γW-box and friends in Dispersion Theory: Certainties and Uncertainties

Misha Gorshteyn

Johannes Gutenberg-Universität Mainz

INT Workshop “Testing the Standard Model in Charged-Weak Decays” - Seattle, January 12-16, 2026

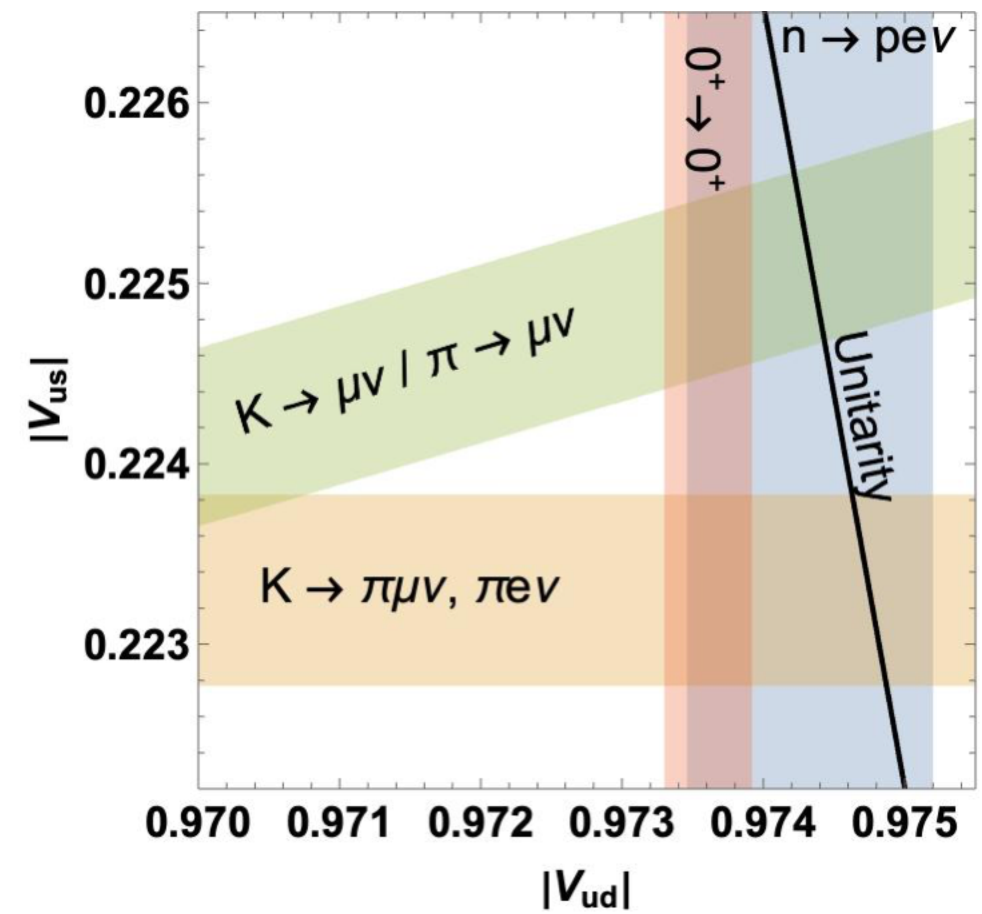
Cabibbo Unitarity:
overconstraining power of SM

Status of Cabibbo unitarity

$$|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 0.9985(6)_{V_{ud}}(4)_{V_{us}}$$

~ 0.95 ~ 0.05 $\sim 10^{-5}$

V_{ud} and V_{us} determinations inconsistent with the SM



Superallowed nuclear β : $|V_{ud}| = 0.9737\,(3)$

At variance with kaon decays + Cabibbo unitarity

$K \to \pi\ell\nu$: $|V_{us}| = 0.2233(5)$

Unitarity $\to |V_{ud}| = \sqrt{1 - |V_{us}|^2} = 0.9747(1)$

$\dfrac{K \to \mu\nu}{\pi \to \mu\nu}$: $|V_{us}/V_{ud}| = 0.2311(5)$

Unitarity $\to |V_{ud}| = [1 + |V_{us}/V_{ud}|^2]^{-1/2} = 0.9743(1)$

PDG $[S = 2.5]$: $|V_{us}| = 0.2243(8)$

Unitarity $\to |V_{ud}| = 0.9745(2)$

But consistent with the free neutron decay: $|V_{ud}| = 0.9743\,(9)$

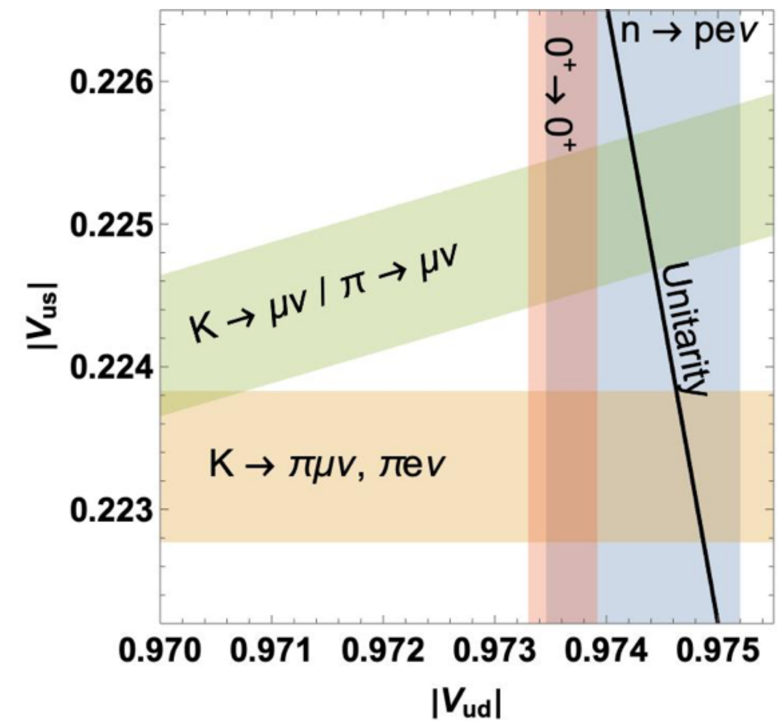
Cabibbo Unitarity - 3 anomalies

SM is overconstraining:
3 observables - 2 unknowns (if unitarity holds - 1 unknown)

$$\Delta^{(1)}_{\rm CKM} = |V_{ud}|^2 + |V^{K_{\ell 3}}_{us}|^2 - 1$$ **= –0.00176(56) –3.1σ**

$$\Delta^{(2)}_{\rm CKM} = |V_{ud}|^2 \left[1 + \left(\left|\frac{V_{us}}{V_{ud}}\right|^{K_{\mu 2}}\right)^2\right] - 1$$ **= –0.00098(58) –1.7σ**

$$\Delta^{(3)}_{\rm CKM} = |V^{K_{\ell 3}}_{us}|^2 \left[\left(\frac{1}{|V_{us}/V_{ud}|^{K_{\mu 2}}}\right)^2 + 1\right] - 1$$ **= –0.0164(63) –2.6σ**



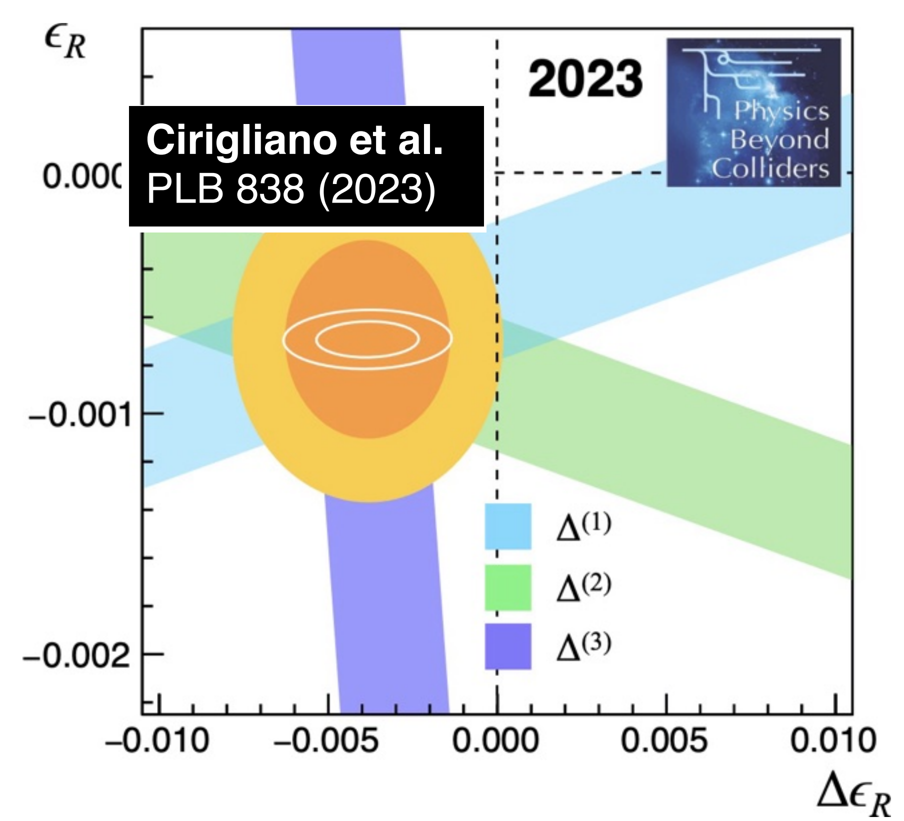
Minimal BSM scenario:
RH SMEFT Op's remove over-constraints of SM
Sensitivity to heavy BSM at ≤ 10TeV

ϵ_R = admixture of RH currents in non-strange sector
ϵ_R + $\Delta\epsilon_R$ = admixture of RH currents in strange sector

$$\Delta^{(1)}_{\rm CKM} = 2\epsilon_R + 2\Delta\epsilon_R V^2_{us}$$
$$\Delta^{(2)}_{\rm CKM} = 2\epsilon_R - 2\Delta\epsilon_R V^2_{us}$$
$$\Delta^{(3)}_{\rm CKM} = 2\epsilon_R + 2\Delta\epsilon_R(2 - V^2_{us})$$

From current fit:

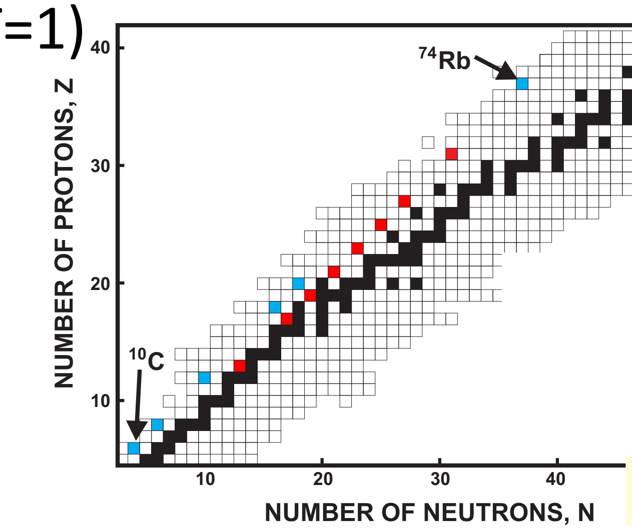
ϵ_R = –0.69(27) × 10^{-3} (2.5σ)
$\Delta\epsilon_R$ = –3.9(1.6) × 10^{-3} (2.4σ)
$\epsilon_R = \Delta\epsilon_R$ = 0 excluded at 3.1σ



Vincenzo's talk

V_{ud} from superallowed $0^+ - 0^+$ nuclear decays

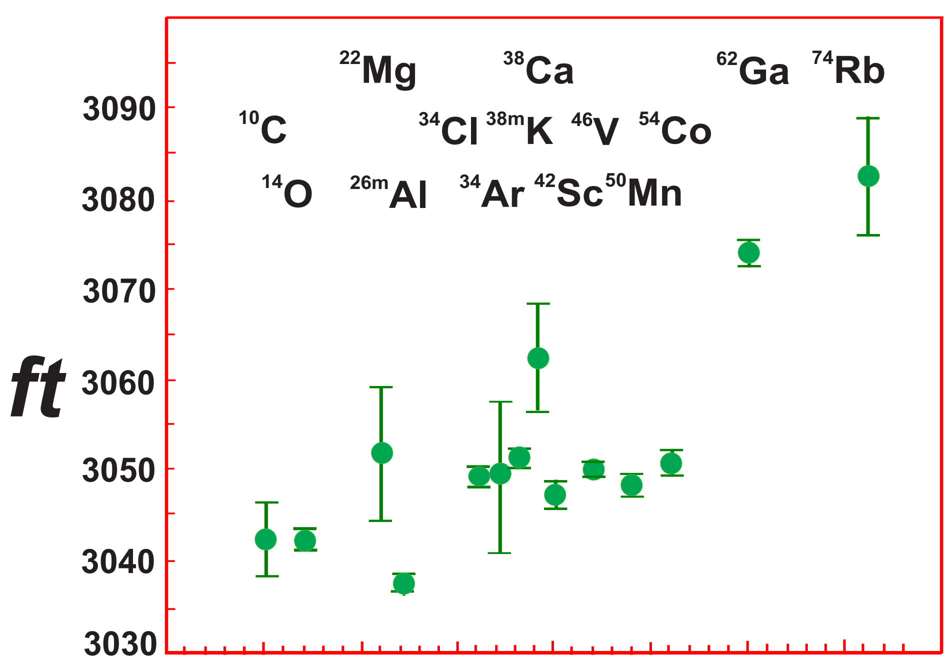
1. Transitions within $J^P=0^+$ isotriplets (T=1)
2. Elementary process: p—>ne$^+\nu$
3. Only conserved vector current
4. 15 measured to better than 0.2%
5. Internal consistency as a check
6. SU(2) good —> corrections ~small



$^{10}_{6}\mathrm{C} \to ^{10}_{5}\mathrm{B}$
$^{14}_{8}\mathrm{O} \to ^{14}_{7}\mathrm{N}$
$^{18}_{10}\mathrm{Ne} \to ^{18}_{9}\mathrm{F}$
$^{22}_{12}\mathrm{Mg} \to ^{22}_{11}\mathrm{Na}$
$^{26}_{14}\mathrm{Si} \to ^{26}_{13}\mathrm{Al}$
$^{30}_{16}\mathrm{S} \to ^{30}_{15}\mathrm{P}$
$^{34}_{18}\mathrm{Ar} \to ^{34}_{17}\mathrm{Cl}$
$^{38}_{20}\mathrm{Ca} \to ^{38}_{19}\mathrm{K}$
$^{42}_{22}\mathrm{Ti} \to ^{42}_{21}\mathrm{Sc}$
$^{46}_{24}\mathrm{Cr} \to ^{46}_{23}\mathrm{V}$
$^{50}_{26}\mathrm{Fe} \to ^{50}_{25}\mathrm{Mn}$
$^{54}_{28}\mathrm{Ni} \to ^{54}_{27}\mathrm{Co}$

