QCD Axion Condensates and Neutron Star Radial Oscillations

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Based on recent work:
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arXiv:2510.16245 (2025)

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In this talk

- Neutron Stars
- QCD axion
- Light QCD axion
- Impact on NS structure
- Stellar oscillations
- Impact on NS radial oscillations
- Conclusions

Neutron Stars

Typical scales:

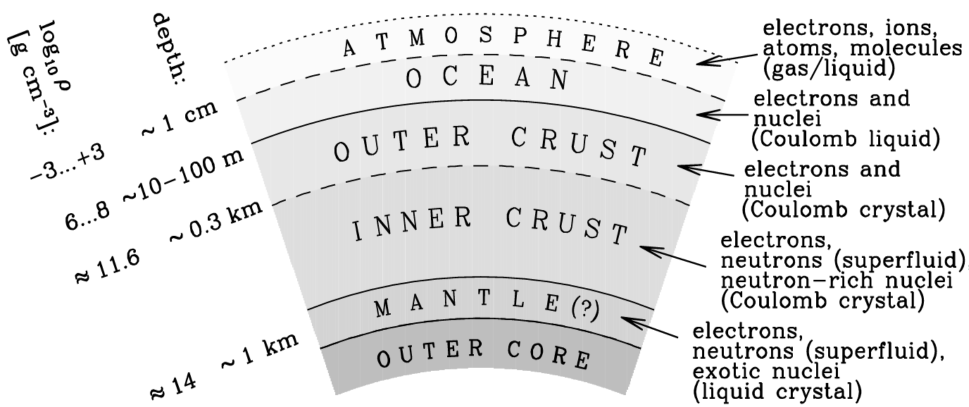
- **Mass**: $1 - 2\ M_\odot$
- **Radius**: $10 - 12\ \mathrm{km}$
- **Density**: $2{,}3 \times 10^{14}\ \mathrm{g/cm^3}$
- **Surface gravity**: $10^{11} g_{\mathrm{Earth}}$
- **Magnetic fields**: $B \sim 10^{11} - 10^{13}\mathrm{G}$ (up to $10^{15}\mathrm{G}$ in magnetars)
- **Rotation**: up to a hundred times per second
- **Population:** over 3000 have been observed

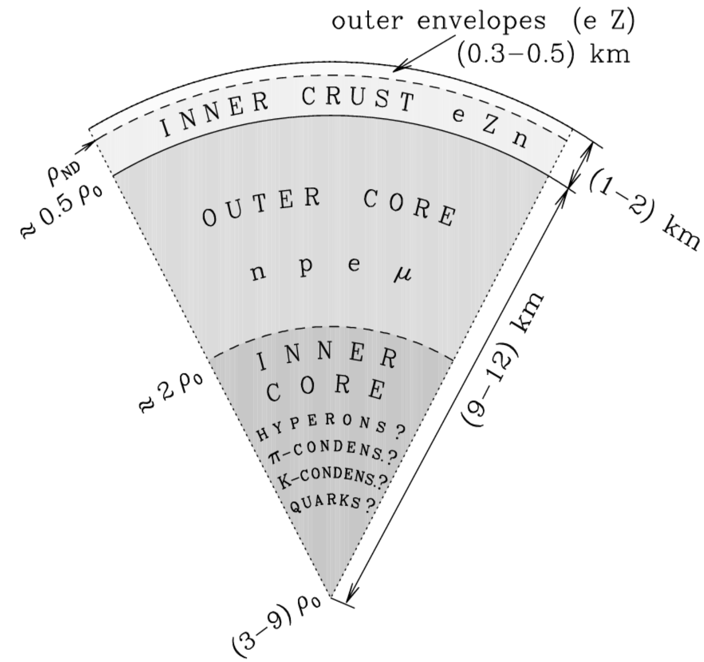
Laboratories for extreme physics:

- Nuclear matter at supranuclear densities
- Strong-field gravity
- Exotic phases: superfluidity, hyperons, quark matter, DM particles

Neutron Stars

- **Main regions**: envelope, outer and inner crust, outer and inner core



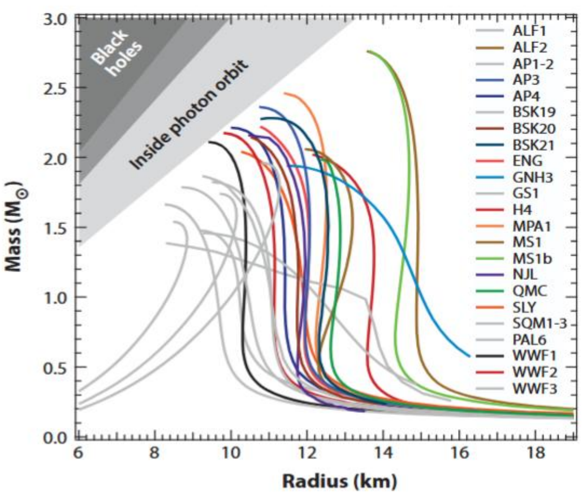


Haensel (2007)

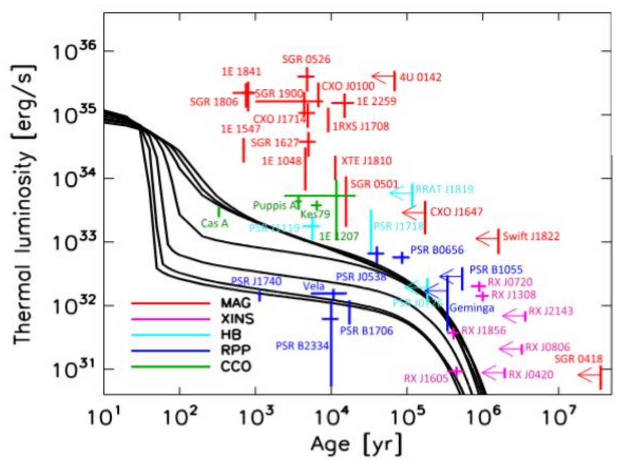
The EOS of matter P(ρ) at core densities is the main unsolved mystery of NS physics

Neutron Stars

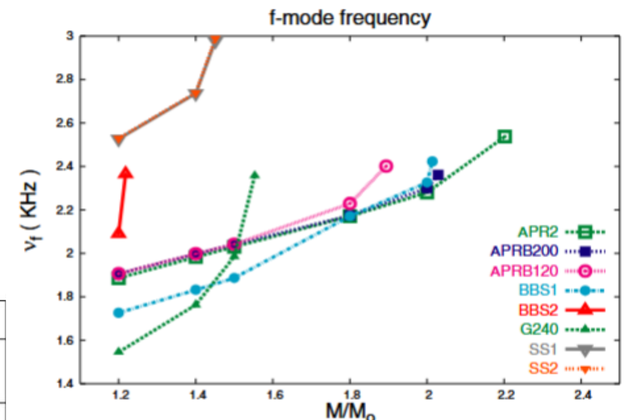
$P(\rho)$



Özel, Freire (2016)



Potekhin, Pons, Page (2017)



Benhar, Ferrari, Gualtieri (2004).

