Neutrons and nuclei as a precision laboratory for Vud and CKM unitarity

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INT workshop “New physics searches at the precision frontier”, May 8, 2023

Outline

The role of β-decays in constructing the Standard Model

Radiative corrections to β-decays: overall setup

EW boxes from dispersion theory and status of Δ_R^V

Dispersion theory of nuclear-structure RC δ_{NS}

Status of isospin-symmetry breaking correction δ_C

Status of V_{us}

BSM solutions to Cabibbo-unitarity puzzle

Open problems and outlook

β-decays as precision tool
for testing the Standard Model

Understanding β Decays: A Cornerstone of the Standard Model

Existence of neutrinos to explain the continuous β spectrum (Pauli, 1930)

Contact theory of β decay (Fermi, 1933)

Parity violation in β decay (Lee, Yang 1956 & Wu 1957)

V - A theory (Sudarshan & Marshak and Gell-Mann & Feynman, 1957)

Radiative corrections to 4-Fermi theory: important step to the Standard Model

RC to muon decay UV finite for V-A —> $G_F = G_\mu = 1.1663788(7) \times 10^{-5} GeV^{-2}$

But RC to neutron decay - log UV divergent!

UV behavior of β decay rate at 1-loop (Sirlin, 1967) $\frac{\alpha}{2\pi} P^0 d^3 p \; 3[1 + 2\bar{Q}] \ln(\Lambda/M)$

$\bar{Q}$: average charge of fields involved: $1 + 2\bar{Q}_{\mu,\nu_\mu} = 0$ but $1 + 2\bar{Q}_{n,p} = 2$

Standard Model with massive W,Z-bosons (Glashow-Salam-Weinberg, 1967)

Precision, Universality and CKM unitarity

In SM the same coupling of W-boson to leptons and hadrons, $G_V = G_\mu$

Before RC were included: $G_V \sim 0.98 G_\mu$

Large $\log(M_Z/M_p)$ in RC for neutron —> $G_V \sim 0.95 G_\mu$

Kaon and hyperon decays? ($\Delta S = 1$) — even lower rates!

Cabibbo: strength shared between 2 generations

$$|G_V^{\Delta S=0}| = \cos\theta_C G_\mu$$

$$|G_V^{\Delta S=1}| = \sin\theta_C G_\mu$$

Cabibbo unitarity: $\cos^2\theta_C + \sin^2\theta_C = 1$

Kobayashi & Maskawa: 3 flavors + CP violation — CKM matrix V

$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix}$$

CKM unitarity - completeness of the SM: $VV^\dagger = \mathbf{1}$

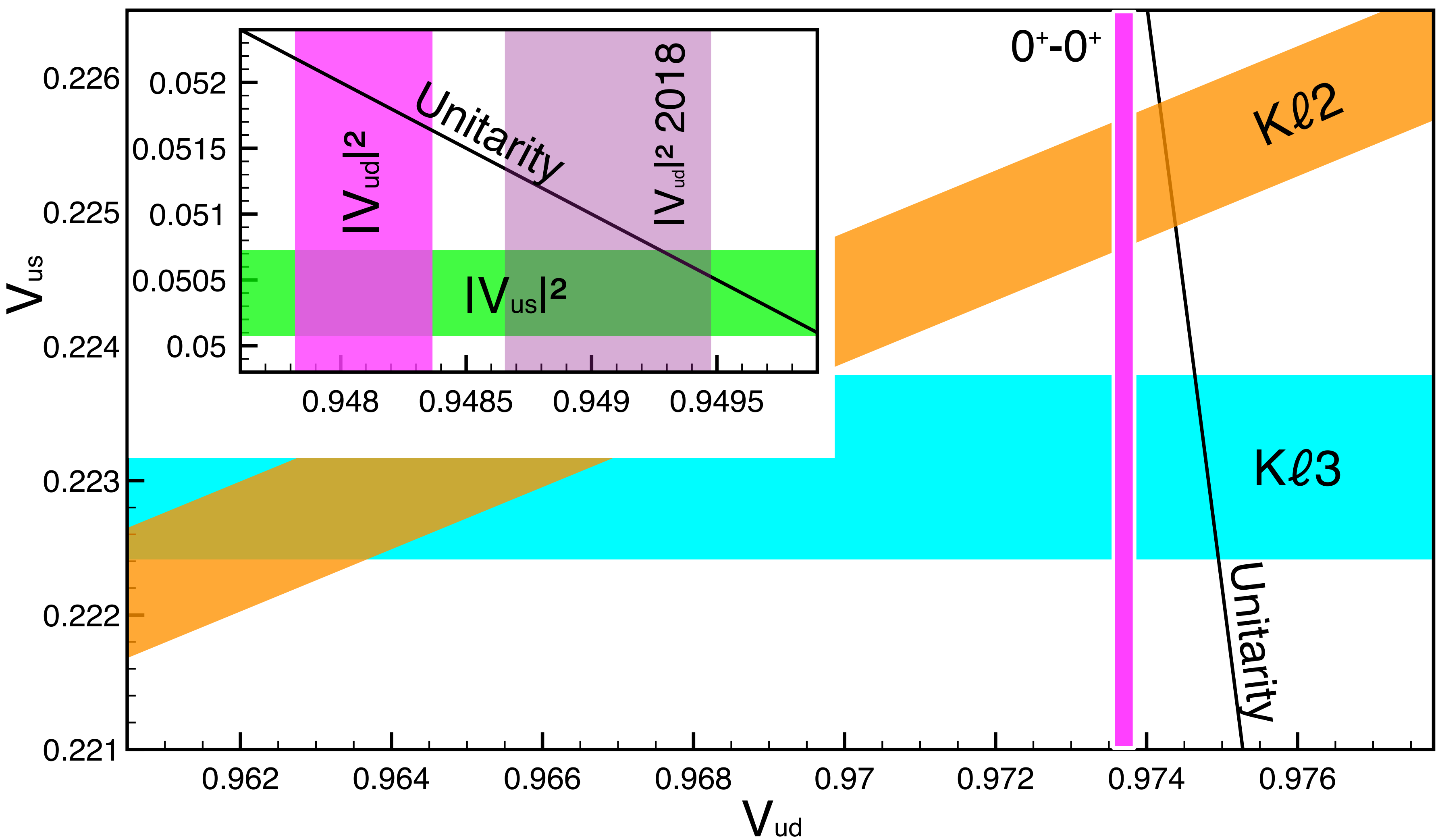
Top row unitarity constraint: $|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 1$

Detailed understanding of β decays largely shaped the Standard Model

Status of top-row CKM unitarity

$$|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 0.9985(6)_{V_{ud}}(4)_{V_{us}}$$

~ 0.95 ~ 0.05 $\sim 10^{-5}$



Inconsistencies between measurements of V_{ud} and V_{us} and SM predictions

Main reason for Cabibbo angle anomaly: significant shift in V_{ud}

Status of V_{ud}

0^+-0^+ nuclear decays: long-standing champion

$$|V_{ud}|^2 = \frac{2984.43s}{\mathscr{F}t(1+\Delta_R^V)}$$

$$|V_{ud}^{0^+-0^+}| = 0.97370\,(1)_{exp,\,nucl}\,(3)_{NS}\,(1)_{RC}[3]_{total}$$

Nuclear uncertainty x 3

Neutron decay: discrepancies in lifetime τ_n and axial charge g_A; competitive!

$$|V_{ud}|^2 = \frac{5024.7\text{ s}}{\tau_n(1+3g_A{}^2)(1+\Delta_R)}$$

Single best measurements only

$$|V_{ud}^{\text{free n}}| = 0.9733\,(2)_{\tau_n}\,(3)_{g_A}\,(1)_{RC}[4]_{total}$$

PDG average

$$|V_{ud}^{\text{free n}}| = 0.9733\,(3)_{\tau_n}\,(8)_{g_A}\,(1)_{RC}[9]_{total}$$

RC not a limiting factor: more precise experiments a-coming

Pion decay $\pi^+ \to \pi^0 e^+ \nu_e$: theoretically cleanest, experimentally tough

$$|V_{ud}|^2 = \frac{0.9799}{(1+\delta)}\frac{\Gamma_{\pi\ell 3}}{0.3988(23)\text{ s}^{-1}}$$

$$|V_{ud}^{\pi\ell 3}| = 0.9739\,(27)_{exp}\,(1)_{RC}$$

Future exp: 1 o.o.m. (PIONEER)

Status of V_{ud}

Major reduction of uncertainties in the past few years

Theory

Universal correction Δ_R^V to free and bound neutron decay

Identified 40 years ago as the bottleneck for precision improvement

Novel approach dispersion relations + experimental data + lattice QCD

$$\Delta_R^V = 0.02467(22)$$

Factor 2 improvement

C-Y Seng et al., Phys.Rev.Lett. 121 (2018) 24, 241804;
C-Y Seng, MG, M.J. Ramsey-Musolf, Phys.Rev. D 100 (2019) 1, 013001;
MG, Phys.Rev.Lett. 123 (2019) 4, 042503;
C-Y Seng, X. Feng, MG, L-C Jin, Phys.Rev. D 101 (2020) 11, 111301;
A. Czarnecki, B. Marciano, A. Sirlin, Phys.Rev. D 100 (2019) 7, 073008

RC to semileptonic pion decay

$$\delta = 0.0332(3)$$

Factor 3 improvement

X. Feng, MG, L-C Jin, P-X Ma, C-Y Seng, Phys.Rev.Lett. 124 (2020) 19, 192002

Experiment

$$g_A = -1.27641(56)$$

Factor 4 improvement

PERKEO-III *B. Märkisch et al, Phys.Rev.Lett. 122 (2019) 24, 242501*

$$g_A = -1.2677(28)$$

aSPECT *M. Beck et al, Phys. Rev. C101 (2020) 5, 055506*

$$\tau_n = 877.75(28)^{+16}_{-12}$$

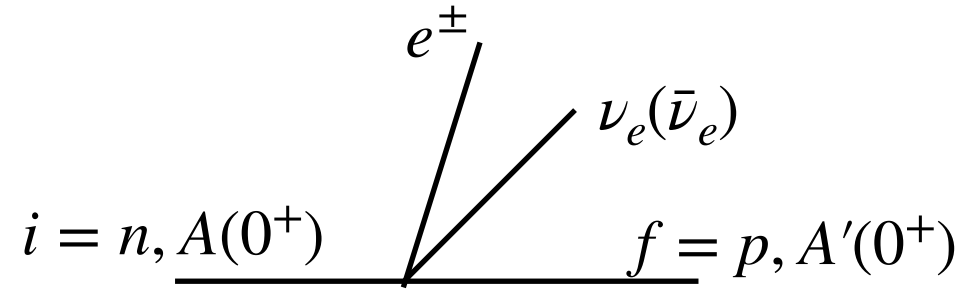
Factor 2-3 improvement

UCNτ *F. M. Gonzalez et al. Phys. Rev. Lett. 127 (2021) 162501*

RC to nuclear beta decay: overall setup

RC to nuclear beta decay: overall setup

Tree-level amplitude



$\sim V_{ud}$

Radiative corrections to tree-level amplitude $\sim \alpha/2\pi \approx 10^{-3}$

Precision goal for V_{ud} extraction 1×10^{-4}

Electron carries away energy E < Q-value of a decay

E-dep RC: $\frac{\alpha}{2\pi}\left(\frac{E}{\Lambda}, \ln\frac{E}{\Lambda}, \ldots\right)$

Energy scales Λ

Λ

Weak boson scale
$M_Z, M_W \sim 90\,\mathrm{GeV}$

Hadronic scale
$\Lambda_{\mathrm{had}} = 300\,\mathrm{MeV}$

Same for all nuclei

Nuclear scale
$\Lambda_{\mathrm{nuc}} = 10 - 30\,\mathrm{MeV}$

Nuclear structure dependent
(QCD)

Decay Q-value (endpoint energy)
$Q_{if} = M_i - M_f = 1 - 10\,\mathrm{MeV}$

Nucleus-specific

Electron mass
$m_e \approx 0.5\,\mathrm{MeV}$

Nuclear structure independent
(QED)

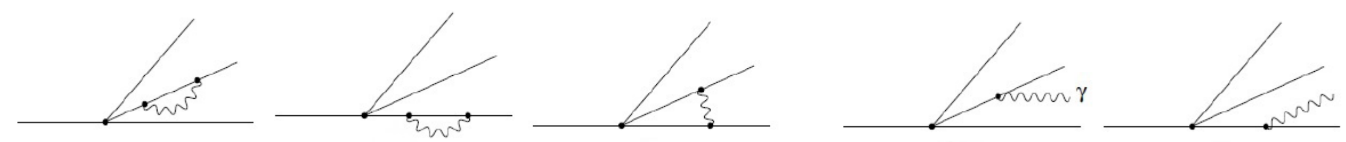
RC to beta decay: overall setup

Generically: only IR and UV extremes feature large logarithms!
Works by Sirlin (1930-2022) and collaborators: all large logs under control

IR: Fermi function + Sirlin function

Fermi function: resummation of $(Z\alpha)^n$ —> Dirac - Coulomb problem

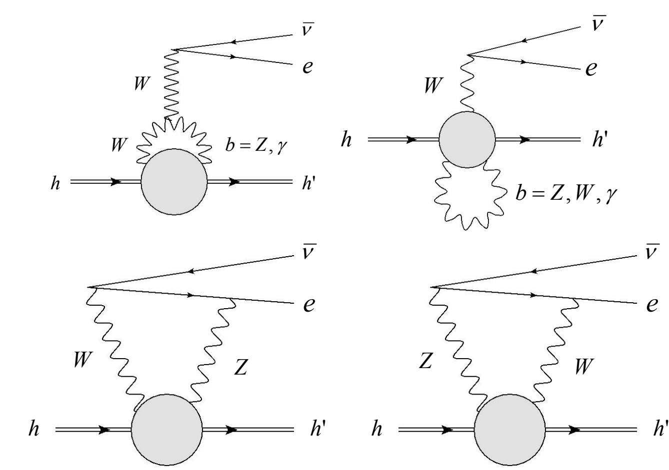
Sirlin function (outer correction):
All IR-div. pieces beyond Coulomb distortion



UV: large EW logs + pQCD corrections

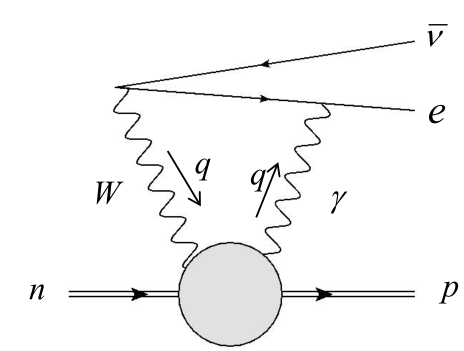
Inner RC:
energy- and model-independent

W,Z - loops
UV structure of SM



γW-box: sensitive to all scales

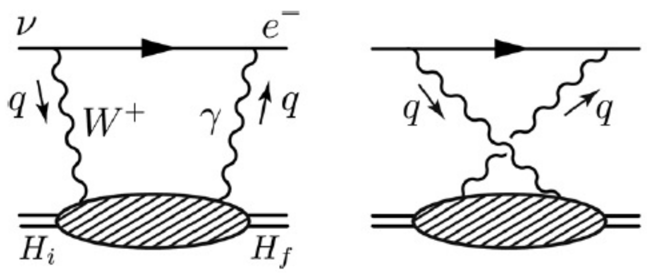
New method for computing EW boxes: dispersion theory
Combine exp. data with pQCD, lattice, EFT, ab-initio nuclear



