Uncertainties in the Quasielastic and Resonance Regions due to Nuclear Effects

Raúl González Jiménez

Grupo de Física Nuclear,
Universidad Complutense & IPARCOS,
Madrid, Spain

MINISTERIO DE CIENCIA E INNOVACIÓN
UNIÓN EUROPEA
FONDO EUROPEO DE DESARROLLO REGIONAL
"Una manera de hacer Europa"
AGENCIA ESTATAL DE INVESTIGACIÓN

Grant PID2021-127098NA-I00 funded by MCIN/AEI/10.13039/501100011033 and by "ERDF A way of making Europe".

IPARCOS

gfn UCM Grupo de Física Nuclear

Theoretical Physics Uncertainties to Empower Neutrino Experiments

Oct 30 to Nov 3, 2023, INT, Seattle, USA

https://indico.fnal.gov/event/57030/

Overview

(Discussed in this talk)

Nuclear Effects as Sources of Uncertainties:

+ Impulse Approximation.

+ Pauli blocking.

+ Final-state interactions: elastic and inelastic.
  Distorted versus plane waves.

+ In-medium modification of the resonance properties.

+ Beyond the impulse approximation: two body currents.

Overview

(Discussed in this talk)
Nuclear Effects as Sources of Uncertainties:
+ Impulse Approximation.

+ Pauli blocking.

+ Final-state interactions: elastic and inelastic.
Distorted versus plane waves.

+ In-medium modification of the resonance properties.

+ Beyond the impulse approximation: two body currents.

(Not discussed in this talk)
Uncertainties in lepton-induced single pion production off (free) nucleon:
+ Unitarity, pion-nucleon FSI.

+ Unphysical predictions of Low-Energy models in the High-W regime (W > 2 GeV).

+ Form factors and couplings, in particular, in the axial current.

+ The free neutron does not exist, one needs to model deuterium.

The **nuclear response** in the intermediate energy regime

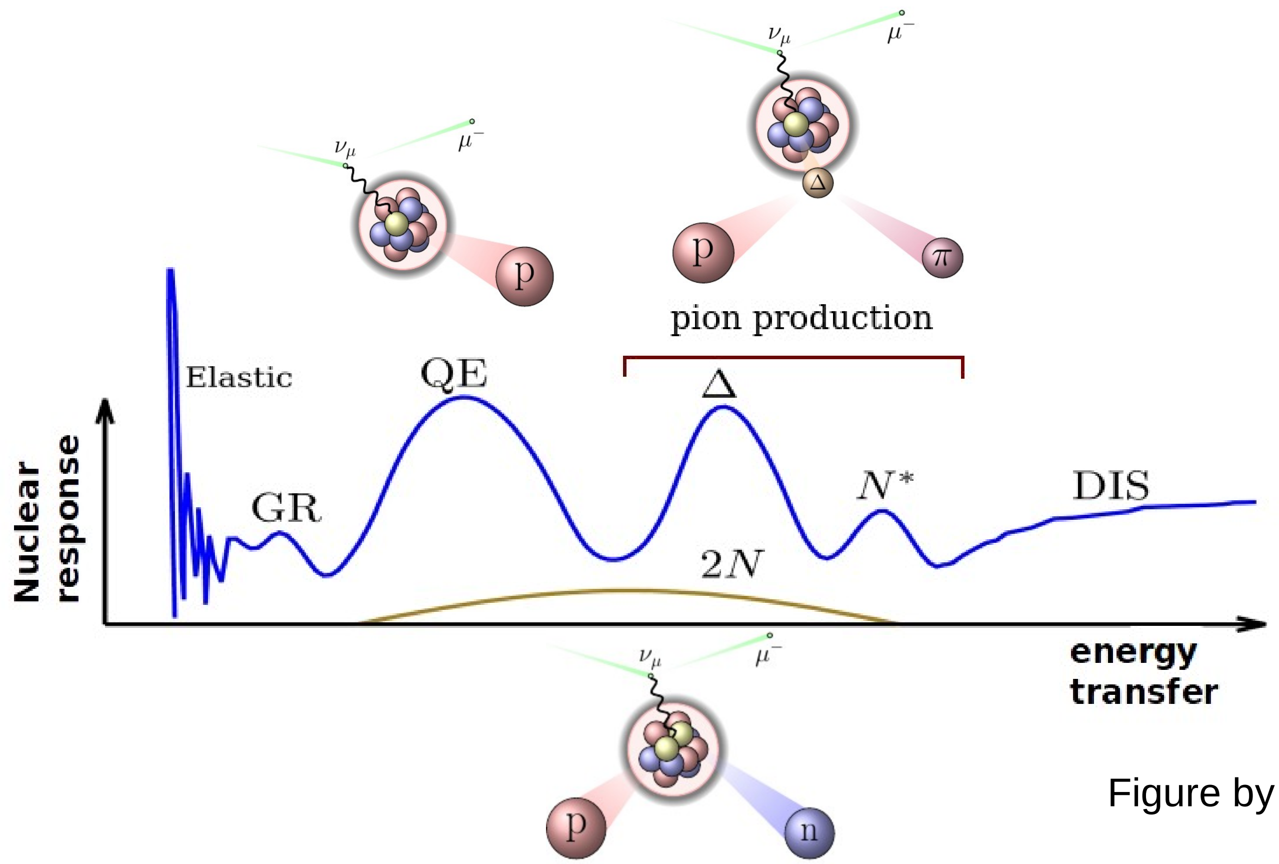
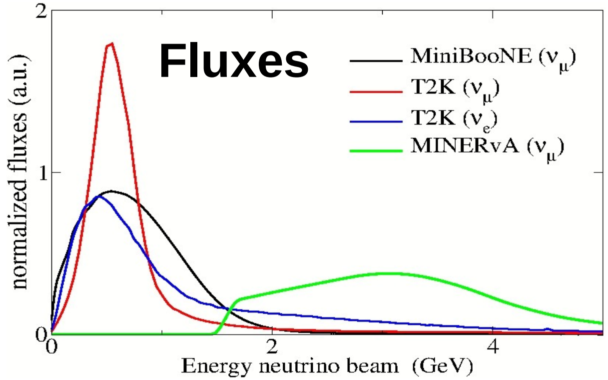
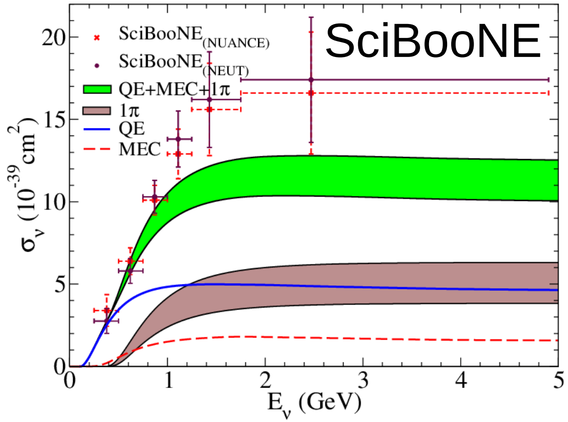


Figure by T. Van Cuyck

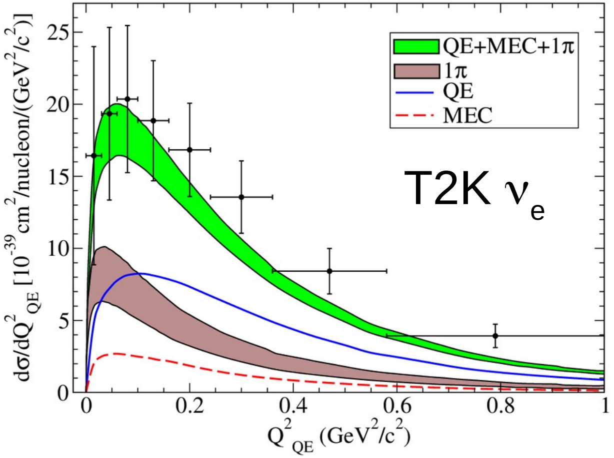
Reaction mechanisms and **neutrino fluxes**



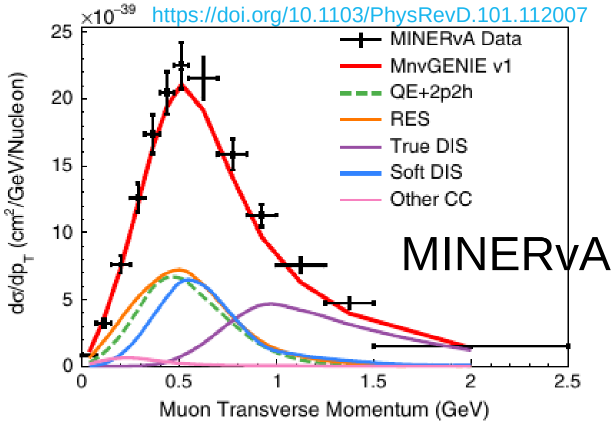
https://doi.org/10.1088/1361-6471/abb128



https://doi.org/10.1088/1361-6471/abb128



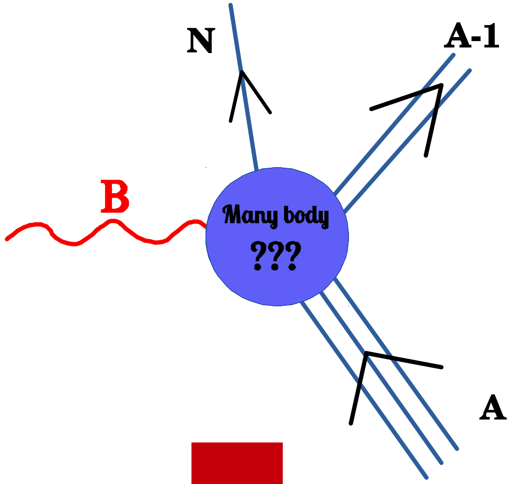
https://doi.org/10.1103/PhysRevD.101.112007



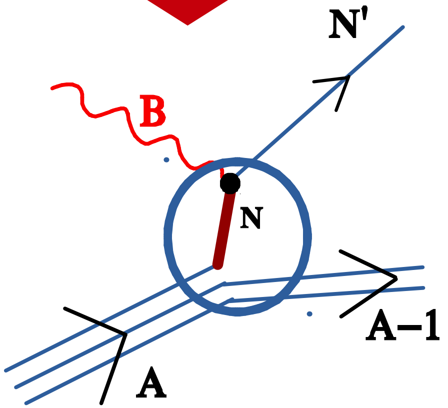
How do we model **quasielastic scattering** and **single-pion production?**

The **IMPULSE APPROXIMATION**

Quasielastic scattering within the IA



Impulse approximation

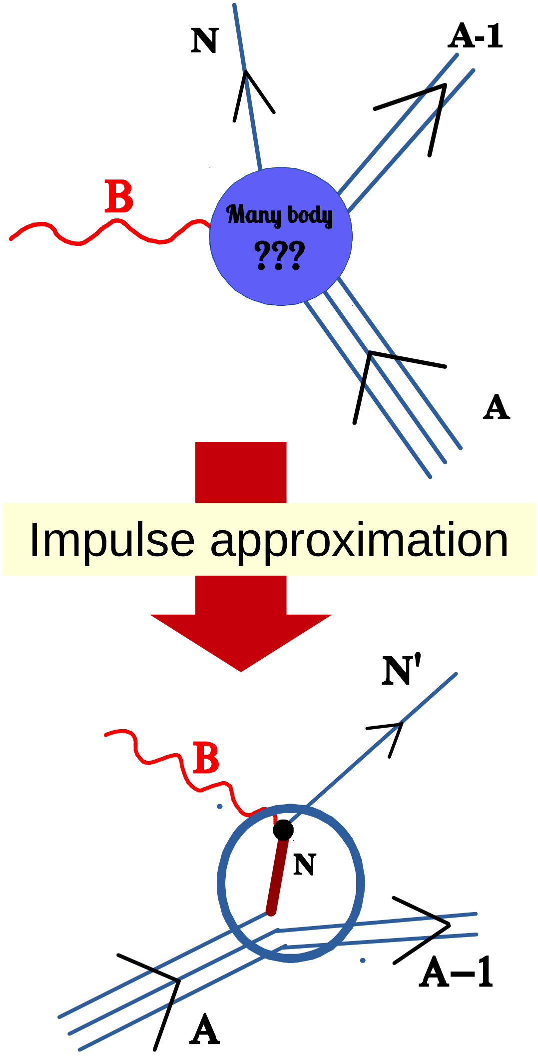


$$J^{\mu}_{had} = \langle N, A-1 | \hat{\mathcal{O}}^{\mu}_{many\text{-}body} | A \rangle$$

Impulse approximation

$$J^{\mu}_{had} = \int d\mathbf{p} \, \overline{\Psi}_F(\mathbf{p}+\mathbf{q}, \mathbf{p}_N) \; \mathcal{O}^{\mu}_{\text{one body}} \; \Psi_B(\mathbf{p})$$

Quasielastic scattering within the IA



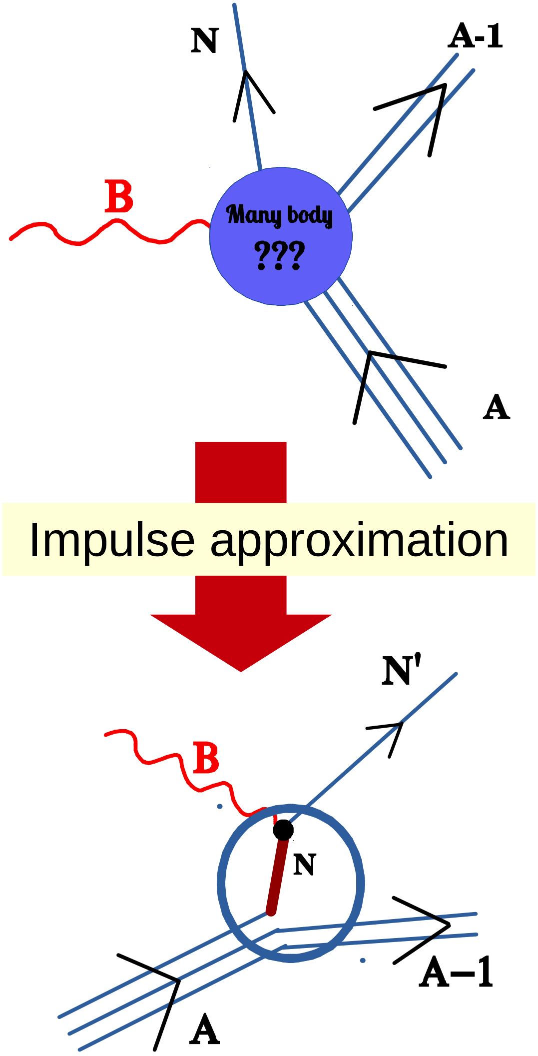
$$J^{\mu}_{had} = \langle N, A-1 | \hat{\mathcal{O}}^{\mu}_{many\text{-}body} | A \rangle$$

Impulse approximation

$$J^{\mu}_{had} = \int d\mathbf{p}\, \overline{\Psi}_F(\mathbf{p}+\mathbf{q}, \mathbf{p}_N)\ \mathcal{O}^{\mu}_{\text{one body}}\ \Psi_B(\mathbf{p})$$

from lepton-(free) nucleon interaction

Quasielastic scattering within the IA



$$J^{\mu}_{had} = \langle N, A-1 | \hat{\mathcal{O}}^{\mu}_{many\text{-}body} | A \rangle$$

Impulse approximation

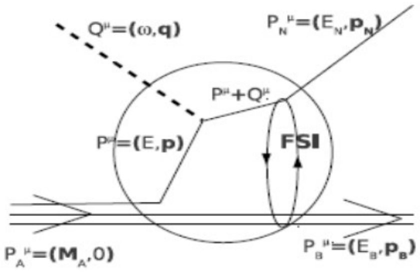
$$J^{\mu}_{had} = \int d\mathbf{p}\, \overline{\Psi}_F(\mathbf{p}+\mathbf{q}, \mathbf{p}_N)\ \mathcal{O}^{\mu}_{\text{one body}}\ \Psi_B(\mathbf{p})$$

from lepton-(free) nucleon interaction

Final and Bound single-particle wave functions

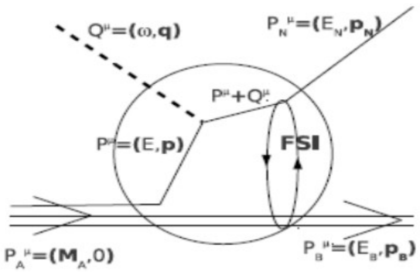
Key concept about the distorted-wave approach:

$$J^{\mu}_{had} = \int d\mathbf{p}\, \overline{\Psi}_F(\mathbf{p}+\mathbf{q}, \mathbf{p}_N)\ \mathcal{O}^{\mu}_{\text{one body}}\ \Psi_B(\mathbf{p})$$



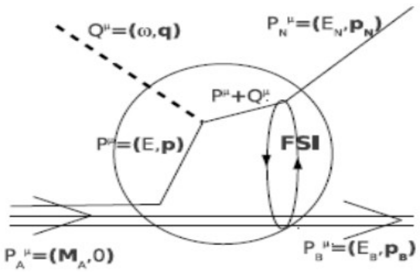
Key concept about the distorted-wave approach:

$$J^{\mu}_{had} = \int d\mathbf{p}\, \overline{\Psi}_F(\mathbf{p}+\mathbf{q}, \mathbf{p}_N)\ \mathcal{O}^{\mu}_{\text{one body}}\ \Psi_B(\mathbf{p})$$



Key concept about the distorted-wave approach:

$$J^{\mu}_{had} = \int d\mathbf{p}\, \overline{\Psi}_F(\mathbf{p}+\mathbf{q}, \mathbf{p}_N)\, \mathcal{O}^{\mu}_{\text{one body}}\, \Psi_B(\mathbf{p})$$

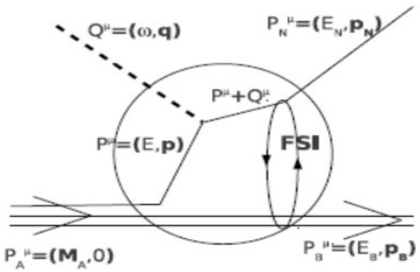


Solution of Dirac eq. with **real potentials**:

+ The **real part** of the potential accounts for the **distortion or "elastic FSI"**.
+ With this approach we can compare with **inclusive cross sections**.

Key concept about the distorted-wave approach:

$$J^{\mu}_{had} = \int d\mathbf{p}\, \boxed{\overline{\Psi}_F(\mathbf{p}+\mathbf{q}, \mathbf{p}_N)}\, \mathcal{O}^{\mu}_{\text{one body}}\, \Psi_B(\mathbf{p})$$



Solution of Dirac eq. with **real potentials**:

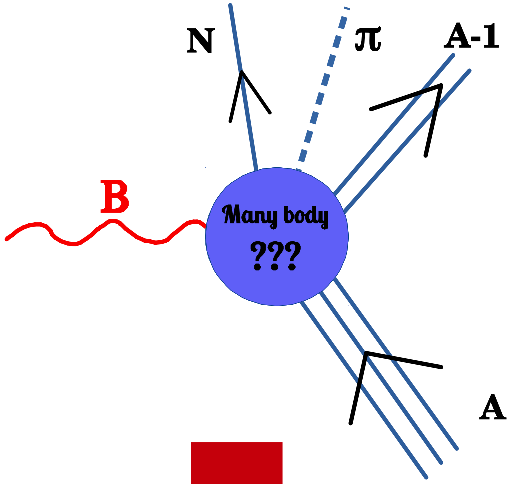
+ The **real part** of the potential accounts for the **distortion or "elastic FSI"**.
+ With this approach we can compare with **inclusive cross sections**.

Solution of Dirac eq. with **complex potentials**:

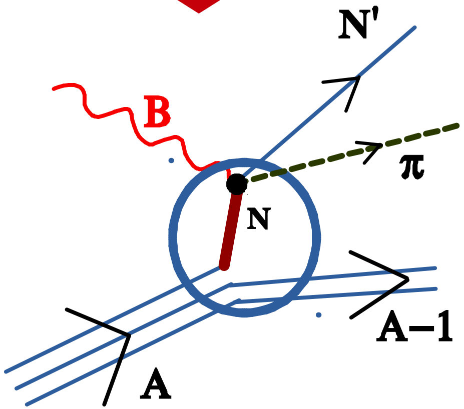
+ The **imaginary part removes the strength** that goes to inelastic channels.
+ With this approach we can compare with **exclusive* cross sections**.

(*) Exclusive: All particles detected, so e.g., in (e,e'p) experiments the missing energy must be below the two-nucleon emission threshold.

Single-Pion Production within the IA



Impulse approximation



$$J^{\mu}_{\text{had}} = \langle N, \pi, A-1 | \mathcal{O}^{\mu}_{\text{many-body}} | A \rangle$$

Impulse approximation

$$J^{\mu}_{\text{had}} \sim \overline{\Psi}_F \, \phi^*_\pi \, \mathcal{O}^{\mu}_{\text{single-nucleon}} \, \Psi_B$$

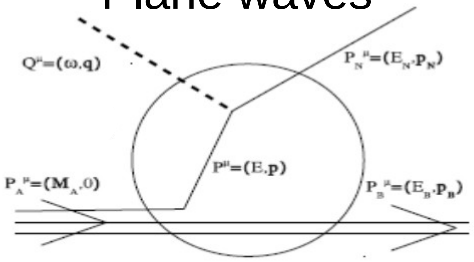
Pion wave function

Pauli blocking and elastic FSI

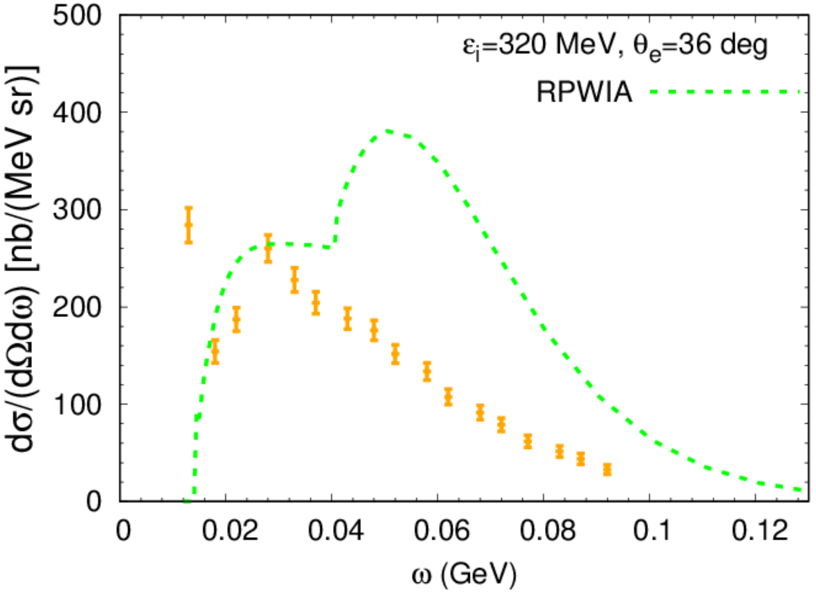
Pauli blocking and elastic FSI

Inclusive electron scattering **at low q:**

Plane waves



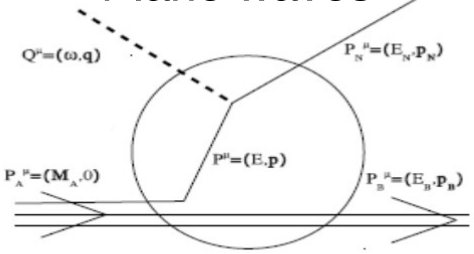
Phys. Rev. C 100 045501 (2019)



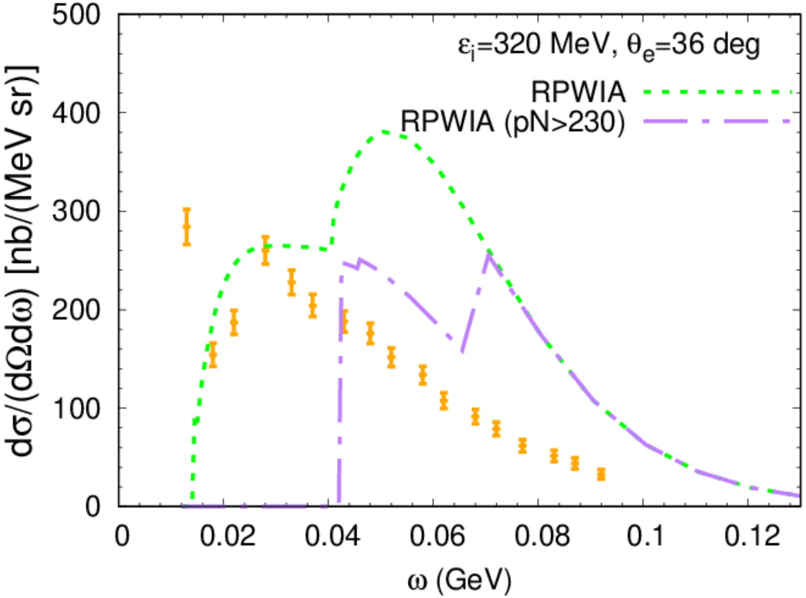
Pauli blocking and elastic FSI

Inclusive electron scattering **at low q:**

Plane waves



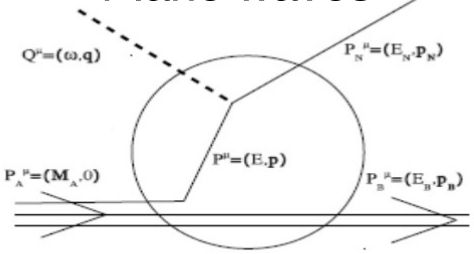
Phys. Rev. C 100 045501 (2019)



