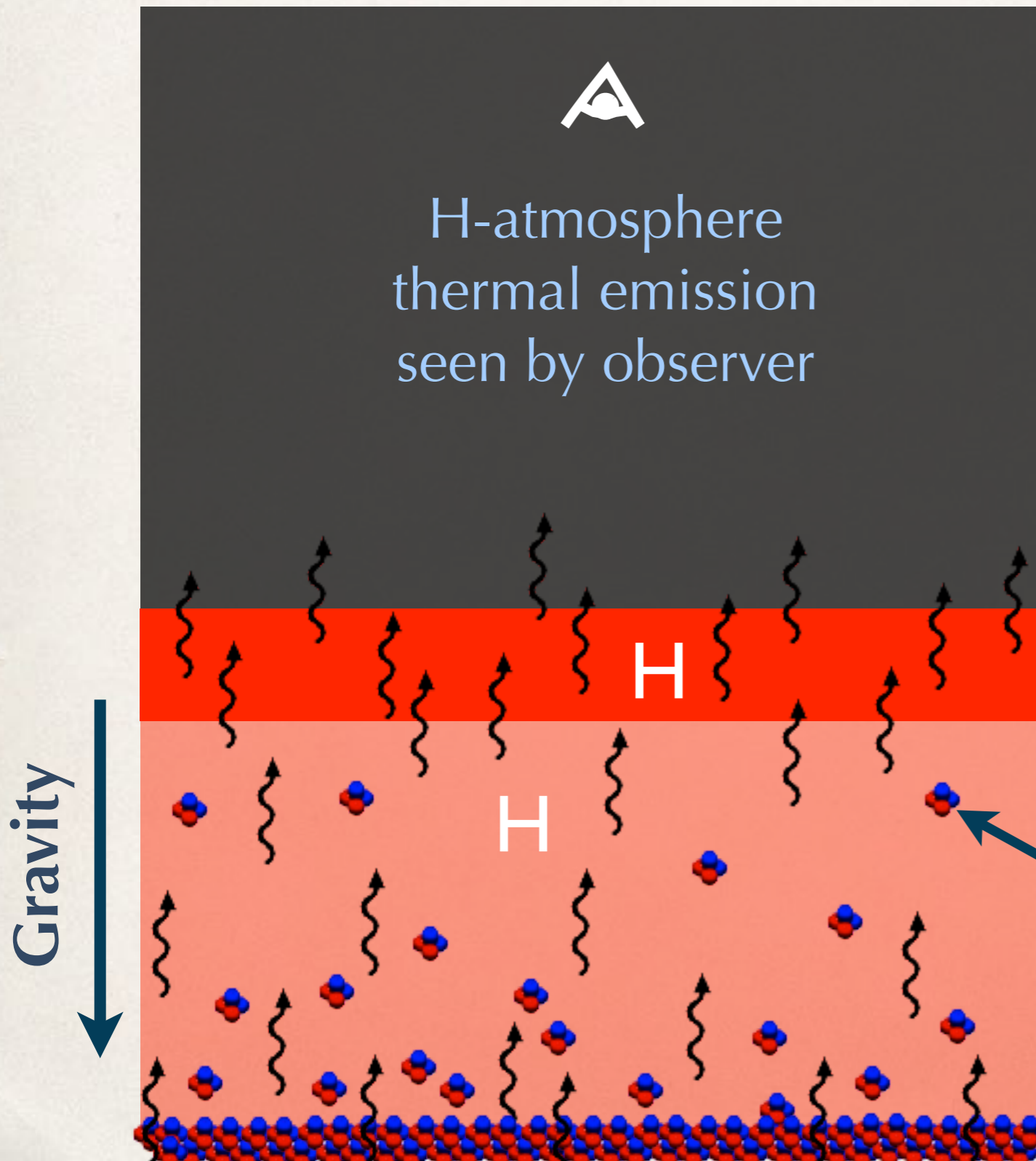
Field LMXB may not return to full quiescence

♦ What is the composition of the neutron star atmosphere ?



Hydrogen, Helium or something else

The atmospheric composition of an accreting neutron star depends on the donor star.