Dispersion Formalism for γW-box

γW-box from dispersion relations

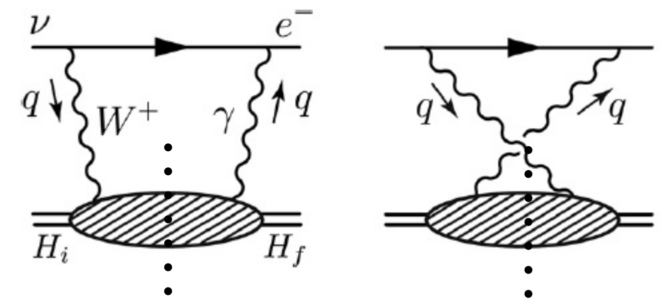
Model-dependent part or RC: γW-box



Generalized Compton tensor
time-ordered product — complicated!

$$\int dx e^{iqx} \langle H_f(p) \,|\, T\{J^{\mu}_{em}(x) J^{\nu,\pm}_{W}(0)\} \,|\, H_i(p) \rangle$$

Generalized (non-diagonal) Compton amplitudes



Commutator (Im part) - only on-shell
hadronic states — related to data

$$\int dx e^{iqx} \langle H_f(p) \,|\, [J^{\mu}_{em}(x), J^{\nu,\pm}_{W}(0)] \,|\, H_i(p) \rangle$$

Interference structure functions

Physics of taming model dependence with dispersion relations:

virtual photon polarizes the nucleon/nucleus;
Long- and intermediate-range part of the box sensitive to hadronic **polarizabilities**
Polarizabilities related to the excitation spectrum via dispersion relation
(Cf. Kramers-Kronig)

Universal RC from dispersion relations

Interference γW structure functions

$$\mathrm{Im}T^{\mu\nu}_{\gamma W} = \ldots + \frac{i\varepsilon^{\mu\nu\alpha\beta}p_\alpha q_\beta}{2(pq)} F_3^{\gamma W}(x, Q^2)$$

After some algebra (isospin decomposition, loop integration)

$$\Box^{b,\mathrm{e}}_{\gamma W}(E_e) = \frac{\alpha}{\pi}\int_0^\infty dQ^2 \frac{M_W^2}{M_W^2+Q^2}\int_{\nu_{\mathrm{thr}}}^\infty \frac{d\nu'}{\nu'}\frac{\nu'+2\sqrt{\nu'^2+Q^2}}{(\nu'+\sqrt{\nu'^2+Q^2})^2}\frac{F_{3,-}(\nu',Q^2)}{Mf_+(0)} + \mathcal{O}(E_e^2)$$

$$\Box^{b,\mathrm{o}}_{\gamma W}(E_e) = \frac{2\alpha E_e}{3\pi}\int_0^\infty dQ^2 \int_{\nu_{\mathrm{thr}}}^\infty \frac{d\nu'}{\nu'}\frac{\nu'+3\sqrt{\nu'^2+Q^2}}{(\nu'+\sqrt{\nu'^2+Q^2})^3}\frac{F_{3,+}(\nu',Q^2)}{Mf_+(0)} + \mathcal{O}(E_e^3)$$

Advantage to previous approach (Marciano & Sirlin):

- Explicit 2-fold integral, isospin decomposition and energy dependence

Nachtmann moments play a role in DIS

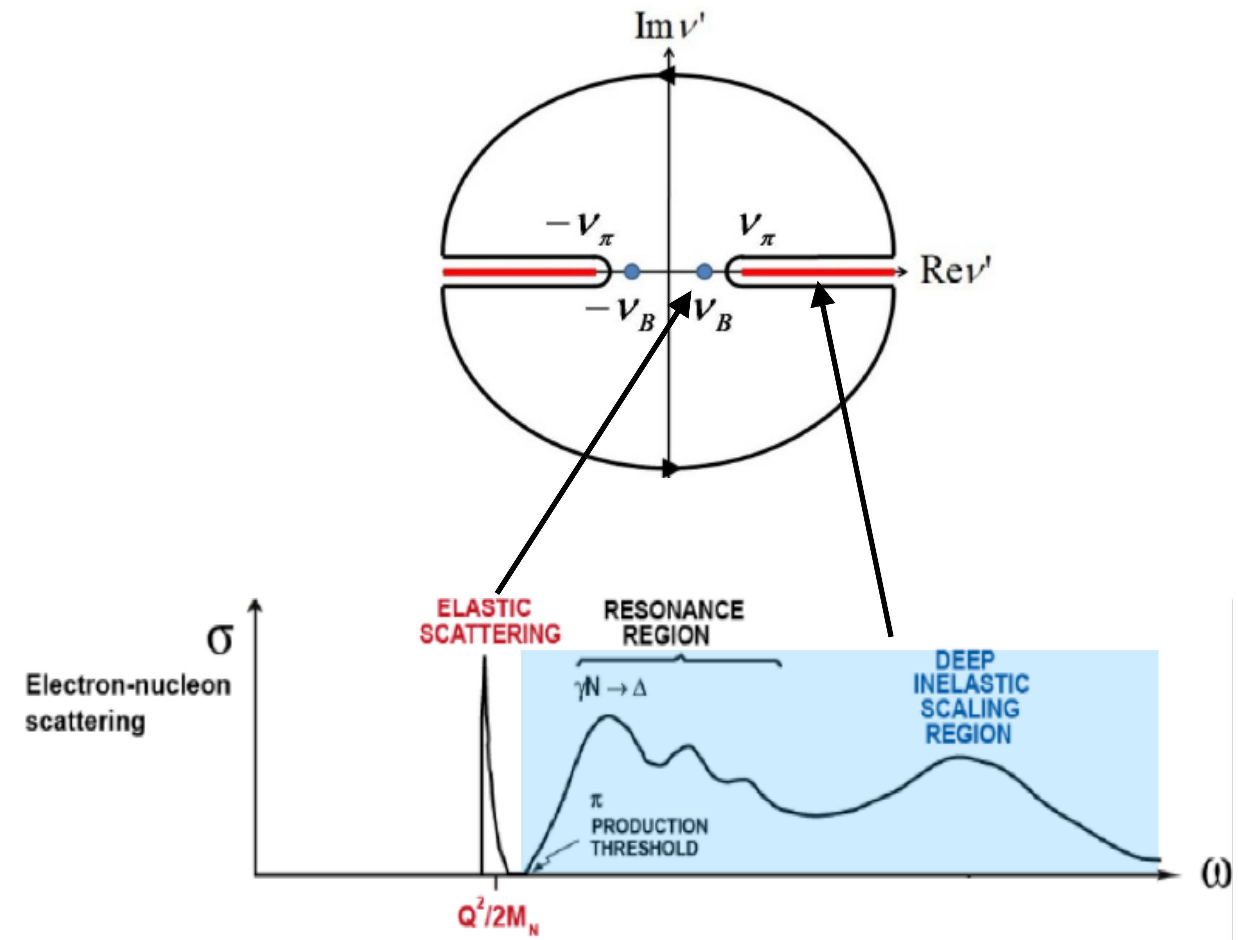
$$M_3(n,Q^2) = \frac{n+1}{n+2}\int_0^1 \frac{dx\,\xi^n}{x^2}\frac{2x(n+1)-n\xi}{n+1}F_3(x,Q^2), \qquad \xi = \frac{2x}{1+\sqrt{1+4M^2x^2/Q^2}}$$

Hiding the nu-integration in the Nachtmann moments:

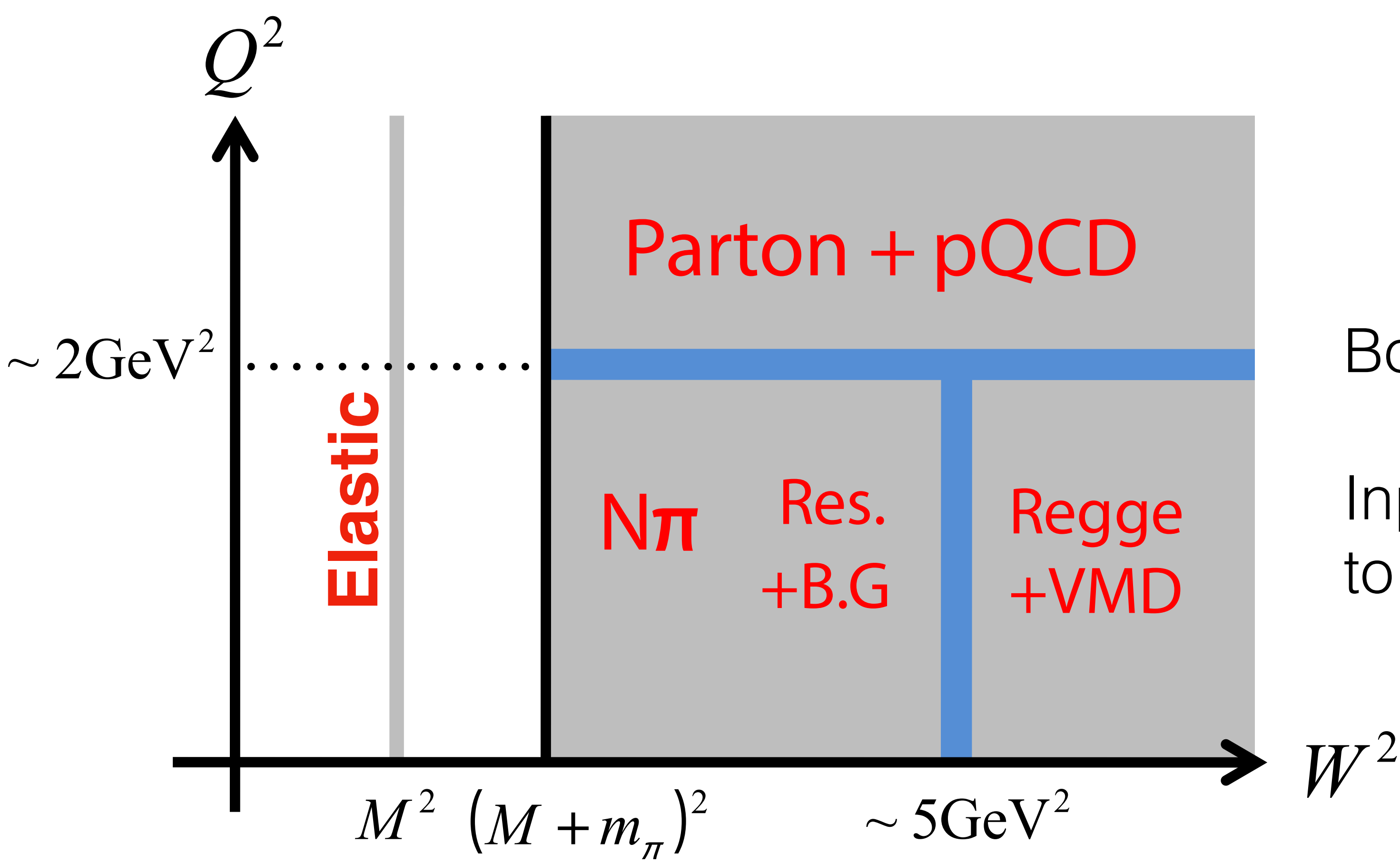
$$\Box^{b}_{\gamma W}(E_e) = \frac{3\alpha}{2\pi}\int_0^\infty \frac{dQ^2}{Q^2}\frac{M_W^2}{M_W^2+Q^2}\left[M_{3,-}(1,Q^2) + \frac{8E_eM}{9Q^2}M_{3,+}(2,Q^2)\right] + \mathcal{O}(E_e^2)$$

Input into dispersion integral

Dispersion in energy: $W^2 = M^2 + 2M\nu - Q^2$
scanning hadronic intermediate states



Dispersion in Q²:
scanning dominant physics pictures



Boundaries between regions - approximate

Input in DR related (directly or indirectly) to experimentally accessible data

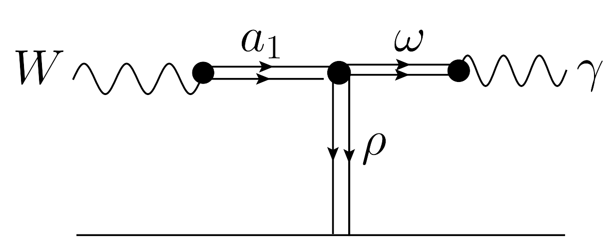
Input into dispersion integral - $\nu/\bar{\nu}$ data

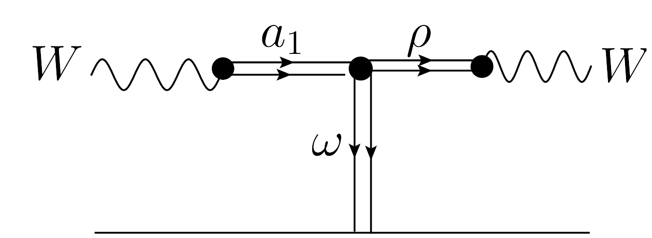
Mixed CC-NC γW SF (no data) <—> Purely CC WW SF (inclusive neutrino data)

Isospin symmetry: vector-isoscalar current related to vector-isovector current

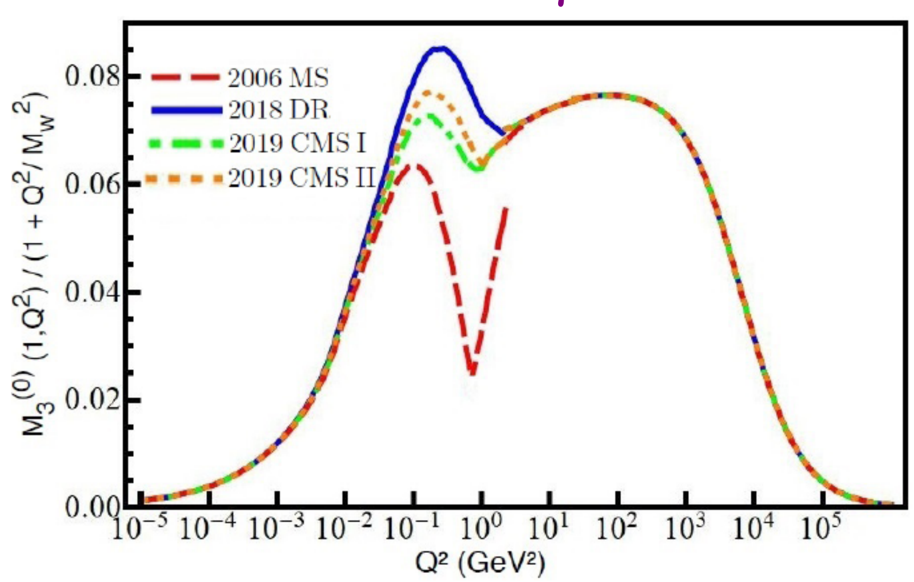
Only useful if we know the physical mechanism (Born, DIS, Regge, Resonance,…)

Were able to identify the missing part with Regge (multiparticle continuum)