$^{26m}_{13}\mathrm{Al} \to ^{26}_{12}\mathrm{Mg}$
$^{34}_{17}\mathrm{Cl} \to ^{34}_{16}\mathrm{S}$
$^{38m}_{19}\mathrm{K} \to ^{38}_{18}\mathrm{Ar}$
$^{42}_{21}\mathrm{Sc} \to ^{42}_{20}\mathrm{Ca}$
$^{46}_{23}\mathrm{V} \to ^{46}_{22}\mathrm{Ti}$
$^{50}_{25}\mathrm{Mn} \to ^{50}_{24}\mathrm{Cr}$
$^{54}_{27}\mathrm{Co} \to ^{54}_{26}\mathrm{Fe}$
$^{62}_{31}\mathrm{Ga} \to ^{62}_{30}\mathrm{Zn}$
$^{66}_{33}\mathrm{As} \to ^{66}_{32}\mathrm{Ge}$
$^{70}_{35}\mathrm{Br} \to ^{70}_{34}\mathrm{Se}$
$^{74}_{37}\mathrm{Rb} \to ^{74}_{36}\mathrm{Kr}$

Exp.: **f** - phase space (Q value)
t - partial half-life ($t_{1/2}$, branching ratio)



ft values: same within ~2% but not exactly!
Reason: SU(2) slightly broken

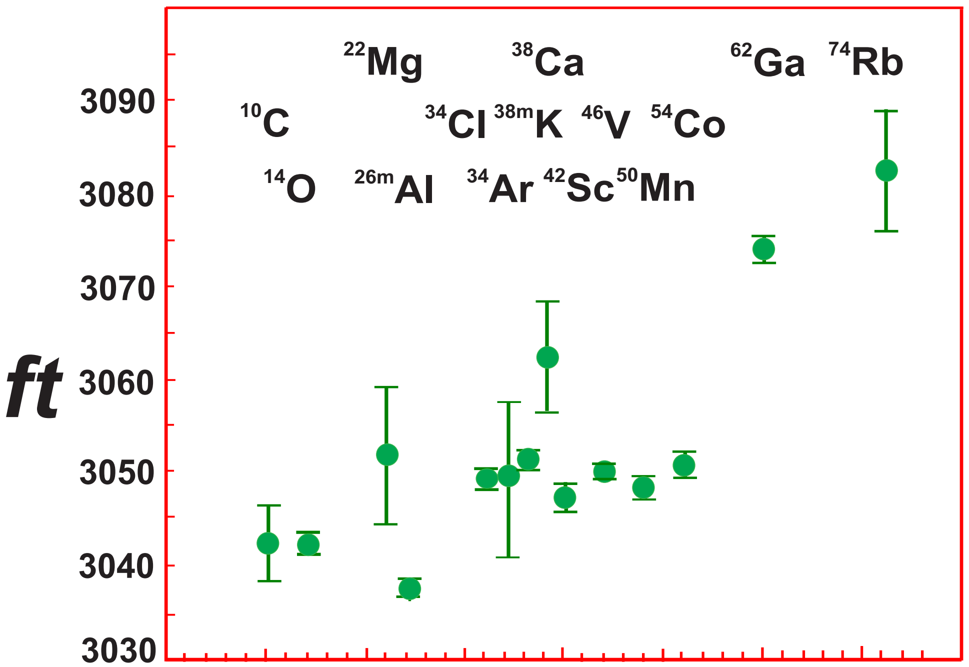
a. RC (e.m. interaction does not conserve isospin)
b. Nuclear WF are not SU(2) symmetric
(proton and neutron distribution not the same)

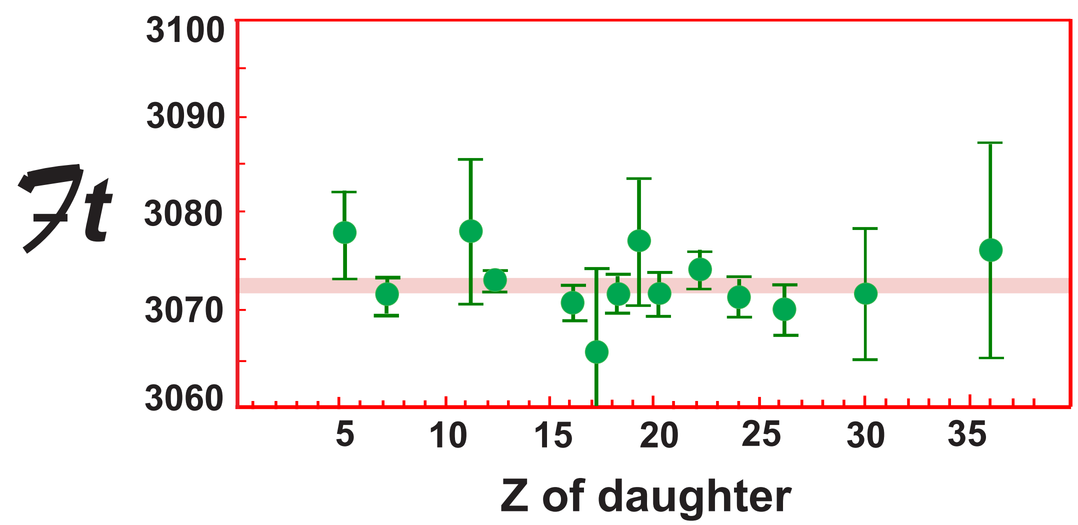
V_{ud} extraction: Universal RC and Universal Ft

To obtain Vud —> absorb all decay-specific corrections into universal **Ft**

$$ft(1 + \mathrm{RC} + \mathrm{ISB}) = \mathscr{F}\mathrm{t}(1 + \Delta_R^V) = \mathrm{ft}(1 + \delta'_R)(1 - \delta_C + \delta_{NS})(1 + \Delta_R^V)$$

~ Measured　　QED　　Isospin-breaking　　Nuclear structure　　Universal RC





Average of 14 decays　　Hardy, Towner 1972 - 2020

Pre-2018: $\overline{\mathscr{F}t} = 3072.1 \pm 0.7\, s$

PDG 2024: $\overline{\mathscr{F}t} = 3072 \pm 2\, s$

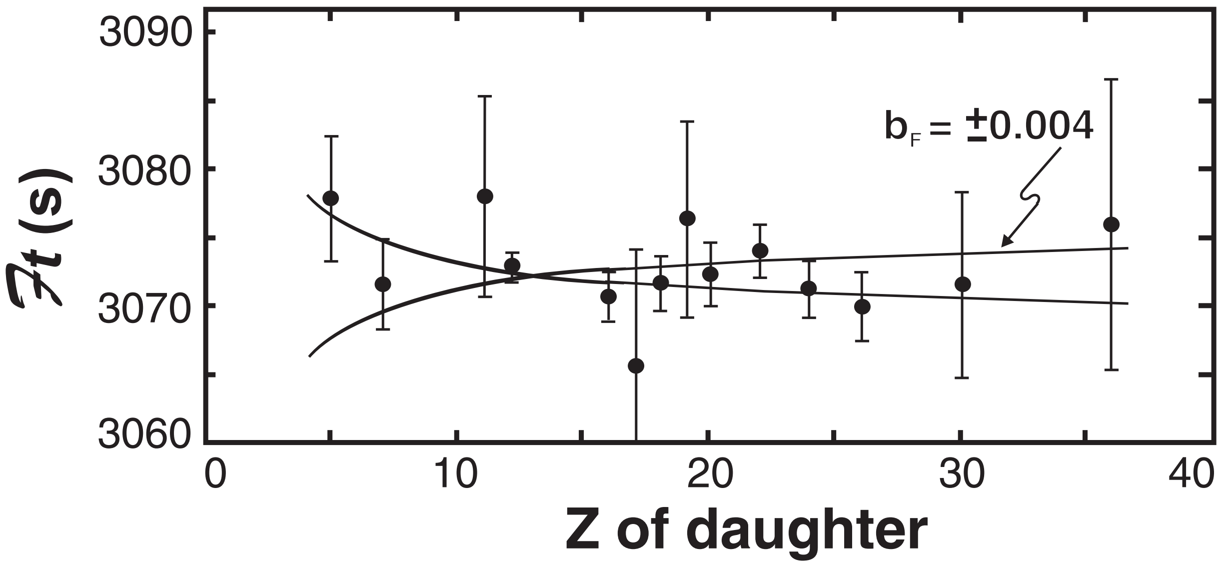
$$|V_{ud}|^2 = \frac{2984.43s}{\mathscr{F}t(1+\Delta_R^V)}$$

$$|V_{ud}^{0^+-0^+}| = 0.9737\,(1)_{exp,\,nucl}(3)_{NS}(1)_{RC}[3]_{total}$$

BSM searches with superallowed beta decays

SM maximally over-constraining in the case of superallowed nuclear beta decays:

Only one unknown with 15 ways to measure it



Induced scalar CC —> Fierz interference bF

$$\mathcal{F}t^{SM} \to \mathcal{F}t^{SM}\left(1 + b_F \frac{m_e}{\langle E_e \rangle}\right)$$

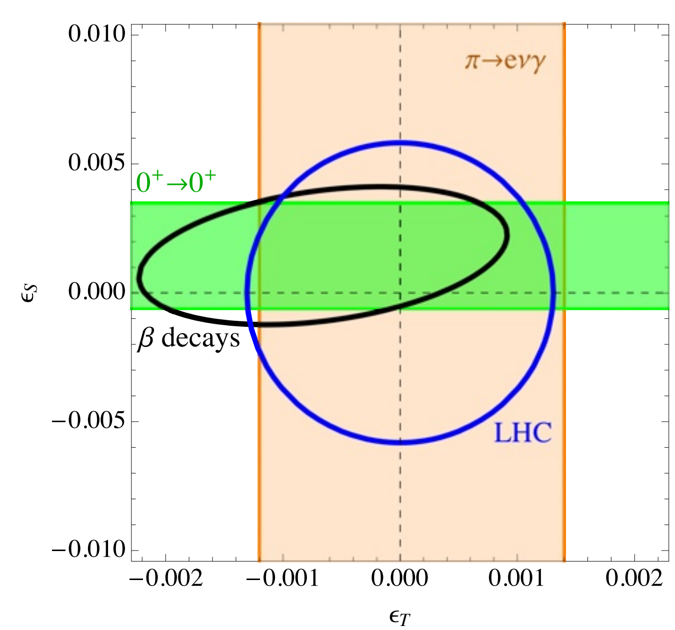
$b_F = -0.0028(26)$ ~ consistent with 0

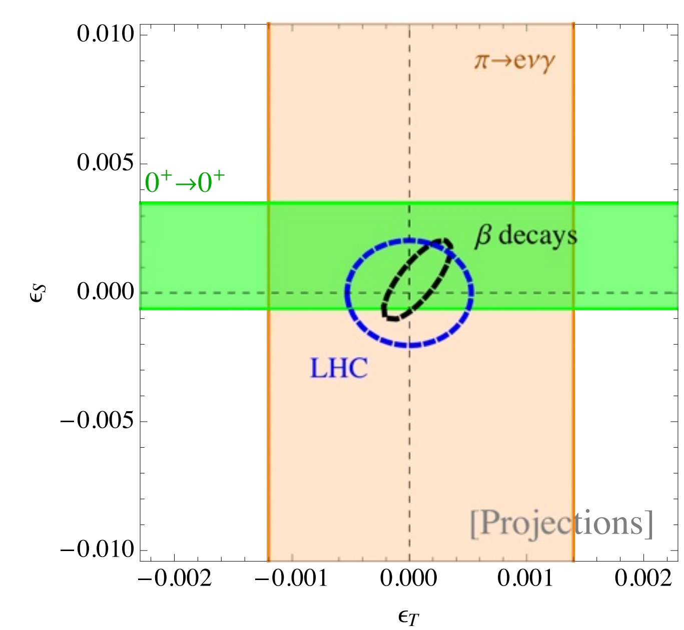
Independently of V_{ud} and CKM unitarity: bounds on BSM via internal consistency of the data base!

S, T interaction flips helicity:
Suppressed at high energy

Beta decay vs. LHC on S,T
Complementarity now and in the future!

Gonzalez-Alonso et al 1803.08732





Precision Tests with Semileptonic Probes:

1-loop Electroweak corrections - set up
Identifying hadronic uncertainties

What enables 0.01% accuracy in SL processes?

At 0.01% level QCD effects likely to obscure the CKM unitarity test
Way out:

- Conserved quantities — no QCD effects at tree level
- Compute SM radiative corrections to $\alpha, \alpha\alpha_s, \alpha\alpha_s^2, \ldots$
- Resum large logs

Symmetries ensure straightforward interpretation
But: symmetry breaking (SU(2) in β decay, SU(3) in K decays)
Non-conserved axial current affects V_{ud}, V_{us} at 1-loop

Decay phase space — hadronic form factors affect total rate
Small phase space —> FF effect small; but only total decay rate measured, integral over phase space usually computed theoretically
Large phase space —> FF effect large, must and can be measured

LQCD + EFT + Data-Driven [dispersion theory, phenomenology]

RC to semileptonic probes: overall setup

Tree-level amplitude

$e^\pm$ $\nu_e(\bar\nu_e)$

$i = n, \pi^\pm, K, A$ $f = p, \pi, A'$

$\sim V_{ud}$

Radiative corrections to tree-level amplitude $\sim \alpha/2\pi \approx 10^{-3}$

Precision goal 1×10^{-4}

Electron carries away energy E < Q-value of a decay

E-dep RC: $\frac{\alpha}{2\pi}\left(\frac{E}{\Lambda}, \ln\frac{E}{\Lambda}, \ldots\right)$

Energy scales Λ

Λ

Weak boson scale

$M_Z, M_W \sim 90\,\mathrm{GeV}$

Hadronic scale
$\Lambda_\mathrm{had} = 300\,\mathrm{MeV}$

Universal

Nuclear scale
$\Lambda_\mathrm{nuc} = 10 - 30\,\mathrm{MeV}$

Nuclear structure dependent
(QCD)

Decay Q-value (endpoint energy)
$Q_{if} = M_i - M_f = 1 - 10\,\mathrm{MeV}$

Nucleus-specific

Electron mass
$m_e \approx 0.5\,\mathrm{MeV}$

Nuclear structure independent
(QED)

Main uncertainty where the scales are poorly separated

How to ID, separate and connect scales?

