# Unitarity Expansion, Pions and Entanglement in $NN$ Scattering

THE GEORGE WASHINGTON UNIVERSITY
WASHINGTON DC

**H. W. Grießhammer**

Institute for Nuclear Studies
The George Washington University, DC, USA

GW INS
Institute for Nuclear Studies
THE GEORGE WASHINGTON UNIVERSITY
WASHINGTON, DC

1. Emergent Phenomena in Nuclear Physics: "Order From Chaos"
2. What Is The Unitarity Limit? And Why Should I Care?
3. Unitarity Expansion With Pions in $NN$ $S$-Waves
4. Concluding Conjecture and Questions

Office of Science
U.S. DEPARTMENT OF ENERGY

**How to root Nuclear Physics in QCD?**
**What is the underlying principle that makes simple structures emerge from complex nuclear dynamics?**

Office of Nuclear Physics
Exploring Nuclear Matter - Quarks to Stars

König/hg/Hammer/van Kolck: *Phys. Rev. Lett.* **118** (2017) 202501 [1607.04623 [nucl-th]]

**Yu Ping Teng/hg: MSc Thesis GW 2023 and [2410.09653 [nucl-th]]**

**Nathan Carter/Oliver Thim/hg: in preparation**

# 1. Emergent Phenomena in Nuclear Physics: "Order From Chaos"

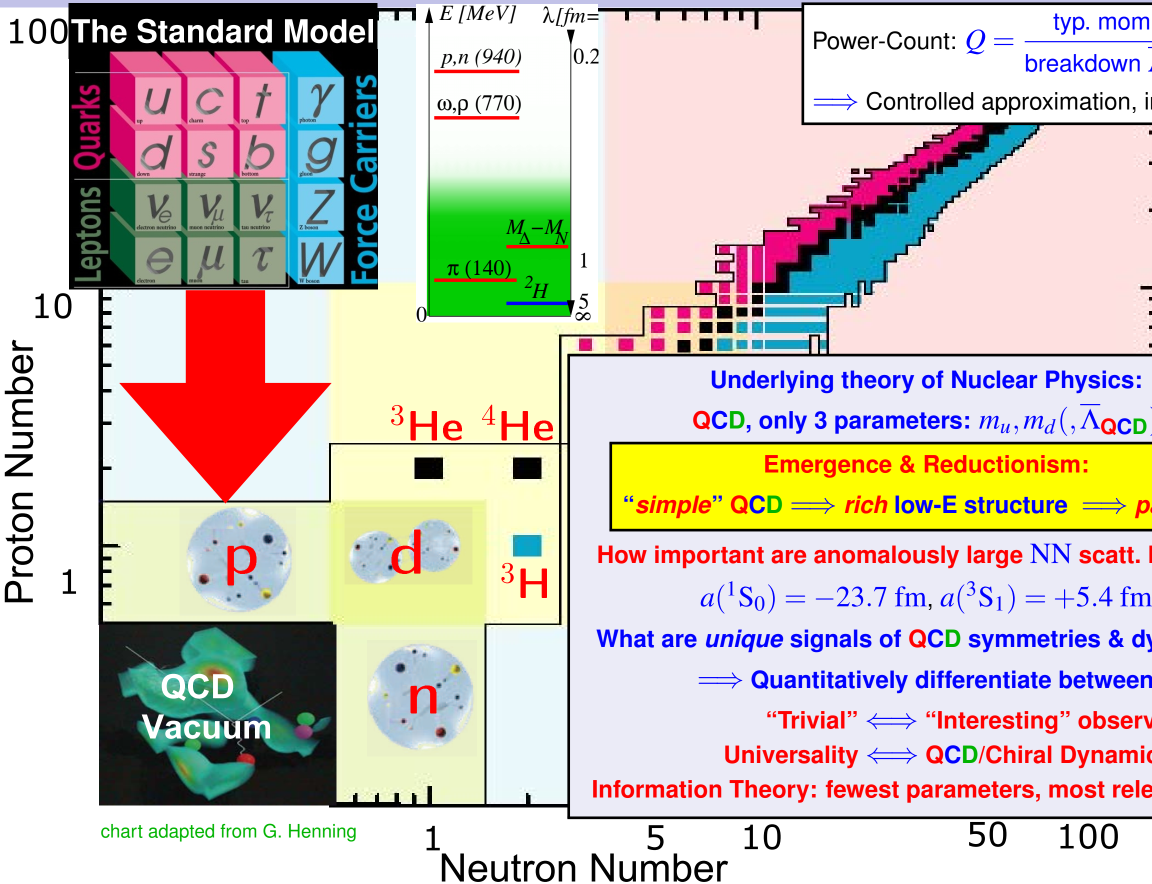



chart adapted from G. Henning

Power-Count: $Q = \frac{\text{typ. mom. } p}{\text{breakdown } \overline{\Lambda}_{\text{EFT}}} < 1$

$\Longrightarrow$ Controlled approximation, improvable.

**Underlying theory of Nuclear Physics:**

**QCD, only 3 parameters:** $m_u, m_d (, \overline{\Lambda}_{\text{QCD}})$

**Emergence & Reductionism:**

**"*simple*" QCD $\Longrightarrow$ *rich* low-E structure $\Longrightarrow$ *patterns***

**How important are anomalously large NN scatt. lengths?:**

$a(^1\text{S}_0) = -23.7 \text{ fm}, \ a(^3\text{S}_1) = +5.4 \text{ fm}$

**What are *unique* signals of QCD symmetries & dynamics?**

$\Longrightarrow$ **Quantitatively differentiate between:**

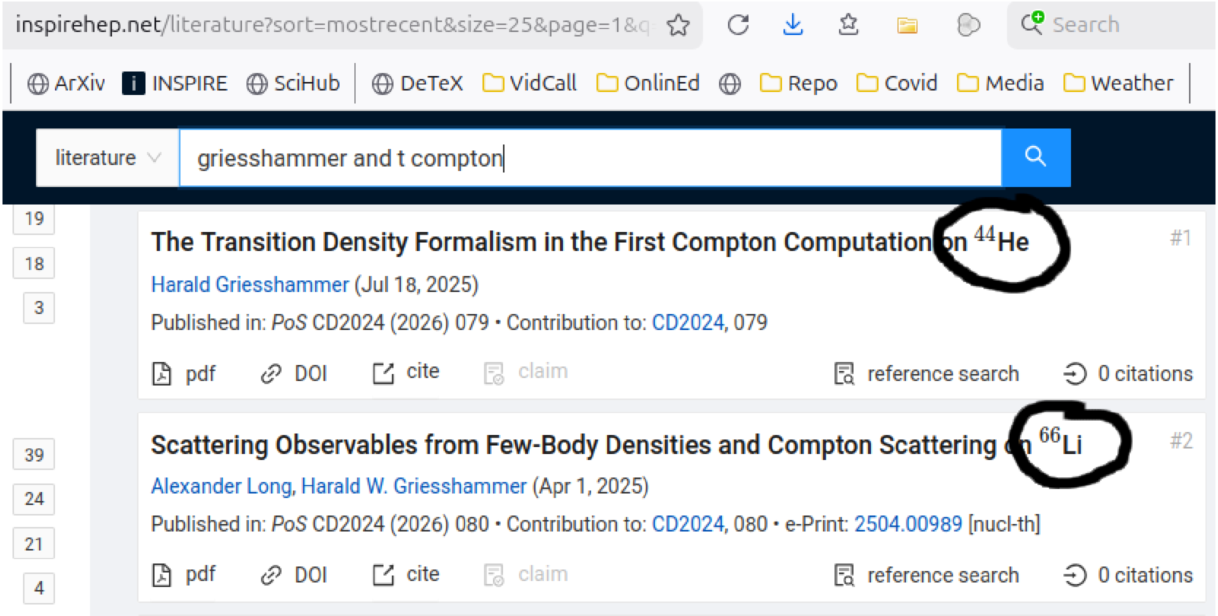
**"Trivial" $\Longleftrightarrow$ "Interesting" observables**

**Universality $\Longleftrightarrow$ QCD/Chiral Dynamics**

**Information Theory: fewest parameters, most relevant info!**

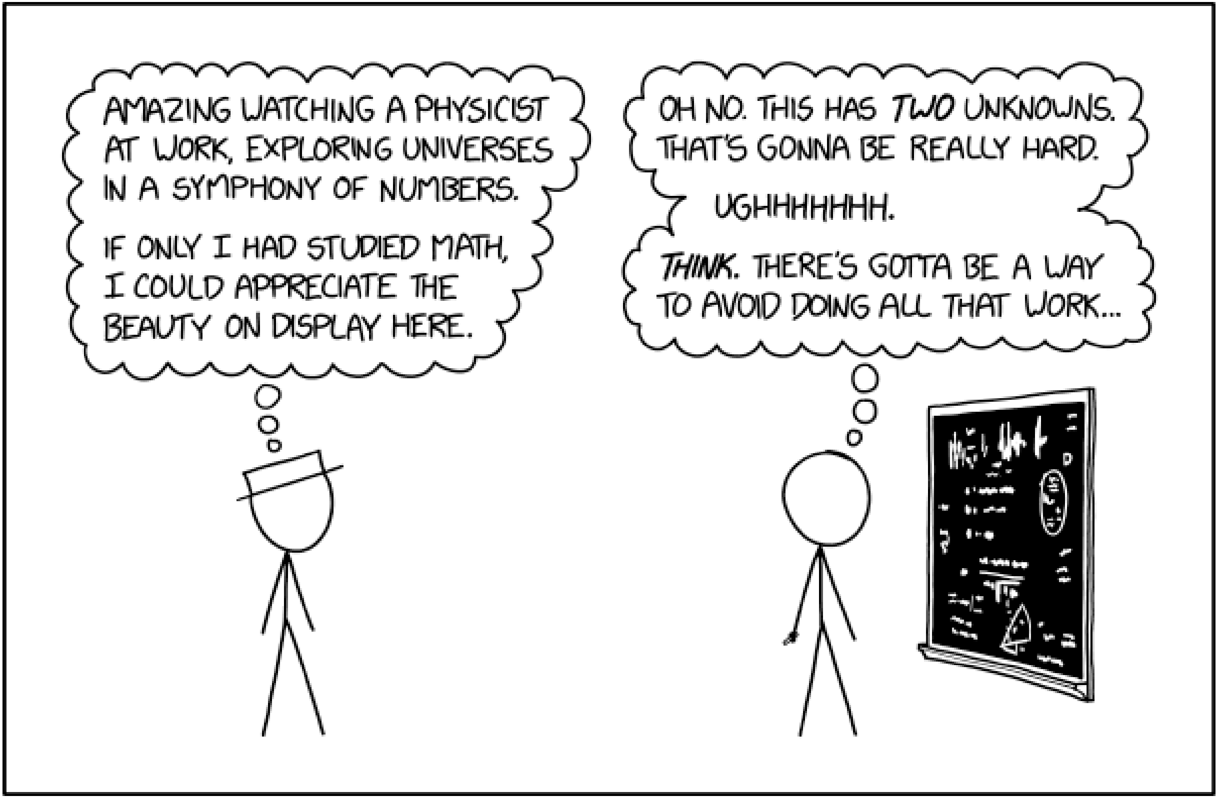
# Despite What INSPIRE Says, This is Not A Pattern We See

Thank you, Hans-Werner!




http://inspirehep.net/literature?sort=mostrecent&size=25&page=1&q=griesshammerandtcompton&ui-citation-summary=true

# (a) 3 Reasons To Simplify: Patterns; Reduce Computational Complexity; And…




xkcd 2019

## (b) Problems/Issues and Paths

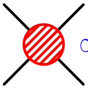
(1) **Do we have a *highly (let alone "most") effcient* organisational scheme?**

- What is the renormalisation-group consistent **power counting**? What is LO? OPE+X?
- Numerical/spin-isospin-momentum **complexity** of interactions increases with order.
- Uncertainty estimate bands should decrease with order.
- Does relative **importance/hierarchy** of few-$\mathrm{N}$ interactions change with $A$? Yes.
- Do we have *all* the **right (approximate) symmetries**? No.
- EFT Promise **Lossless Compression**: Encode information in smallest number of parameters at given level of accuracy: importance sampling.
- Christian Drischler: **"Superfluous information in $\chi$iral interactions"**;
  cf. eigenvector continuation, emulators,...

(2) **Do more "Strict Perturbation" about a LO Non-perturbative Interaction!**

- Better Physics.
- Convolute only operators, not strengths: $< \alpha\mathcal{O} >= \alpha < \mathcal{O} >$ vs. $< \frac{1}{1-\alpha\mathcal{O}} >$
- Numerically/computationally simpler/less complex than "count potential and iterate".
- More efficient use of CPU time, emulators,...
- Clearer convergence patterns; fewer cutoff artefacts/deeply-bound states.
- Wider cutoff variation. $\Longrightarrow$ Better tests of proper renormalisation.