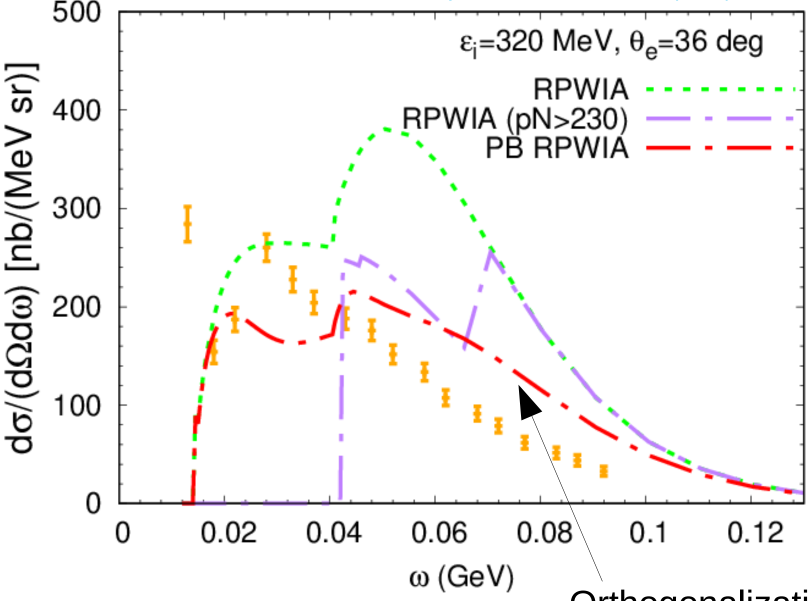
Pauli blocking and elastic FSI

Inclusive electron scattering **at low q:**

Plane waves



Phys. Rev. C 100 045501 (2019)



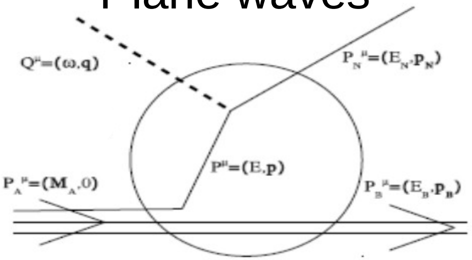
Orthogonalization

$$|\Psi^{s_N}(\mathbf{p}_N)\rangle = \left|\psi_{pw}^{s_N}(\mathbf{p}_N)\right\rangle - \sum_{\kappa,m_j}\left[C_\kappa^{m_j,s_N}(\mathbf{p}_N)\right]^\dagger \left|\psi_\kappa^{m_j}\right\rangle$$

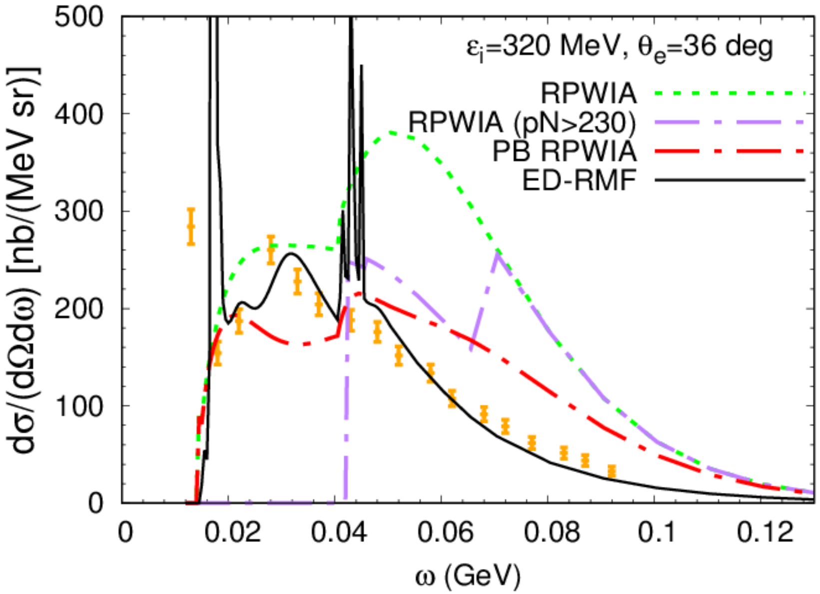
Pauli blocking and elastic FSI

Inclusive electron scattering **at low q:**

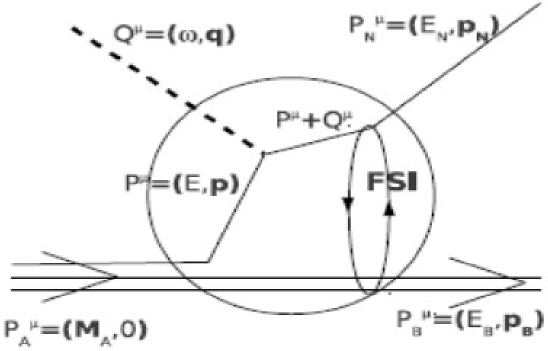
Plane waves



Phys. Rev. C 100 045501 (2019)



Distorted waves



Pauli blocking and elastic FSI

Inclusive electron scattering **at low q:**

Pauli blocking in the Spectral Function Approach (SFA), inspired by what is done in a local Fermi gas*:

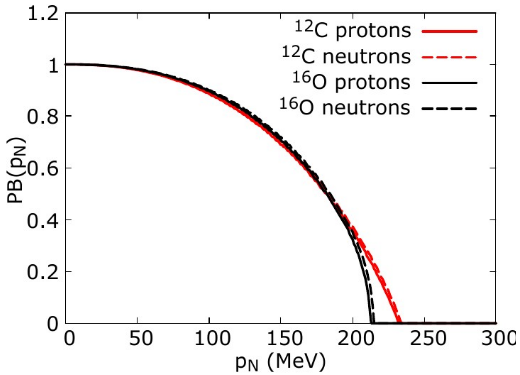
(*) Ref. Ankowski, Benhar and Sakuda PRD 91, 033005 (2015).

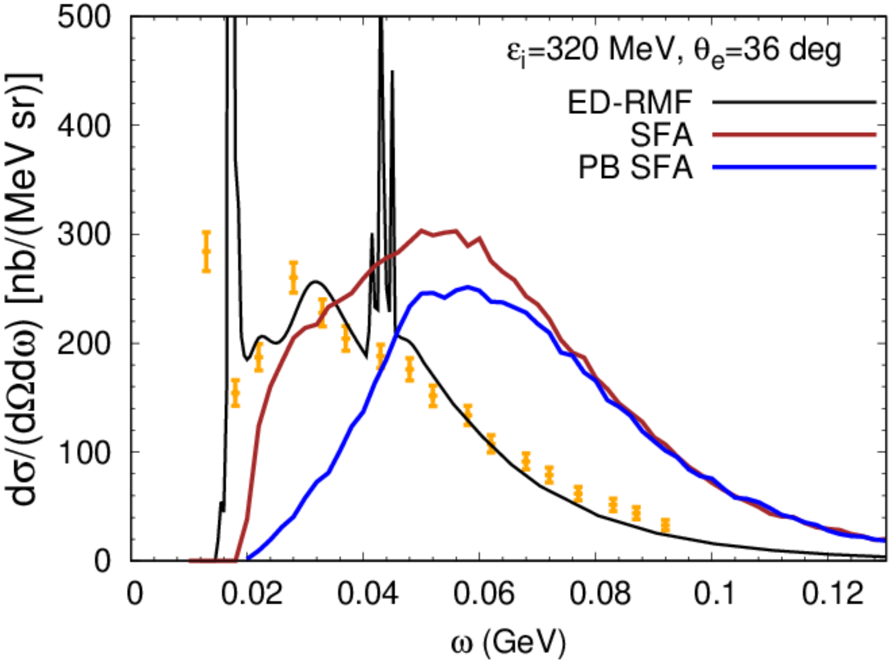
$$\left.\frac{d^6\sigma}{d\mathbf{k}_\mu d\mathbf{p}_N}\right|_{\text{Pauli blocked}} = \frac{d^6\sigma}{d\mathbf{k}_\mu d\mathbf{p}_N}\left(1 - \int d\mathbf{r}\ \rho_{\mathcal{N}}\ \theta[k_F(r) - p_N]\right)$$

$\rho_{\mathcal{N}}$ is the proton or neutron density

$$k_F(r) = \left(\frac{3\pi^2}{2}\mathcal{N}\rho_{\mathcal{N}}(r)\right)^{1/3}$$

Pauli blocking function (it depends on mom. of final nucleon).

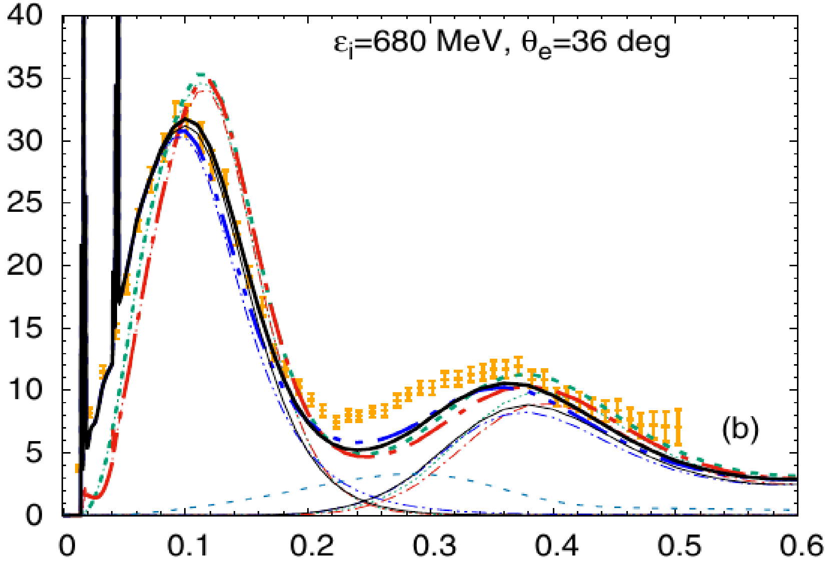


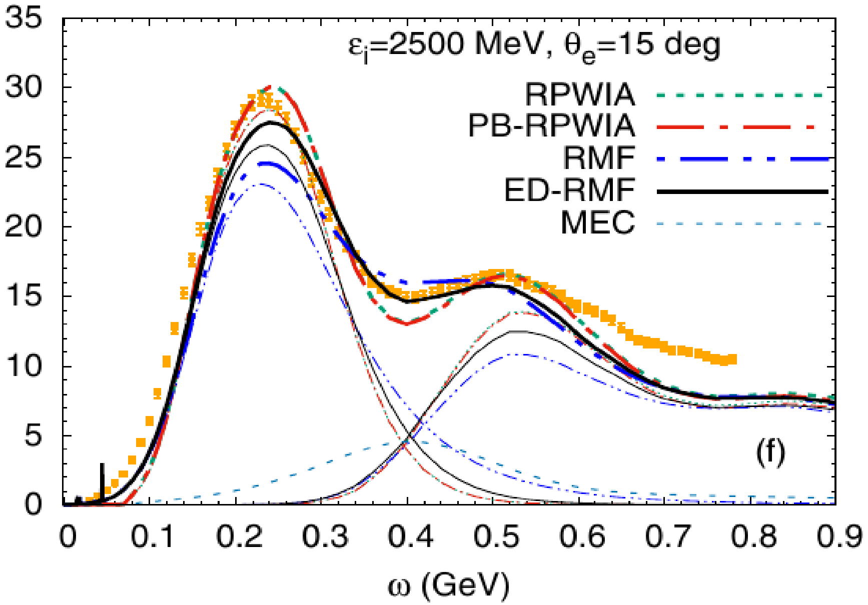


Pauli blocking and elastic FSI

Inclusive electron scattering **at intermediate q:**

Phys. Rev. C 100 045501 (2019)





Distortion of the outgoing nucleon (elastic FSI in a Quantum Mechanical way) is important at intermediate energies too !!!

IMPORTANT:

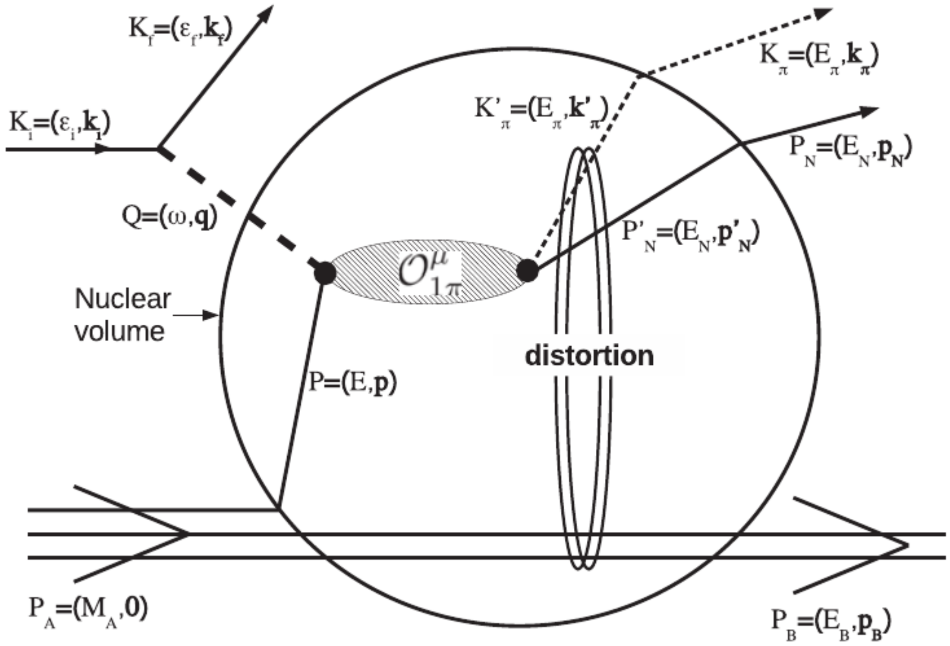
Classical CASCADE models do NOT affect the inclusive* cross section, therefore, it is important to use models of the primary vertex that provide realistic predictions of the inclusive cross section.

For consistency, **the model of the primary vertex should provide full information on the hadron(s),** which will propagate through nucleus via cascade.

*inclusive = only the scattered lepton is detected.

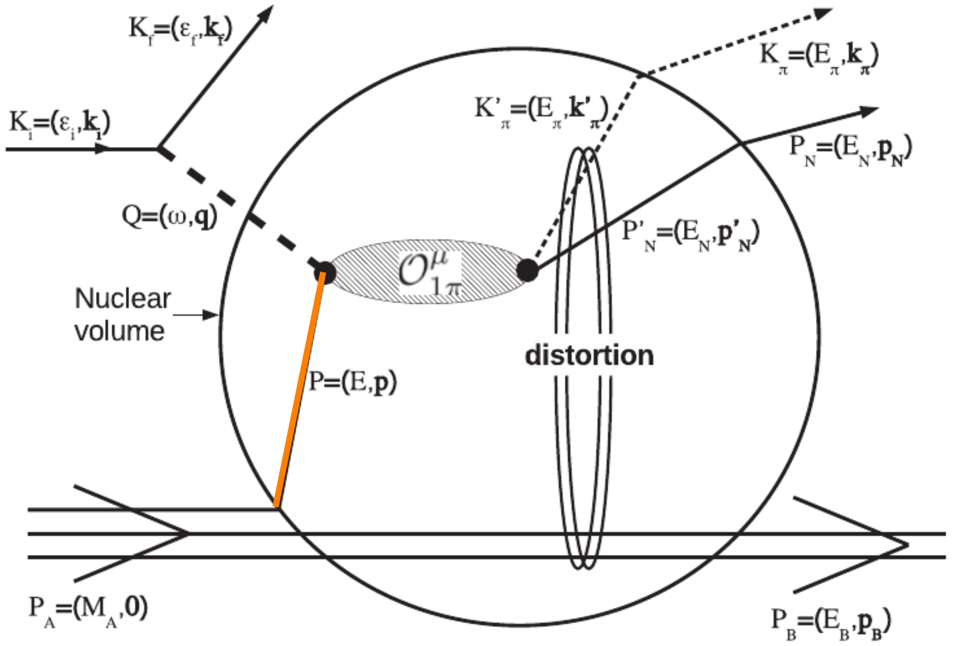
Single-Pion Production
(in the Impulse Approximation)

Single-Pion Production
(in the Impulse Approximation)



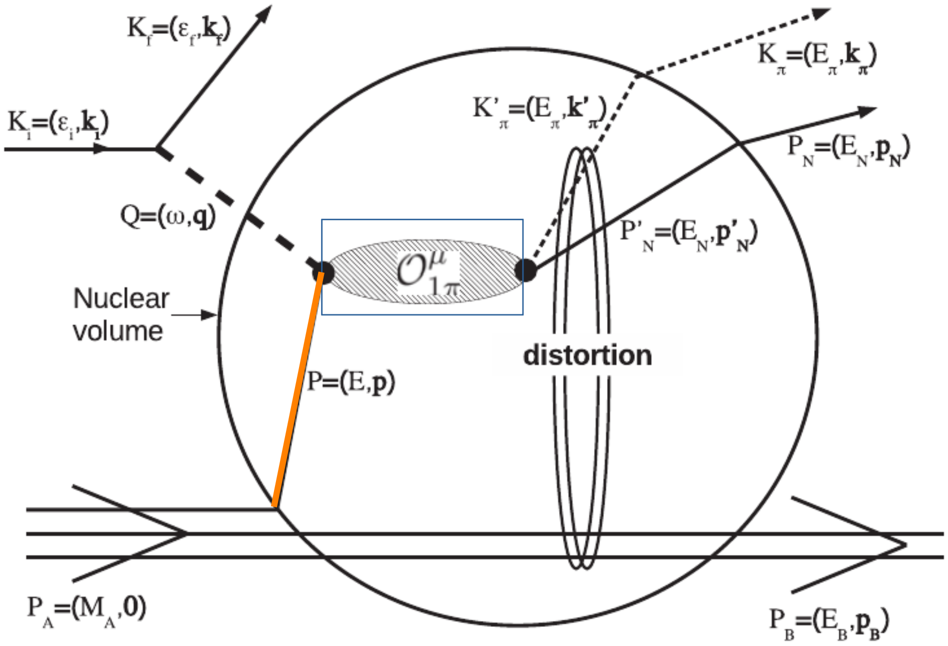
$$J^{\mu}_{\rm had} = \int d\mathbf{p} \int d\mathbf{p}'_N \overline{\Psi}_F(\mathbf{p}'_N, \mathbf{p}_N)\phi^*_{\pi}(\mathbf{p}+\mathbf{q}-\mathbf{p}'_N, \mathbf{k}_{\pi})\mathcal{O}_{1\pi}(Q, P'_N, P)\Psi_B(\mathbf{p})$$

Single-Pion Production
(in the Impulse Approximation)



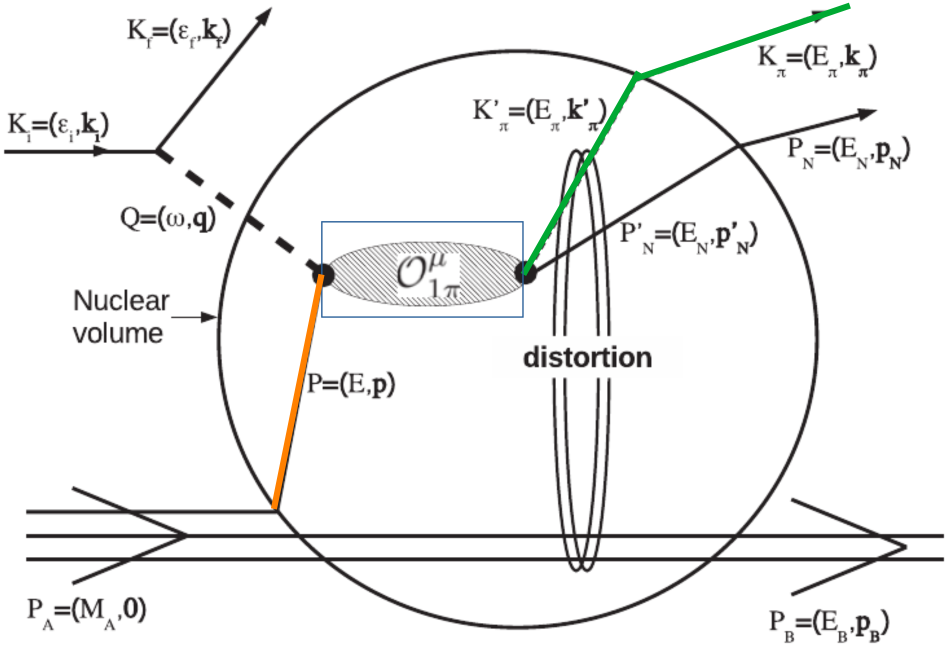
$$J^\mu_{\rm had} = \int d\mathbf{p} \int d\mathbf{p}'_N \overline{\Psi}_F(\mathbf{p}'_N,\mathbf{p}_N)\phi^*_\pi(\mathbf{p}+\mathbf{q}-\mathbf{p}'_N,\mathbf{k}_\pi)\mathcal{O}_{1\pi}(Q,P'_N,P)\Psi_B(\mathbf{p})$$

Single-Pion Production
(in the Impulse Approximation)



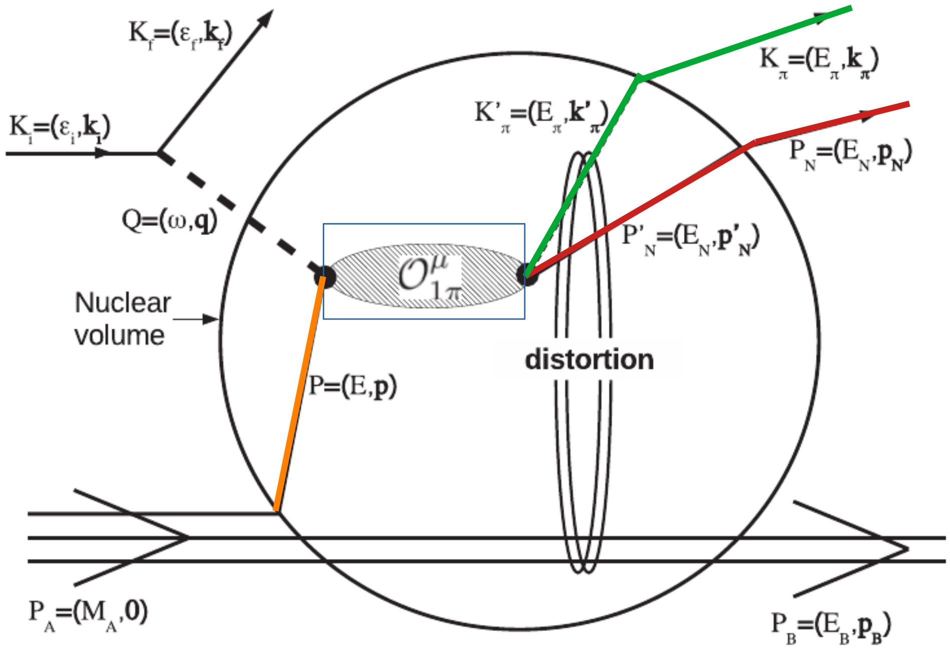
$$J^{\mu}_{\rm had} = \int d\mathbf{p} \int d\mathbf{p}'_N \overline{\Psi}_F(\mathbf{p}'_N, \mathbf{p}_N)\phi^*_\pi(\mathbf{p}+\mathbf{q}-\mathbf{p}'_N, \mathbf{k}_\pi)\mathcal{O}_{1\pi}(Q, P'_N, P)\Psi_B(\mathbf{p})$$

