

Lattice-QCD Calculation of the $0\nu\beta\beta$ Contact Term



UNIVERSITY OF
MARYLAND

Anthony V. Grebe

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Dirac or Majorana?

- Oscillation experiments $\rightarrow m_\nu > 0$
- Neutrinos could gain Dirac mass term through Higgs coupling

$$-m_D \bar{\nu}_L \nu_R$$

- Could also include Majorana mass term

$$-M \nu_R^T \nu_R$$

- Violates lepton number by 2 units
- Seesaw mechanism: m_ν naturally small (if $M \sim M_{\text{Pl}}$)

$$m_\nu \propto \frac{(Y_\nu)^2}{M} < 1 \text{ eV}$$

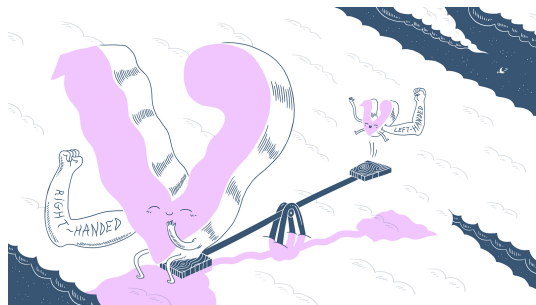
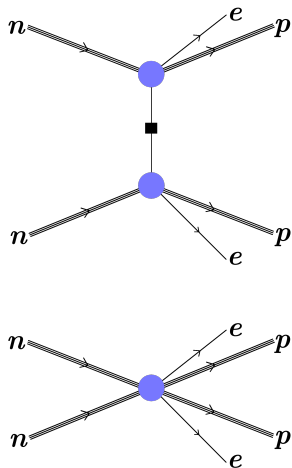


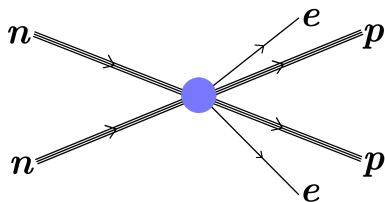
Image credit: Kova (Symmetry Magazine, Sandbox Studio)

Nuclear EFT for $0\nu\beta\beta$



- Neutrino energy can be hard or soft
- Low-energy contribution factorises into two SM weak currents
 - Can be computed from existing experimental data
- High-energy intermediate ν outside of EFT validity
- Need contact term g_{NN}^ν to absorb high-energy behavior
(Cirigliano et al., PRC 97, 065501 (1710.01729), PRL 120, 202001 (1802.10097))
- Contact term promoted to leading order in EFT

Nuclear EFT for $0\nu\beta\beta$



- EFT contact term g_{NN}^ν unique to $0\nu\beta\beta$
 - No experimental data!
 - Cannot be computed from $2\nu\beta\beta$
- Can be estimated using dispersive relations (generalized Cottingham formula) ([Cottingham, AP 25, 424 \(1963\)](#); [Cirigliano et al., JHEP 05, 289 \(2102.03371\)](#))
 - Likely correct to within 40% but requires model assumptions
 - Ongoing work to refine calculation ([Van Goffrier, PhD thesis \(2023\)](#))
- Calculate simple system with lattice QCD, match to EFT

Lattice QCD

- Discretize equations of QCD on 4-dimensional space-time lattice
- Non-perturbative (works for large coupling constants)
- First-principles, model-independent solution to hadronic physics
- Only input = Lagrangian of QCD ($\{m_q\}, \alpha_s$)
- Systematically controllable errors

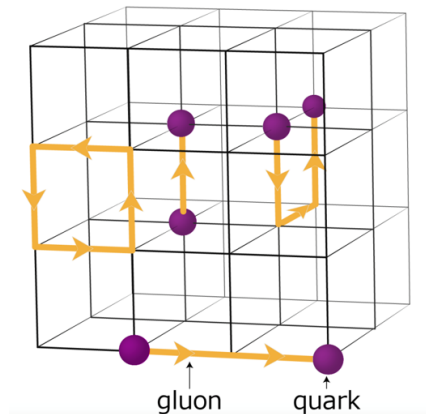
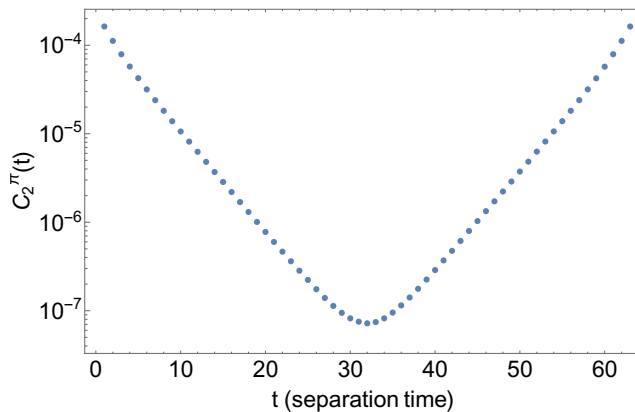