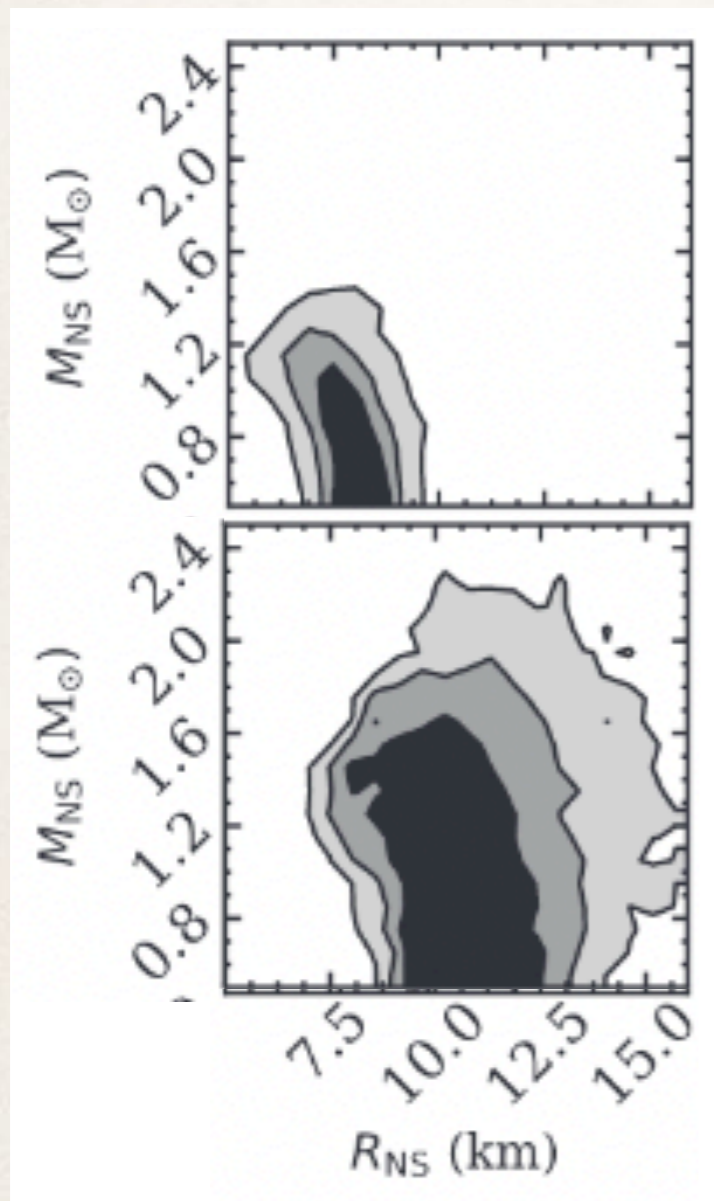


Photosphere ≈ 1 cm

Helium

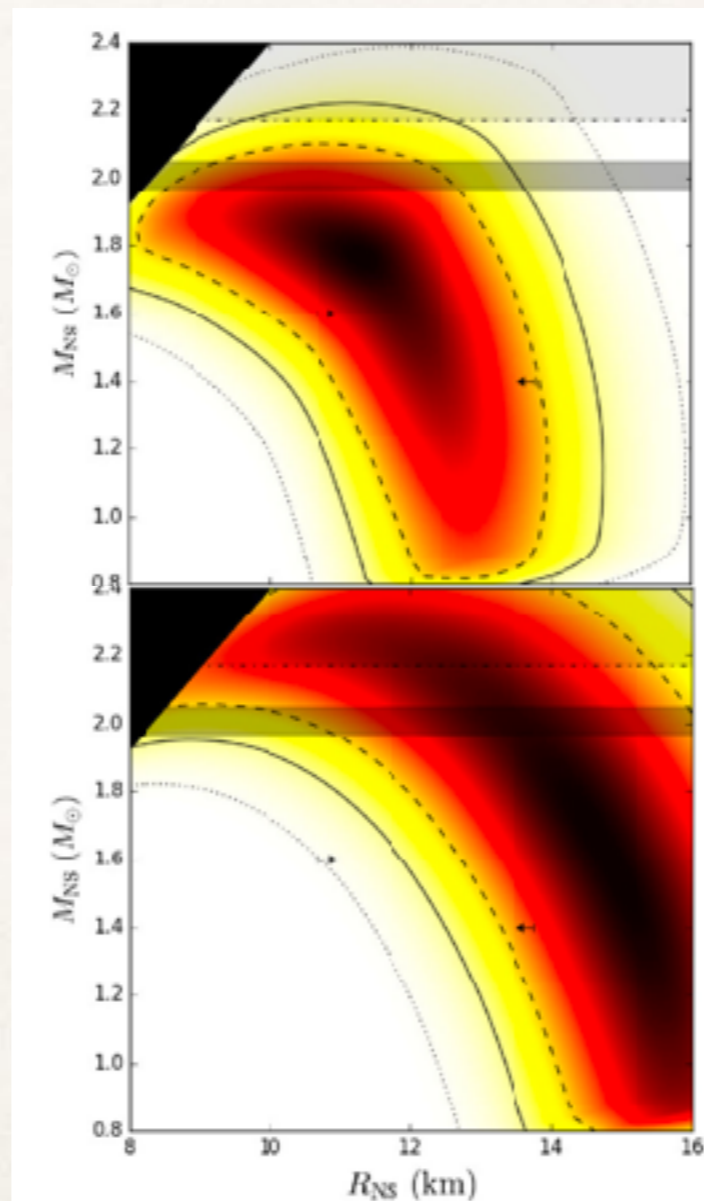
Assuming the wrong composition may severely bias the result.