# 2. What Is The Unitarity Limit? And Why Should I Care?

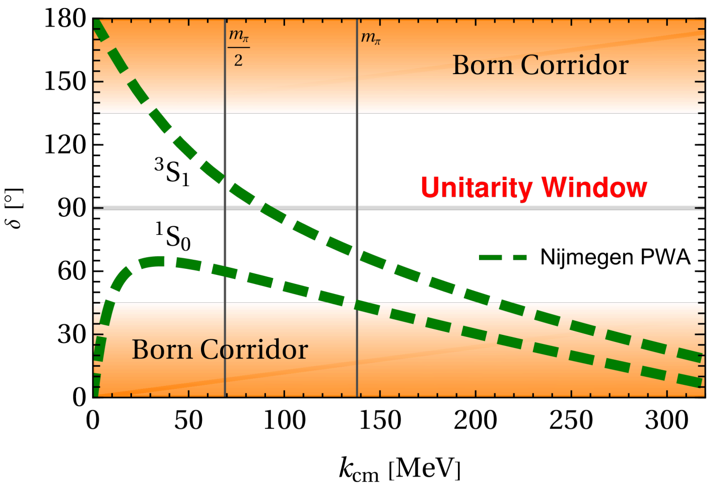


$\propto \frac{1}{\underbrace{k\cot\delta}_{\text{interaction}} - \mathrm{i}k} \to \left\{ \begin{array}{ll} \frac{1}{k\cot\delta}\left[1+\frac{\mathrm{i}}{\cot\delta}+\dots\right] & \text{for } \cot\delta \gg |\mathrm{i}| \text{, i.e. } \delta\to 0 \implies \\ \frac{1}{-\mathrm{i}k}\left[1+\frac{\cot\delta}{\mathrm{i}}+\dots\right] & \text{for } \cot\delta \ll |\mathrm{i}| \text{, i.e. } \delta\to 90^\circ \implies \end{array}\right.$

interaction ↓ Unitarity

**Born Approximation:**
interactions small & perturbative,
their details & scales drive $A_{\mathrm{NN}}$
no bound states

***Unitarity Limit implies Universality:***
**interaction strong: *non-perturbative*,**
**squeezed against prob. conservation:**
**details irrelevant, unitarity drives $A_{\mathrm{NN}}$;**
***Unitarity Expansion at LO:***
**no scales in $A_{\mathrm{NN}}$, bound state at $k=0$.**




**Unitarity Window**: $|\cot\delta| \lesssim 1$ ($45^\circ \lesssim \delta \lesssim 135^\circ$)

$\implies$ LO NN nonperturbative in $^1\mathrm{S}_0$ & $^3\mathrm{S}_1$ for

$30\ \mathrm{MeV} \lesssim k_{\mathrm{cm}} \lesssim [1.5\dots2]m_\pi$

Outside: **Born Corridors**

LO perturbative for $|\cot\delta| \gtrsim 1$ ($|\delta| \lesssim 45^\circ$)

**How much of Nuclear Physics does really depend on details of QCD?**

**How much just from (corrections to) universal aspects around Unitarity?**

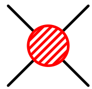
# (a) Symmetries in the Unitarity Limit

(1) **Amplitude saturated at Unitarity Limit:** $\sigma = \dfrac{4\pi}{k^2}$ maximal (probability conservation).

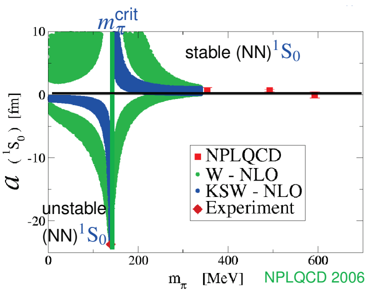
(2) **Scale Invariance:** $\vec{k} \to e^{\lambda}\,\vec{k}$. inside nonrel. Conformal/Schrödinger Symmetry. . . Jackiw, Niederer, Hagen 1972 Mehen/Stewart/Wise 2000 Nishida/Son 2007

(3) **Wigner-$\mathrm{SU}(4)$ Symmetry of combined spin-isospin rotations** $\begin{pmatrix} p\uparrow \\ p\downarrow \\ n\uparrow \\ n\downarrow \end{pmatrix} \to U \begin{pmatrix} p\uparrow \\ p\downarrow \\ n\uparrow \\ n\downarrow \end{pmatrix}$ Wigner, Hund 1937 for heavy nuclei cf. Mehen/Stewart/Wise 1999

In NN: ${}^1\mathrm{S}_0$-${}^3\mathrm{S}_1$ forms Wigner-super-sextuplet $\mathbf{6}_A(L=0)$

if $a({}^3\mathrm{S}_1) = a({}^1\mathrm{S}_0)$:  $= \dfrac{4\pi}{M}\dfrac{1}{-\frac{1}{a} - ik} = A_{\mathrm{NN}}({}^3\mathrm{S}_1) = A_{\mathrm{NN}}({}^1\mathrm{S}_0)$

**Theorists love Unitarity Limit as Nontrivial Fixed Point characterised by high symmetry:**
**Wigner-$\mathrm{SU}(4)+$ scale-invariance close to FP protected in renormalisation.**
**What About Nature?**

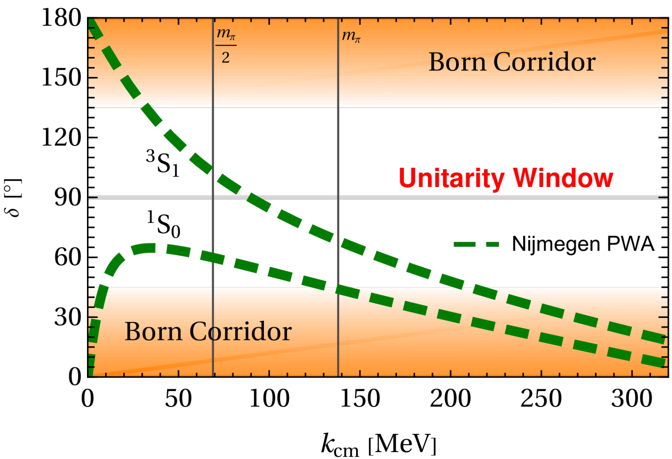


NPLQCD 2006

χEFT allows for but cannot explain anomalous scatt. lengths/shallow binding: Worlds with $a \lesssim \dfrac{1}{m_\pi}$!

**Accident or Symmetry?**

# (b) $\chi$EFT vs. Unitarity Expansion: Clash Of The Symmetries




NN S waves well in **Unitarity Window** $|\cot\delta| < 1$
for $30\ \text{MeV} \lesssim k_{cm} \lesssim [1.5\dots2]m_\pi$.

Upper limit close to $\overline{\Lambda}_{NN} = \frac{16\pi f_\pi^2}{g_A^2 M} \approx 300\ \text{MeV}$
where OPE becomes nonperturbative KSW 1999 FMS 2000.

$\Longrightarrow$ **How to merge Unitarity and $\chi$iral symmetry?**

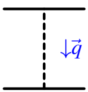
**Problem: Both fundamental & incompatible?!**

Pion breaks Unitarity: breaks scaling by $f_\pi, m_\pi$, splits Wigner by SD mixing.

Unitarity breaks Chiral Symmetry: tensor-OPE$\rightarrow 0$.

**"Wigner-breaking"**: split inside $\mathbf{6}_A(L=0)$, or mix with $L \neq 0$

**Unitarity dictates *absence/irrelevance*!**




$$: \ -\frac{g_A^2}{4f_\pi^2}\frac{1}{\vec{q}^2+m_\pi^2}\left[\underbrace{\frac{1}{3}(\vec{\tau}_1\cdot\vec{\tau}_2)(\vec{\sigma}_1\cdot\vec{\sigma}_2)\vec{q}^2}_{\text{central: Wigner-symmetric}} + \underbrace{(\vec{\tau}_1\cdot\vec{\tau}_2)\left((\vec{\sigma}_1\cdot\vec{q})(\vec{\sigma}_2\cdot\vec{q})-\frac{1}{3}(\vec{\sigma}_1\cdot\vec{\sigma}_2)\vec{q}^2\right)}_{\text{tensor: Wigner-splitting, mixes } S\leftrightarrow D,\ D\rightarrow D}\right]$$

**Chiral Symmetry dictates *presence* and relative size!**

**Each symmetry hidden from other: *"accidental"* $\Longrightarrow$ How do they manifest/emerge together in observables?**

Explore transition "no$\longrightarrow$nonperturbative pions" via Perturbative ("KSW") Pions (only undisputedly consistent $\chi$EFT).

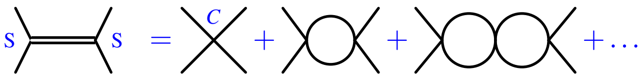
# 3. Unitarity Expansion With Pions in NN S-Waves

## (a) Perturbative Pions at N$^2$LO: $Q \sim \frac{1}{ka}, \frac{k, m_\pi}{\Lambda_{NN}} \ll 1$

based on Rupak/Shoresh [nucl-th/9902077] ($^1S_0$), Fleming/Mehen/Stewart [nucl-th/9911001] ($^1S_0$, $^3S_1$)
mod. for unitarity Teng/hg MSc thesis GW 2023, [2410.09653]

$\mathcal{O}(Q^{-1})$ **(LO):** Nonperturbative; no scale, perfect Wigner, pure S wave.

$$A^{(S)}_{-1} = \frac{4\pi i}{M}\,\frac{1}{k} =$$

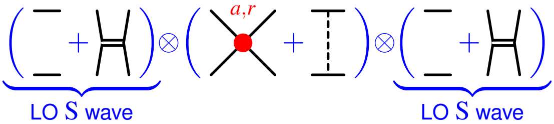



from structureless contacts $C\;(N^\dagger N)^2$

$\mathcal{O}(Q^{0})$ **(NLO):** Scaling and Wigner broken by contacts to reproduce PWA values of $a, r$.

Non-iterated OPE: central only, does not split Wigner but breaks scaling.

$$A^{(S)}_{0} =$$



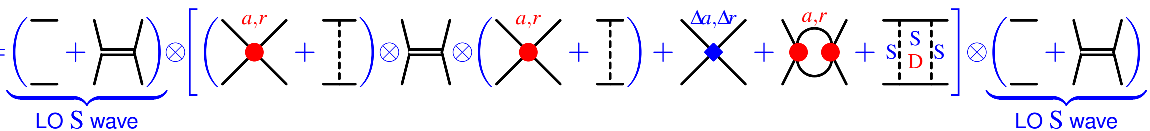

$\Longrightarrow$ **Unitarity, Wigner-SU(4) spin-isospin symmetry align naturally for Perturbative Pions at NLO.**

$\mathcal{O}(Q^{1})$ **(N$^2$LO):** Contacts adjusted to keep $a, r$ at PWA values; multiplied with non-iterated OPE (central only).

Once-iterated OPE: $A_{1\text{sym}}$: Central $S \to S \to S$: no Wigner-split but scaling broken: identical in $^1S_0$ and $^3S_1$.

$A_{1\text{split}}$: Tensor $S \to D \to S$ splits Wigner, breaks scaling: only in $^3S_1$.

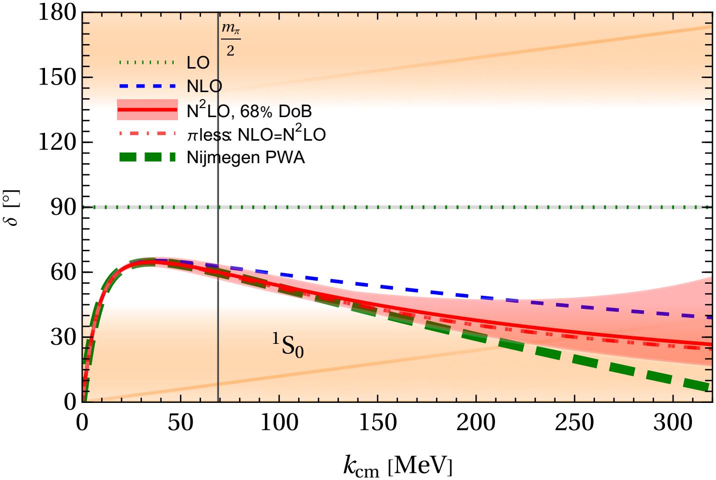
$$A^{(S)}_{1} =$$




$\Longrightarrow$ **Is splitting of Wigner-SU(4) spin-isospin symmetry for Perturbative Pions at N$^2$LO indeed small?**

# (b) Perturbative Pions at $N^2LO$: $^1S_0$

perturbative pions to $N^2LO$: Rupak/Shoresh 2000, Fleming/Mehen/Stewart 2000
unitarity for them: Teng/hg MSc Thesis GW 2023, [2410.09653]




$^1S_0$: central OPE $\Longrightarrow$ Wigner-symmetric.

$f_\pi, m_\pi$ break scaling.

$^1S_0$ is **"boring"** partial wave: no tensor int.

Bayesian truncation uncertainty at $68\%$ DoB.

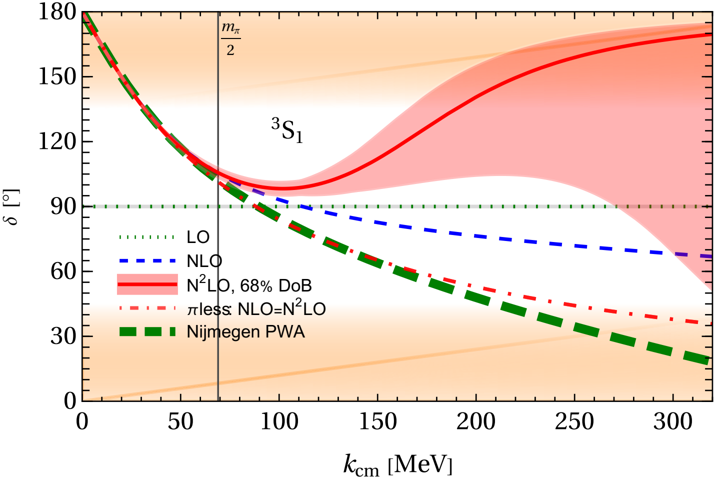
$\Longrightarrow$ Converges order-by-order $\lesssim 300$ MeV.

Agrees within uncertainties with PWA for $\lesssim 250$MeV (even outside Unitarity Window).

Compare to EFT($\not\pi$): minuscule impact of $\pi$.

# (c) Perturbative Pions at N$^2$LO: $^3S_1$

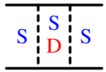
perturbative pions to N$^2$LO: Fleming/Mehen/Stewart 2000
unitarity for them: Teng/hg MSc Thesis GW 2023, [2410.09653]




$^3S_1$: pions split Wigner-$SU(4)$, break scale inv.

$^3S_1$ is **"interesting"** partial wave:

tensor-OPE $\Longrightarrow$ SD mixing from




$\Longrightarrow$ Terrible convergence (already in FMS):

Converges order-by-order $\lesssim 80$ MeV.