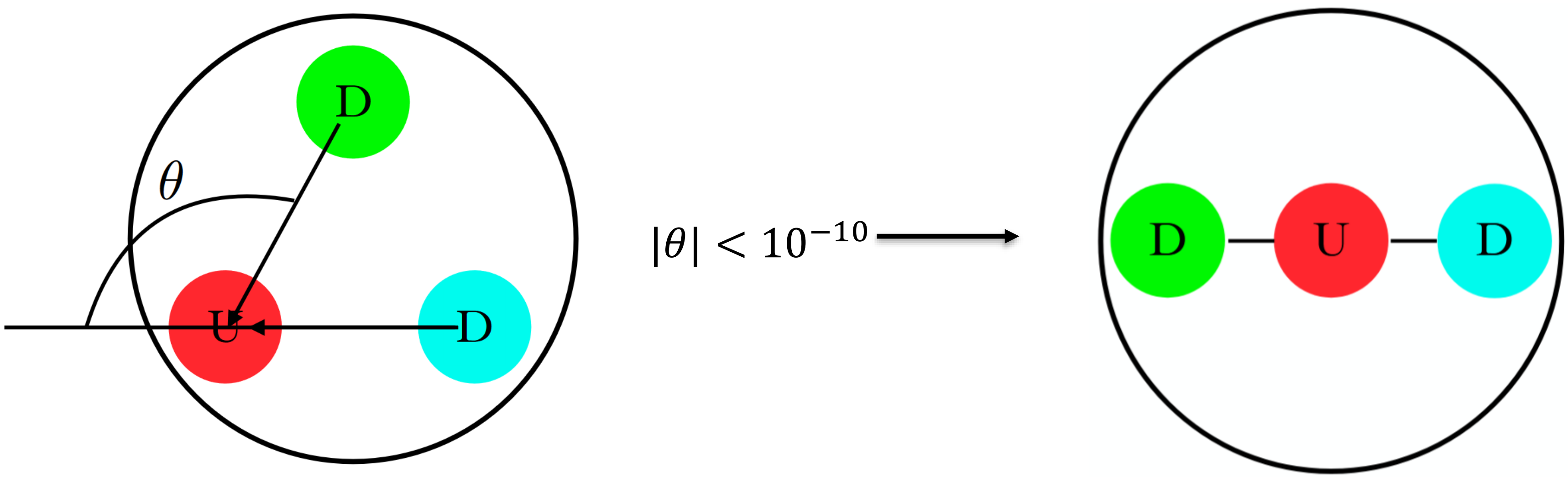
Could new physics change these scenarios?

QCD Axion

- QCD Axion: Strong CP solution + DM candidate

$$\mathcal{L} \supset \theta \frac{g_s}{32\pi^2} \mathrm{Tr}\left(\mathrm{G}\widetilde{\mathrm{G}}\right) \to |d_n| \sim 10^{-16} |\theta| \, \mathrm{e} \cdot \mathrm{cm}$$

$$|d_n{}^{\mathrm{exp}}| < 10^{-26} \, \mathrm{e} \cdot \mathrm{cm}$$



Hook (2018)

QCD Axion

- QCD vacuum energy is minimized for $\theta = 0$:

$$V(\theta) = m_\pi^2 f_\pi^2 [1 - g(\theta)] \qquad g(\theta) = \sqrt{1 - \frac{4 m_u m_d}{(m_u + m_d)^2} \sin^2\left(\frac{\theta}{2}\right)}$$

$$m_a = \frac{m_\pi f_\pi}{f_a} \frac{\sqrt{m_u m_d}}{(m_u + m_d)}$$

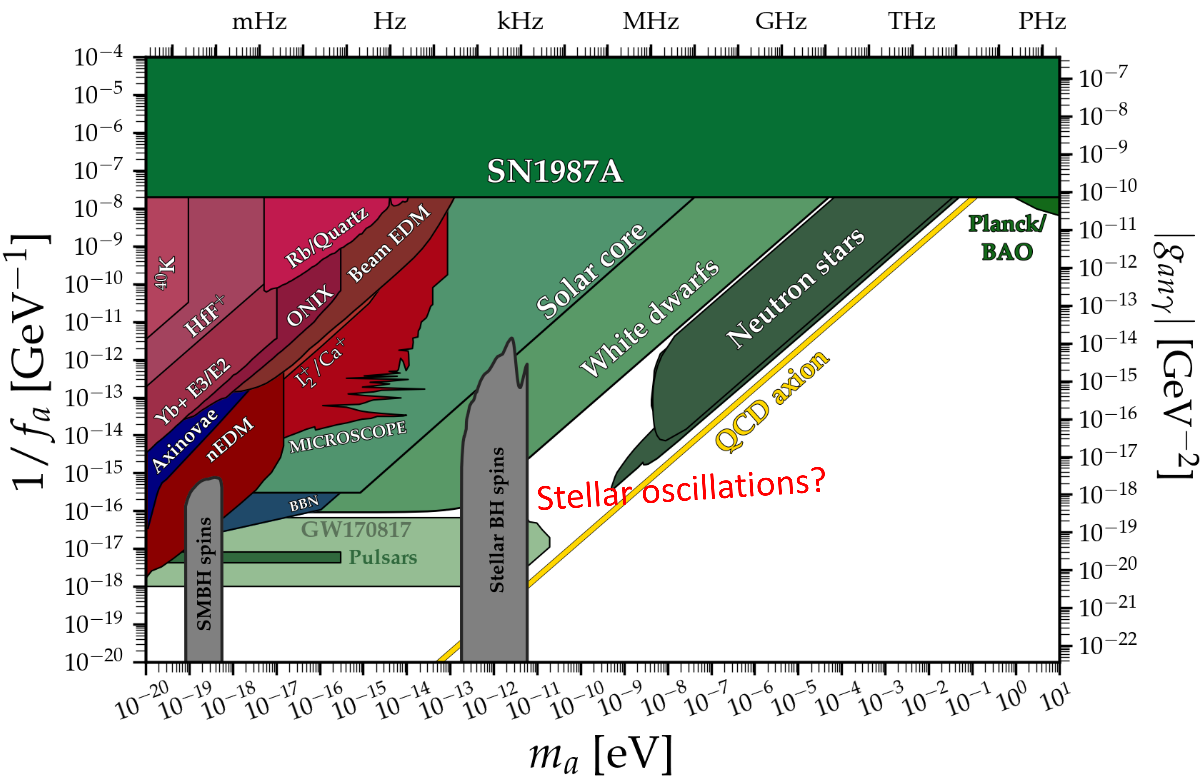
- Let θ be a dynamical field, with potential $V(\theta)$
- Consequence: θ will minimize its energy by going to 0