Single-Pion Production
(in the Impulse Approximation)



$$J^{\mu}_{\rm had} = \int d\mathbf{p} \int d\mathbf{p}'_N \overline{\Psi}_F(\mathbf{p}'_N, \mathbf{p}_N)\phi^*_\pi(\mathbf{p}+\mathbf{q}-\mathbf{p}'_N, \mathbf{k}_\pi)\mathcal{O}_{1\pi}(Q, P'_N, P)\Psi_B(\mathbf{p})$$

Single-Pion Production
(in the Impulse Approximation)



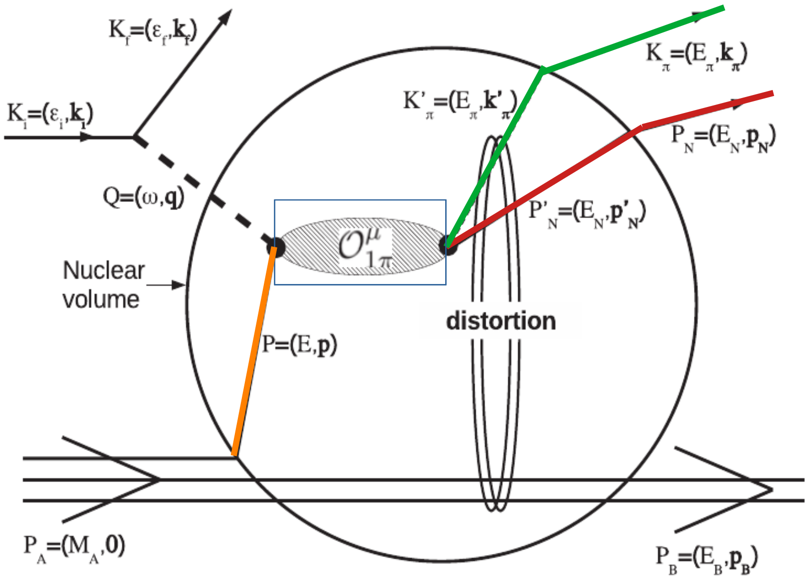
$$J^\mu_{\text{had}} = \int d\mathbf{p} \int d\mathbf{p}'_N \overline{\Psi}_F(\mathbf{p}'_N,\mathbf{p}_N)\phi^*_\pi(\mathbf{p}+\mathbf{q}-\mathbf{p}'_N,\mathbf{k}_\pi)\mathcal{O}_{1\pi}(Q,P'_N,P)\Psi_B(\mathbf{p})$$

From complex to simple

Amaro et al. https://doi.org/10.1088/1361-6471/abb128
Nikolakopoulos et al. https://doi.org/10.1103/PhysRevD.107.053007

Most general case: **all** particles as **distorted waves**

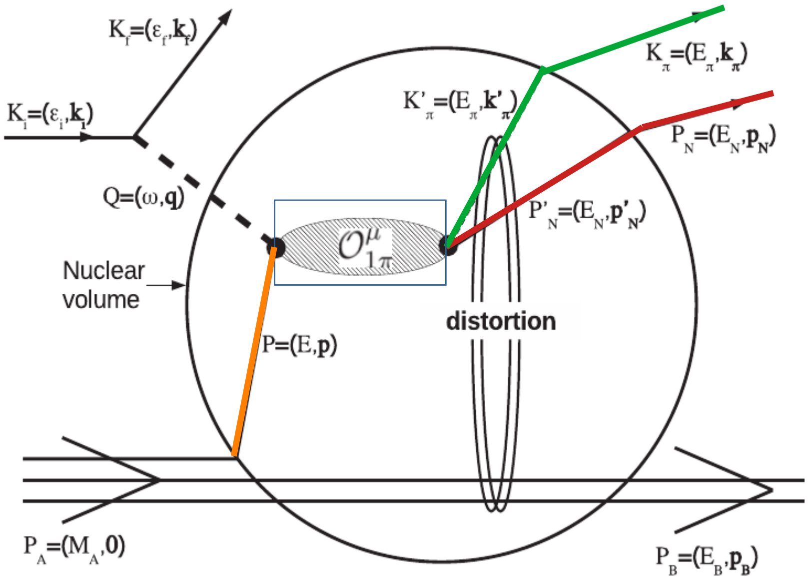
$$J^{\mu}_{\text{had}} = \int d\mathbf{p} \int d\mathbf{p}'_N \overline{\Psi}_F(\mathbf{p}'_N, \mathbf{p}_N) \phi^*_\pi(\mathbf{p}+\mathbf{q}-\mathbf{p}'_N, \mathbf{k}_\pi) \mathcal{O}_{1\pi}(Q, P'_N, P) \Psi_B(\mathbf{p})$$



$$J^{\mu}_{\text{had}} = \int d\mathbf{p} \int d\mathbf{p}'_N \overline{\Psi}_F(\mathbf{p}'_N, \mathbf{p}_N) \phi^*_\pi(\mathbf{p}+\mathbf{q}-\mathbf{p}'_N, \mathbf{k}_\pi) \mathcal{O}_{1\pi}(Q, P'_N, P) \Psi_B(\mathbf{p})$$

From complex to simple

Amaro et al. https://doi.org/10.1088/1361-6471/abb128
Nikolakopoulos et al. https://doi.org/10.1103/PhysRevD.107.053007



$$J^{\mu}_{\text{had}} = \int d\mathbf{p} \int d\mathbf{p}'_N \overline{\Psi}_F(\mathbf{p}'_N, \mathbf{p}_N) \phi^*_\pi(\mathbf{p}+\mathbf{q}-\mathbf{p}'_N, \mathbf{k}_\pi) \mathcal{O}_{1\pi}(Q, P'_N, P) \Psi_B(\mathbf{p})$$

Most general case: **all** particles as **distorted waves**

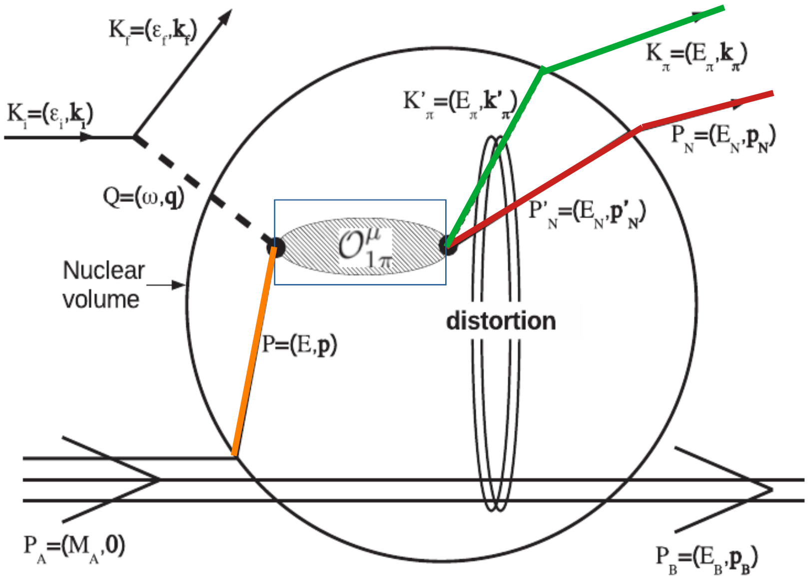
$$J^{\mu}_{\text{had}} = \int d\mathbf{p} \int d\mathbf{p}'_N \overline{\Psi}_F(\mathbf{p}'_N, \mathbf{p}_N) \phi^*_\pi(\mathbf{p}+\mathbf{q}-\mathbf{p}'_N, \mathbf{k}_\pi) \mathcal{O}_{1\pi}(Q, P'_N, P) \Psi_B(\mathbf{p})$$

Asymptotic (or local) operator approximation

$$J^{\mu}_{\text{had}} = \int d\mathbf{p} \int d\mathbf{p}'_N \overline{\Psi}_F(\mathbf{p}'_N, \mathbf{p}_N) \phi^*_\pi(\mathbf{p}+\mathbf{q}-\mathbf{p}'_N, \mathbf{k}_\pi) \mathcal{O}^{\mu}_{\text{asymp.}}(Q, K_\pi, P_N) \Psi_B(\mathbf{p})$$

From complex to simple

Amaro et al. https://doi.org/10.1088/1361-6471/abb128
Nikolakopoulos et al. https://doi.org/10.1103/PhysRevD.107.053007



$$J^\mu_{\text{had}} = \int d\mathbf{p} \int d\mathbf{p}'_N \overline{\Psi}_F(\mathbf{p}'_N, \mathbf{p}_N) \phi^*_\pi(\mathbf{p}+\mathbf{q}-\mathbf{p}'_N, \mathbf{k}_\pi) \mathcal{O}_{1\pi}(Q, P'_N, P) \Psi_B(\mathbf{p})$$

Most general case: **all** particles as **distorted waves**

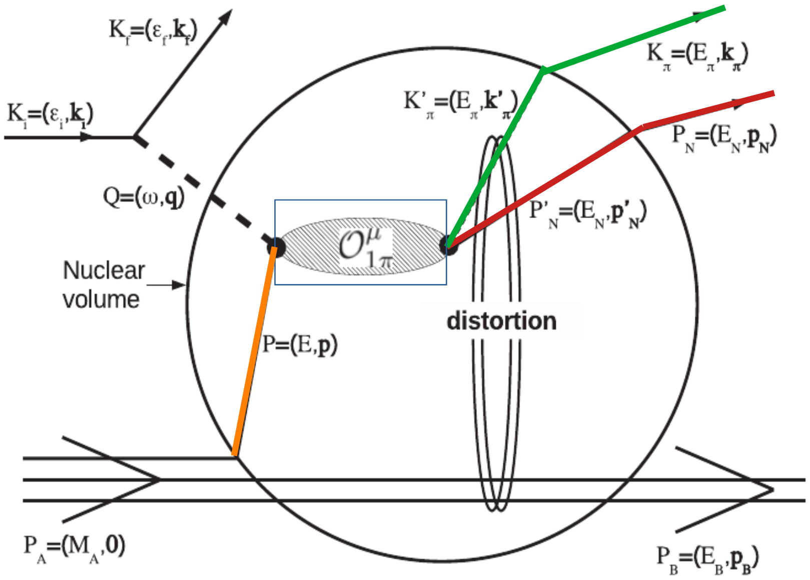
$$J^\mu_{\text{had}} = \int d\mathbf{p} \int d\mathbf{p}'_N \overline{\Psi}_F(\mathbf{p}'_N, \mathbf{p}_N) \phi^*_\pi(\mathbf{p}+\mathbf{q}-\mathbf{p}'_N, \mathbf{k}_\pi) \mathcal{O}_{1\pi}(Q, P'_N, P) \Psi_B(\mathbf{p})$$

Asymptotic (or local) operator approximation
In coordinate space

$$J^\mu_{\text{had}} = \int d\mathbf{r} \int d\mathbf{r}' \overline{\Psi}_F(\mathbf{r}', \mathbf{p}_N) \phi^*_\pi(\mathbf{r}', \mathbf{k}_\pi) \mathcal{O}^\mu_{\text{local}}(Q, K_\pi, P_N) \Psi_B(\mathbf{r}) e^{i\mathbf{q}\cdot\mathbf{r}}$$

From complex to simple

Amaro et al. https://doi.org/10.1088/1361-6471/abb128
Nikolakopoulos et al. https://doi.org/10.1103/PhysRevD.107.053007



$$J^{\mu}_{\text{had}} = \int d\mathbf{p} \int d\mathbf{p}'_N \overline{\Psi}_F(\mathbf{p}'_N, \mathbf{p}_N) \phi^*_\pi(\mathbf{p}+\mathbf{q}-\mathbf{p}'_N, \mathbf{k}_\pi) \mathcal{O}_{1\pi}(Q, P'_N, P) \Psi_B(\mathbf{p})$$

Most general case: all particles as distorted waves

$$J^{\mu}_{\text{had}} = \int d\mathbf{p} \int d\mathbf{p}'_N \overline{\Psi}_F(\mathbf{p}'_N, \mathbf{p}_N) \phi^*_\pi(\mathbf{p}+\mathbf{q}-\mathbf{p}'_N, \mathbf{k}_\pi) \mathcal{O}_{1\pi}(Q, P'_N, P) \Psi_B(\mathbf{p})$$

Asymptotic (or local) operator approximation

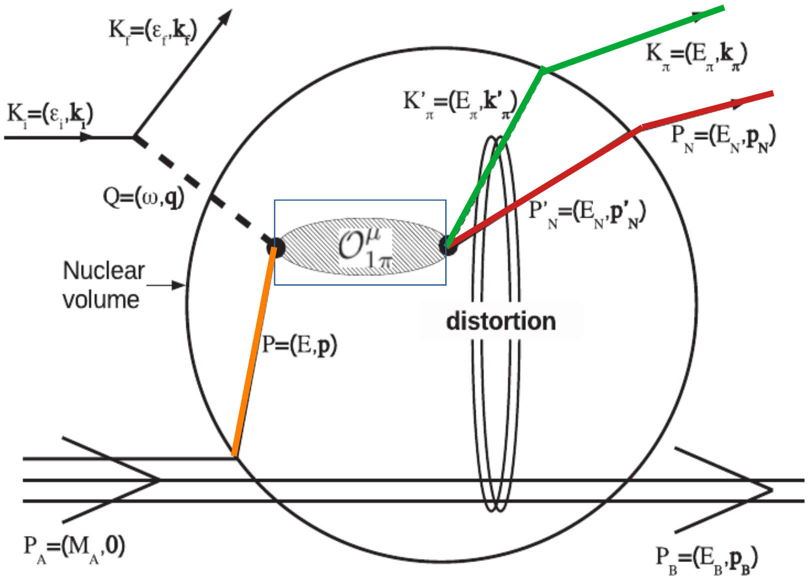
$$J^{\mu}_{\text{had}} = \int d\mathbf{p} \int d\mathbf{p}'_N \overline{\Psi}_F(\mathbf{p}'_N, \mathbf{p}_N) \phi^*_\pi(\mathbf{p}+\mathbf{q}-\mathbf{p}'_N, \mathbf{k}_\pi) \mathcal{O}^{\mu}_{\text{asymp.}}(Q, K_\pi, P_N) \Psi_B(\mathbf{p})$$

Pion as a plane wave

$$J^{\mu}_{\text{had}} = \frac{1}{\sqrt{2E_\pi}} \int d\mathbf{p} \overline{\Psi}_F(\mathbf{q}+\mathbf{p}-\mathbf{k}_\pi, \mathbf{p}_N) \mathcal{O}^{\mu}_{1\pi}(Q, P, K_\pi) \Psi_B(\mathbf{p})$$

From complex to simple

Amaro et al. https://doi.org/10.1088/1361-6471/abb128
Nikolakopoulos et al. https://doi.org/10.1103/PhysRevD.107.053007



$$J^\mu_{\rm had} = \int d\mathbf{p} \int d\mathbf{p}'_N \overline{\Psi}_F(\mathbf{p}'_N,\mathbf{p}_N) \phi^*_\pi(\mathbf{p}+\mathbf{q}-\mathbf{p}'_N,\mathbf{k}_\pi) \mathcal{O}_{1\pi}(Q,P'_N,P) \Psi_B(\mathbf{p})$$

Most general case: **all** particles as **distorted waves**

$$J^\mu_{\rm had} = \int d\mathbf{p} \int d\mathbf{p}'_N \overline{\Psi}_F(\mathbf{p}'_N,\mathbf{p}_N) \phi^*_\pi(\mathbf{p}+\mathbf{q}-\mathbf{p}'_N,\mathbf{k}_\pi) \mathcal{O}_{1\pi}(Q,P'_N,P) \Psi_B(\mathbf{p})$$

Asymptotic (or local) operator approximation

$$J^\mu_{\rm had} = \int d\mathbf{p} \int d\mathbf{p}'_N \overline{\Psi}_F(\mathbf{p}'_N,\mathbf{p}_N) \phi^*_\pi(\mathbf{p}+\mathbf{q}-\mathbf{p}'_N,\mathbf{k}_\pi) \mathcal{O}^\mu_{\rm asymp.}(Q,K_\pi,P_N) \Psi_B(\mathbf{p})$$

Pion as a plane wave

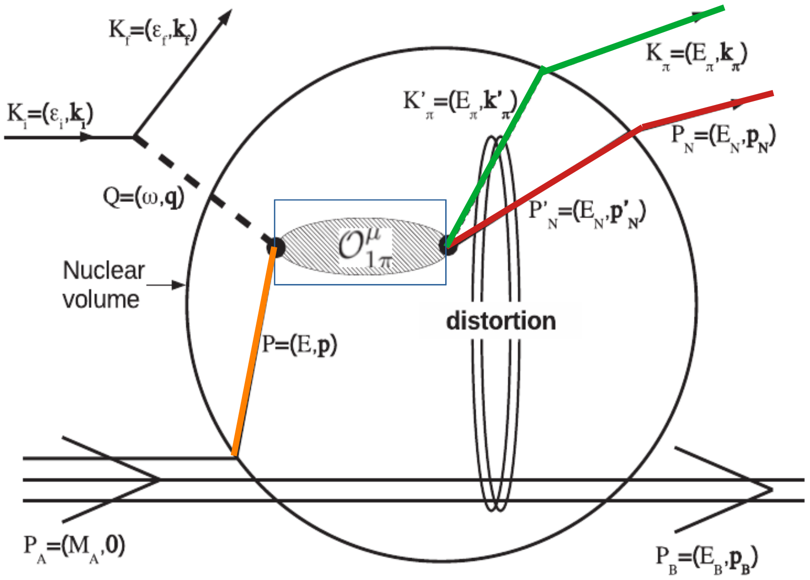
$$J^\mu_{\rm had} = \frac{1}{\sqrt{2E_\pi}} \int d\mathbf{p} \overline{\Psi}_F(\mathbf{q}+\mathbf{p}-\mathbf{k}_\pi,\mathbf{p}_N) \mathcal{O}^\mu_{1\pi}(Q,P,K_\pi) \Psi_B(\mathbf{p})$$

Pion and final nucleon as plane waves

$$J^\mu_{\rm had} = (2\pi)^{3/2} \sqrt{\frac{M_N}{2E_\pi E_N}}\, \overline{u}(\mathbf{p}_N,s_N) \mathcal{O}^\mu_{1\pi}(Q,P_N,K_\pi) \Psi_B(\mathbf{p}_N+\mathbf{k}_\pi-\mathbf{q})$$

From complex to simple

Amaro et al. https://doi.org/10.1088/1361-6471/abb128
Nikolakopoulos et al. https://doi.org/10.1103/PhysRevD.107.053007



$$J^{\mu}_{\rm had} = \int d\mathbf{p} \int d\mathbf{p}'_N \overline{\Psi}_F(\mathbf{p}'_N, \mathbf{p}_N) \phi^*_{\pi}(\mathbf{p}+\mathbf{q}-\mathbf{p}'_N, \mathbf{k}_{\pi}) \mathcal{O}_{1\pi}(Q, P'_N, P) \Psi_B(\mathbf{p})$$

Most general case: **all** particles as **distorted waves**

$$J^{\mu}_{\rm had} = \int d\mathbf{p} \int d\mathbf{p}'_N \overline{\Psi}_F(\mathbf{p}'_N, \mathbf{p}_N) \phi^*_{\pi}(\mathbf{p}+\mathbf{q}-\mathbf{p}'_N, \mathbf{k}_{\pi}) \mathcal{O}_{1\pi}(Q, P'_N, P) \Psi_B(\mathbf{p})$$

Asymptotic (or local) operator approximation

$$J^{\mu}_{\rm had} = \int d\mathbf{p} \int d\mathbf{p}'_N \overline{\Psi}_F(\mathbf{p}'_N, \mathbf{p}_N) \phi^*_{\pi}(\mathbf{p}+\mathbf{q}-\mathbf{p}'_N, \mathbf{k}_{\pi}) \mathcal{O}^{\mu}_{\rm asymp.}(Q, K_{\pi}, P_N) \Psi_B(\mathbf{p})$$

Pion as a plane wave

$$J^{\mu}_{\rm had} = \frac{1}{\sqrt{2E_{\pi}}} \int d\mathbf{p} \overline{\Psi}_F(\mathbf{q}+\mathbf{p}-\mathbf{k}_{\pi}, \mathbf{p}_N) \mathcal{O}^{\mu}_{1\pi}(Q, P, K_{\pi}) \Psi_B(\mathbf{p})$$

Pion and final nucleon as plane waves

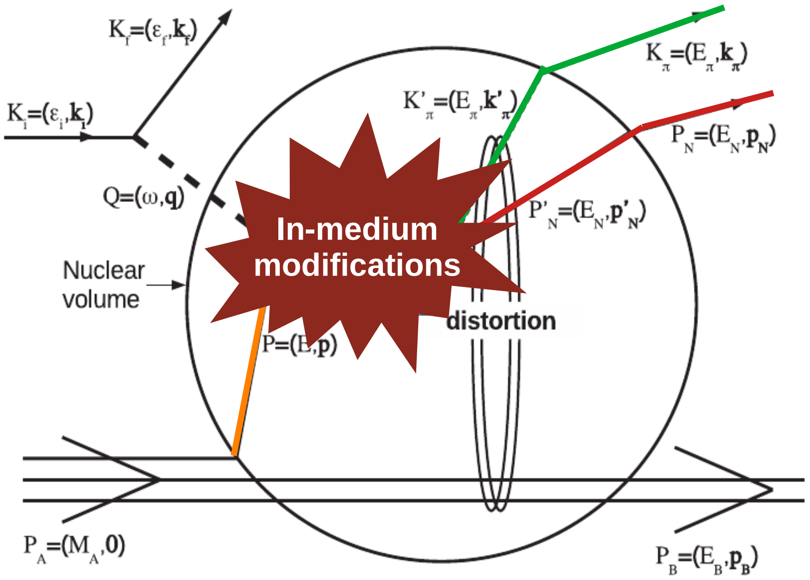
$$J^{\mu}_{\rm had} = (2\pi)^{3/2} \sqrt{\frac{M_N}{2E_{\pi}E_N}}\, \overline{u}(\mathbf{p}_N, s_N) \mathcal{O}^{\mu}_{1\pi}(Q, P_N, K_{\pi}) \Psi_B(\mathbf{p}_N + \mathbf{k}_{\pi} - \mathbf{q})$$

All particles as **plane waves**

$$J^{\mu}_{\rm had} = (2\pi)^3 \delta^3(\mathbf{p}_N + \mathbf{k}_{\pi} - \mathbf{q} - \mathbf{p}) \sqrt{\frac{M_N^2}{2E_{\pi}E_N E}}\, \overline{u}(\mathbf{p}_N, s_N) \mathcal{O}^{\mu}_{1\pi}(Q, P_N, K_{\pi}) u(\mathbf{p}, s)$$

From complex to simple

Amaro et al. https://doi.org/10.1088/1361-6471/abb128
Nikolakopoulos et al. https://doi.org/10.1103/PhysRevD.107.053007



$$J^\mu_{\rm had} = \int d\mathbf{p} \int d\mathbf{p}'_N \overline{\Psi}_F(\mathbf{p}'_N,\mathbf{p}_N)\phi^*_\pi(\mathbf{p}+\mathbf{q}-\mathbf{p}'_N,\mathbf{k}_\pi)\mathcal{O}_{1\pi}(Q,P'_N,P)\Psi_B(\mathbf{p})$$

Most general case: **all** particles as **distorted waves**

$$J^\mu_{\rm had} = \int d\mathbf{p} \int d\mathbf{p}'_N \overline{\Psi}_F(\mathbf{p}'_N,\mathbf{p}_N)\phi^*_\pi(\mathbf{p}+\mathbf{q}-\mathbf{p}'_N,\mathbf{k}_\pi)\mathcal{O}_{1\pi}(Q,P'_N,P)\Psi_B(\mathbf{p})$$

Asymptotic (or local) operator approximation

$$J^\mu_{\rm had} = \int d\mathbf{p} \int d\mathbf{p}'_N \overline{\Psi}_F(\mathbf{p}'_N,\mathbf{p}_N)\phi^*_\pi(\mathbf{p}+\mathbf{q}-\mathbf{p}'_N,\mathbf{k}_\pi)\mathcal{O}^\mu_{\rm asymp.}(Q,K_\pi,P_N)\Psi_B(\mathbf{p})$$

Pion as a plane wave

$$J^\mu_{\rm had} = \frac{1}{\sqrt{2E_\pi}} \int d\mathbf{p}\overline{\Psi}_F(\mathbf{q}+\mathbf{p}-\mathbf{k}_\pi,\mathbf{p}_N)\mathcal{O}^\mu_{1\pi}(Q,P,K_\pi)\Psi_B(\mathbf{p})$$

Pion and final nucleon as plane waves

$$J^\mu_{\rm had} = (2\pi)^{3/2}\sqrt{\frac{M_N}{2E_\pi E_N}}\,\overline{u}(\mathbf{p}_N,s_N)\mathcal{O}^\mu_{1\pi}(Q,P_N,K_\pi)\Psi_B(\mathbf{p}_N+\mathbf{k}_\pi-\mathbf{q})$$

All particles as **plane waves**

$$J^\mu_{\rm had} = (2\pi)^3\delta^3(\mathbf{p}_N+\mathbf{k}_\pi-\mathbf{q}-\mathbf{p})\sqrt{\frac{M_N^2}{2E_\pi E_N E}}\,\overline{u}(\mathbf{p}_N,s_N)\mathcal{O}^\mu_{1\pi}(Q,P_N,K_\pi)u(\mathbf{p},s)$$

Free nucleon

versus

RFG
(it accounts for **Fermi motion** but all particles are **plane waves**)

versus

RPWIA
(**bound nucleon is a shell-model state**, so Fermi motion and the shell structure are accounted for, the **pion and final nucleon** are plane waves)

Praet et al. (2009), https://doi.org/10.1103/physrevc.79.044603

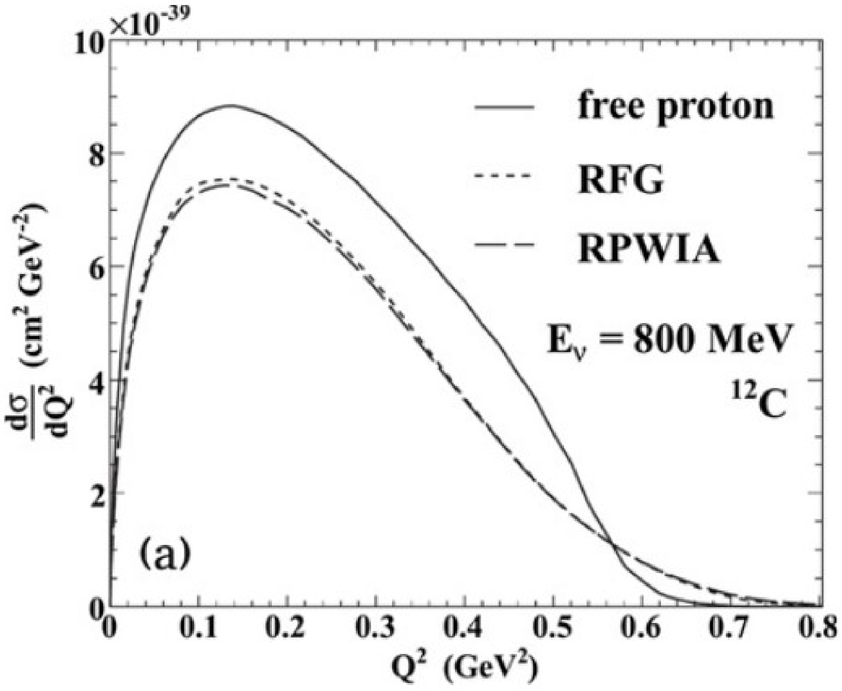


Figure: CC neutrino-^{12}C induced SPP.