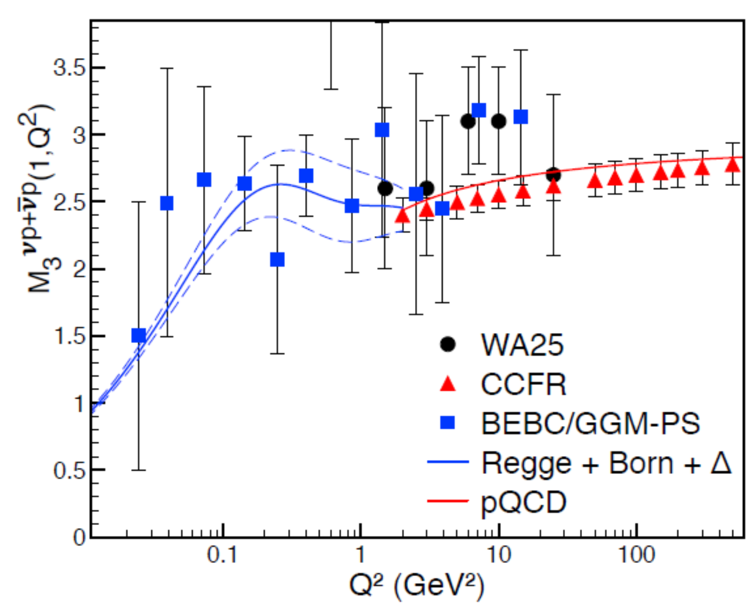




Free neutron γW box



Neutrino scattering data

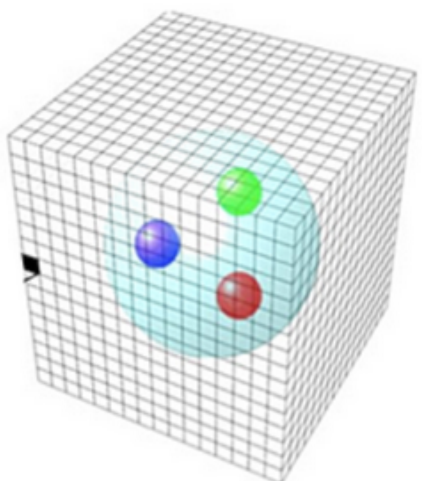


Marciano, Sirlin 2006: $\Delta_R^V = 0.02361(38)$ —> $|V_{ud}| = 0.97420(10)_{Ft}(18)_{RC}$

DR (Seng et al. 2018): $\Delta_R^V = 0.02467(22)$ —> $|V_{ud}| = 0.97370(10)_{Ft}(10)_{RC}$

γW-box from DR + Lattice QCD input

Currently available neutrino data at low Q^2 - low quality;
Look for alternative input — compute Compton amplitude on the lattice



$$\mathcal{H}^{VA}_{\mu\nu}(x) = \langle \pi^0(p) \big| T[J^{\rm em}_\mu(x) J^{W,A}_\nu(0)] \big| \pi^-(p) \rangle$$

$$M_\pi(Q^2) = -\frac{1}{6\sqrt{2}} \frac{\sqrt{Q^2}}{m_\pi} \int d^4x \omega(Q,x) \epsilon_{\mu\nu\alpha 0} x_\alpha \mathcal{H}^{VA}_{\mu\nu}(x)$$

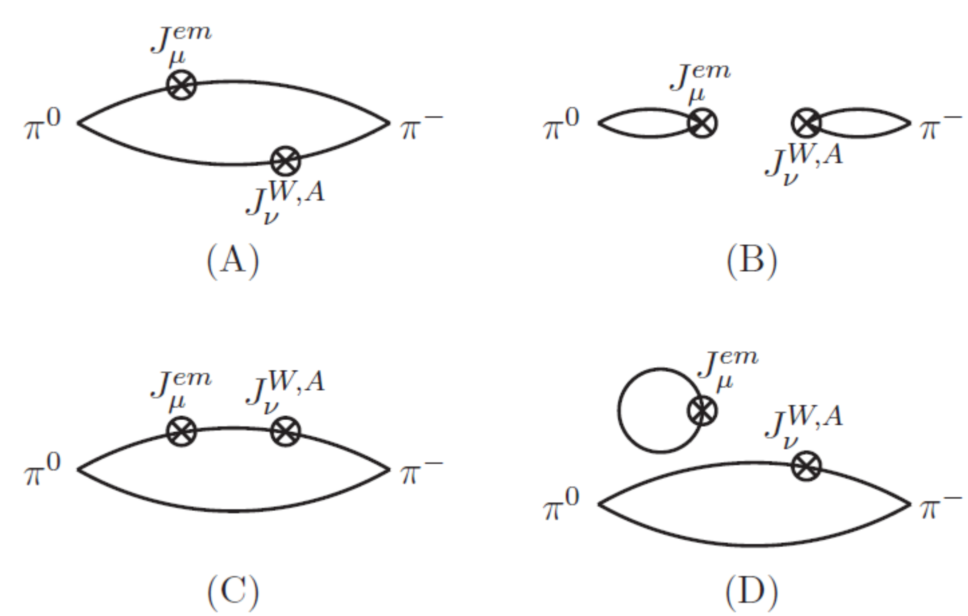
Direct LQCD computation for $\pi^- \to \pi^0 e^- \nu_e$

Feng, MG, Jin, Ma, Seng 2003.09798

5 LQCD gauge ensembles at physical pion mass
Generated by RBC and UKQCD collaborations
w. 2+1 flavor domain wall fermion

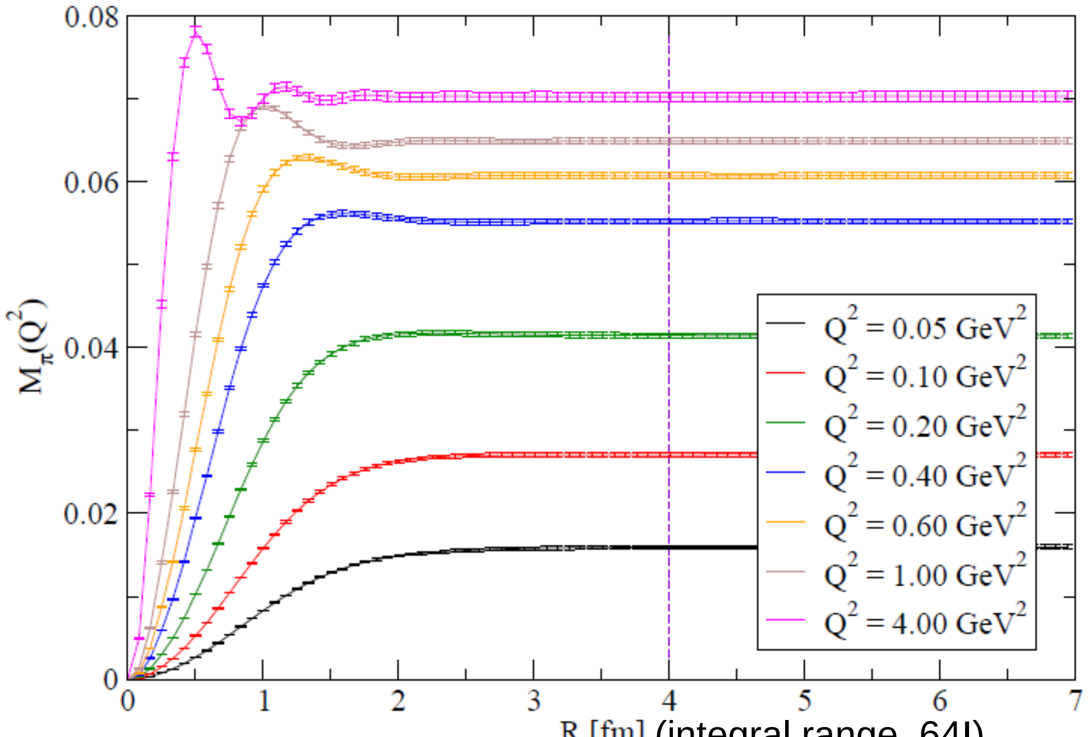
Ensemble	m_π [MeV]	L	T	a^{-1} [GeV]	$N_{\rm conf}$	N_r	$\Delta t/a$
24D	141.2(4)	24	64	1.015	46	1024	8
32D	141.4(3)	32	64	1.015	32	2048	8
32D-fine	143.0(3)	32	64	1.378	71	1024	10
48I	135.5(4)	48	96	1.730	28	1024	12
64I	135.3(2)	64	128	2.359	62	1024	18

Blue: DSDR
Red : Iwasaki

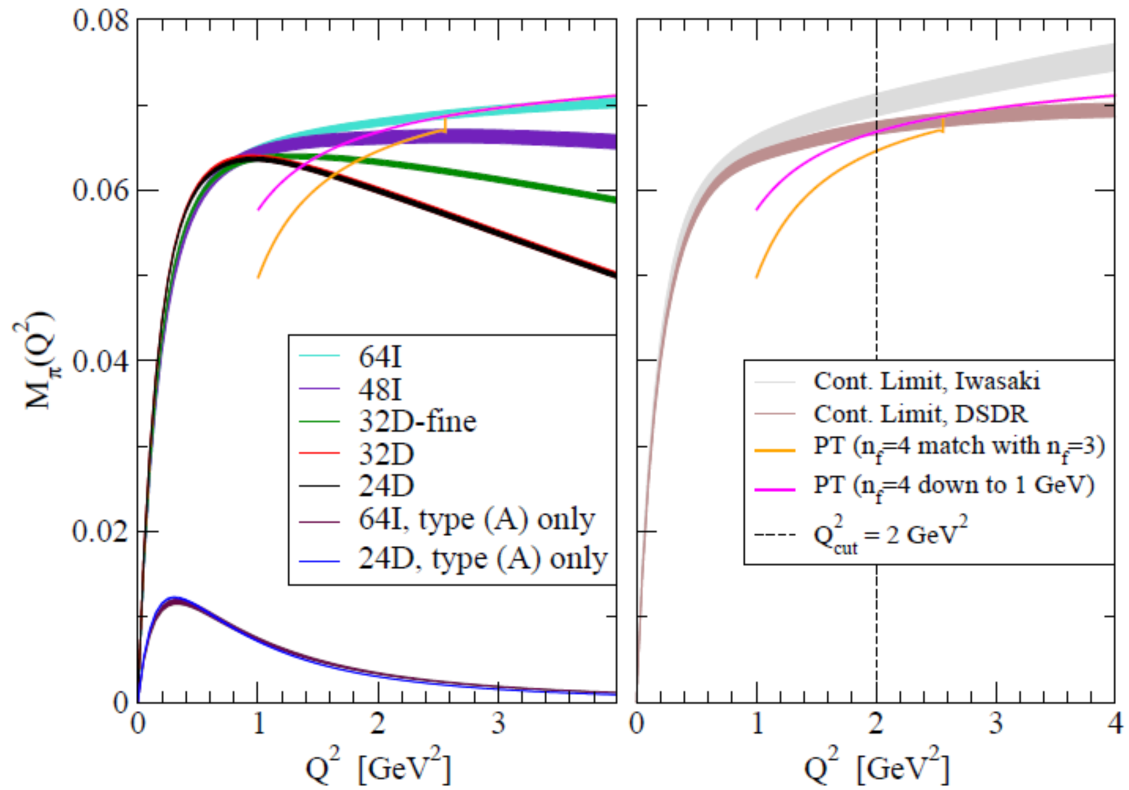


Quark contraction diagrams

First lattice QCD calculation of γW-box



(integral range, 64I)



(before cont. extrapolation) (after cont. extrapolation)

Estimate of major systematic effects:

- **Lattice discretization effect**: Estimated using the discrepancy between DSDR and Iwasaki
- **pQCD calculation**: Estimated from the difference between 3-loop and 4-loop results
- **Higher-twist effects at large Q^2**: Estimated from lattice calculation of type (A) diagrams

Direct impact for pion decay $\pi^+ \to \pi^0 e^+ \nu_e$

$$|V_{ud}|^2 = \frac{0.9799}{(1+\delta)} \frac{\Gamma_{\pi\ell 3}}{0.3988(23)\,\mathrm{s}^{-1}}$$

Previous calculation of δ — in ChPT

Cirigliano, Knecht, Neufeld and Pichl, EPJC 2003

Significant reduction of the uncertainty!

$$\delta: \quad 0.0334(10)_{\mathrm{LEC}}(3)_{\mathrm{HO}} \to 0.0332(1)_{\gamma W}(3)_{\mathrm{HO}}$$

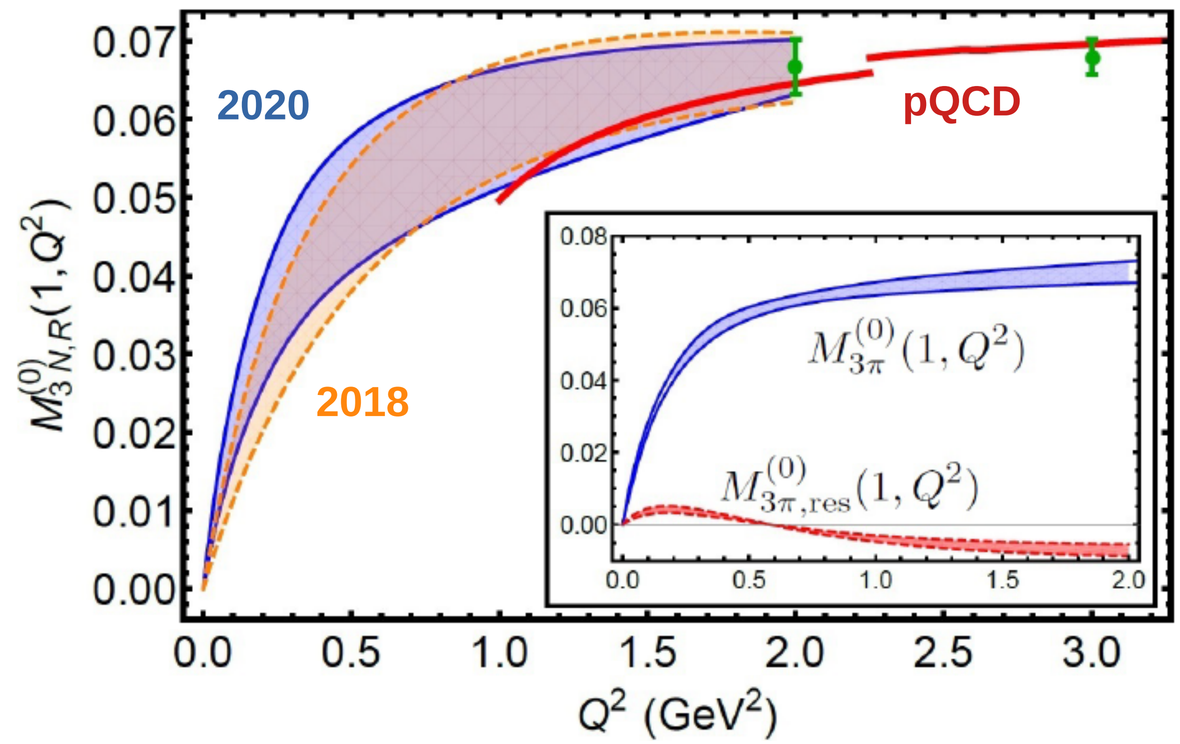
Implications for the free nucleon γW-box

Seng, MG, Feng, Jin, 2003.11264

Indirectly constrains the free neutron γW-box

Independent confirmation of the empirical DR result AND uncertainty

$$\Delta_R^V = 0.02467(22)_{\text{DR}} \rightarrow 0.02477(24)_{\text{LQCD+DR}}$$



Free-n RC in agreement by several groups & methods

Method	Δ_R^V
DR with neutrino data (1)	0.02467(22)
DR with neutrino data (2)	0.02471(18)
DR with indirect lattice data	0.02477(24)
Non-DR (1)	0.02426(32)
Non-DR (2)	0.02473(27)