- The QCD axion has couplings to other fields (they depend on UV completion):

$$\frac{g_{a\gamma\gamma}}{4} \frac{a}{f_a} F\tilde{F} \qquad C_{af}\, \frac{\partial_\mu a}{2 f_a} \bar{f} \gamma^\mu \gamma_5 f$$

QCD Axion

- Astrophysical constraints still provide the most stringent bounds



cajohare/AxionLimits: AxionLimits

- WD MR relations:
  - Balkin et al. (2024)
- NS cooling:
  - Kumamoto et al. (2025)
  - Gómez-Bañón et al. (2024)
- LVK merger observations:
  - Zhang et al. (2021)
- BH spins:
  - Baryakhtar et al. (2021)

Yellow line:

$$m_a = \frac{m_\pi f_\pi}{f_a} \frac{\sqrt{m_u m_d}}{(m_u + m_d)}$$

(Light) QCD Axion in NS

- Finite nuclear density corrections to the axion potential: Hook, Huang (2018); Balkin et al. (2021)

$$V(\theta) = \epsilon m_\pi^2 f_\pi^2 [1 - g(\theta)]$$

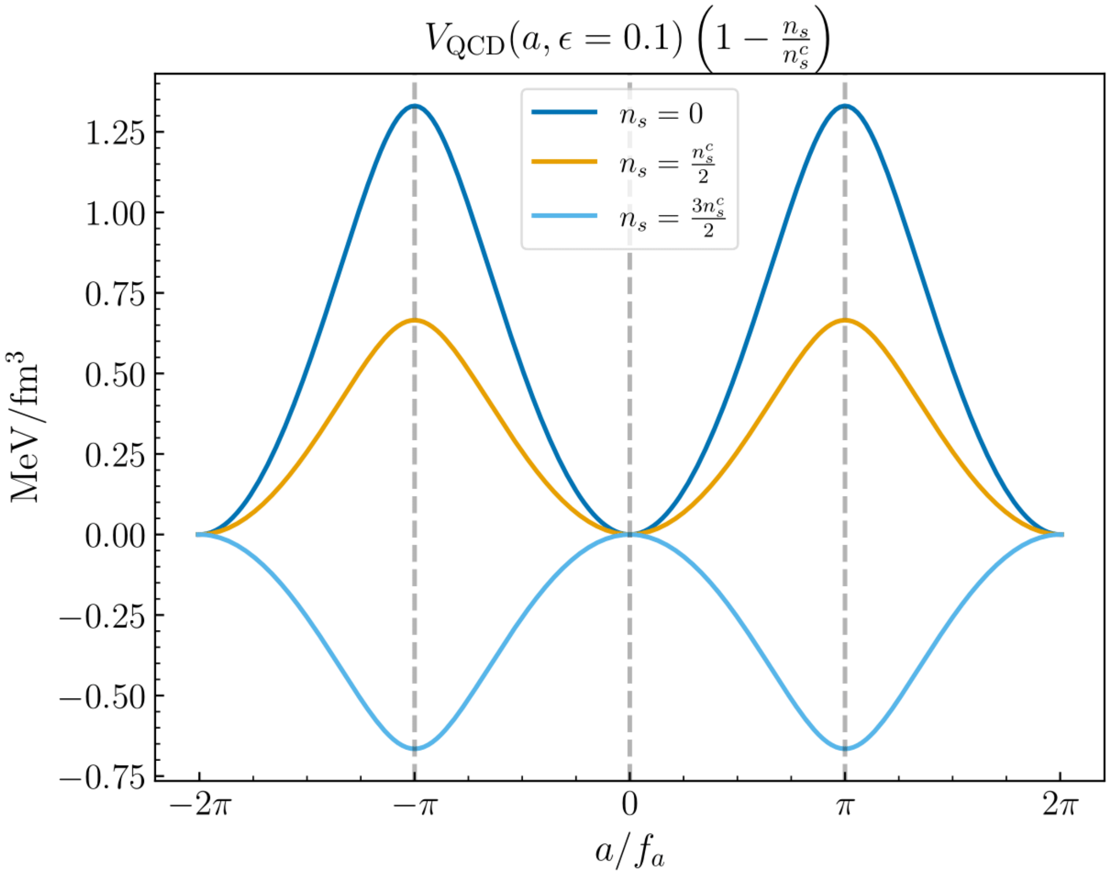
$$\mathcal{L} \supset \sigma_N n_s [1 - g(\theta)]$$

$$\downarrow$$

$$U(\theta, n_s) = \epsilon m_\pi^2 f_\pi^2 \left(1 - \frac{n_s \sigma_N}{\epsilon m_\pi^2 f_\pi^2}\right) [1 - g(\theta)]$$

(Light) QCD Axion in NS

For large enough n_s , $\theta = \pm\pi$ become the stable minima:



$$n_c \equiv \frac{\epsilon m_\pi^2 f_\pi^2}{\sigma_N}$$

$$n_c \sim \epsilon(0{,}4\ \mathrm{fm}^{-3})$$

Achieved in NS, but

$$m_a R \gtrsim 1$$

is also needed.

(Light) QCD axion in NS

- Finite θ also affects nuclear matter:
  - If $\theta = \pm\pi$ nucleon masses are reduced by $\sim 32\ \mathrm{MeV}$ ($m_\pi(\theta)$ is also reduced: Kumamoto et al. (2025))