Image credit: JICFuS, Tsukuba

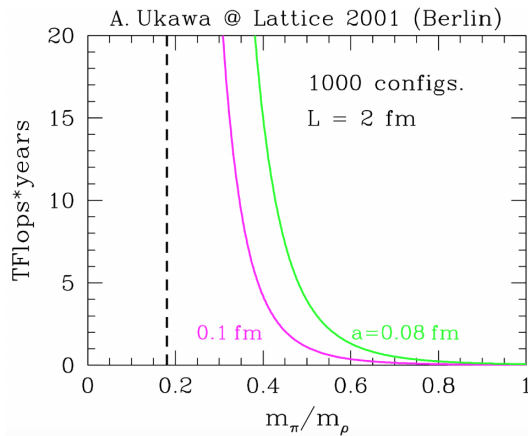
Behavior of Correlators

$$C_2(t) = \sum Z_n e^{-E_n t} \rightarrow Z_0 e^{-E_0 t}$$



Challenges of Lattice QCD

- Continuum limit: naïve cost is a^{-4} , in practice a^{-7} or worse
- Physical quark masses: historically quoted as m_π^{-6}



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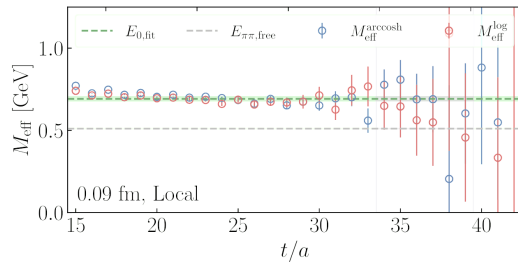


Figure credit: Bazavov, AVG, et al., PRL 135, 011901 (2412.18491)

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- Physical quark masses: historically quoted as m_π^{-6}
- Signal-to-noise ratio: exponentially bad at large (Euclidean) time separations
- Substantial algorithmic and computational work over last few decades (Clark, hep-lat/0610048; Clark et al., CPC 181, 1517 (0911.3191); Frommer et al., 1303.1377)

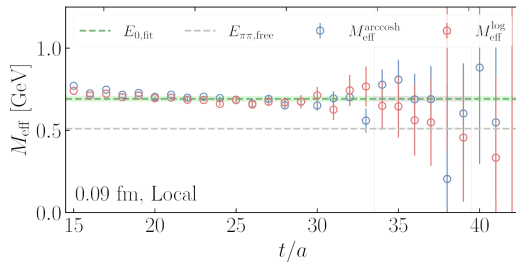


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Brute Force is the Last Resort of the Incompetent

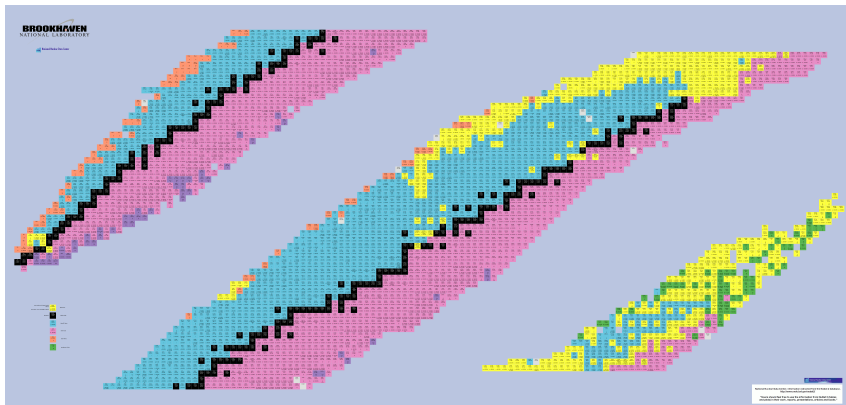


Image credit: Oak Ridge National Laboratory

Compute cost of LQCD calculation can be >\$10 million

Nuclear Physics

- Nuclear structure and interactions in principle computable from lattice QCD (+ QED)
- In practice, costs are prohibitively high for large A
- Focus on few-nucleon systems ($A \leq 4$), match to EFT descriptions



Historical Overview

- Lattice QCD collaborations (NPLQCD, CalLat, HALQCD) started computing multi-nucleon systems about 15–20 years ago
- Calculations expensive – reduce cost with asymmetric correlation functions (different source/sink)
 - NPLQCD used local operator at source (“hexaquark”), bi-local operator at sink (“dibaryon”)
- Correlation functions in LQCD equal to sum of exponentials

$$C(t) = \sum_n A_n e^{-E_n t} \rightarrow A_0 e^{-E_0 t}$$

- Effective mass ($-\ln C'(t)$) approaches E_0 from above for symmetric correlators ($A_n \geq 0$) but not for asymmetrics

Challenges

- Signal-to-noise problem: Degrades exponentially as $\exp\left[-A\left(m_N - \frac{3}{2}m_\pi\right)t\right]$
 - Milder at heavy pion masses
- Cost of contractions (naïvely) increases factorially in A
- EFT coefficients often sensitive to difference between nucleus and individual nucleons
 - Binding energies are $\lesssim 1\%$ of mass (0.1% for d)
 - Resolving $E_d - 2m_N$ from 0 requires permille precision on E_d
 - Matrix elements often need $\lesssim 5\%$ errors