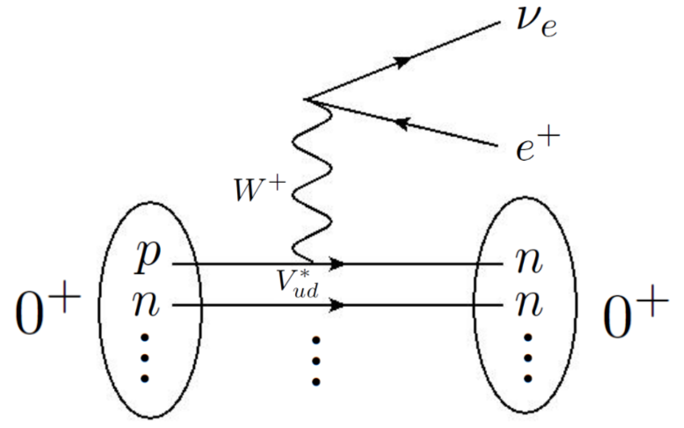
C-Y Seng et al., Phys.Rev.Lett. 121 (2018) 24, 241804;
C-Y Seng, MG, M.J. Ramsey-Musolf, Phys.Rev. D 100 (2019)
Shiells, Blunden, Melnitchouk, Phys.Rev.D 104 (2021) 3, 033003
Seng, MG, Feng, Jin, 2003.11264
Czarnecki, Marciano, Sirlin, Phys.Rev. D 100 (2019) 7, 073008
Hayen, Phys.Rev.D 103 (2021) 11, 113001

Status of δ_{NS}

Splitting the γW-box into Universal and Nuclear Parts

Vud from superallowed nuclear decays

$$|V_{ud}|^2 = \frac{2984.43s}{\mathcal{F}t(1+\Delta_R^V)}$$



Experiment: half-life; branching ratio; Q-value —> decay-specific **ft**-value

To obtain Vud —> absorb all decay-specific corrections into universal **Ft**

$$ft(1 + \mathrm{RC} + \mathrm{ISB}) = \mathcal{F}\mathrm{t}(1 + \Delta_\mathrm{R}^\mathrm{V}) = \mathrm{ft}(1 + \delta_\mathrm{R}')(1 - \delta_\mathrm{C} + \delta_\mathrm{NS})(1 + \Delta_\mathrm{R}^\mathrm{V})$$

Outer: QED — Isospin-breaking — Nuclear structure — Universal inner

NS correction reflects extraction of the free box $\delta_\mathrm{NS} = 2[\,\Box_{\gamma W}^{\mathrm{VA,\,nucl}} - \Box_{\gamma W}^{\mathrm{VA,\,free\,n}}\,]$

Splitting the γW-box into Universal and Nuclear Parts

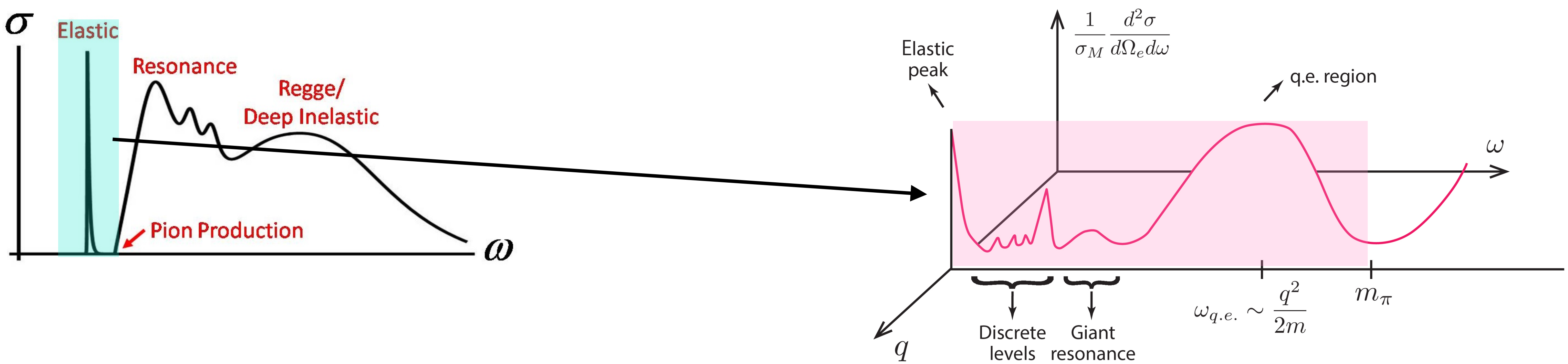
RC on a free neutron
$$\Delta_R^V \propto F_3^{\text{free n}} \propto \int dx e^{iqx} \sum_X \langle p | J_{em}^{\mu,(0)}(x) | X \rangle \langle X | J_W^{\nu,+}(0) | n \rangle$$

RC on a nucleus
$$\Delta_R^V + \delta_{NS} \propto F_3^{\text{Nucl.}} \propto \int dx e^{iqx} \sum_{X'} \langle A' | J_{em}^{\mu,(0)}(x) | X' \rangle \langle X' | J_W^{\nu,+}(0) | A \rangle$$

Nuclear modification in the lower part of the spectrum

Input in the DR for the universal RC

Input in the DR for the RC on a nucleus



δ_{NS} from DR with energy dependence averaged over the spectrum

$$\delta_{NS} = \frac{2\alpha}{\pi M} \int_0^{\text{few GeV}^2} dQ^2 \int_{\nu_{thr}}^{\nu_\pi} \frac{d\nu}{\nu} \left[\frac{\nu + 2q}{(\nu+q)^2} \left(F_3^{(0)\,Nucl.} - F_3^{(0),B} \right) + \frac{2\langle E \rangle}{3} \frac{\nu + 3q}{(\nu+q)^3} F_3^{(-)\,Nucl.} \right]$$

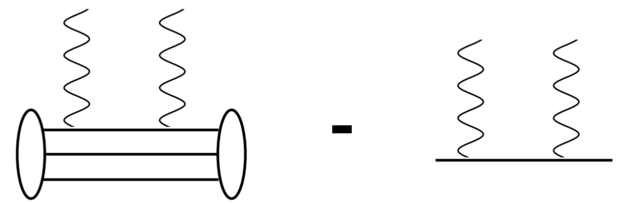
Splitting the γW-box into Universal and Nuclear Parts

Need to know the full nuclear Green's function
indices k, l count the nucleon d.o.f. in a nucleus

$$T_{\mu\nu}^{\gamma W\ \mathrm{nuc}} \sim \sum_{k,\ell} \langle f | J_\mu^W(k)\, G_{\mathrm{nuc}}\, J_\nu^{\mathrm{EM}}(\ell) | i \rangle$$

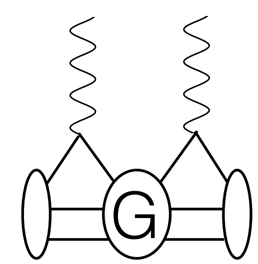
$\delta_{\mathrm{NS}} =$

(A) same active nucleon



Modified Born

(B) two nucleons correlated by G



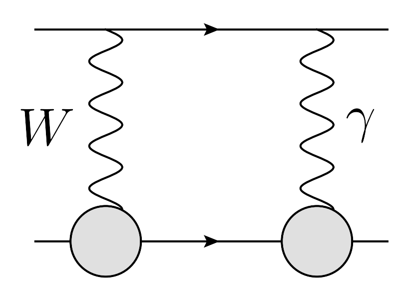
Specifically nuclear effect

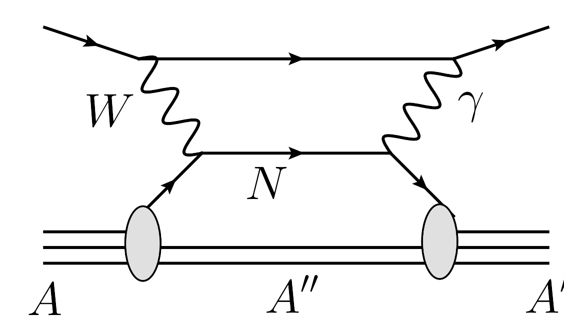
Case (A): non-interacting (=on-shell) neutron propagating between interaction vertices
Case (B): all two-nucleon contributions (QE 2p2h and nuclear excitations)

Insert on-shell intermediate states:

$$T_{\mu\nu}^{A} \to \sum_{k} \langle f \,|\, J_\mu^W(k)[S_F^N \otimes G_{nuc}^{A''}] J_\nu^{EM}(k) \,|\, i \rangle$$

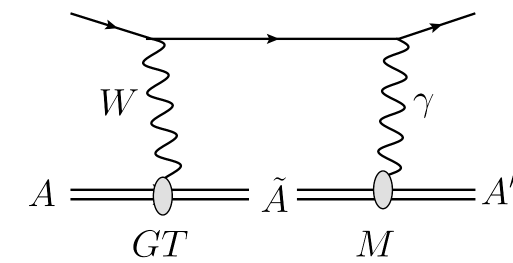
The elastic nucleon box
is replaced by a single N QE knockout





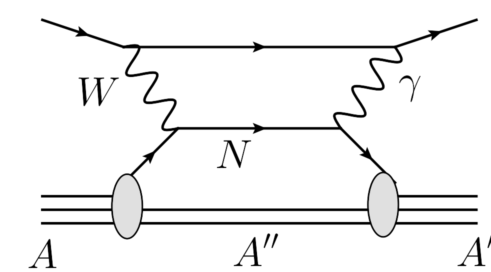
Universal vs. Nuclear Corrections

Towner 1994 and ever since: quenching $\Box_{\gamma W}^{\text{quenched Born}} - \Box_{\gamma W}^{\text{Born}} = [q_S^{(0)} q_A - 1]\,\Box_{\gamma W}^{\text{Born}}$



Numerical impact on Ft values $\mathscr{F}t = 3072.1(7)s$

$$[\delta\mathscr{F}t]^{quenched\,Born} \approx -1.8(4)\,\text{s}$$



From DR perspective: misidentified!

Excited nuclear state, not modified box on free nucleon!

Correct estimate: QE 1-nucleon knockout

QE contribution from DR: $\delta_{\text{NS}}^{\text{QE}} = \delta_{\text{NS}}^{\text{QE},\,0} + \langle E\rangle \delta_{\text{NS}}^{\text{QE},\,1}$

$$\delta_{NS} = \frac{2\alpha}{\pi NM}\int_0^{\text{few GeV}^2} dQ^2 \int_{\nu_{thr}}^{\nu_\pi} \frac{d\nu}{\nu}\left[\frac{\nu+2q}{(\nu+q)^2}\left(F_3^{(0)\,QE} - F_3^{(0),\,B}\right) + \frac{2\langle E\rangle}{3}\frac{\nu+3q}{(\nu+q)^3}F_3^{(-)\,QE}\right]$$

HT value 2018:

$\mathscr{F}t = 3072.1(7)s$

Old estimate: $\delta\mathscr{F}t = -(1.8\pm 0.4)s \quad + \quad (0\pm 0)s$

New estimate: $\delta\mathscr{F}t = -(3.5\pm 1.0)s \quad + \quad (1.6\pm 0.5)s$

C-Y Seng, MG, M J Ramsey-Musolf 1812.03352

MG 1812.04229

Nuclear structure uncertainty tripled! $\mathscr{F}t = (3072 \pm 2)s$

Ab-Initio $\delta_{\rm NS}$

Only a naive warm-up calculation — ab-initio $\delta_{\rm NS}$ necessary!

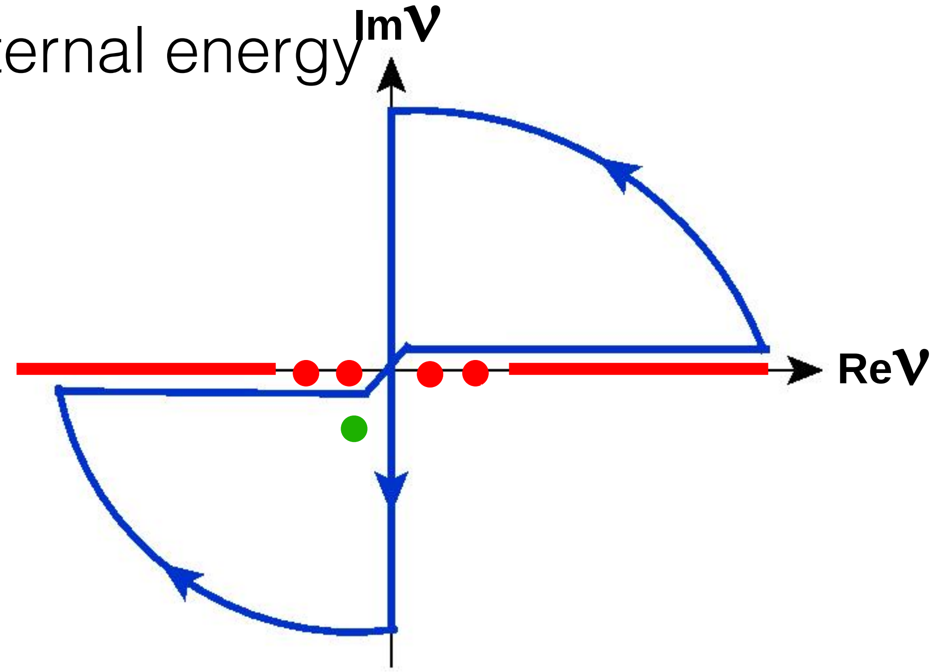
Dispersion theory of $\delta_{\rm NS}$: isospin structure + multipole expansion

Seng, MG 2211.10214

Interesting effects detected:

Mixed isospin structure due to 2B currents (absent for n, $\pi e3$)

Residue contribution if 0^+ state is not g.s.: anomalous threshold
Normal threshold: nuclear excitation spectrum separated from external state by finite energy gap — only virtual;
if there are states below — can go on-shell even without external energy



Residue contribution: contains parts singular at $E_e = 0$
—> should contribute to outer correction δ_R'

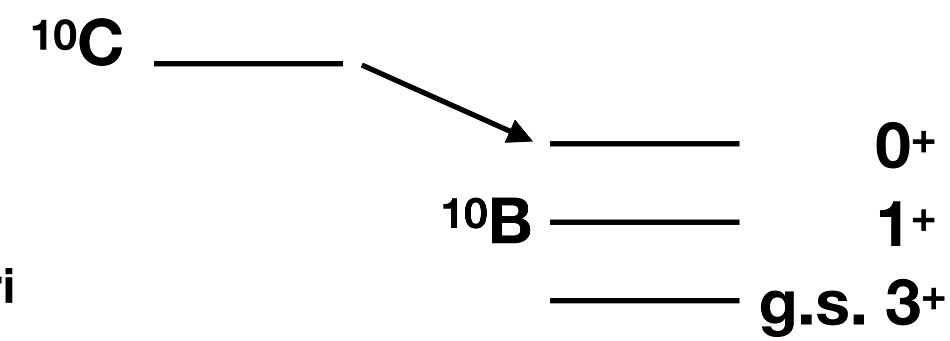
Currently, effort on light systems C-10, O-14

Accessible to NCSM, GFMC, CC, …
Important cross checks should become possible soon (?)

Michael Gennari, Petr Navratil, Garrett King

Ab-Initio δ_{NS}: what to expect?