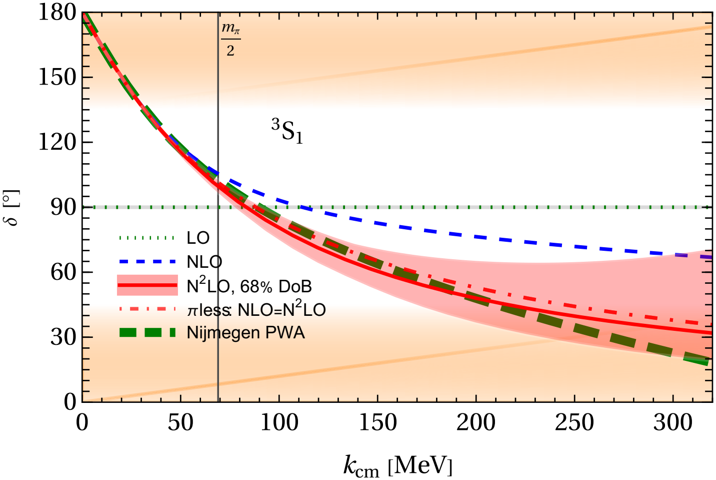
Agrees within uncertainties with PWA only for $\lesssim 70$ MeV (not even in Unitarity Window).

Compare to EFT($\not\pi$): huge impact of pion.

Source of discrepancy: tensor int. (via SD).

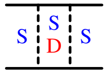
(central is identical in $^3S_1$&$^1S_0$)

# (c) Perturbative Pions at N$^2$LO: $^3S_1$




$^3S_1$: pions split Wigner-$SU(4)$, break scale inv.

$^3S_1$ is **"interesting"** partial wave:

tensor-OPE $\Longrightarrow$ SD mixing from 


**Broken Wigner-$SU(4)$ spoils convergence!**

**Idea:** Use Wigner-$SU(4)$-symmetric pion part.

$\Longrightarrow$ Set tensor OPE to zero.

$\Longrightarrow$ Only $^1S_0$-$^3S_1$ differences of $a$ & $r$ break Wigner-$SU(4)$.

RG-invariant, mildly $\chi$symmetry-breaking.

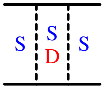
$\Longrightarrow$ Converges order-by-order $\gtrsim 300$ MeV.

Agrees within uncertainties with PWA for $\gtrsim 300$ MeV (even outside Unitarity Window).

Compare to EFT($\not\pi$): tiny impact of pion.

$\Longrightarrow$ **All very similar to $^1S_0$.**

# 4. Concluding Conjecture and Questions




in $^1S_0$: $V_C(r)$ —- in $^3S_1$: $\begin{pmatrix} V_C(r) & \cancel{\sqrt{8}V_T(r)} \\ \cancel{\sqrt{8}V_T(r)} & V_C(r) \cancel{-2V_T(r)} \end{pmatrix} \Longrightarrow$ Wigner-symm. is only $V_C(r)$
Wigner-$\mathrm{SU}(4)$ split by $V_T(r)$

**$\chi$EFT with Perturbative Pions in Unitarity Expansion $Q \sim \frac{1}{ka}, \frac{kr}{2}, \frac{m_\pi}{\Lambda_{\mathrm{NN}}} \ll 1$: needs $\delta \to \frac{\pi}{2} \Longrightarrow {}^1S_0, {}^3S_1$ only!**

**Chiral Physics:** $m_\pi, f_\pi, (\vec{\sigma}_1 \cdot \vec{q})(\vec{\sigma}_2 \cdot \vec{q})(\tau_1 \cdot \tau_2)$ seem opposed to Wigner, but $\mathrm{NN}$/few-$\mathrm{N}$ projection forces into it.

**Conjecture (at least for Perturbative Pions): Tensor/Wigner-$\mathrm{SU}(4)$ symmetry-splitting part of One-Pion Exchange is *super-perturbative* in few-$\mathrm{N}$ systems, i.e. does *not enter before* N$^3$LO.**

**$\Longleftrightarrow$ *Persistence:* Footprint of Symmetries in Unitarity Limit extends far into $p_{\mathrm{typ}} \gtrsim m_\pi$, more relevant than $\chi$iral symmetry in few-$\mathrm{N}$?!**

**$\Longleftrightarrow$ Information Theory: Compress to relevant info: fewest parameters at given accuracy!**

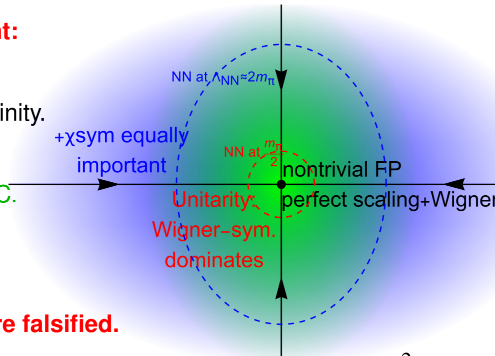
**Appeal: Fine-Tuning $\Longrightarrow$ High Symmetry at Nontrivial Fixed Point:**

Universality/scaling $+$ **Wigner-$\mathrm{SU}(4)$**

protected in renormalisation at FP $\Longrightarrow$ weakly split/broken in vicinity.

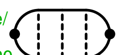
$\chi$iral symmetry not explicit at FP: less protected? $\Longrightarrow$ **Quantify!**

No Wigner in meson/1N sector $\Longrightarrow$ no change to $\chi$PT, HB$\chi$PT PC.




**"Coincidence":** N$^2$LO Perturbative Pions overpredict
$^3SD_1$ mixing, $^3D_1$ $\Longrightarrow$ Zero without tensor int. at N$^2$LO.

**Some Crucial Tests: If either fails without good reason, Conjecture falsified.**

N$^3$LO cf. Beane/Kaplan/Vuorinen 2009, Kaplan 2020 

$d\pi \to d\pi$, $\gamma d \to \pi d$ cf. Borasoy/hg 2003

$\mathrm{Nd}$ scattering cf. Bedaque/hg 2000

Nonperturbative Pions to N$^2$LO in strict perturbation LO: hg 2023

# (a) $\mathrm{NN}$ Scenario: What Is LO and Is It Perturbative? Depends on Partial Wave...

**S-Waves**

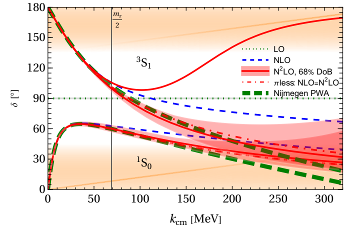
Perturbative about Unitarity. $\Longrightarrow$ Interaction details irrelevant.

Pions of little impact in Unitarity Window (but likely in probes!).

LO: only $1$ momentum-indep. contact $C_S\,(\mathrm{N}^\dagger\mathrm{N})^2$ so that $\frac{1}{a}=0$.

Plus central OPE? – tensor-OPE only at high orders...

$\Longrightarrow$ Overall patterns of Nuclear Physics.



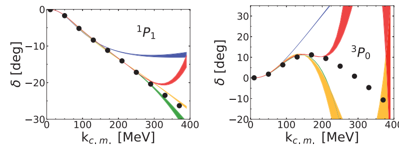

**P/D-Waves**

Unitarity irrelevant. $\Longrightarrow$ Interaction details important.

LO perturbative or nonperturbative? Birse, Long/..., Kaplan, ...

Pions may be important... $\Longrightarrow$ Full OPE at LO?

$\Longrightarrow$ Overall more details in patterns of Nuclear Physics.




Wu/Long [1807.04407]

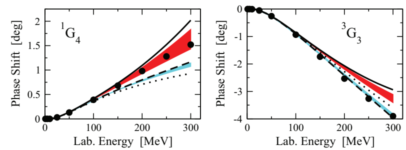
**Higher Waves**

Unitarity irrelevant. $\Longrightarrow$ Interaction details important.

LO perturbative: Born approximation Kaiser/Brockmann/Weise

OPE details important, but not overall.

$\Longrightarrow$ Details of Nuclear Physics.




Epelbaum/...[nucl-th/0304037]

**Simplify interactions, reduce computational complexity, concentrate on essentials.**

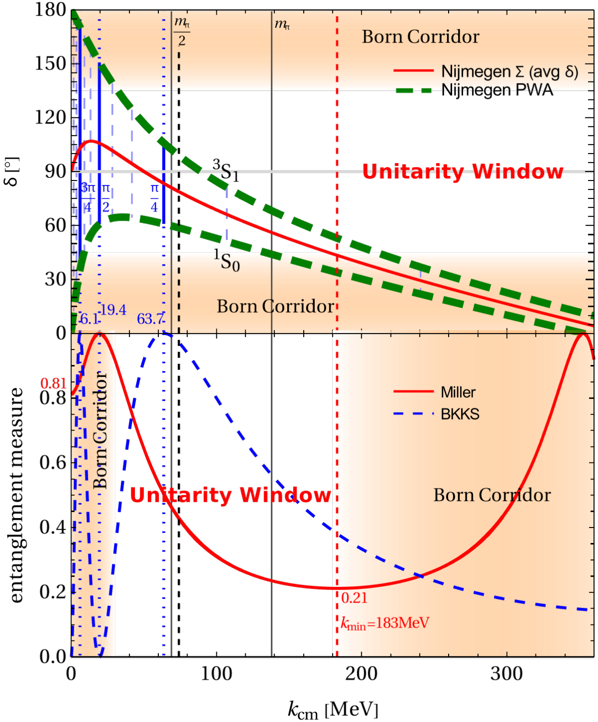
$\Longrightarrow$ **3$\mathrm{N}$ interactions, interactions with probes,...**

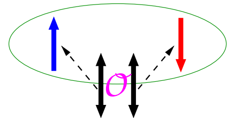
$\Longrightarrow$ **Explore few-/many-$\mathrm{N}$ systems, also with probes: Use Perturbation Theory beyond LO: DWBA, Born.**

# (b) What is the Small Parameter?: QM Entanglement?

**Need expansion parameter related to Wigner-$\mathrm{SU}(4)$ to characterise tensor suppression near Fixed Point.**

**Entanglement $\widehat{=}$ A-Causal Influence:** Deviation of QM multi-particle *states* from "classical" direct product $\bigotimes\limits_n |\,n\rangle$.




Scattering in $^1S_0$-$^3S_1$ entangles product states $|\!\uparrow\downarrow\rangle, |\!\downarrow\uparrow\rangle$:

$$S = e^{2i\Sigma}\left[\mathbb{1}\cos\Delta + i\,SW_\sigma\,\sin\Delta\right],$$

$\Sigma$ : phase avg.
$\Delta$ : phase diff.

$$SW_\sigma := \tfrac{1}{2}(\mathbb{1} + \vec{\sigma}_1\cdot\vec{\sigma}_2)\text{: spin swap } 1 \leftrightarrow 2$$

Wigner-$\mathrm{SU}(4)$: $S(\Delta=0) = e^{2i\Sigma}\,\mathbb{1} \Longrightarrow$ not entangling: classical!

**How to Define Entanglement Power of *Operator*?**:

$$\mathcal{E}_{\rm BKKS} = \sin^2[2\Delta]$$ Beane/Kaplan/Klco/Savage 2019

$$\mathcal{E}_{\rm Miller} = \frac{\sin^2\Delta(1+\cos^2\Delta - 2\cos\Delta\,\cos 2\Sigma)}{(1-\cos\Delta\,\cos 2\Sigma)^2}$$ Miller 2023
Cavallin/…2025

How to find $\mathcal{E}$ before computation??

# (c) QM Entanglement From Operator Geometry: $^1S_0$-$^3S_1$ Wave

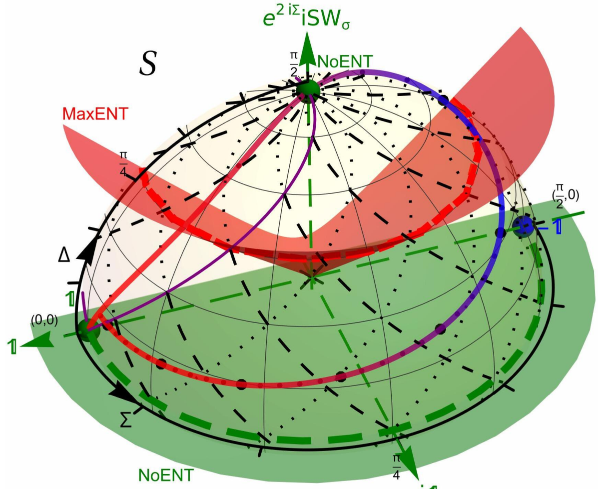
$$S = e^{2i\Sigma}\cos\Delta\,\mathbb{1} + e^{2i\Sigma}\cos\Delta\, iSW_\sigma$$

Non-entangling $\mathcal{O}\begin{pmatrix}\uparrow\downarrow\\ \downarrow\uparrow\end{pmatrix} \propto \begin{pmatrix}\uparrow\downarrow\\ \downarrow\uparrow\end{pmatrix}$ or $\begin{pmatrix}\downarrow\uparrow\\ \uparrow\downarrow\end{pmatrix}$, $SW_\sigma^2 = \mathbb{1}$.

$\mathrm{NoENT} := \{|A|\,e^{i\alpha}\,\mathbb{1};\ |B|\,e^{i\beta}\,SW_\sigma\} \supset \{e^{2i\Sigma}\,\mathbb{1}\}_{\mathrm{Wigner}}$!

$\mathrm{NoENT}^n \subset \mathrm{NoENT}$ subgroup: bounces on axes.

$\Longrightarrow$ Interpret $S$-matrix as $S^2 \times S^1$ manifold in complex-$2$-dimensional **Operator Space** $\mathcal{H}_{\mathrm{Op}}$ with axes $\{\mathbb{1}; SW_\sigma\}$ orthonormal under Hilbert-Schmidt scalar product $\langle\mathcal{O}_1;\mathcal{O}_2\rangle := \frac{1}{2}\mathrm{tr}[\mathcal{O}_1^\dagger\mathcal{O}_2]$.