1. Identifying relevant scales relies on “measure of relevance” - arbitrary?

2. Scale separation central to reliability of a method
   EFT relies on large log dominance — best for well-separated scales
   DR uses unitarity, analytical structure and general features of scattering data
   Lattice does NOT separate scales but has to stay away from IR and UV

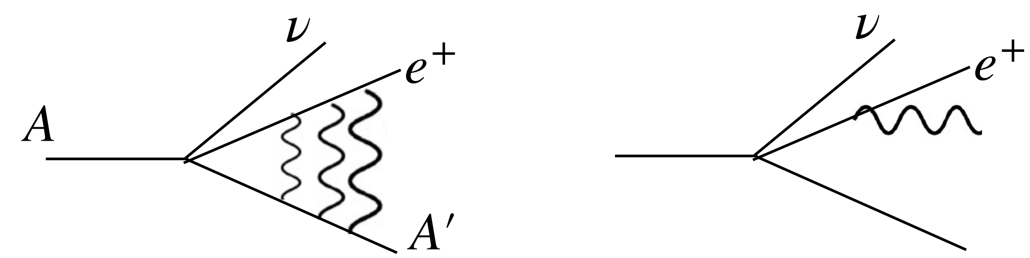
3. Once separated, reconnect scales guided by a general principle:
   EFT - RGE running + matching
   DR - analyticity

4. In the past, details of scale separation often neglected when putting things back together

5. Estimate uncertainties inherent to the method
   EFT: power counting + counter terms
   Lattice: errors statistical + systematical (finite volume + discretization)
   DR: statistical if data available; model errors if not

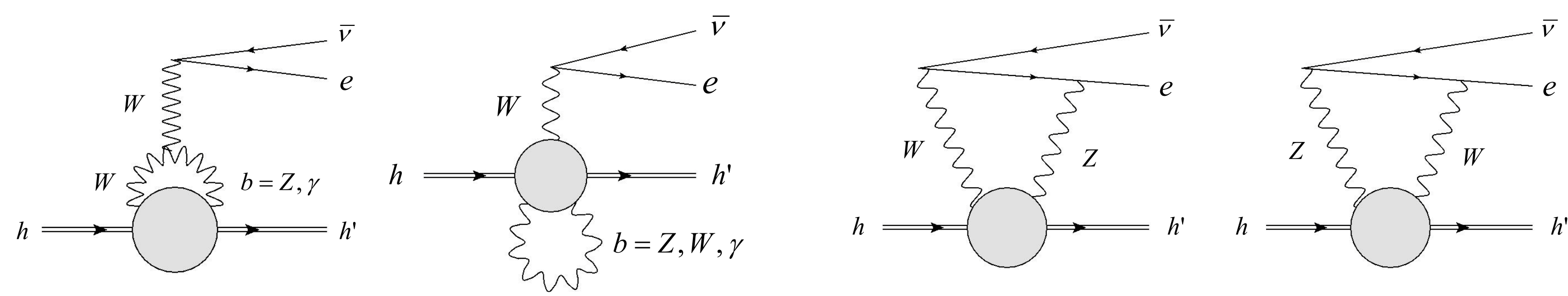
Radiative Corrections to β decay: sensitivity to scales from IR to UV

IR: Fermi function (Dirac-Coulomb problem)
+ **Sirlin function** (soft Bremsstrahlung)



UV: large EW logs + pQCD corrections

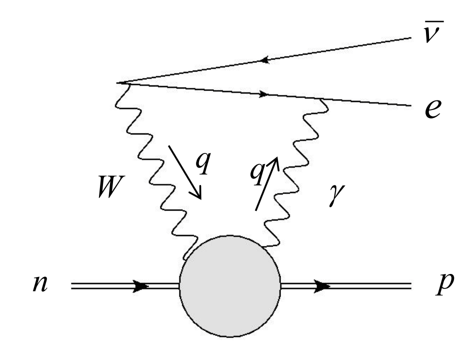
Inner RC:
energy- and model-independent



γW-box: sensitive to all scales

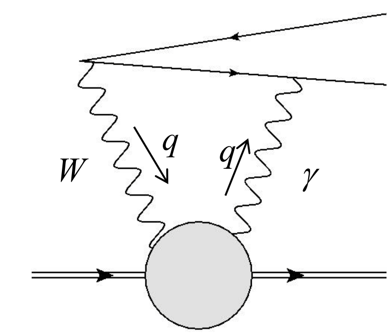
UV-sensitive γW-box on free neutron Δ_R^V: Sirlin, Marciano, Czarnecki 1967 - 2006

$$\Delta_R^V = \frac{\alpha}{2\pi}\left\{3\ln\frac{M_Z}{M_p} + \ln\frac{M_Z}{M_W} + \tilde{a}_g\right\} + \delta_{\text{QED}}^{HO} + 2\,\Box_{\gamma W}$$



All non-enhanced terms $\sim \alpha/2\pi \sim 10^{-3}$ — only need to ~10% — doable with modern methods!

γW-box



Box at zero momentum transfer* (but with energy dependence)

$$T_{\gamma W} = \sqrt{2}e^2 G_F V_{ud} \int \frac{d^4q}{(2\pi)^4} \frac{\bar{u}_e \gamma^\mu (\not{k} - \not{q} + m_e)\gamma^\nu (1-\gamma_5) v_\nu}{q^2[(k-q)^2 - m_e^2]} \frac{M_W^2}{q^2 - M_W^2} T_{\mu\nu}^{\gamma W}$$

*Precision goal: 10^{-4}; RC ~ $\alpha/2\pi$ ~ 10^{-3}; recoil on top - negligible

Hadronic tensor: two-current correlator

$$T^{\mu\nu}_{\gamma W} = \int dx e^{iqx} \langle f \,|\, T[J^\mu_{em}(x) J^{\nu,\pm}_W(0)] \,|\, i \rangle$$

General gauge-invariant decomposition of a spin-independent tensor

$$T^{\mu\nu}_{\gamma W} = \left(-g^{\mu\nu} + \frac{q^\mu q^\nu}{q^2}\right) T_1 + \frac{1}{(p\cdot q)} \left(p - \frac{(p\cdot q)}{q^2} q\right)^\mu \left(p - \frac{(p\cdot q)}{q^2} q\right)^\nu T_2 + \frac{i\epsilon^{\mu\nu\alpha\beta} p_\alpha q_\beta}{2(p\cdot q)} T_3$$

Loop integral with generally unknown forward amplitudes

$$T_{\gamma W} = -\frac{\alpha}{2\pi}\sqrt{2} G_F V_{ud} \int \frac{d^4 q M_W^2}{q^2(M_W^2 - q^2)} \bar{u}_e \gamma_\beta (1-\gamma_5) u_\nu \sum_i C_i^\beta(E,\nu,q^2) T_i^{\gamma W}(\nu, q^2)$$

$$p^\mu = (M, \vec{0})$$

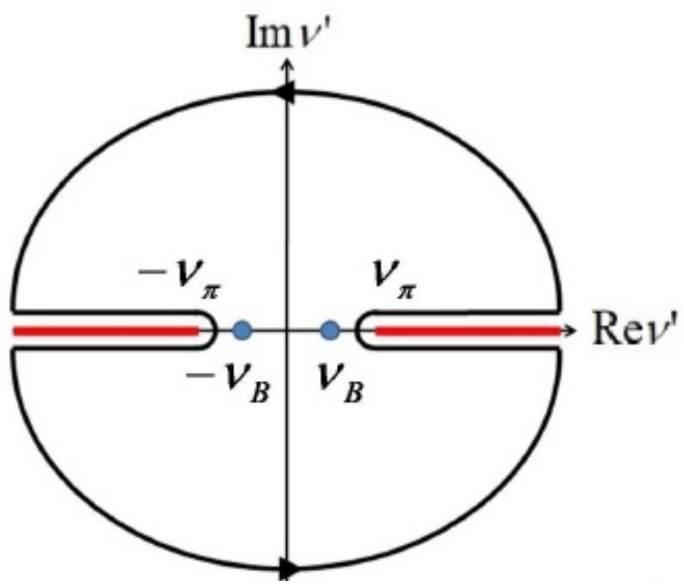
$$E = (pk)/M$$

$$\nu = (pq)/M$$

Known functions of external energy E and loop variables ν, q^2

γW-box from Dispersion Relations

γW-box from Dispersion Relations

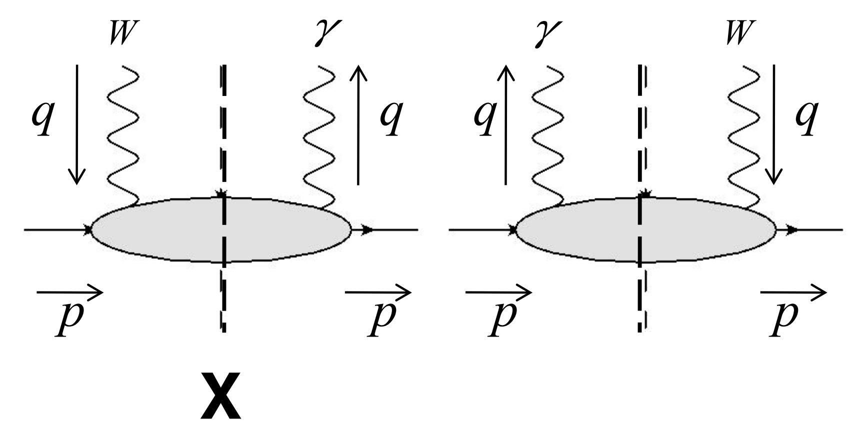


$T_{1,2,3}$ - analytic functions inside the contour C in the complex ν-plane determined by their singularities on the real axis - poles + cuts

$$T_i^{\gamma W}(\nu, Q^2) = \frac{1}{2\pi i}\oint dz \frac{T_i^{\gamma W}(z, Q^2)}{z-\nu}, \;\; \nu \in C$$

Forward amplitudes T_i - unknown;
Their absorptive parts can be related to production of on-shell intermediate states
—> a γW-analog of structure functions $F_{1,2,3}$

$$\mathrm{Im}\, T_i^{\gamma W}(\nu, Q^2) = 2\pi F_i^{\gamma W}(\nu, Q^2)$$

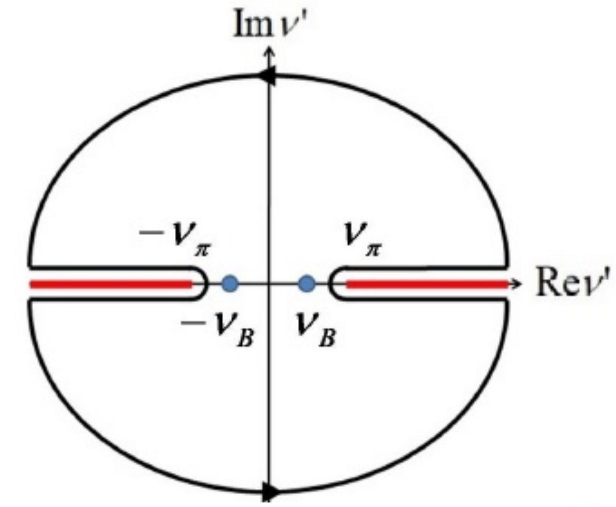


X = inclusive on-shell physical states

Structure functions $F_i^{\gamma W}$ are NOT data — but can be related to data

γW-box from Dispersion Relations

Crossing behavior: relate the left and right hand cut
Mismatch between the initial and final states - asymmetric;
Symmetrize - γ is a mix of I=0 and I=1



$$T_i^{\gamma W,a} = T_i^{(0)}\tau^a + T_i^{(-)}\frac{1}{2}[\tau^3, \tau^a]$$

$$T_i^{(I)}(-\nu, Q^2) = \xi_i^{(I)} T_i^{(I)}(\nu, Q^2)$$

$$\xi_1^{(0)} = +1, \;\; \xi_{2,3}^{(0)} = -1; \quad \xi_i^{(-)} = -\xi_i^{(0)}$$

Two types of dispersion relations for scalar amplitudes

$$T_i^{(I)}(\nu, Q^2) = 2\int_0^\infty d\nu' \left[\frac{1}{\nu' - \nu - i\epsilon} + \frac{\xi_i^{(I)}}{\nu' - \nu - i\epsilon}\right] F_i^{(I)}(\nu', Q^2)$$

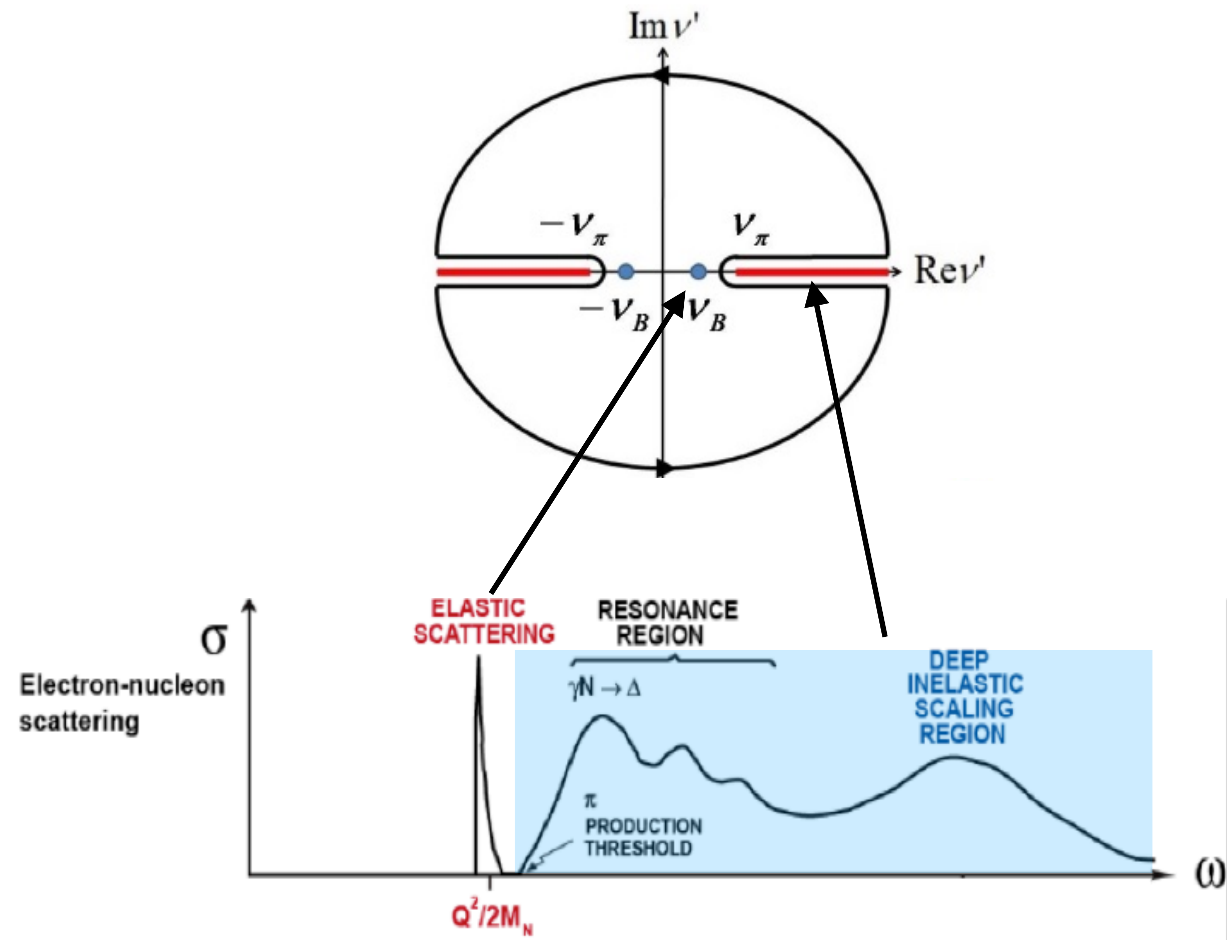
Substitute into the loop and calculate leading energy dependence

$$\mathrm{Re}\,\Box_{\gamma W}^{even} = \frac{\alpha}{\pi N}\int\limits_0^\infty dQ^2 \int\limits_{\nu_{thr}}^\infty d\nu \frac{F_3^{(0)}}{M\nu}\frac{\nu + 2q}{(\nu+q)^2} + O(E^2)$$

