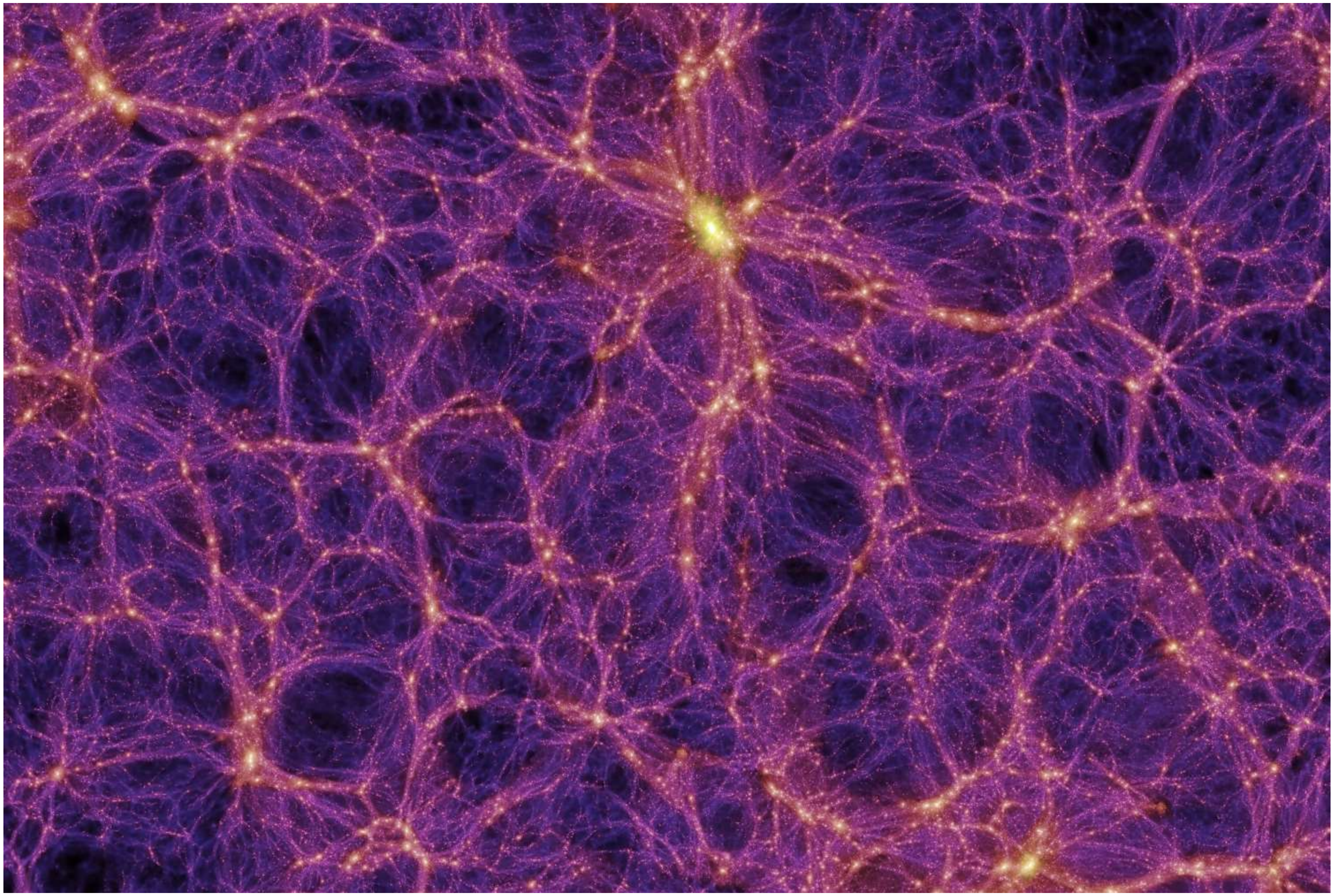

DISTRIBUTION FUNCTIONS OF DARK MATTER HALOS

Image Credit: Millennium Simulation Project

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In Collaboration with Yong-Zhong Qian and Zhaozhou Li

The NFW Profile

- In 1997, Navarro, Frenk, and White showed that dark matter halos can be universally fit by a single functional form:

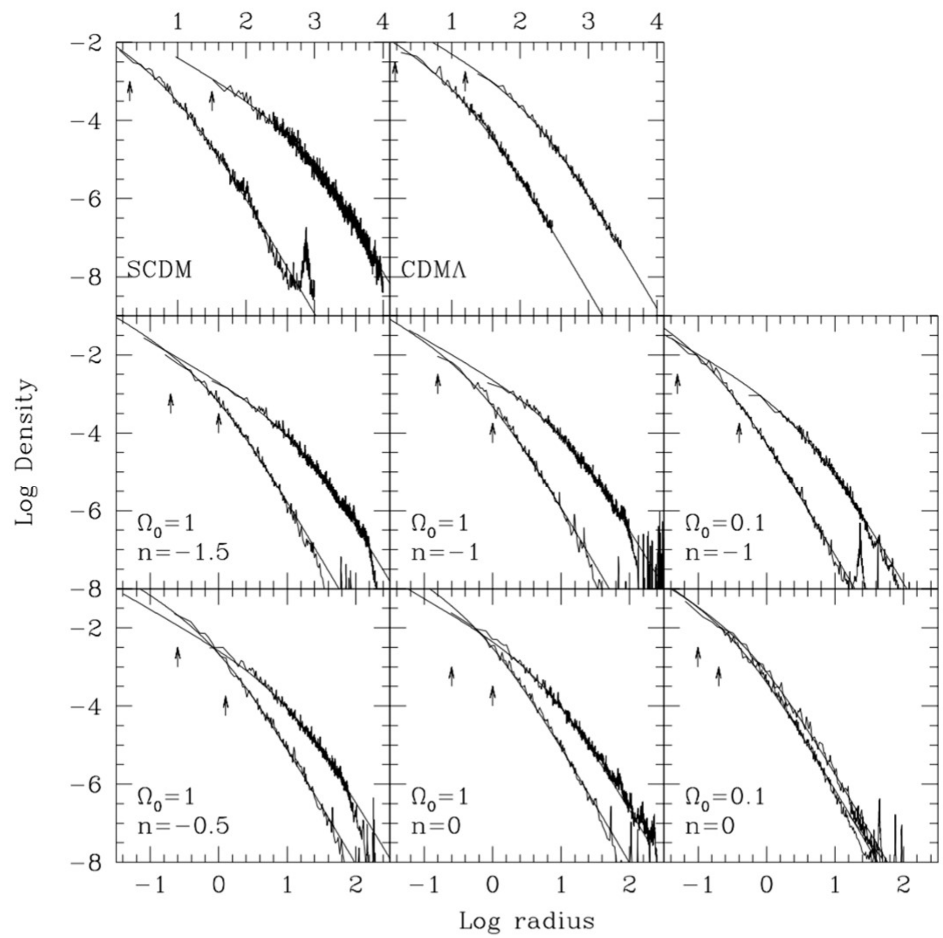


Image Credit: Navarro, Frenk, White, 1997

The NFW Profile (2)

- In 1997, Navarro, Frenk, and White showed that dark matter halos can be universally fit by a single functional form: $\rho(r) = \dfrac{\rho_0}{\frac{r}{R_S}\left(1+\frac{r}{R_S}\right)^2}$
- Because this is divergent, it is common to define an edge of the halo so the average density is $200\rho_c$, and the concentration as the ratio of the virial and scale radius

$$R_{vir} = \left(\frac{3M}{4\pi\,(200\rho_c)}\right)^{1/3} \qquad R_{vir} = cR_s$$

$$M = 4\pi\rho_0 R_s^3\left[\ln\left(1+\frac{R_{vir}}{R_s}\right) - \frac{R_{vir}}{R_{vir}+R_s}\right] \qquad (M, c) \Leftrightarrow (R_s, \rho_0)$$

The DarkEXP Profile

- In 2010, Hjorth and Williams presented "DarkEXP", a profile obtained through equilibrium statistical mechanics which fits simulation data:

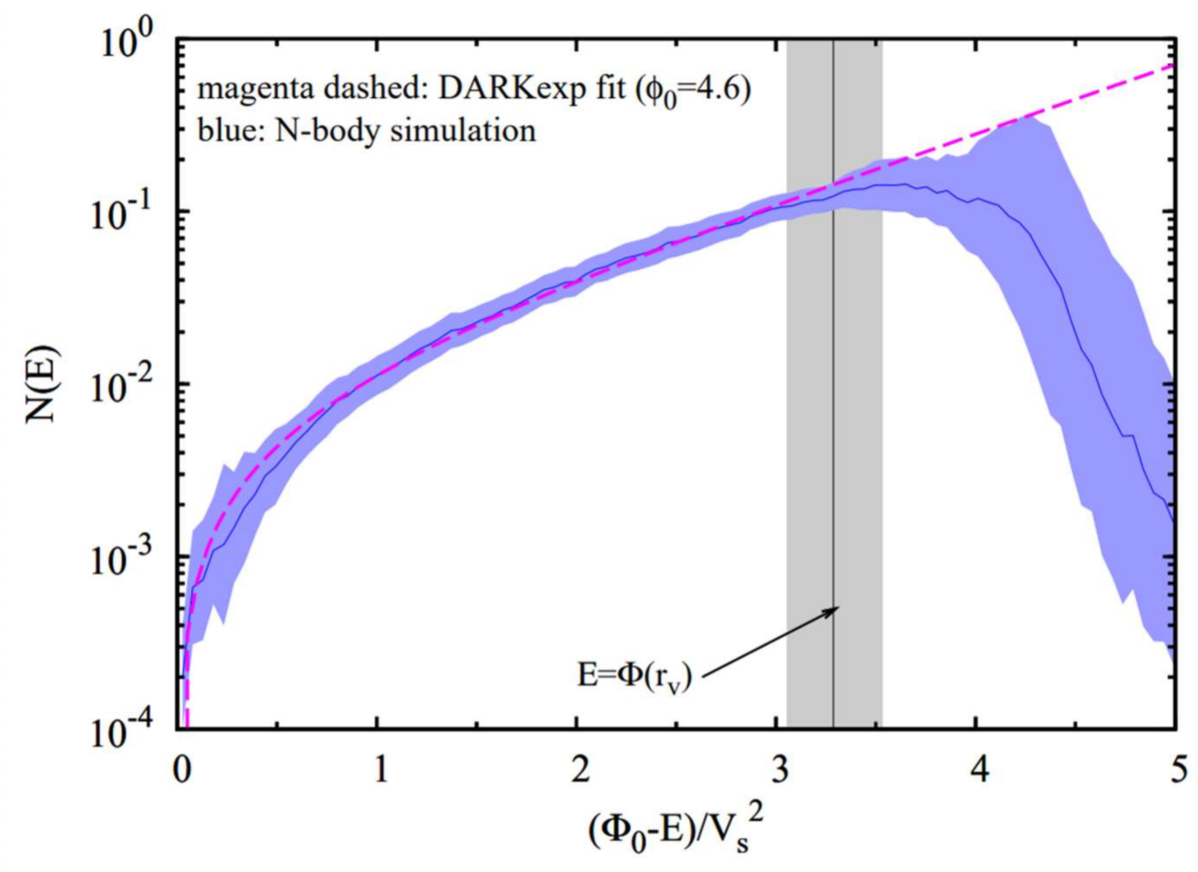


Image Credit: Hjorth, Williams, 2010

The DarkEXP Profile (2)

- In 2010, Hjorth and Williams presented "DarkEXP", a profile obtained through equilibrium statistical mechanics which fits simulation data:

$$N(E) = A(e^{-(E-\phi)/E_s} - 1)$$

- ϕ is the central potential of the system, such that $N(E) \to 0$ at E_{min}.
- A is a normalization constant such that $\int N(E) = 1$
- E_s represents a characteristic energy scale.

Comparison of Profiles: Main Premise

- Both profiles are shown to match simulation, so there should be an approximate relation between the parameters:

NFW

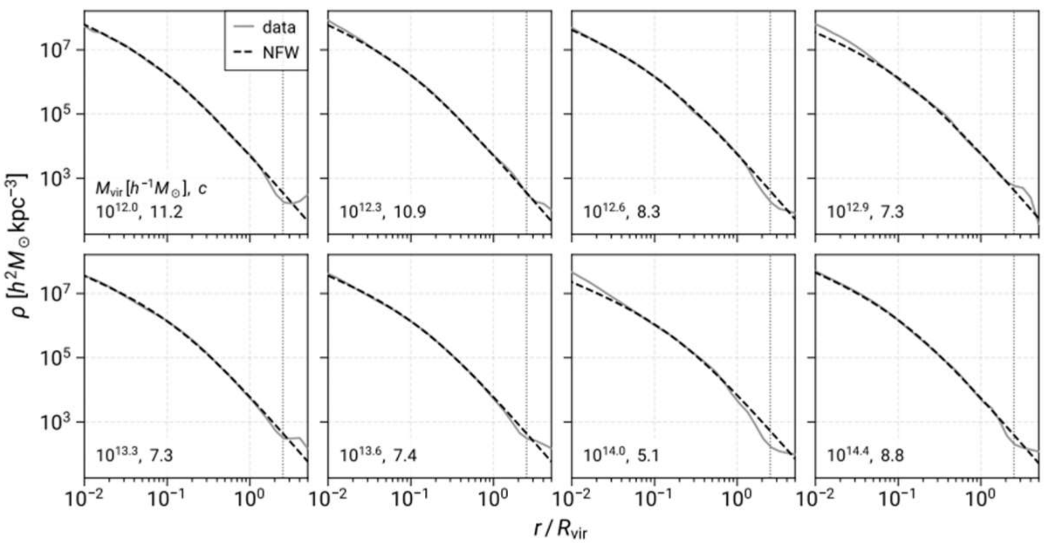
$$\rho(r) = \frac{\rho_0}{\frac{r}{R_S}\left(1 + \frac{r}{R_S}\right)^2}$$