qLMXB in M30



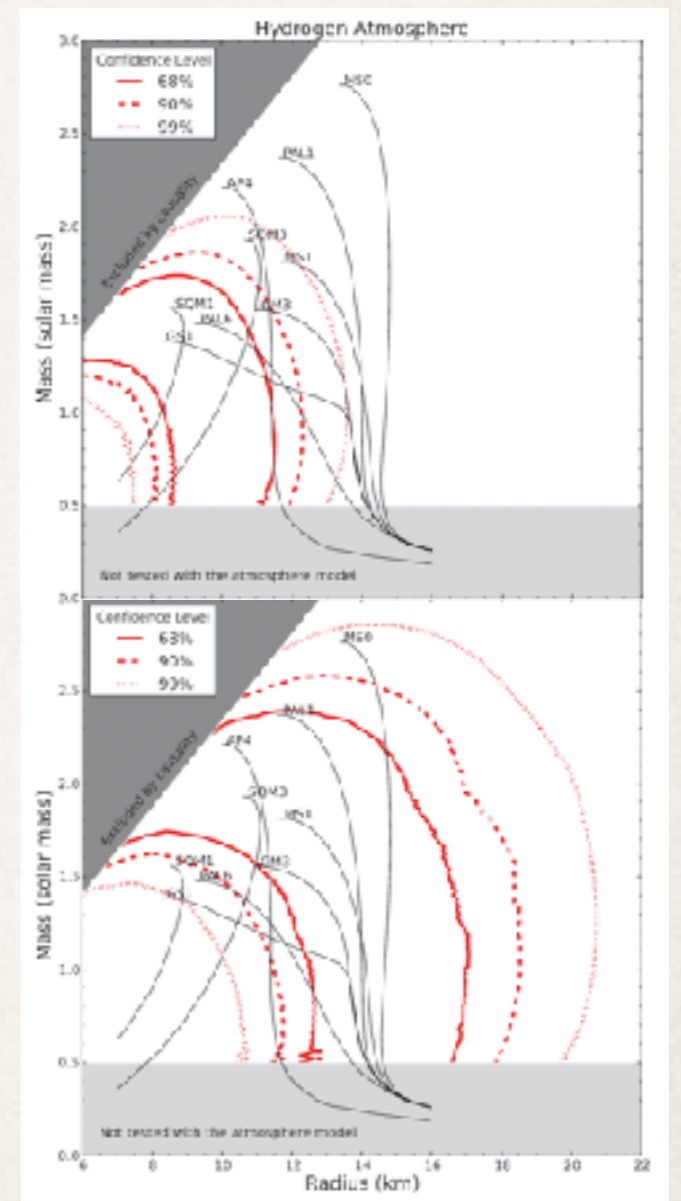
Echiburú, SG et al. 2020

qLMXB in M13



Shaw et al. 2018

qLMXB in M28



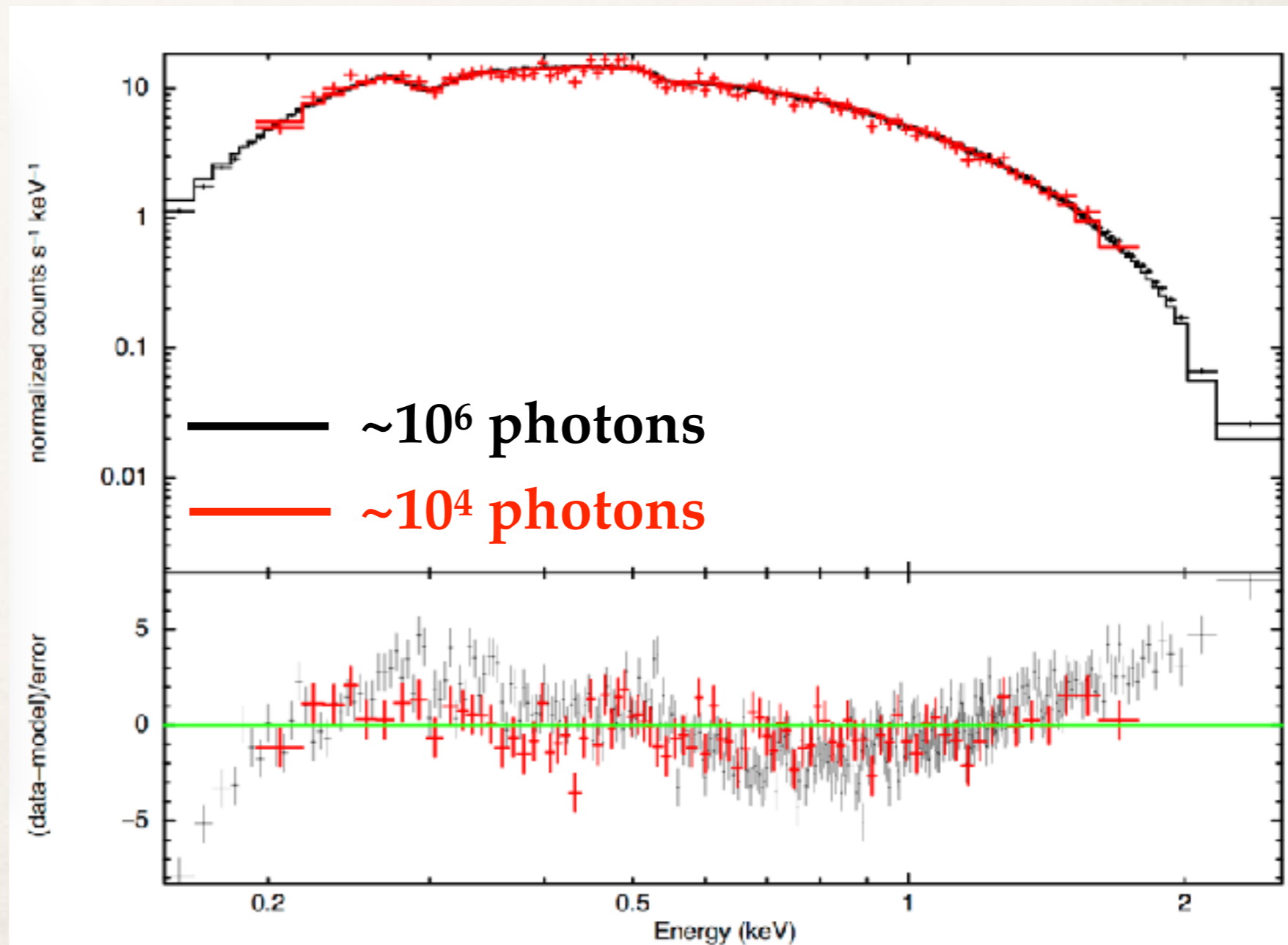
Shaw et al. 2012

Can we tell if a neutron star atmosphere is composed of H or He?

Extremely high S/N spectra permit detection of the subtle variations between H and He atmospheres

