



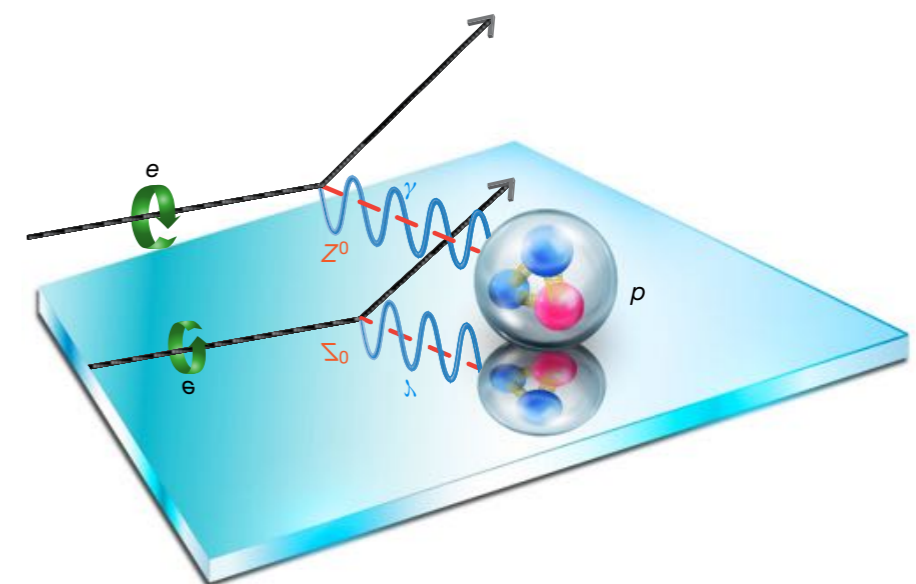
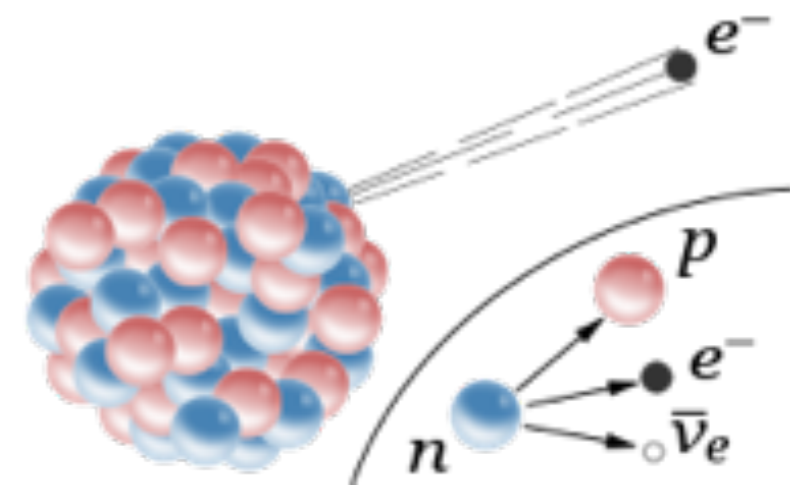
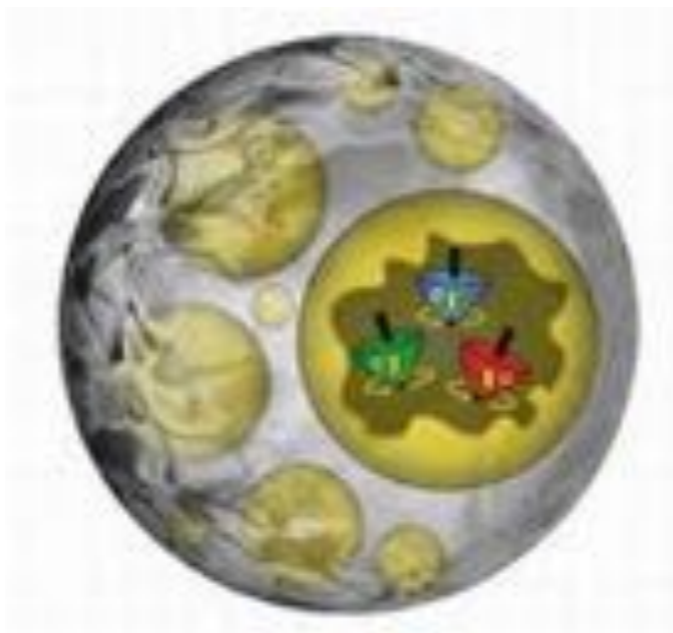
Box Diagrams for Elastic PVES and Relevance for DIS

Misha Gorshteyn

Johannes Gutenberg-Universität Mainz

Collaborators:

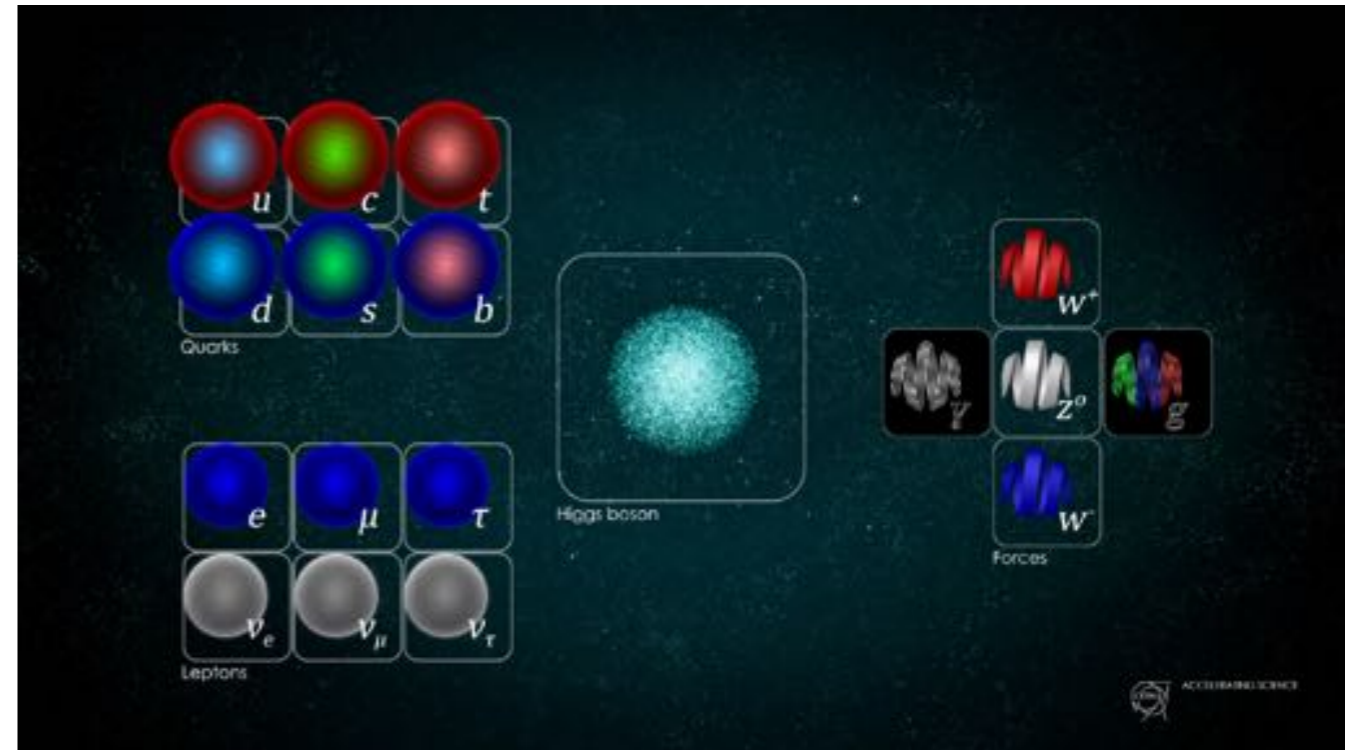
Chien-Yeah Seng
Michael Ramsey-Musolf
Chuck Horowitz
Hubert Spiesberger
Jens Erler
Oleksandr Koshchii
Xu Feng
Lu-Chang Jin
Peng Ma



Standard Model: success story with open questions

Standard Model:

- Renormalizable non-Abelian gauge field theory
- Incorporates observed symmetries
- Symmetry breaking
- “small” number of parameters



Together with gravity describes how things work on Earth, in the solar system, in stars

Yet it is also notoriously insufficient (Dark Matter & Dark Energy, Matter-Antimatter asymmetry in the Universe, Hierarchy problem, Fine tuning of SM parameters, ...)

Searches for beyond Standard Model (BSM) particles and interaction along three frontiers: High-Energy frontier; Astrophysics frontier; Precision frontier

Low-Energy precision tests: compare accurate experimental measurements with equally accurate theory calculations and deduce information about BSM from this comparison

Precision tests of SM at low energies - basics

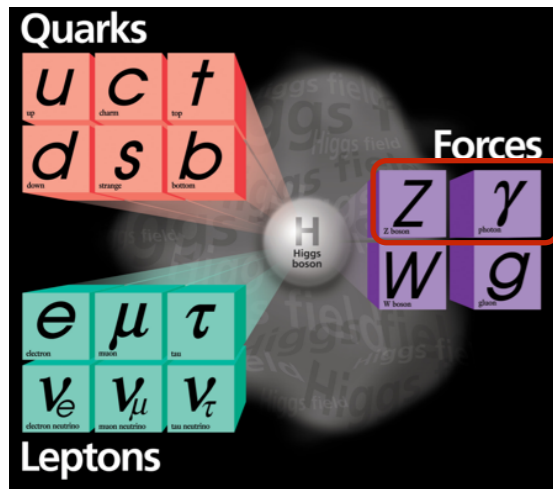
- SM parameters: charges, masses, mixing
- At low energy quarks are bound in hadrons - how can we access their fundamental properties through hadronic mess?
- A **charge associated with a conserved current** is not renormalized by strong interaction - the charge of a composite = \sum charges of constituents
- Strong interaction may modify observables at NLO in $\alpha_{em}/\pi \approx 2 \cdot 10^{-3}$
- Experiment + pure EW RC - accuracy at 10^{-4} level or better
- This accuracy corresponds to ~ 50 TeV scale of heavy BSM particles
- In many low-energy tests hadron structure effects is the main limitation!

Precise determination of SM mixing parameters:

Weak mixing angle $\sin^2 \theta_W$
Cabibbo angle V_{ud}

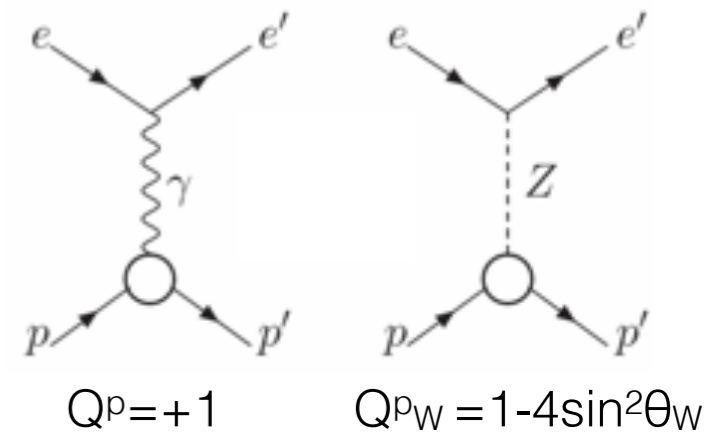
Precision measurements of weak mixing angle

Weak mixing angle - mixing of the NC gauge fields

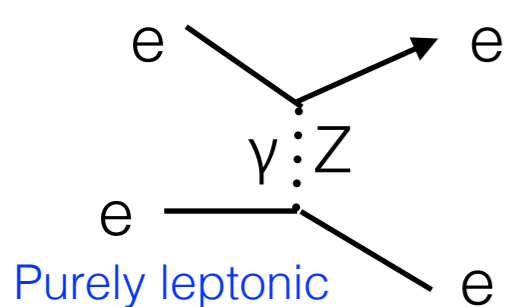


$$\begin{pmatrix} \gamma \\ Z^0 \end{pmatrix} = \begin{pmatrix} \cos \theta_W & \sin \theta_W \\ -\sin \theta_W & \cos \theta_W \end{pmatrix} \begin{pmatrix} B^0 \\ W^0 \end{pmatrix}$$

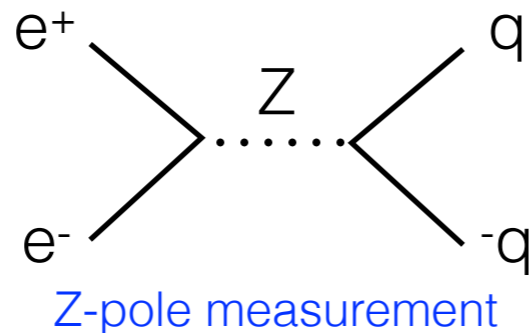
WMA determines the relative strength of the weak NC vs. e.-m. interaction