Spectroscopy at $m_\pi \approx 800$ MeV (c. 2012)

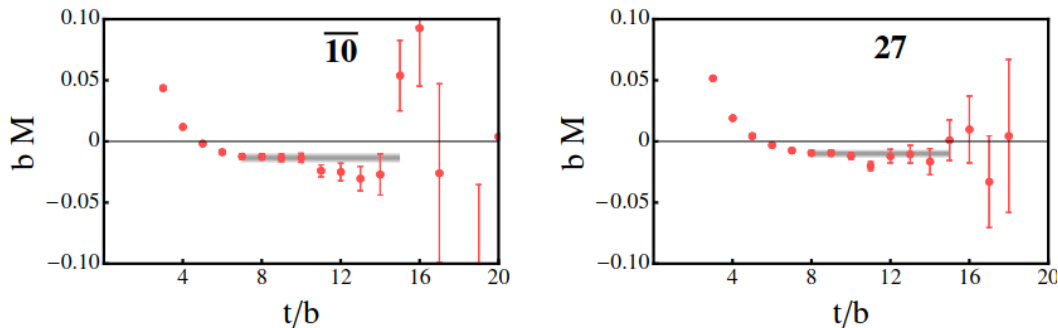


Figure credit: Beane et al. (NPLQCD), PRD 87, 034506 (1206.5219)

- Bound states (~ 15 MeV) for np and nn
- Corroborated by CalLat, conflicting results from HALQCD
- Called into question by some more recent calculations
 - Others confirm bound state at larger-than-physical masses

Proton-Proton Fusion

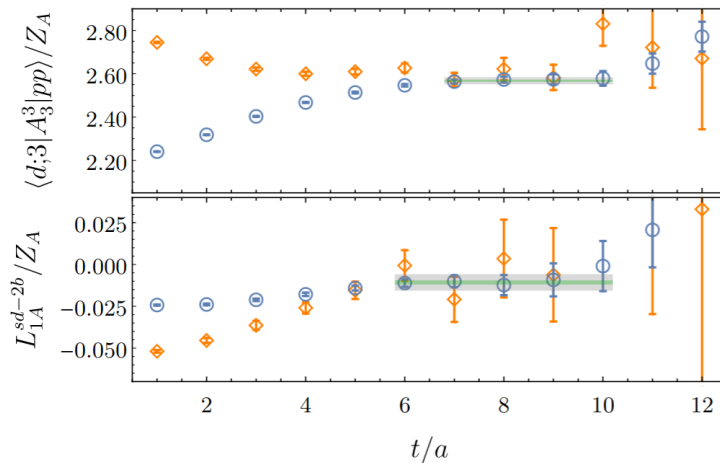


Figure credit: Savage et al. (NPLQCD), PRL 119, 062002 (1610.04545)

Tritium Beta Decay

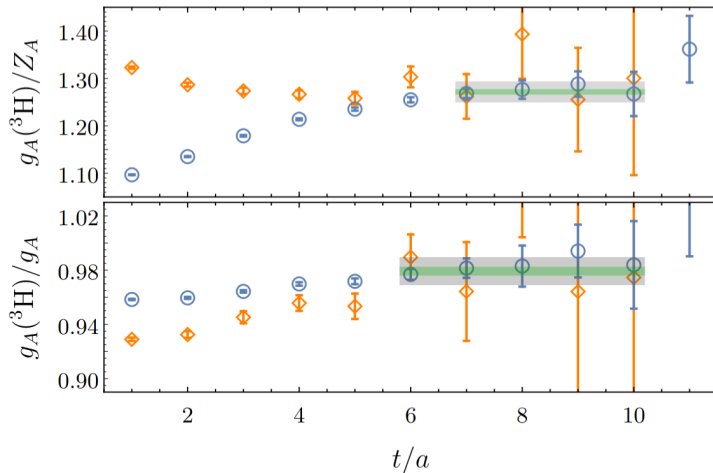


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Neutrinoless Double-Beta Decay ($2\nu\beta\beta$)

- Rarest observed Standard Model process
- Experimental data used as inputs or tests of nuclear models of $0\nu\beta\beta$ ([Engel, Menéndez, RPP 80, 046301 \(1610.06548\)](#))

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- Computed for $nn \rightarrow pp$ transition from lattice QCD (Shanahan et al., PRL 119, 062003 (1701.03456); Tiburzi et al., PRD 96, 054505 (1702.02929))
 - Single lattice spacing and $m_\pi = 800$ MeV
 - Computed matrix element to $\sim 2\%$ uncertainty (stat.) and extracted $2\nu\beta\beta$ counterterm

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 - Single lattice spacing and $m_\pi = 800$ MeV
 - Computed matrix element to $\sim 2\%$ uncertainty (stat.) and extracted $2\nu\beta\beta$ counterterm
- No intermediate ν prop – weak currents decouple
 - Background field method – quark propagators computed in presence of uniform weak field (Fucito et al., PLB 115, 148; Martinelli et al., PLB 116, 434; Bernard et al., PRL 49, 1076)