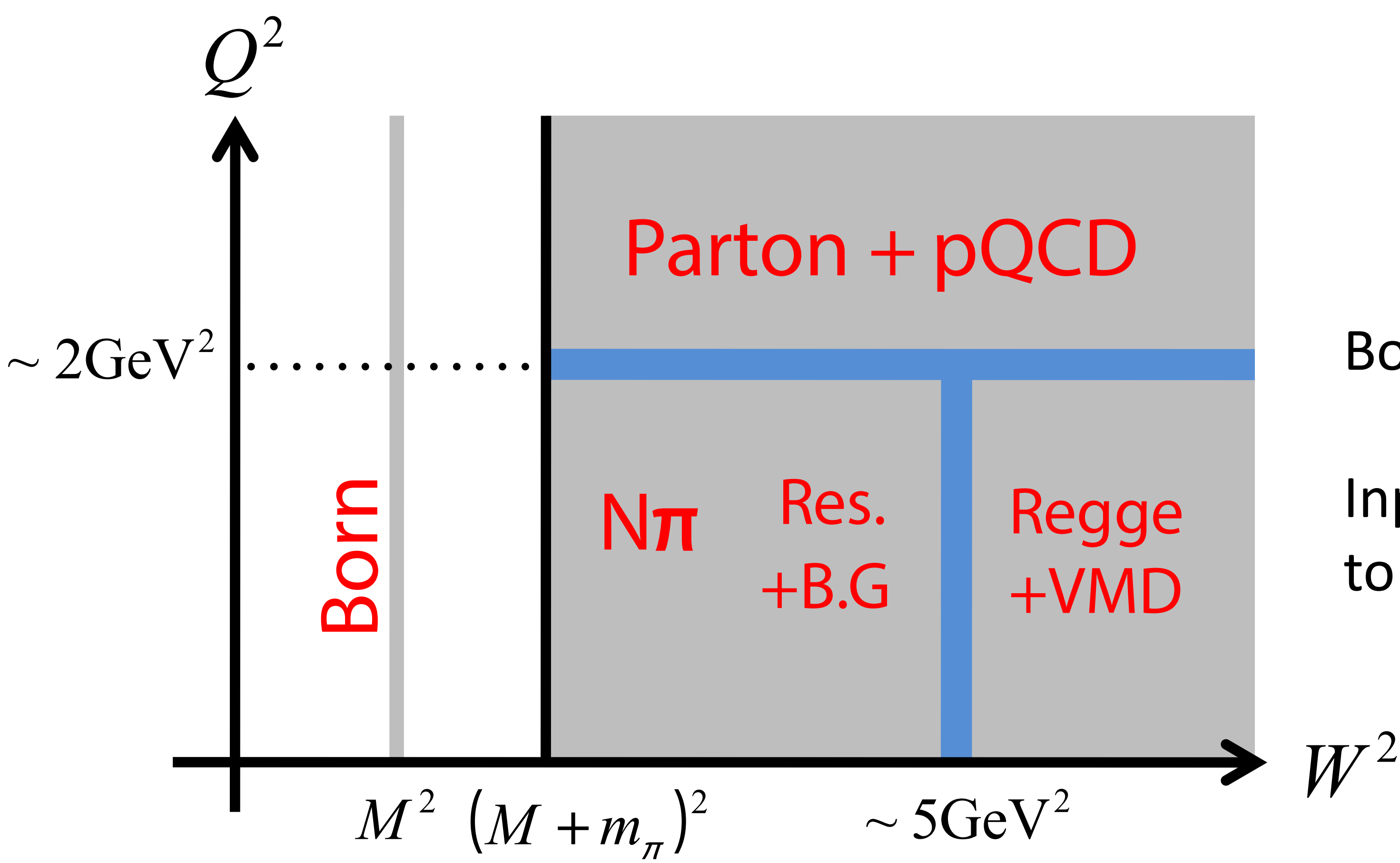
$$\mathrm{Re}\,\Box_{\gamma W}^{odd}(E) = \frac{8\alpha E}{3\pi N M}\int\limits_0^\infty dQ^2 \int\limits_{\nu_{thr}}^\infty \frac{d\nu}{(\nu+q)^3}\left[\mp F_1^{(0)} \mp \left(\frac{3\nu(\nu+q)}{2Q^2} + 1\right)\frac{M}{\nu}F_2^{(0)} + \frac{\nu+3q}{4\nu}F_3^{(-)}\right] + O(E^3)$$

Input into dispersion integral

Dispersion in energy: $W^2 = M^2 + 2M\nu - Q^2$
scanning hadronic intermediate states



Dispersion in Q²:
scanning dominant physics pictures



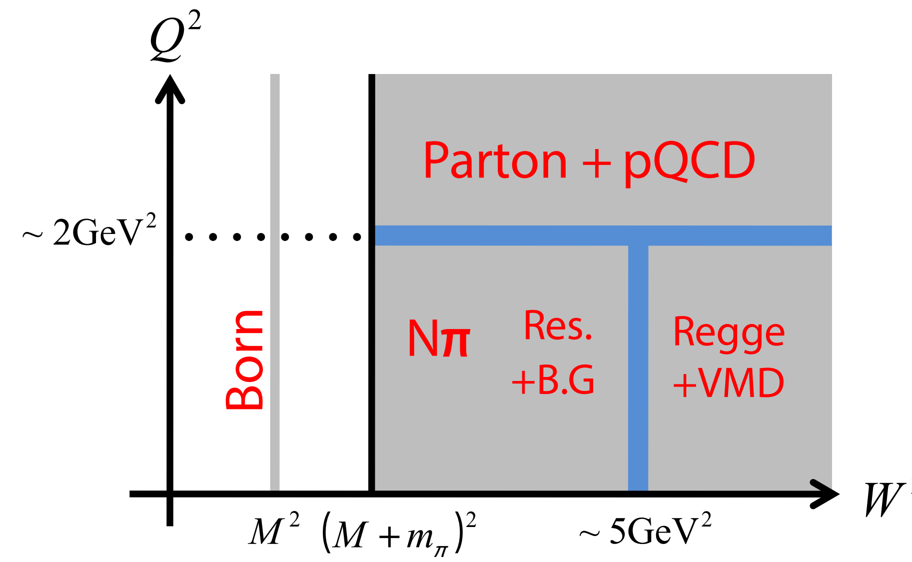
Boundaries between regions - approximate

Input in DR related (directly or indirectly) to experimentally accessible data

Input into dispersion integral

$$F_3^{(0)} \propto \int dx e^{iqx} \langle p \,|\, [J^{\mu,(0)}_{em}(x), J^{\nu,+}_{W}(0)] \,|\, n \rangle \sim \int dx e^{iqx} \sum_X \langle p \,|\, J^{\mu,(0)}_{em}(x) \,|\, X \rangle \langle X \,|\, J^{\nu,+}_{W}(0) \,|\, n \rangle$$

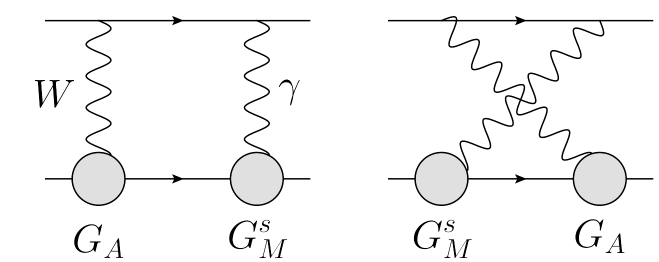
Parametrization of the needed SF follows from this diagram



$$F_3^{(0)} = F_{\rm Born} + \begin{cases} F_{\rm pQCD}, & Q^2 \gtrsim 2\ \mathrm{GeV}^2 \\ F_{\pi N} + F_{\rm res} + F_{\mathbb{R}}, & Q^2 \lesssim 2\ \mathrm{GeV}^2 \end{cases}$$

Born: elastic FF from e$^-$, ν scattering data — in DR language present at all Q^2 (NOT in EFT!)

$$\Box^{VA,\mathrm{Born}}_{\gamma W} = -\frac{\alpha}{\pi} \int_0^\infty dQ \frac{2\sqrt{4M^2+Q^2}+Q}{\left(\sqrt{4M^2+Q^2}+Q\right)^2} G_A(Q^2) G_M^S(Q^2)$$



πN: relativistic ChPT calculation plus nucleon FF

Resonances: axial excitation from PCAC (Lalakulich et al 2006) - used in neutrino event generators
isoscalar photoexcitation (PWA MAID and PDG) - electron and γ inelastic scattering

Above resonance region: multiparticle continuum described by Regge exchanges

Input into dispersion integral

Unfortunately, no data can be obtained for $F_3^{\gamma W\,(0)}$
But: data exist for the pure CC processes

$$\frac{d^2\sigma^{\nu(\bar\nu)}}{dxdy} = \frac{G_F^2 ME}{\pi}\left[xy^2F_1 + \left(1 - y - \frac{Mxy}{2E}\right)F_2 \pm x\left(y - \frac{y^2}{2}\right)F_3\right]$$

$$\sigma^{\nu p} - \sigma^{\bar\nu p} \sim F_3^{\nu p} + F_3^{\bar\nu p} = u_v^p(x) + d_v^p(x)$$

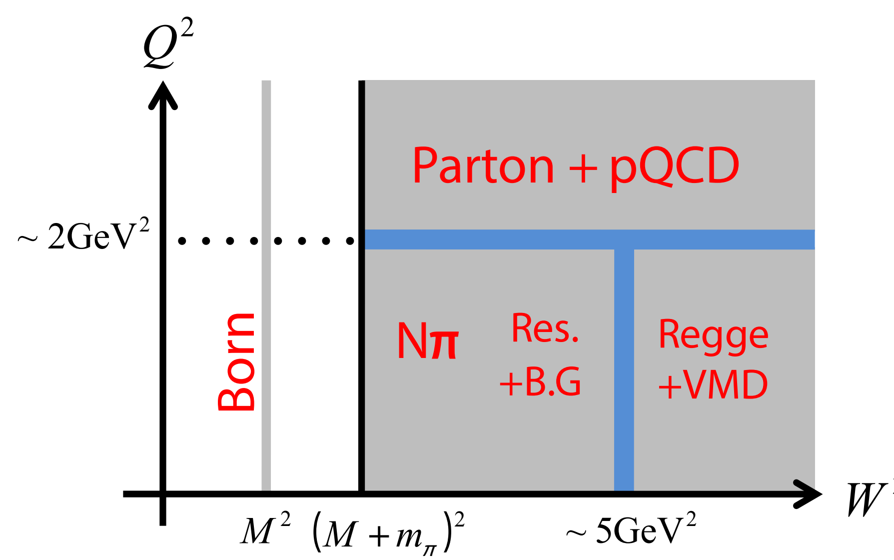
Gross-Llewellyn-Smith (number) sum rule $\int_0^1 dx(u_v^p(x) + d_v^p(x)) = 3$

Build the model for CC process; apply an isospin rotation to obtain γW

$$F_{3,\,\mathrm{low}-Q^2}^{\nu p+\bar\nu p} = F_{3,\,el.}^{\nu p+\bar\nu p} + F_{3,\,\pi N}^{\nu p+\bar\nu p} + F_{3,\,R}^{\nu p+\bar\nu p} + F_{3,\,\mathrm{Regge}}^{\nu p+\bar\nu p}$$

Low-W part of spectrum:
neutrino data from MiniBooNE, Minerva, …
- axial FF, resonance contributions, pi-N continuum

High-W: Regge behavior $F_3 \sim q^{\nu} \sim x^{-\alpha}$, $\alpha \sim 0.5$-0.7



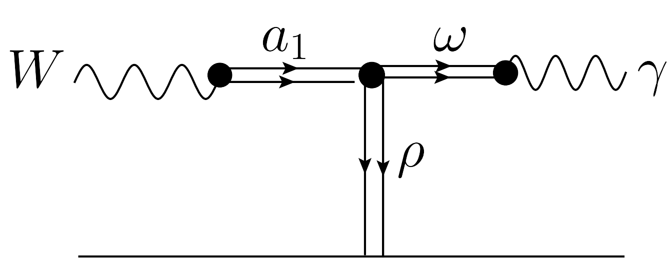
Input into dispersion integral

Scattering at high energy can be very effectively described by Regge exchanges

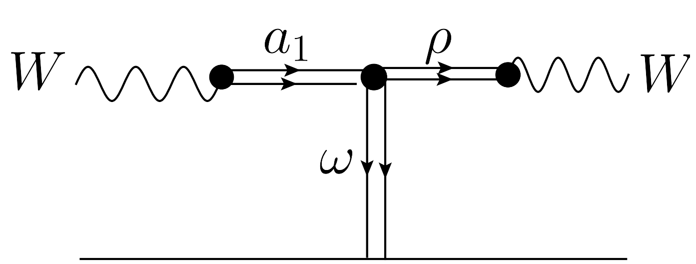
$$F_3^{(0),\mathrm{Regge}}(\nu, Q^2) = C_R(Q^2)\left(\frac{\nu}{\nu_0}\right)^{\alpha_\rho}$$

Regge behavior in EW processes: hadron-like behavior of HE electroweak probes - Vector/Axial Vector Dominance is the proper language

γW-box: conversion of $W^\pm$ (charged, I=1, axial) to γ (neutral, vector, I=0)
requires charged vector exchange w. I=1 - $\rho^\pm$
effective a_1 - ρ - ω vertex



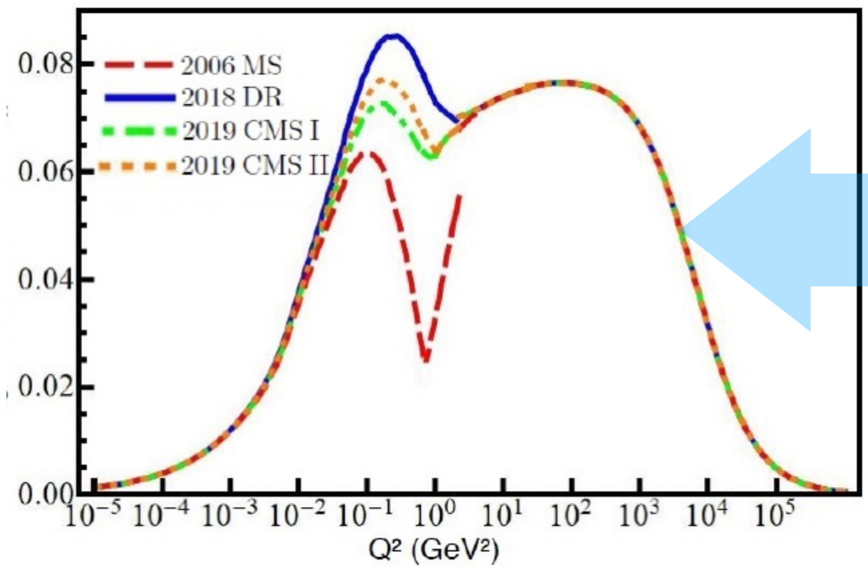
Inclusive ν scattering: conversion of $W^\pm$ (axial) to $W^\pm$ (vector)
requires neutral vector exchange w. I=0 - ω
effective a_1 - ω - ρ vertex



Minimal model for both reactions - check with data.