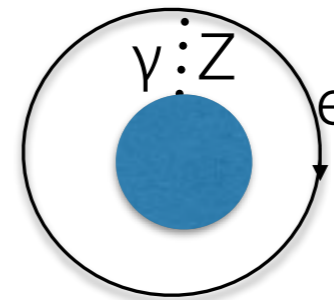
Møller scattering



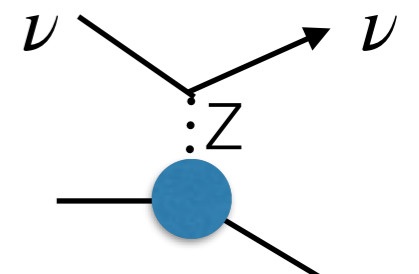
Colliders



Atomic PV

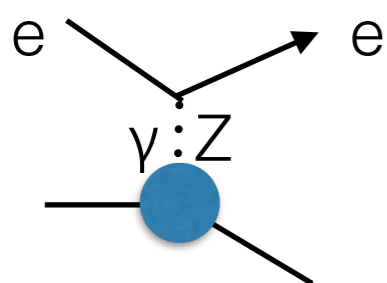


CEvNS

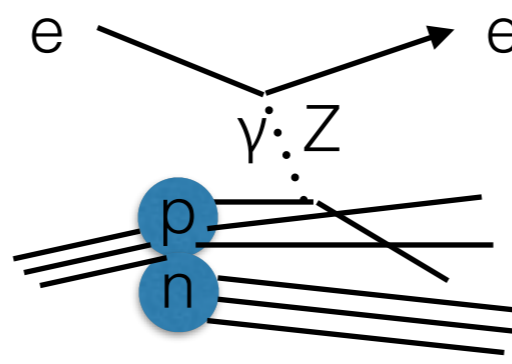


P2 MESA @ Mainz

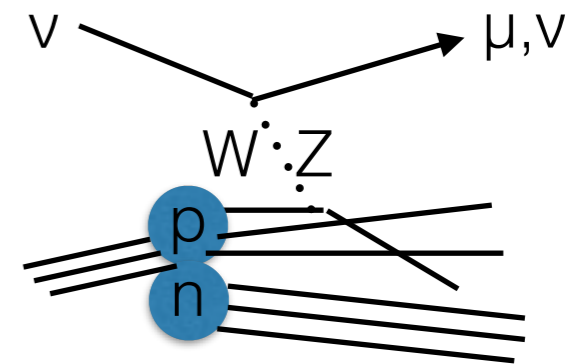
Q-Weak @ JLab



e-DIS @ JLab, EIC



nu-DIS @ NuTeV

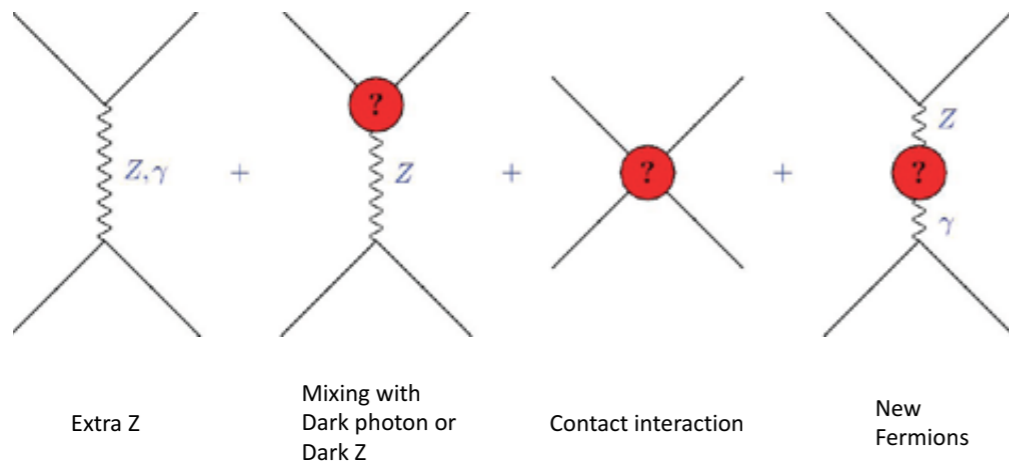


SM running of the weak mixing angle

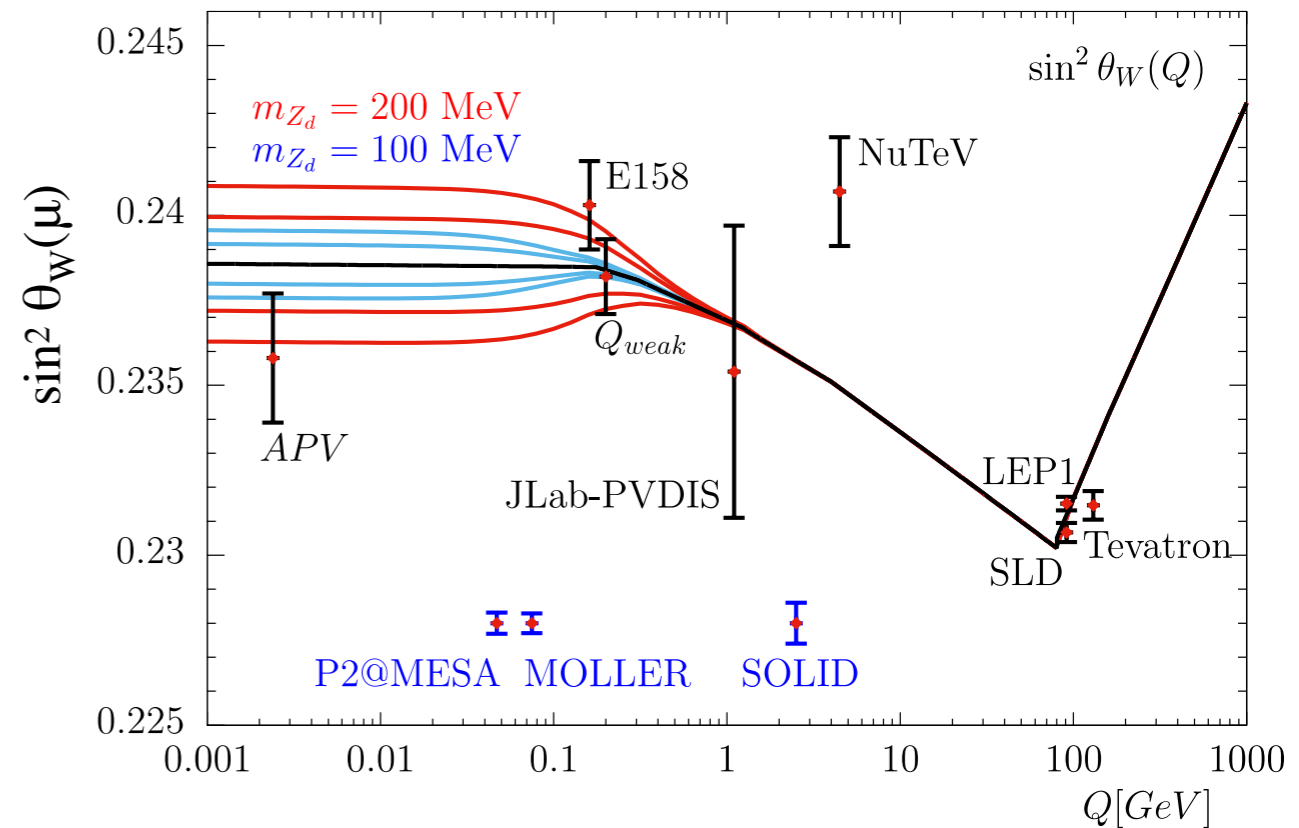
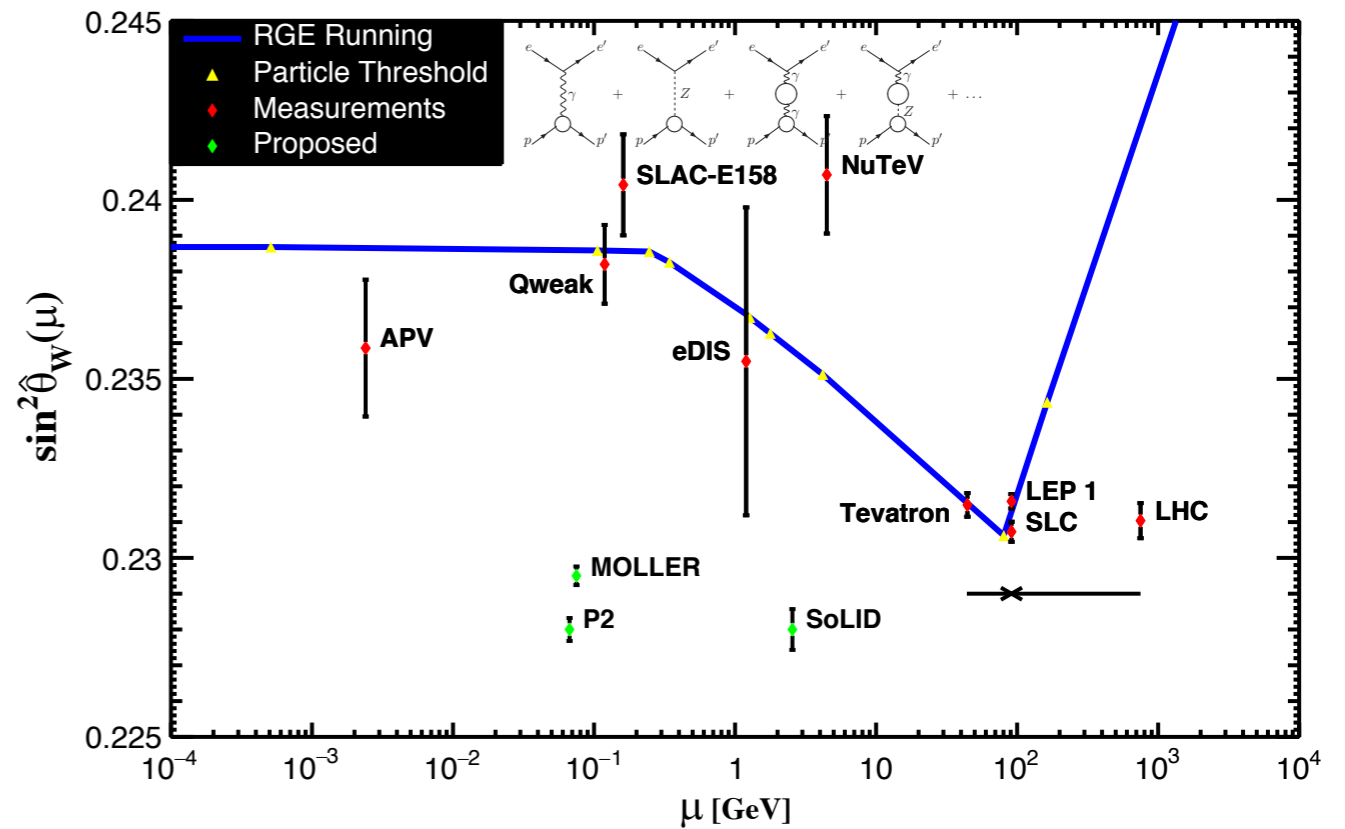
Universal quantum corrections can be absorbed into running, scale-dependent $\sin^2\theta_W(\mu)$

SM uncertainty: few $\times 10^{-5}$

Universal running - clean prediction of SM Deviation anywhere - BSM signal

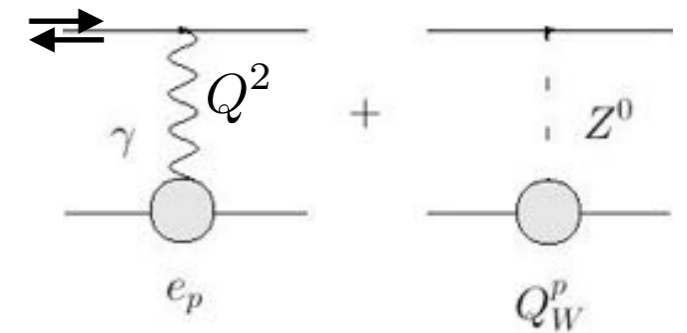


Heavy BSM reach: up to 50 TeV
 Light dark gauge sector: down to 70 MeV
Complementary to colliders



Hadronic weak charges from PVES

Elastic scattering of longitudinally polarized electrons off unpolarized nuclei at low momentum transfer



$$A^{PV} = \frac{\sigma_{\rightarrow} - \sigma_{\leftarrow}}{\sigma_{\rightarrow} + \sigma_{\leftarrow}} = - \frac{G_F Q^2}{4\sqrt{2}\pi\alpha} \frac{Q_W}{Z} (1 + \Delta)$$

Nuclear weak charge $Q_W(Z, N) = -N + Z(1 - 4 \sin^2 \theta_W)$

Proton ($Z=1, N=0$): $Q_W^p = 1 - 4 \sin^2 \theta_W \approx 0.07$ in SM

Qweak@JLab: $Q^2 \sim 0.03 \text{ GeV}^2$ $A^{PV} = - (226.5 \pm 9.3) \text{ ppb}$ $Q_W^p = 0.0718 \pm 0.0044$ (rel. 6%)

D. Androic et al [Qweak Coll.], Nature 557 (2018), 207

P2 @ MESA/Mainz: go down to $Q^2 \sim 0.005 \text{ GeV}^2$ – tiny asymmetry to 1.5-2%

Talk by M. Wilfert

$$Q_W^{p, 1\text{-loop}} = (1 + \Delta_\rho + \Delta_e)(1 - 4 \sin^2 \hat{\theta}_W + \Delta'_e) + \square_{WW} + \square_{ZZ} + \square_{\gamma Z}$$

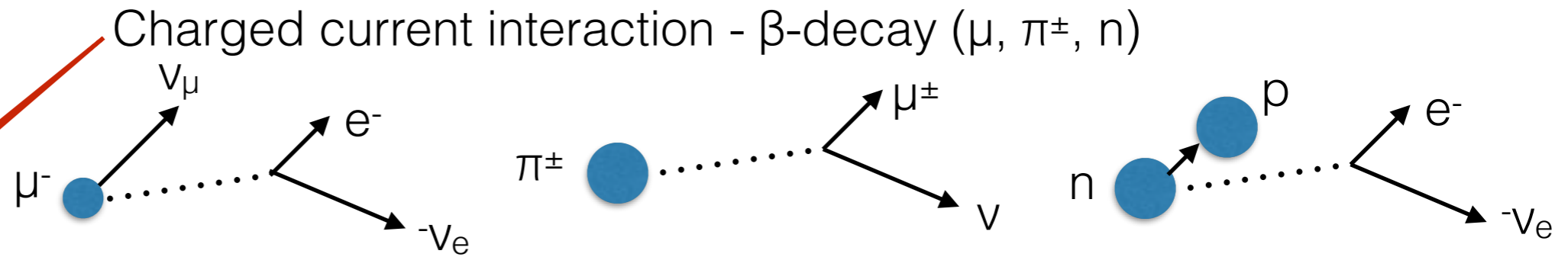
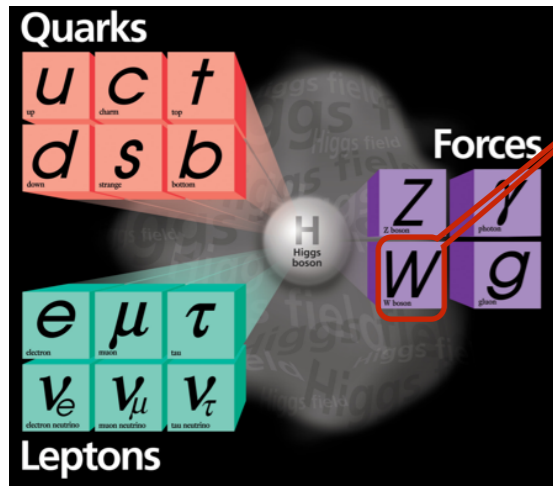
Marciano, Sirlin PRD 1984