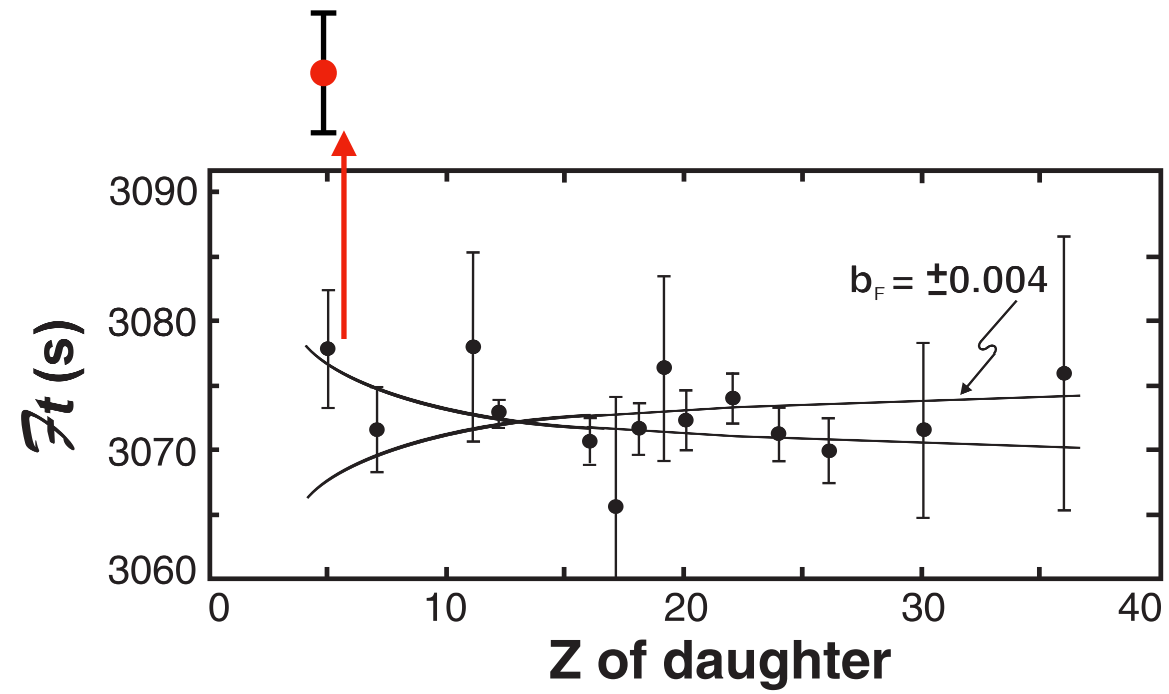
At present only preliminary results for C-10;



residue due to B-10 levels numerically large (1%) needs confirmation!

Michael Gennari

Prize is high: if confirmed - nonzero Fierz!



But there are many more questions to raise!

δ_{NS} from H & T: negative for light nuclei

Parent nucleus	$\delta_{NS}(\%)$ Quenched	$\delta_{NS}(\%)$ Adopted
$T_z = -1$:		
^{10}C	−0.357	−0.360(35)
^{14}O	−0.295	−0.250(50)
^{18}Ne	−0.325	−0.290(35)

Hardy, Towner 2002 review

DR + isospin symmetry:

$$\Box_{\gamma W}^{Nucl} \propto F_{3,\gamma W} \propto F_{3,WW} \propto \frac{d\sigma^{\nu A}}{dxdy} - \frac{d\sigma^{\bar{\nu} A}}{dxdy}$$

Common knowledge: ν cross sections always higher than $\bar{\nu}$!
Can this pattern be tested experimentally? Is δ_{NS} positive/negative definite?

Status of δ_C

Isospin symmetry breaking in superallowed β-decay

Tree-level Fermi matrix element

$$M_F = \langle f | \tau^+ | i \rangle$$

τ^+ — Isospin operator

$|i\rangle, |f\rangle$ — members of I=1 isotriplet

If isospin symmetry were exact, $M_F \to M_0 = \sqrt{2}$

Isospin symmetry is broken in nuclear states (e.g. Coulomb, nucleon mass difference, …)

In presence of isospin symmetry breaking (ISB):

$$|M_F|^2 = |M_0|^2(1 - \delta_C)$$

ISB correction is crucial for V_{ud} extraction

TABLE X. Corrections δ'_R, δ_{NS}, and δ_C that are applied to experimental ft values to obtain $\mathcal{F}t$ values.

Parent nucleus	δ'_R (%)	δ_{NS} (%)	δ_{C1} (%)	δ_{C2} (%)	δ_C (%)
$T_z = -1$					
^{10}C	1.679	−0.345(35)	0.010(10)	0.165(15)	0.175(18)
^{14}O	1.543	−0.245(50)	0.055(20)	0.275(15)	0.330(25)
^{18}Ne	1.506	−0.290(35)	0.155(30)	0.405(25)	0.560(39)
^{22}Mg	1.466	−0.225(20)	0.010(10)	0.370(20)	0.380(22)
^{26}Si	1.439	−0.215(20)	0.030(10)	0.405(25)	0.435(27)
^{30}S	1.423	−0.185(15)	0.155(20)	0.700(20)	0.855(28)
^{34}Ar	1.412	−0.180(15)	0.030(10)	0.665(55)	0.695(56)
^{38}Ca	1.414	−0.175(15)	0.020(10)	0.745(70)	0.765(71)
^{42}Ti	1.427	−0.235(20)	0.105(20)	0.835(75)	0.940(78)
$T_z = 0$					
^{26m}Al	1.478	0.005(20)	0.030(10)	0.280(15)	0.310(18)
^{34}Cl	1.443	−0.085(15)	0.100(10)	0.550(45)	0.650(46)
^{38m}K	1.440	−0.100(15)	0.105(20)	0.565(50)	0.670(54)
^{42}Sc	1.453	0.035(20)	0.020(10)	0.645(55)	0.665(56)
^{46}V	1.445	−0.035(10)	0.075(30)	0.545(55)	0.620(63)
^{50}Mn	1.444	−0.040(10)	0.035(20)	0.610(50)	0.645(54)
^{54}Co	1.443	−0.035(10)	0.050(30)	0.720(60)	0.770(67)
^{62}Ga	1.459	−0.045(20)	0.275(55)	1.20(20)	1.48(21)
^{66}As	1.468	−0.060(20)	0.195(45)	1.35(40)	1.55(40)
^{70}Br	1.486	−0.085(25)	0.445(40)	1.25(25)	1.70(25)
^{74}Rb	1.499	−0.075(30)	0.115(60)	1.50(26)	1.62(27)

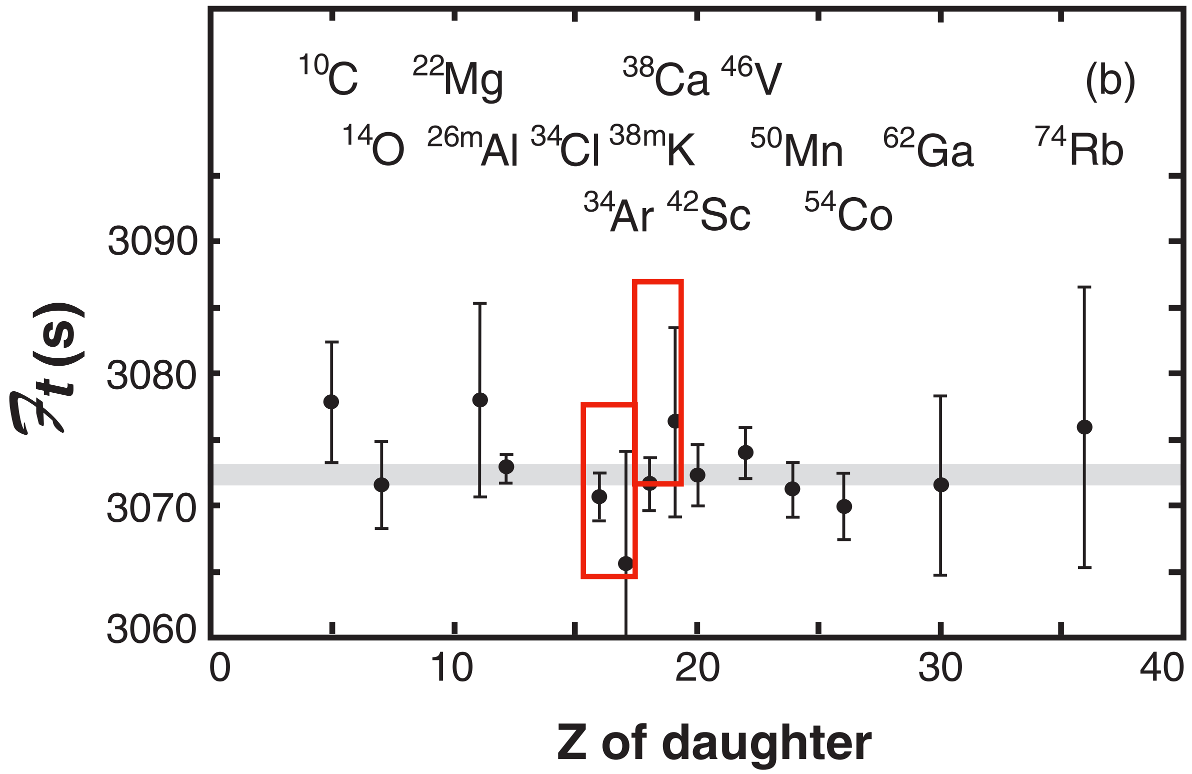
J. Hardy, I. Towner, Phys.Rev. C 91 (2014), 025501

HT: calculate $\delta_{C1,C2}$ in shell model with *phenomenological* Woods-Saxon potential locally adjusted:

- Masses of the isobaric multiplet T=1, 0^+
- Neutron and proton separation energies
- Known proton radii of stable isotopes

ISB in superallowed β-decay BSM scalar interactions

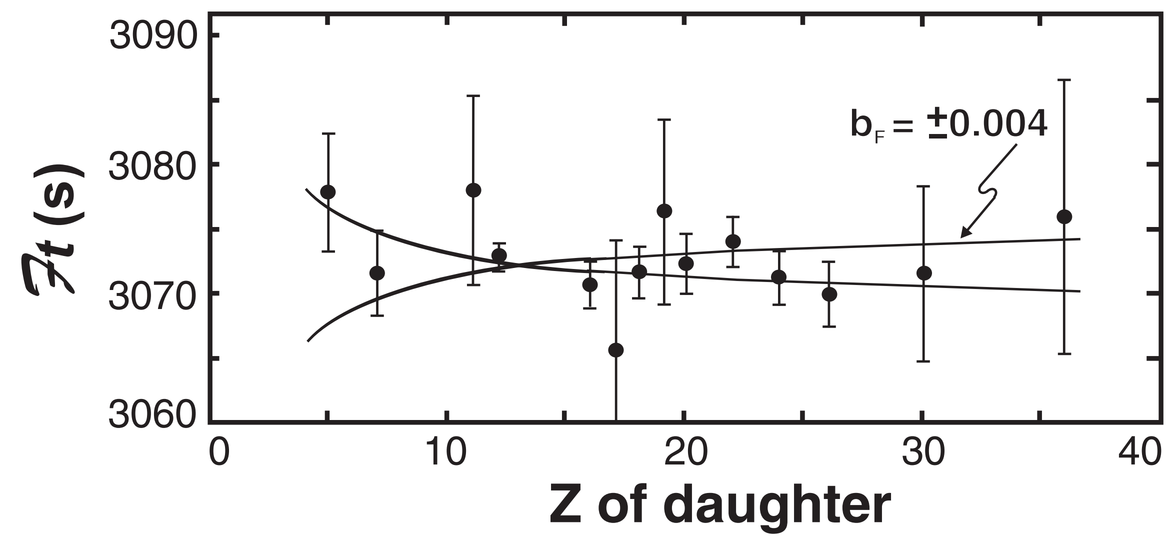
Conserved vector current —> Ft constant



If Ft were not constant:

Presence of scalar currents - BSM

Fierz interference term $\sim b_F m_e / E_e$



Fit to 14 transitions:

Ft constant within 2×10^{-4} and $b_F = -0.0028(26)$

However: to achieve this precision the model was adjusted locally in each iso-multiplet

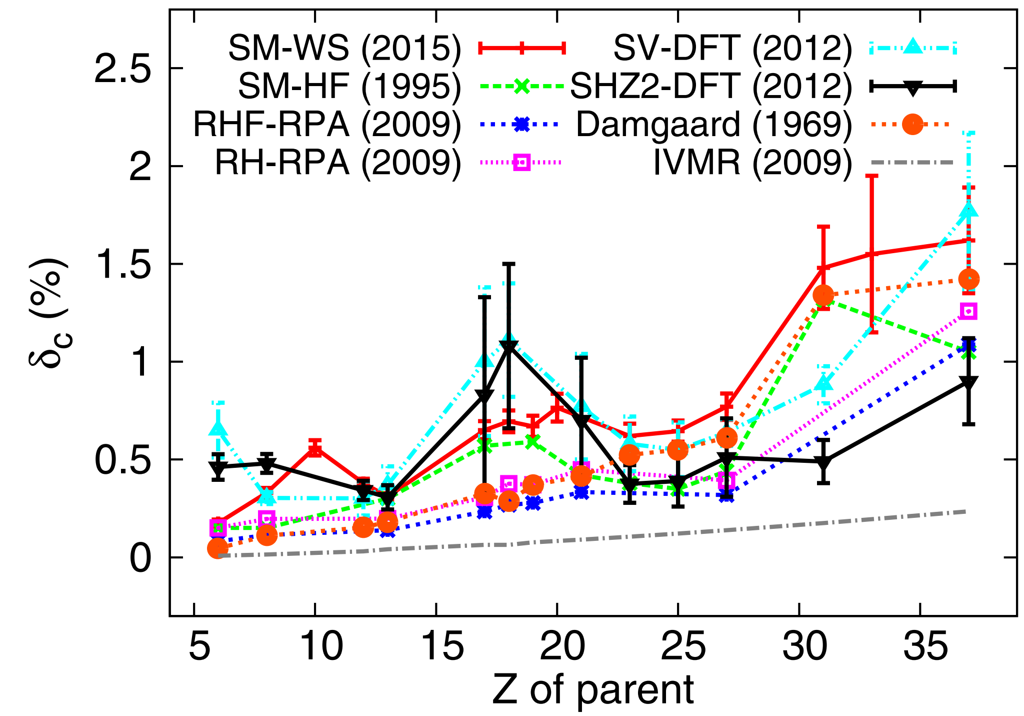
- Is this formalism the right tool to assess consistency amongst all the measurements?
- Red squares: even within one iso-multiplet ($^{34}Ar - {}^{34}Cl - {}^{34}S$, $^{38}Ca - {}^{38m}K - {}^{38}Ar$) discrepancies between central values may be larger than the total uncertainty
- Shell model does not cover all the model space (e.g. continuum)
- HT method criticized for using incorrect isospin formalism (G. Miller, A. Schwenk)
- Ab initio methods do not warrant such high precision