$3_\mathbb{R}$-dim. projection of $2_\mathbb{C}$-dim. torus

$\Longrightarrow$ **Entanglement Power of $\mathcal{O}$**
$\widehat{=}$ **"Combined (Mis-)Alignment relative to all NoENT axes.**
Independent of strength $\langle\mathcal{O},\mathcal{O}\rangle$.

$$\mathrm{CMA}[\mathcal{O}] := \underbrace{4}_{\text{max is } 1}\ \underbrace{\frac{|\langle\mathbb{1},\mathcal{O}\rangle|^2}{\langle\mathcal{O},\mathcal{O}\rangle}}_{\cos^2\theta_H(\mathcal{O};\mathbb{1})}\ \underbrace{\frac{|\langle SW_\sigma,\mathcal{O}\rangle|^2}{\langle\mathcal{O},\mathcal{O}\rangle}}_{\cos^2\theta_H(\mathcal{O};SW_\sigma)} = \begin{cases} 0 & \mathrm{NoENT}\\ 1 & \mathrm{MaxENT}\end{cases}$$

align with $\mathbb{1}$ align with $SW_\sigma$

$\Longrightarrow$ Max. mis-alignment with all axes $\widehat{=}$ maximal entanglement power: $\mathcal{O} \in \mathrm{MaxENT} := \{|A|\,[e^{i\alpha}\,\mathbb{1} + e^{i\beta}\,SW_\sigma]\}$: $\mathrm{MaxENT}^n \not\subset \mathrm{MaxENT}$

**Constructed Entanglement Power $\mathrm{CMA}[\mathcal{O}]$ from operator properties only (no action on states).**
**Systematisation by geometric interpretation: *a-priori* estimate of Operator Entanglement Power.**

hgrie forthcoming

**$S$-Matrix (BKKS)**

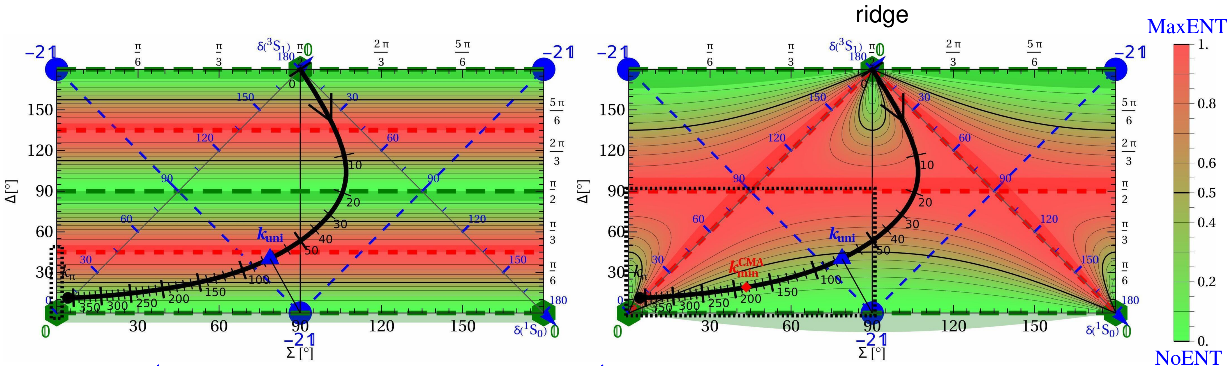
$\mathcal{E}_{\text{BKKS}} = \sin^2[2\Delta]$ entangled also without interaction

$\mathcal{E}_{\text{BKKS}}(k \to 0) = 0$ uniform

**Scatt. Amplitude $S - \mathbb{1}$ (Miller; Cavallin/...)**

$\mathcal{E}_{\text{Miller}} = \dfrac{\sin^2\Delta(1+\cos^2\Delta - 2\cos\Delta\cos 2\Sigma)}{(1-\cos\Delta\,\cos 2\Sigma)^2}$ entangled by interaction only

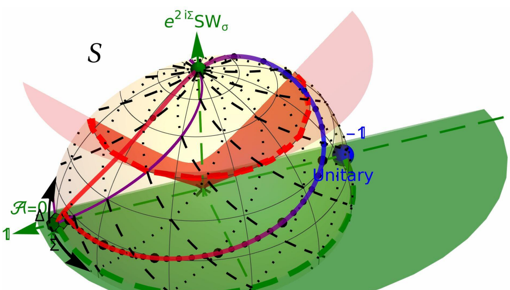
$\overset{k\to 0}{=} \left(\frac{a_t^2 - a_s^2}{a_t^2 + a_s^2}\right)^2 \approx 0.8113$ not uniform, near MaxENT




⬢: Unity $\mathcal{A} = \mathbb{0}$ (no scattering) ●: Unitarity $\mathcal{A} \propto -\mathbb{1}$: non-trivial Fixed Point

Nijmegen trajectory in landscapes: not particularly close to Unitarity FP, not particularly deep in SmallENT territory.

NoENT & MaxENT lines change as $S$-matrix manifold moves w.r.t. coordinate axes $\widehat{=}$ differ by interpretation of "scattering".



# (d) Nijmegen PWA Trajectory in the Entanglement Landscapes

hgrie forthcoming

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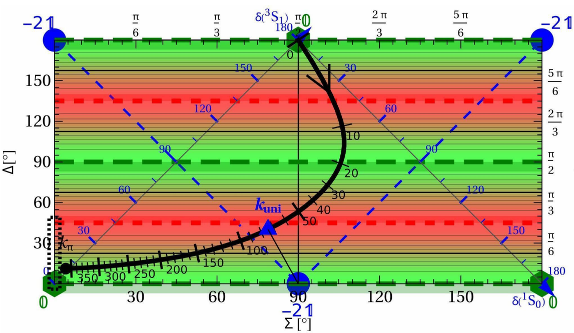


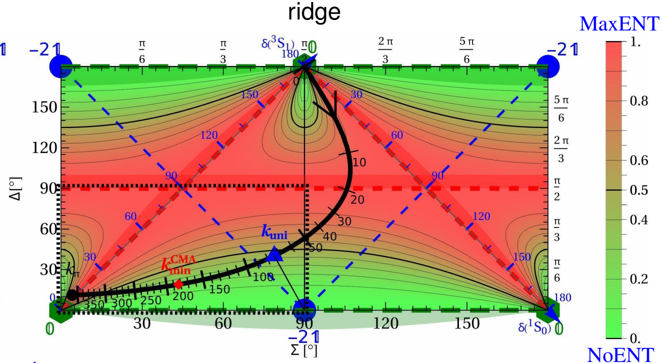


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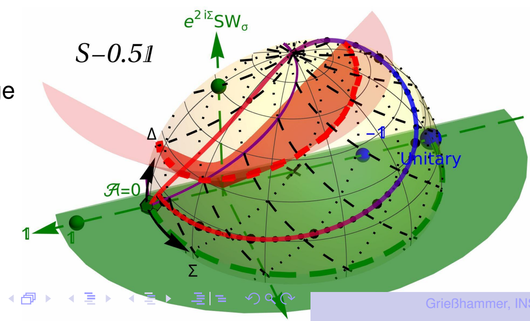

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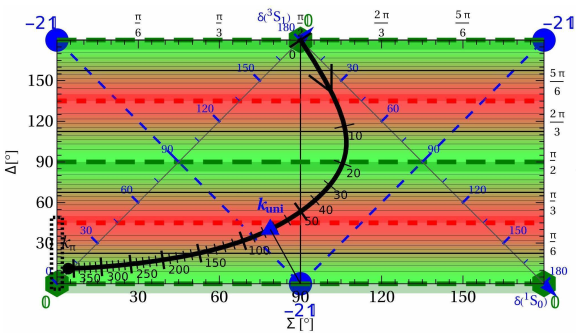


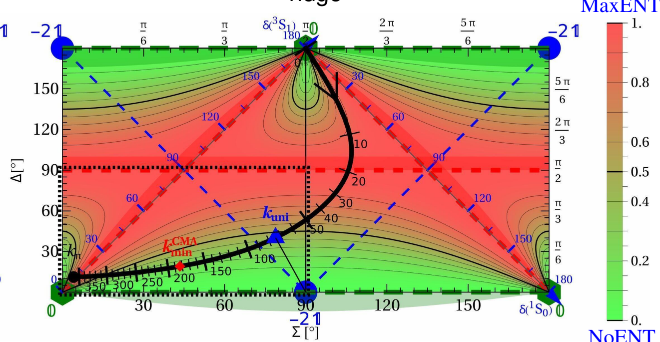


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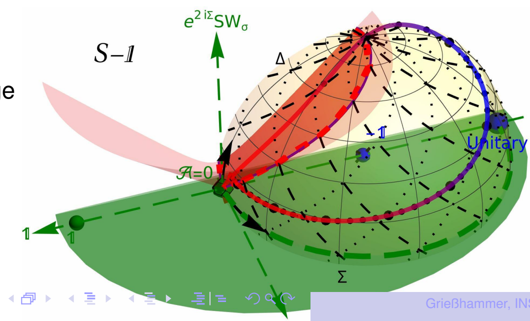
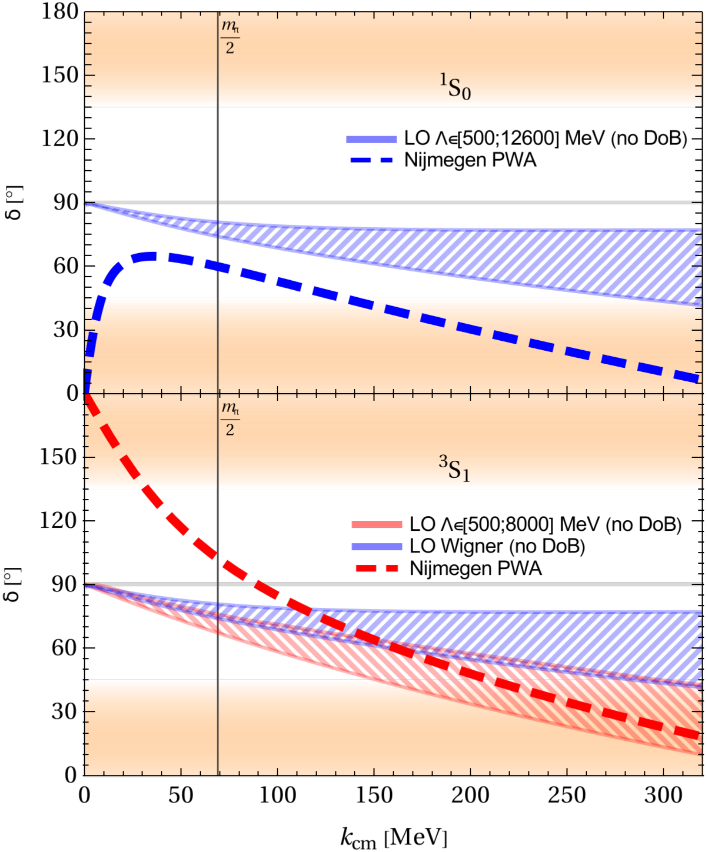


# (e) Nonperturbative Pions at LO: Maybe Not Hopeless




LO, $1$ mom.-indep. CT, Gaußian regulator.

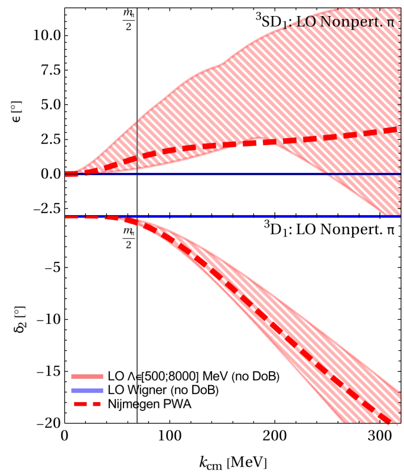
Already deviates from Unitarity $\delta = 90^\circ$.

$\Longrightarrow$ Explicit scale breaking at LO,

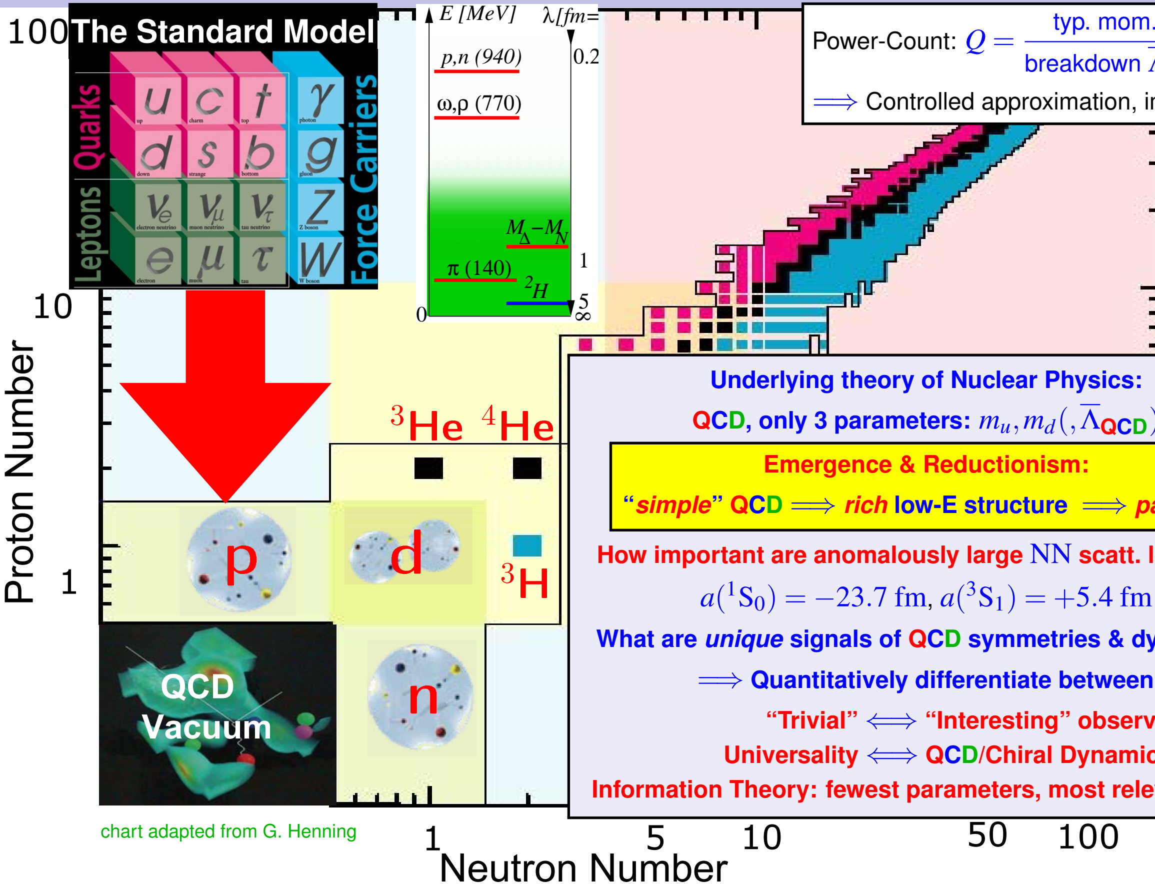
$$r = \begin{cases} {}^1S_0/\text{Wigner } [1\dots2]\text{fm}; {}^3S_1\ [1.2\dots2.5]\text{fm} \\ \text{PWA } 2.767(9)\text{fm} \quad 1.852(2)\text{fm} \end{cases}$$

Tensor/Wigner-splitting less compatible with unitarity than central/Wigner-symmetric.