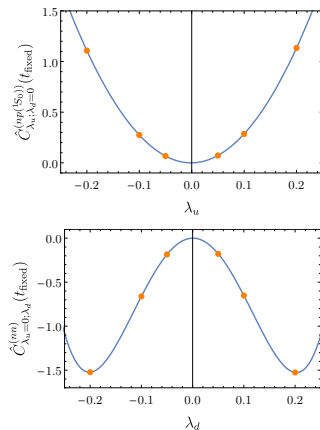


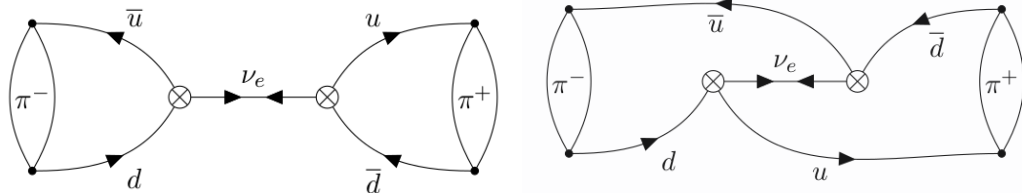
Figure credit: 1702.02929

Neutrinoless Double-Beta Decay ($2\nu\beta\beta$)

$$i\mathcal{C}_{nn \rightarrow pp} =$$

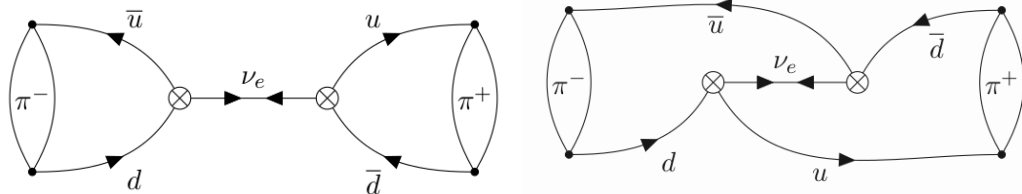
The diagrams represent the two-current contact term in the effective theory for $nn \rightarrow pp$. The first row shows five diagrams with two external wavy lines (neutrinos) and two internal wavy lines (pions). The second row shows two diagrams, including a contact term with a square box, followed by $+ \mathcal{O}(\lambda^4)$.

Two-current contact term: $\mathbb{H}_{2,S} = 4.7(1.3)(1.8)$ fm
 (Tiburzi et al., PRD 96, 054505 (1702.02929))

$0\nu\beta\beta$ for $\pi^- \rightarrow \pi^+$ 

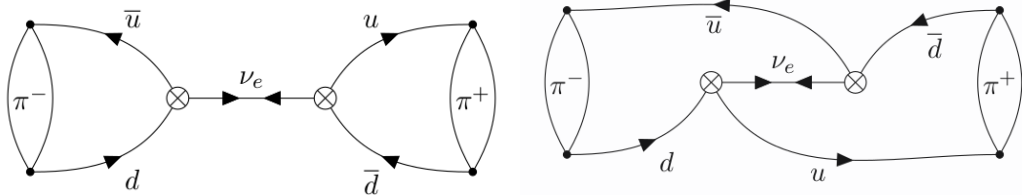
$$C_{\pi^- \rightarrow \pi^+} = \sum_{\mathbf{x}, \mathbf{y}} \int \frac{d^4 q}{(2\pi)^4} \frac{e^{iq \cdot (x-y)}}{q^2} \langle \mathcal{O}_{\pi^+}(t_+) J_\mu(x) J_\mu(y) \mathcal{O}_{\pi^-}^\dagger(t_-) \rangle$$

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 - Naïve cost: L^6 (expensive)
 - FFT convolution theorem reduces cost to $O(L^3 \log L)$

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- Final integration over $t = x_4 - y_4$ required for matrix element

$0\nu\beta\beta$ for $\pi^- \rightarrow \pi^+$

$$\langle \pi^+ | J^\mu J_\mu | \pi^- \rangle \propto 1 + \frac{m_\pi^2}{8\pi^2 f_\pi^2} \left(3 \log \left(\frac{\mu^2}{m_\pi^2} \right) + \frac{7}{2} + \frac{\pi^2}{4} + \frac{5}{6} g_\nu^{\pi\pi}(\mu) \right)$$

- Matrix element completely determined up to $g_\nu^{\pi\pi}$ by χ PT

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- $g_\nu^{\pi\pi}(\mu = m_\rho)$ measured by two groups with domain-wall fermions, extrapolated to physical point
 - $-10.9(8)$ (Tuo, Feng, Jin, PRD 100, 094511 (1909.13525))
 - $-10.8(5)$ (Detmold, Murphy, 2004.07404)