Free nucleon

versus

RFG
(it accounts for **Fermi motion** but all particles are **plane waves**)

versus

RPWIA
(**bound nucleon is a shell-model state**, so Fermi motion and the shell structure are accounted for, the **pion and final nucleon** are plane waves)

Praet et al. (2009), https://doi.org/10.1103/physrevc.79.044603

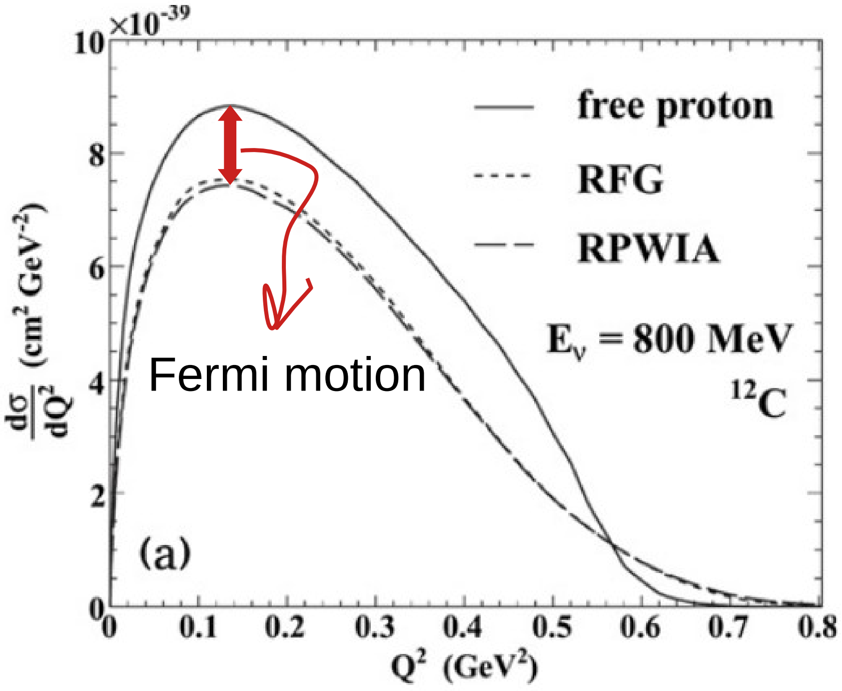


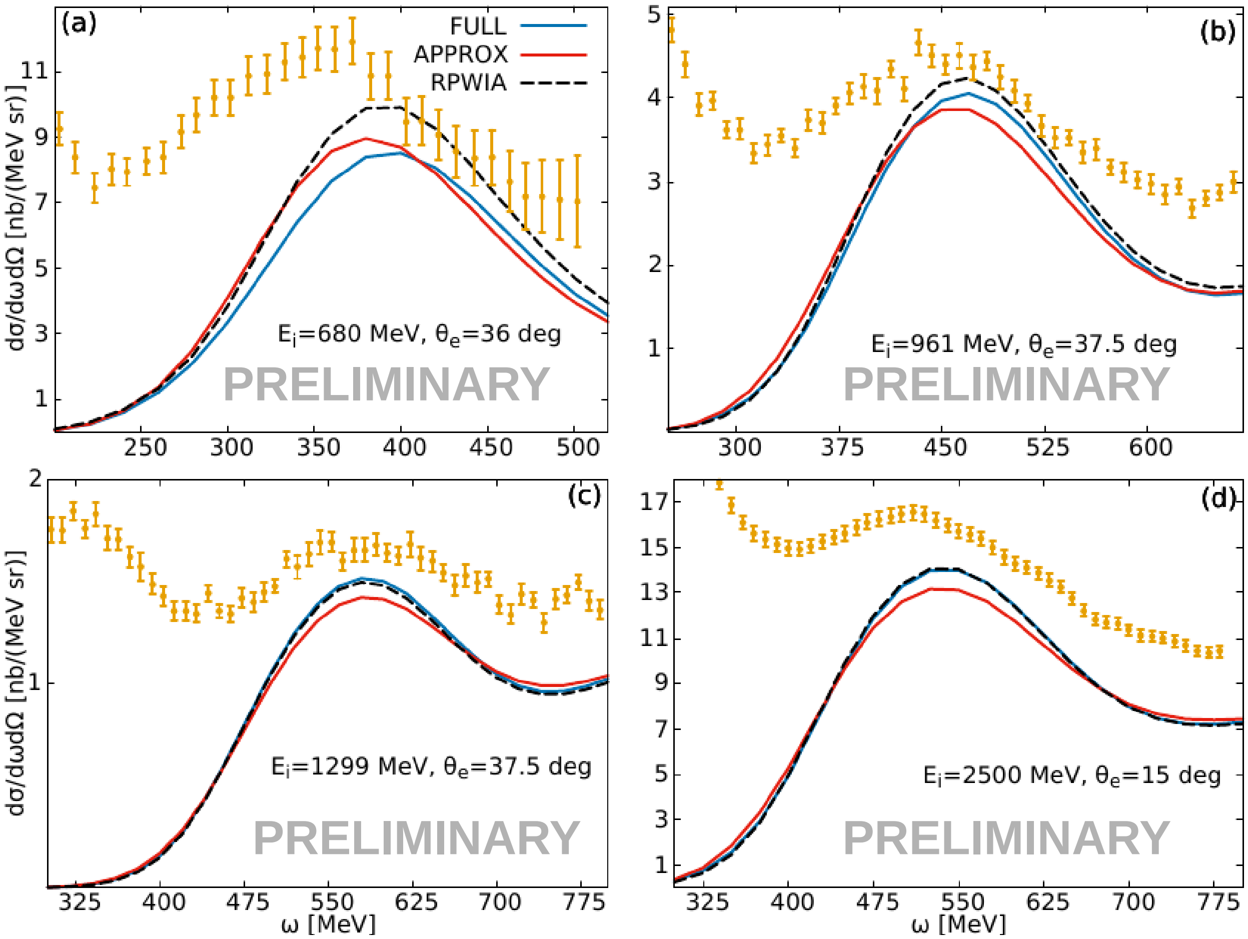
Figure: CC neutrino-^{12}C induced SPP.

J. García-Marcos et al., Towards a more complete description of nucleon distortion in lepton-induced single-pion production at low-Q2
https://doi.org/10.48550/arXiv.2310.18056

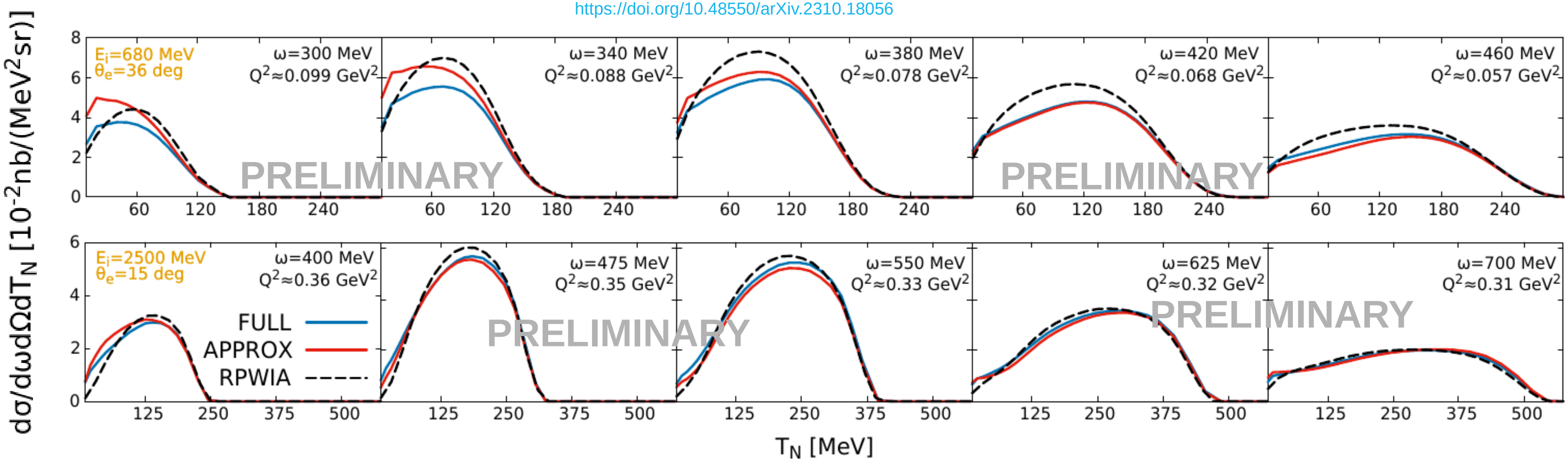
Distortion of the nucleon wave function
(or Elastic FSI of the nucleon)

and

Asymptotic approximation for the SPP operator
(or local versus non-local operator)



J. García-Marcos et al., Towards a more complete description of nucleon distortion in lepton-induced single-pion production at low-Q2
https://doi.org/10.48550/arXiv.2310.18056



Distortion of the pion wave function
(or Elastic FSI of the pion)

There are some works on **photoproduction** and a few on **electroproduction** but they usually include the distortion of the nucleon and pion all together and compare it to the case of plane-wave approach. That makes it difficult to isolate the effect of distortion of the pion.

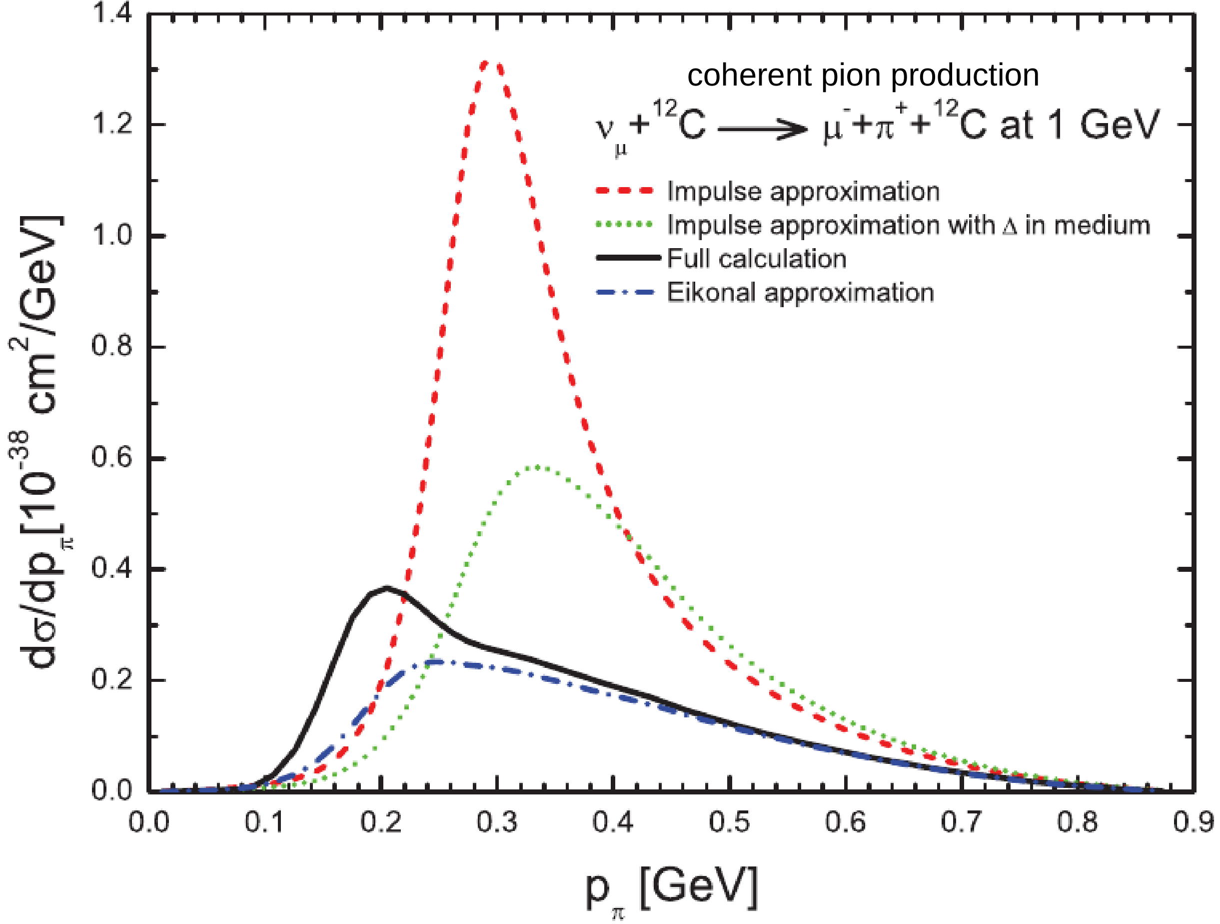
Instead, the study of **coherent pion production** is a clean way to study this effect:

$$\gamma + A \text{ -----> } \pi^0 + A$$

or

$$\text{lepton} + A \text{ -----> } \text{lepton}' + \pi + A$$

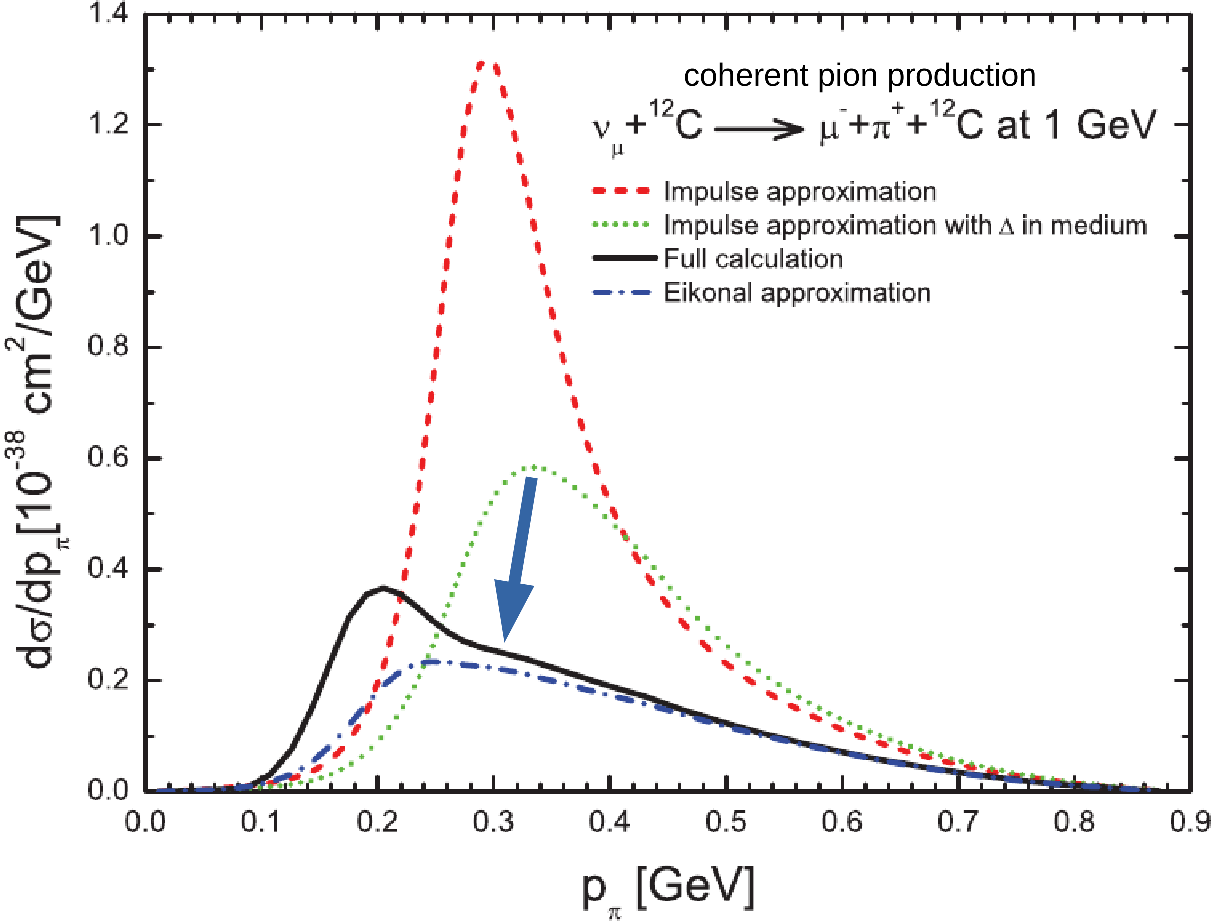
Alvarez-Ruso et al. (2007), http://dx.doi.org/10.1103/PhysRevC.75.055501



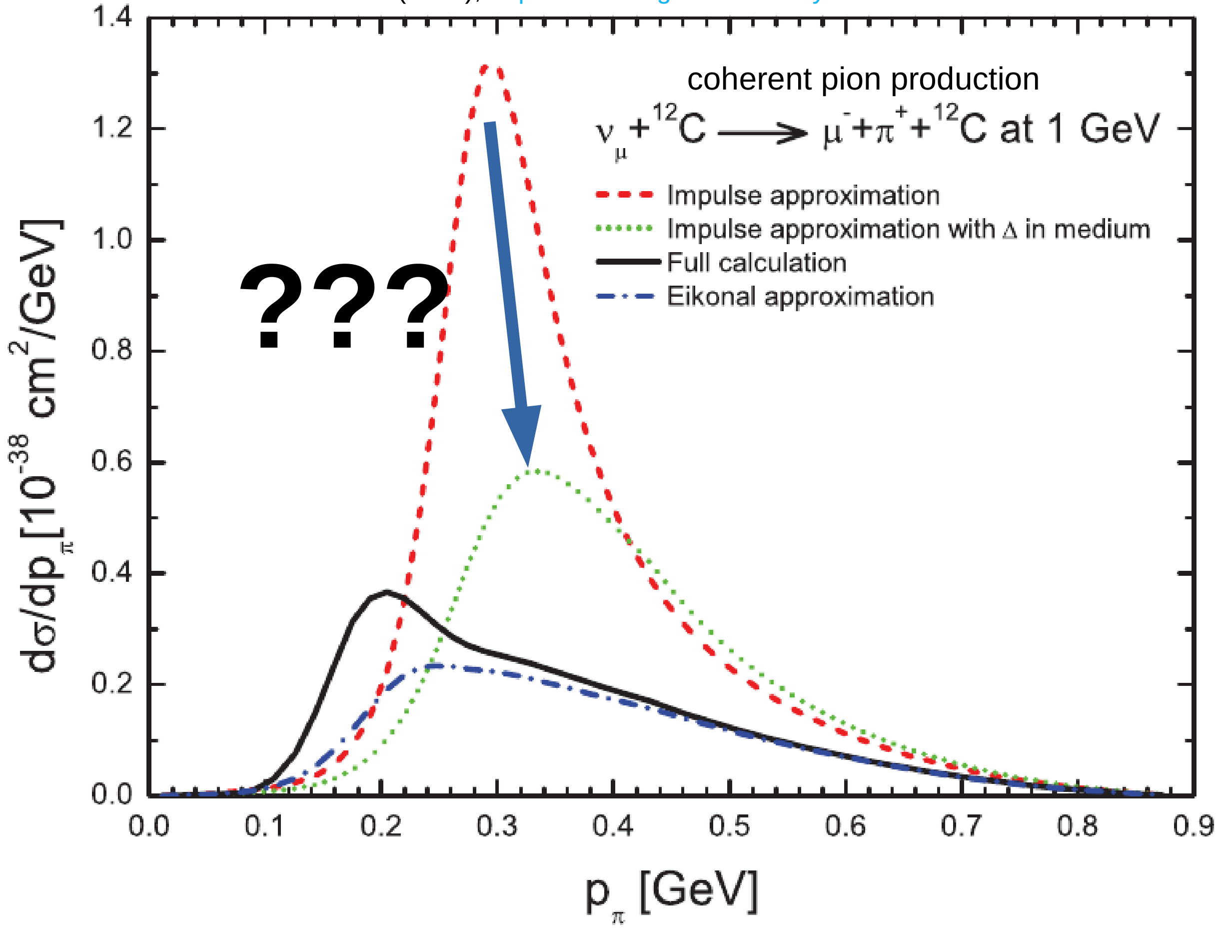
Distortion of the pion wave function
(or Elastic FSI of the pion)

Distortion of the pion wave function
(or Elastic FSI of the pion)

Alvarez-Ruso et al. (2007), http://dx.doi.org/10.1103/PhysRevC.75.055501

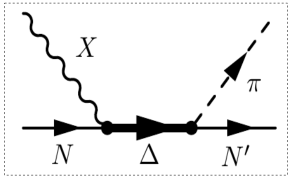


Alvarez-Ruso et al. (2007), http://dx.doi.org/10.1103/PhysRevC.75.055501



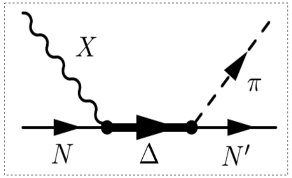
In-medium modification of the resonance properties

In-medium modification of the resonance properties: *à la* Oset



Oset and Salcedo,
https://doi.org/10.1016/0375-9474(87)90185-0

In-medium modification of the resonance properties: *à la* Oset



Delta propagator:

$$S_{\Delta,\alpha\beta} = \frac{-(\not{K}_\Delta + M_\Delta)}{K_\Delta^2 - M_N^2 + iM_\Delta \Gamma_{\text{width}}} \left(g_{\alpha\beta} - \frac{1}{3}\gamma_\alpha\gamma_\beta - \frac{2}{3M_\Delta^2} K_{\Delta,\alpha} K_{\Delta,\beta} - \frac{2}{3M_\Delta}(\gamma_\alpha K_{\Delta,\beta} - K_{\Delta,\alpha}\gamma_\beta) \right)$$

Replace the **free decay width** by an ***in-medium*** *one*:

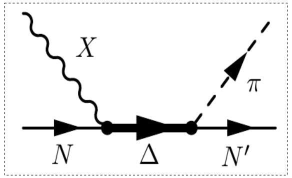
$$\Gamma_{\text{width}}^{\text{free}} \longrightarrow \Gamma_{\text{width}}^{\text{in-medium}} = \Gamma_{\text{Pauli}} - 2\Im(\Sigma_\Delta)\,, \quad M_\Delta^{\text{free}} \longrightarrow M_\Delta^{\text{in-medium}} = M_\Delta^{\text{free}} + \Re(\Sigma_\Delta)\,.$$

+ Γ_{Pauli}: some nucleons from Δ-decay are Pauli blocked (the Δ-decay width decreases).
+ The parametrization of $\Im(\Sigma_\Delta)$ and $\Re(\Sigma_\Delta)$ is given in terms of the nuclear density ρ:

$$\begin{aligned} -\Im(\Sigma_\Delta) &= C_{QE}\,(\rho/\rho_0)^\alpha + C_{A2}\,(\rho/\rho_0)^\beta + C_{A3}\,(\rho/\rho_0)^\gamma\,, \\ \Re(\Sigma_\Delta) &= 40\text{ MeV}\,(\rho/\rho_0)\,. \end{aligned}$$

Oset and Salcedo, https://doi.org/10.1016/0375-9474(87)90185-0

In-medium modification of the resonance properties: *à la* Oset



$$-\Im(\Sigma_\Delta) = C_{QE}\,(\rho/\rho_0)^\alpha + C_{A2}\,(\rho/\rho_0)^\beta + C_{A3}\,(\rho/\rho_0)^\gamma$$

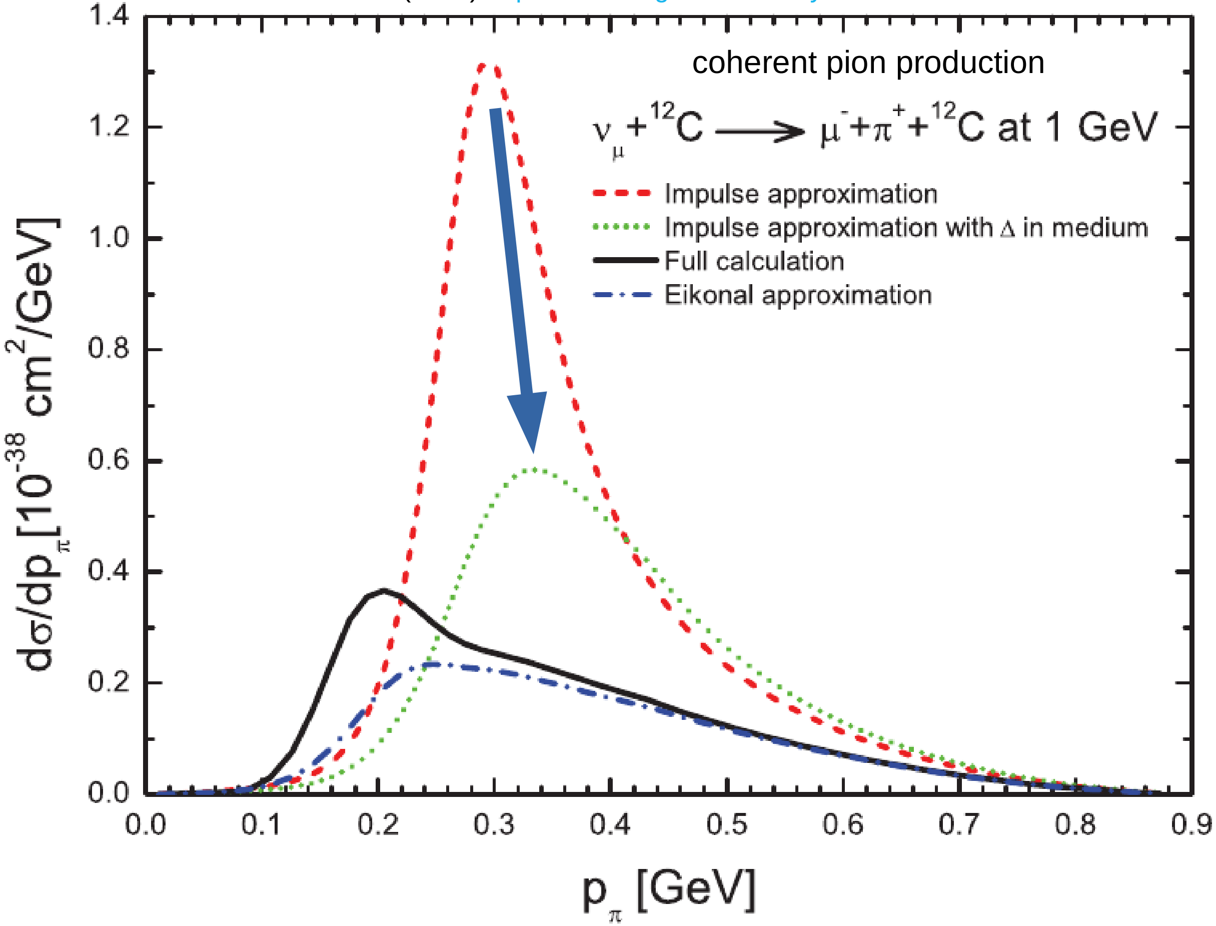
Each contribution corresponds to a different process:

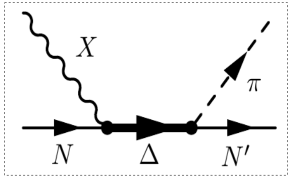
- QE $\Longrightarrow \Delta N \rightarrow \pi NN$ (still one pion in the final state)
- A2 $\Longrightarrow \Delta N \rightarrow NN$ (no pions in the final state)
- A3 $\Longrightarrow \Delta NN \rightarrow NNN$ (no pions in the final state)

Oset and Salcedo, https://doi.org/10.1016/0375-9474(87)90185-0

In-medium modification of the resonance properties: *à la* Oset

Alvarez-Ruso et al. (2007) http://dx.doi.org/10.1103/PhysRevC.75.055501

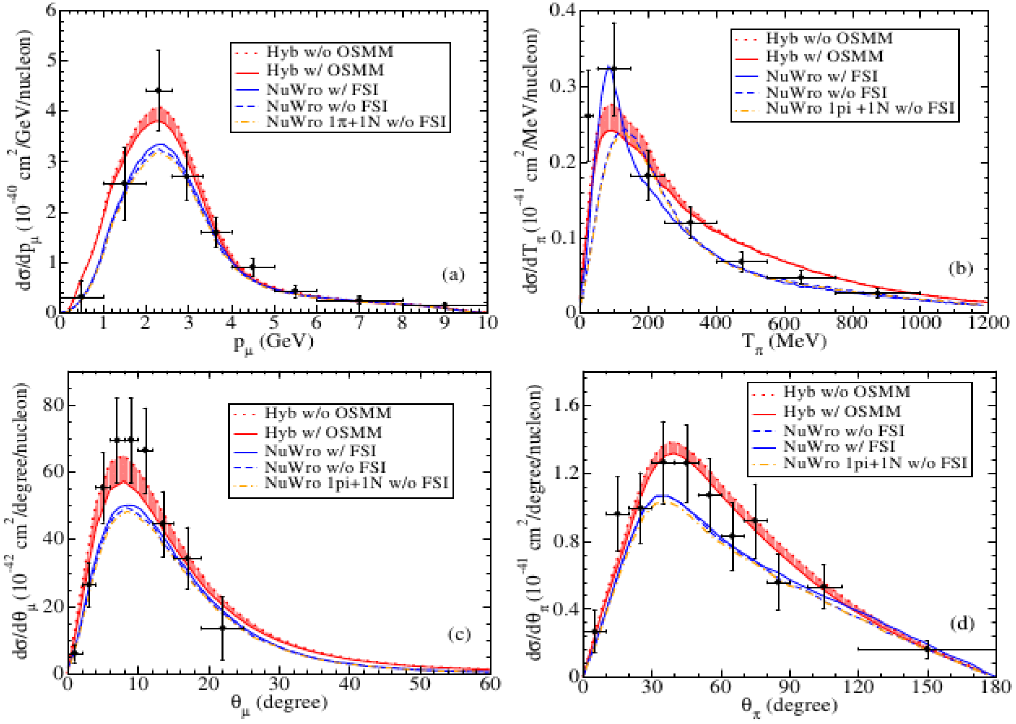




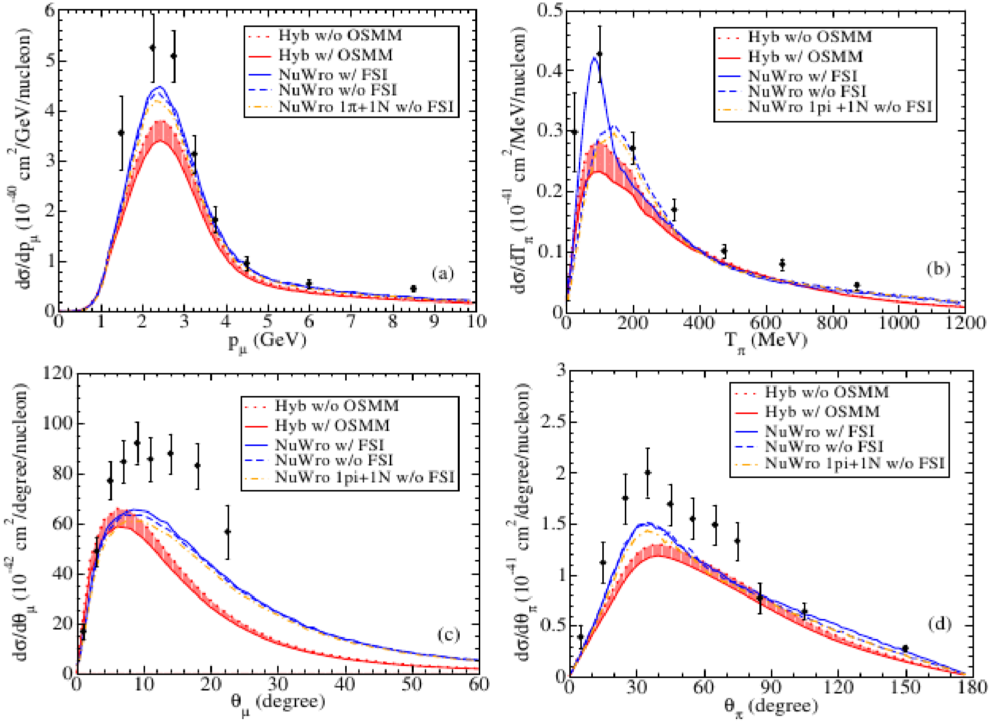
In-medium modification of the resonance properties: *à la* Oset

Lower (upper) bound of the red band corresponds to with (without) medium modification of the delta decay width.

MINERvA antineutrino CC 1π^0.



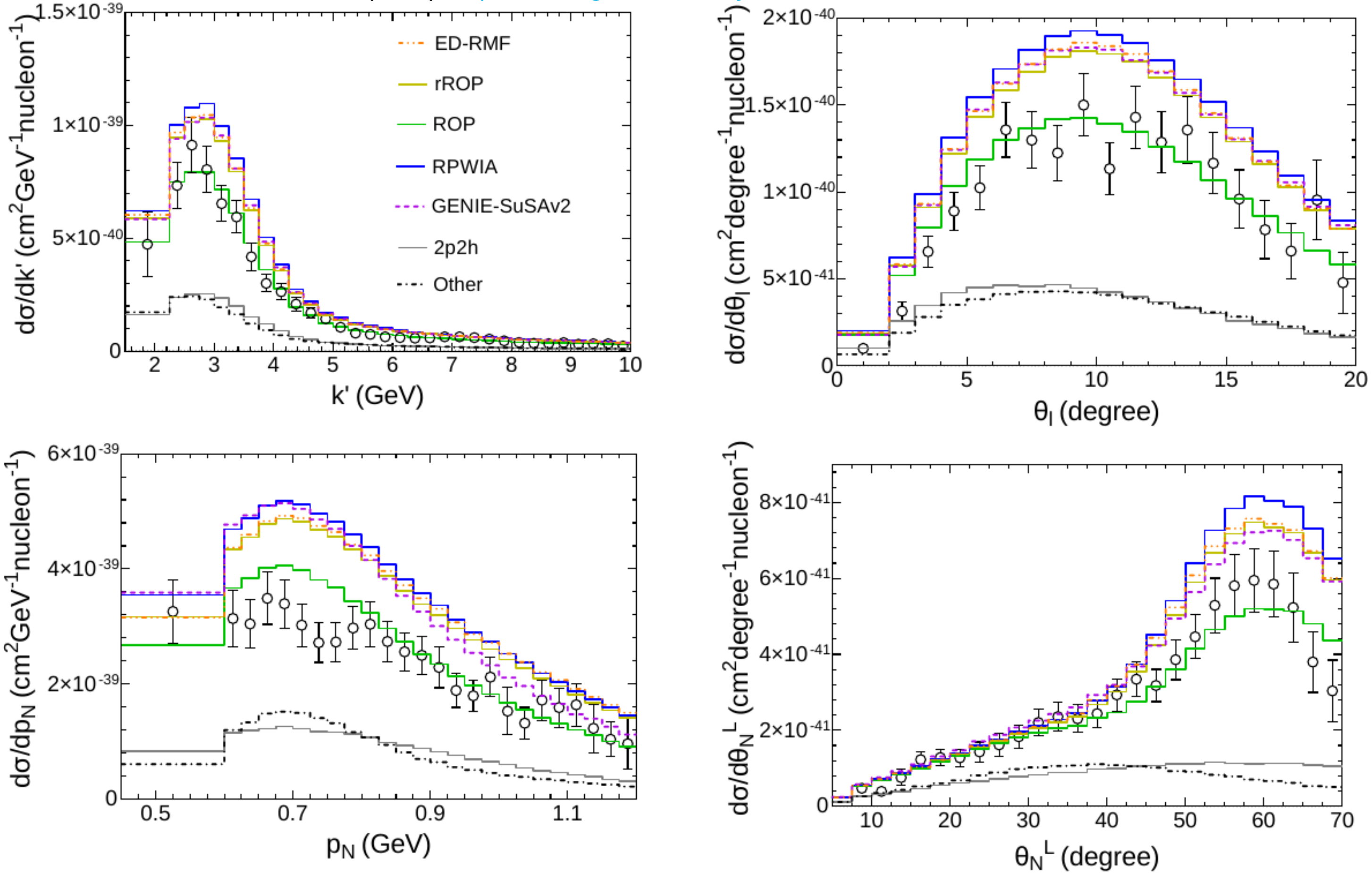
MINERvA neutrino CC 1π^0.



Nikolakopoulos et al. (2018) https://doi.org/10.1103/PhysRevD.97.093008

Contribution of the *pionless delta decay channel(s)* to QE-like signal

Franco-Patino et al. (2022), https://doi.org/10.1103/PhysRevD.106.113005



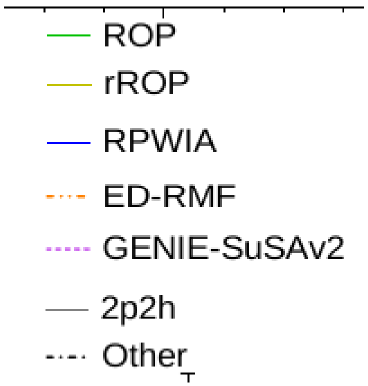
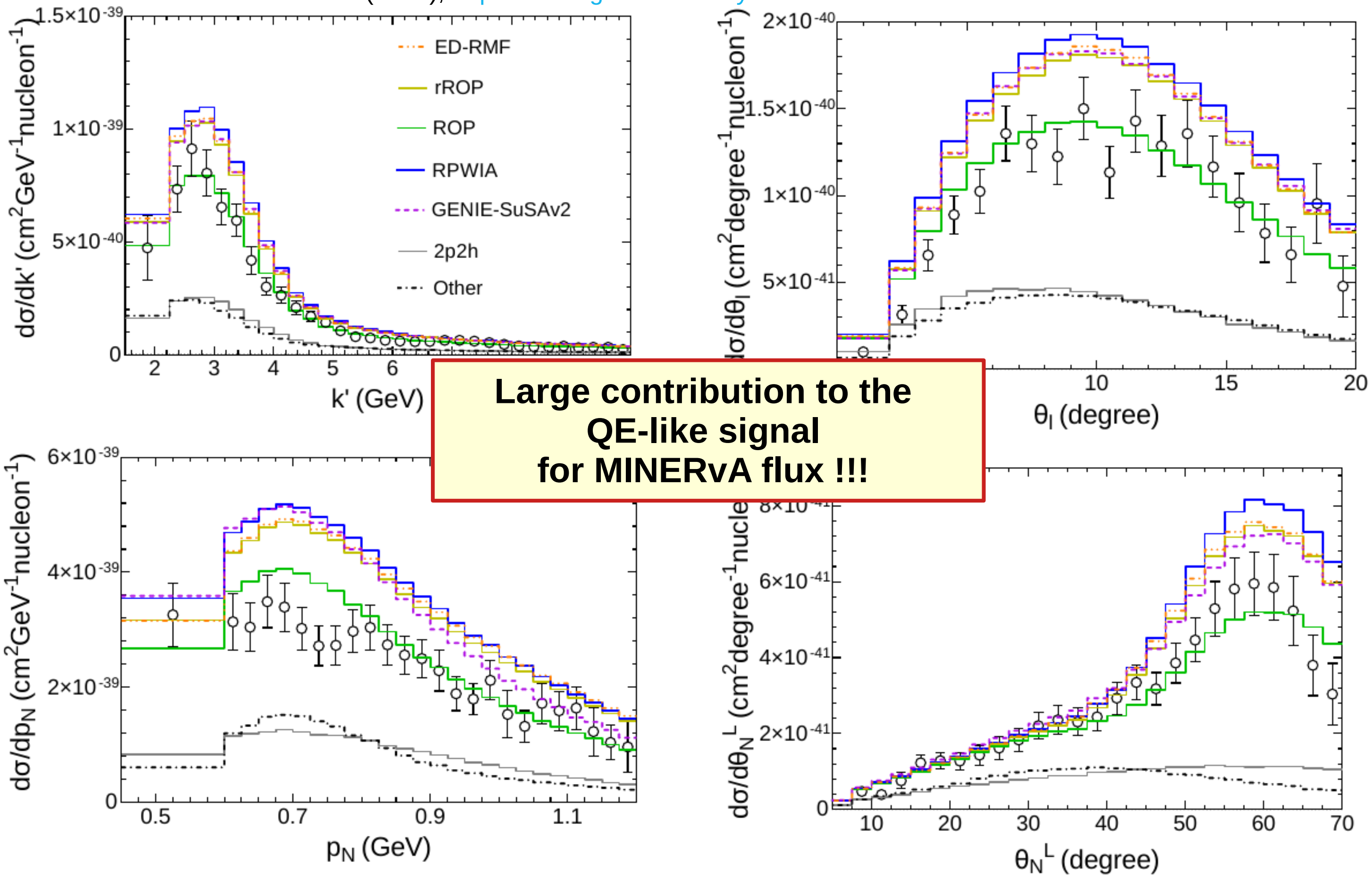


Figure: MINERνA semi-inclusive ν_μ-^{12}C cross section as function of the final muon momentum and scattering angle (top) and as function of the final proton momentum and polar angle (bottom). All curves include the 2p2h and pion absorption contribution (shown separately), evaluated using GENIE.

Contribution of the *pionless delta decay channel(s)* to QE-like signal

Franco-Patino et al. (2022), https://doi.org/10.1103/PhysRevD.106.113005



Large contribution to the QE-like signal for MINERvA flux !!!

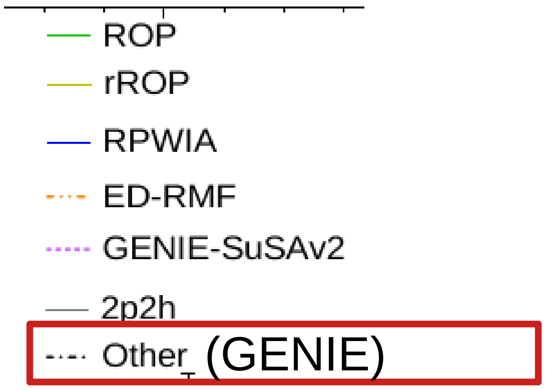


Figure: MINERvA semi-inclusive ν_μ-^{12}C cross section as function of the final muon momentum and scattering angle (top) and as function of the final proton momentum and polar angle (bottom). All curves include the 2p2h and **pion absorption contribution (shown separately), evaluated using GENIE**.

In-medium modification of the resonance properties: a different approach

Medium effects in coherent photo- and electroproduction on 4He and 12C.
Drechsel et al. Nuclear Physics A 660 (1999) 423-438

Oset and Salcedo, https://doi.org/10.1016/0375-9474(87)90185-0

Drechsel et al. Nuclear Physics A 660 (1999) 423-438

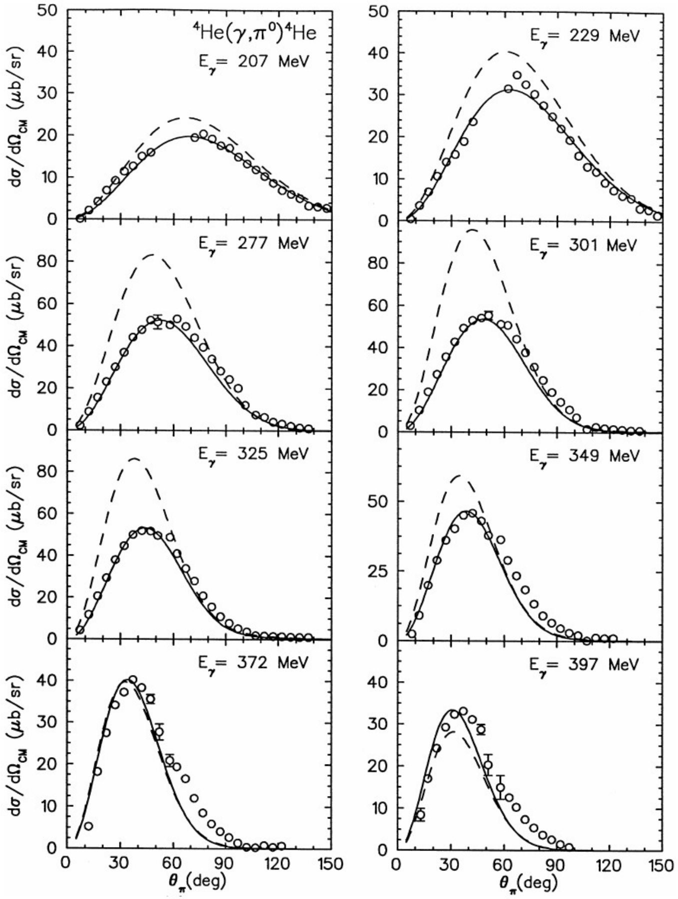


Fig. 3. The differential cross sections for the $^4He(\gamma,\pi^0)^4He$ reaction. The dashed curves are the DWIA results. The solid curves are the results obtained with the F-type (28) parametrizations for the Δ self-energy. Experimental data are from Ref. [38].

1. Start with a specific pion-nucleus optical potential.
2. Parametrize the **in-medium correction** to the delta decay width.

As pointed out in Section 1, we are looking for a phenomenological parametrization of Σ_Δ which should be simple, common for all nuclei and able to describe the available data. For this purpose we shall test two types of parametrization

$$\Sigma_\Delta(E_\gamma, q^2) = V_1(E_\gamma)\, F(q^2), \qquad F(q^2) = e^{-\beta q^2}, \tag{28}$$

$$\Sigma_\Delta(E_\gamma, r) = (A-1)\, V_2(E_\gamma)\, \rho(r)/\rho_0, \qquad \rho_0 = 0.17\,\mathrm{fm}^{-3}, \tag{29}$$

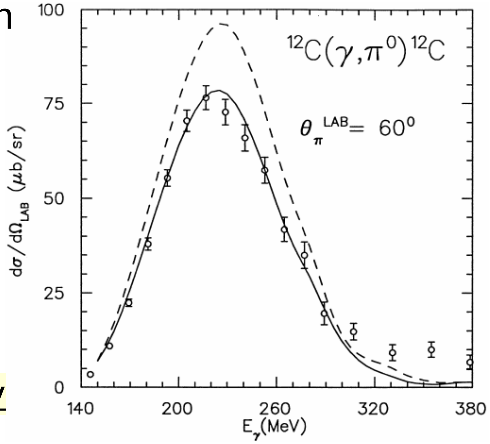
where $V_{1,2}$ is a (complex and energy-dependent) free parameter and $q = |\boldsymbol{k}_\gamma - \boldsymbol{k}_\pi|$ is

3. Fit it to reproduce $^4He(\gamma,\pi^0)^4He$ experimental data.

4. The obtained in-medium correction function is used to predict data on different nuclei, in particular on ^{12}C.

And… it works very well, good!