Corridors: $\Lambda$-variation, *Not* Bayesian DoBs.



# 4. Concluding Conjecture and Questions




Power-Count: $Q = \frac{\text{typ. mom. } p}{\text{breakdown } \overline{\Lambda}_{\text{EFT}}} < 1$

$\Longrightarrow$ Controlled approximation, improvable.

**Underlying theory of Nuclear Physics:**

**QCD, only 3 parameters:** $m_u, m_d (, \overline{\Lambda}_{\text{QCD}})$

**Emergence & Reductionism:**

**"*simple*" QCD $\Longrightarrow$ *rich* low-E structure $\Longrightarrow$ *patterns***

**How important are anomalously large NN scatt. lengths?:**

$$a(^1\mathrm{S}_0) = -23.7\ \mathrm{fm},\ a(^3\mathrm{S}_1) = +5.4\ \mathrm{fm}$$

**What are *unique* signals of QCD symmetries & dynamics?**

$\Longrightarrow$ **Quantitatively differentiate between:**

**"Trivial" $\Longleftrightarrow$ "Interesting" observables**

**Universality $\Longleftrightarrow$ QCD/Chiral Dynamics**

**Information Theory: fewest parameters, most relevant info!**

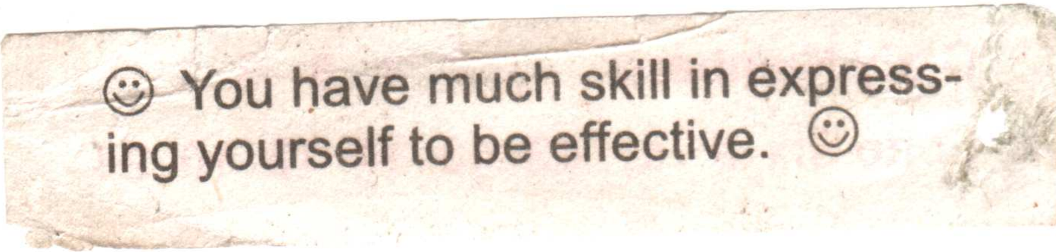
# (e) Nonperturbative Pions at LO: Maybe Not Hopeless

(1) **Do we have a *highly (let alone "most") effcient* organisational scheme?**

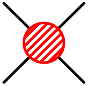
- What is the renormalisation-group consistent **power counting**? What is LO? OPE+X?
- Numerical/spin-isospin-momentum **complexity** of interactions increases with order.
- Uncertainty estimate bands should decrease with order.
- Does relative **importance/hierarchy** of few-N interactions change with $A$? Yes.
- Do we have *all* the **right (approximate) symmetries**? No.
- EFT Promise **Lossless Compression**: Encode information in smallest number of parameters at given level of accuracy: importance sampling.
- Christian Drischler: **"Superfluous information in $\chi$iral interactions"**;
  cf. eigenvector continuation, emulators,...

(2) **Do more "Strict Perturbation" about a LO Non-perturbative Interaction!**

- Better Physics.
- Convolute only operators, not strengths: $< \alpha\mathcal{O} >= \alpha < \mathcal{O} >$ vs. $< \frac{1}{1-\alpha\mathcal{O}} >$
- Numerically/computationally simpler/less complex than "count potential and iterate".
- More efficient use of CPU time, emulators,...
- Clearer convergence patterns; fewer cutoff artefacts/deeply-bound states.
- Wider cutoff variation. $\Longrightarrow$ Better tests of proper renormalisation.


☺ You have much skill in express-
ing yourself to be effective. ☺

# (a) Unitarity Expansion in EFT($\not\pi$)

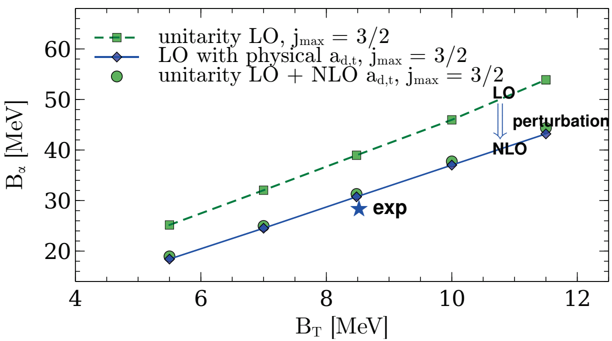
**EFT($\not\pi$)/ERE:**  $\propto \frac{1}{-ik}\left[1+\frac{k\cot\delta = -\frac{1}{a}+\frac{r}{2}k^2+\dots}{ik}+\dots\right] \to \underbrace{\frac{1}{-ik}}_{\text{LO}}\left[1+i\underbrace{\left(\underbrace{\frac{1}{ka}}_{<1?!}-\underbrace{\frac{kr}{2}}_{<1?!}\right)}_{\text{NLO correction}}+\dots\right]$

***a priori* justified if** $0 \leftarrow \frac{1}{a}$ (inverse scatt. length/ NN system size/ NN binding momentum) $\ll$ **typ. momentum** $k \ll \Lambda_{\not\pi} \overset{???}{\sim} m_\pi \sim \frac{1}{r}$ (breakdown/ resolution) **scale.**

**LO: No NN scale. $\Longrightarrow$ Nuclear Physics correlated to *just one* 3N *RG scale fixed by* $B_3$ *via Efimov effect*.**

**PARADIGM SHIFT: *Unitarity* de-emphasises details of NN & pions, emphasises 3N scale & Universality.**

**$\Longrightarrow$ Explore *Sweet Spot* for patterns, unique signals of QCD:**
**bound weakly enough to be insensitive to interaction details ($\frac{kr}{2} \ll 1$),**
**but strongly enough to be insensitive to detailed size of large system ($ka \gg 1$).**




$B_{^3\mathrm{H}} - B_{^3\mathrm{He}}$: NLO: $[0.92 \pm 0.18]$ MeV; exp: $0.764$ MeV

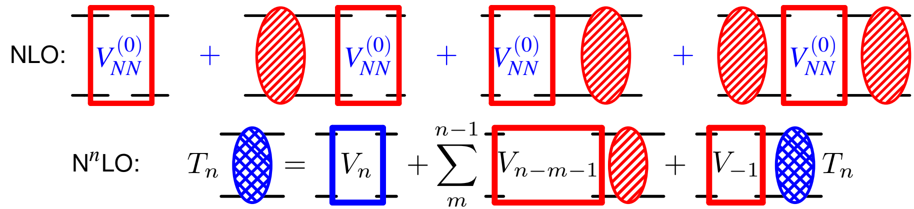
| | Fermion Unitarity LO $\to$ NLO | exp $^4$He/$^3$H |
|---|---|---|
| ground: $B_4/B_3$ | $4.6 \to 3.8 \pm 0.2$ | $3.66$ |
| excited: $B_4^*/B_3$ | $\sim 1.1 \to \sim 0.98 \pm 0.05$ | $0.96$ |

| **Symm. Nucl. Matter** | $\rho_0$ [fm$^{-3}$] | $B/A$ [MeV] | $E_{\mathrm{sym}}$ [MeV] | $L$[MeV] slope of $E_{\mathrm{sym}}$ | $K_\infty$[MeV] compressib. |
|---|---|---|---|---|---|
| Kievsky/... EFT($\not\pi$)-inspired | $0.15$ | $-16$ | $35$ | $70$ | $251$ |
| exp | $0.16$ | $-16$ | $\approx 30$ | $[40\dots60]$ | $210$ |

# (b) Do Contributions to *Observables* Decrease With Increasing Order?

$\Longrightarrow$ Find radius of convergence $k \lesssim \overline{\Lambda}_{\text{EFT}}$, systematically estimate truncation error (Bayes) –
and *only then* compare to data: beware of confirmation bias.

**Corrections in $Q \ll 1$ by "strict perturbation" about LO (Distorted-Wave Born; efficient way Vanasse 1305.0283):**
**$\Longrightarrow$ Power-counting of amplitudes (observables); simple, no resummation artefacts.**




**Use/Develop More Strict-Perturbation Methods!** cf. hg notes Trento 2018-21;

$\rightarrow$ Oliver Thim; $\Delta\mathcal{O}_0 = \dfrac{\mathcal{O}[V_{-1}+\epsilon\, V_0]-\mathcal{O}[V_{-1}]}{\epsilon}$ with $\epsilon \to 0$ Shi/.../Long/... PRC 106 (2022) 015505; [2205.02000] ...

**Contrast to Popular "Partially-Resummed Perturbation"** Weinberg 1990

Power-count $V_{NN}$ & iterate $\Longrightarrow T = \dfrac{V_{\text{LO}}+V_{\text{NLO}}+\dots}{1-(V_{\text{LO}}+V_{\text{NLO}}+\dots)\,G_{NN}}$.

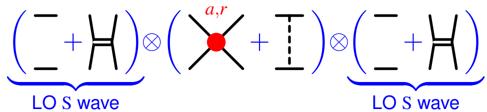
$\Longrightarrow$ Obscures PC in observables, unphysical poles around $\overline{\Lambda}_{\text{EFT}}$: artefacts, wrong causal structure.

$\Longrightarrow$ Limited to small cutoff variation range $\Lambda \approx \overline{\Lambda}_{\text{EFT}} \pm 20\%$, implementation & numerics more difficult.

Works *under assumption that* expansion indeed small, e.g. $1+x+x^2+x^3-\dfrac{1}{1-x} = x^4+\mathcal{O}(x^5)$ *if* $|x|<1$.
But resummed version loses control over convergence test & over which interactions needed for
$\Lambda$-independence (UV changed!). May still provide some guidance/insight – but beware of missed CTs!

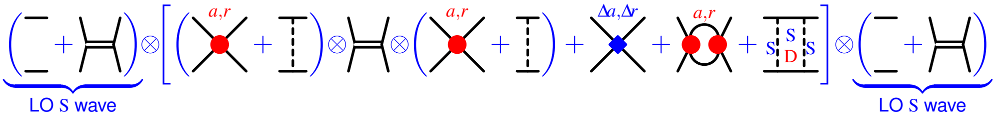
**LO:** $A^{(\mathrm{S})}_{-1}(k) = \frac{4\pi\mathrm{i}}{M}\,\frac{1}{k}$ is only $\mathrm{S}$ wave


**NLO:** $A^{(\mathrm{S})}_{0}(k) = -\frac{4\pi}{Mk}\left(\frac{1}{ka}-\frac{kr}{2}\right) - \frac{g_A^2}{4f_\pi^2}\left(1-\frac{m_\pi^2}{4k^2}\overbrace{\ln[1+\frac{4k^2}{m_\pi^2}]}^{1\pi\text{ cut}}\right)$




Non-iterated OPE does not split Wigner.

**N$^2$LO:**




Once-iterated OPE splits Wigner: $\mathrm{S}\to\mathrm{D}\to\mathrm{S}$

$$A_1^{(^1\mathrm{S}_0)}(k) \equiv A^{(\mathrm{S})}_{1\mathrm{sym}}(k) = \frac{\left[A^{(\mathrm{S})}_0(k)\right]^2}{A^{(\mathrm{S})}_{-1}(k)} + \frac{g_A^2}{2f_\pi^2}\left[\frac{4}{3am_\pi} - \frac{m_\pi}{k}\left(\frac{1}{ka}-\frac{kr}{2}\right)\right]$$

$$- \frac{g_A^2 M m_\pi}{16\pi f_\pi^2}\left\{A^{(\mathrm{S})}_0(k)\,\frac{m_\pi}{k}\,\underbrace{\arctan[\frac{2k}{m_\pi}]}_{1\pi\text{ cut}} + \frac{g_A^2}{2f_\pi^2}\left[\frac{1}{12}+\left(\frac{m_\pi^2}{4k^2}-\frac{1}{3}\right)\ln 2 - \underbrace{F_\pi(\frac{k}{m_\pi})}_{1,2\pi\text{ cut}}\right]\right\}$$

$$A_1^{(^3\mathrm{S}_1)}(k) = A^{(\mathrm{S})}_{1\mathrm{sym}}(k) + A^{(\mathrm{S})}_{1\mathrm{split}}(k)$$