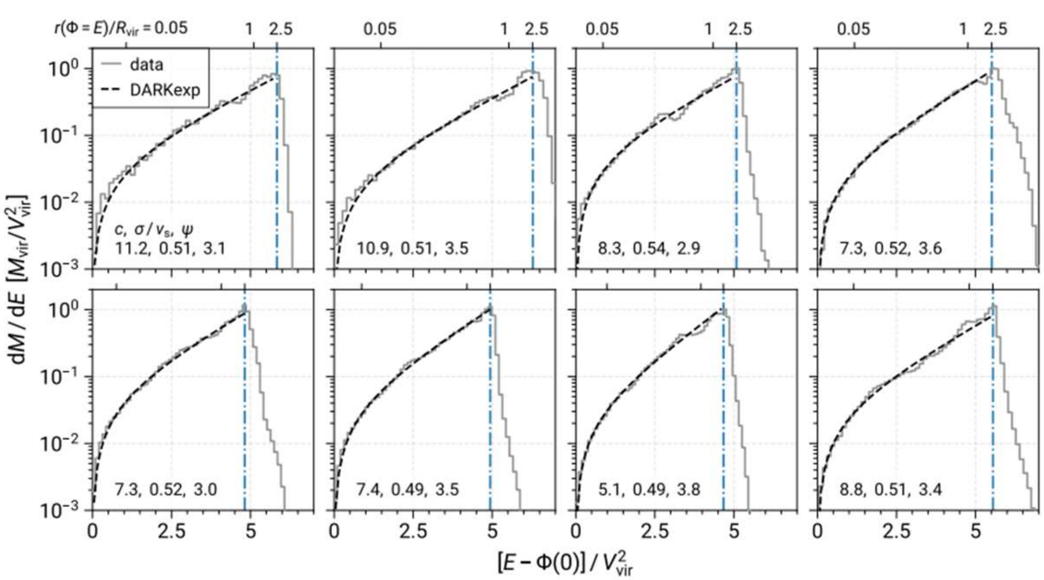
$$\Longleftrightarrow$$

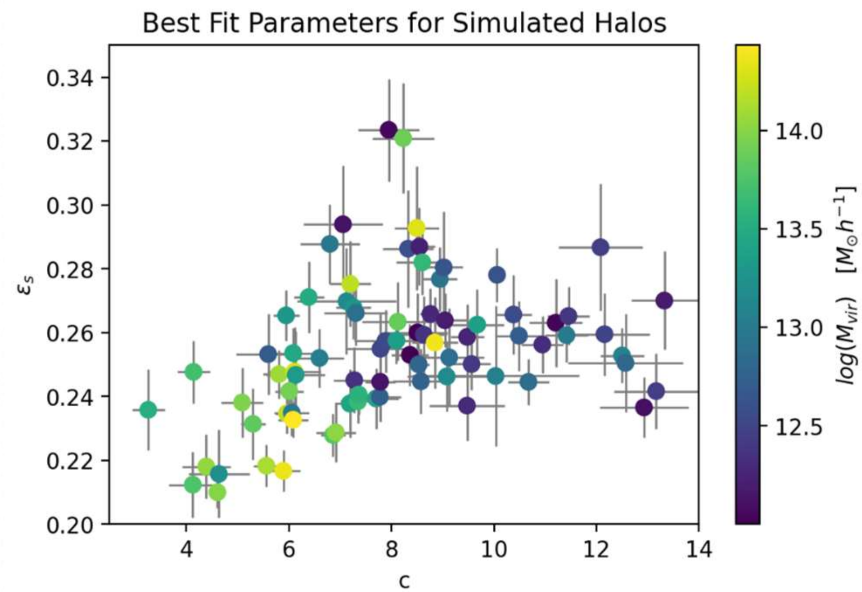
DarkEXP

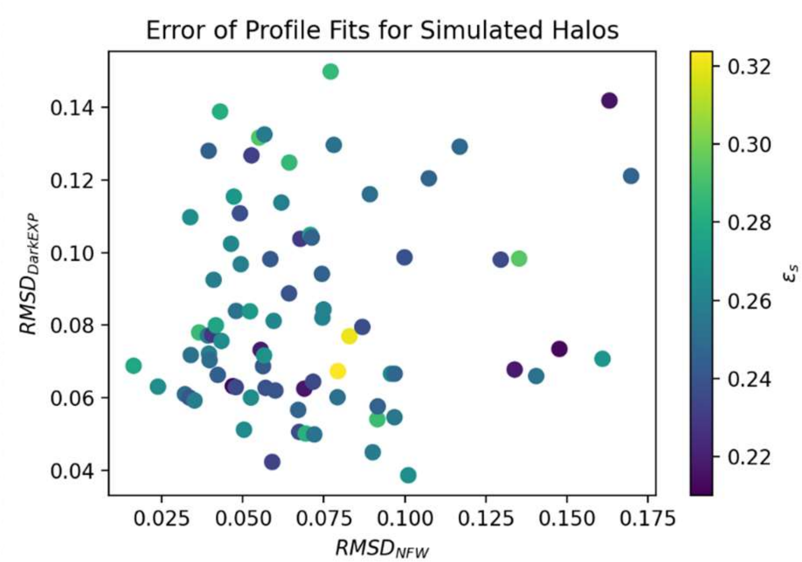
$$N(E) = A(e^{-(E-\phi)/E_s} - 1)$$

Comparison using N-Body Simulations









Relating the Two Profiles

- We define the distribution function:

$$\int d^3x d^3v f(\vec{x}, \vec{v}, t) = 1 \qquad dP = d^3x d^3v f(\vec{x}, \vec{v}, t)$$

- Conservation of probability gives the collisionless Boltzmann Equation:

$$\frac{\partial f}{\partial t} + \mathrm{v} \cdot \frac{\partial f}{\partial \boldsymbol{x}} - \frac{\partial \phi}{\partial \boldsymbol{x}} \cdot \frac{\partial f}{\partial \boldsymbol{v}} = 0$$

- Define an integral of motion (IOM) to be:

$$\frac{d}{dt} I[x(t), v(t)] = 0$$

$$\frac{\partial I}{\partial \boldsymbol{x}} \cdot \frac{\partial \boldsymbol{x}}{\partial t} + \frac{\partial I}{\partial \boldsymbol{v}} \cdot \frac{\partial \boldsymbol{v}}{\partial t} = 0 \qquad \Rightarrow \qquad \boldsymbol{v} \cdot \frac{\partial I}{\partial \boldsymbol{x}} - \frac{\partial \phi}{\partial \boldsymbol{x}} \cdot \frac{\partial I}{\partial \boldsymbol{v}} = 0$$

Relating the Two Profiles (2)

- Jeans Theorem: Any steady-state solutions must depend on the phase space coordinates through only IOM, and any function of IOM must be a steady state solution:

$$\frac{d}{dt} f(I_1, \ldots, I_n) = \sum_{m=1}^{n} \frac{\partial f}{\partial I_m} \frac{dI_m}{dt} = 0$$

- In spherical symmetry, integrals of motion are E and $\vec{\mathrm{L}}$.
- A distribution function that only depends on E is called ergodic

$$f(E) = f(\frac{v^2}{2} + \phi(r))$$

Relating the Two Profiles (3)

- If you have the distribution function, it is straightforward to determine the density:

$$\rho(r) = M \int d^3v\, f\left(\frac{1}{2}v^2 + \phi(r)\right) = 4\pi M \int dE\; f(E)\sqrt{2(E - \phi(r))}$$