Δ : hadronic structure (size, spin, strangeness; suppressed at low Q^2 ; trivial E-dependence)

Non-universal RC: boxes ($\alpha_{em}/\pi \sim 10^{-3}$; unsuppressed by Q^2 ; nontrivial E-dependence)

Talk by P. Blunden

Precise beta decays: universality of weak interaction



Rates close but not the same: CKM mixing matrix + Radiative Corrections
Crucial ingredients for establishing the Standard Model

$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix} = V_{CKM} \begin{pmatrix} d \\ s \\ b \end{pmatrix}$$

CKM - Determines the relative strength of the weak CC interaction of quarks vs. that of leptons

CKM unitarity - measure of completeness of the SM: $|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 1$

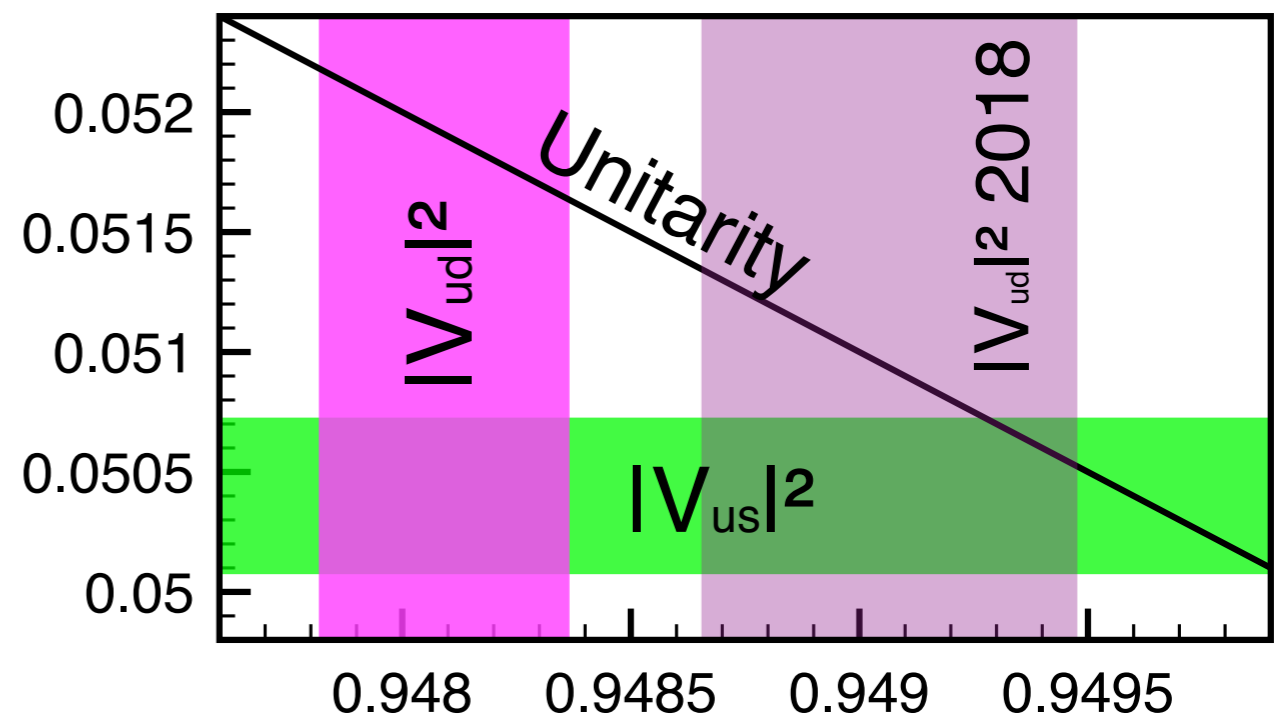
Neglecting $V_{ub} \sim 10^{-3}$: Cabibbo angle θ_C

$$|V_{ud}| = \cos \theta_C, |V_{us}| = \sin \theta_C$$

PDG 2022:

$$\cos^2 \theta_C + \sin^2 \theta_C = 0.9985(3) V_{ud}(4) V_{us}$$

Reason: re-evaluation of SM RC to V_{ud} (γW -box)



BSM searches at low energies vs. colliders

WMA measurements on the Z-pole most straightforward:

Z on-shell, corrections non-resonant

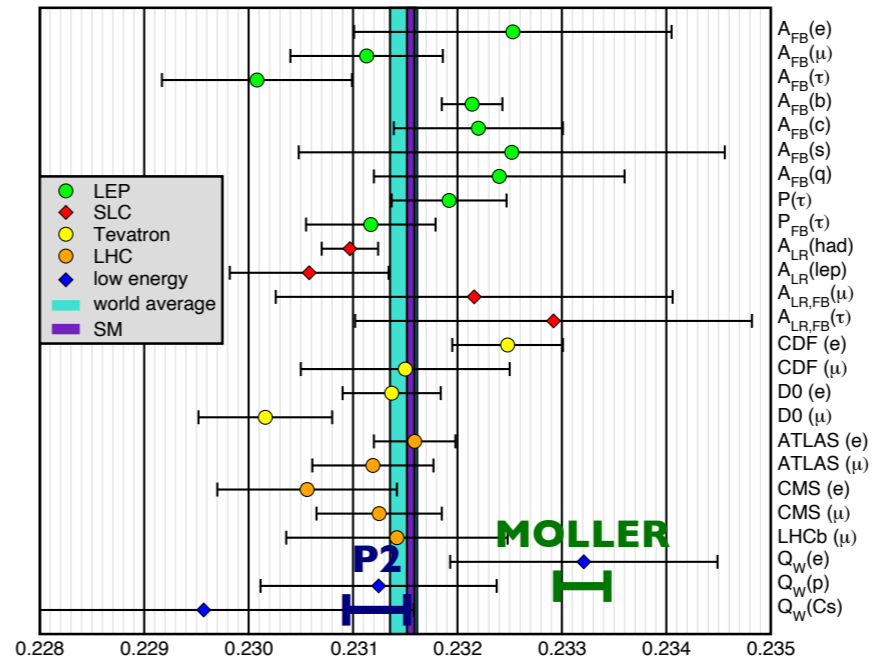
Downside: BSM enters quadratically
SM amplitude imaginary,
BSM amplitude real

Low energies:
BSM in linear interference with SM
Sensitivity to heavy and light BSM

**BSM searches:
LHC vs β -decays**

Constraints on non-standard
Scalar/Tensor CC interactions
Complementarity to LHC
Now and in the future

Weak mixing angle measurements



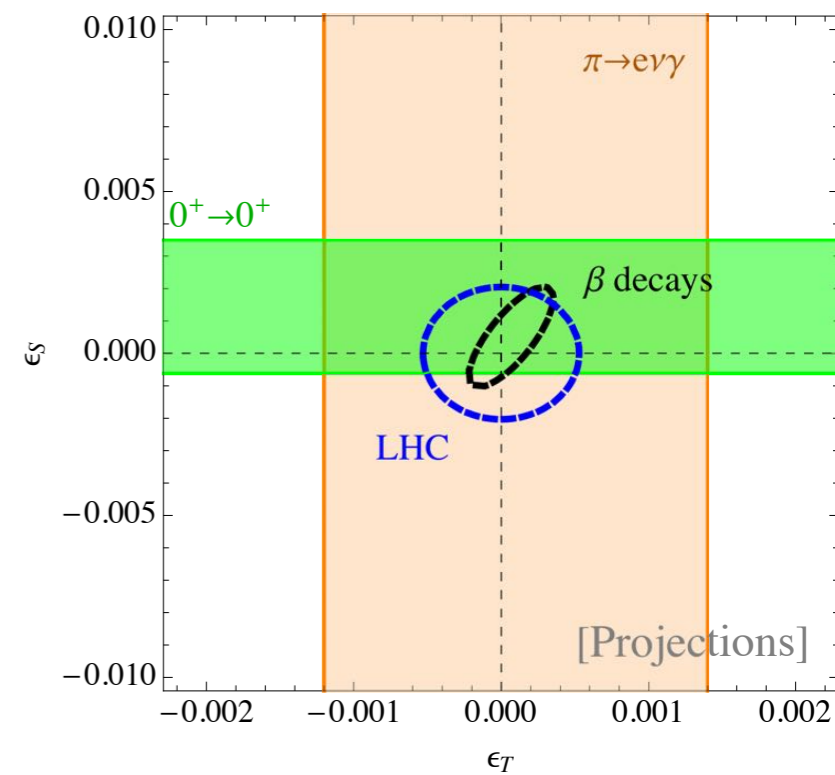
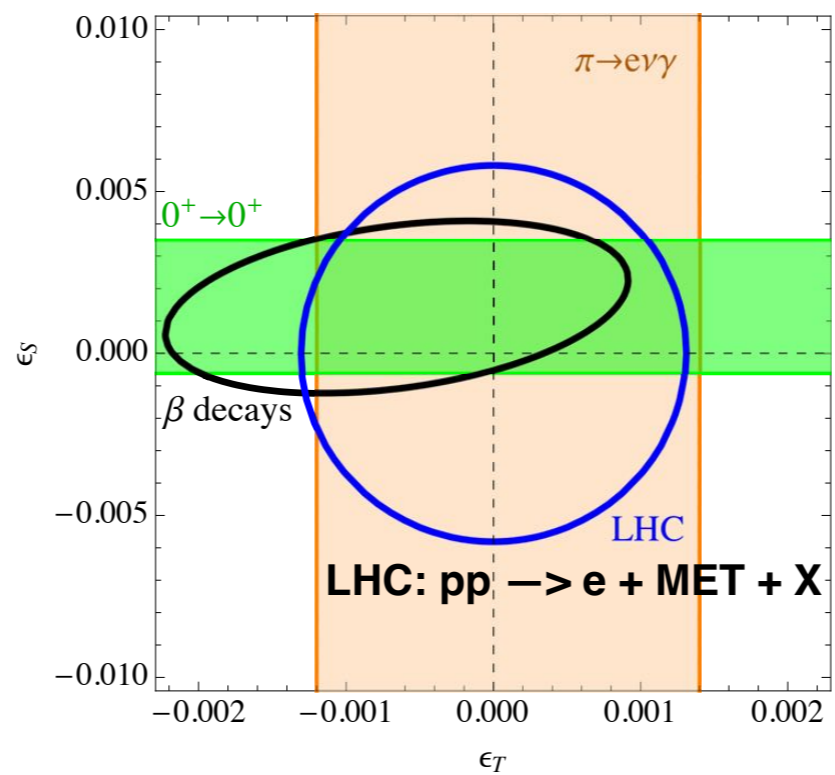
LEP & SLC:
 0.23151 ± 0.00016

Tevatron:
 0.23148 ± 0.00033

LHC:
 0.23129 ± 0.00033

average direct
 0.23148 ± 0.00013

global fit
 0.23153 ± 0.00004



Gonzalez Alonso et al, PPNP 2019

γZ and γW boxes from dispersion relations

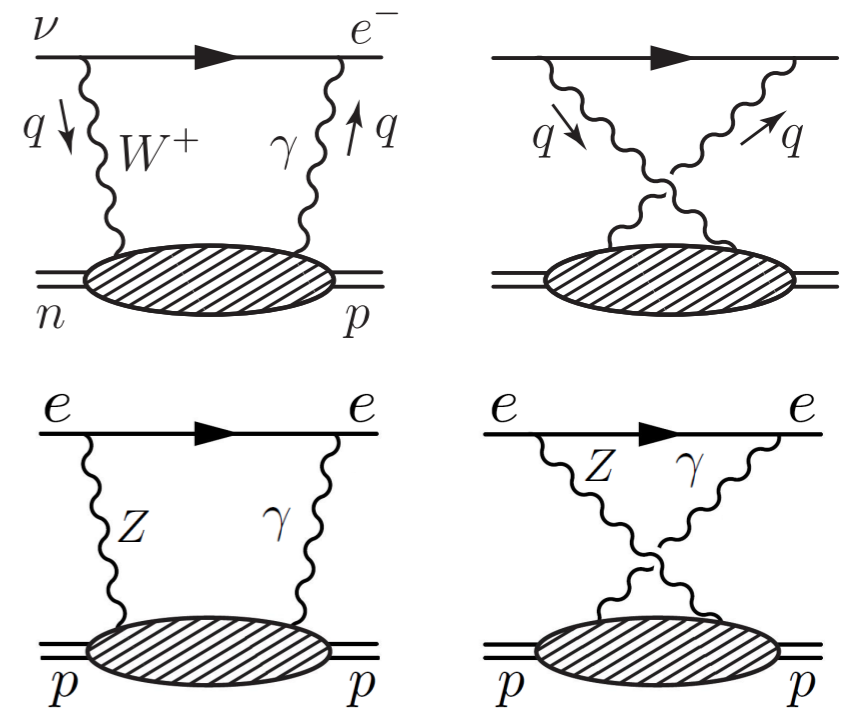
Unified approach in PVES and β decays

Dispersion theory of electroweak boxes

Evaluate the box at zero momentum transfer

E.g., for γW -box (γZ -box analogous)

$$T_{\gamma W} = -\sqrt{2}e^2 G_F V_{ud} \int \frac{d^4 q}{(2\pi)^2} \frac{\bar{u}_e \gamma^\mu (\not{k} - \not{q} + m_e) \gamma^\nu (1 - \gamma_5) v_\nu T_{\mu\nu}^{\gamma W}}{q^2 [(k - q)^2 - m_e^2] [1 - q^2/M_W^2]}$$