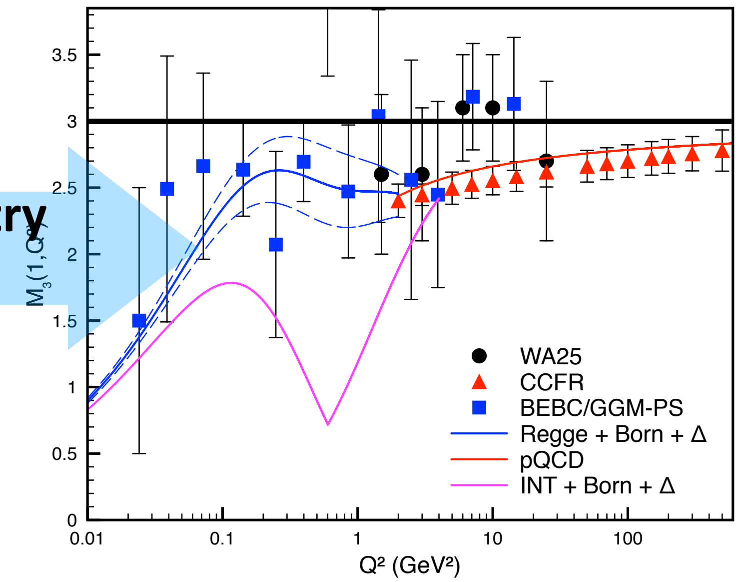
Using $\nu/\bar{\nu}$ data to constrain input

Free γW-box = area under the curve



Isospin symmetry + hadronic data

Neutrino scattering data



Gross-Llewellyn-Smith (number) sum rule

Marciano, Sirlin 2006: $\Delta_R^V = 0.02361(38)$ —> $|V_{ud}| = 0.97420(10)_{Ft}(18)_{RC}$

DR (Seng et al. 2018): $\Delta_R^V = 0.02467(22)$ —> $|V_{ud}| = 0.97370(10)_{Ft}(10)_{RC}$

Seng, MG, Ramsey-Musolf, 1807.10197; 1812.03352

Shift upwards by 3σ + reduction of uncertainty by factor 2 —> confirmed in LQCD

LQCD on pion + pheno: $\Delta_R^V = 0.02477(24)_{\mathrm{LQCD}^\pi+\mathrm{pheno}}$

Feng et al, 2003.09798
Seng et al, 2003.11264
Yoo et all, 2305.03198

LQCD on neutron: $\Delta_R^V = 0.02439(19)_{\mathrm{LQCD}^n}$

Ma et al 2308.16755

A Comment on Lattice Evaluation

Matching of the LQCD-computed integrand to pQCD

Discretization effects preclude one from going to arbitrarily high scales

pQCD prediction reliable above $Q^2 \approx 1 - 2\,\mathrm{GeV}^2$
LQCD reliable below — stitch the two together

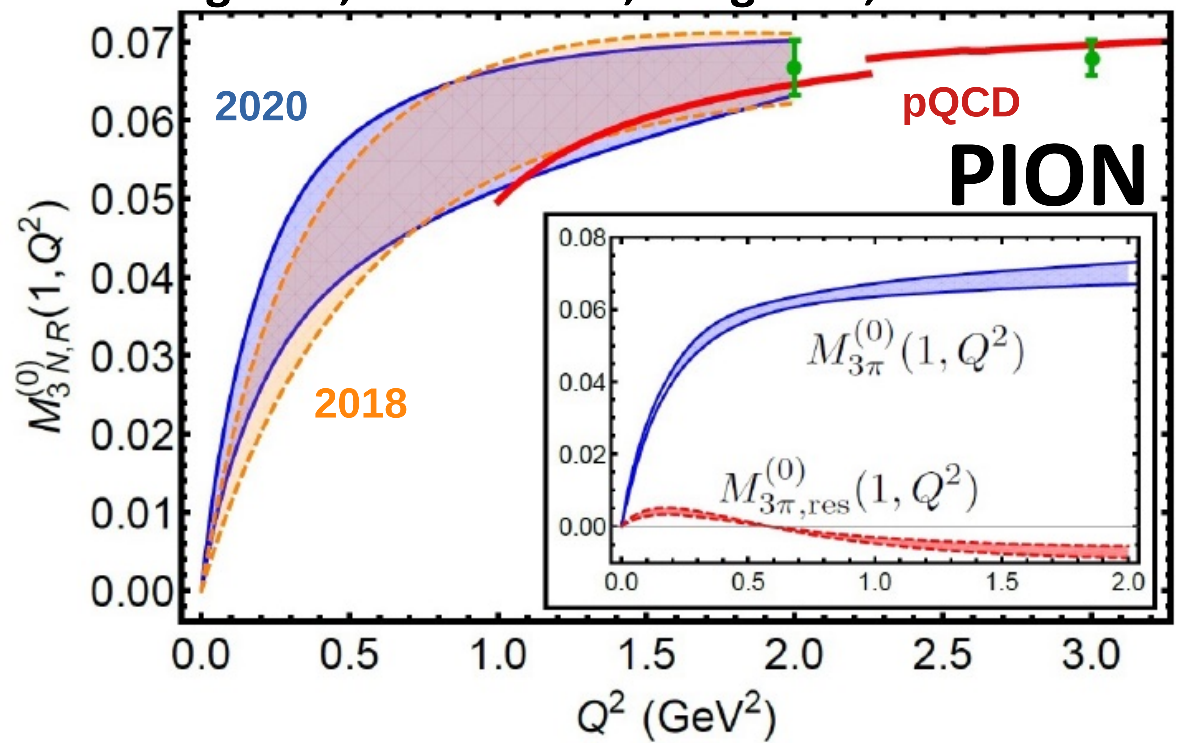
Phenomenologically:
at $Q^2 \approx 1 - 2\,\mathrm{GeV}^2$ dominated by Regge;
Regge factorizes and is universal across hadronic processes

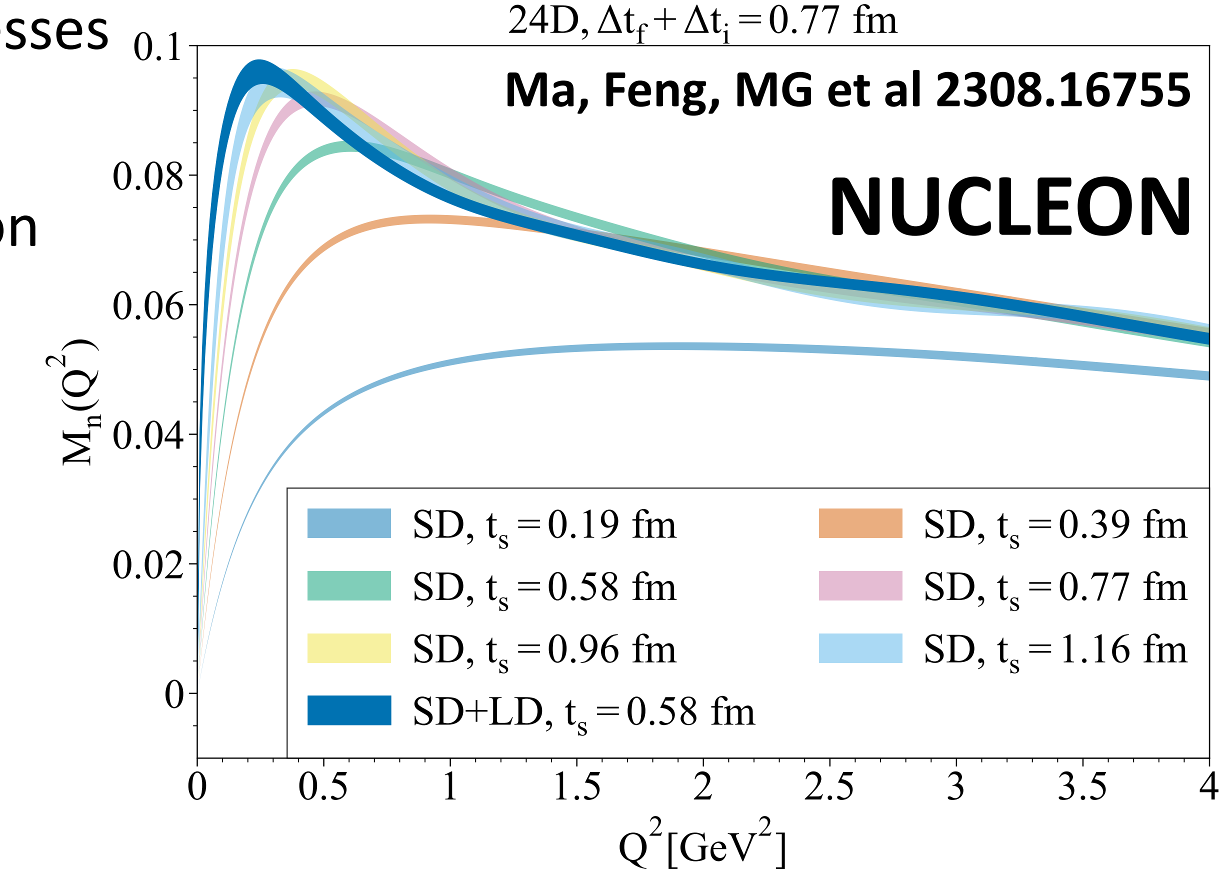
However, apparently the matching for pion and nucleon work quite differently:

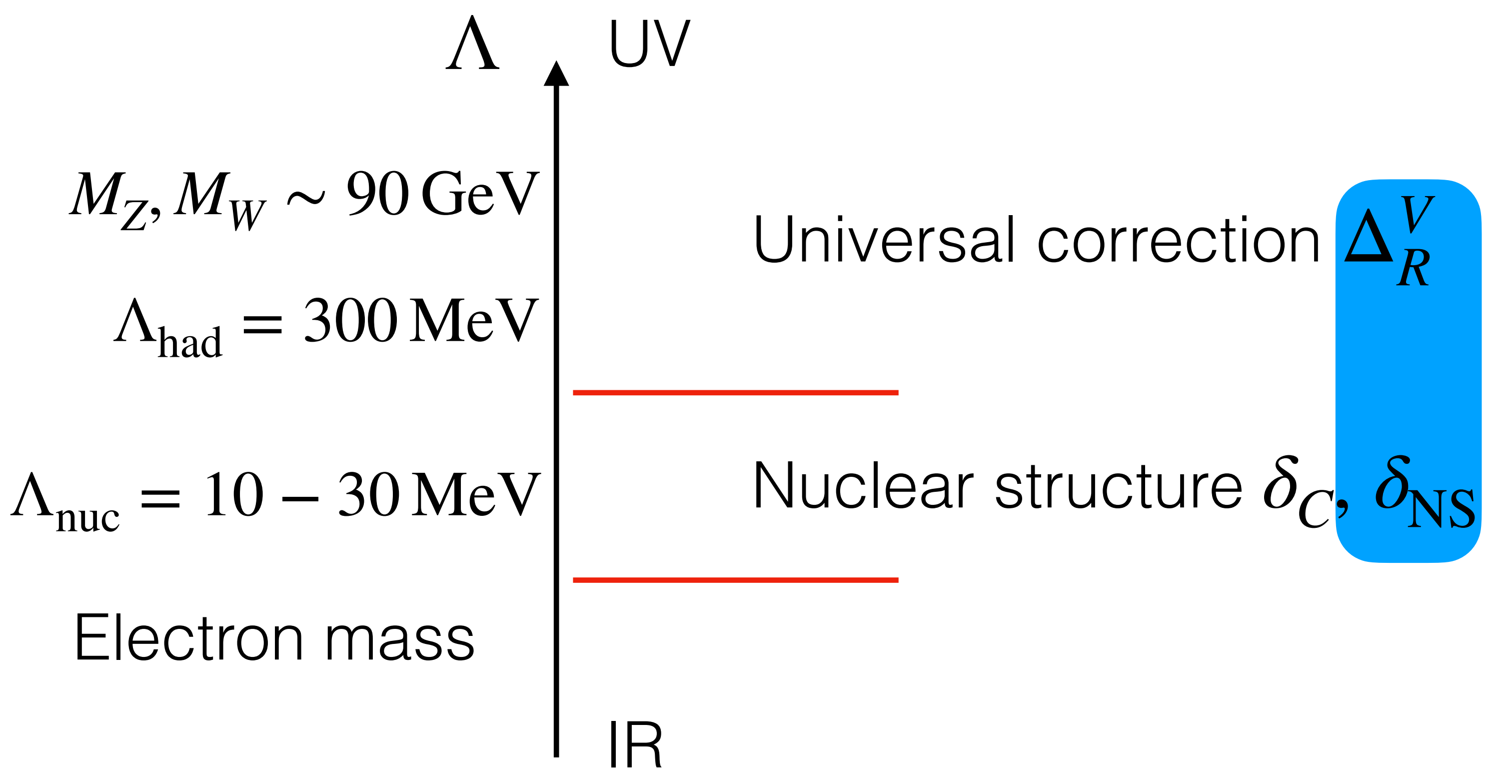
In pQCD increase as function of Q2
Nicely observed for pion
For nucleon keeps decreasing

Unknown lattice systematics/artifacts?