$$m_n{}^*(\theta) = m_n - \sigma_N[1 - g(\theta)]$$

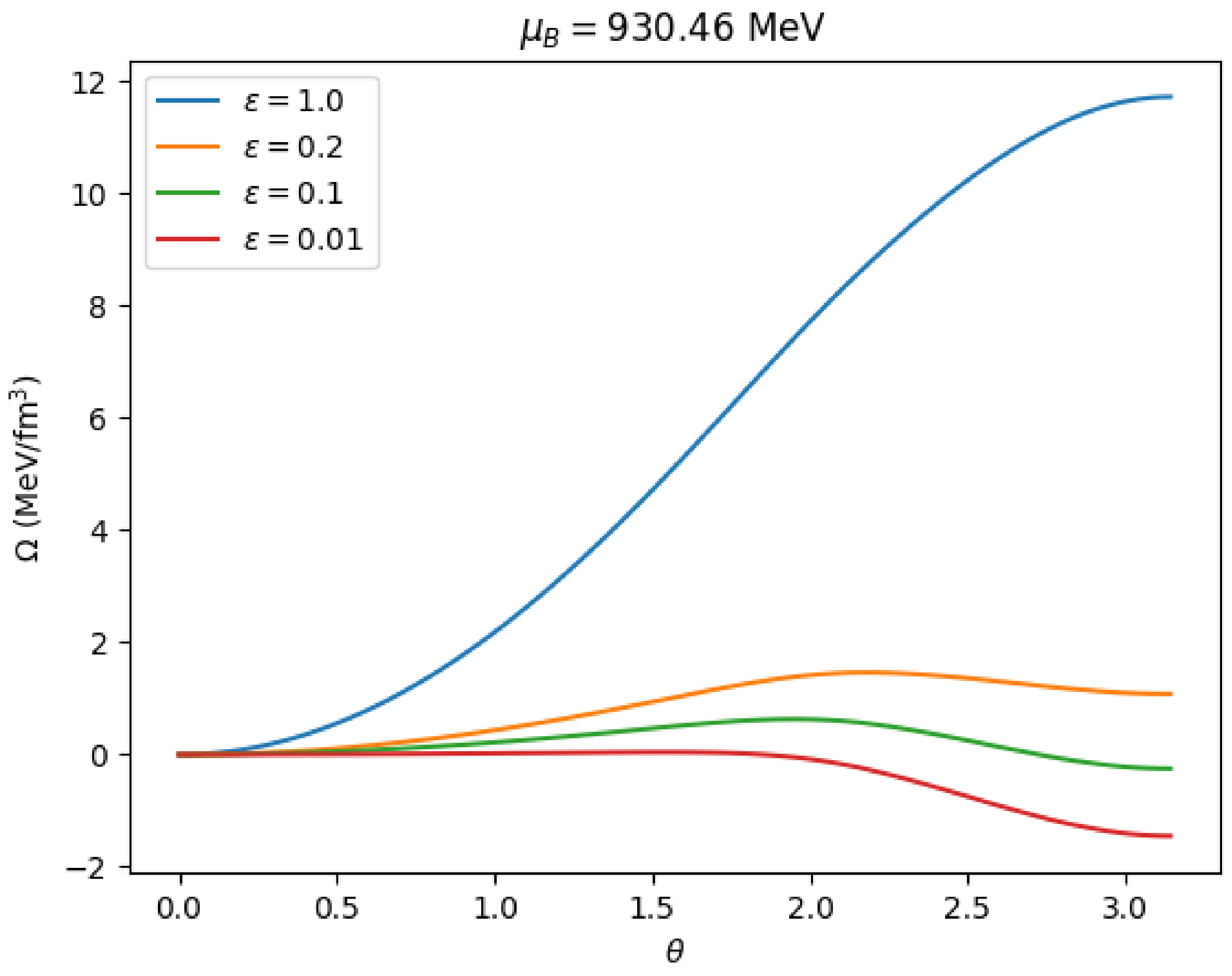
- Cheapest way of introducing finite θ in the matter energy density ε:

$$\varepsilon(n_b, \theta) = \varepsilon(n_b, 0) - n_b \sigma_N [1 - g(\theta)]$$

$$\varepsilon(n_b, \theta) + V(\theta) = \varepsilon(n_b, 0) - n_b \sigma_N [1 - g(\theta)] + \epsilon m_\pi^2 f_\pi^2 [1 - g(\theta)]$$

- For $\epsilon < 0{,}1$: NGS of nuclear matter with $\theta = \pm\pi$ is realized in interacting NS EOS (Kumamoto et al. (2025), Gómez-Bañón et al. (2024))

(Light) QCD axion in NS



NS Structure with QCD Axion

$$p' = -(\varepsilon + p)\,\Phi' - n_s \frac{\partial m^*}{\partial \theta}\theta',$$

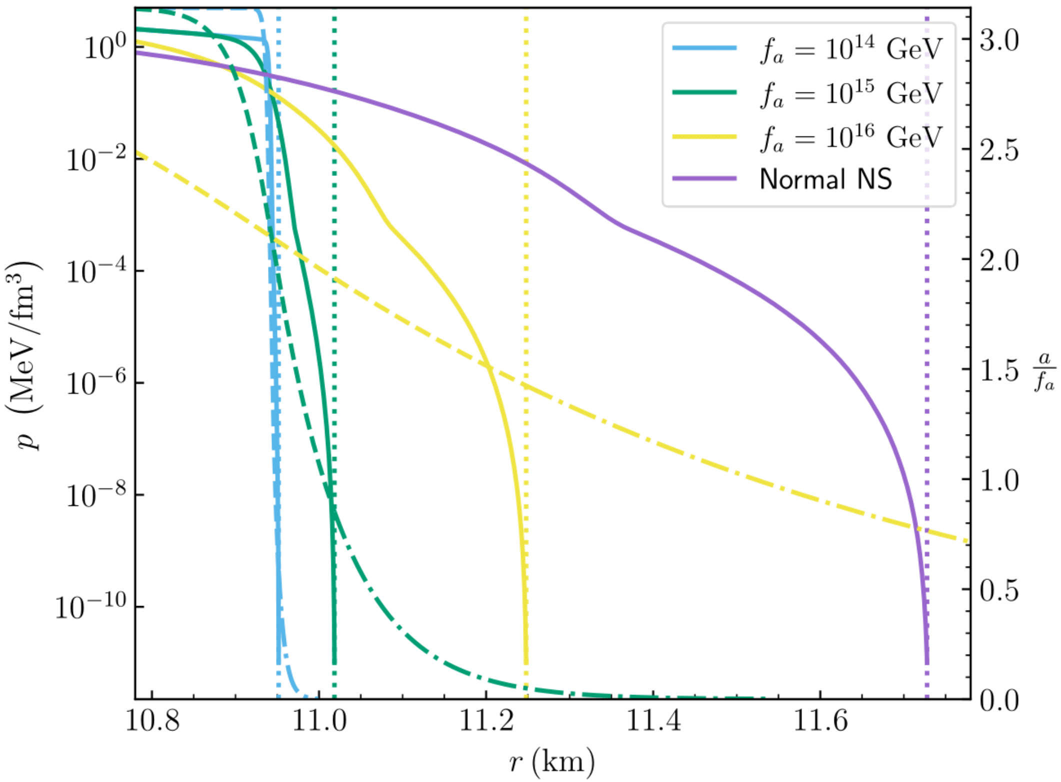
$$\Phi' = \frac{1}{r^2}\left(1 - \frac{2GM}{r}\right)^{-1}\left\{GM + 4\pi G r^3\left[p - V(\theta) + f_a^2 \frac{(\theta')^2}{2}\left(1 - \frac{2GM}{r}\right)\right]\right\},$$

$$M' = 4\pi r^2\left[\varepsilon + V(\theta) + f_a^2 \frac{(\theta')^2}{2}\left(1 - \frac{2GM}{r}\right)\right],$$

$$\theta''\left(1 - \frac{2GM}{r}\right) + 2\frac{\theta'}{r}\left(1 - \frac{GM}{r} - 2\pi G r^2\left(\varepsilon - p + 2V(\theta)\right)\right) = \frac{1}{f_a^2}\left(\frac{\partial V}{\partial \theta} + n_s \frac{\partial m^*}{\partial \theta}\right).$$

- In the limit $f_a \to 0$, the axion profile becomes a step at $n_b \sim n_c$
- The condensed profile compresses the NS and modifies structure
- Shrink of exterior layers causes anomalously fast cooling: Kumamoto et al. (2025), Gómez-Bañón et al. (2024).