Generalized Compton tensor (lower blob):

incorporate all symmetries; consider spin-independent part only (vector charges)

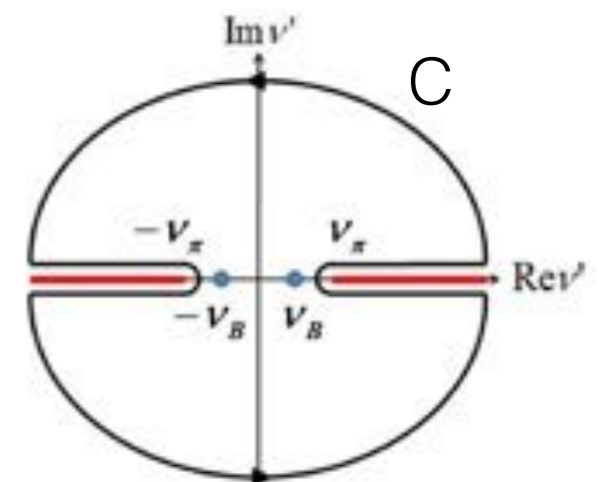
$$T_{\gamma W}^{\mu\nu} = \left[-g^{\mu\nu} + \frac{q^\mu q^\nu}{q^2} \right] T_1 + \frac{\hat{p}^\mu \hat{p}^\nu}{(p \cdot q)} T_2 + \frac{i\epsilon^{\mu\nu\alpha\beta} p_\alpha q_\beta}{2(p \cdot q)} T_3$$

To evaluate the loop — need to know forward Compton amplitudes

$T_{1,2,3}$ - analytic functions inside C in the complex v -plane determined by singularities on the real axis: poles + cuts

Forward box: only ν -analyticity needed;

if finite t also t -analyticity generally required - complicated



Dispersion theory of electroweak boxes

Forward amplitudes T_i - unknown;
 Their absorptive parts can be related to
 production of on-shell intermediate states
 —> a γW (γZ) structure functions $F_{1,2,3}$

$$\text{Im } T_i^{\gamma W}(\nu, Q^2) = 2\pi F_i^{\gamma W}(\nu, Q^2)$$

$$\text{Im } T_i^{\gamma Z}(\nu, Q^2) = 2\pi F_i^{\gamma Z}(\nu, Q^2)$$

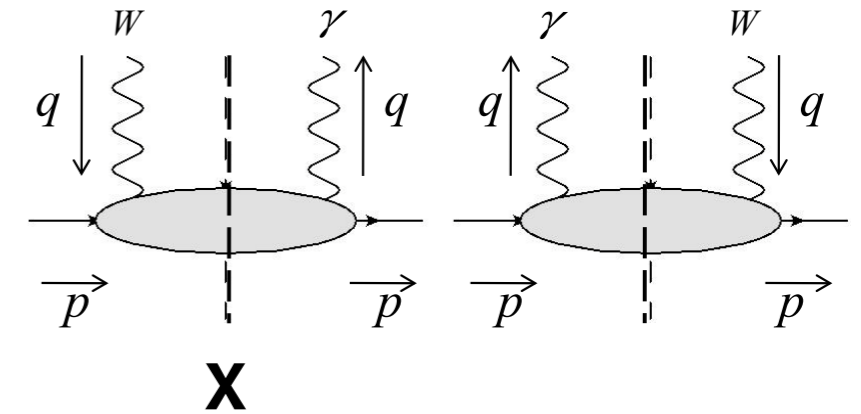
Structure functions $F_i^{\gamma Z}$ are data

Structure functions $F_i^{\gamma W}$ are NOT data but can be related to data

Define box corrections according to

$$T_W + T_{\gamma W} = -\sqrt{2}G_F V_{ud} \bar{u}_e \not{p}(1 - \gamma_5)v_\nu(1 + \square_{\gamma W})$$

$$T_Z + T_{\gamma Z} = -\frac{G_F}{2\sqrt{2}} \bar{u}_e \not{p}(1 - \gamma_5)u_e F_{\text{weak}}(Q^2)[Q_W + \square_{\gamma Z}(E, Q^2)]$$

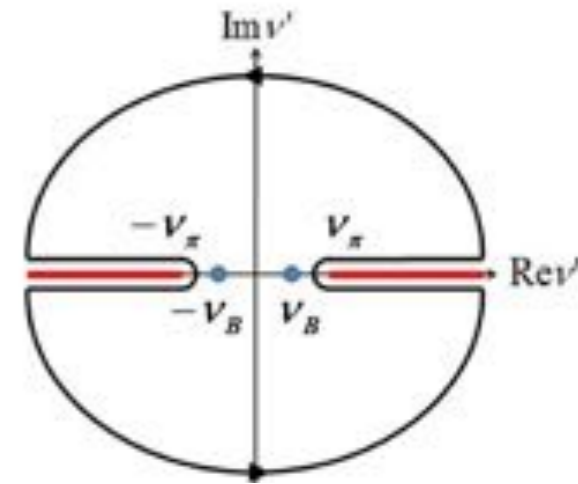


X
 X = inclusive strongly-interacting
 on-shell physical states

Dispersion representation of $\square_{\gamma W/\gamma Z}$

Dispersion representation of Compton amplitudes:
Combine left-hand and right-hand singularities

$$\text{Re } T_i(\nu, Q^2) = \frac{1}{\pi} \int_0^\infty d\nu' \left[\frac{1}{\nu' - \nu} \pm \frac{1}{\nu' + \nu} \right] \text{Im } T_i(\nu', Q^2)$$



Crossing behavior

γZ : same initial and final state \rightarrow definite crossing: T_1 even, $T_{2,3}$ odd function of ν

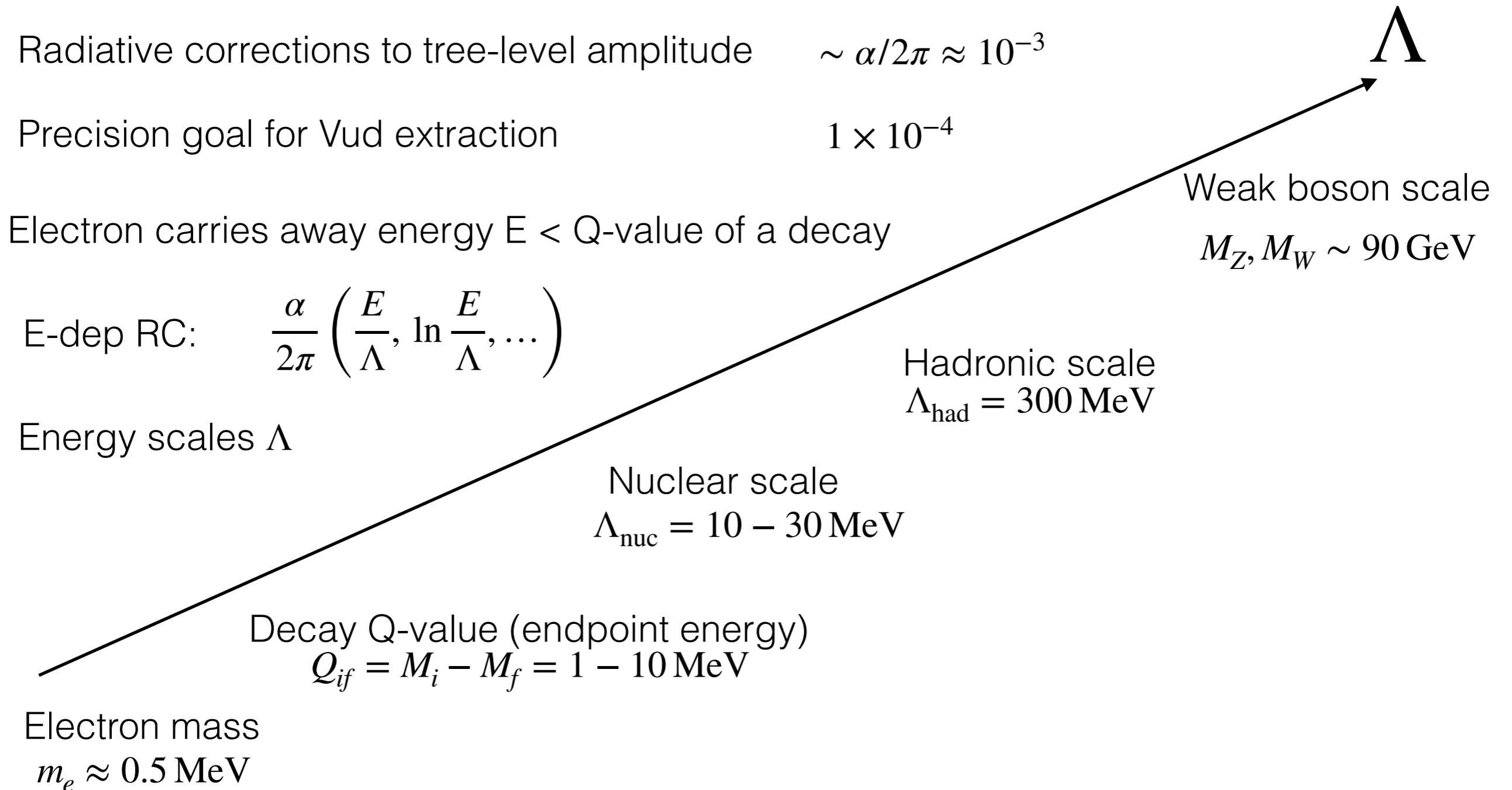
γW : initial and final state different \rightarrow mixed crossing, depends on the isospin of γ

$$T_i^{\gamma W} = T_i^{(I_\gamma=0)} + T_i^{(I_\gamma=1)}$$

with $T_1^{(0)}$, $T_{2,3}^{(1)}$ even,

$T_1^{(1)}$, $T_{2,3}^{(0)}$ odd function of ν

Dispersion theory of $\Box_{\gamma W}$: relevant scales

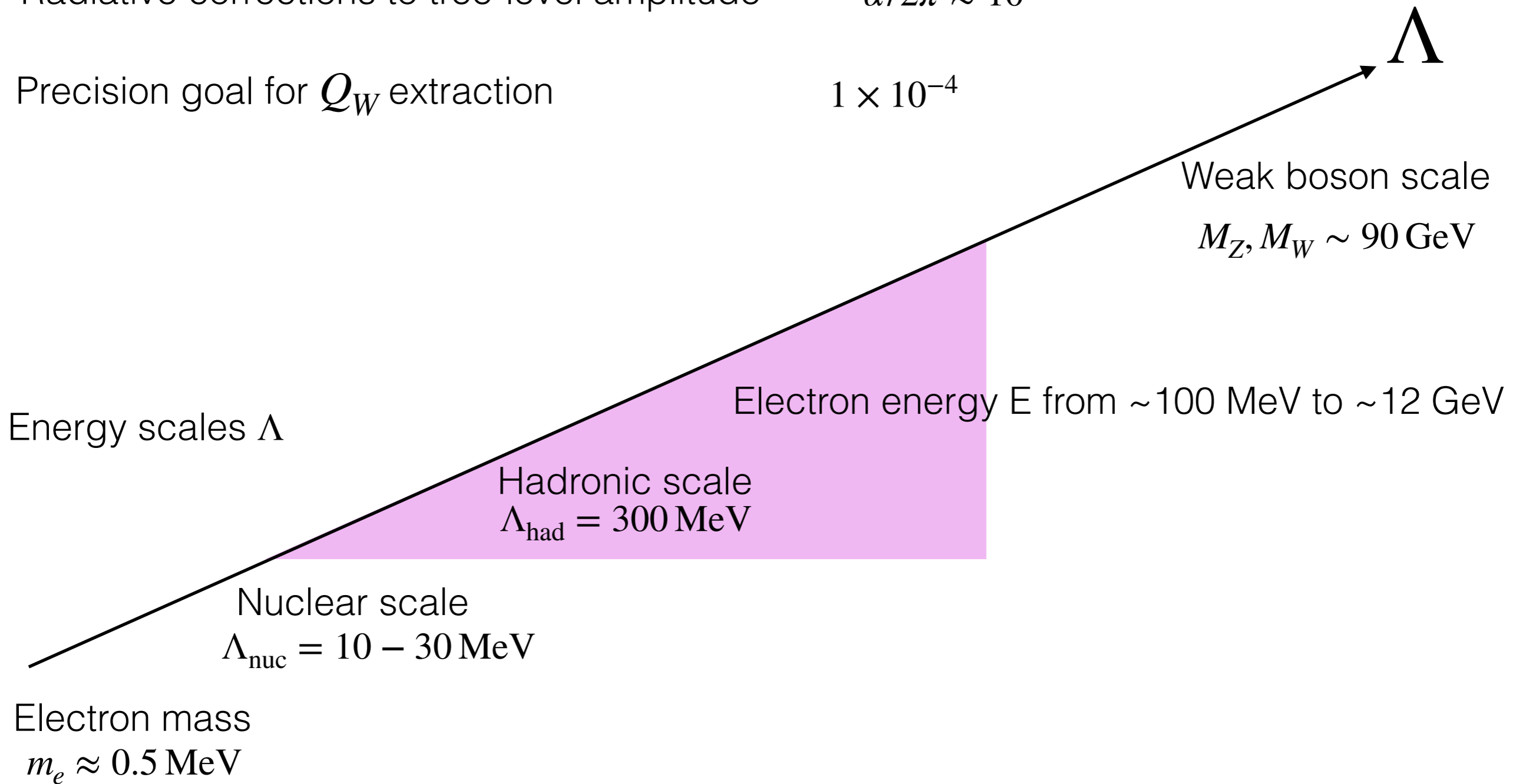


Leading energy behavior (E^0, E^1) sufficient

Dispersion theory of $\Box_{\gamma Z}$: relevant scales

Radiative corrections to tree-level amplitude $\sim \alpha/2\pi \approx 10^{-3}$

Precision goal for Q_W extraction 1×10^{-4}



Full energy dependence necessary!