Feng et al, 2003.09798; Seng et al, 2003.11264







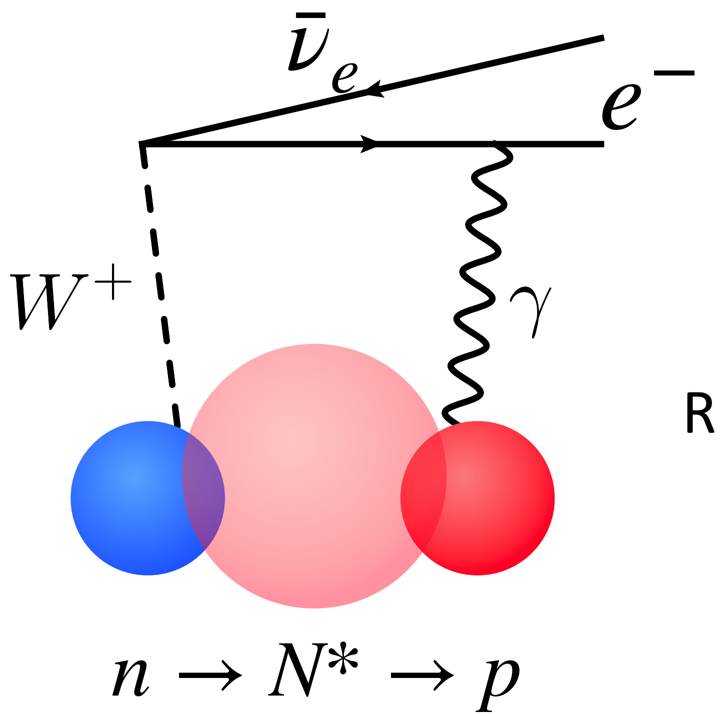
Unified Formalism for Δ_R^V and δ_{NS}

δ_{NS} : the low-energy part of γW-box

NS correction reflects extraction of the free box
DR: a framework to control this subtraction!

$$\Delta_R^V \propto 2\,\Box_{\gamma W}^{\mathrm{VA,\,free\,n}}$$

$$\Delta_R^V + \delta_{\mathrm{NS}} \propto 2\,\Box_{\gamma W}^{\mathrm{VA,\,nucl}}$$

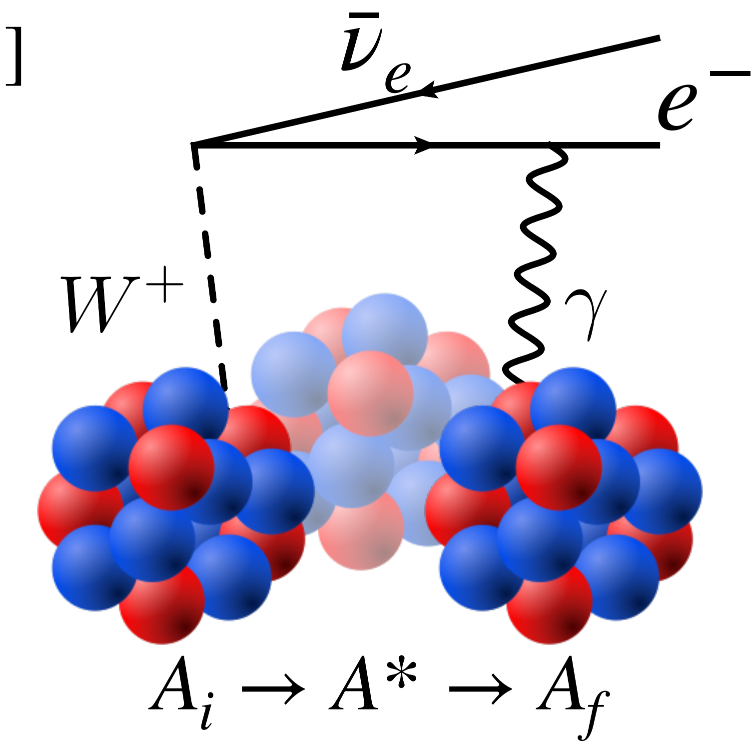


$$\delta_{\mathrm{NS}} = 2[\,\Box_{\gamma W}^{\mathrm{VA,\,nucl}} - \Box_{\gamma W}^{\mathrm{VA,\,free\,n}}\,]$$

Differences due to:

Richer excitation spectrum in nuclei

Different quantum numbers
(spin, isospin)



Early insights from DR:
reduction of “elastic γW-box” in nuclei underestimated
significant energy dependence due to nuclear polarization

Seng et al, 1812.03352
MG, 1812.04229

Ab initio nuclear theory for δ_{NS} with controlled uncertainty: several groups active!

*** Ab initio calculations do not use DR: structure functions more complicated than their moments**

δ_{NS} in ab-initio nuclear theory

Low-momentum part of the loop: account for nucleon d.o.f. only

Modern framework: ab initio methods

NN interaction derived from chiral effective field theory (χEFT)

Pions integrated out: low energies, pions not dynamical, only nucleons

Low-energy coefficients (LEC) of χEFT fitted to NN-scattering data (scattering phase, length, effective range, …)

- Nuclear interactions from Chiral EFT:
  - NN-N^{4}LO+3N$_{lnl}$
  - NN-N^{4}LO+3N*$_{lnl}$

Entem, Machleidt and Nosyk, 2017 PRC;
Gysbers et al., 2019 Nature;
Kravvaris, Navrátil, Quaglioni, Hebborn and Hupin, 2023 PLB

Systematically improvable calculations, controlled uncertainty estimates

Various methods are being developed:

No-Core Shell Model (NCSM)

Quantum Monte Carlo

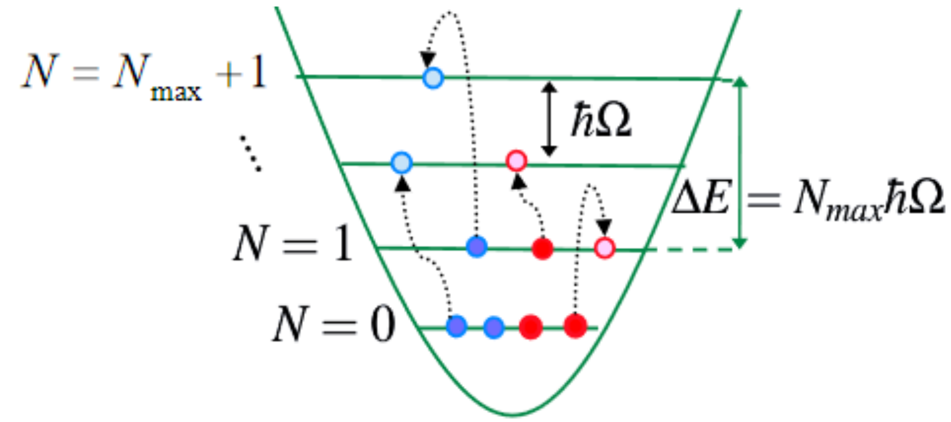
Coupled Cluster

In-Medium Similarity Renormalization Group

δ_{NS} for $^{10}C \rightarrow {}^{10}B$ in NCSM

- Nuclear interactions from Chiral EFT:
  - NN-N^{4}LO+3N$_{lnl}$
  - NN-N^{4}LO+3N*$_{lnl}$

Entem, Machleidt and Nosyk, 2017 PRC;
Gysbers et al., 2019 Nature;
Kravvaris, Navrátil, Quaglioni, Hebborn and Hupin, 2023 PLB

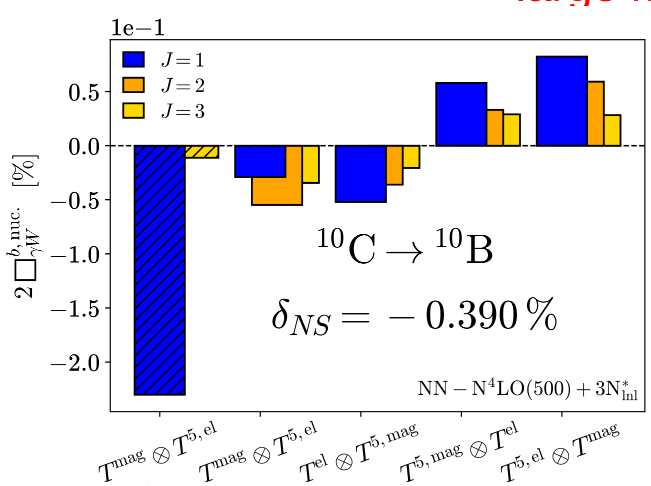


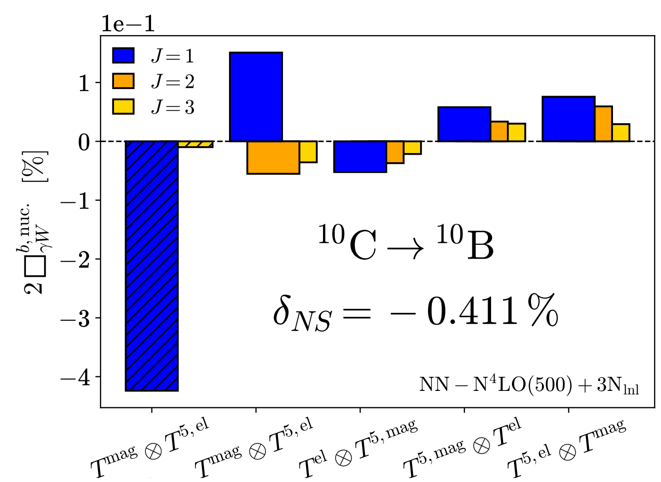
Evaluate the m.e. of nuclear Green's function

$$G(z) \equiv \frac{1}{z - H_0}$$

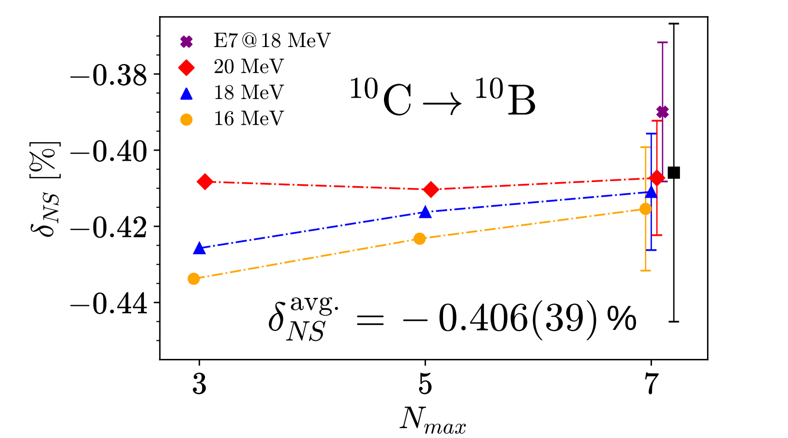
Difficulty: Inverting a large matrix!

Lanczos continuous fraction method





Check **Ω-independence** and **convergence w.r.t. N$_{max}$**



$$\delta_{\rm NS} = -\,0.406(39)\,\%$$

Gennari et al, 2405.19281

Compare to Hardy-Towner (old-fashion Shell Model)

$\delta_{\rm NS} = -\,0.347(35)\,\%$ HT (2014) → Dispersion formalism: correct account for quasielastic knockout and energy dependence → $\delta_{\rm NS} = -\,0.400(50)\,\%$ HT (2020)

Seng et al, 1812.03352; MG 1812.04229

Ab-initio δ_{NS} for $^{10}C \to {}^{10}B$ and $^{14}O \to {}^{14}N$ transitions in QMC

Ab initio QMC calculation for $^{10}C \to {}^{10}B$ $\delta_{\rm NS} = -0.429(73)\,\%$ **King et al 2509.07310**

Compare to NCSM $\delta_{\rm NS} = -0.406(39)\,\%$ **Gennari et al, 2405.19281**

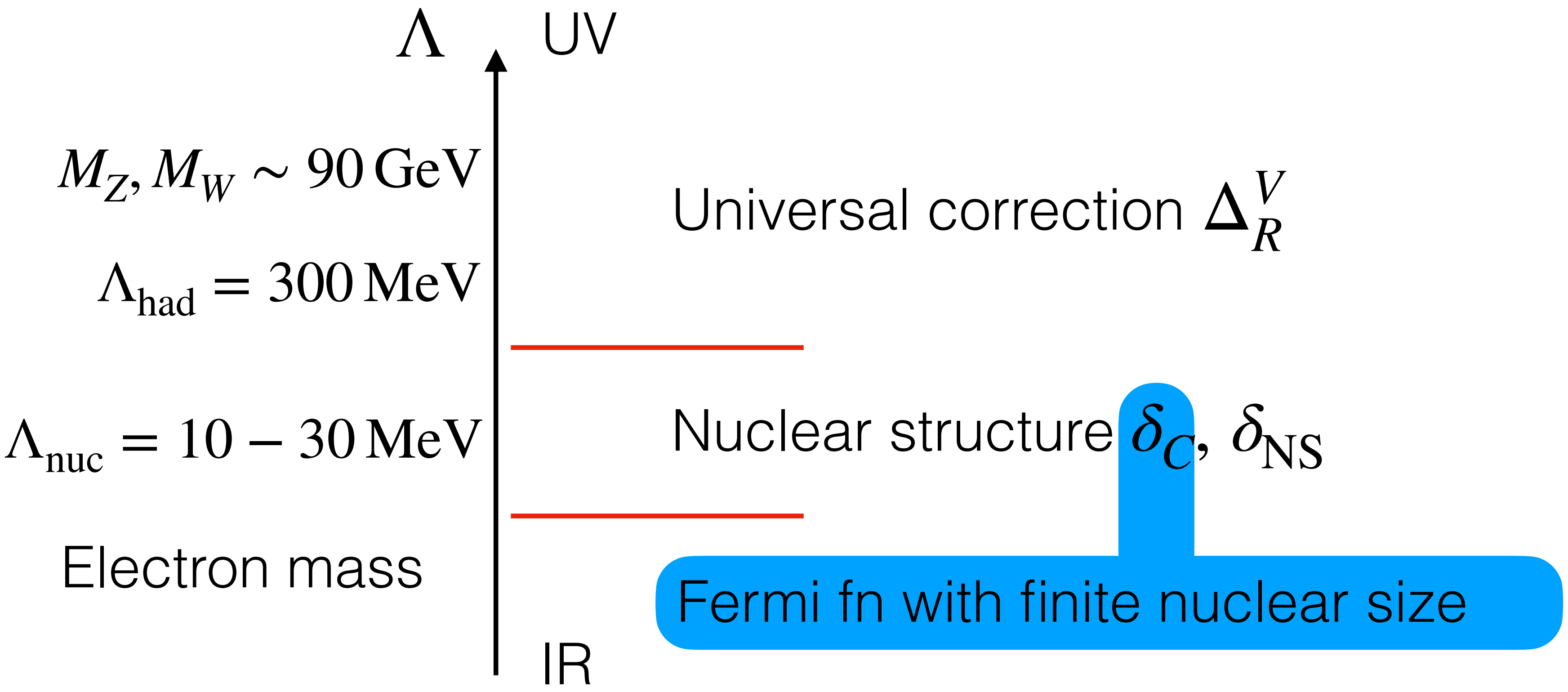
Compare to a previous shell model estimate $\delta_{\rm NS} = -0.400(50)\,\%$ **Hardy, Towner, PRC 2020**

First ab initio QMC calculation for $^{14}O \to {}^{14}N$ $\delta_{\rm NS} = -0.187(88)\,\%$ **Cirigliano et al, 2405.18469**

Compare to Hardy-Towner 2020: $\delta_{\rm NS} = -0.196(50)\,\%$

NCSM (Gennari et al) — in progress, stay tuned!