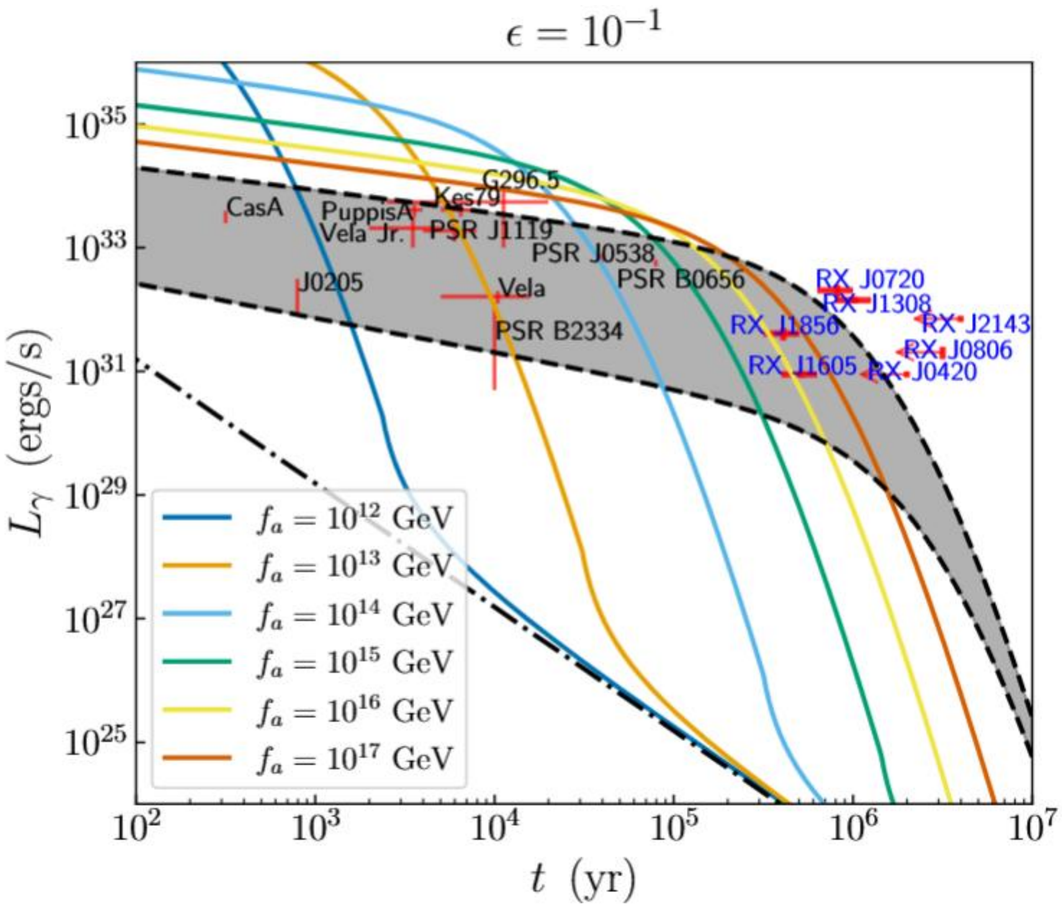
NS Structure with QCD Axion



$$p' = -\left(\varepsilon + p\right)\Phi' - n_s \frac{\partial m^*}{\partial \theta}\theta'$$