Dispersion representation of $\square_{\gamma W/\gamma Z}$

γZ -box:

$$\text{Even: } \square_{\gamma Z}^A(E) = \frac{1}{2\pi M E} \int_0^\infty dQ^2 \frac{\nu_e(Q^2) \alpha(Q^2)}{1 + Q^2/M_Z^2} \int_0^\infty \frac{d\nu}{\nu} \left[\ln \left| \frac{E + E_m}{E - E_m} \right| + \frac{\nu}{2E} \ln \frac{|E^2 - E_m^2|}{E_m^2} \right] F_3^{\gamma Z}$$

$$\text{Odd: } \square_{\gamma Z}^V(E) = \frac{\alpha}{\pi} \int_0^\infty \frac{dQ^2}{1 + \frac{Q^2}{M_Z^2}} \int_0^\infty d\nu \left\{ \left[\frac{1}{2E} \ln \left| \frac{E + E_m}{E - E_m} \right| - \frac{1}{E_m} \right] \frac{F_1^{\gamma Z}}{M E} + \frac{F_2^{\gamma Z}}{2E\nu E_m} \right.$$

$$\left. + \left[\left(1 - \frac{Q^2}{4E^2} \right) \ln \left| \frac{E + E_m}{E - E_m} \right| + \frac{\nu}{E} \ln \frac{|E^2 - E_m^2|}{E_m^2} \right] \frac{F_2^{\gamma Z}}{\nu Q^2} \right\}$$

$$E_m = \frac{\nu + \sqrt{\nu^2 + Q^2}}{2}$$

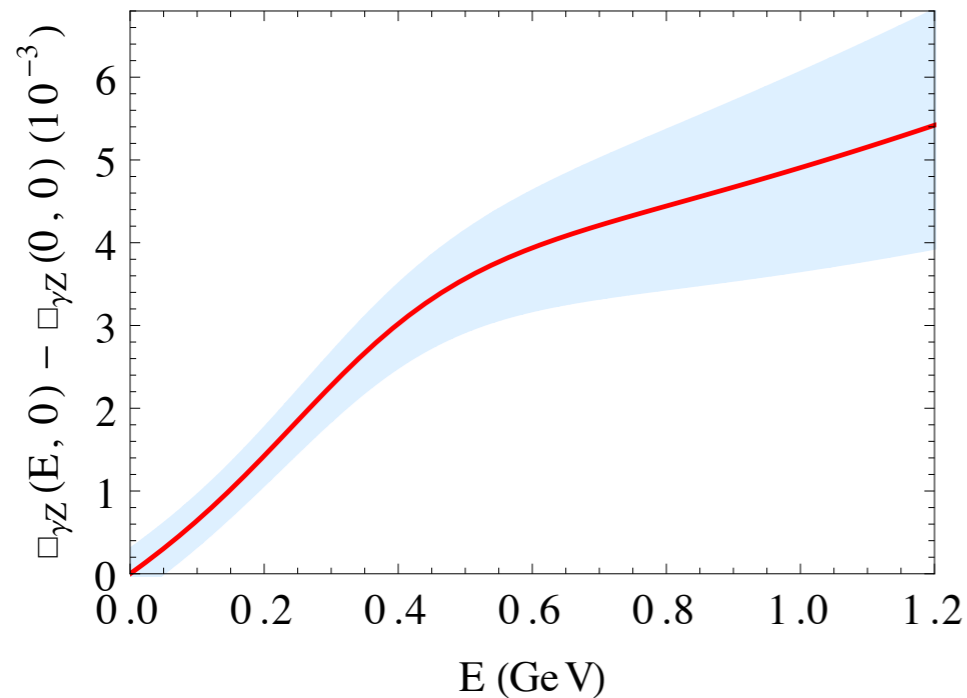
$$\gamma W\text{-box: } \square_{\gamma W}^{\text{even}} = \frac{\alpha}{\pi M} \int_0^\infty dQ^2 \int_{\nu_{thr}}^\infty d\nu \frac{\nu + 2q}{\nu(\nu + q)^2} F_3^{(0)}(\nu, Q^2) + O(E^2)$$

$$\square_{\gamma W}^{\text{odd}} = \frac{2\alpha E}{3\pi M} \int_0^\infty dQ^2 \int_{\nu_{thr}}^\infty d\nu \frac{\nu + 3q}{\nu(\nu + q)^3} F_3^{(1)}(\nu, Q^2) + O(E^3)$$

Only E = 0 (EVEN) results were included in the original RC analysis of Marciano & Sirlin

Energy dependence of the γZ -box

Reference value: 1-loop SM $Q_W^{p,SM} = 0.0713(7)$ Erler, Ferro-Hernandez, arXiv:1712.09146



MG, Horowitz, PRL 102 (2009) 091806;

Nagata, Yang, Kao, PRC 79 (2009) 062501;

Tjon, Blunden, Melnitchouk, PRC 79 (2009) 055201;

Zhou, Nagata, Yang, Kao, PRC 81 (2010) 035208;

Sibirtsev, Blunden, Melnitchouk, PRD 82 (2010) 013011;

Rislow, Carlson, PRD 83 (2011) 113007;

MG, Horowitz, Ramsey-Musolf, PRC 84 (2011) 015502;

Blunden, Melnitchouk, Thomas, PRL 107 (2011) 081801;

Rislow, Carlson PRD 85 (2012) 073002;

Blunden, Melnitchouk, Thomas, PRL 109 (2012) 262301;

Hall et al., PRD 88 (2013) 013011;

Rislow, Carlson, PRD 88 (2013) 013018;

Hall et al., PLB 731 (2014) 287;

MG, Zhang, PLB 747 (2015) 305;

Hall et al., PLB 753 (2016) 221;

MG, Spiesberger, Zhang, PLB 752 (2016) 135;

Erler, MG, Koshchii, Seng, Spiesberger, PRD100 (2019), 053007

Steep energy dependence observed - added strong motivation for P2 @ MESA

Talk by P. Blunden

$\sigma_{\gamma Z}$ by different groups (parametrizations & uncertainty treatments) closely agree
(Theory is) looking forward to new measurements of WMA at low energies!

γW box from dispersion relations:

Taming the uncertainties

Dispersion representation of $\square_{\gamma W}$

γW -box at zero energy

$$\square_{\gamma W}^{even} = \frac{\alpha}{\pi M} \int_0^\infty \frac{dQ^2 M_W^2}{M_W^2 + Q^2} \int_0^\infty \frac{d\nu}{\nu} \frac{\nu + 2q}{(\nu + q)^2} F_3^{(0)}(\nu, Q^2)$$

$$\square_{\gamma W}^{odd}(E = 0) = 0$$

Rewrite in terms of the first Nachtmann moment of F_3

$$M_3^{(0)}(1, Q^2) = \frac{4}{3} \int_0^1 dx \frac{1 + 2\sqrt{1 + 4M^2 x^2 / Q^2}}{(1 + \sqrt{1 + 4M^2 x^2 / Q^2})^2} F_3^{(0)}(x, Q^2) \quad x = \frac{Q^2}{2M\nu}$$

$$\square_{\gamma W}^{even} = \frac{3\alpha}{2\pi} \int_0^\infty \frac{dQ^2 M_W^2}{Q^2(M_W^2 + Q^2)} M_3^{(0)}(1, Q^2)$$

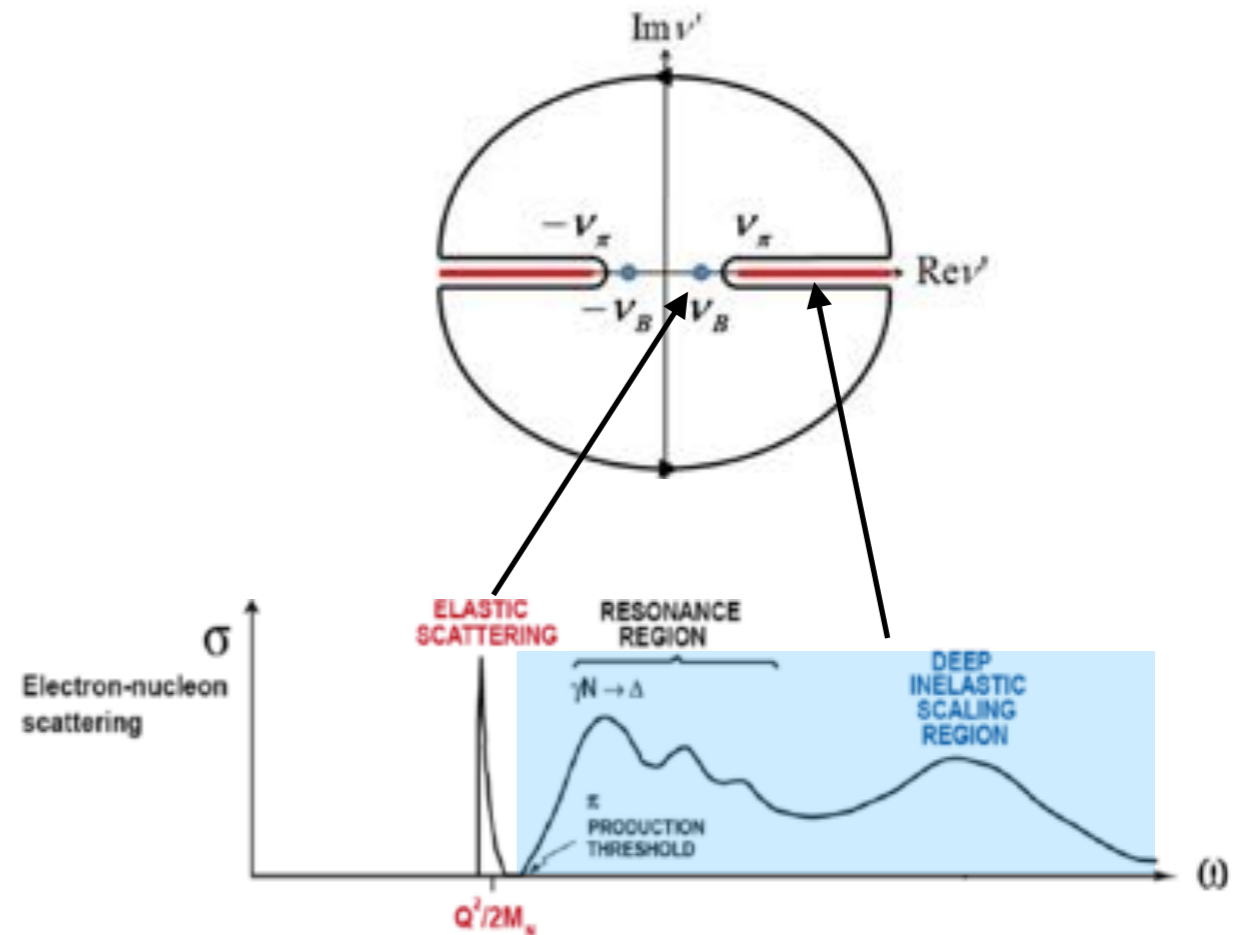
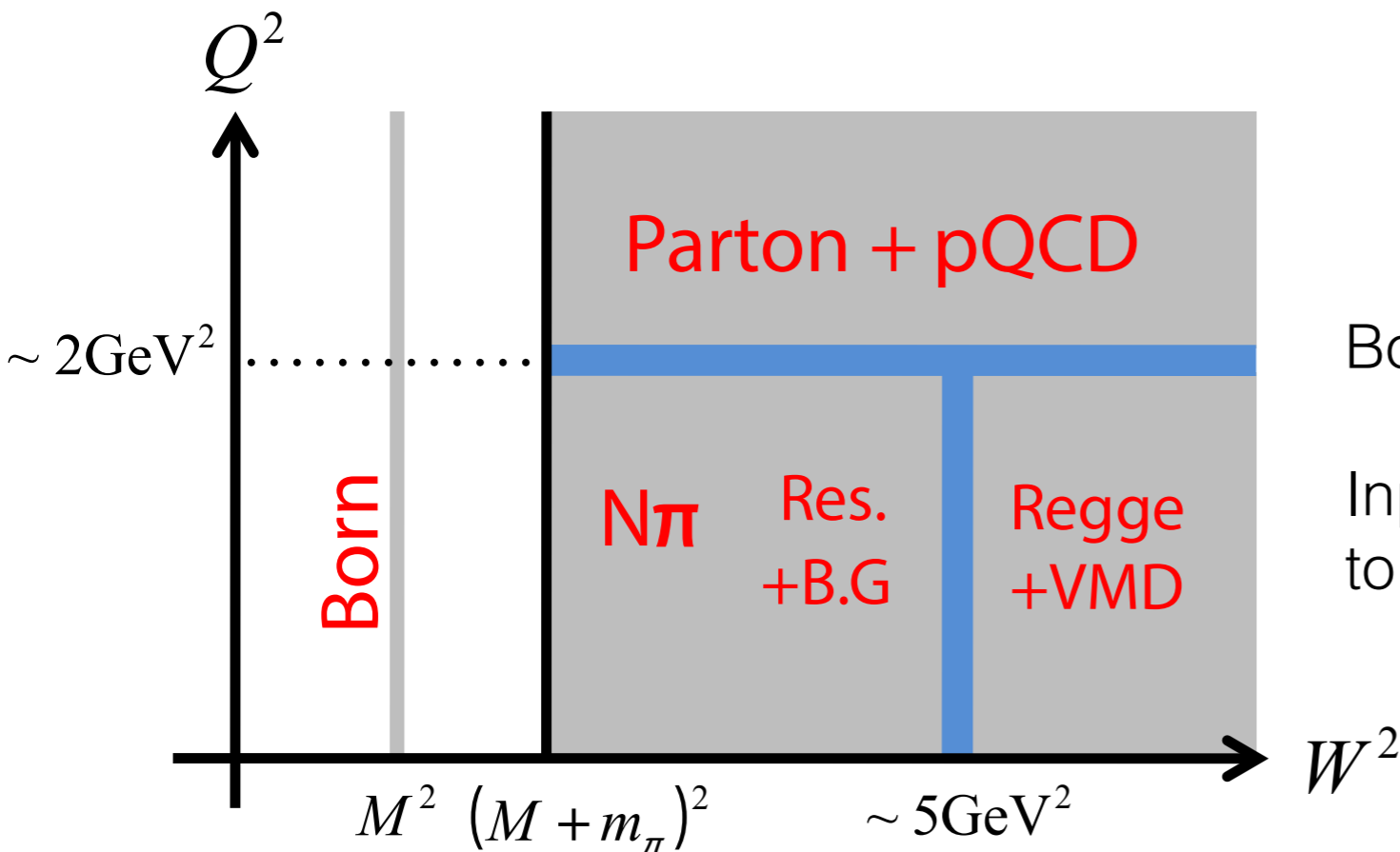
First Nachtmann moment of F_3 was studied extensively in 1980-1990's in $\nu/\bar{\nu}$ scattering in the context of the Gross-Llewellyn-Smith (GLS) sum rule.