NS simulated with He-atmosphere, and fitted with H-atmosphere

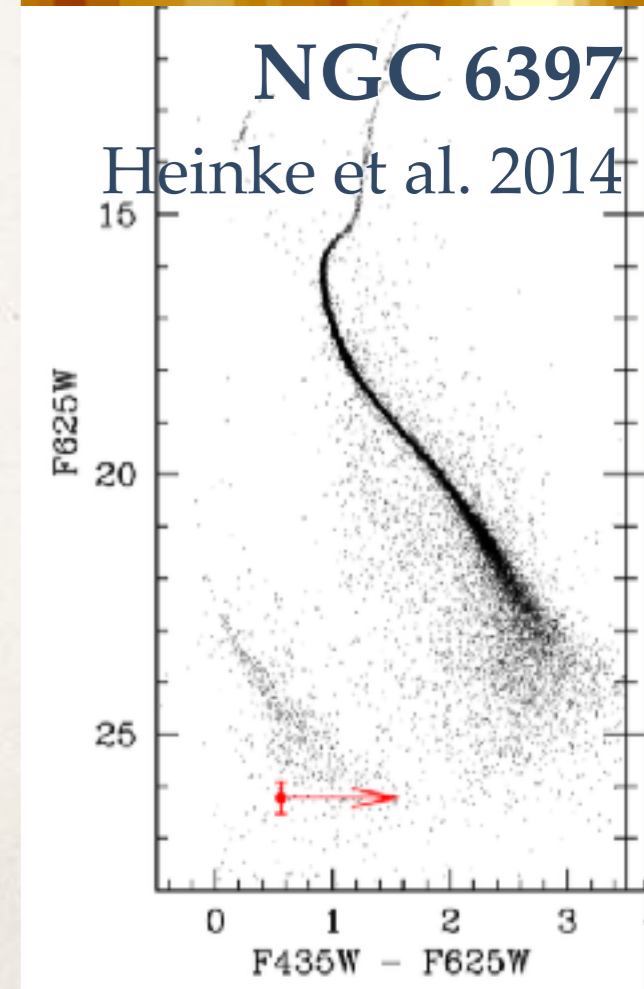
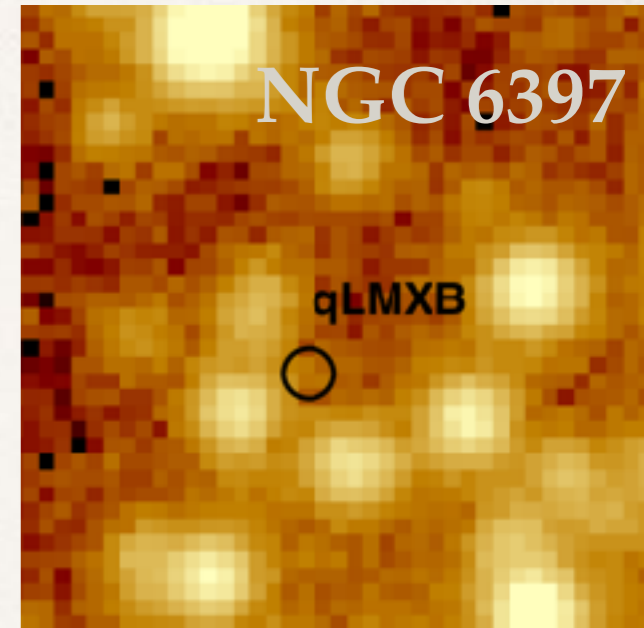
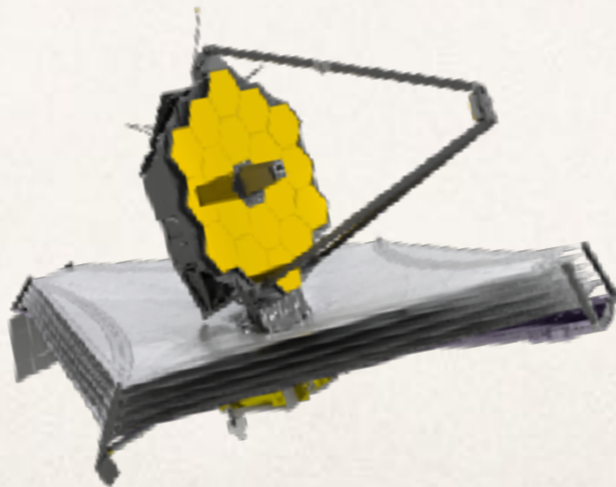
Simulations for proposed mission Lynx



Can we tell if a neutron star atmosphere is composed of H or He?

Identifying the donor star in the crowded environments of globular clusters

- ◆ Very difficult with ground based (e.g., VLT), even with AO
- ◆ Difficult with Hubble Space Telescope
- ◆ Easier with JWST



There remains some discussion points and possible caveats!



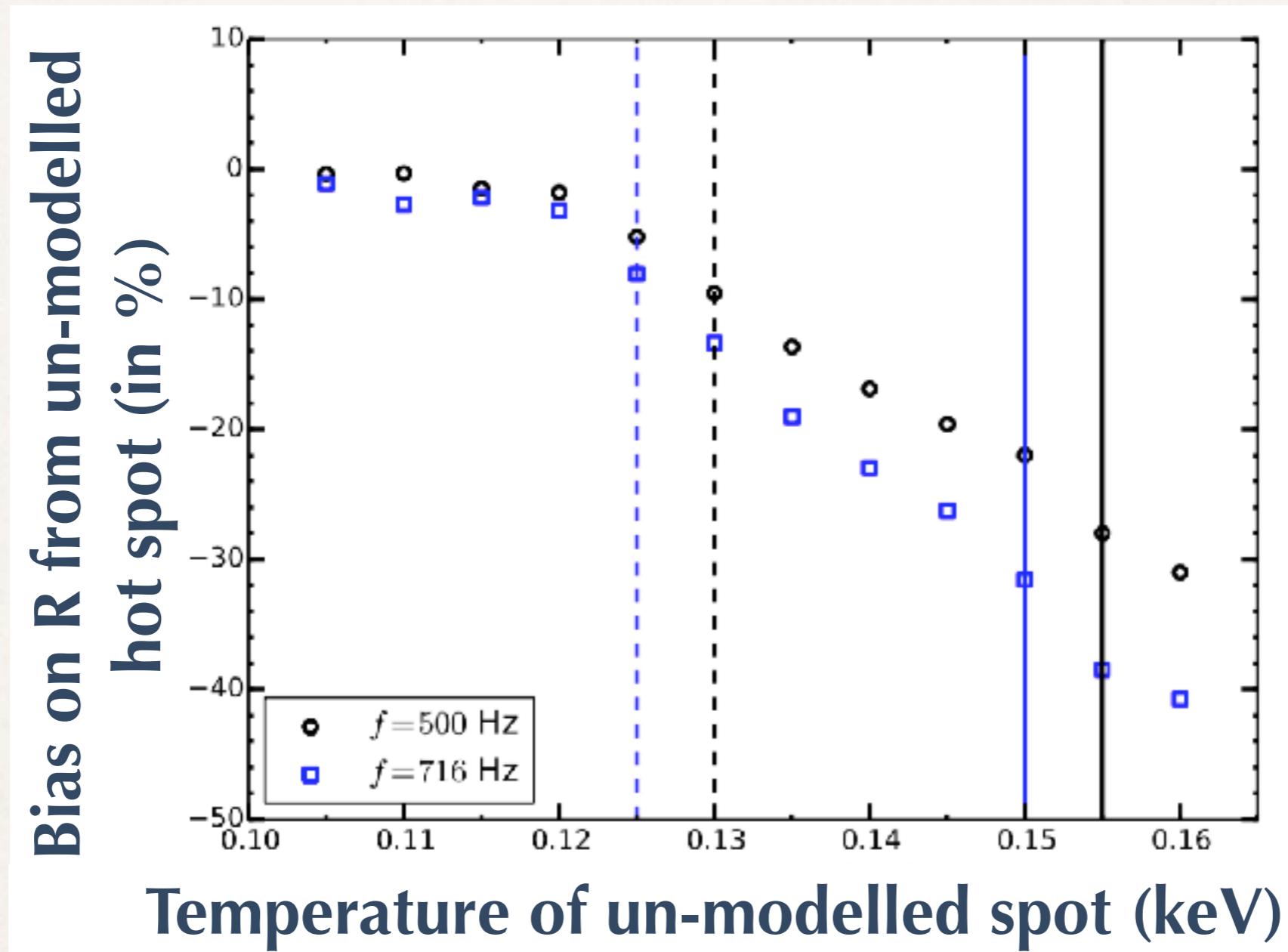
- ◆ Why only use qLMXBs in globular clusters ? → Field LMXB may not return to full quiescence
- ◆ What is the composition of the neutron star atmosphere ? → Hydrogen, Helium or something else
- ◆ Is the surface magnetic field really negligible ? → No measurement, but expected for LXMBs