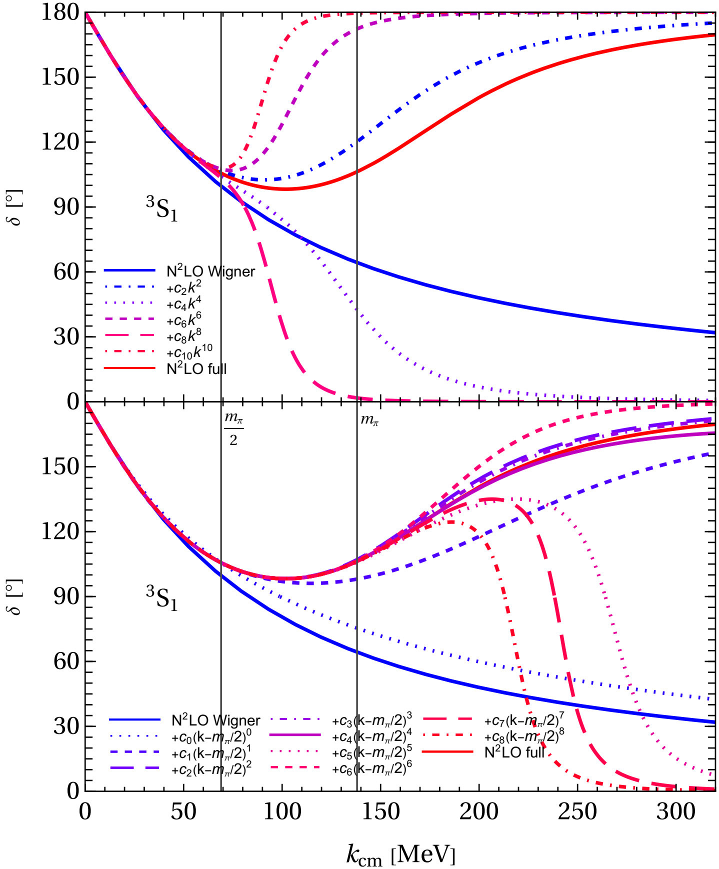
$$A^{(\mathrm{S})}_{1\mathrm{split}}(k) = -\frac{\left[A^{(\mathrm{SD})}_0(k)\right]^2}{A^{(\mathrm{S})}_{-1}} + \frac{g_A^2}{f_\pi^2}\,\frac{g_A^2 M m_\pi}{16\pi f_\pi^2}\left\{\frac{571-352\ln 2}{210} - \left(1+\frac{3m_\pi^2}{2k^2}+\frac{9m_\pi^4}{16k^4}\right)\overbrace{F_\pi(\frac{k}{m_\pi})}^{1,2\pi\text{ cut}} + \frac{2m_\pi^2}{5k^2}(\ln 4-1)\right.$$

$$\left. + \frac{3m_\pi^4}{16k^4} - \frac{3}{2}\left[\frac{k}{m_\pi}+\frac{m_\pi}{k}-\frac{m_\pi^3}{8k^3}-\frac{3m_\pi^5}{16k^5}\right]\overbrace{\arctan[\frac{k}{m_\pi}]}^{2\pi\text{ cut}} + \frac{3}{16}\left(\frac{m_\pi^4}{k^4}+\frac{3m_\pi^6}{4k^6}\right)\overbrace{\ln[\frac{16(k^2+m_\pi^2)}{4k^2+m_\pi^2}]}^{1,2\pi\text{ cut}}\right\}$$

$$F_\pi(x) := \frac{1}{8x^3}\left(\underbrace{\arctan[2x]\ln[1+4x^2] - \mathrm{Im}\left[\mathrm{Li}_2[\frac{2\mathrm{i}x+1}{2\mathrm{i}x-1}]\right.}_{1\pi\text{ cut}} \left. -2\,\underbrace{\mathrm{Li}_2[\frac{1}{2\mathrm{i}x-1}]}_{2\pi\text{ cut}}\right]\right)$$

# (d) Whence the Hockey Stick in ${}^3\mathrm{S}_1$?

Teng/hg [2410.09653]




Expand Wigner-splitting in Taylor:

$$A^{(S)}_{\mathrm{sym}}(k) + \sum \frac{1}{n!}[A^{(S)}_{\mathrm{split}}]^{(n)}(k_0)(k-k_0)^n$$

**Expand about $0$:**

$k \lesssim \frac{m_\pi}{2}$: convergent, Wigner-splitting tiny

$k \gtrsim \frac{m_\pi}{2}$: no convergence

$\Longrightarrow$ ERE not the problem.

Sorry, no Cohen/Hansen 1999.

**Expand about 1st branch point scale $\frac{m_\pi}{2}$:**

$k \lesssim \frac{m_\pi}{\sqrt{2}}$: convergent, Wigner-splitting tiny

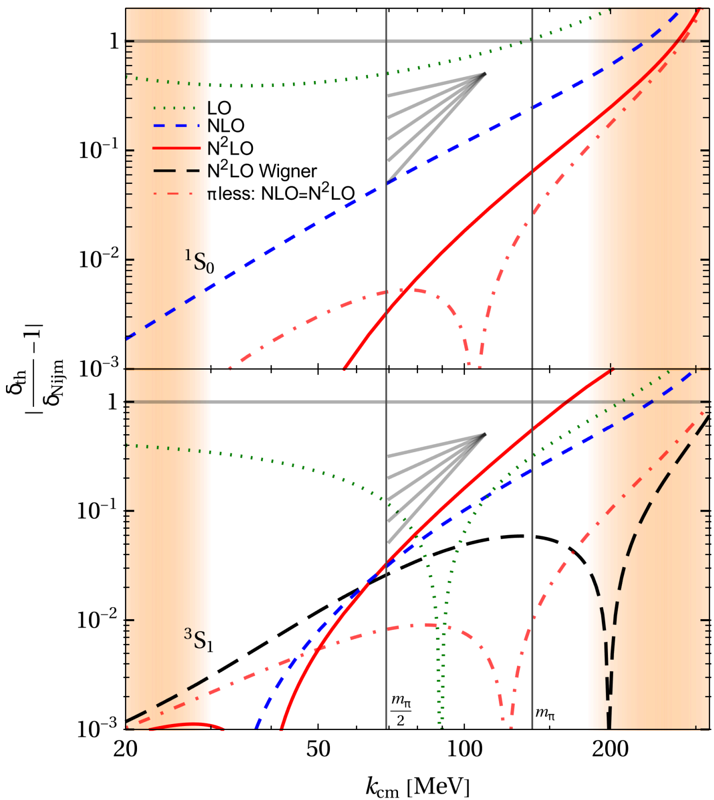
(larger distance to branch point)

$k \lesssim \frac{3}{2}m_\pi$ ($>$2nd br. pt. scale): convergent

$k \gtrsim \frac{3}{2}m_\pi$: asymptotic (optimal: incl. $k^4$)

## (e) Convergence to Data

Landau/Páez/Bordeianu: Comp. Phys., Lepage 1997
Teng/hg [2410.09653]




$$\frac{\delta(\text{N}^n\text{LO}) - \delta(\text{PWA})}{\delta(\text{PWA})} \sim \left(\frac{k, m_\pi}{\overline{\Lambda}}\right)^{n+1}$$

at N$^n$LO with empirical breakdown scale $\overline{\Lambda}$.

$^1S_0$ and Wigner-symmetric $^3S_1$:

consistent slopes and

$\overline{\Lambda} \approx 270\ \text{MeV} \approx \overline{\Lambda}_{NN}$ OPE scale.

Full $^3S_1$:

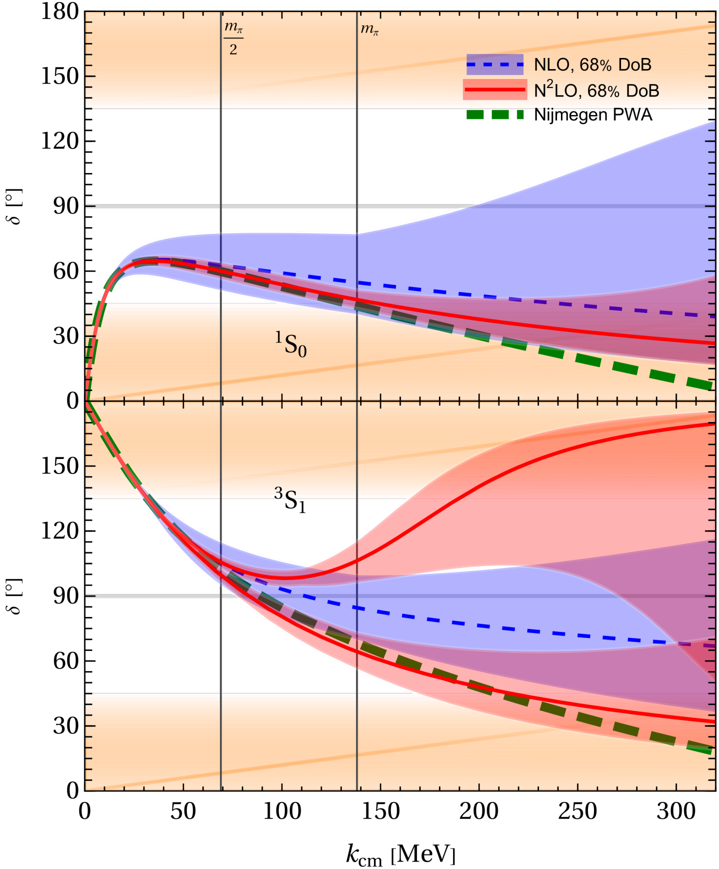
N$^2$LO worse than NLO for $\gtrsim 70\ \text{MeV}$.

Picture obscured by points where

theory & PWA identical ("artificial zero"),

or PWA close to zero ("artificial $\infty$").

# (f) NLO & N$^2$LO Bayesian Truncation Uncertainties




Apply "max" criterion to $\cot\delta$ order-by-order:

Unitarity: $k\cot\delta_{\text{LO}} = 0 \Rightarrow$ "$-ik$" sets scale.

Bayesian N$^2$LO truncation uncertainty at $k$:

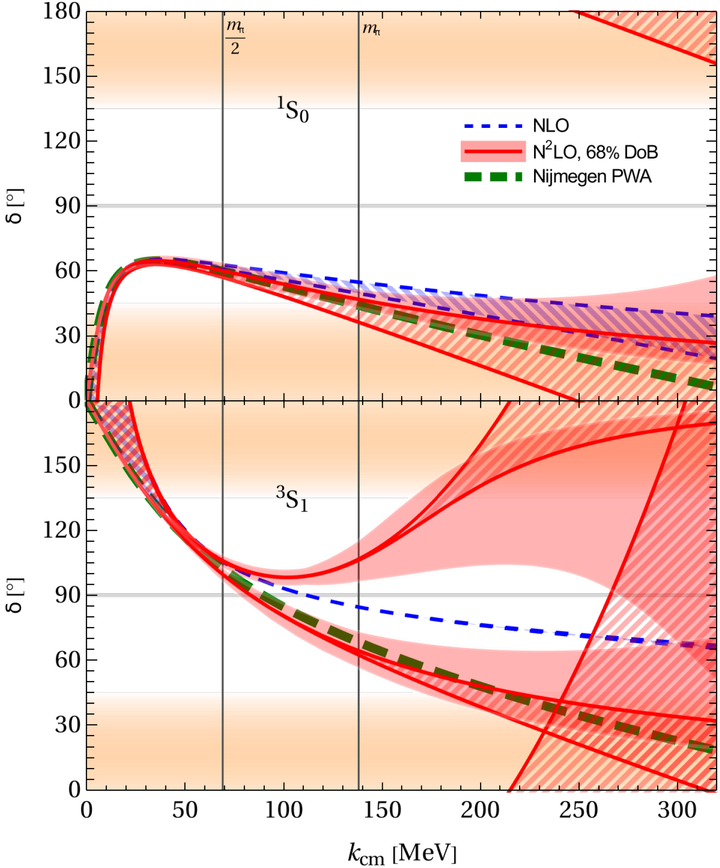
$$\pm Q^3 \max\left\{\frac{\cot\delta_0(k)-\cot\delta_0(0)}{Q};\frac{\cot\delta_1(k)}{Q^2}\right\}$$

with $Q = \dfrac{\max\{k;m_\pi\}}{\overline{\Lambda}_{\text{NN}} \sim 300\ \text{MeV}}$

NLO: rescaled to 68% DoB,
assuming uniform&log-uniform prior.

Only Wigner-symmetric forms have
N$^2$LO uncertainties consistent with NLO,
and NLO&N$^2$LO consistent with PWA.

# (g) Different Ways To Extract Phase Shifts at NLO and N$^2$LO




So far:

$$k\cot\delta = 0_{\text{LO}} + k\cot\delta_{\text{NLO}} + k\cot\delta_{\text{N}^2\text{LO}}$$
$$= -\frac{4\pi}{M}\left[\frac{A_0}{A_{-1}^2} - \frac{A_0^2}{A_{-1}^3} + \frac{A_1}{A_{-1}^2}\right]$$

is fundamental, derive $\delta(k)$ from it.

$$\xrightarrow{k\to 0} 0_{\text{LO}} + \left(-\frac{1}{a} + \frac{r}{2}k^2\right)_{\text{N}^{1+2}\text{LO}} + \mathcal{O}(k^4)$$

constructed to reproduce ERE.

Hatched: difference to

directly from amplitude KSW 1999,FMS 2000

$$\delta(k) = \frac{\pi}{2}\Big|_{-1} + \left(\frac{1}{ka} - \frac{kr}{2}\right)\Big|_0 + \dots$$

$\to\infty$ for $k\to 0$ outside Unitarity Window.