Conclusion: "Comparing ^4He and ^{12}C we find no A-dependence of the potential and conclude that the Δ-nucleus interaction saturates already for ^4He."



Drechsel et al. Nuclear Physics A 660 (1999) 423-438

1. Start with a specific pion-nucleus optical potential.
2. Parametrize the **in-medium correction** to the delta decay width.

ooking for a phenomenological parametrization
or all nuclei and able to describe the available
es of parametrization

$= e^{-\beta q^2}$, (28)

$\rho_0 = 0.17\,\mathrm{fm}^{-3}$, (29)

parameter and $q = |\boldsymbol{k}_\gamma - \boldsymbol{k}_\pi|$ is

erimental data.

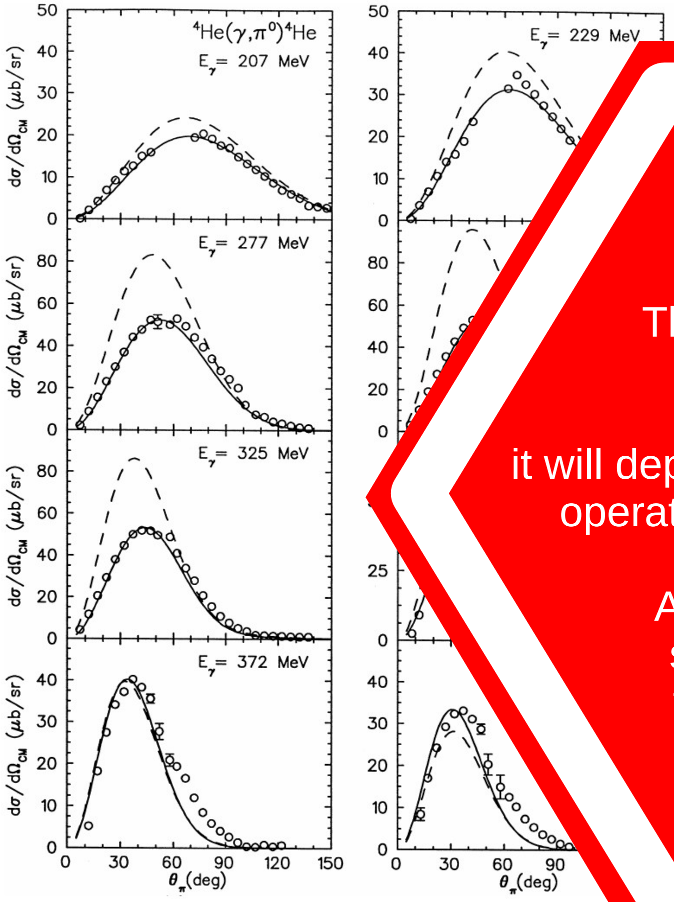


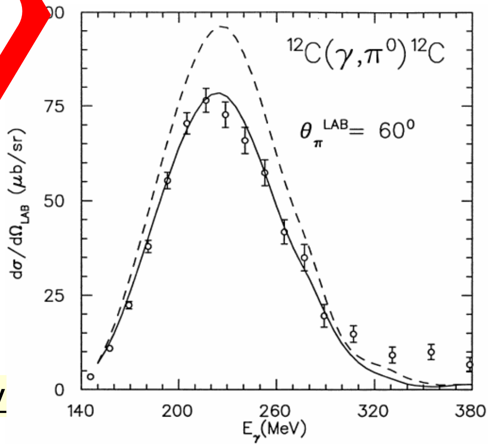
Fig. 3. The differential cross sections for the ^{4}He(γ,π^0)^{4}He reaction. The dashed curv
results. The solid curves are the results obtained with the F-type (28) parametrizations for th
Experimental data are from Ref. [38].

WARNING!!!

This **in-medium correction function** is very much **model dependent**, it will depend, e.g., on the pion-production operator and on the optical potential.

As soon as one changes a single piece of the model this in-medium correction **won't work anymore**…

and
e of the
the Δ-
tes already



In-medium modification of the resonance properties

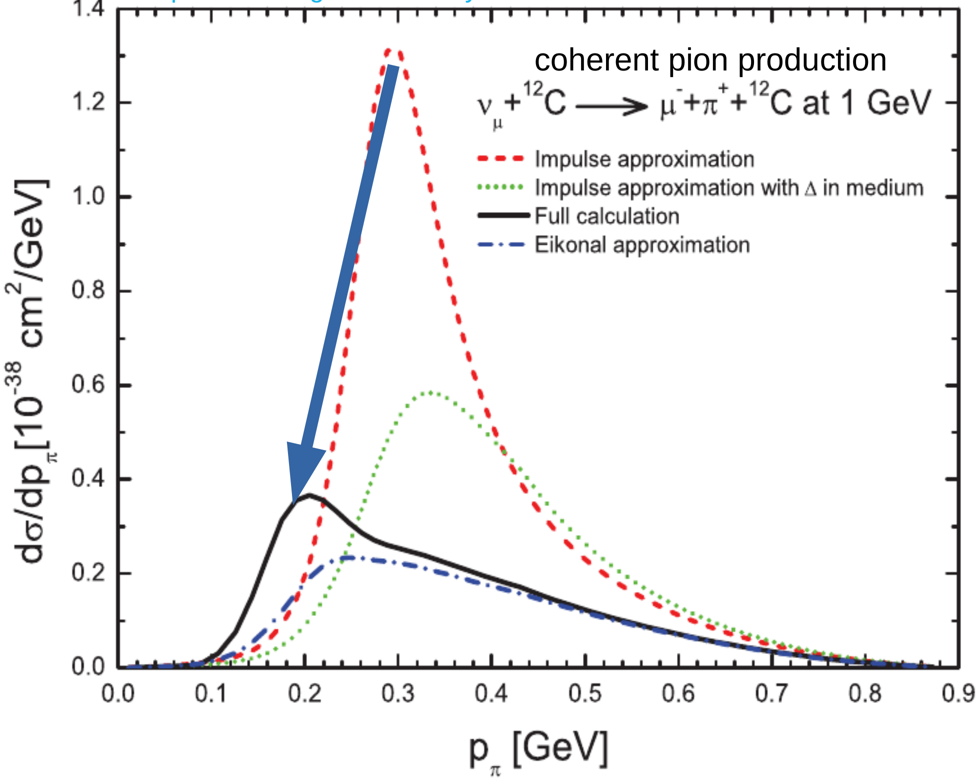
An energy-dependent optical potential fit to **elastic pion-nucleus scattering** data should (by construction) account for the loss of pions

In-medium modification of the resonance properties

An energy-dependent optical potential fit to **elastic pion-nucleus scattering** data should (by construction) account for the loss of pions

why not???

Alvarez-Ruso et al. (2007)
http://dx.doi.org/10.1103/PhysRevC.75.055501



In-medium modification of the resonance properties

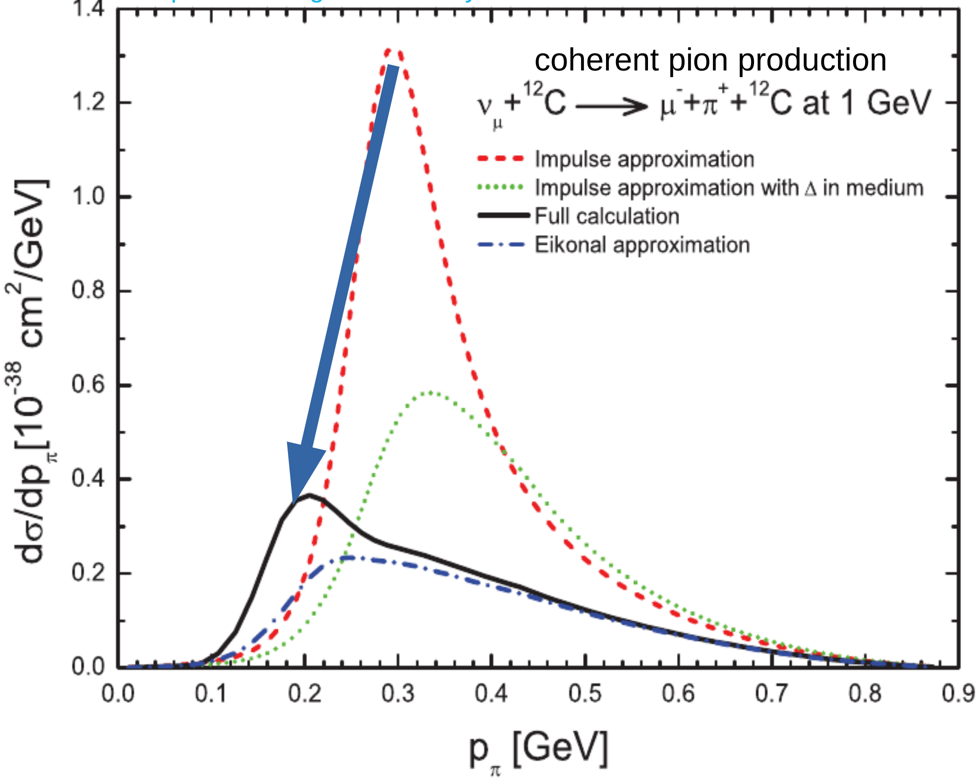
An energy-dependent optical potential fit to **elastic pion-nucleus scattering** data should (by construction) account for the loss of pions

why not???

It would provide solid predictions for 1π production with **minimum nuclear uncertainties.** It would correspond to the process in which the pion only interacts elastically (it passes through the cascade without interactions)

Useful to benchmark cascade models. Analogously to the idea proposed for the QE case in Nikolakopoulos et al. (2022)
https://doi.org/10.1103/PhysRevC.105.054603

Alvarez-Ruso et al. (2007)
http://dx.doi.org/10.1103/PhysRevC.75.055501



In-medium modification of the resonance properties

There is no delta medium modification in this model, just an optical potential.

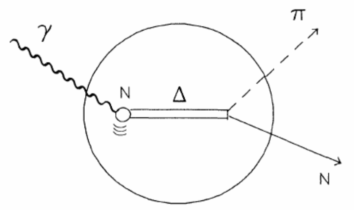


FIG. 1. Diagram of the reaction $A(\gamma, \pi N)B$ in the Δ region. The background Born terms are not shown.

Li, Wright and Benhold (1993), https://doi.org/10.1103/PhysRevC.48.816

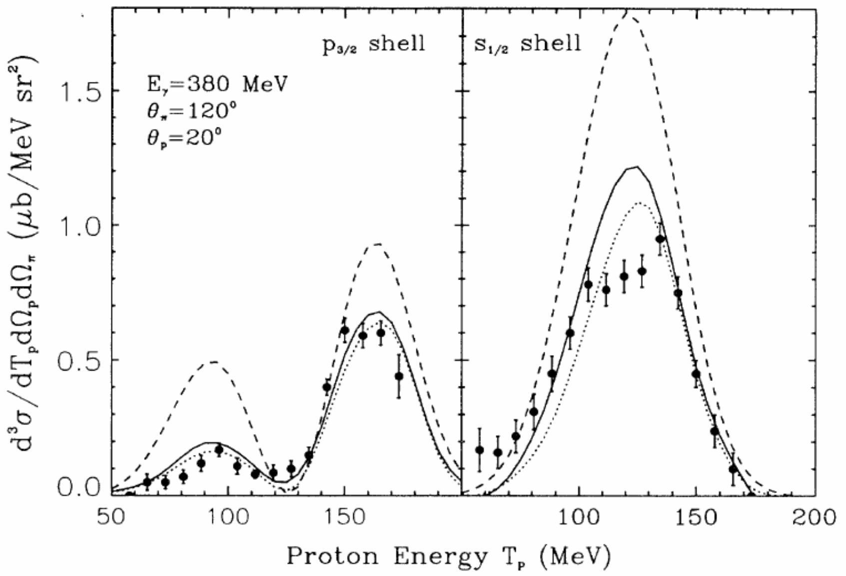
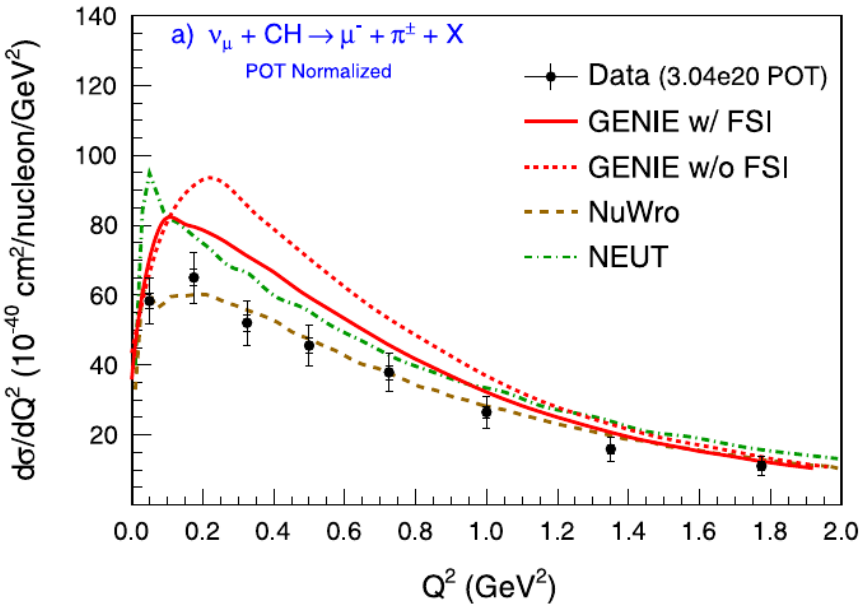


FIG. 2. Proton energy dependence of the triple coincidence cross section from $p_{3/2}$ and $s_{1/2}$ shell neutrons in $^{12}C(\gamma, \pi^- p)^{11}C$ for fixed E_γ, θ_π, and θ_p. Theoretical curves are calculated in PWIA (dashed line), local DWIA (dotted line), and nonlocal DWIA (solid line). Data are taken from Ref. [13].

Inelastic final-state interactions

Necessary to make predictions about the hadron multiplicity in the final state.

Inelastic final-state interactions: "*In Cascade we trust*"



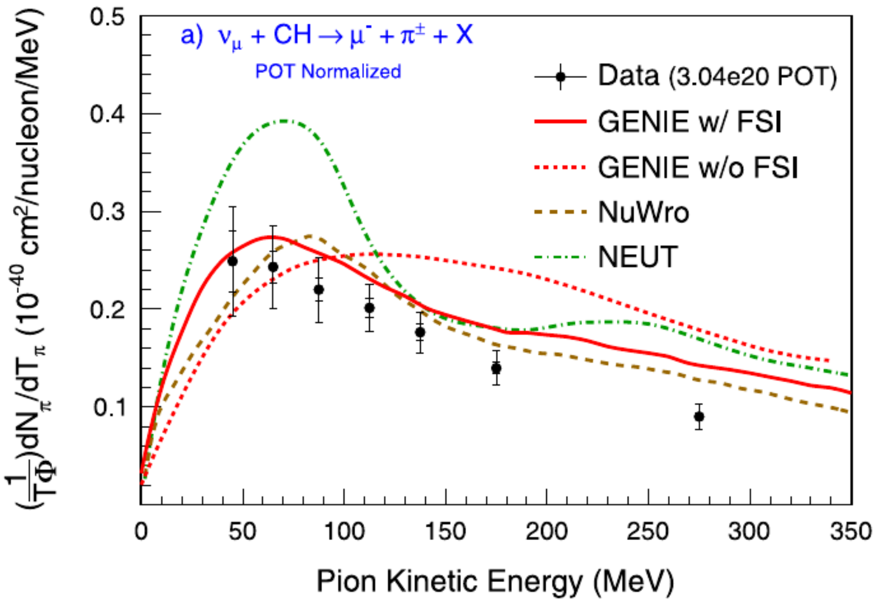


Fig. 13. Comparisons of event generator calculations with MINERνA $\nu_\mu CH$ CC π^+ data [290] (left) Q^2 and (right) kinetic energy. Both results include resonances at $W < 1.8$ GeV.

Inelastic final-state interactions: "*In GiBUU we trust*"

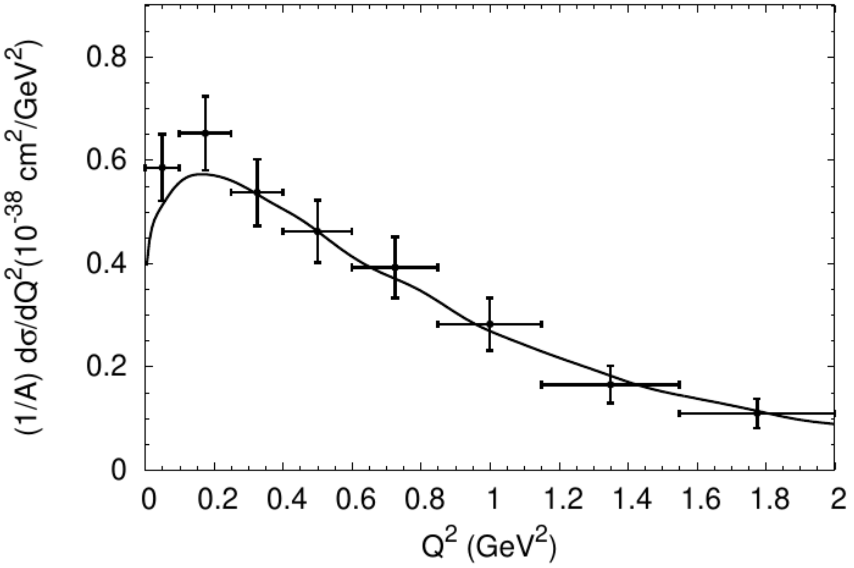


FIG. 12. Q^2 distribution of multiple charged pions in the MINERvA flux for a CH target with $W_{\rm rec} < 1.8$ GeV. Data are from [10]

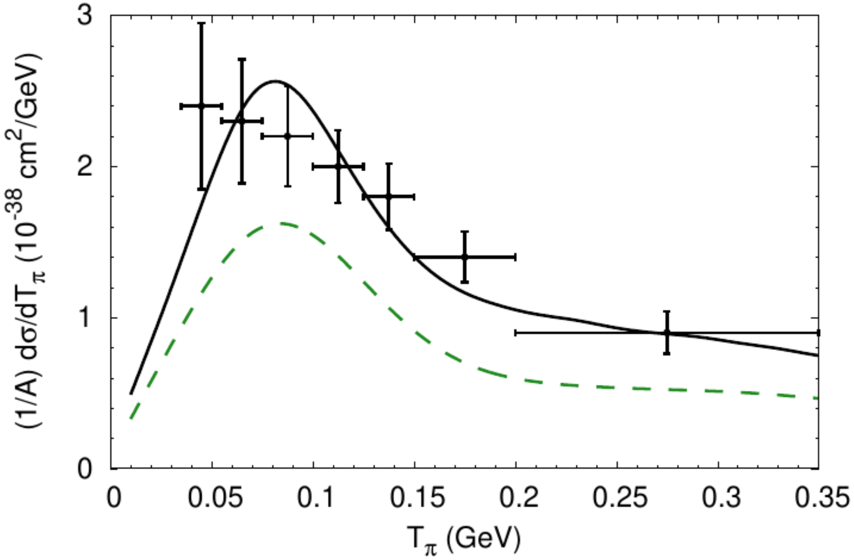


FIG. 8. Kinetic energy spectrum per nucleon of multiple charged pions in the MINERvA flux for a CH target with $W_{\rm rec} < 1.8$ GeV (solid line). The dashed line gives the 1-pion contribution. Data are from [10]

Beyond Impulse Approximation: two-body currents in the 1p-1h sector

https://arxiv.org/abs/2203.09996v2
https://arxiv.org/abs/2306.10823

Beyond Impulse Approximation: two-body currents in the 1p-1h sector

$$J^\mu_{had} = \int d\mathbf{p}\,\overline{\Psi}_F(\mathbf{p}+\mathbf{q},\mathbf{p}_N)\,\left(\mathcal{O}^\mu_{\text{one body}} + \mathcal{O}^\mu_{\text{two body}}\right)\,\Psi_B(\mathbf{p})$$

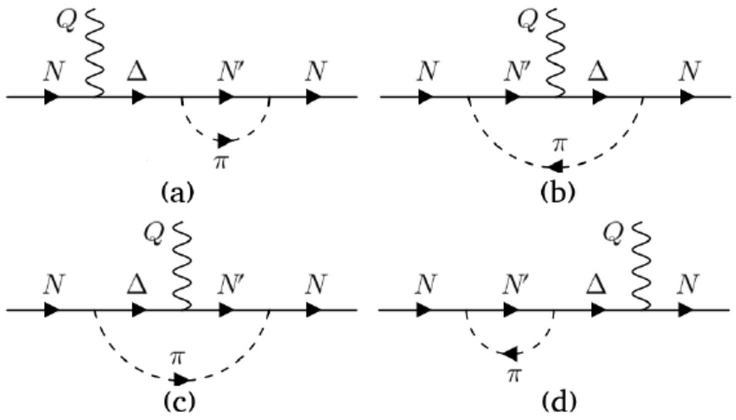


FIG. 1. Delta contributions.