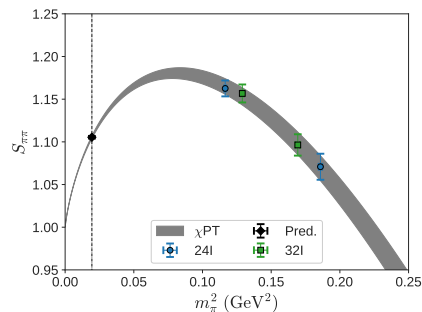
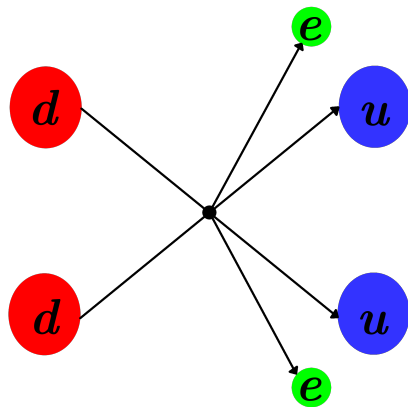


Figure credit: 2004.07404

Short-Distance Mechanism

$$\mathcal{O} = (\bar{d}\Gamma_i u) (\bar{d}\Gamma_j u)$$

- Contact interactions at scale of QCD
- Basis of 9 operators
 - 5 scalar operators ($\Gamma^i \Gamma^j = s$):
 $\mathcal{O}_1, \mathcal{O}_2, \mathcal{O}_3, \mathcal{O}'_1, \mathcal{O}'_2$
 - 4 vector operators ($\Gamma^i \Gamma^j = v^\mu$): $\mathcal{V}_1, \mathcal{V}_2, \mathcal{V}_3, \mathcal{V}_4$
- Coefficients determined by BSM theories
 - Compute matrix elements of 9 operators separately
- Scalar operator matrix elements calculated for $\pi^- \rightarrow \pi^+$ by CalLat (Nicholson et al., PRL 121, 172501 (1805.02634)) and NPLQCD (Detmold et al., PRD 107, 094501 (2208.05322))



Dimension-9 $0\nu\beta\beta$ Coefficients

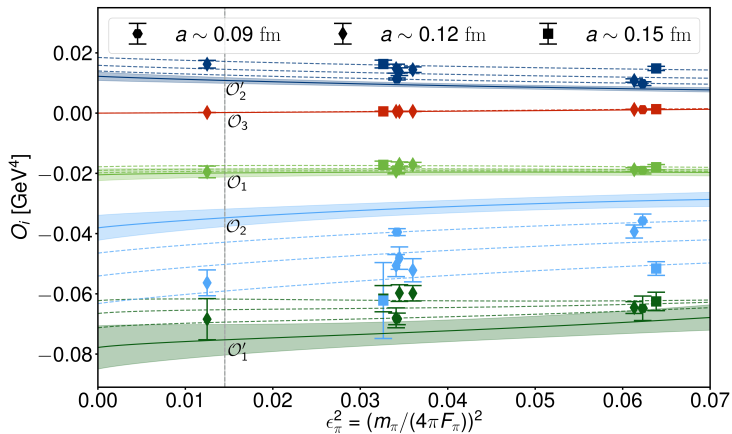


Figure credit: Nicholson et al., PRL 121, 172501 (1805.02634)

Challenges for $0\nu\beta\beta$ in $nn \rightarrow pp$

$$C_{nn \rightarrow pp} = \sum_{\mathbf{x}, \mathbf{y}} \int \frac{d^4 q}{(2\pi)^4} \frac{e^{iq \cdot (x-y)}}{q^2} \langle \mathcal{O}_{pp}(t_+) J_\mu(x) J_\mu(y) \mathcal{O}_{nn}^\dagger(t_-) \rangle$$

- Current insertions coupled by ν propagator
 - Cannot use background field method

Davoudi, Detmold, Fu, **AVG**, Jay, Murphy, Oare, Shanahan, Wagman (NPLQCD), PRD 109, 114514 (2402.09362)

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- Complexity of contractions $\propto N_q!$
 - $N_c!^4 N_u! N_d! = 6^4 24^2 \approx 10^6$ contractions needed

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Dinucleon Interpolating Operators

- Dibaryon (bi-local) operators – good signal quality but computationally expensive
 - Require cost reduction techniques, e.g. sparsening (Detmold et al., PRD 104, 034502 (1908.07050), Amarasinghe et al., PRD 107, 094508 (2108.10835)), distillation (Peardon et al., PRD 80, 054506 (0905.2160); Hörz et al., PRC 103, 014003 (2009.11825))

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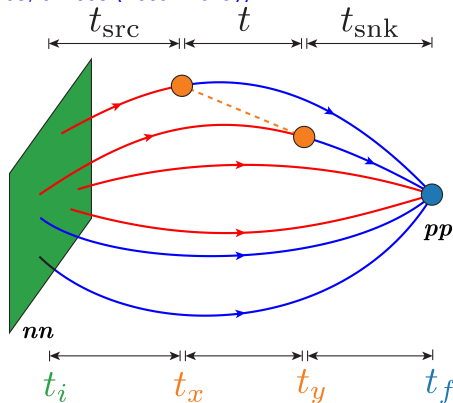
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- Variational analysis – expensive