In a nutshell: the use of those data allowed to improve the $\square_{\gamma W}$ calculation

Input into dispersion integral

Dispersion in energy: $W^2 = M^2 + 2M\nu - Q^2$
 scanning hadronic intermediate states

Dispersion in Q^2 :
 scanning dominant physics pictures



Boundaries between regions - approximate

Input in DR related (directly or indirectly)
 to experimentally accessible data

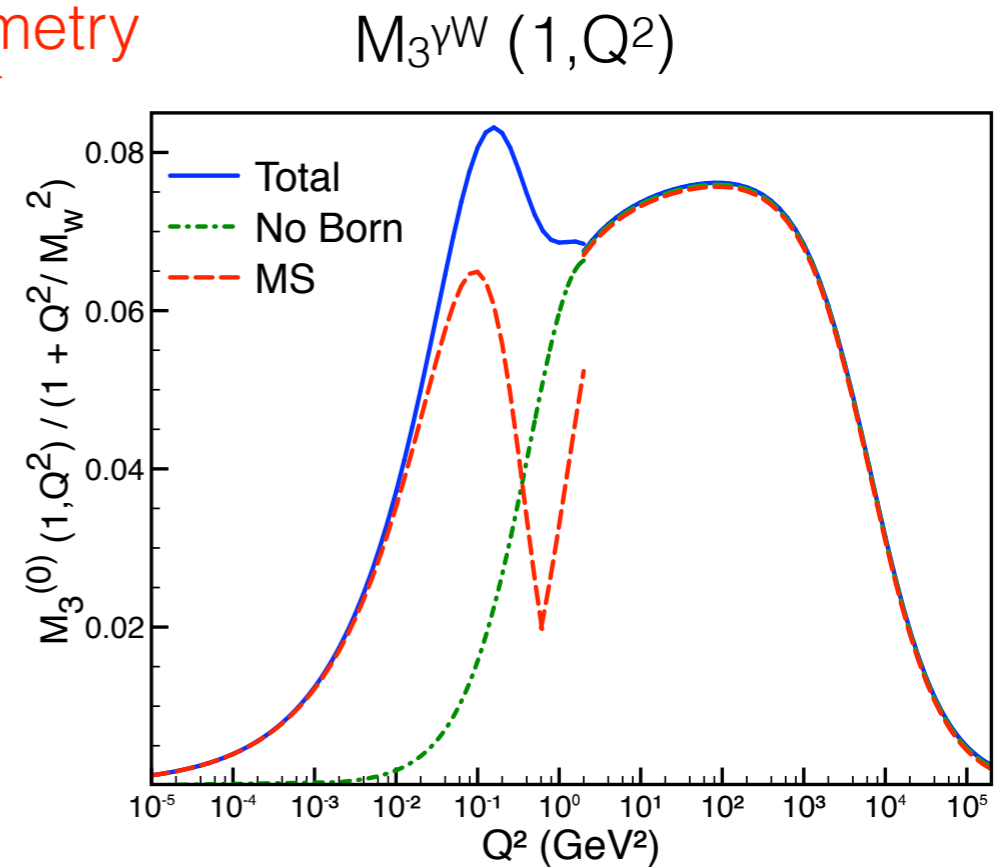
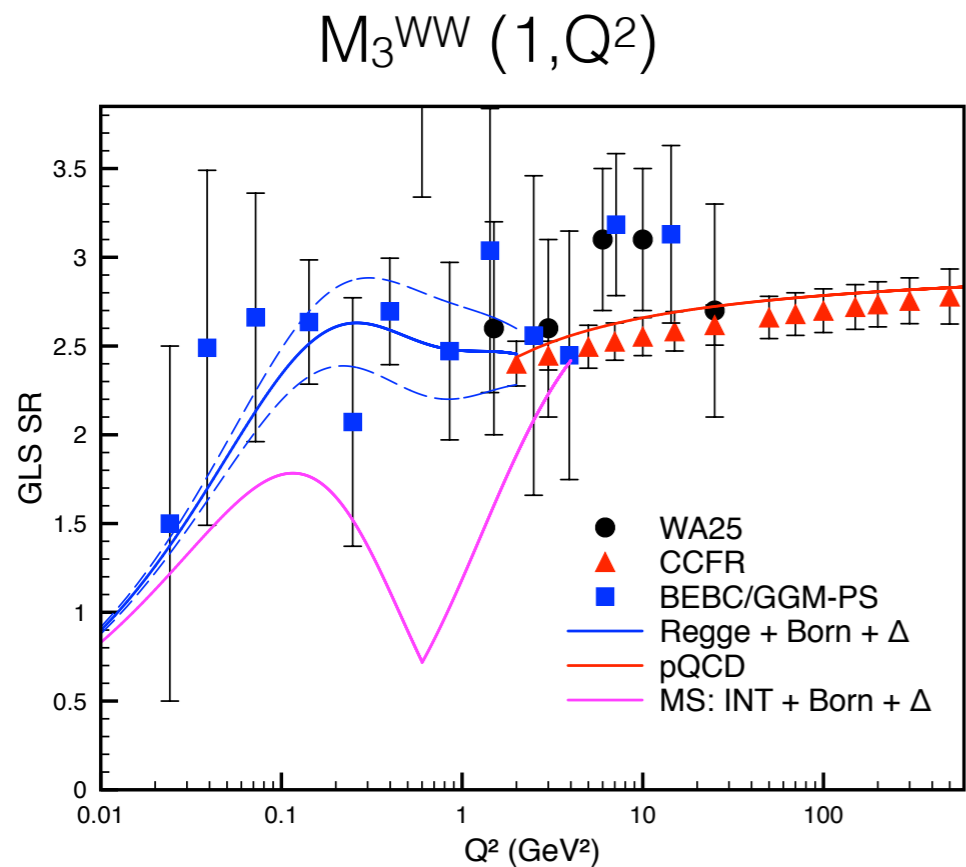
Input into dispersion integral

Unfortunately, no data can be obtained for $F_3^{\gamma W(0)}$

Data exist for the pure CC processes $\sigma^{\nu p} - \sigma^{\bar{\nu} p} \sim F_3^{\nu p} + F_3^{\bar{\nu} p} = u_v^p(x) + d_v^p(x)$

Gross-Llewellyn-Smith sum rule $\int_0^1 dx (u_v^p(x) + d_v^p(x)) = 3$

Isospin symmetry \longrightarrow



Previous calculation by Marciano&Sirlin '06:
No use of neutrino data, ad-hoc connection
of low and high scales

M & S : $\square_{\gamma W}^{(0)} = 0.00324 \pm 0.00018$
 New DR : $\square_{\gamma W}^{(0)} = 0.00379 \pm 0.00010$

First lattice QCD calculation of γW -box

For low $Q^2 \leq 2 \text{ GeV}^2$: direct lattice calculation of the generalized Compton tensor

Feng, MG, Jin, Ma, Seng 2003.09798

Main executors: Xu Feng (Peking U.), Lu-Chang Jin (UConn/RIKEN BNL)

Supercomputers: Blue Gene/Q Mira computer (Argonne, USA),

Tianhe 3 prototype (Tianjin, China)

$$\mathcal{H}_{\mu\nu}^{VA}(t, \vec{x}) \equiv \langle H_f(P) | T [J_{\mu}^{em}(t, \vec{x}) J_{\nu}^{W,A}(0)] | H_i(P) \rangle$$

$$M_{3\pi}^{\gamma W(0)}(Q^2) = -\frac{1}{6\sqrt{2}} \frac{Q}{m_{\pi}} \int d^4x \omega(Q, x) \varepsilon_{\mu\nu\alpha 0} x_{\alpha} \mathcal{H}_{\mu\nu}^{VA}(x)$$

Lattice setup:

5 LQCD gauge ensembles at physical pion mass

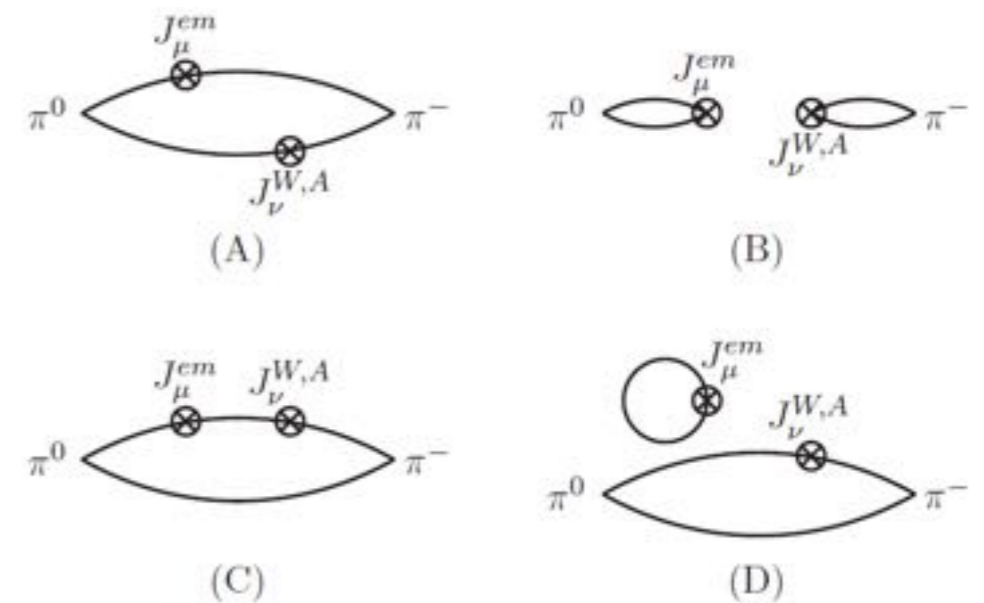
Generated by RBC and UKQCD collaborations

w. 2+1 flavor domain wall fermion

Ensemble	m_{π} [MeV]	L	T	a^{-1} [GeV]	N_{conf}	N_r	$\Delta t/a$
24D	141.2(4)	24	64	1.015	46	1024	8
32D	141.4(3)	32	64	1.015	32	2048	8
32D-fine	143.0(3)	32	64	1.378	71	1024	10
48I	135.5(4)	48	96	1.730	28	1024	12
64I	135.3(2)	64	128	2.359	62	1024	18

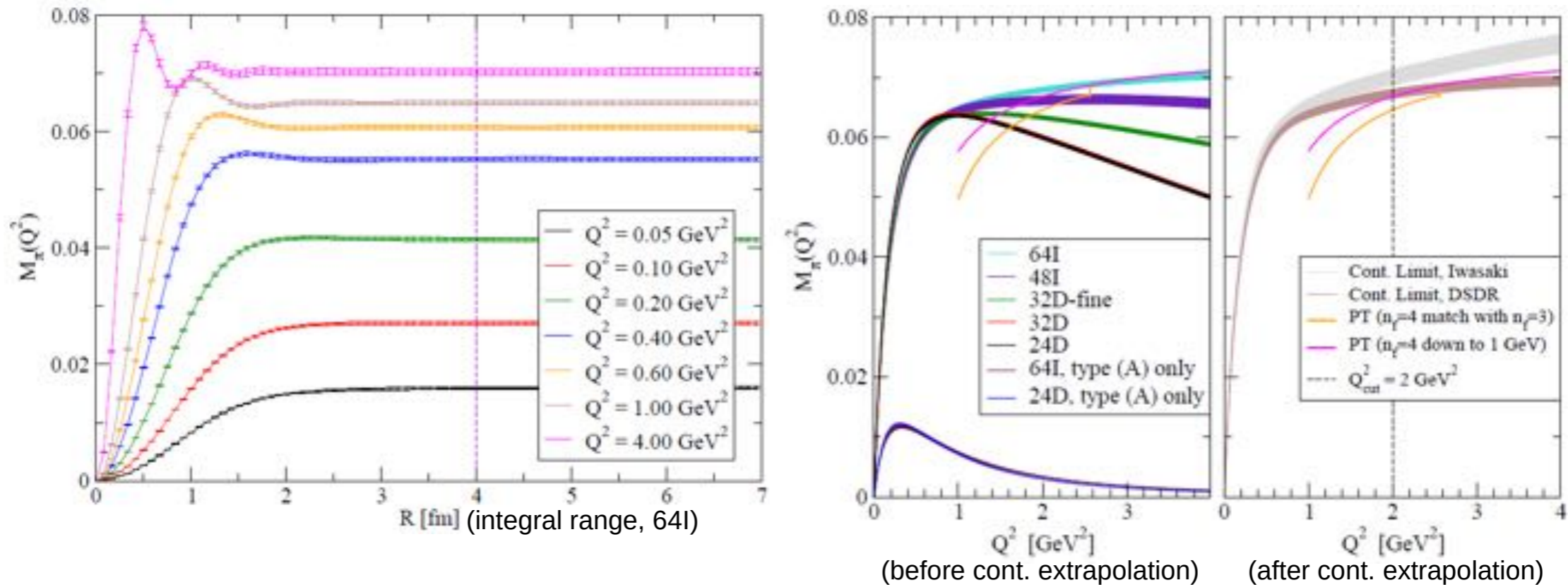
Blue: DSDR

Red : Iwasaki



Quark contraction diagrams

First lattice QCD calculation of γW -box



Estimate of major systematic effects:

- **Lattice discretization effect:** Estimated using the discrepancy between DSDR and Iwasaki
- **pQCD calculation:** Estimated from the difference between 3-loop and 4-loop results
- **Higher-twist effects at large Q^2 :** Estimated from lattice calculation of type (A) diagrams