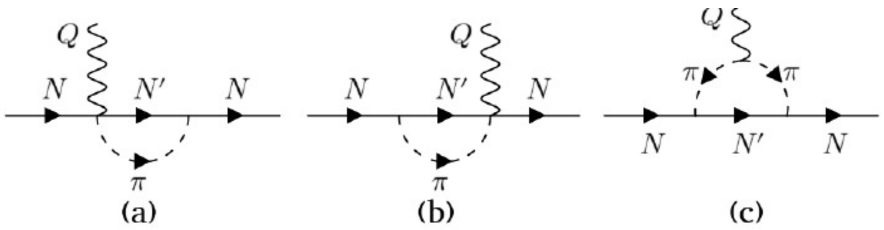
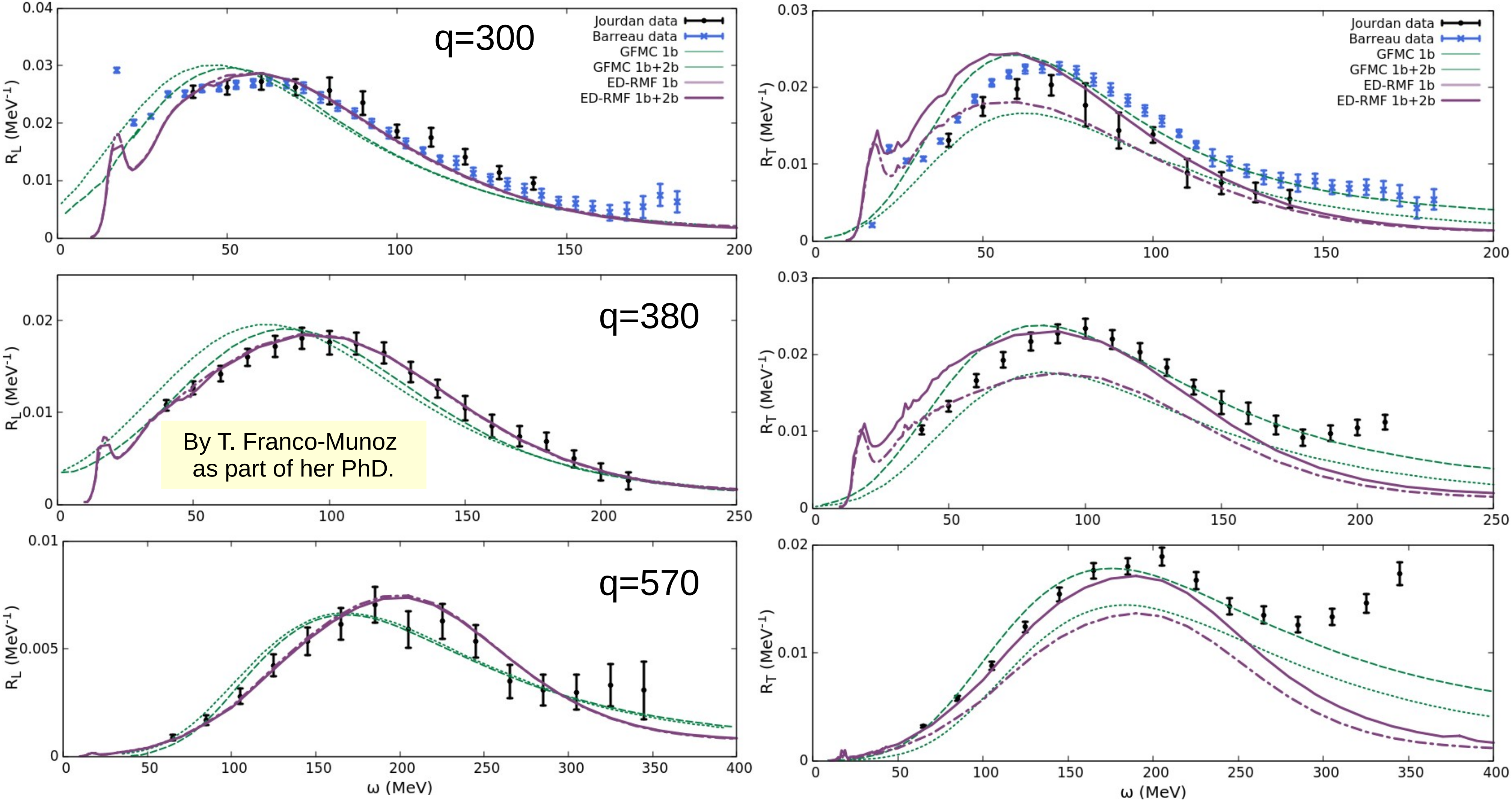


FIG. 2. Background contributions: seagull or contact [CT, (a) and (b)] and pion-in-flight [PF, (c)].

https://arxiv.org/abs/2203.09996v2
https://arxiv.org/abs/2306.10823

Carbon 12 responses

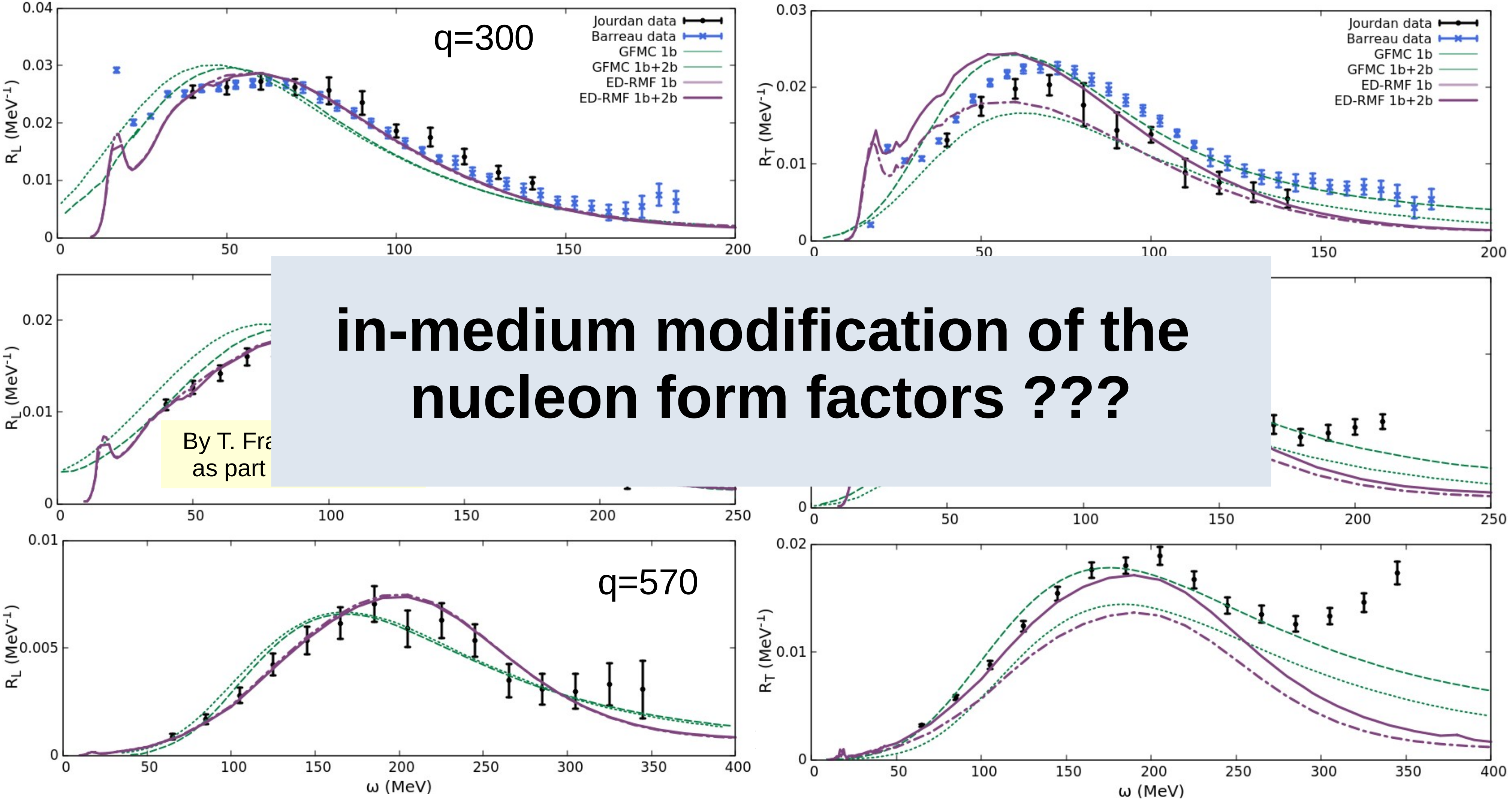
green lines from Lovato et al.
PRL 117, 082501 (2016)



https://arxiv.org/abs/2203.09996v2
https://arxiv.org/abs/2306.10823

Carbon 12 responses

green lines from Lovato et al.
PRL 117, 082501 (2016)



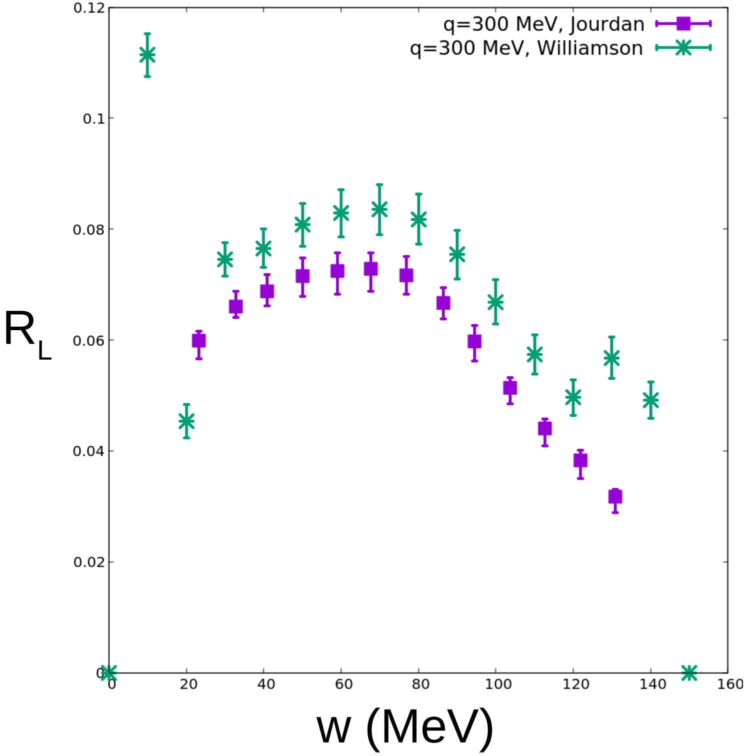
in-medium modification of the nucleon form factors ???

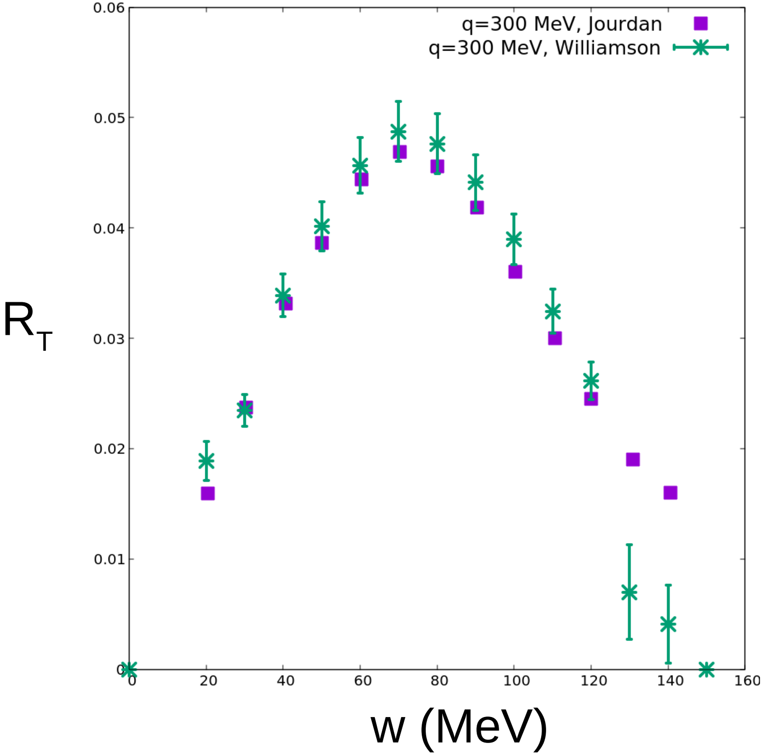
Heavier nuclei?

Calcium 40
(same amount of nucleons as our favorite nucleus: **Argon 40**)

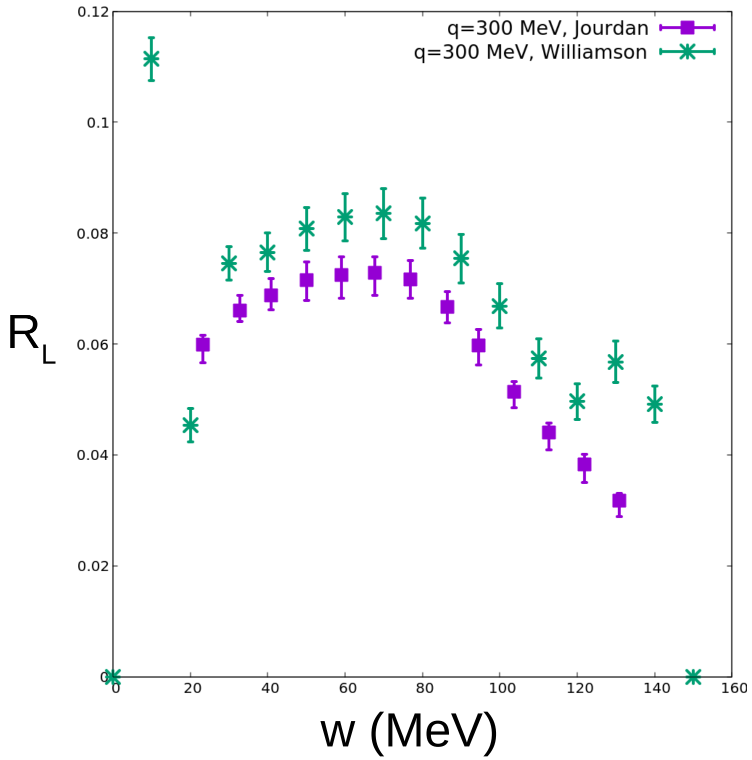
Data from:
Jourdan, NPA 603, 117-160 (1996).
Meziani et al., PRL52, 2130 (1984).
Meziani et al., PRL52, 1233 (1985).
Williamson et al., PRC56, 3152 (1997).

L and T Responses: Rosenbluth separation

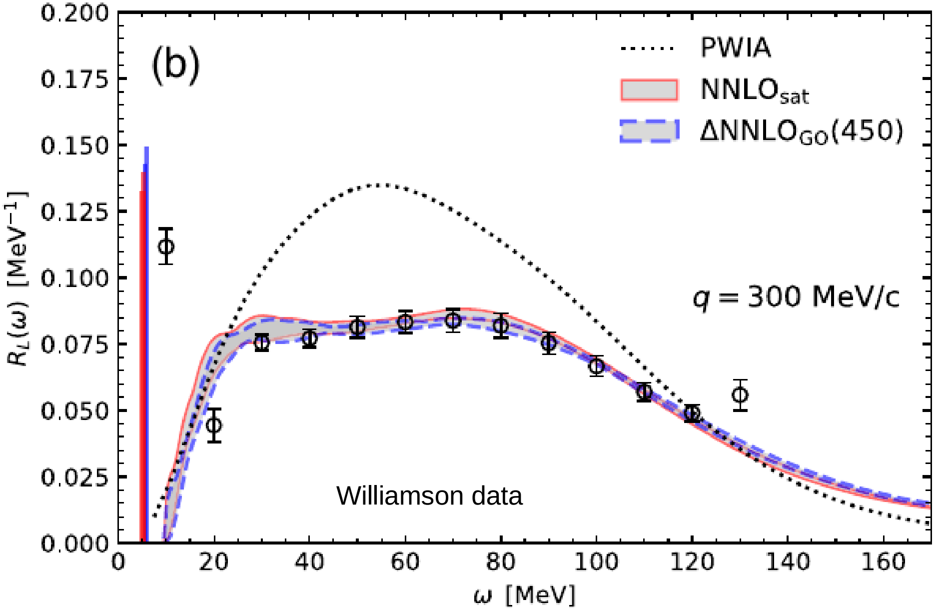




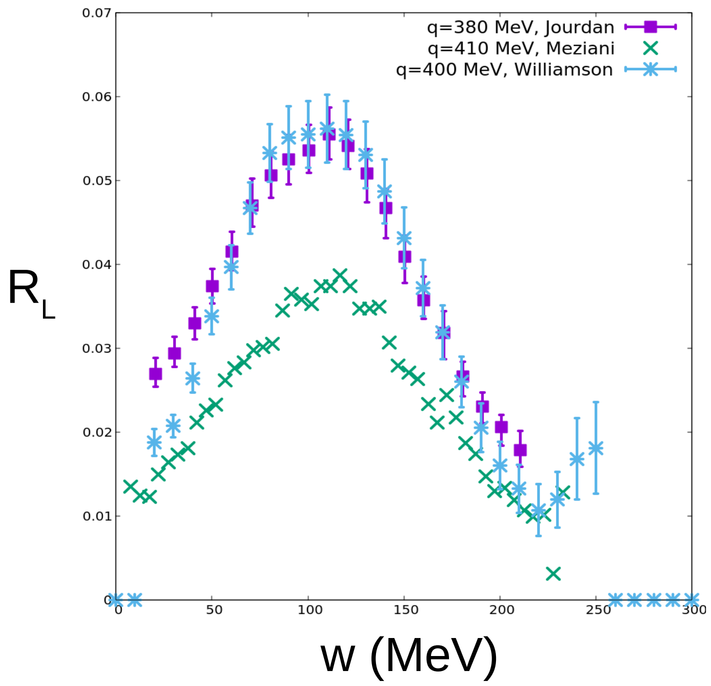
L and T Responses: Rosenbluth separation

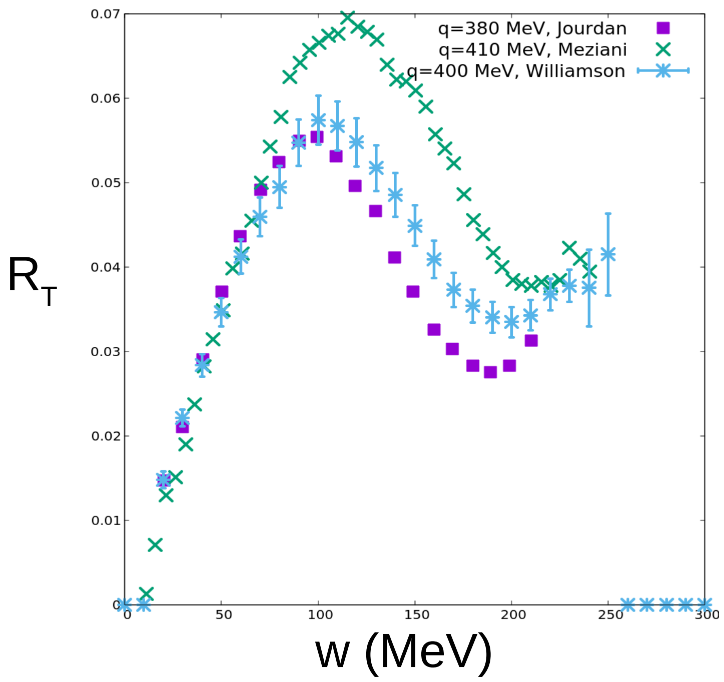


Sobczyk et al. https://doi.org/10.1103/PhysRevLett.127.072501

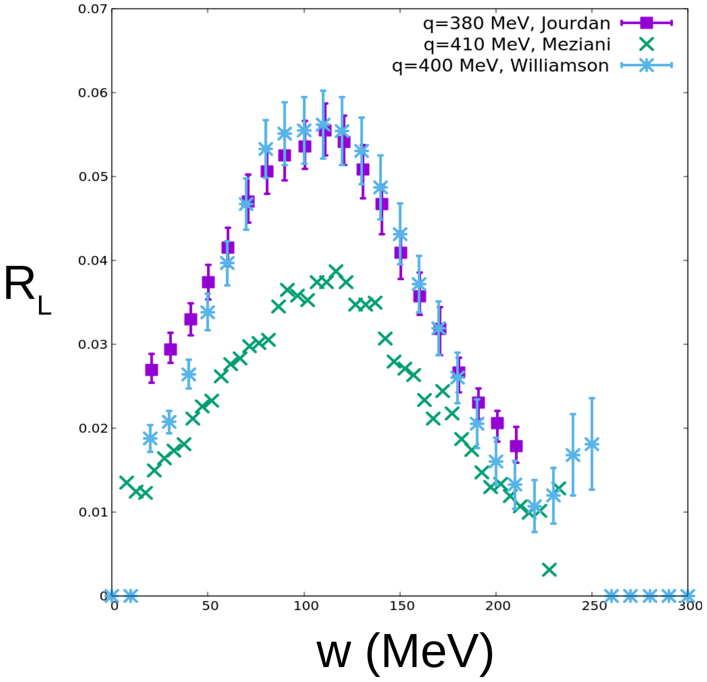


L and T Responses: Rosenbluth separation

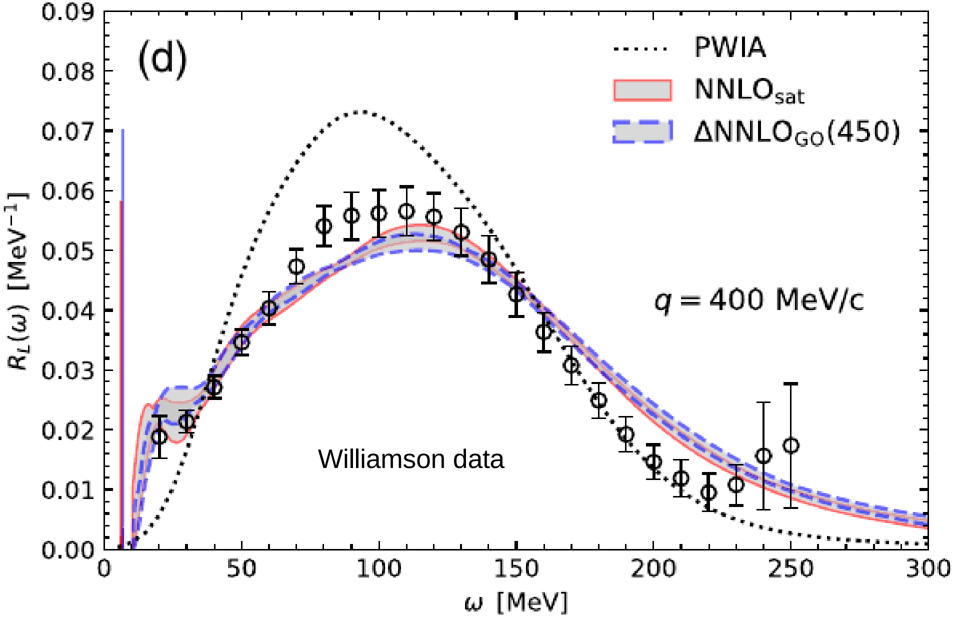




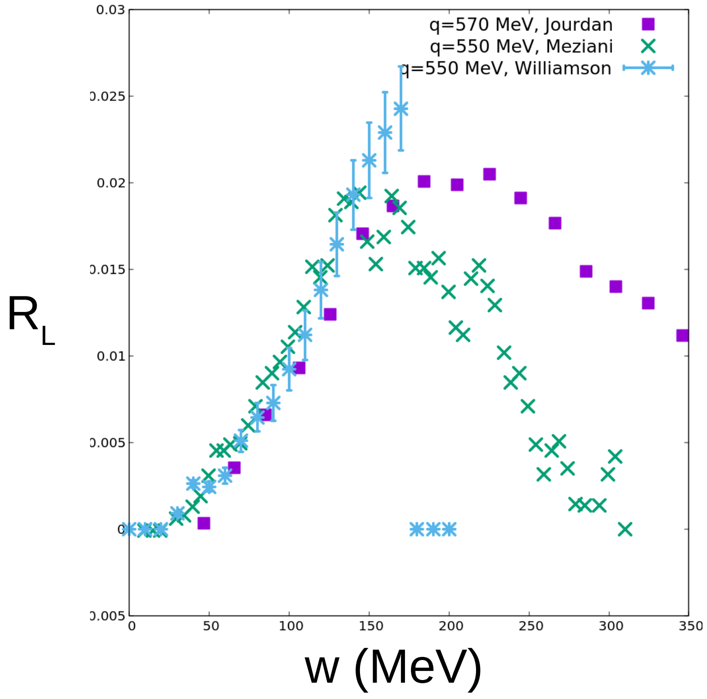
L and T Responses: Rosenbluth separation

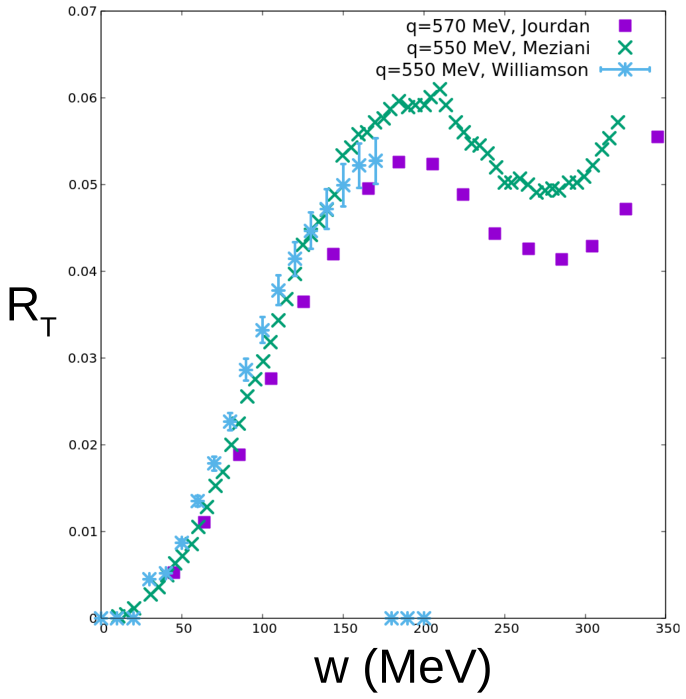


Sobczyk et al. https://doi.org/10.1103/PhysRevLett.127.072501



L and T Responses: Rosenbluth separation





L and T Responses: Rosenbluth separation

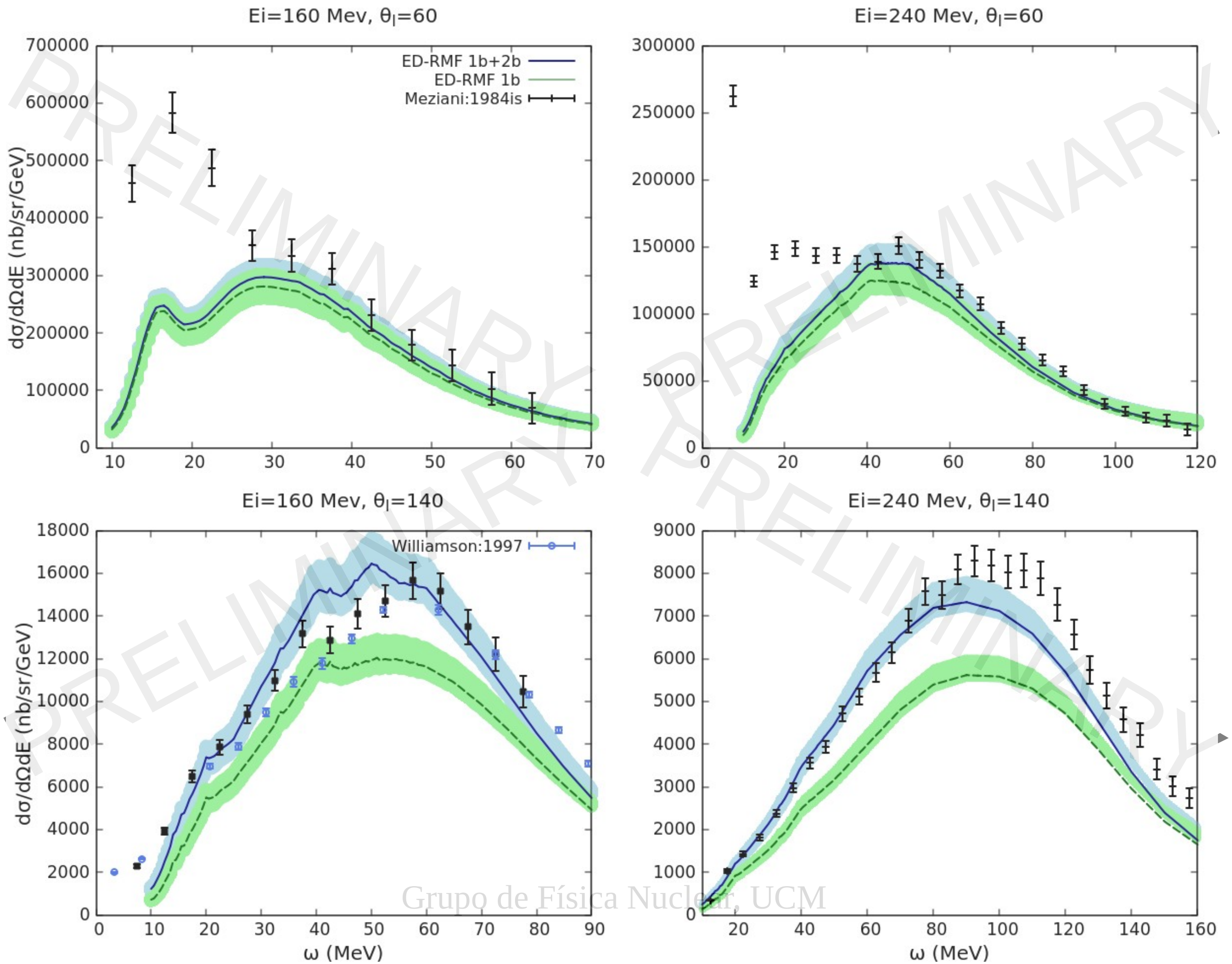
Main problem: **Coulomb distortion** of the electron **breaks** the factorization

lepton tensor x **hadron tensor** (it’s no longer valid)

Heavier nuclei (more protons) means larger Coulomb effects.