δ_{NS} in the EFT language: unknown LEC limit the accuracy of predictions



Finite nuclear size effects: β spectrum and δ_C

QED + FNS corrections to β-spectrum

$$f = m_e^{-5} \int_{m_e}^{E_0} dE_e \, |\vec{p}_e| E_e (E_0 - E_e)^2 F(E_e) C(E_e) Q(E_e) R(E_e) r(E_e)$$

Unperturbed beta spectrum

Depend on finite nuclear size

Fermi function F: e⁺ in Coulomb field

Shape factor C: spatial distribution of decay probability

Pure QED

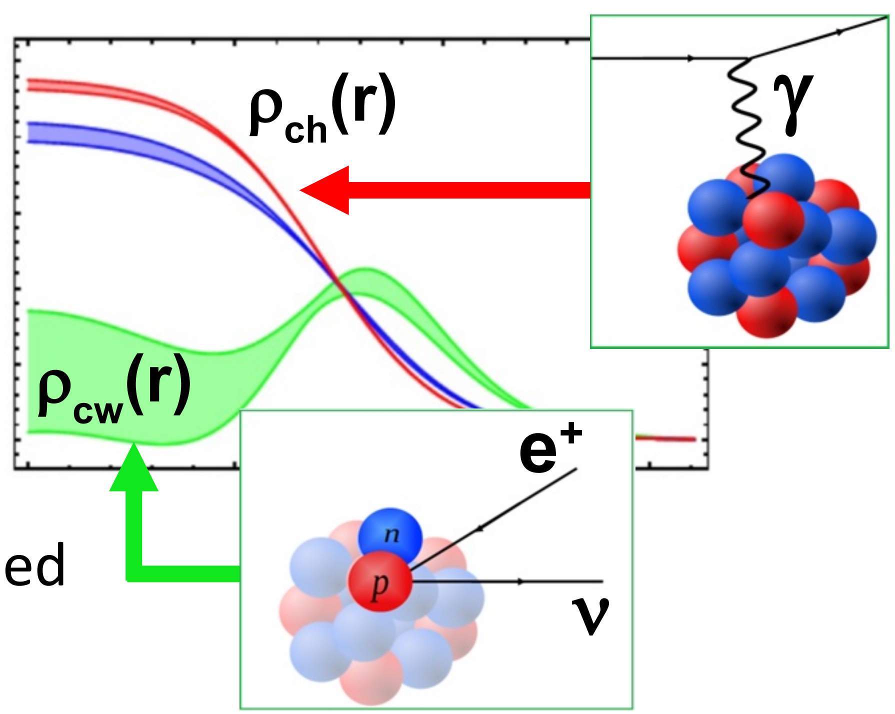
Traditionally: assumed decay probability equally distributed across the nucleus, $\rho_{\rm cw} \approx \rho_{\rm ch}$

But: Isospin symmetry + known charge distributions of T=1 members implies

Seng, 2212.02681
MG, Seng 2311.16755

$$0^+, T=1, T_z=-1 \;\; \rho_{ch}^{T_z=-1} \xrightarrow{\rho_{cw}} 0^+, T=1, T_z=0 \;\; \rho_{ch}^{T_z=0} \xrightarrow{\rho_{cw}} 0^+, T=1, T_z=1 \;\; \rho_{ch}^{T_z=1}$$

$$\rho_{cw} = Z_0 \rho_{ch}^{T_z=0} - Z_1 \rho_{ch}^{T_z=1} = \frac{1}{2}\left[Z_{-1} \rho_{ch}^{T_z=-1} - Z_1 \rho_{ch}^{T_z=1} \right]$$



Photon probes the entire nuclear charge

Only outer protons can decay: all neutron states in the core occupied

Transition density has larger radius

Impact of precise nuclear radii on Ft and V_{ud}

Recent measurement at ISOLDE

Plattner et al, arXiv: 2310.15291 $R_c(^{26m}\mathrm{Al}) = 3.130(15)\ \mathrm{fm}$

Previously guessed by Hardy and Towner

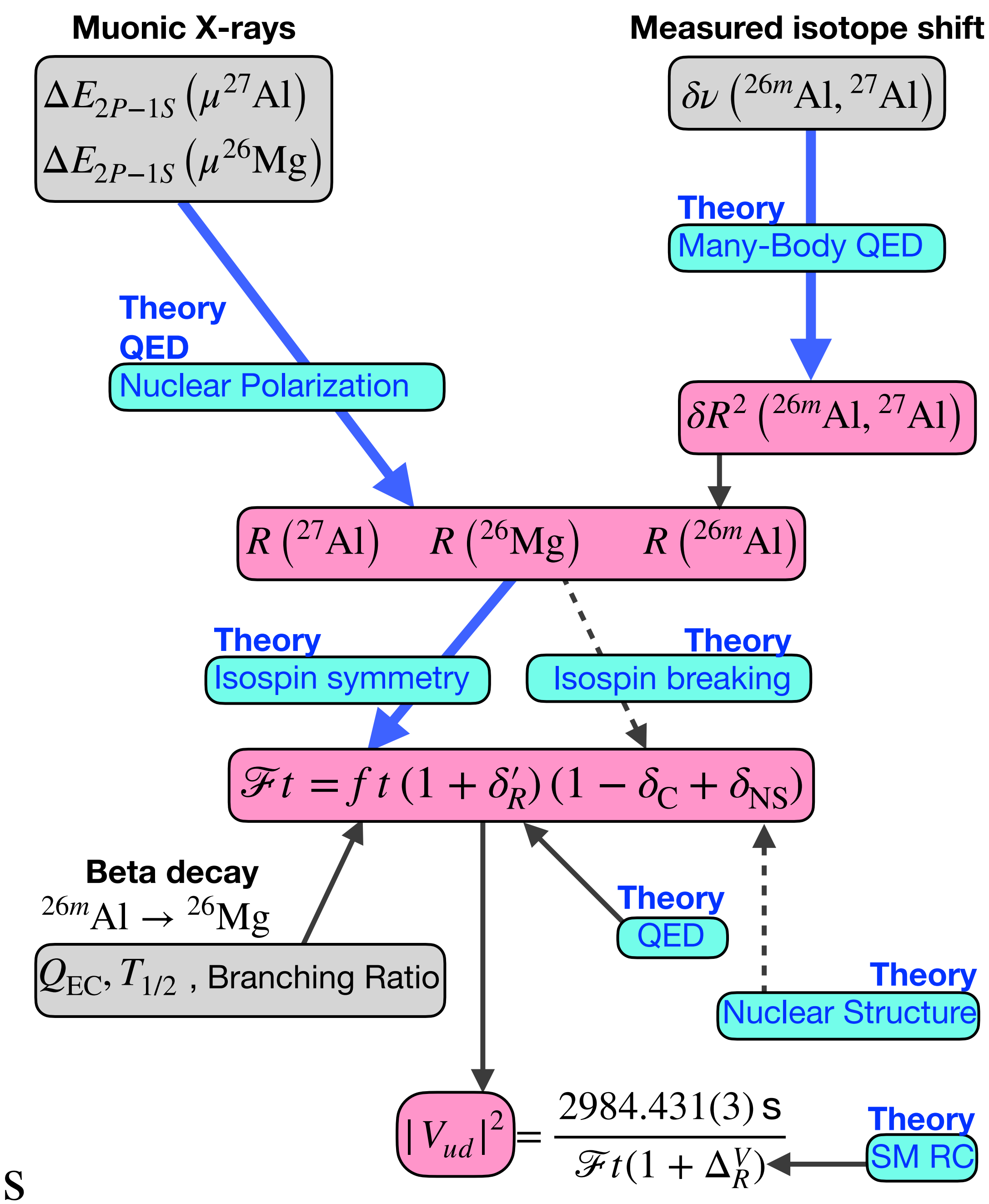
$$R_c(^{26m}Al) = 3.040(20)\,\mathrm{fm}$$

Re-examined ~ALL ingredients **MG et al, arXiv: 2502.17070**

Careful reevaluation of f-value (QED)

isotope shift factors F, M (Many-body QED for e-atoms)

charge radii of Al-27, Mg-26 (Nuclear theory for μ-atoms)



Major impact on Ft value uncovered

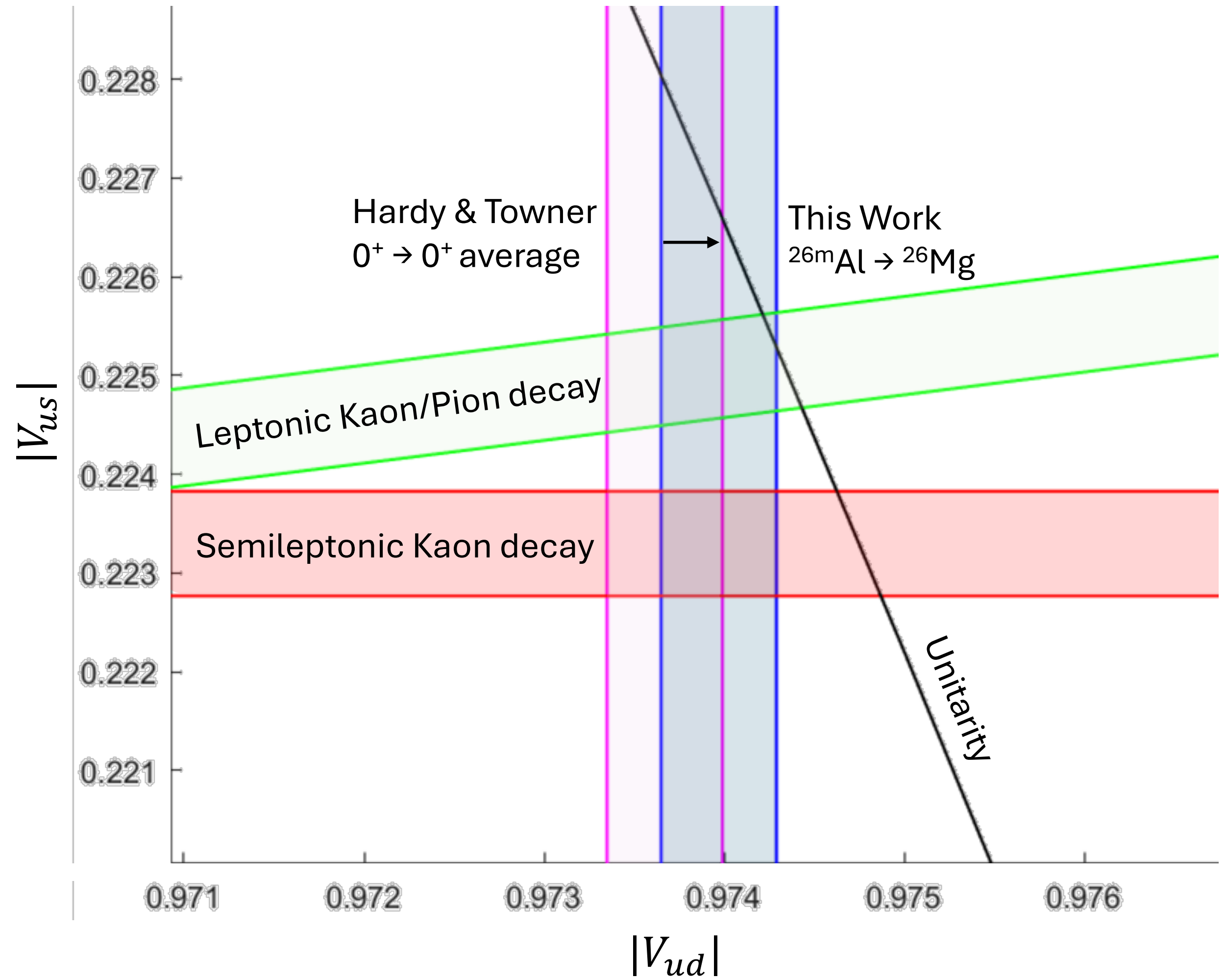
$$\mathscr{F}t[^{26m}\mathrm{Al} \rightarrow {}^{26}\mathrm{Mg}] = 3072.4(1.1)_{\mathrm{stat}}\,\mathrm{s} \rightarrow 3070.0(1.2)_{\mathrm{stat}}\,\mathrm{s}$$

Al-26m—> Mg-26 is the most precisely measured transition —> impacts the V_{ud} determination!

One radius makes a difference in BSM search!

Cabibbo anomaly disappears? — 2.5σ to 1.3σ

$$|V_{ud}|^2 + |V_{us}|^2 = 0.9985(7) \longrightarrow |V_{ud}|^2 + |V_{us}|^2 = 0.9991(7)$$



MG et al, arXiv: 2502.17070

But: only f was revisited; need to check δ_{NS} and δ_C

Isospin-breaking correction δ_C in nuclear models

ISB correction δ_C changes by factor ~10 from light to heavy
Crucial for Ft-alignment!