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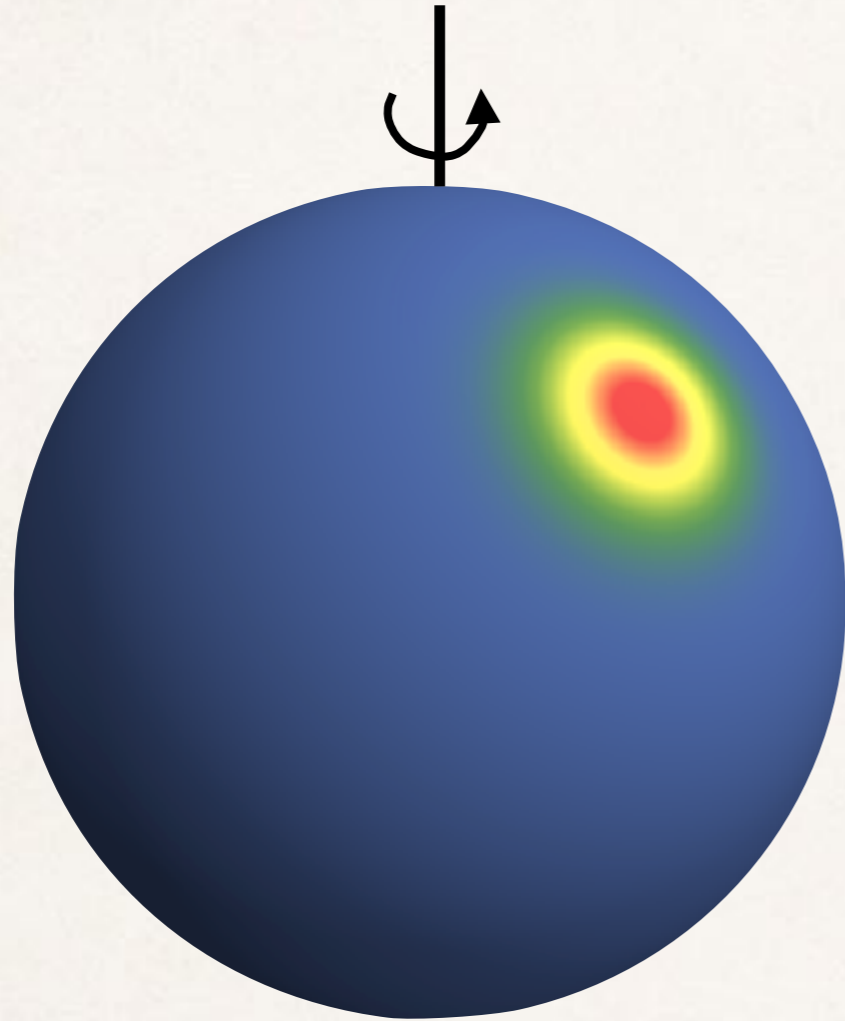


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- ◆ Is the emission really from the entire surface ? → No constraints exist, but ...

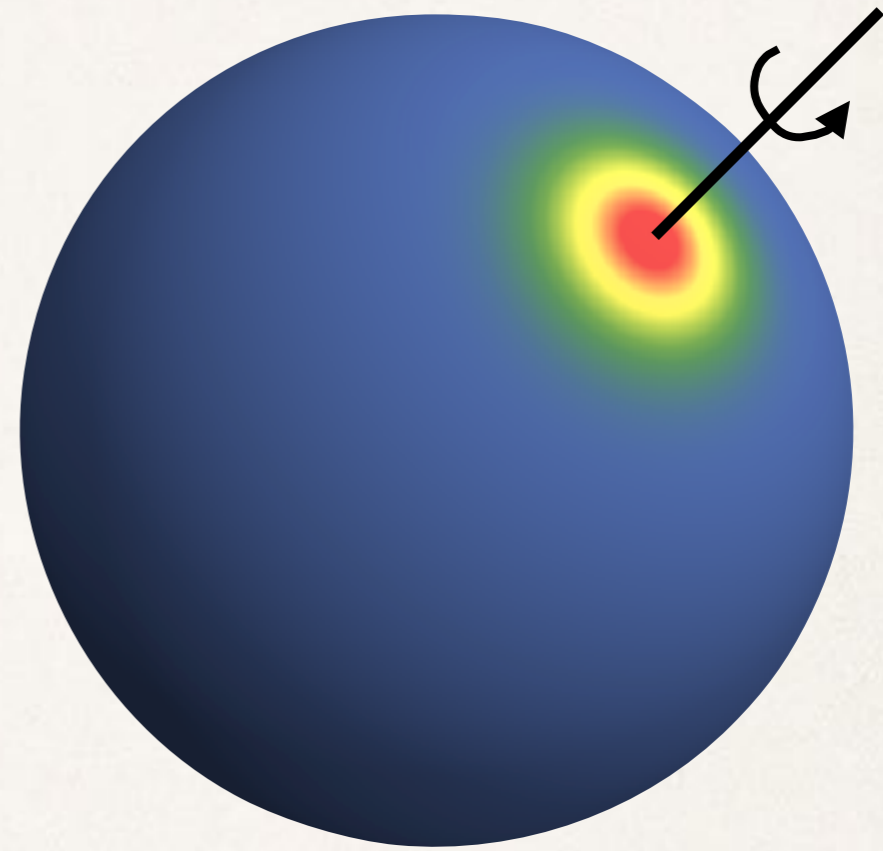
Assuming a uniform surface temperature may bias the radius measurement.