ISB in superallowed β-decay: nuclear model comparison

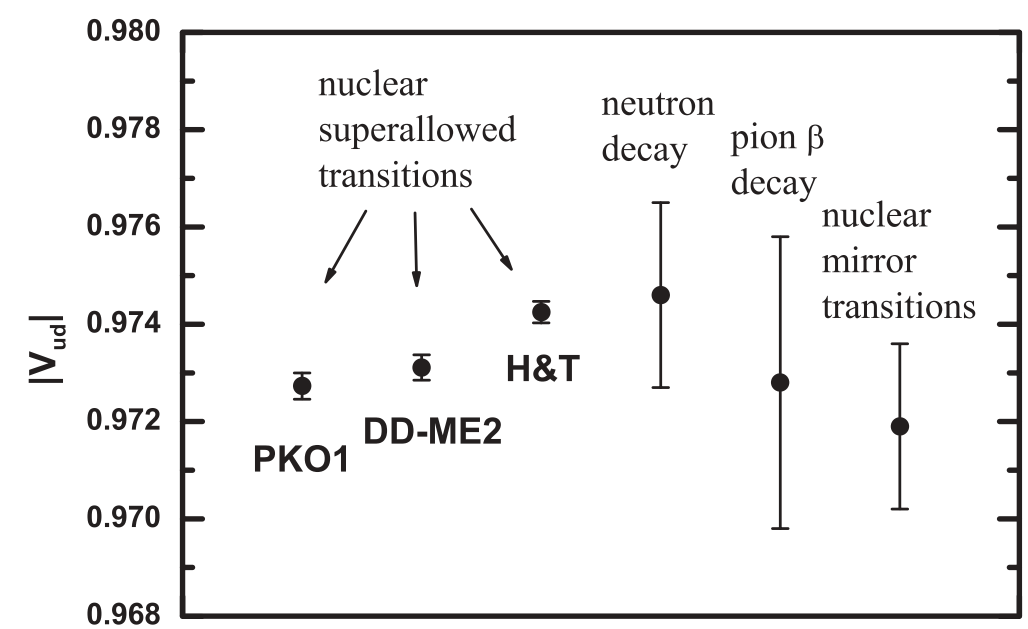
TABLE XI. Recent δ_C calculations (in percent units) based on models labeled SM-WS (shell-model, Woods-Saxon), SM-HF (shell-model, Hartree-Fock), RPA (random phase approximation), IVMR (isovector monopole resonance), and DFT (density functional theory). Also given is the χ^2/ν, χ^2 per degree of freedom, from the confidence test discussed in the text. *J. Hardy, I. Towner, Phys.Rev. C 91 (2014), 025501*

			RPA				
	SM-WS	SM-HF	PKO1	DD-ME2	PC-F1	IVMR[a]	DFT
$T_z = -1$							
^{10}C	0.175	0.225	0.082	0.150	0.109	0.147	0.650
^{14}O	0.330	0.310	0.114	0.197	0.150		0.303
^{22}Mg	0.380	0.260					0.301
^{34}Ar	0.695	0.540	0.268	0.376	0.379		
^{38}Ca	0.765	0.620	0.313	0.441	0.347		
$T_z = 0$							
^{26m}Al	0.310	0.440	0.139	0.198	0.159		0.370
^{34}Cl	0.650	0.695	0.234	0.307	0.316		
^{38m}K	0.670	0.745	0.278	0.371	0.294	0.434	
^{42}Sc	0.665	0.640	0.333	0.448	0.345		0.770
^{46}V	0.620	0.600					0.580
^{50}Mn	0.645	0.610					0.550
^{54}Co	0.770	0.685	0.319	0.393	0.339		0.638
^{62}Ga	1.475	1.205					0.882
^{74}Rb	1.615	1.405	1.088	1.258	0.668		1.770
χ^2/ν	1.4	6.4	4.9	3.7	6.1		4.3[b]

L. Xayavong, N.A. Smirnova, Phys.Rev. C 97 (2018), 024324



Latsamy Xayavong



HT: χ^2 as criterion to prefer SM-WS;

V_{ud} and limits on BSM strongly depend on nuclear model

Nuclear community embarked on ab-initio δ_C calculations

Especially interesting for light nuclei accessible to different techniques!

Electroweak radii constrain ISB in superallowed β-decay

δ_C generally expected to be dominated by Coulomb repulsion between protons (hence C)

In this picture we can connect δ_C to measurable quantities: charge and weak nuclear radii!

Seng, MG 2208.03037; 2304.03800
Seng 2212.02681

Nuclear Hamiltonian with ISB potential: $H = H_0 + V_{\rm ISB} \approx H_0 + V_C$

Coulomb potential for uniformly charged sphere $\quad V_C \approx -\frac{Ze^2}{4\pi R_C^3}\sum_{i=1}^{A}\left(\frac{1}{2}r_i^2 - \frac{3}{2}R_C^2\right)\left(\frac{1}{2} - \hat{T}_z(i)\right)$

ISB due to IV monopole, $V_{\rm ISB} \approx \frac{Ze^2}{8\pi R^3}\sum_i r_i^2 \hat{T}_z(i) = \frac{Ze^2}{8\pi R^3}\hat{M}_0^{(1)}$

Same op generates nuclear radii, $\quad R_{p/n,\phi} = \sqrt{\frac{1}{X}\langle\phi|\sum_{i=1}^{A} r_i^2\left(\frac{1}{2} \mp \hat{T}_z(i)\right)|\phi\rangle}$

Construct ISB-sensitive combinations of radii: directly related to electroweak form factors!

$$\Delta M_A^{(1)} \equiv \langle f|M_{+1}^{(1)}|i\rangle + \langle f|M_0^{(1)}|f\rangle \qquad \Delta M_B^{(1)} \equiv \frac{1}{2}\left(Z_1 R_{p,1}^2 + Z_{-1}R_{p,-1}^2\right) - Z_0 R_{p,0}^2$$

Electroweak radii constrain ISB in superallowed β-decay

Employ the correct isospin formalism by **Schwenk, Miller 0805.0603; 0910.2790**

δ_C expressed via the same set of matrix elements!

$$\delta_\mathrm{C} = \frac{1}{3}\sum_a \frac{|\langle a;0||V||g;1\rangle|^2}{(E_{a,0}-E_{g,1})^2} + \frac{1}{2}\sum_{a\neq g} \frac{|\langle a;1||V||g;1\rangle|^2}{(E_{a,1}-E_{g,1})^2} - \frac{5}{6}\sum_a \frac{|\langle a;2||V||g;1\rangle|^2}{(E_{a,2}-E_{g,1})^2} + \mathcal{O}(V^3)$$

$$\Delta M_A^{(1)} = \frac{1}{3}\Gamma_0 + \frac{1}{2}\Gamma_1 + \frac{7}{6}\Gamma_2 + \mathcal{O}(V^2)$$

$$\Delta M_B^{(1)} = \frac{2}{3}\Gamma_0 - \Gamma_1 + \frac{1}{3}\Gamma_2 + \mathcal{O}(V^2),$$

$$\Gamma_T = -\sum_a \frac{|\langle a;T||V||g;1\rangle|^2}{E_{a,T}-E_{g,1}}$$

Different scaling with ISB: $\delta_C \sim \mathrm{ISB}^2,\ \Delta M_A^{(1)} \sim \mathrm{ISB}^1,\ \Delta M_B^{(1)} \sim \mathrm{ISB}^3$

Transitions	δ_C (%)					$\Delta M_A^{(1)}$ (fm^2)					$\Delta M_B^{(1)}$ (fm^2)				
	WS	DFT	HF	RPA	Micro	WS	DFT	HF	RPA	Micro	WS	DFT	HF	RPA	Micro
$^{26m}\mathrm{Al} \to {}^{26}\mathrm{Mg}$	0.310	0.329	0.30	0.139	0.08	-2.2	-2.3	-2.1	-1.0	-0.6	-0.12	-0.12	-0.11	-0.05	-0.03
$^{34}\mathrm{Cl} \to {}^{34}\mathrm{S}$	0.613	0.75	0.57	0.234	0.13	-5.0	-6.1	-4.6	-1.9	-1.0	-0.17	-0.21	-0.16	-0.06	-0.04
$^{38m}\mathrm{K} \to {}^{38}\mathrm{Ar}$	0.628	1.7	0.59	0.278	0.15	-5.4	-14.6	-5.1	-2.4	-1.3	-0.15	-0.42	-0.15	-0.07	-0.04
$^{42}\mathrm{Sc} \to {}^{42}\mathrm{Ca}$	0.690	0.77	0.42	0.333	0.18	-6.2	-6.9	-3.8	-3.0	-1.6	-0.15	-0.17	-0.09	-0.07	-0.04
$^{46}\mathrm{V} \to {}^{46}\mathrm{Ti}$	0.620	0.563	0.38	/	0.21	-5.8	-5.3	-3.6	/	-2.0	-0.12	-0.11	-0.08	/	-0.04
$^{50}\mathrm{Mn} \to {}^{50}\mathrm{Cr}$	0.660	0.476	0.35	/	0.24	-6.4	-4.6	-3.4	/	-2.4	-0.12	-0.09	-0.06	/	-0.04
$^{54}\mathrm{Co} \to {}^{54}\mathrm{Fe}$	0.770	0.586	0.44	0.319	0.28	-7.8	-5.9	-4.4	-3.2	-2.8	-0.13	-0.10	-0.07	-0.05	-0.05

Can discriminate model predictions for ΔM_A from measured radii —> test models for δ_C

Electroweak radii constrain ISB in superallowed β-decay

Conversely: predict transition weak radius R^2_{CW} from known charge radii across isotriplet

Daughter charge radius used for recoil corrections to ft — but from isospin symmetry

$$R^2_{\rm CW} = R^2_{\rm Ch,1} + Z_0(R^2_{\rm Ch,0} - R^2_{\rm Ch,1}) = R^2_{\rm Ch,1} + \frac{Z_{-1}}{2}(R^2_{\rm Ch,-1} - R^2_{\rm Ch,1})$$

Seng 2212.02681

A	$R_{\rm Ch,-1}$ (fm)	$R_{\rm Ch,0}$ (fm)	$R_{\rm Ch,1}$ (fm)	$R^2_{\rm Ch,1}$ (fm^2)	$R^2_{\rm CW}$ (fm^2)
10	$^{10}_{6}$C	$^{10}_{5}$B(ex)	$^{10}_{4}$Be: 2.3550(170)[a]	5.546(80)	N/A
14	$^{14}_{8}$O	$^{14}_{7}$N(ex)	$^{14}_{6}$C: 2.50 25(87)[a]	6.263(44)	N/A
18	$^{18}_{10}$Ne: 2.9714(76)[a]	$^{18}_{9}$F(ex)	$^{18}_{8}$O: 2.77 26(56)[a]	7.687(31)	13.40(53)
22	$^{22}_{12}$Mg: 3.0691(89)[b]	$^{22}_{11}$Na(ex)	$^{22}_{10}$Ne: 2.9525(40)[a]	8.717(24)	12.93(71)
26	$^{26}_{14}$Si	$^{26m}_{13}$Al	$^{26}_{12}$Mg: 3.0337(18)[a]	9.203(11)	N/A
30	$^{30}_{16}$S	$^{30}_{15}$P(ex)	$^{30}_{14}$Si: 3.1336(40)[a]	9.819(25)	N/A
34	$^{34}_{18}$Ar: 3.3654(40)[a]	$^{34}_{17}$Cl	$^{34}_{16}$S: 3.2847(21)[a]	10.789(14)	15.62(54)
38	$^{38}_{20}$Ca: 3.467(1)[c]	$^{38m}_{19}$K: 3.437(4)[d]	$^{38}_{18}$Ar: 3.4028(19)[a]	11.579(13)	15.99(28)
42	$^{42}_{22}$Ti	$^{42}_{21}$Sc: 3.5702(238)[a]	$^{42}_{20}$Ca: 3.5081(21)[a]	12.307(15)	21.5(3.6)
46	$^{46}_{24}$Cr	$^{46}_{23}$V	$^{46}_{22}$Ti: 3.6070(22)[a]	13.010(16)	N/A
50	$^{50}_{26}$Fe	$^{50}_{25}$Mn: 3.7120(196)[a]	$^{50}_{24}$Cr: 3.6588(65)[a]	13.387(48)	23.2(3.8)
54	$^{54}_{28}$Ni: 3.738(4)[e]	$^{54}_{27}$Co	$^{54}_{26}$Fe: 3.6933(19)[a]	13.640(14)	18.29(92)
62	$^{62}_{32}$Ge	$^{62}_{31}$Ga	$^{62}_{30}$Zn: 3.9031(69)[b]	15.234(54)	N/A
66	$^{66}_{34}$Se	$^{66}_{33}$As	$^{66}_{32}$Ge	N/A	N/A
70	$^{70}_{36}$Kr	$^{70}_{35}$Br	$^{70}_{34}$Se	N/A	N/A
74	$^{74}_{38}$Sr	$^{74}_{37}$Rb: 4.1935(172)[b]	$^{74}_{36}$Kr: 4.1870(41)[a]	17.531(34)	19.5(5.5)

Potential systematic shift by ~ 0.001 to most ft values —> would alleviate unitarity deficit

Theory strategy: compute all radii AND δ_C — check pattern, compare to available data, motivate exp.

Outlook for V_{ud}

Axial charge g_A outlook

$g_A = -1.2723(23)$ → $g_A = -1.2764(6)$ **But** $g_A = -1.2677(28)$

pre-2018 — PERKEO-III (big A) — aSPECT (little a)

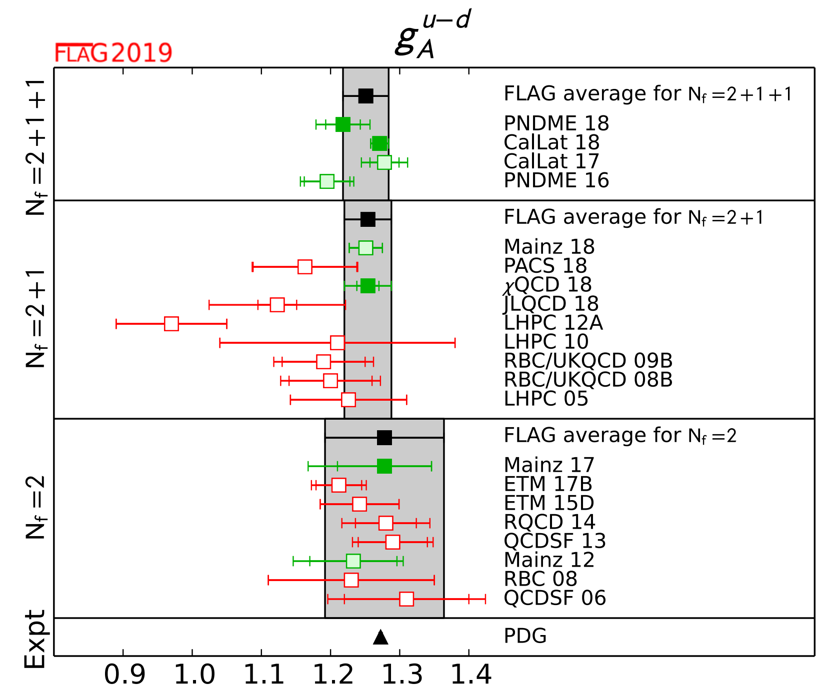
PERKEO-III $\delta g_A / g_A \approx 0.04\,\%$ PERC, Nab, UCNA, ESS, … $\delta g_A / g_A < 0.01\,\%$

g_A on the lattice

$g_A^{\text{FLAG 2019}} = -1.251(33)$

$g_A^{\text{CalLat18}} = -1.271(12)$ *Chang et al., 1805.12130, Nature*

$g_A^{\text{CalLat22}} = -1.264(9)$ *Andre Walker-Loud — preliminary*



g_V not renormalized by strong interaction: tests of EW SM

g_A is renormalized — precision tests of QCD

RC to g_A to compare lattice to experiment:

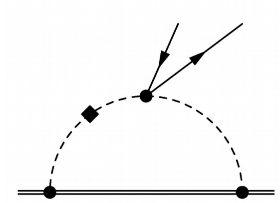
Hayen, PRD 103 (2021) 11, 113001
MG, C-Y Seng, JHEP 10 (2021) 153

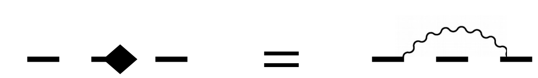
No surprises from γW-box

Unexpectedly large vertex correction ~1% !!!