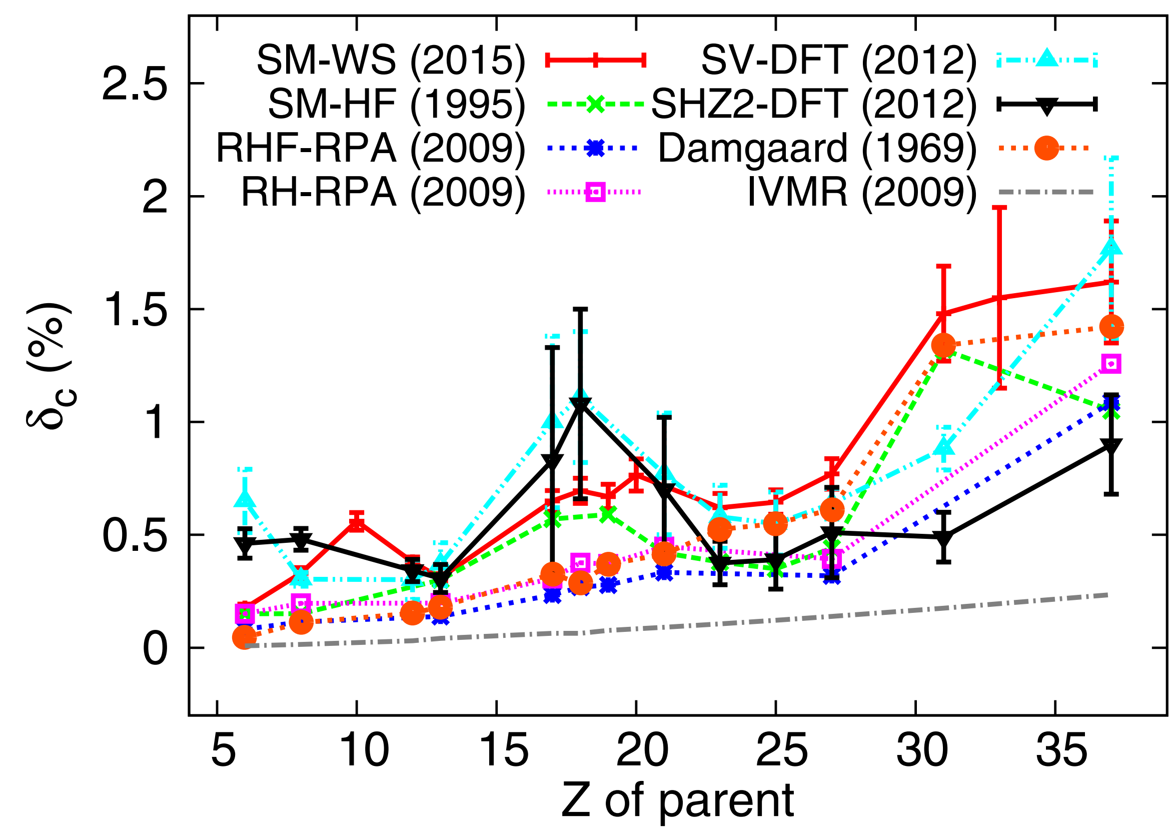
Nadezhda's talk

	SM-WS	SM-HF	RPA PKO1	RPA DD-ME2	RPA PC-F1	IVMR[a]	DFT
$T_z = -1$							
^{10}C	0.175	0.225	0.082	0.150	0.109	0.147	0.650
^{14}O	0.330	0.310	0.114	0.197	0.150		0.303
^{22}Mg	0.380	0.260					0.301
^{34}Ar	0.695	0.540	0.268	0.376	0.379		
^{38}Ca	0.765	0.620	0.313	0.441	0.347		
$T_z = 0$							
^{26m}Al	0.310	0.440	0.139	0.198	0.159		0.370
^{34}Cl	0.650	0.695	0.234	0.307	0.316		
^{38m}K	0.670	0.745	0.278	0.371	0.294	0.434	
^{42}Sc	0.665	0.640	0.333	0.448	0.345		0.770
^{46}V	0.620	0.600					0.580
^{50}Mn	0.645	0.610					0.550
^{54}Co	0.770	0.685	0.319	0.393	0.339		0.638
^{62}Ga	1.475	1.205					0.882
^{74}Rb	1.615	1.405	1.088	1.258	0.668		1.770
χ^2/ν	1.4	6.4	4.9	3.7	6.1		4.3[b]

Hardy, Towner, Phys.Rev. C 91 (2014), 025501

δ_C plagued by large model dependence!

Xayavong, Smirnova, 1708.00616



HT: χ^2 as criterion to prefer SM-WS;
—> V_{ud} and BSM intertwined with nuclear models!

Nuclear theory community embarked on ab-initio δ_C calculations
Complement with independent test: data-driven approach to benchmark model calculations

Data-driven δ_C from nuclear radii

ISB-sensitive combinations of nuclear radii across isotriplet

Seng, MG 2208.03037; 2304.03800; 2212.02681

$$\Delta M_B^{(1)} \equiv \frac{1}{2}\left(Z_1 R_{p,1}^2 + Z_{-1} R_{p,-1}^2\right) - Z_0 R_{p,0}^2$$

$\Delta M_B^{(1)} = 0$ used for f-value in isospin limit

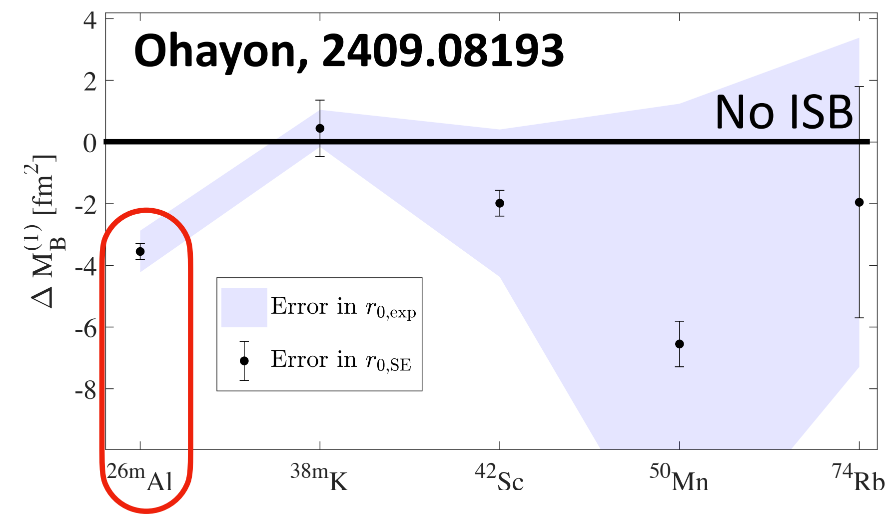
Many radii not known: use phenomenological information from known mirror radii

5 isotriplets to test the IS assumption

No ISB for A=38, 42, 50, 74

Large ISB for A=26!

Precision needs to be improved to test



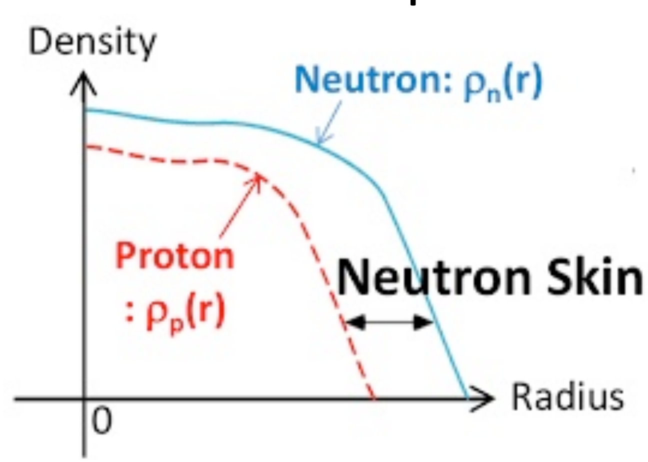
Another ISB-sensitive combination involves radii of neutron and proton distributions

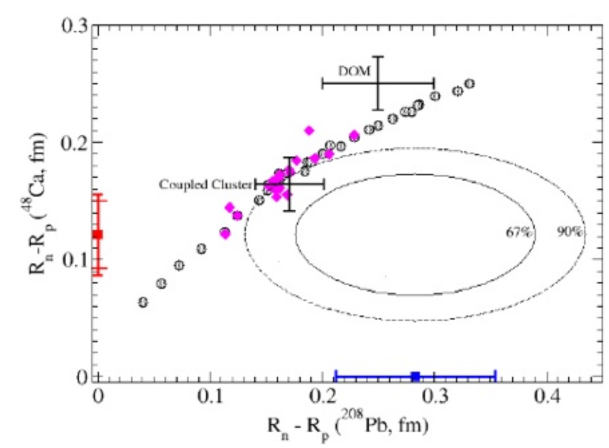
$$\Delta M_A^{(1)} \equiv -\langle r_{\rm CW}^2\rangle + \left(\frac{N_1}{2}\langle r_{n,1}^2\rangle - \frac{Z_1}{2}\langle r_{p,1}^2\rangle\right)$$

Neutron radius accessible with PV e-scattering

PV asymmetry ~ $\langle r_{n,1}^2\rangle - \langle r_{p,1}^2\rangle$ - neutron skin

Studies in neutron rich nuclei <—> neutron stars



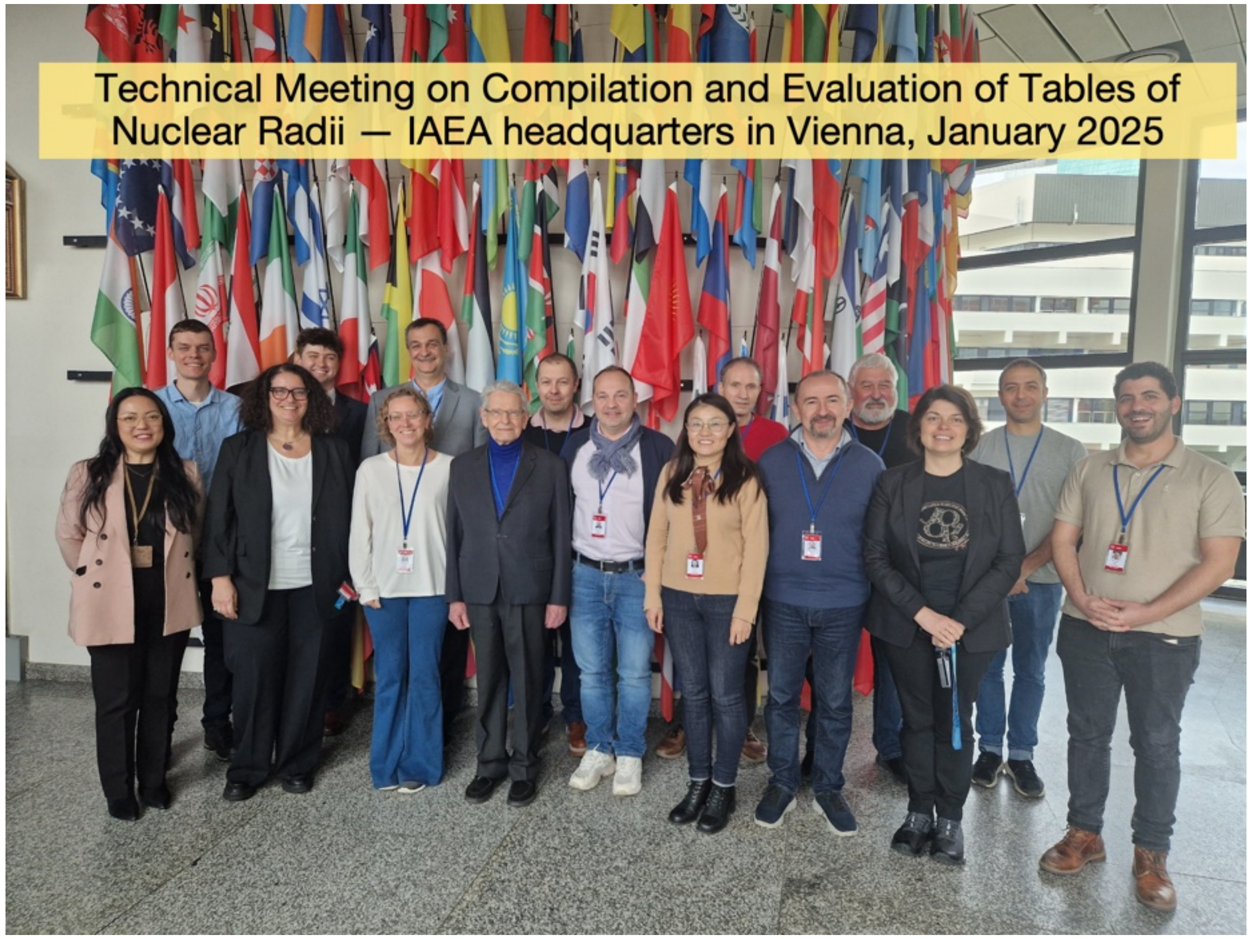


Upcoming exp. program at Mainz (MREX):
neutron skins of stable daughters (e.g. Mg-26, Ca-42, Fe-54)
Sub-% measurement of R_n feasible (case study C-12)

N. Cargioli et al, 2407.09743

New endeavor: updated tables of nuclear radii

Work on update of Angeli-Marinova tables initiated under umbrella of IAEA



Summary report online:
https://nds.iaea.org/publications/indc/indc-nds-0918/

Initiative group working on the White Paper with recommendations for update
—> will be proposed to the community for endorsement

RADIANT (Radii Analysis and Data for InterActive Nuclear Table)
project within HORIZON EUROPE (European network application) - awaiting approval

Upcoming workshops in 2026

“Precise nuclear radii and beyond” MPIK Heidelberg, January 26-30
https://plan.events.mpg.de/event/544/overview

NREC-2026 (Nuclear Radius Extraction Collaboration), Stony Brook U., April 13-17
https://indico.cfnssbu.physics.sunysb.edu/event/515/overview

MITP program “Tensions in the CKM Paradigm: From B Decays to the Cabibbo Anomaly"
Capri, May 18-29
https://indico.mitp.uni-mainz.de/event/440/overview

ECT* workshop “From Nuclear Structure to New Physics”,
Trento, August 3-7
https://www.ectstar.eu/workshops/from-nuclear-structure-to-new-physics/

Apply!

International workshop on Electroweak Precision InterseCtions EPIC 2024

September 22-27 2024, Cala Serena Beach Resort (Geremeas)

https://indico.cern.ch/event/1400714/

Brought together different communities:

Particle, Nuclear, Atomic, Neutrino, Astro, GW

Study existing synergies & elaborate new ones!

Pre-workshop school for PhD students, poster and pitch-talk prizes

Excellent infrastructure to bring along your family

STAY TUNED!

2nd EPIC workshop planned in fall 2027

~last week of September — Exact dates to be confirmed

Conclusions & Outlook

- Tests of Cabibbo unitarity at 0.01% require hadronic corrections to 10%

- Interplay of experiment, LQCD, EFT, ab initio nuclear theory and data-driven methods
  - EFTs: overarching language from IR to UV (control ALL large logs)
  - LQCD: non-perturbative input away from extremes (finite spacing&volume)
  - Dispersion theory: unitarity and analyticity to connect scales, LQCD and EFT
  - Nuclear theory community embarked on re-evaluation of nuclear structure corrections with modern ab initio methods
  - New connections with atomic physics (nuclear radii) and PVES (neutron skins) IDed & explored

- Experimental programs: new results to be expected in near future
  - Improved neutron lifetime (bottle: UCNτ, τSPECT, PENeLOPE, HOPE; beam: NIST, JPARC)
  - Improved λ (Nab, pNAB, PERC) in near future
  - Competitive V_{ud} from pion beta decay (PIONEER) in ~10 years
  - Improved superallowed (IGISOL, TRIUMF, UW, …)
  - Improved charge radii (ISOLDE, TRIUMF, FRIB) and neutron skins (MESA)