Can we tell whether the surface temperature is uniform or not?



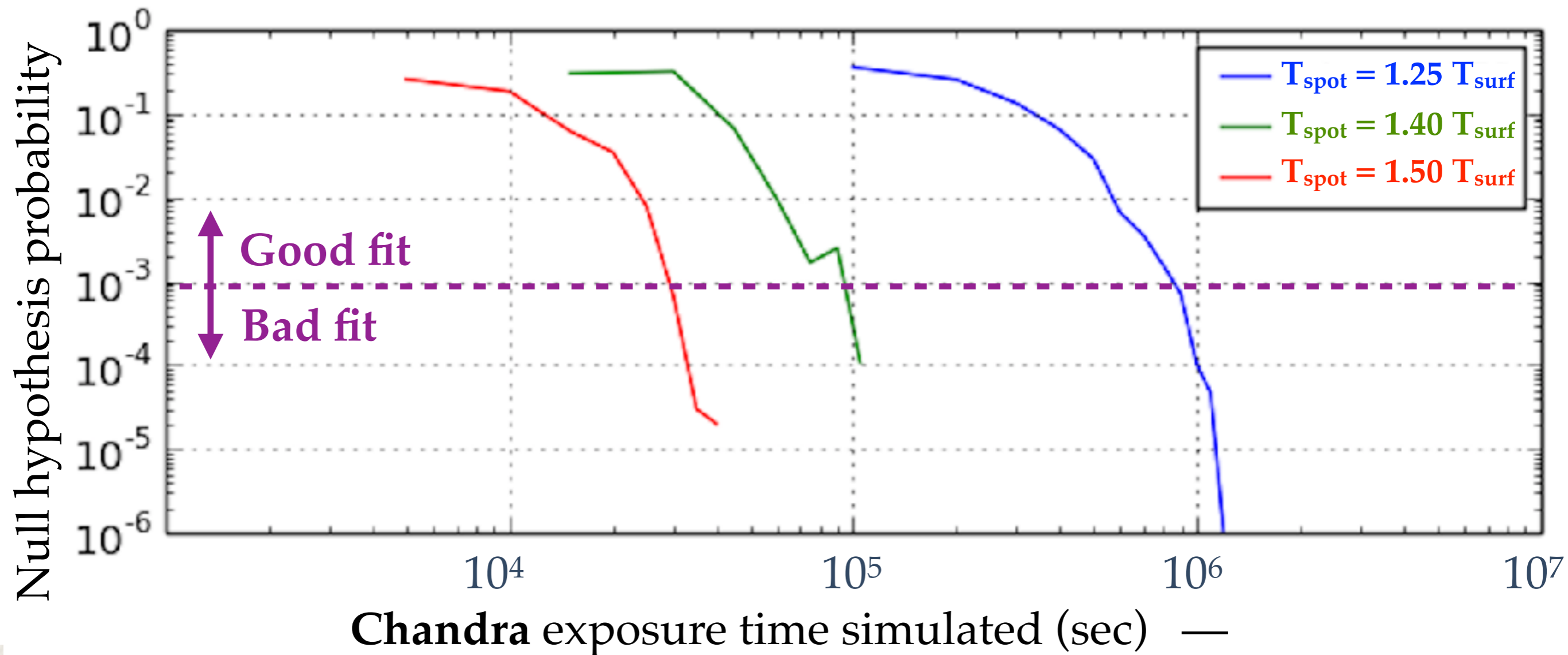
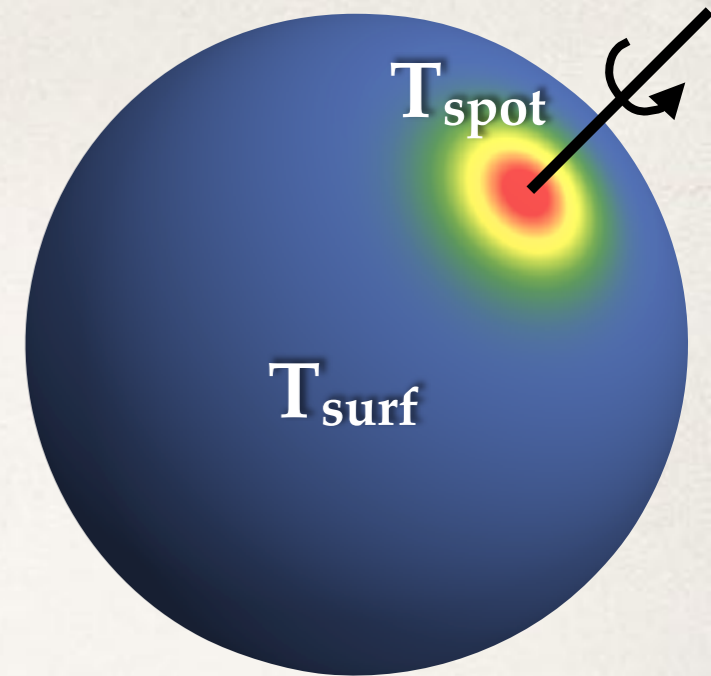
**Non-uniform surface
manifests as X-ray pulsations**



**No X-ray pulsations for
some specific geometries**

Can we tell from the X-ray spectra?

A hot spot that does not generate X-ray pulsations may be detected spectrally.