Final result: $\square_{\gamma W}^{VA} \Big|_{\pi} = 2.830(11)_{\text{stat}}(26)_{\text{syst}} \times 10^{-3}$

Significant reduction of the uncertainty! $\delta : 0.0334(10)_{\text{LEC}}(3)_{\text{HO}} \rightarrow 0.0332(1)_{\gamma W}(3)_{\text{HO}}$

Cleanest way to access V_{ud} theoretically: $|V_{ud}| = 0.9740(28)_{\text{exp}}(1)_{\text{th}}$

Next-gen experiments: aim at 1 o.o.m. exp. uncertainty improvement

Implications for the free nucleon γW -box

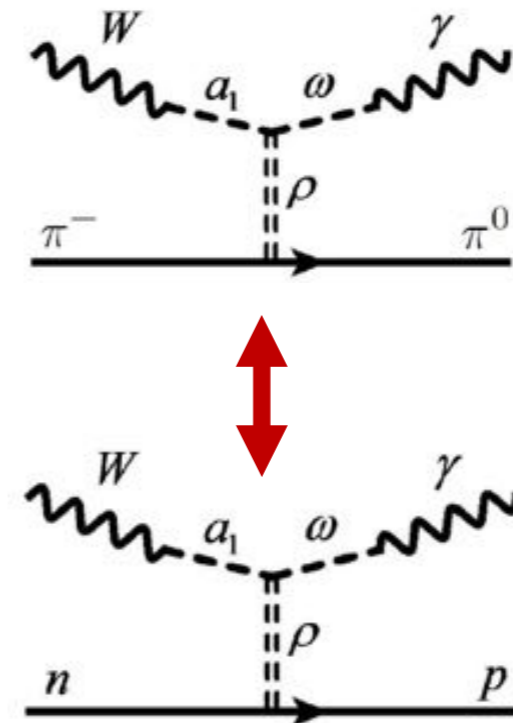
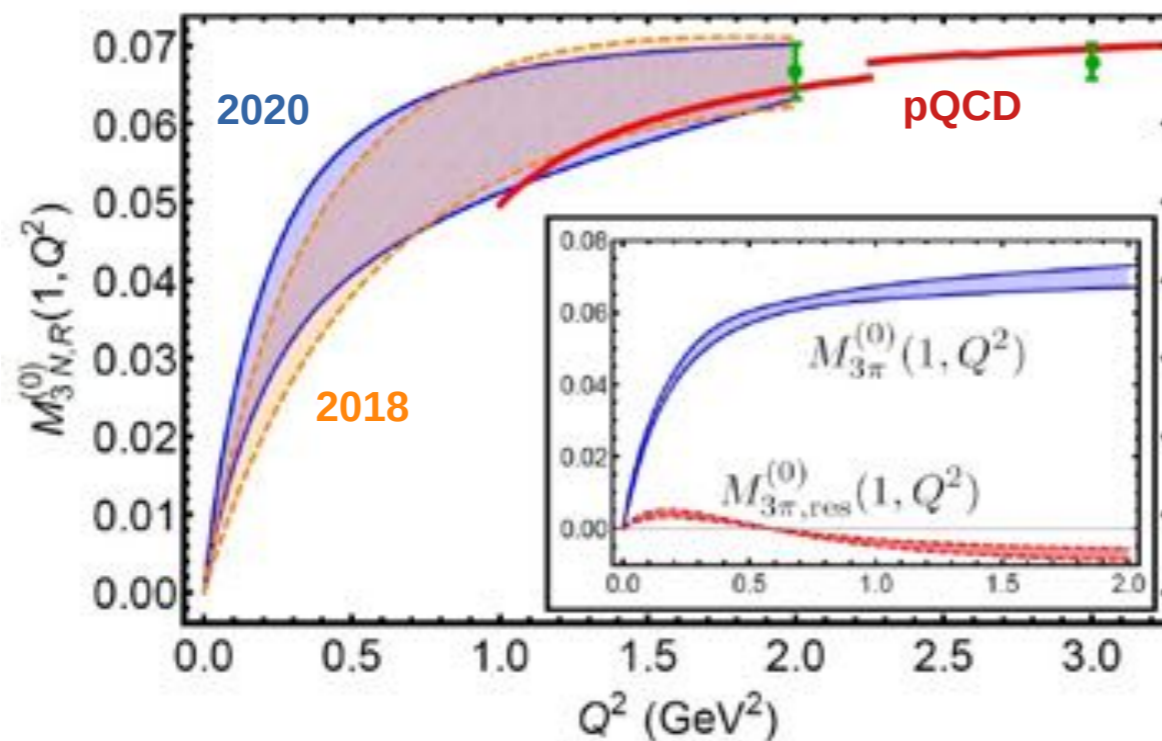
Main uncertainty of the DR calculation of the free neutron γW -box:

Poorly constrained parameters of the Regge contribution which dominates the Nachtmann moment at $Q^2 \sim 1 - 2 \text{ GeV}^2$

Use the Regge universality and a body of $\pi\pi$, πN , NN scattering data.

$$\frac{T_{W^+ + \pi^- \rightarrow \gamma + \pi^0}^\rho}{T_{W^+ + n \rightarrow \gamma + p}^\rho} = \frac{T_{\pi\pi \rightarrow \pi\pi}^\rho}{T_{\pi N \rightarrow \pi N}^\rho} = \frac{T_{\pi N \rightarrow \pi N}^\rho}{T_{NN \rightarrow NN}^\rho}$$

Seng, MG, Feng, Jin, 2003.11264



Independent confirmation of the empirical DR result AND uncertainty

$$\Delta_R^V = 0.02467(22)_{\text{DR}} \rightarrow 0.02477(24)_{\text{LQCD+DR}}$$

V_{ud} extraction and CKM unitarity

Marciano, Sirlin 2006: $\Delta_R^V = 0.02361(38) \rightarrow |V_{ud}| = 0.97420(10)_{Ft}(18)_{RC}$

DR (Seng et al. 2018): $\Delta_R^V = 0.02467(22) \rightarrow |V_{ud}| = 0.97370(10)_{Ft}(10)_{RC}$

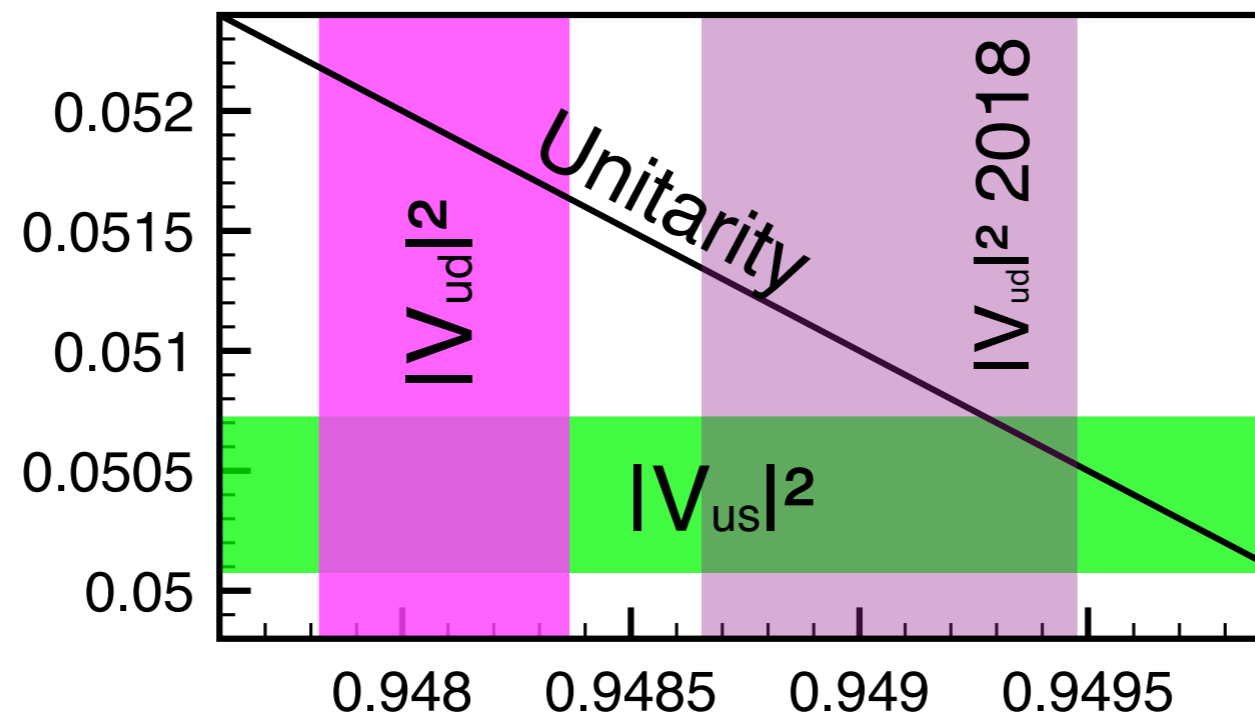
In July 2019 Czarnecki, Marciano and Sirlin published an update **1907.06737**

$$\Delta_R^V = 0.02421(32) \rightarrow |V_{ud}| = 0.97391(10)_{Ft}(15)_{RC}$$

DR (Shiells, Blunden, Melnitchouk) **2012.01580** — closely agrees with Seng et al.

$$\Delta_R^V = 0.02472(18) \rightarrow |V_{ud}| = 0.97368(14)$$

With new RC $\sim 3\sigma$ unitarity deficit (a.k.a. Cabibbo angle anomaly)



V_{ud} in Presence of Nuclear Structure Effects

0⁺-0⁺ nuclear decays $|V_{ud}|^2 = \frac{2984.43s}{\mathcal{F}t(1 + \Delta_R^V)}$

$$\Delta_R^V = \frac{\alpha}{2\pi} \left[3 \ln \frac{M_Z}{M} + \ln \frac{M_Z}{M_W} \right] + 2\Box_{\gamma W}^V$$

General structure of RC for nuclear decay

$$ft(1 + RC) = Ft(1 + \delta'_R)(1 - \delta_C + \delta_{NS})(1 + \Delta_R^V)$$

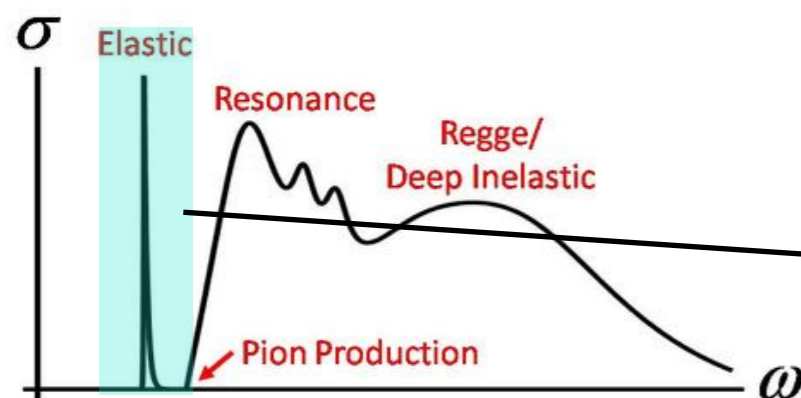


NS correction reflects this extraction of the free box

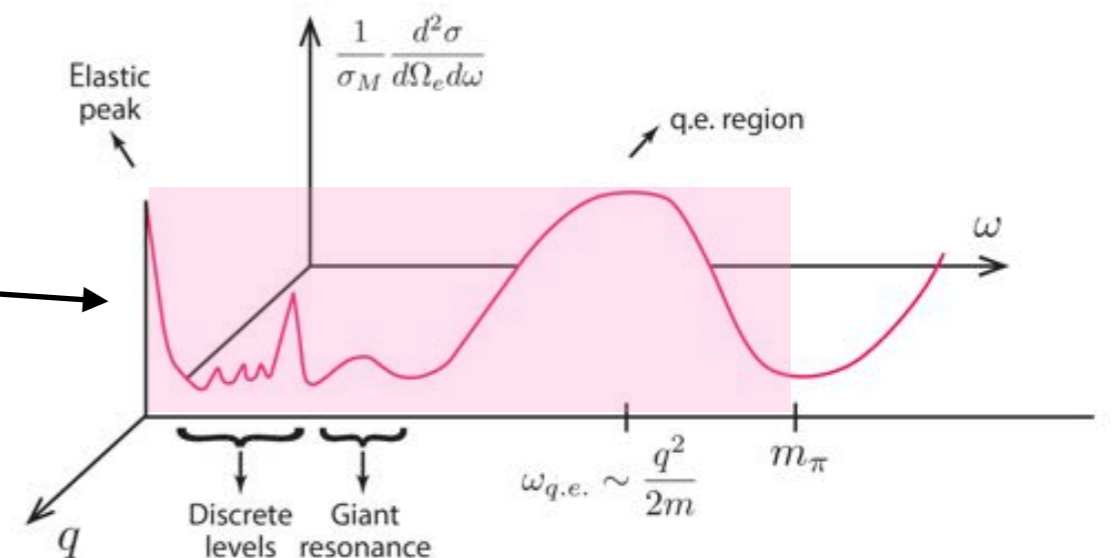
$$\Box_{\gamma W}^{VA, \text{Nucl.}} = \Box_{\gamma W}^{VA, \text{free n}} + \left[\Box_{\gamma W}^{VA, \text{Nucl.}} - \Box_{\gamma W}^{VA, \text{free n}} \right]$$

Nuclear modification in the lower part of the spectrum

Input in the DR for the universal RC



Input in the DR for the RC on a nucleus



Nuclear Structure Modification

C-Y Seng, MG, M J Ramsey-Musolf, arXiv: 1812.03352

MG, arXiv: 1812.04229

δ_{NS} from DR with energy dependence averaged over the spectrum

$$\delta_{NS} = \frac{2\alpha}{\pi NM} \int_0^{1 \text{ GeV}^2} dQ^2 \int_{\nu_{thr}}^{\nu_{\pi}} \frac{d\nu}{\nu} \left[\frac{\nu + 2q}{(\nu + q)^2} \left(F_3^{(0) Nucl.} - F_3^{(0), B} \right) + \frac{2\langle E \rangle}{3} \frac{\nu + 3q}{(\nu + q)^3} F_3^{(-) Nucl.} \right]$$

Hardy & Towner 1994 on: ad hoc description by nuclear quenching of spin operators

Compare the effect on the average Ft value:

HT value 2018:

$$\mathcal{F}t = 3072.1(7)s$$

Old estimate:

$$\delta\mathcal{F}t = - (1.8 \pm 0.4)s + (0 \pm 0)s$$

New estimate:

$$\delta\mathcal{F}t = - (3.5 \pm 1.0)s + (1.6 \pm 0.5)s$$

Two 2σ corrections that cancel each other;

The cancellation is delicate: the two terms are highly correlated

Larger E-dep. term will correspond to a smaller negative E-indep. term and vv.