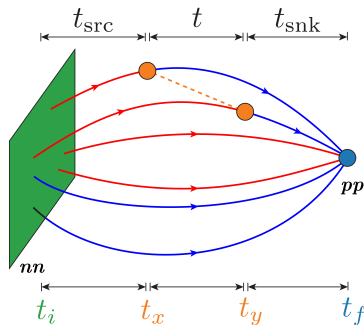
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- Compromise: Wall source, point sink
 - Improve signal with sparse (4^3) grid at sink



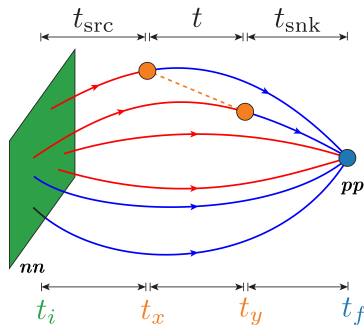
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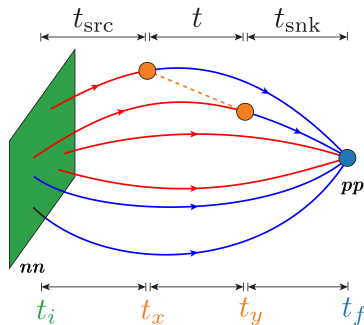
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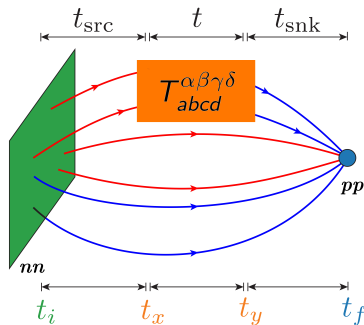
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- Sparsening at operator $\rightarrow$ additional systematics



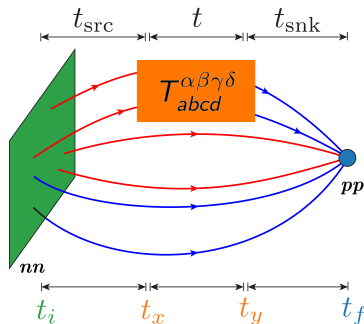
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- ~~Sparsening at operator~~ $\rightarrow$ additional systematics
- Decouple operator position sum from nuclear contractions
 - Sum 4-quark tensor $T_{abcd}^{\alpha\beta\gamma\delta}$ over x, y
 - Reduces work to $(N_c N_s)^4 V \log V + 10^6 \sim 10^{10}$



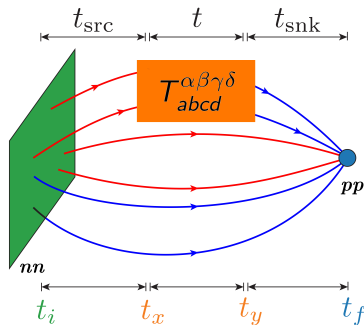
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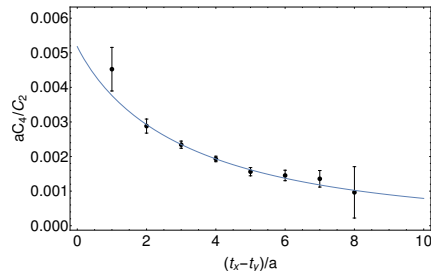
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- Total cost of $O(10^9)$ prop multiplications/sink location/ (t_x, t_y, T)
 - ~ 200 CPU core-hours/config



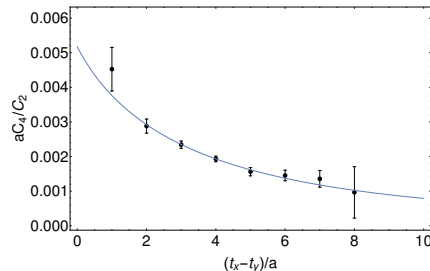
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 - Corresponds to large temporal separation between operators
 - Difficult to control (signal-to-noise problem)
- Solution: Use zero-mode subtracted propagator (Davoudi and Kadam, PRL 126, 152003 (2012.02083))

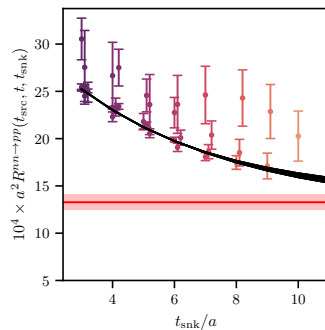


$$S_\nu(\tau, \mathbf{z}) = \frac{m_{\beta\beta}}{2L^3} \sum_{\mathbf{q} \in \frac{2\pi}{L}\mathbb{Z}^3 \setminus \{0\}} \frac{e^{i\mathbf{q} \cdot \mathbf{z}}}{|\mathbf{q}|} e^{-|\mathbf{q}|\tau}$$

- Contribution falls off exponentially in t
- Match to zero-mode removed EFT amplitude