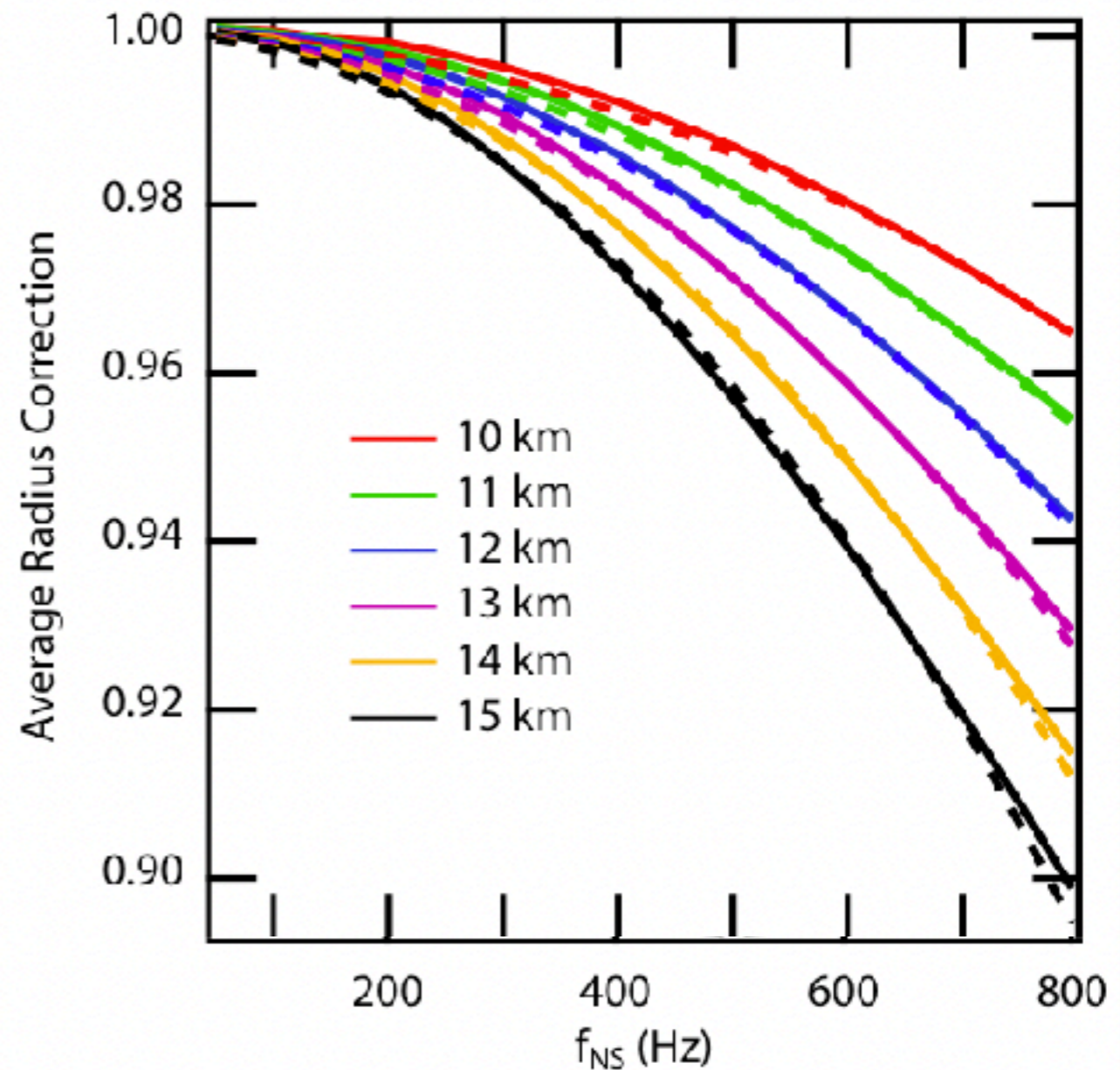
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- ◆ Is the emission really from the entire surface ? → No constraints exist, but ...
- ◆ What are the effects of assuming slowly rotating neutron stars? → Fast rotation may bias the R measurement

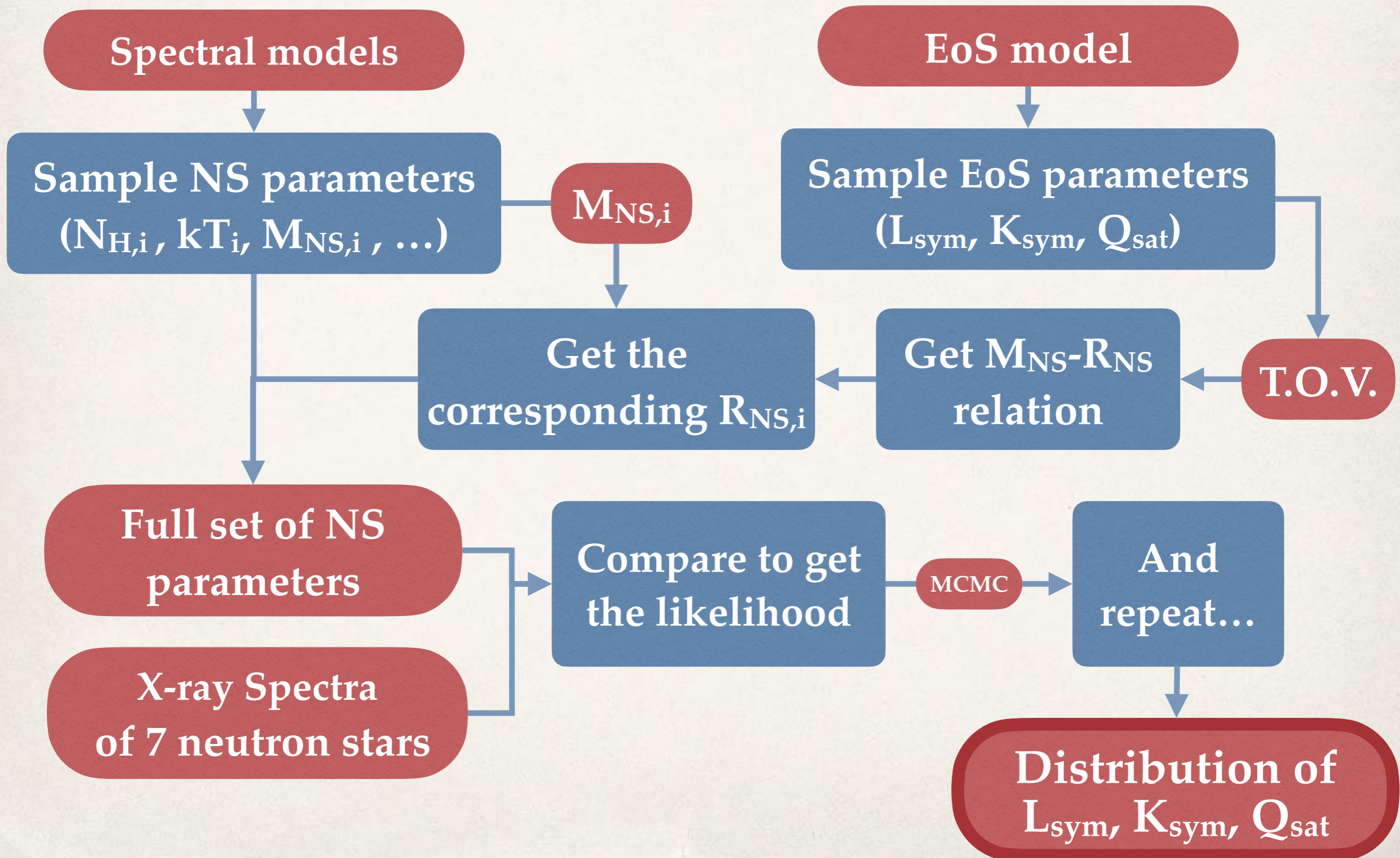
Rotation broadens the spectrum and neglecting it may bias the radius.