Isospin breaking from $\pi^{\pm} - \pi^0$ mass difference

However: unknown counterterm





Cirigliano et al, 2202.10439

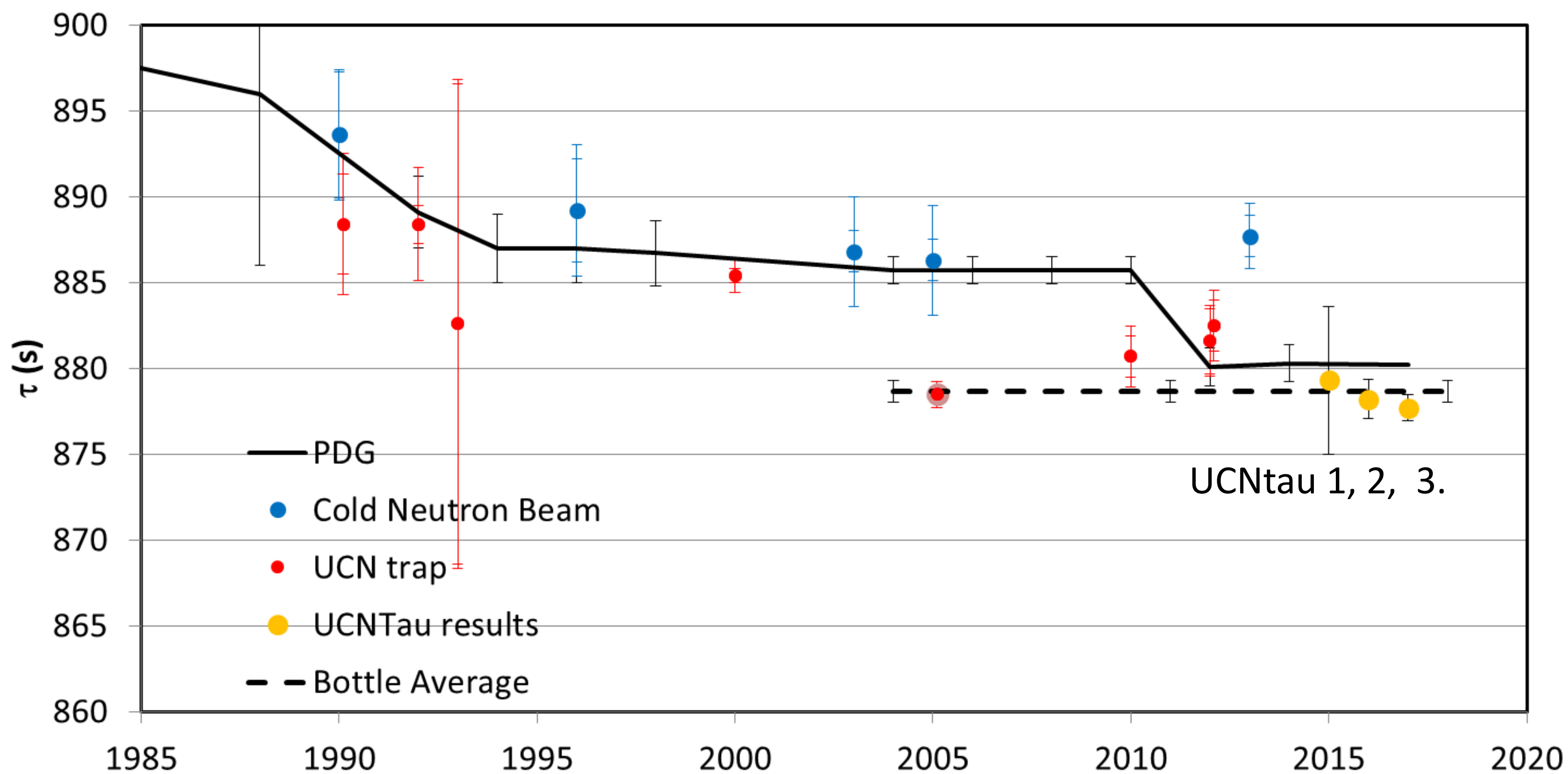
Neutron lifetime τ_n outlook

Bottle (ultra-cold neutrons)

UCNτ 3 $\tau_n = 877.75(28)^{+16}_{-12}$

Plans (2023 on) $\delta\tau_n = 0.1$s

Current limitation:
Beam-bottle discrepancy



Beam (cold neutrons)

BL1 (NIST) $\tau_n = 887.7(2.3)$

Yue et al, PRL 111 (2013) 222501

BL2 (2023 on) —> $\delta\tau_n < 2$s

BL3 (2026 on) —> $\delta\tau_n < 0.3$s

Superallowed (nuclear and pion) Outlook

Superallowed nuclear:
Experiment — not critical (FRIB, ISOLDE…)

$$|V_{ud}^{0^+-0^+}| = 0.97370\,(1)_{exp,\,nucl}\,(3)_{NS}\,(1)_{RC}$$

NS uncertainty currently largest — work necessary and ongoing
- Dispersion formalism applicable to nuclear calculations *Seng, MG, 2211.10214*
- Collaboration started for light nuclei (C-10, O-14)
- Pastore & Co [Green-Function Monte Carlo]
- Navratil & Co [No-Core Shell Model]

ISB uncertainty may be underestimated — work ongoing
- Related to charge and weak radii of the superallowed isotriplet *Seng, MG, 2208.03037*
- Direct ab-initio calculations (e.g. coupled clusters) - for medium nuclei

Semileptonic pion (superallowed meson):
Theory in great shape!

$$|V_{ud}|_{\pi e3} = 0.9740(28)_{BR}(1)_{th}$$

Experiment — future PIONEER @ PSI: o.o.m. improvement! *PIONEER Coll. 2203.01908*

Phase I 2029 on
Phase II: improve BR($\pi e3$) by factor 3
Phase III: improve BR($\pi e3$) by factor 10

Status of V_{us}

V_{us} Status and Outlook

$$\Gamma(K_{\ell 3(\gamma)}) = \frac{C_K^2 G_F^2 m_K^5}{192\pi^3} S_{\rm EW} |V_{us}|^2 |f_+^{K^0\pi^-}(0)|^2 I_{K\ell}(\lambda_{K\ell}) \left(1 + 2\Delta_K^{SU(2)} + 2\Delta_{K\ell}^{\rm EM}\right)$$

with $K \in \{K^+, K^0\}$; $\ell \in \{e, \mu\}$, and:

C_K^2 1/2 for K^+, 1 for K^0

$S_{\rm EW}$ Universal SD EW correction (1.0232)

Inputs from experiment:

$\Gamma(K_{\ell 3(\gamma)})$ Rates with well-determined treatment of radiative decays:

- Branching ratios
- Kaon lifetimes

$I_{K\ell}(\{\lambda\}_{K\ell})$ Integral of form factor over phase space: λs parameterize evolution in t

Inputs from theory:

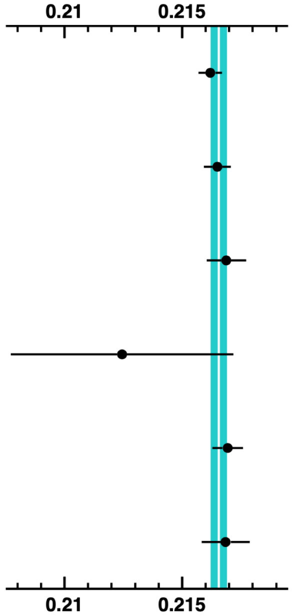
$f_+^{K^0\pi^-}(0)$ Hadronic matrix element (form factor) at zero momentum transfer ($t = 0$)

$\Delta_K^{SU(2)}$ Form-factor correction for $SU(2)$ breaking

$\Delta_{K\ell}^{\rm EM}$ Form-factor correction for long-distance EM effects

V_{us} from Kl3 decays

$|V_{us}| f_+(0)$ from world data: 2022 update

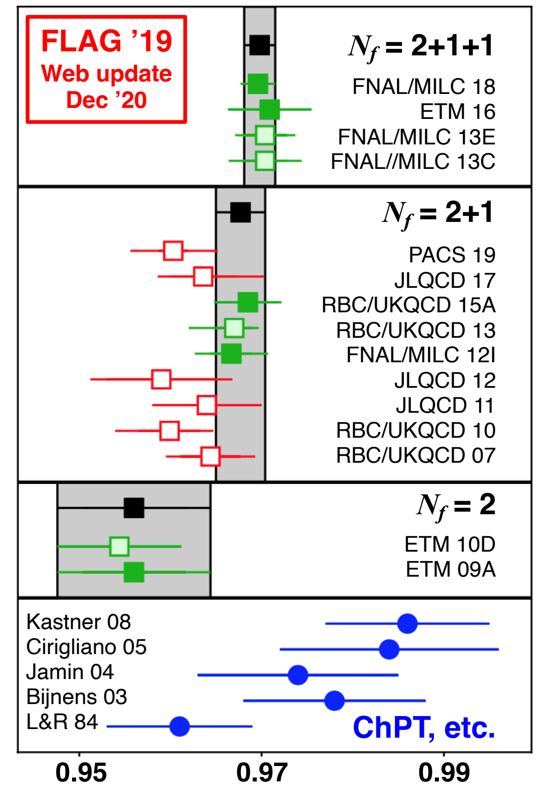


Approx. contrib. to % err from:

		% err	BR	τ	Δ	Int
$K_L e3$	0.2162(5)	0.23	0.09	0.20	0.02	0.05
$K_L \mu 3$	0.2165(6)	0.26	0.15	0.18	0.02	0.07
$K_S e3$	0.2169(8)	0.39	0.38	0.02	0.02	0.05
$K_S \mu 3$	0.2125(47)	2.2	2.2	0.02	0.02	0.08
$K^\pm e3$	0.2169(6)	0.30	0.27	0.06	0.11	0.05
$K^\pm \mu 3$	0.2168(10)	0.47	0.45	0.06	0.11	0.08

Average: $|V_{us}| f_+(0)$ = 0.21656(35) χ^2/ndf = 1.89/5 (86%)

Evaluations of $f_+(0)$



FLAG '21 averages:

N_f = 2+1+1 $f_+(0)$ = 0.9698(17)

Uncorrelated average of:

FNAL/MILC 18: HISQ, 5sp, $m_\pi \to$ 135 MeV, new ensembles added to FNAL/MILC 13E

ETM 16: TwMW, 3sp, $m_\pi \to$ 210 MeV, full q^2 dependence of f_+, f_0

N_f = 2+1 $f_+(0)$ = 0.9677(27)

Uncorrelated average of:

FNAL/MILC 12I: HISQ, $m_\pi \sim$ 300 MeV

RBC/UKQCD 15A: DWF, $m_\pi \to$ 139 MeV

JLQCD 17 not included because only single lattice spacing used

ChPT $f_+(0)$ = 0.970(8)

Ecker 15, Chiral Dynamics 15: Calculation from Bijnens 03, with new LECs from Bijnens, Ecker 14

$K_{\mu 3}$

$f_+(0)$ = 0.9698(17)

N_f = 2+1+1

$$V_{us} = 0.22330(35)_{exp}(39)_{lat}(8)_{IB}$$

$$\Delta^{(1)}_{CKM} = -0.00176(16)_{exp+IB}(17)_{lat}(51)_{ud} \quad = -3.1\sigma$$

V_{us} / V_{ud} from Kl2 decays

$$\frac{|V_{us}|}{|V_{ud}|}\frac{f_K}{f_\pi} = \left(\frac{\Gamma_{K_{\mu 2(\gamma)}} m_{\pi^\pm}}{\Gamma_{\pi_{\mu 2(\gamma)}} m_{K^\pm}}\right)^{1/2} \frac{1-m_\mu^2/m_{\pi^\pm}^2}{1-m_\mu^2/m_{K^\pm}^2}\left(1-\frac{1}{2}\delta_{\rm EM}-\frac{1}{2}\delta_{SU(2)}\right)$$

Inputs from experiment:

From $K^\pm$ BR fit:

BR($K^\pm_{\mu 2(\gamma)}$) = 0.6358(11)

$\tau_{K\pm}$ = 12.384(15) ns

From PDG:

BR($\pi^\pm_{\mu 2(\gamma)}$) = 0.9999

$\tau_{\pi\pm}$ = 26.033(5) ns

Inputs from theory:

$\delta_{\rm EM}$ Long-distance EM corrections

$\delta_{SU(2)}$ Strong isospin breaking

$f_K/f_\pi \rightarrow f_{K\pm}/f_{\pi\pm}$

f_K/f_π Ratio of decay constants

Cancellation of lattice-scale uncertainties from ratio

NB: Most lattice results already corrected for $SU(2)$-breaking: $f_{K\pm}/f_{\pi\pm}$

V_{us} / V_{ud} from Kl2 decays

Giusti et al.
PRL 120 (2018)

First lattice calculation of EM corrections to P_{l2} decays

- Ensembles from ETM
- N_f = 2+1+1 Twisted-mass Wilson fermions

$\delta_{SU(2)} + \delta_{EM} = -0.0122(16)$

- Uncertainty from quenched QED included (0.0006)

Compare to ChPT result from Cirigliano, Neufeld '11:

$\delta_{SU(2)} + \delta_{EM} = -0.0112(21)$

Di Carlo et al.
PRD 100 (2019)

Update, extended description, and systematics of Giusti et al.