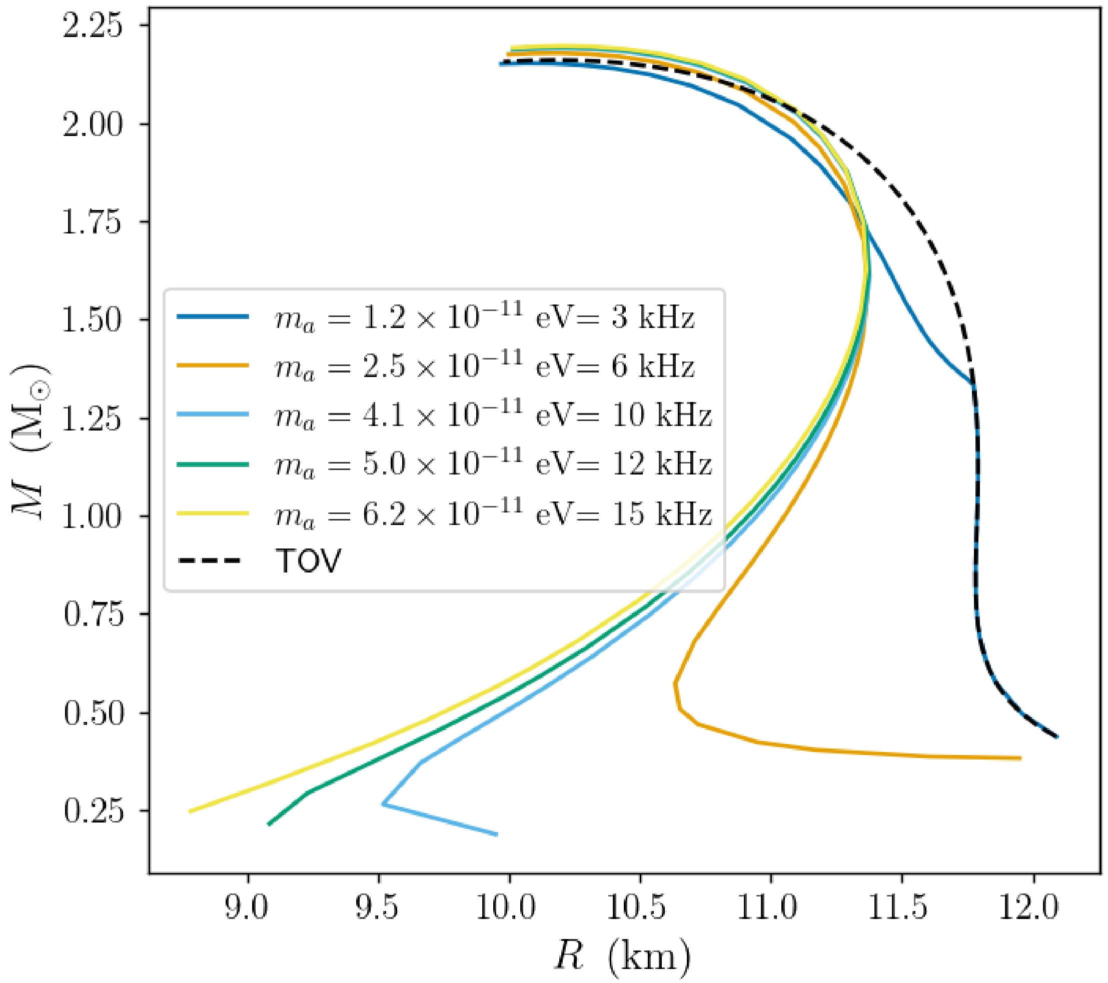
NS Structure with QCD Axion

- The extra pressure gradient from the axion field shrinks the NS envelope



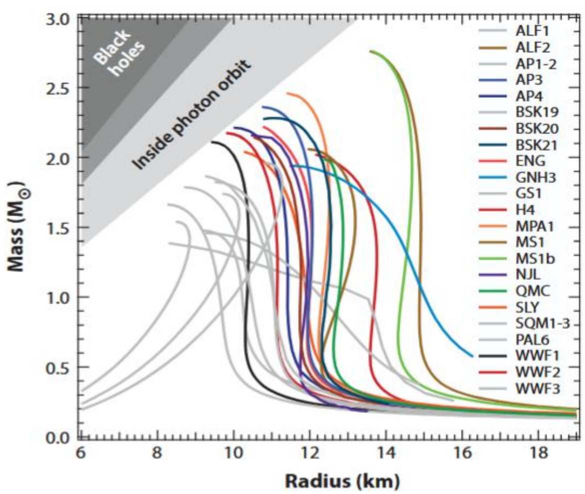
$$p' = -\left(\varepsilon + p\right)\Phi' - n_s \frac{\partial m^*}{\partial \theta}\theta'$$

NS Structure with QCD Axion

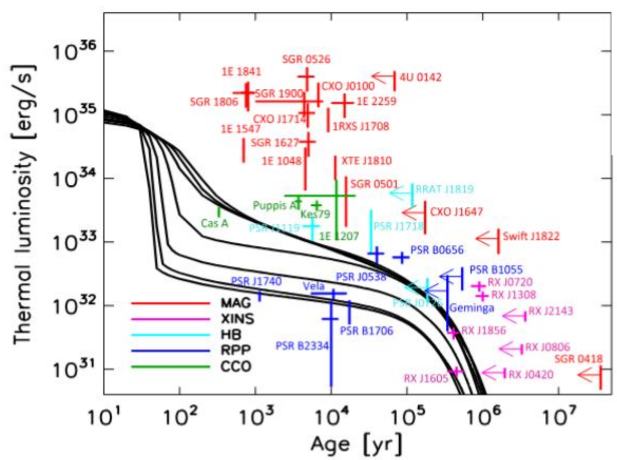


Neutron Stars

$P(\rho)$



Özel, Freire (2016)



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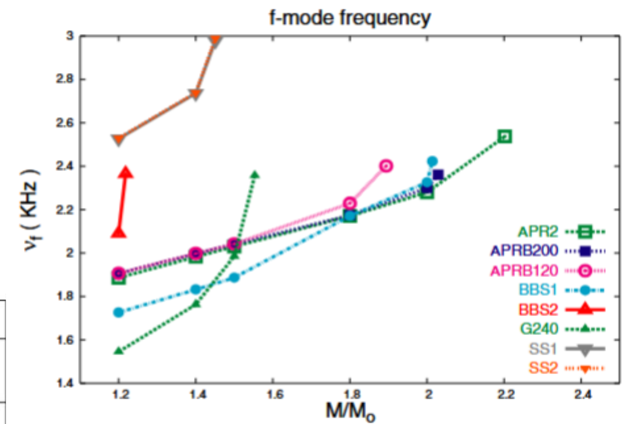
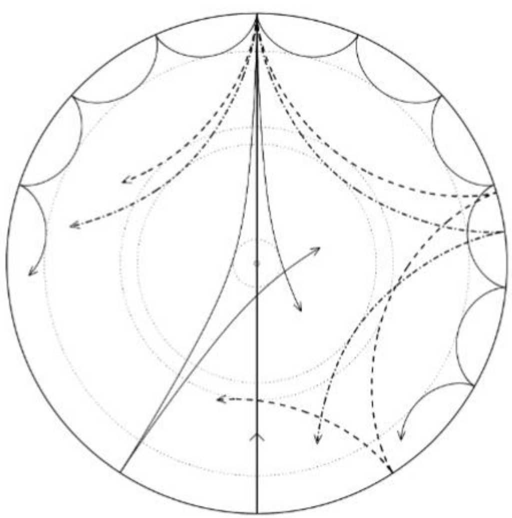


Benhar, Ferrari, Gualtieri (2004).

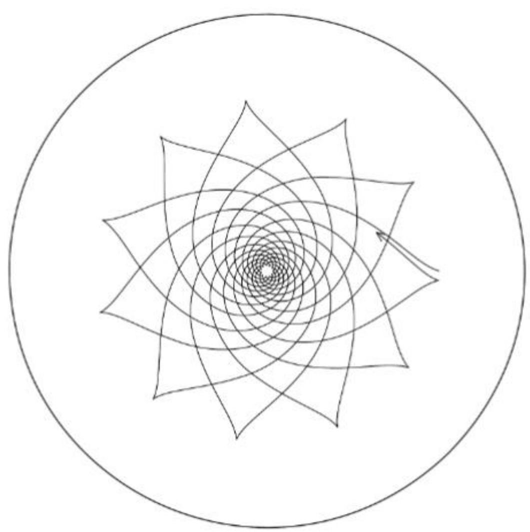
What are the effects of an axion condensate on the oscillation modes of a NS?