- ◆ No measurement of spin in qLMXB since no X-ray or radio pulsations.
- ◆ Other LMXBs have spin frequencies in the range $\sim 200\text{--}600$ Hz
- ◆ Ignoring rotation will bias the radius by a few % at most for a 12 km NS



Bauböck et al. 2015

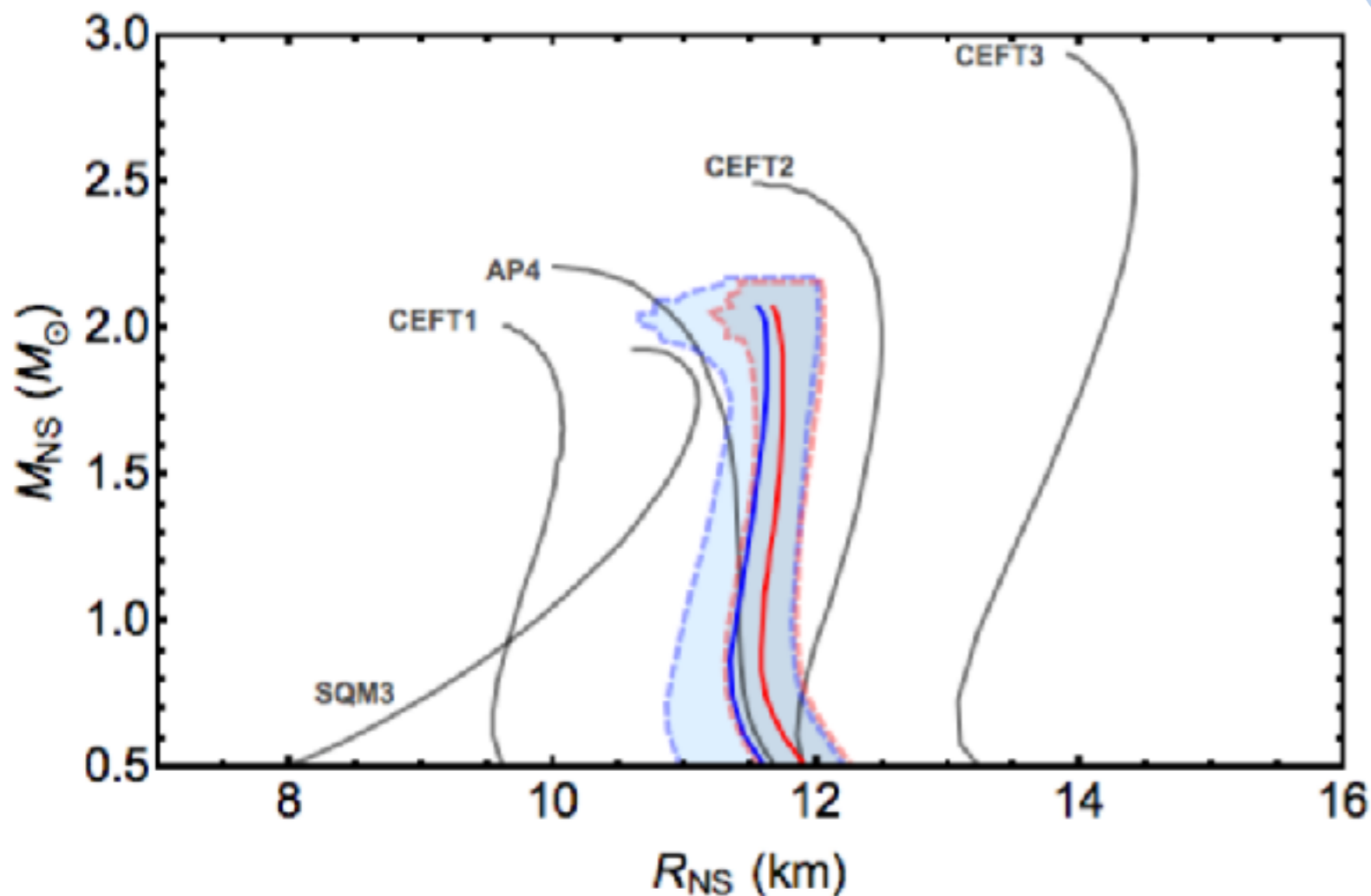
The MCMC samples the parameters space to fits the observed X-ray spectra.



The ATHENA X-ray Observatory will drastically improve constraints!

$$\left. \frac{\Delta R_{\text{NS}}}{R_{\text{NS}}} \right|_{1.4 M_{\odot}} = \pm 1.7\%$$

7 qLMXBs with ATHENA



R-SCIOBJ-331



“Athena shall constrain the equation of state of neutron stars by obtaining X-ray spectra of seven quiescent low mass X-ray binaries with a good distance estimate.”

M30 at ATHENA's resolution