Fitting Procedure

- Asymmetric excited state contamination from source and sink
 - More severe from point sink than wall source
- Extrapolate $t_{\text{src}}, t_{\text{snk}} \rightarrow \infty$ at given operator separation t



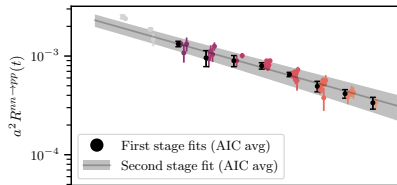
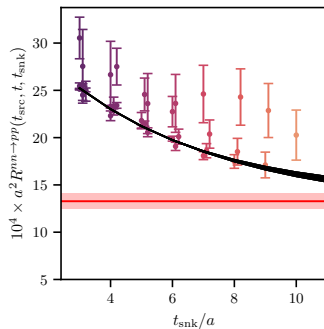
Fitting Procedure

- Asymmetric excited state contamination from source and sink
 - More severe from point sink than wall source
- Extrapolate $t_{\text{src}}, t_{\text{snk}} \rightarrow \infty$ at given operator separation t
- Fit t dependence to exponential and integrate:

$$\begin{aligned} \langle pp | JJ | nn \rangle &\propto 2m_{nn} \int_{-\infty}^{\infty} dt \frac{C_4(t, \tau)}{C_2(\tau)} \\ &= 0.14(3) \text{ GeV}^2 \text{ (stat.)} \end{aligned}$$

- Need high stats (5M total sources) to resolve dependence on $t, t_{\text{src}}, t_{\text{snk}}$

Thanks to XSEDE/ACCESS, TACC, and RCAC for compute time!



Difficulties in Extracting g_{NN}

$$\frac{\langle pp|JJ|nn\rangle}{2m_{nn}} \frac{1}{\mathcal{R}(E)\mathcal{M}(E)^2} = (1 + 3g_A^2)(J^\infty + \delta J^V) - \frac{m_n^2}{8\pi^2} \tilde{g}_\nu^{NN}$$

- $\langle pp|JJ|nn\rangle = 0\nu\beta\beta$ amplitude from LQCD
- $\tilde{g}_\nu^{NN} \propto g_\nu^{NN}$ = EFT counterterm of interest
- Known functions of NN interactions:
 - $\mathcal{M}$ = NN scattering (from effective-range expansion)
 - $\mathcal{R}$ = Lellouch-Lüscher residue
 - J^∞ = contribution from soft ν exchange
 - δJ^V = FV correction

Kaplan et al., PLB 424, 390 (nucl-th/9801034); Lellouch and Lüscher, CMP 219, 31 (hep-lat/0003023);
 Davoudi and Kadam, PRD 102, 114521 (2007.15542), PRL 126, 152003 (2012.02083), PRD 105, 094502
 (2111.11599)

Difficulties in Extracting g_{NN}

$$\mathcal{M}(E) = -\frac{4\pi}{m_N} \frac{1}{1/a - rp^2/2 + ip}$$

- Inputs required:
 - a = scattering length
 - r = effective range
 - $E = p^2/2m_N$ = FV energy shift
- Difficult to determine at $m_\pi = 800$ MeV
 - Values for $\mathcal{M}$, $\mathcal{R}$ very different for bound vs. scattering states
- Well determined from experiment ($a = 23.5$ fm, $r = 2.75$ fm)

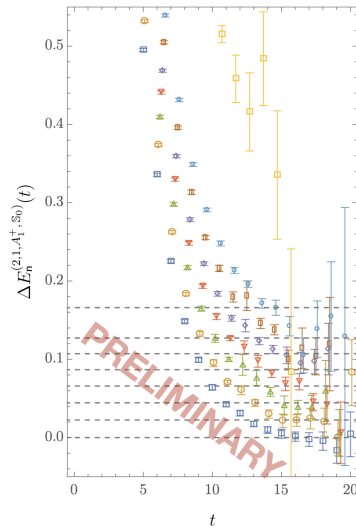
Physical Point

- EFT matching is more straightforward
- Lattice calculation more difficult
 - More expensive propagators
 - Worse signal-to-noise problem

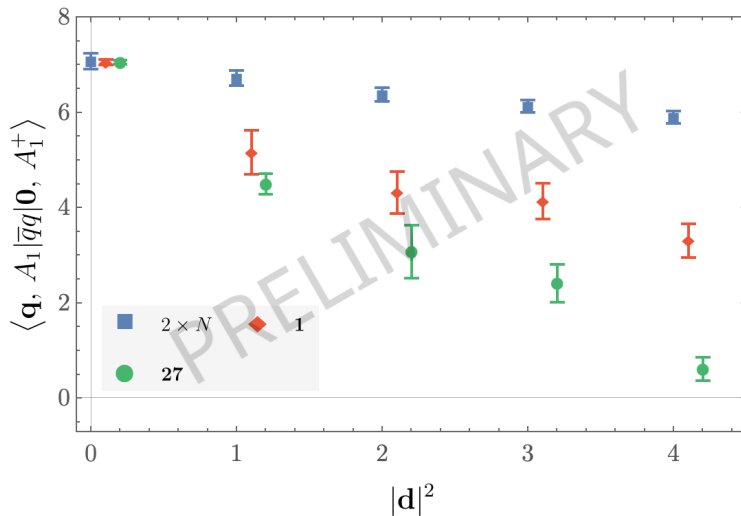
Physical Point

- EFT matching is more straightforward
- Lattice calculation more difficult
 - More expensive propagators
 - Worse signal-to-noise problem
- Goal: Find good interpolating operator(s) at physical point, use these for $0\nu\beta\beta$
 - $t > 2$ fm difficult to resolve $\rightarrow$ need to reduce excited states