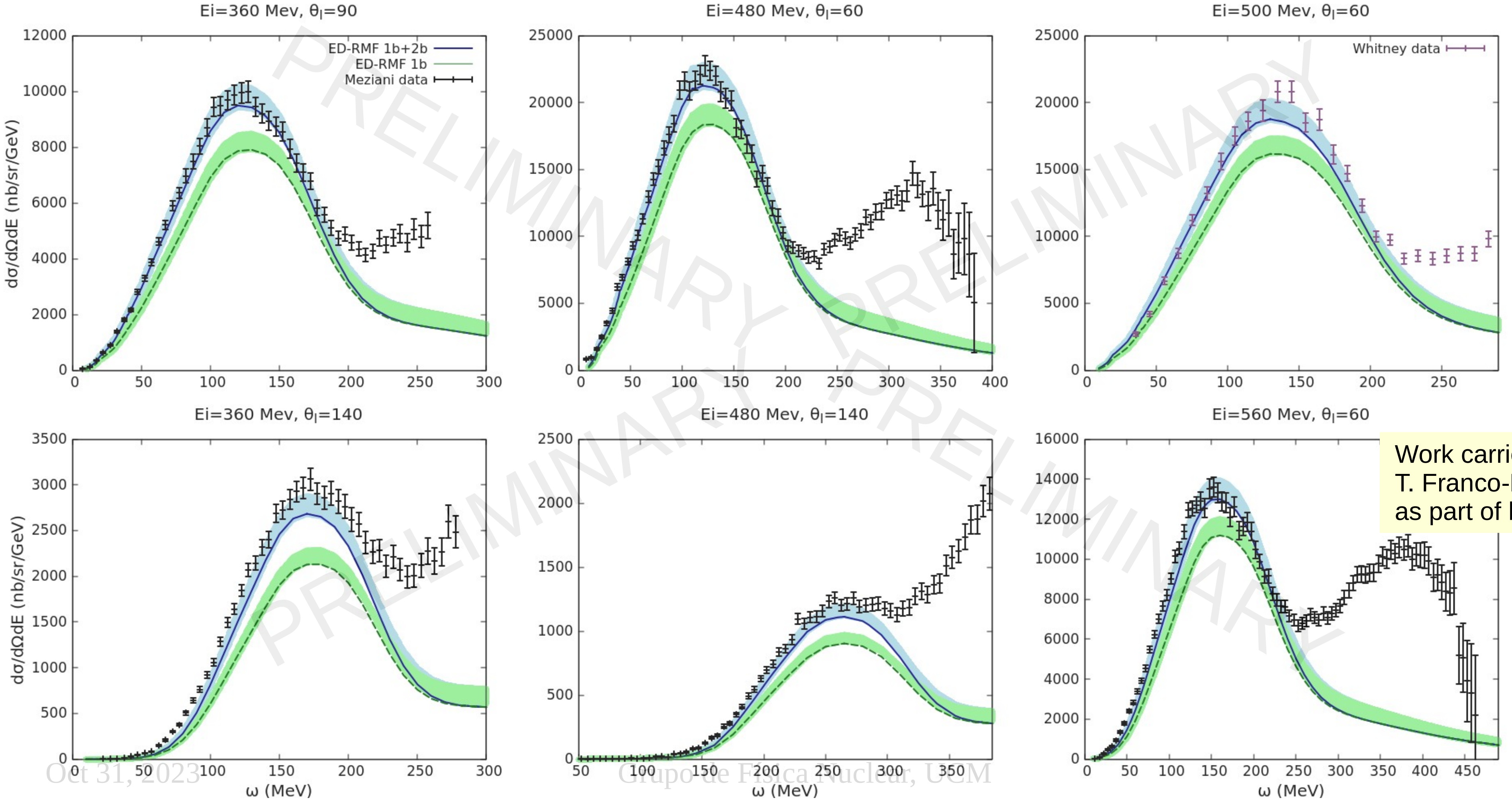
Cross sections are generally more reliable than responses.

Calcium 40 cross sections



Work carried out by T. Franco-Munoz as part of her PhD.

Calcium 40 cross sections



Work carried out by T. Franco-Munoz as part of her PhD.

Final remarks: Pauli blocking and Elastic FSI

Classical CASCADE models do NOT affect the inclusive cross section, therefore, it is important to use models of the primary vertex that provide realistic predictions of the inclusive cross section.

For consistency, **the model of the primary vertex should provide full information on the hadron(s),** which will propagate through nucleus via cascade.

+ A quantum mechanical treatment is essential to reproduce the features of the inclusive cross section, due to Pauli blocking and distortion effects, and corrections beyond the impulse approximation (two-body currents).

Final remarks: Pauli blocking and Elastic FSI

Classical <u>CASCADE models do NOT affect the inclusive cross section</u>, therefore, it is important to use models of the primary vertex that provide realistic predictions of the inclusive cross section.

For consistency, **the model of the primary vertex should provide full information on the hadron(s),** which will propagate through nucleus via cascade.

+ A <u>quantum mechanical treatment</u> is essential to reproduce the features of the inclusive cross section, due to Pauli blocking and distortion effects, and corrections beyond the impulse approximation (two-body currents).

Final remarks: in-medium modification of the resonance properties

Pions that one expects should be there are not there.

<u>How to model it?</u>
Very much model dependent: Nature only informs us about initial and final states (the rest is up to you...)

IMPORTANT: Try to be consistent and avoid double-counting.

ADDITIONAL SLIDES

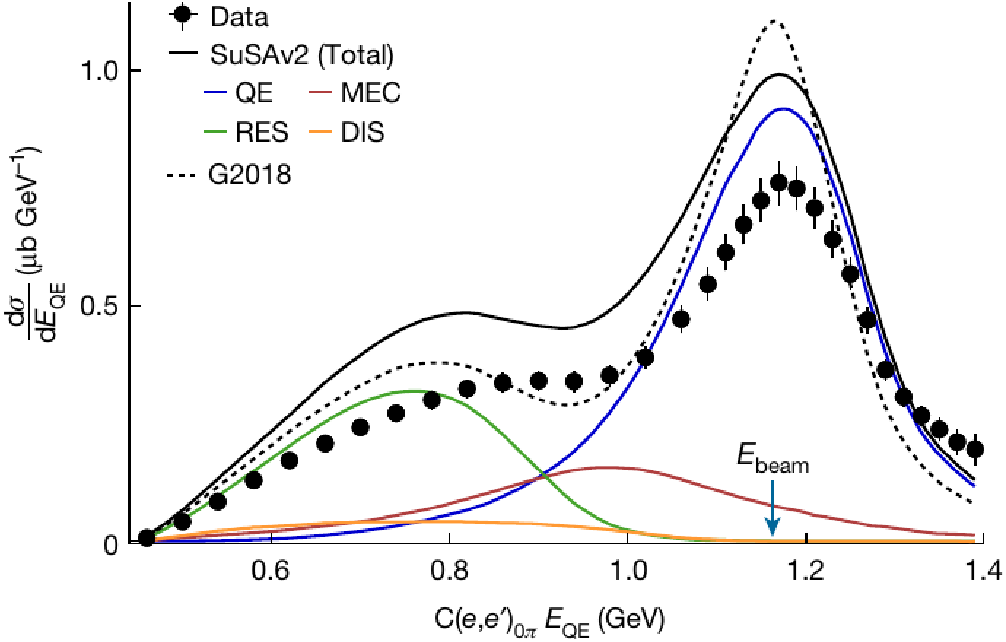
Article

Electron-beam energy reconstruction for neutrino oscillation measurements

https://doi.org/10.1038/s41586-021-04046-5

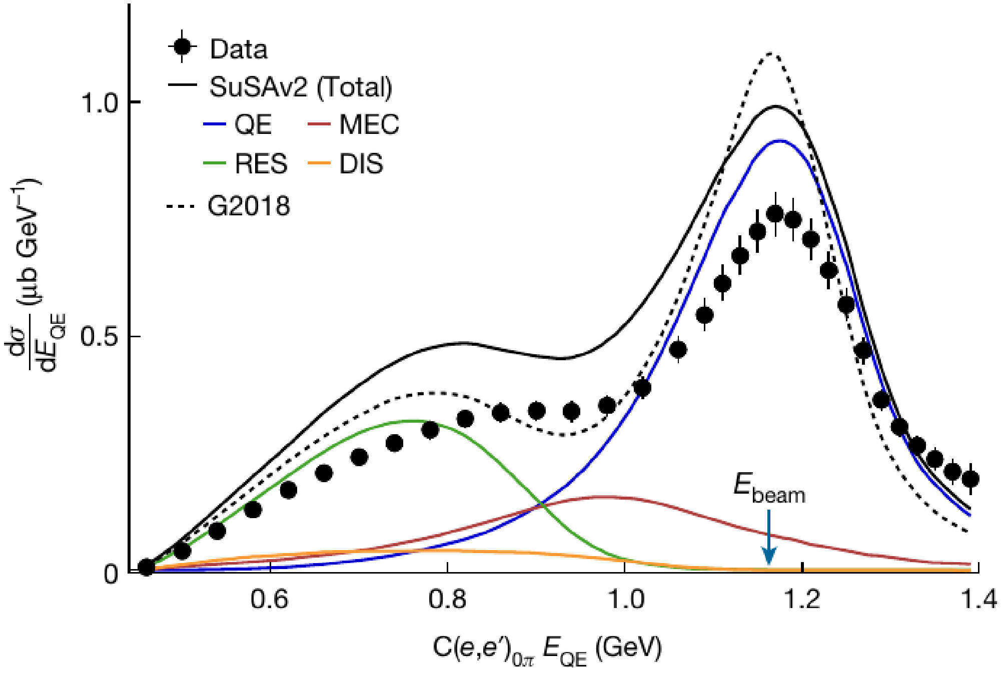
Received: 29 June 2020

M. Khachatryan[1,56], A. Papadopoulou[2,56], A. Ashkenazi[2✉], F. Hauenstein[1,2], L. B. Weinstein[1], O. Hen[2], E. Piasetzky[3], the CLAS Collaboration* & e4v Collaboration*



E(beam) = 1.159 GeV
scattering angle: 15° ≤ θe ≤ 45°

e4nu collaboration (June 2020)
https://doi.org/10.1038/s41586-021-04046-5

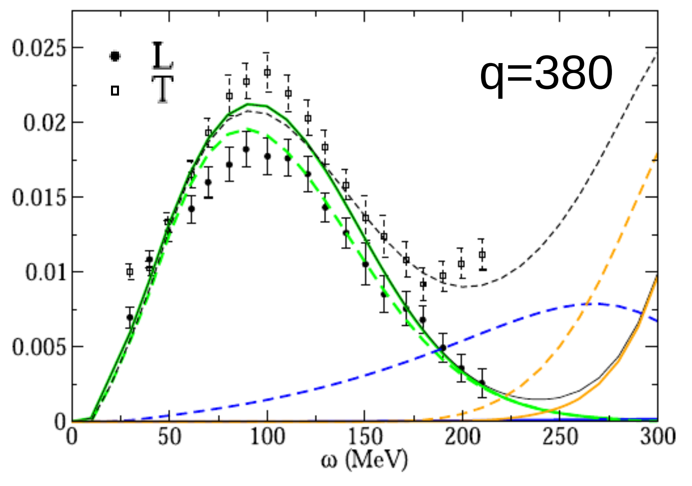


Ebeam = 1.159, and angles 15° ≤ θe ≤ 45°

(I'll focus on the QE peak.)
So far, SuSAv2+MEC has proven to be able to reproduce quite well all inclusive (e,e') data.
So, <u>what's going on here</u>? A possibility:

The **Longitudinal** response plays an important role in this sample, and SuSAv2 overestimates the L response for carbon 12

(https://doi.org/10.1088/1361-6471/abb128)



Longitudinal: solid lines.
Transverse: dashed lines.

J. Phys. G: Nucl. Part. Phys. 47 (2020) 124001

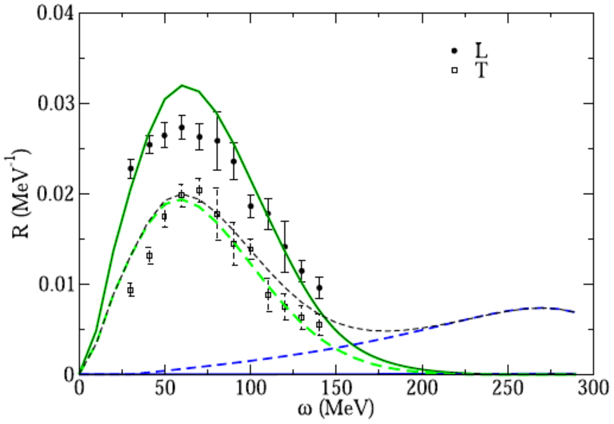
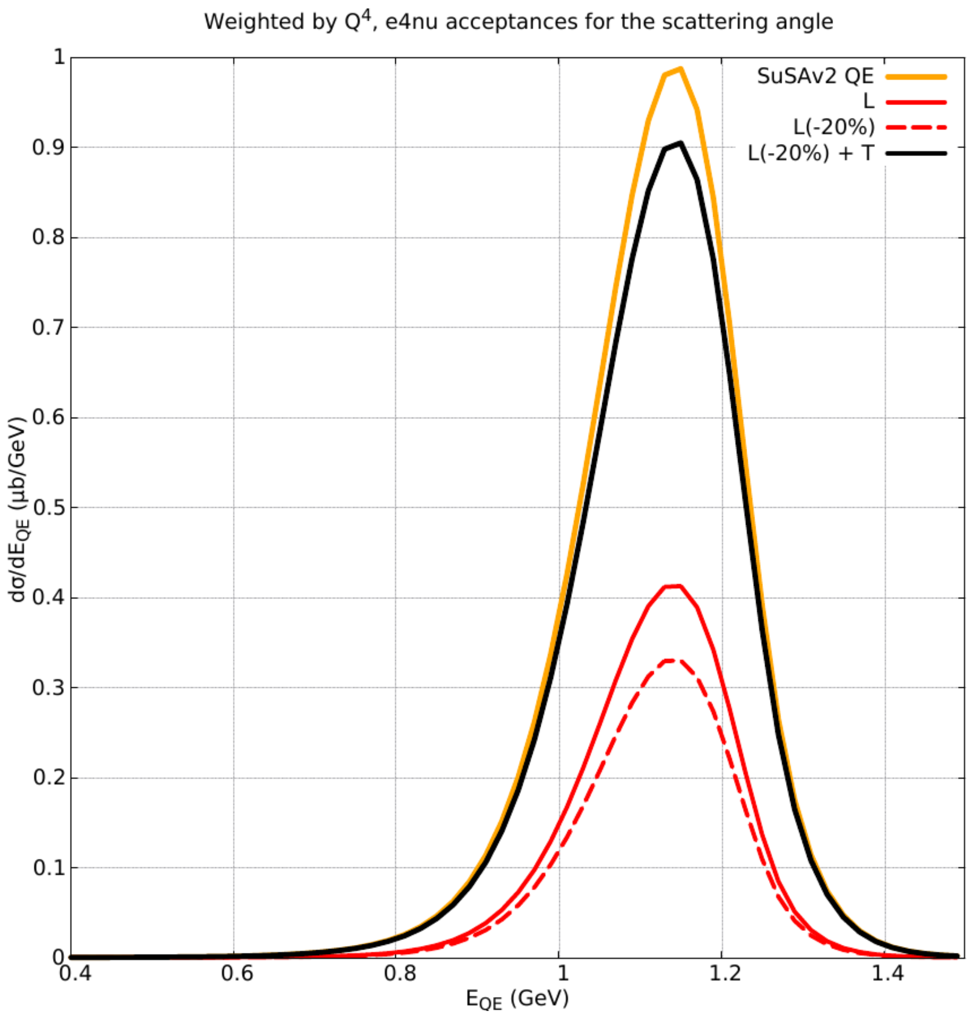


Figure 36. Analysis of the longitudinal (solid lines) and transverse responses (dashed lines) in (e, e′) scattering at $q = 300$ MeV/c (left panel), $q = 380$ MeV/c (middle panel) and $q = 570$ MeV/c (right panel). QE, MEC and inelastic contributions are shown, respectively, as green, blue and orange lines. The total response is shown by the black lines. Data taken from [21].

Rescaling the L response by ~20% down means that the QE prediction is ~10% lower.



In-medium modification of the resonance properties: *à la* Oset

C. Praet PhD Thesis (2009), Ghent University
https://biblio.ugent.be/publication/734583

$$\Delta\Gamma = \widetilde{\Gamma} - \Gamma,$$

$$\widetilde{\Gamma} = \Gamma_{\text{Pauli}} - 2\,\Im(\Sigma_\Delta),$$

$$-\Im(\Sigma_\Delta) = C_{QE}\left(\frac{\rho}{\rho_0}\right)^{\alpha} + C_{A2}\left(\frac{\rho}{\rho_0}\right)^{\beta} + C_{A3}\left(\frac{\rho}{\rho_0}\right)^{\gamma},$$

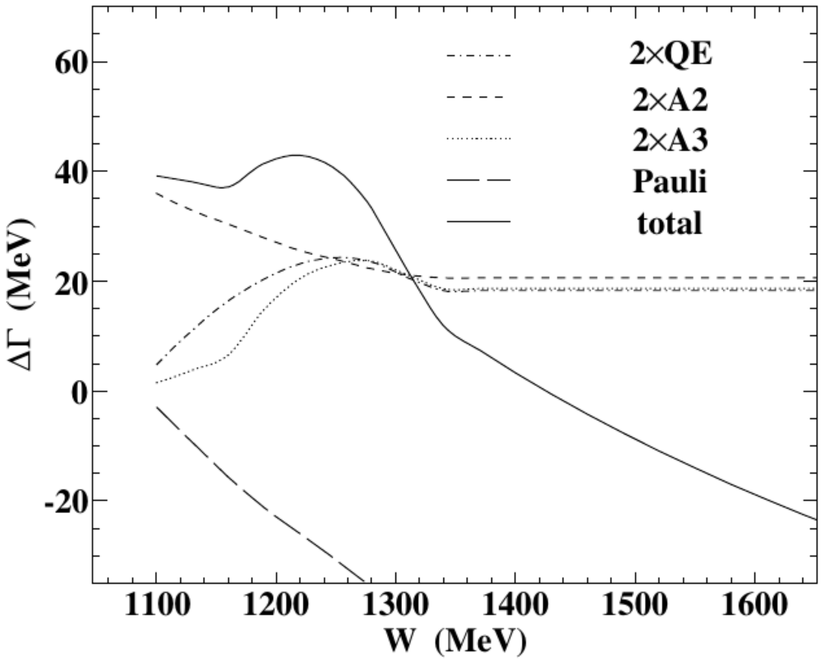


Figure 3.10: Overview of medium corrections to the free Δ width, using the parameterizations in Refs. [29, 176] for $\rho = 0.75\rho_0$.

In-medium modification of the resonance properties: *à la* Oset

Nikolakopoulos et al. (2018) https://doi.org/10.1103/PhysRevD.97.093008

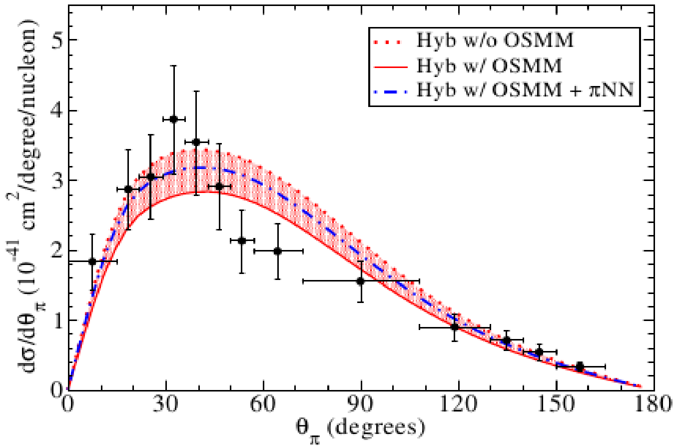


FIG. 3: MINERvA ν-induced $1\pi^+$ production sample [5] compared with RPWIA predictions. Solid (dotted) line is the result with (without) medium modification of the Delta width. The dash-dotted line is the result with OSMM when the contribution from the $\Delta N \to \pi N N$ channel is added to the cross section. The results were computed with the Hybrid model (see Sec. IV B).

L and T Responses: Rosenbluth separation

Udías et al. (1993)
https://doi.org/10.1103/PhysRevC.48.2731

See also works by Giusti, Pacati and coll., e.g.,NPA (1987),
https://doi.org/10.1016/0375-9474(87)90276-4

$$\frac{d^4\sigma}{d\epsilon_f d\Omega_f dE_F d\Omega_F} = \frac{\delta(\epsilon_i + E_A - \epsilon_f - E_F - E_{A-1})}{(2\pi)^5} \times 4\alpha^2 \epsilon_f^2 E_F |\mathbf{P}_F| \overline{\sum} |W_{if}|^2 , \quad (2.1)$$

where $\overline{\Sigma}$ indicates sum (average) over final (initial) polarizations and

$$W_{if} = \int d\mathbf{x} \int d\mathbf{y} \int \frac{d\mathbf{q}}{(2\pi)^2} j_\mu^e(\mathbf{x}) e^{-i\mathbf{q}\cdot(\mathbf{x}-\mathbf{y})} \frac{(-1)}{q_\mu^2} J_N^\mu(\mathbf{y}) \; . \quad (2.2)$$

In this expression j_μ^e and J_N^μ stand for the electron and nuclear currents, respectively. The electron current is given by the well-known pointlike Dirac particle expression:

$$j_e^\mu(\mathbf{r}) = \bar{\psi}_f^e(\mathbf{r}) \gamma^\mu \psi_i^e(\mathbf{r}) \; , \quad (2.3)$$

where ψ_i^e, ψ_f^e stand for initial and final electron wave functions. In IA and within an independent particle model picture, the nuclear current can be written in terms of the nucleon current operator $\hat{J}_N^\mu$:

$$J_N^\mu(\mathbf{r}) = \bar{\psi}_F^N(\mathbf{r}) \hat{J}_N^\mu \psi_B^N(\mathbf{r}) \; , \quad (2.4)$$

with ψ_B^N, ψ_F^N the wave functions for the initial bound nucleon and final nucleon, respectively, and $\hat{J}_N^\mu$ a nucleon current operator to be specified later.