Stellar oscillations (or asteroseismology)

- Asteroseismology originated from the study of pulsating variable stars: Cepheids, RR Lyrae
- Oscillations can be classified into two categories:
  - Radial (or breathing modes)
  - Non-radial



p-modes ($l = 2, 20, 25, 75$)



g-mode ($l = 5$)

Mode propagation between turning points where $k_r = 0$

[M. S. Cunha *et al.* (2007) Astron. Astrophys. Rev. **14**, 217]

"What appliance can pierce through the outer layers of a star and test the conditions within?"
Eddington (1926)

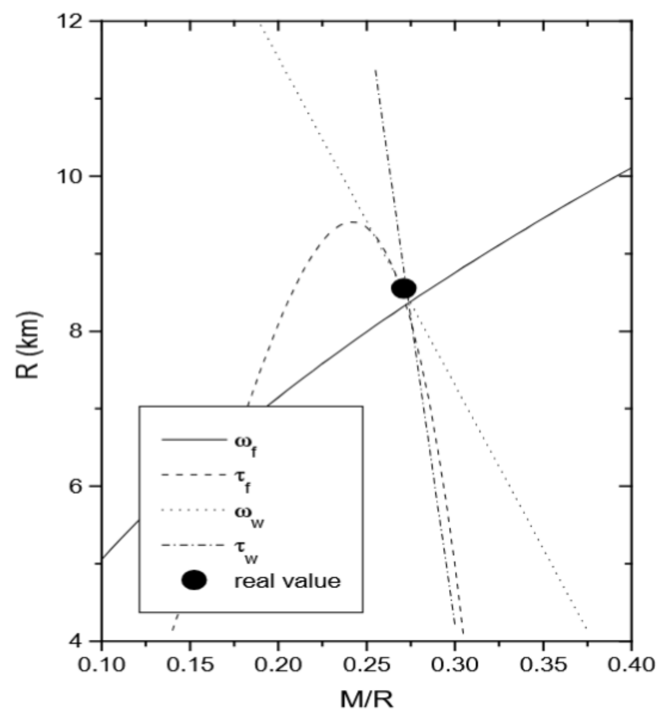
The Sun's ${c_s}^2$ profile has been determined to few ppm

NS oscillations

- The NS oscillation problem has been studied for long: Chandrasekhar (1964); Thorne, Campolattaro (1967); Glass, Lindblom (1983); Lindblom, Detweiler (1983).

- Physical properties of NS can be related to different types of oscillation modes:
  - Composition
  - Magnetic fields
  - Crust elasticity
  - Superfluidity

- GW from NS carry information about the NS EOS



Kokkotas et al. (2001)

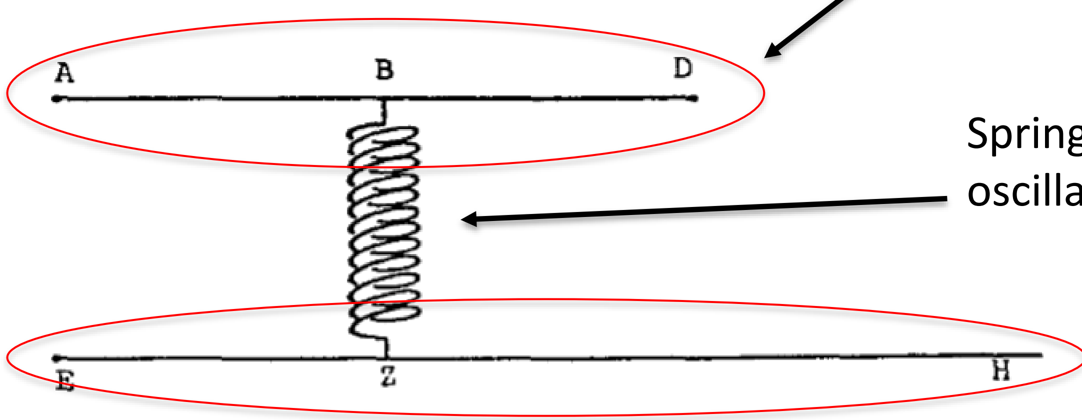
NS oscillations

- In the case of NS, non-radial modes have complex oscillation frequencies (QNM):

$$e^{-i\omega t} = e^{-i\omega_R t} e^{\omega_I t}$$

- These are associated to the emission of GW.

This is a NS



Spring only acts for NS quadrupole oscillations

GW emission

Kokkotas and Schutz (1985)

This is spacetime

Axion-condensed NS oscillations

- As a first step, we investigate radial oscillation modes.
- They do not emit GWs but are simple enough to get some insights.
- **Main difference**: the NS is now coupled to another propagating field θ

$$\begin{array}{c} \boxed{n_b \longleftrightarrow \theta} \\ \big\downarrow \\ \boxed{\text{Spherically symmetric fluid oscillations will perturb } \theta} \end{array}$$

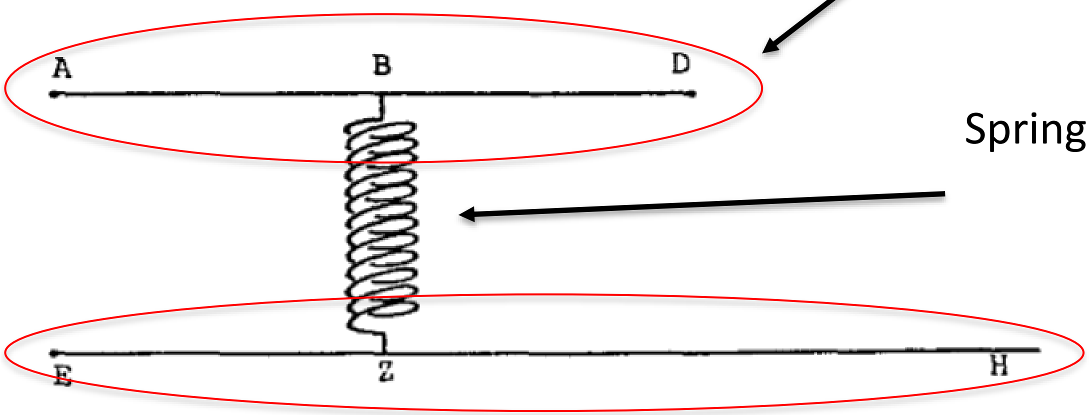
- In the presence of a QCD axion condensate, **radial NS oscillations will become QNMs.**
- This is analogous to the emission of GW by quadrupole oscillations.
- **Direct coupling** between the **scalar field and the fluid**, not mediated by gravity.