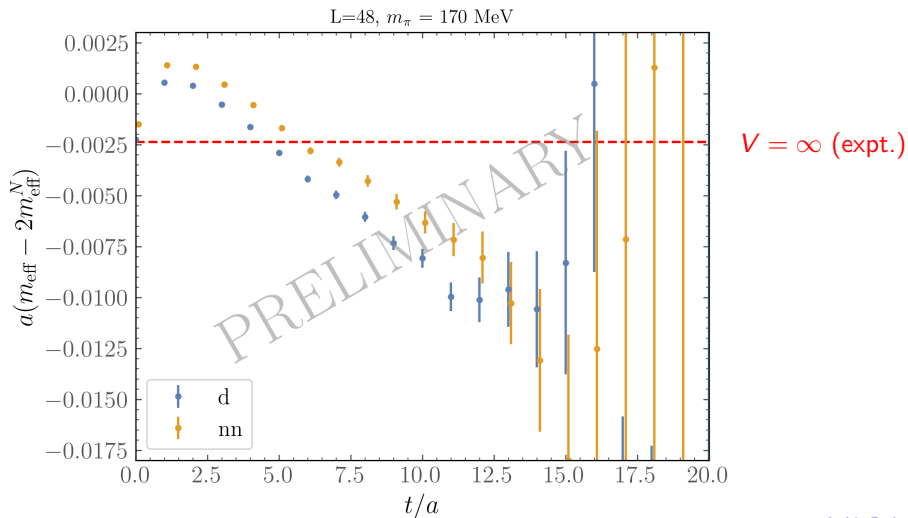
Figure credit: Davoudi et al. (NPLQCD), unpublished



Scalar Hyperon Form Factors ($m_\pi = m_K \approx 800$ MeV)

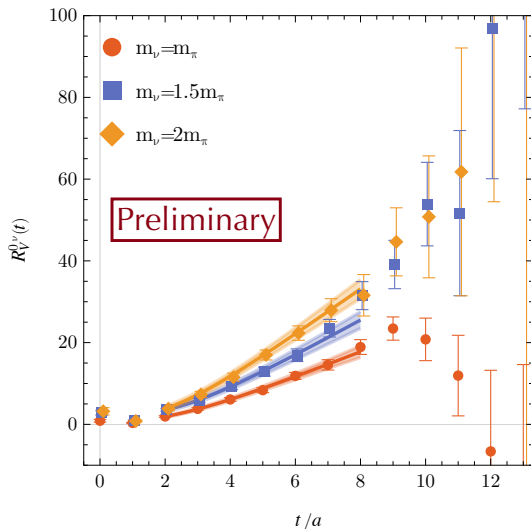


Deuteron Binding Energy



Symmetric $0\nu\beta\beta$ Calculation

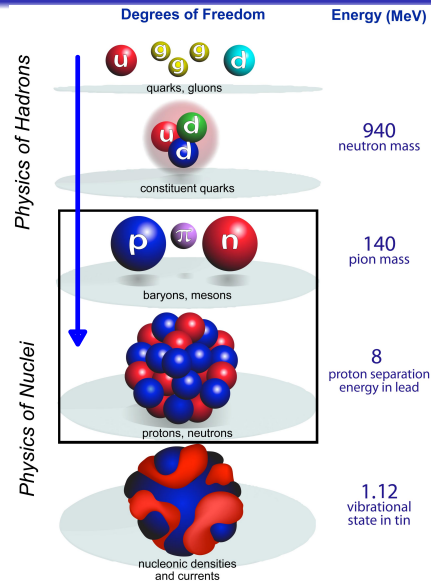
- Progress toward $0\nu\beta\beta$ at $m_\pi = 432$ MeV
- Use bi-local interpolators at source and sink to suppress excited states
- Use unphysical $m_\nu \sim m_\pi$ to suppress large- t tail and FV corrections
- Stochastic noise vectors to represent neutrino propagator
- Summation method – read contribution to $0\nu\beta\beta$ from slope versus t



Future Plans

- Single-current matrix element calculations
- $2\nu\beta\beta$ and short-distance (dim-9) $0\nu\beta\beta$ matrix elements
- First physical-mass calculation of long-distance $0\nu\beta\beta$ matrix element
- Continuum extrapolation, control of systematic uncertainties

Figure credit: DOE/NSF NSAC (0809.3137)



Triton Binding Energy