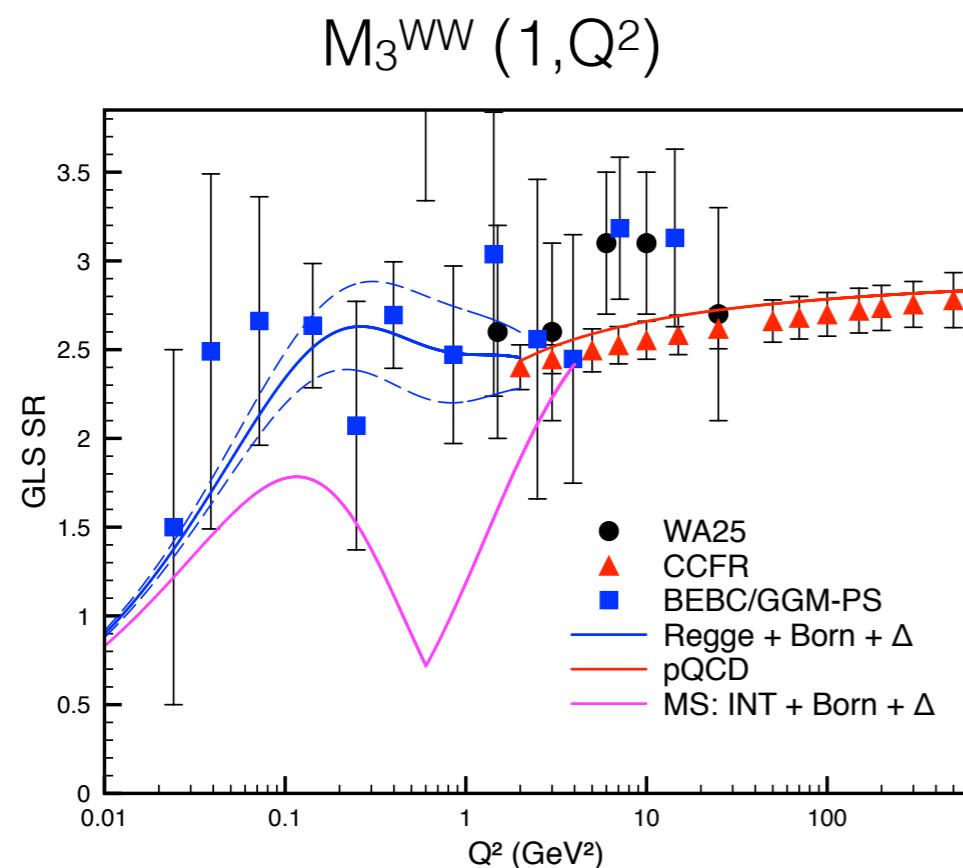
Conservative uncertainty estimate: 100%

$$\mathcal{F}t = (3072 \pm 2)s$$

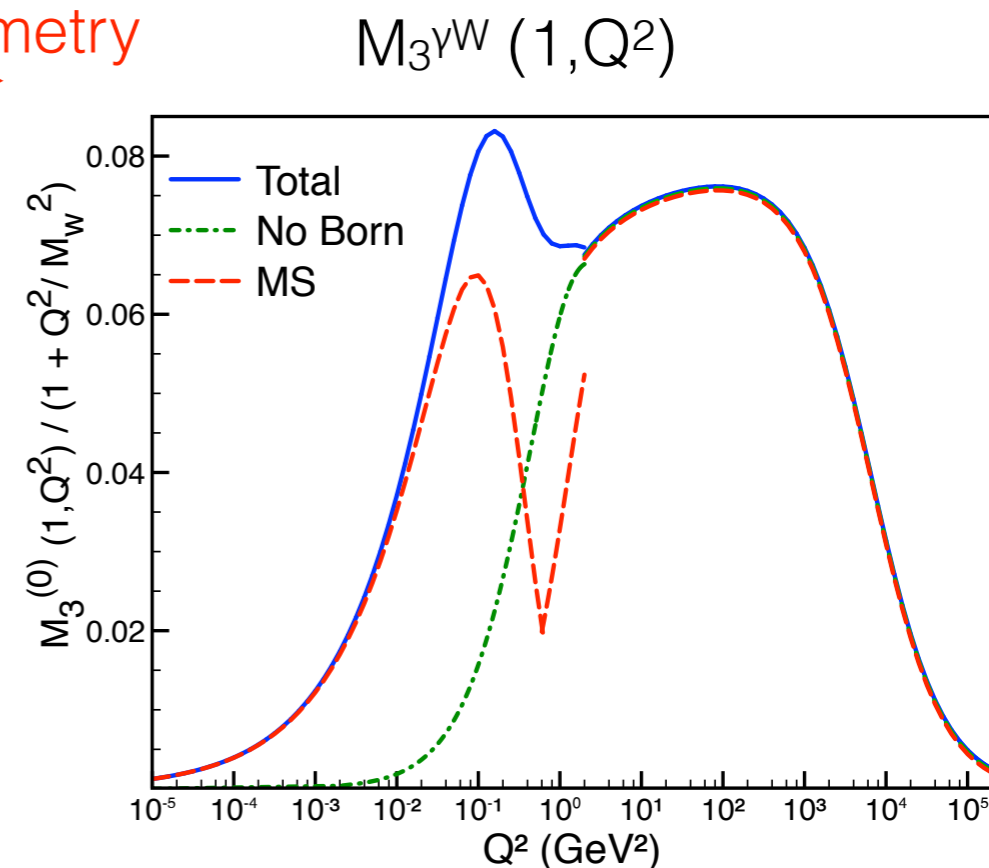
Future developments

Improvement in $\square_{\gamma Z}$ and $\square_{\gamma W}$ achieved thanks to:

Use of dispersion relations + data + isospin symmetry + lattice QCD — TEST!



Isospin symmetry



Each calculation of $\square_{\gamma Z}$ and $\square_{\gamma W}$ contains a prediction for EW structure functions

Can and should be tested by confronting with new, high quality data!

NC program @ JLab, EIC

Neutrino program @ DUNE (bubble chambers, H/D targets)

Snowmass white papers arXiv: 2203.11298, 2203.11319

Nuclear modification of structure functions and their moments

New Data: may be a challenge!

Significant interest in new/better data on PV structure function F_3

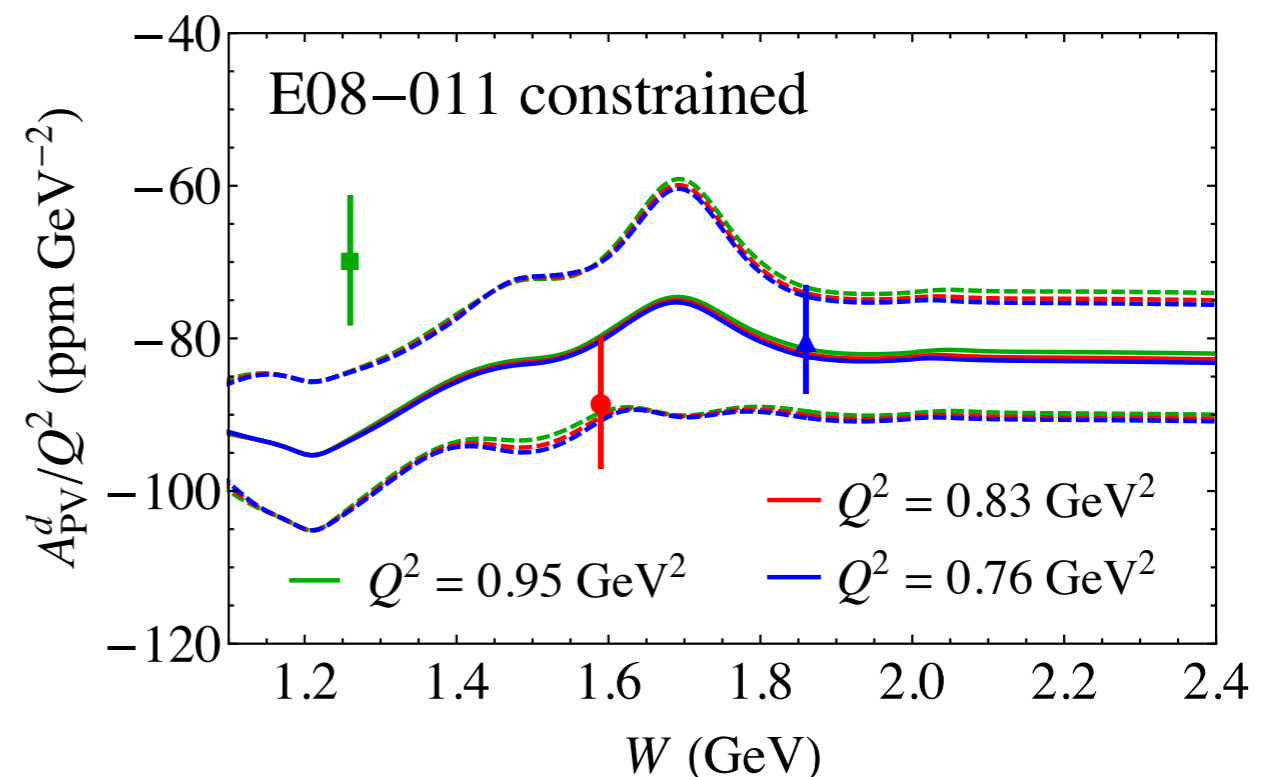
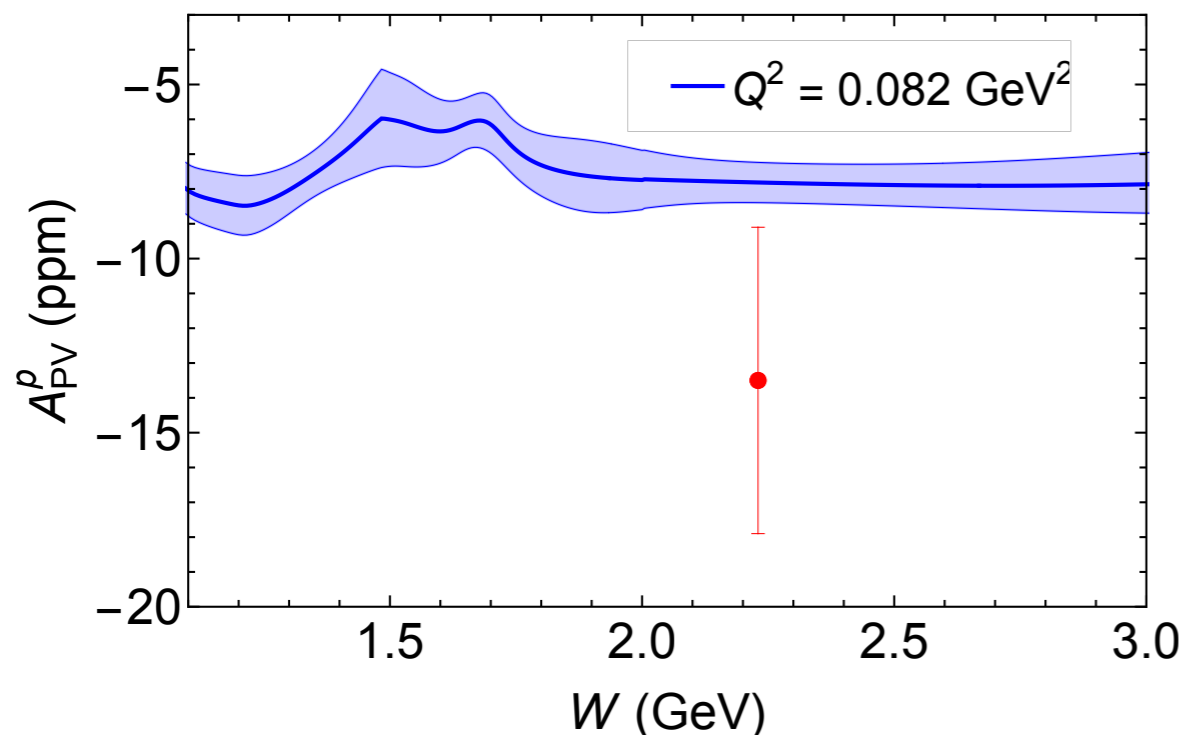
This SF is C-parity odd: opposite sign for particles and antiparticles

In neutrino scattering: need ν and $\bar{\nu}$ beam — challenging (systematics)

In PVES can be deduced from e^+/e^- data? However suppressed by small g_V^e

In general: how should we use new data?

Recent example: inelastic PV asymmetry in PVDIS and QWeak



Conclusions

- Sensitive tests of SM and beyond with PVES and beta decays
- Need for a reliable calculation of box diagrams for a 10^{-4} precision goal
- Consistent dispersive treatment of the γW -box correction to neutron and nuclear β -decay, and the γZ -box correction to PVES and weak charges
- pQCD, hadronic and nuclear contributions treated in a unified framework
- γW -box calculation confirmed by a first-ever direct lattice calculation
- New more precise data on EW structure functions $F_{1,2,3}^{\gamma Z}$ (JLab, EIC)
- New more precise data on EW structure functions $F_{1,2,3}^{WW}$ (DUNE)
- Test of Lattice QCD predictions for moments at low Q^2
- Test of pQCD running at high Q^2
- What if model and data disagree?