$\delta_{SU(2)} + \delta_{EM} = -0.0126(14)$

$$|V_{us}/V_{ud}| \times f_K/f_\pi = 0.27679(28)_{BR}(20)_{corr}$$

Lattice results for f_K/f_π

N_f = 2+1+1 — $f_K/f_\pi = 1.1978(22)$ $S = 1.1$

Average is problematic with correlations assumed by FLAG, dominated by FNAL/MILC17 (symmetrized)

ETM 21 New!	**1.1995(44)(7)**
TM quarks, 3sp, $m_\pi \to$ physical Not yet in FLAG '21 average! Replaces ETM 14E in our average	
Miller 20	1.1964(44)
FNAL/MILC17	$1.1980(^{+13}_{-19})$
HPQCD13A	1.1948(15)(18)

(Miller 20, FNAL/MILC17, HPQCD13A:) Share ensembles. Partially correlated uncertainties using FLAG prescription

N_f = 2+1 — $f_K/f_\pi = 1.1946(34)$*

QCDSF/UKQCD17	1.192(10)(13)
BMW16	1.182(10)(26)
RBC/UKQCD14B	1.1945(45)
BMW10	1.192(7)(6)
HPQCD/UKQCD07	1.198(2)(7)

* MILC10 omitted from average because unpublished

$K_{\mu 2}$

$f_K/f_\pi = 1.1978(22)$
$N_f = 2+1+1$

$$V_{us}/V_{ud} = 0.23108(23)_{exp}(42)_{lat}(16)_{IB}$$

$$V_{us} = 0.22504(28)_{exp}(41)_{lat}(06)_{ud}$$

$$\Delta^{(2)}_{CKM} = -0.00098(13)_{exp}(19)_{lat}(53)_{ud} \quad = -1.8\sigma$$

$$\Delta V_{us}(K_{\mu 3} - K_{\mu 2}) = -0.0174(73) \quad -2.4\sigma$$

Existing data from BNL865, KTeV, ISTRA+, KLOE, NA48, NA48/2
Upcoming data from KLOE-2 and NA62

Cabibbo Angle Anomaly as a BSM Signal

Cabibbo Angle Anomaly as a BSM Signal

Leptonic interactions

$$\mathcal{L}_{CC}^{(\mu)} = -\frac{G_F^{(0)}}{\sqrt{2}} \left(1 + \epsilon_L^{(\mu)}\right) \bar{e}\gamma^\rho(1-\gamma_5)\nu_e \cdot \bar{\nu}_\mu \gamma_\rho (1-\gamma_5)\mu \; + \; \ldots$$

Semi-leptonic interactions

$$\frac{G_F^{(\mu)} V_{ud}}{\sqrt{2}} \left(1 - \epsilon_L^{(\mu)}\right)$$

$$\begin{aligned}
\mathcal{L}_{\rm CC} \;=\; -\frac{G_F^{(0)} V_{ud}}{\sqrt{2}} \times \Big[& \left(\delta^{ab} + \epsilon_L^{ab}\right) \; \bar{e}_a \gamma_\mu (1-\gamma_5)\nu_b \cdot \bar{u}\gamma^\mu (1-\gamma_5) d \\
& + \; \epsilon_R^{ab} \; \bar{e}_a \gamma_\mu (1-\gamma_5)\nu_b \cdot \bar{u}\gamma^\mu (1+\gamma_5) d \\
& + \; \epsilon_S^{ab} \; \bar{e}_a (1-\gamma_5)\nu_b \cdot \bar{u} d \\
& - \; \epsilon_P^{ab} \; \bar{e}_a (1-\gamma_5)\nu_b \cdot \bar{u}\gamma_5 d \\
& + \; \epsilon_T^{ab} \; \bar{e}_a \sigma_{\mu\nu} (1-\gamma_5)\nu_b \cdot \bar{u}\sigma^{\mu\nu} (1-\gamma_5) d \Big] + {\rm h.c.}
\end{aligned}$$

$\varepsilon_i \sim (v/\Lambda)^2$

For global analysis of beta decays in this framework see:

Falkowski, Gonzalez-Alonso, Naviliat-Cuncic, 2010.13797

Cabibbo Angle Anomaly as a BSM Signal

Connect beta decays to UV physics via EFT: Wilson coeffs. of 4-fermion operators

$$
\begin{aligned}
|\bar{V}_{ud}|^2_{0^+\to 0^+} &= |V_{ud}|^2 \left(1 + 2\,(\epsilon_L^{ee} + \epsilon_R - \epsilon_L^{(\mu)}) + c_{0^+}^S(Z)\ \epsilon_S^{ee}\right) \\
|\bar{V}_{ud}|^2_{n\to pe\bar{\nu}} &= |V_{ud}|^2 \left(1\ +\ 2\,(\epsilon_L^{ee} + \epsilon_R - \epsilon_L^{(\mu)}) + c_n^S \epsilon_S^{ee} + c_n^T \epsilon_T^{ee}\right) \\
|\bar{V}_{us}|^2_{Ke3} &= |V_{us}|^2 \left(1 + 2\,(\epsilon_L^{ee(s)} + \epsilon_R^{(s)} - \epsilon_L^{(\mu)})\right) \\
|\bar{V}_{ud}|^2_{\pi e3} &= |V_{ud}|^2 \left(1 + 2\,(\epsilon_L^{ee} + \epsilon_R - \epsilon_L^{(\mu)})\right) \\
|\bar{V}_{us}|^2_{K_{\mu 2}} &= |V_{us}|^2 \left(1 + 2\,(\epsilon_L^{\mu\mu(s)} - \epsilon_R^{(s)} - \epsilon_L^{(\mu)}) - 2\frac{B_0}{m_\ell}\,\epsilon_P^{\mu\mu(s)}\right) \\
|\bar{V}_{ud}|^2_{\pi_{\mu 2}} &= |V_{ud}|^2 \left(1 + 2\,(\epsilon_L^{\mu\mu} - \epsilon_R - \epsilon_L^{(\mu)}) - 2\frac{B_0}{m_\ell}\,\epsilon_P^{\mu\mu}\right)
\end{aligned}
$$

Three distinct Cabibbo unitarity deficits may be defined

$$\Delta^{(1)}_{\rm CKM} = \left|V_{ud}^{\beta}\right|^2 + \left|V_{us}^{K_{\ell 3}}\right|^2 - 1$$

V_{us} from $K_{\ell 3}$ + V_{ud} from β decays

$$\Delta^{(2)}_{\rm CKM} = \left|V_{ud}^{\beta}\right|^2 + \left|V_{us}^{K_{\ell 2}/\pi_{\ell 2},\beta}\right|^2 - 1$$

V_{us}/V_{ud} from $K_{\mu 2}$ + V_{ud} from β decays

$$\Delta^{(3)}_{\rm CKM} = \left|V_{ud}^{K_{\ell 2}/\pi_{\ell 2},\,K_{\ell 3}}\right|^2 + \left|V_{us}^{K_{\ell 3}}\right|^2 - 1$$

V_{us} from $K_{\ell 3}$ + V_{us}/V_{ud} from $K_{\mu 2}$

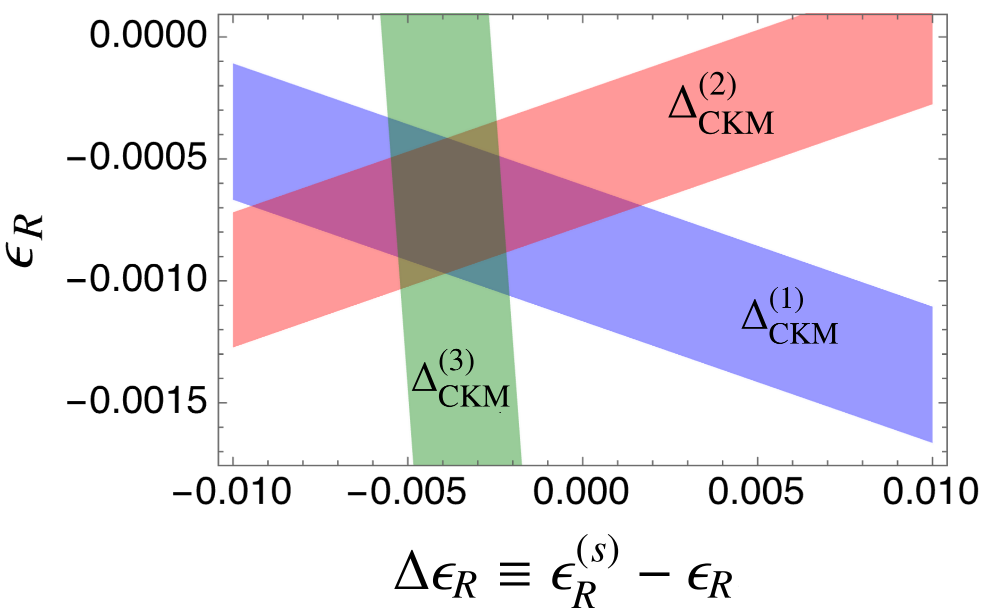
Cabibbo Angle Anomaly as a BSM Signal

RH currents in ud- and us-sectors

V_{ud}, V_{us}, V_{ud}/V_{us} overconstrained, can solve all tensions

$$
\begin{aligned}
|\bar{V}_{ud}|^2_{0^+\to 0^+} &= |V_{ud}|^2 \left(1 + 2\,\epsilon_R\right) \\
|\bar{V}_{ud}|^2_{n\to pe\bar{\nu}} &= |V_{ud}|^2 \left(1 + 2\,\epsilon_R\right) \\
|\bar{V}_{us}|^2_{Ke3} &= |V_{us}|^2 \left(1 + 2\,\epsilon_R^{(s)}\right) \\
|\bar{V}_{ud}|^2_{\pi e3} &= |V_{ud}|^2 \left(1 + 2\,\epsilon_R\right) \\
|\bar{V}_{us}|^2_{K\mu 2} &= |V_{us}|^2 \left(1 - 2\,\epsilon_R^{(s)}\right) \\
|\bar{V}_{ud}|^2_{\pi\mu 2} &= |V_{ud}|^2 \left(1 - 2\,\epsilon_R\right)
\end{aligned}
$$

Cirigliano et al, 2208.11707



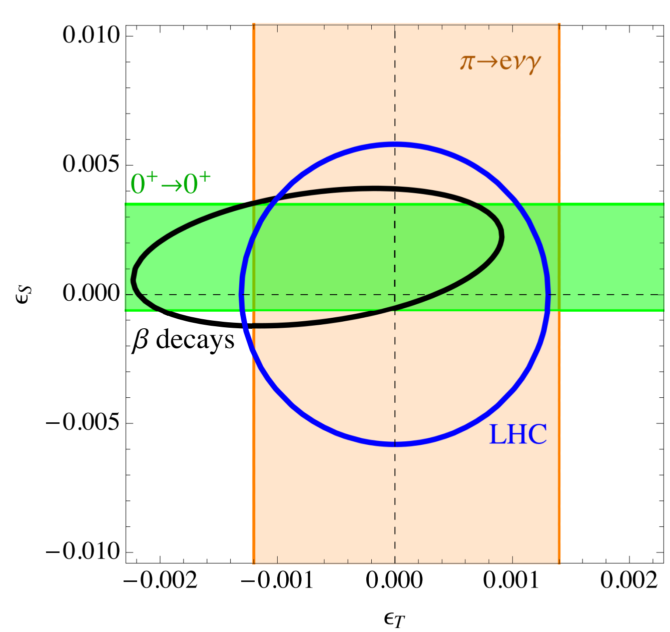
$$
\begin{aligned}
\Delta^{(1)}_{\rm CKM} &= 2\epsilon_R + 2\Delta\epsilon_R V_{us}^2, \\
\Delta^{(2)}_{\rm CKM} &= 2\epsilon_R - 2\Delta\epsilon_R V_{us}^2, \\
\Delta^{(3)}_{\rm CKM} &= 2\epsilon_R + 2\Delta\epsilon_R (2 - V_{us}^2)
\end{aligned}
$$

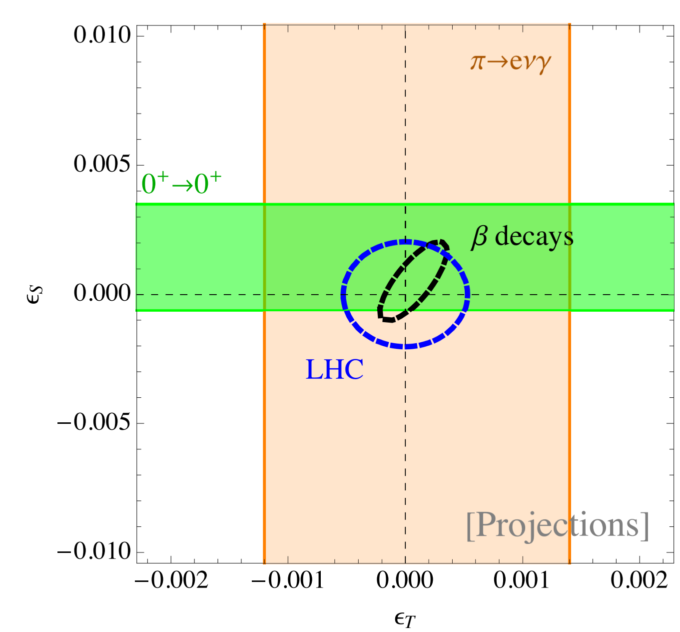
$$
\begin{aligned}
\epsilon_R &= -0.69(27)\times 10^{-3} \\
\Delta\epsilon_R &= -3.9(1.6)\times 10^{-3}
\end{aligned}
$$

$\Lambda_R \sim$ 5-10 TeV   2.5σ effect

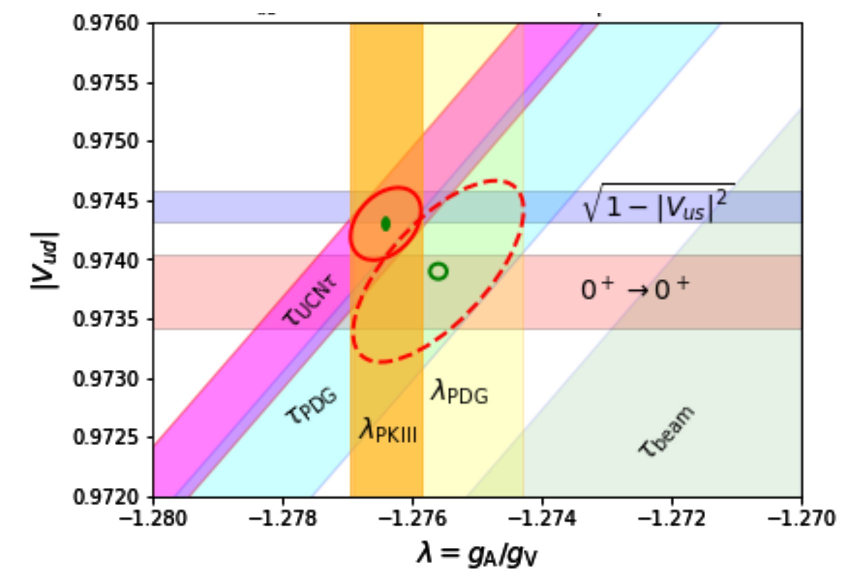
Beta decay vs. LHC on S,T
Complementarity now and in the future!

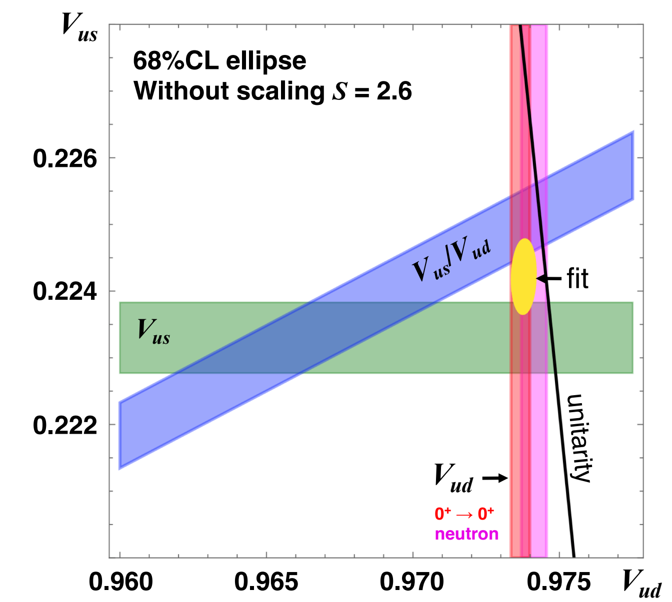
Gonzalez-Alonso et al 1803.08732





Summary: Status of V_{ud} and top-row CKM unitarity





3-sigma CKM unitarity deficit established

Significant shift in V_{ud} due to shift in Δ_R^V

EW boxes: DR + Exp. + Lattice QCD+ ChPT +…

Calculation for Δ_R^V confirmed by several groups

Formalism applied to $K\ell 3$ decays;

Puzzles: $K\ell 2$ - $K\ell 3$, Beam-Bottle n-lifetime

Unified universal RC Δ_R^V and nuclear correction $\delta_{\rm NS}$

Both SM (V_{ud}) and BSM (b_F) tests depend on δ_C and $\delta_{\rm NS}$

Direct lattice QCD evaluation of the γW-box

Modern ab initio theory of δ_C and $\delta_{\rm NS}$ underway!

BSM: RH currents across light and strange quarks may resolve all puzzles