- We can also relate the distribution function to the number density:

$$g(E) = \int d^3r d^3v\, \delta\left(\frac{1}{2}v^2 + \phi(r) - E\right) = 16\pi^2 \int dr\, r^2\sqrt{2(E - \phi(r))}$$

$$N(E) = g(E)f(E)$$

- Thus, we can relate N(E) and ρ(r)

$$\rho(r) = 4\pi M \int dE\; \frac{N(E)}{g(E)}\sqrt{2(E - \phi(r)}$$

- Density must also determine the potential

$$\nabla^2\phi = 4\pi G\rho$$

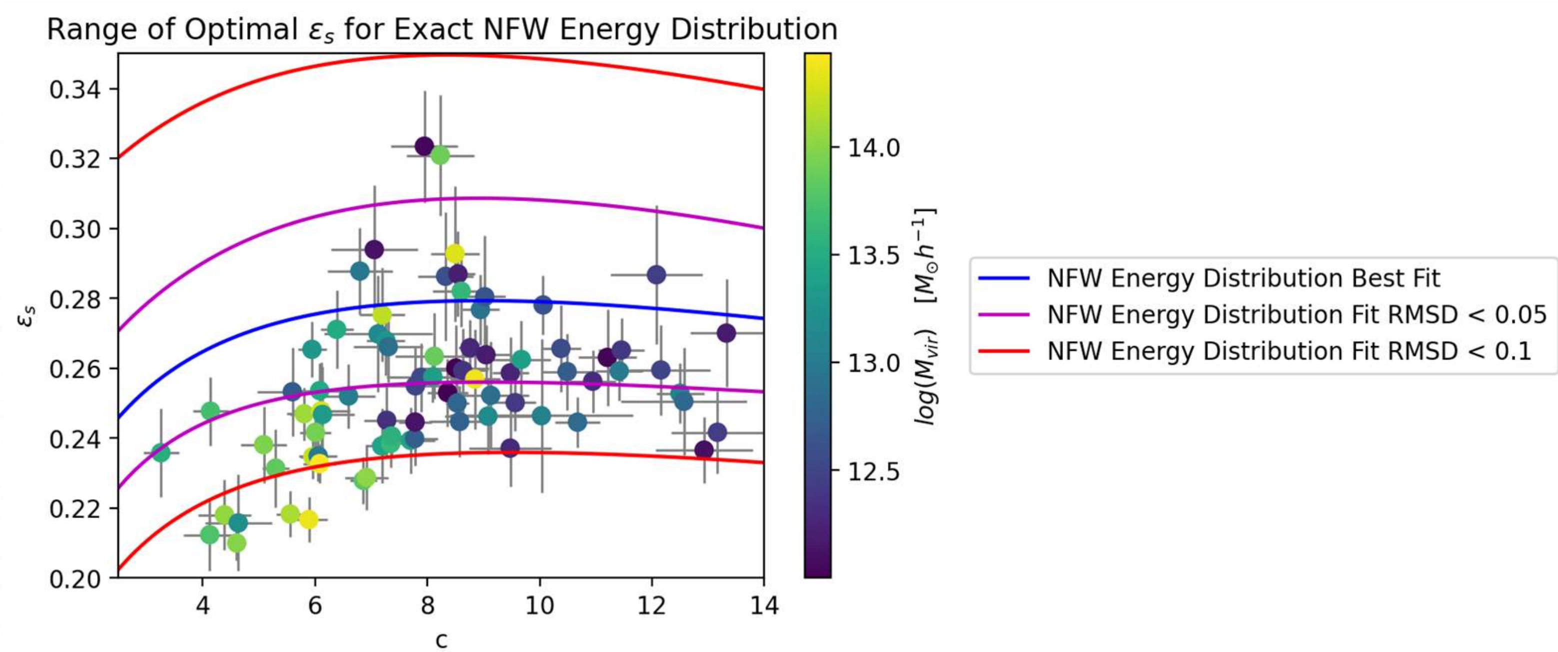
A Self Consistent Energy Distribution

- The distribution function can be calculated through an Eddington inversion:

$$f(E) = \frac{1}{\sqrt{8}\pi^2 M} \frac{d}{dE} \int \frac{d\phi}{\sqrt{\phi - E}} \frac{d\rho}{d\phi}$$

- After inversion of the NFW profile, the density of states can be used to find N(E), and this can be fit to the DarkEXP profile.
- We should accept fits where root mean squared deviation is of the same order as the errors in the fits to simulated halos.

Comparison between Profiles



Conclusions

- The NFW and DarkEXP profiles both provide a good description of dark matter halos, albeit in different parameter spaces.
- Simulations show a relationship between the parameters of these profiles.
- We can understand this relationship theoretically by connecting the profiles by means of the distribution function, which is the fundamental descriptor of the system.