Methods agree inside Unitarity Window

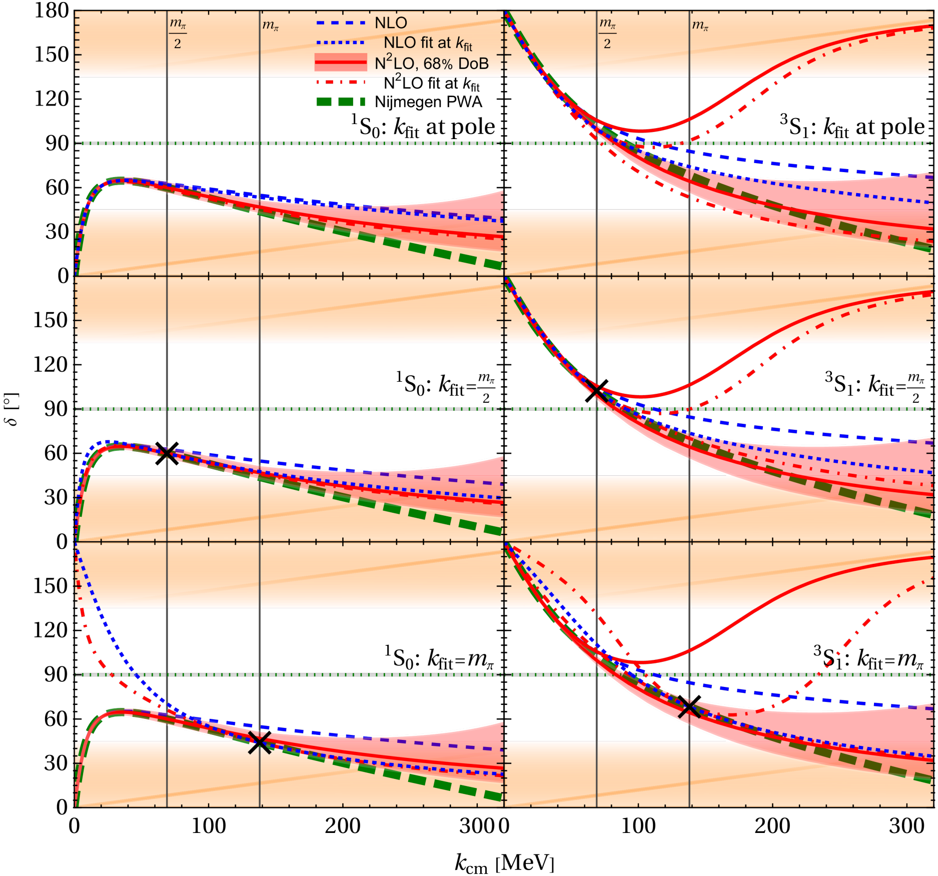
$\frac{1}{ka}, \frac{kr}{2} < 1$ (must in centre $|\cot\delta| \to 0$):

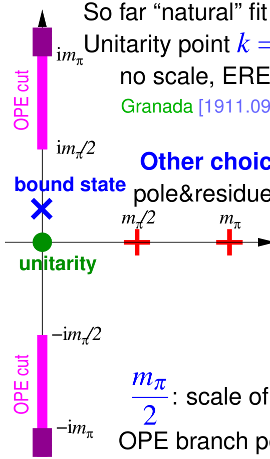
Independent assessment of

truncation uncertainty, consistent with Bayes.

# (h) Different Renormalisation/Parameter-Determination Points

Teng/hg [2410.09653]




So far "natural" fit at Unitarity point $k=0$: no scale, ERE

Granada [1911.09637]

**Other choices:**

pole&residue: ok

$\frac{m_\pi}{2}$: scale of 1st OPE branch point

No cure to hockey-stick.

Uncertainties & breakdown scale very similar.

$m_\pi$: 2nd OPE branch point

No cure to hockey-stick.

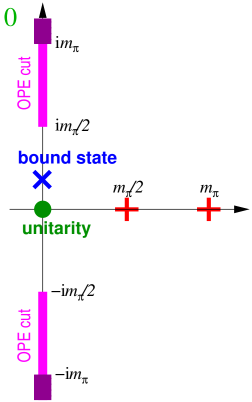
Low-$k$ $^1$S$_0$ bad inside Unitarity Window.

# (i) Virtual/Real Bound-State Pole Positions and Residues

Teng/hg [2410.09653]

$$\frac{1}{A_{-1}(\mathrm{i}\gamma_{-1}+\mathrm{i}\gamma_0+\mathrm{i}\gamma_1+\dots)+A_0(\mathrm{i}\gamma_{-1}+\mathrm{i}\gamma_0+\mathrm{i}\gamma_1+\dots)+A_1(\mathrm{i}\gamma_{-1}+\mathrm{i}\gamma_0+\mathrm{i}\gamma_1+\dots)+\dots} \stackrel{!}{=} 0$$

$$\frac{1}{Z} = \mathrm{i}\,\frac{\mathrm{d}}{\mathrm{d}k}\left(\mathrm{k}\cot\delta(k) - \mathrm{i}k\right)\Big|_{k=\mathrm{i}\gamma_{-1}+\mathrm{i}\gamma_0+\mathrm{i}\gamma_1+\dots}$$




For $k_{\text{fit}} = 0$, pions cannot correct $a$, $r$ since we force the ERE values Granada [1911.09637]

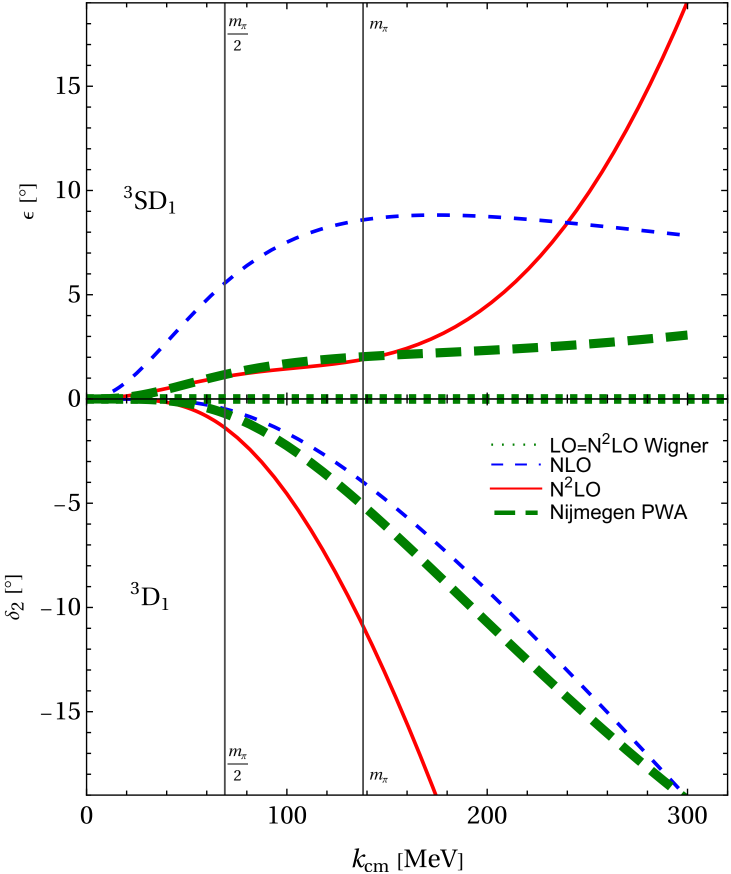
$\Longrightarrow$ pole at binding momentum $\mathrm{i}\gamma = \frac{\mathrm{i}}{a}\left(1 + \frac{r}{2a} + \frac{r^2}{2a^2} + \mathcal{O}(\frac{r^3}{a^3})\right)$

with residue $Z = 1 + \frac{r}{a} + \frac{3r^2}{2a^2} + \mathcal{O}(\frac{r^3}{a^3})$.

For general $k_{\text{fit}}$, match to $\mathrm{k}\cot\delta_{\text{PWA}}(k_{\text{fit}})$, $\frac{\mathrm{d}}{\mathrm{d}k}\mathrm{k}\cot\delta_{\text{PWA}}(k_{\text{fit}})$. $\Longrightarrow$ Predict $a, r$.

| | $^1S_0$ | | | $^3S_1$ | | |
|---|---|---|---|---|---|---|
| $k_{\text{fit}}$ | scatt. length $a$ [ fm] | eff. range $r$ [ fm] | (bind. mom.,residue) $(\gamma$ [ MeV], $Z)$ | scatt. length $a$ [ fm] | eff. range $r$ [ fm] | (bind. mom.,residue) $(\gamma$ [ MeV], $Z)$ |
| ERE | $-23.735(6)*$ | $2.673(9)*$ | $(-7.892, 0.9034)$ | $5.435(2)*$ | $1.852(2)*$ | $(+47.7023, 1.689)*$ |
| pole | $-23.7104$ | $2.7783$ | | $5.6128$ | $2.3682$ | |
| $\frac{m_\pi}{2}$ NLO | $-38.988$ | $3.3270$ | $(-4.86, 0.925)$ | $4.9310$ | $2.4966$ | $(+55., 1.9)$ |
| $\frac{m_\pi}{2}$ $N^2LO$ | $-25.428$ | $2.7281$ | $(-7.34, 0.910(2))$ | $4.7768$ | $2.4492$ | $(+57(3)., 1.9(2))$ |
| $\frac{m_\pi}{2}$ sym. | | | | $5.4625$ | $1.6124$ | $(+43.0(5), 1.42(4))$ |
| $m_\pi$ NLO | $+9.2856$ | $4.2285$ | $(+28., 1.8)$ | $3.3442\dagger$ | $3.1886\dagger$ | $(+114., 3.)$ ⚡ |
| $m_\pi$ $N^2LO$ | $+34.3335$ | $2.8956$ | $(+6.01, 1.10)$ | $1.8376\dagger$ | $3.3741\dagger$ | $(+387(330)., 7(9).)$ ⚡ |
| $m_\pi$ sym. | | | | $4.5344$ | $1.7006$ | $(+54(1)., 1.5(1))$ |

Bayesian $N^2LO$ uncertainties $\quad *$: input $\quad$ ⚡: cannot converge: $Q \sim \frac{1}{ka}, \frac{kr}{2} \ll 1 \Longrightarrow \frac{r}{2a} \ll 1$ ⚡




No other channels close to Unitarity Window:

$|\delta_{l\geq 1}| < 25°$ ($|\cot\delta_{l\geq 1} > 2|$).

$^3SD_1$ mixing only by tensor/Wigner-splitting.

In Unitarity Expansion very similar to FMS:

$k \gtrsim 70$ MeV:

No order-by-order convergence,

convergence to PWA elusive.

Zero by Wigner at N$^2$LO.

Natural size at N$^3$LO at $k \approx m_\pi$:

$90° \times (Q \approx 0.5)^3 \approx 10°$

$\Longleftrightarrow$ PWA: $\lesssim 10°$.

$\Longrightarrow$ Not inconsistent.

SD & DD contacts at N$^3$LO

$\Longrightarrow$ Reproducing PWA possible.

# (k) What is the Small Parameter?: Large-$N_c$ Limit of QCD?

't Hooft 1974

**Need expansion parameter related to Wigner-$\mathrm{SU}(4)$ to characterise tensor suppression near Fixed Point.**

**Candidate Expansion of QCD for a large number $N_C \to \infty$ of colours:**

Kaplan/Savage [hep-ph/9509371]
Kaplan/Manohar [nucl-th/9612021]
Calle Cordón/Ruiz Arriola [0807.2918]

Predicts that all $V_{\mathrm{NN}}$ in $\mathrm{S}$ waves are suppressed against central (Wigner-$\mathrm{SU}(4)$) – **except** tensor↯.

**Way out?:** Wigner-$\mathrm{SU}(4)$ only realised in long-range parts, strongly broken in short-range?? Calle Cordón/Ruiz Arriola [0807.2918]

Here: Wigner-$\mathrm{SU}(4)$ splitting only in LECs: short-range – long-range ($k \to 0$) still Wigner-$\mathrm{SU}(4)$ symmetric.

**Way out?!:** $1/N_c$ expansion assumes that coefficients "of natural size".

Wigner-$\mathrm{SU}(4)$/proximity to Unitarity *forces* leading-$1/N_c$ coefficient of tensor-$V_{\mathrm{NN}}$ to be exact zero.

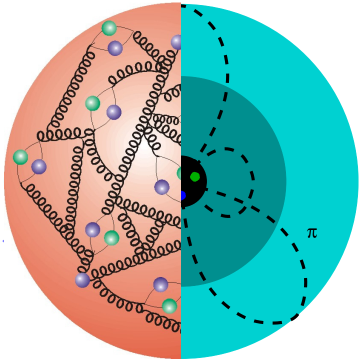
Advantage: Guaranteed to survive renormalisation by Unitarity FP symmetry.

 Wilson, Weinberg, ...

## (a) Chiral Effective Field Theory of Nuclear Physics

**At low energies, quarks & gluons rearrange into new,**
***effective* low-energy degrees of freedom: Nucleons, Pions, $\Delta(1232)$.**

$$\mathcal{L}_{\chi\text{EFT}} = (D_\mu \pi^a)(D^\mu \pi^a) - m_\pi^2\, \pi^a \pi^a + \dots$$
$$+ N^\dagger[\mathrm{i}\, D_0 + \frac{\vec{D}^2}{2M} + \frac{g_A}{2f_\pi}\, \vec{\sigma}\cdot\vec{D}\pi + \dots]N + C_0\left(N^\dagger N\right)^2 + H_0\left(N^\dagger N\right)^3 + \dots$$

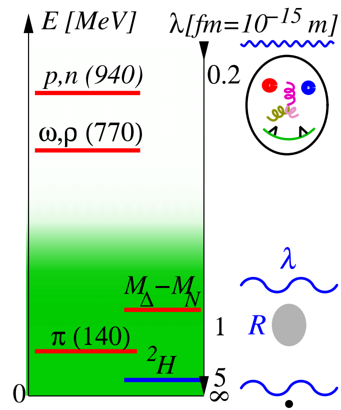



**Correct long-range + symmetries: Chiral SSB, gauge, iso-spin,...**
$\Longrightarrow$ ***Write most general Interaction Lagrangean permitted.***
**Short-range: ignorance into *minimal* parameter-set at given order.**
**Coefficients from experiment or QCD or...**

***"The Power Counting":***

**Systematic ordering** in $Q = \dfrac{\text{typ. momentum } \sim m_\pi}{\text{breakdown scale } \sim 1\text{ GeV}} \approx \dfrac{1}{5\dots7}$.

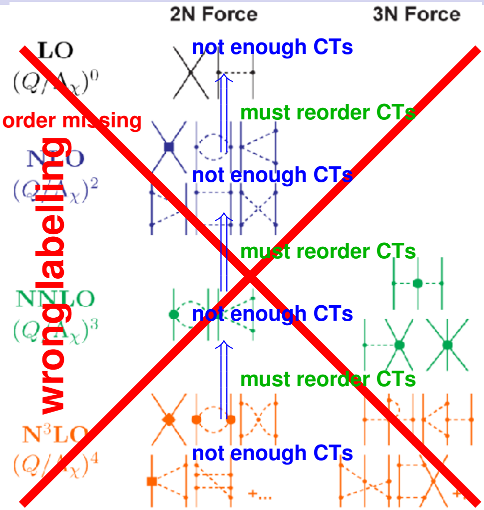
**Controlled approximation:** model-independent, error-estimate.
**Space for improvement.**




**$\Longrightarrow$ Chiral Effective Field Theory $\chi$EFT $\equiv$ low-energy QCD**

**Pragmatic, widely used ("Everybody Does It").**

**But *conceptually inconsistent*:**




underlying plot by Epelbaum, butchered by hg

– Not renormalised in low partial waves with attractive tensor. Nogga/Timmermans/van Kolck 2005

– ${}^1\mathrm{S}_0$: $m_\pi^2$-dependence of CT and divergence do not match.

$$V_C \to \frac{m_\pi^2}{\pi^2\, r} \text{ but } C_0 \sim m_\pi^0 \text{ bites you for } m_\pi \neq 140\ \mathrm{MeV}!$$

Kaplan/Savage/Wise 1996, Beane/Bedaque/Savage/van Kolck 2002

**Not Just LO Reg/Ren Problem: ricochets through orders.**

$\Longrightarrow$ WPP underestimates number of CTs per order.

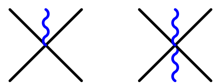
$\Longrightarrow$ WPP at alleged order $Q^n$ not as accurate as thought:

Accurate only to lower order $Q^{n-1,2,3,\dots}$.

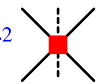
Fitting may obscure the problem...

**Not Just $\mathrm{NN}$ Problem: $2\mathrm{N}$ currents promoted.** Phillips/Valderrama 2015

$\Longrightarrow$ Gauged & gauge-invariant currents earlier, e.g. at LO $\vec{p}\cdot\vec{p}\,'C_P \to$



$\Longrightarrow$ Chiral-gauged $\mathrm{NN}$ currents earlier: $D_2 m_\pi^2$ in LO ${}^1\mathrm{S}_0 \xrightarrow{\chi\mathrm{sym}} D_2\pi^2$



LO in $\pi$Nucleus.

**We may be unable to say whose PC is right, but we have evidence whose is wrong. WPP is; it's *In-Effective*.**

**Still, use it pragmatically to develop numerics & first glimpses at final theory – with *caveat on systematics*!**

# (c) NN $\chi$EFT Power Counting Comparison

prepared for Orsay Workshop by Grießhammer 7.3.2013
based on and approved by the authors in private communications

**Derived with explicit & implicit assumptions; contentious issue.**

*All but WPP: RGE as construction principle, but different approximations at short-range lead to variant interpretations.*

**Proposed order $Q^n$ at which counter-term enters *differs*. $\Longrightarrow$ Predict *different* accuracy, # of parameters.**

| wave | order | Yang/Long PRC**86**(2012) 024001 etc. | Pavon Valderrama PRC**74** (2006) 054001 etc. | Birse PRC**74** (2006) 014003 |
|---|---|---|---|---|
| $^1S_0$ | LO | | $-1$ | |
| | NLO | | $0$ | |
| | N$^2$LO | $1$ | $2$ | |
| $^3S_1$ | LO | | $-1$ | |
| | NLO | $1$ | $2$ | $\frac{1}{2}$ |
| $^3SD_1$ | LO | $1$ | $-\frac{1}{2}$ | $-1$ |
| | NLO | | $2$ | $\frac{1}{2}$ |
| $^3D_1$ | LO | | $-\frac{1}{2}$ | $-1$ |
| | NLO | | $2$ | $\frac{1}{2}$ |
| $^3P_0$ (attr. triplet) | LO | $-1$ | | $-\frac{1}{2}$ |
| | NLO | $1$ | $2$ | $\frac{3}{2}$ |
| TPE | LO | $1$ | $2$ | |
| | NLO | $2$ | $3$ | |
| # of param. at $Q^{-1}$ | | 2 | 3 | 4 |
| # of param. at $Q^{0}$ | | 4 | 6 | 6 |
| # of param. at $Q^{1}$ | | **8** | **6** | **9** |

**Weinberg:** LO: $2$; NLO: $+0$; N$^2$LO: $+7=9$ – different channels; consistency questioned Beane/...2002; Nogga/...2005

**With same $\chi^2$/d.o.f., the *self-consistent* proposal with least parameters *wins*: minimum information bias.**

**Still, use any *pragmatically* to develop numerics & first glimpses at final theory – with caveat on systematics!**

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| order | **Weinberg** (Pragm. Prop.) PL**B251** (1990) 288 etc. | **Birse** PR**C74** (2006) 014003 etc. | **Pavon Valderrama et al.** PR**C74** (2006) 054001 etc. | **Long/Yang** PR**C86**(2012) 024001 etc. |
|---|---|---|---|---|
| $Q^{-1}$ | LO of $^1S_0$, $^3S_1$, OPE | LO of $^1S_0$, $^3S_1$, OPE plus $^3D_1$, $^3SD_1$ | LO of $^1S_0$, $^3S_1$, OPE plus $^3P_{0,2}$, $^3D_2$ | LO of $^1S_0$, $^3S_1$, OPE plus $^3P_{0,2}$ |
| $Q^{-\frac{1}{2}}$ | none | LO of $^3P_{0,1,2}$, $^3PF_2$, $^3F_2$, $^3D_2$ | LO of $^3SD_1$, $^3D_1$, $^3PF_2$, $^3F_2$ | none |
| $Q^0$ | none | NLO of $^1S_0$ | NLO of $^1S_0$ | NLO of $^1S_0$ |
| $Q^{\frac{1}{2}}$ | none | NLO of $^3S_1$, $^3D_1$, $^3SD_1$ | none | none |
| $Q^1$ | LO of $^3SD_1$, $^1P_1$, $^3P_{0,1,2}$; NLO of $^1S_0$, $^3S_1$ | none | none | LO of $^3SD_1$, $^1P_1$, $^3P_1$, $^3PF_2$; NLO of $^3S_1$, $^3P_0$, $^3P_2$; N$^2$LO of $^1S_0$ |
| # at $Q^{-1}$ | $2$ | $4$ | $5$ | $4$ |
| # at $Q^0$ | $+0$ | $+7$ | $+5$ | $+1$ |
| # at $Q^1$ | $+7$ | $+3$ | $+0$ | $+8$ |
| total at $Q^1$ | $9$ | $14$ | $10$ | $13$ |

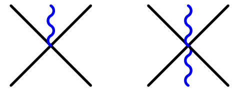
**With same $\chi^2/$d.o.f., the *self-consistent* proposal with least parameters *wins*: minimum information bias.**

**Still, use any *pragmatically* to develop numerics & first glimpses at final theory – with caveat on systematics!**

# (d) EFT and Information Theory: Lossless Compression vs. Data Reproduction

**Number of parameters at $Q^1$ for some attractive partial waves:**

| wave | **Weinberg** (Pragm. Prop.) PLB**251** (1990) 288 etc. | **Birse** PRC**74** (2006) 014003 etc. | **Pavon Valderrama et al.** PRC**74** (2006) 054001 etc. | **Long/Yang** PRC**86**(2012) 024001 etc. |
|---|---|---|---|---|
| $^3P_2$-$^3F_2$ | $1$, **very small** | $3$ **of similar size** | $3$ **of different orders** | $2$ **of different orders** |
| $^3P_0$ | $1$, **very small** | $1$ **just below LO** | $1$ **non-perturbative (LO)** | $2$ **of different orders** |

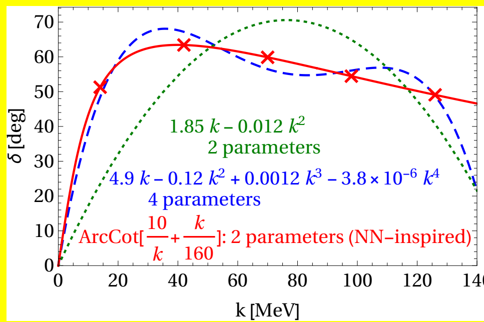
Predict different importance also for gauge currents: $\vec{p}\cdot\vec{p}\,'C_P \;\rightarrow$ 

**Information-Theory Aspect of the EFT Promise:**

**Encode information about unresolved short-range**
**at given resolution and *at given order***
**into smallest number of independent CTs:**

**minimal set of parameters for *lossless compression*.**

$\Longrightarrow$ **Falsifiability; robust predictions to uncover**
**new Physics, Alternative Worlds, hidden symmetries**
**(unitarity, large-$N_c$ Schindler/Springer 2018-,...).**



# (e) Not An Ivory Tower Exercise: Beyond-SM for $0\nu\beta\beta$, EDM and Dark Matter

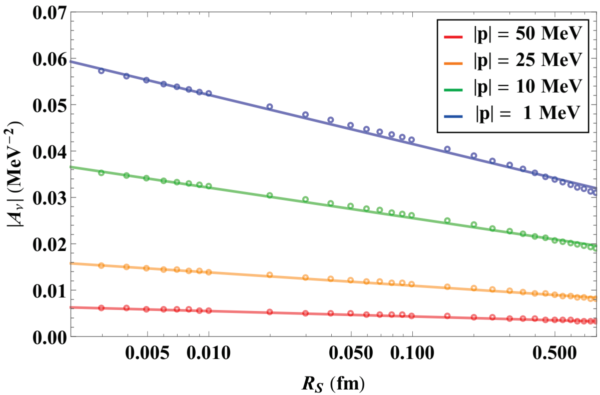
$\Longrightarrow$ "Unexpected" $2\mathrm{N}$ currents to absorb cutoff-dependence/restore RG-invariance & symmetries.

**LO In Nuclear Matrix Elements for Dark Matter Detection:** Hoferichter/Klos/Schwenk PLB 746 (2015) 410 [1503.04811]
de Vries/Köber/Nogga/Shain [2310.11343]

**NLO In Nuclear Matrix Elements for Strong-$CP$ Violation:** de Vries/Gnech/Shain PRC 103 (2021) 012501 [2007.04927]

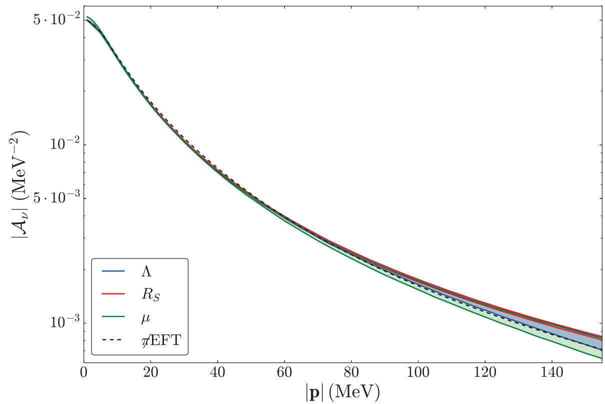
**LO In Nuclear Matrix Elements for Neutrinoless Double-Beta Decay Detection:**

Cirigliano/Dekens/de Vries/Graesser/Mereghetti/Pastore/van Kolck PRL120 (2018) 202001 [1802.10097]




RG/cutoff-dependence at fixed $k$ without CT

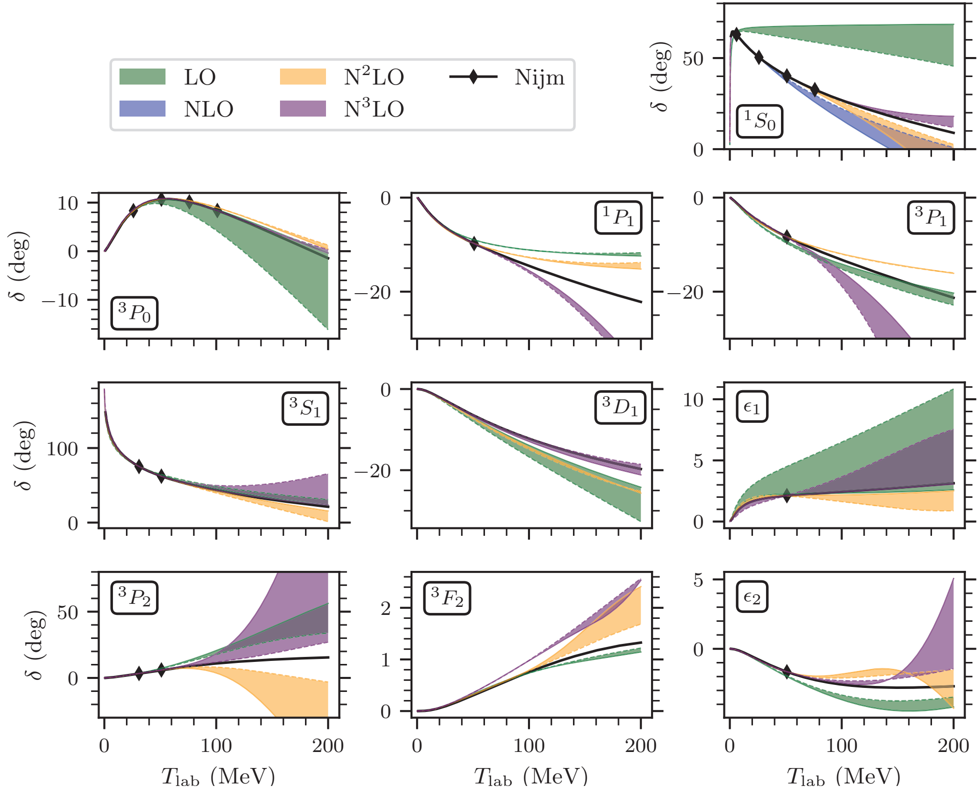
(Weinberg's Pragmatic Proposal)




RG/cutoff-corridor with $k$ with CT

**Multi-million-$ stakes! Community acknowledgment: Snowmass 2021 White Paper, INT & ECT* Workshops.**

Thim/Ekström/Forssén PRC 109 (2024) 064001
[2402.15325] → Oliver Thim's talk




Order-by-order convergence ✓ —- Uncertainty corridors from $\Lambda \in [500; 2500]$ MeV