Axion-condensed NS oscillations

This is a NS

Spring acts for any perturbation in n_b

Axion emission



Kokkotas and Schutz (1985)

This is the QCD axion field

Axion-condensed NS oscillations

- Pick a static, spherically symmetric solution of the TOV-KG equations
- Perturb all quantities to first order, assuming a harmonic time dependence $e^{-i\omega t}$:

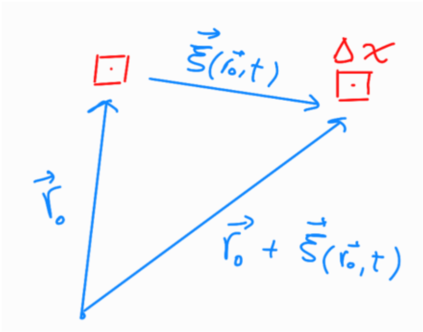
$$\delta X = \delta\Phi, \delta\lambda, \delta p, \delta\varepsilon, \delta \mathrm{n}_b, \delta\theta$$

$$\delta X(t,r) = \sum \delta\tilde{X}(r) e^{-i\omega t}$$

- Thermodynamical quantities will be related via the EOS: $p(\varepsilon, \theta)$

$$\xi(t,r) = \sum \tilde{\xi}(r) e^{-i\omega t}$$

- ξ indicates displacement of matter from the background position



Axion-condensed NS oscillations

- KG equation

$$e^{-2\lambda}\delta a'' + e^{-2\lambda}\delta a'\left(\Phi' - \lambda' + \frac{2}{r}\right)$$

$$-2\delta\lambda\left(\frac{\partial V}{\partial a} + n_b\frac{\partial m^*}{\partial a}\right) + e^{-2\lambda}a'\left(\delta\Phi' - \delta\lambda'\right)$$

$$-\left(\frac{\partial^2 V}{\partial a^2} + n_b\frac{\partial^2 m^*}{\partial a^2} - \omega^2 e^{-2\Phi}\right)\delta a = \delta n_b\frac{\partial m^*}{\partial a},$$

- Einstein equations

$$\delta\Phi' - \delta\lambda\left(\frac{1}{r} + 2\Phi'\right) = 4\pi G r e^{2\lambda}\left[\delta p - \frac{\partial V}{\partial a}\delta a - \delta\lambda e^{-2\lambda}(a')^2 + e^{-2\lambda}a'(\delta a)'\right],$$

$$\delta\lambda = -4\pi G r e^{2\lambda}\left[(\varepsilon + p)\xi^r - e^{-2\lambda}a'\delta a\right],$$

- TOV equation

$$(\delta p)' + (\delta\varepsilon + \delta p)\,\Phi' + (\varepsilon + p)\,\delta\Phi' - \omega^2\,(\varepsilon + p)\,e^{2(\lambda-\Phi)}\xi^r =$$

$$= -\frac{\partial m^*}{\partial a}\left(\delta n_b a' + n_b(\delta a)'\right) - n_b\frac{\partial^2 m^*}{\partial a^2}a'\delta a,$$

- Baryon conservation

$$\left[(\varepsilon + p)\,\xi^r\right]' + (\varepsilon + p)\,\xi^r\left(\frac{2}{r} + \lambda' + \Phi'\right) = -\left(\frac{\delta p}{c_s^2} + (\varepsilon + p)\,\delta\lambda\right).$$

Axion-condensed NS oscillations

- Boundary conditions (asteroseismology):

$$\Delta p(R) = 0$$

(Hydrostatic equilibrium at the NS surface)

- QCD Axion (outgoing boundary condition):

$$\left(1 - \frac{2GM}{r}\right)\delta a'' + \frac{2}{r}\left(1 - \frac{GM}{r}\right)\delta a' - \left(m_a^2 - \frac{\omega^2}{1 - 2GM/r}\right)\delta a = 0,$$

$$\delta a \sim \frac{A}{r}e^{ikr} + \frac{B}{r}e^{-ikr},$$

Outgoing condition:

$$\delta a'(\infty) = +ik\delta a(\infty)$$
$$k = \sqrt{\omega^2 - {m_a}^2}$$

- Asymptotic power series
- Phase-amplitude method (Andersson et al. (1995)

Axion-condensed NS oscillations (Simplified)

- Ignore metric perturbations, spherical coordinates, background gradients…

$$\delta\tilde{p}'' + \frac{\bar{\omega}^2}{c_s^2}\delta\tilde{p} = -\lambda^2\alpha^2\delta\tilde{p} - \alpha\left(\bar{\omega}^2 - 1 + \alpha\right)\delta\theta,$$ (TOV equation)

$$\delta\theta'' + \left(\bar{\omega}^2 - 1 + \alpha\right)\delta\theta = -\lambda^2\alpha\delta\tilde{p}.$$ (KG equation)

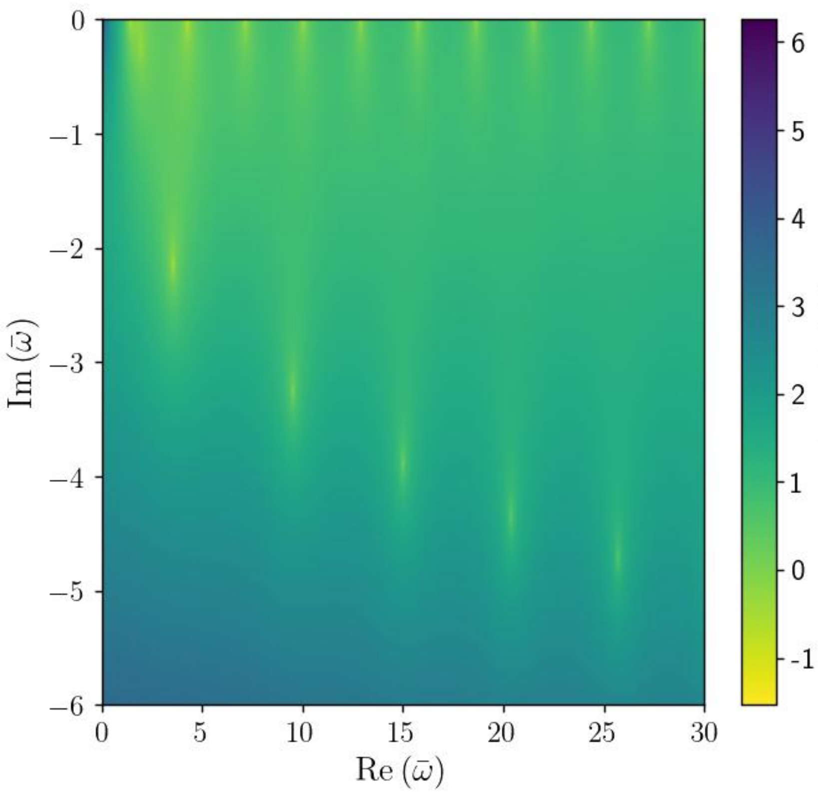
$$\alpha \equiv \frac{n_b}{n_c}, \overline{\omega} \equiv \frac{\omega}{m_a}, \overline{k} \equiv \frac{k}{m_a}, \lambda^2 = \frac{{m_a}^2 {f_a}^2}{(\varepsilon+p)c_s^2}$$

- Outside the NS: $$\delta\theta'' + \left(\bar{\omega}^2 - 1\right)\delta\theta = 0$$

Axion-condensed NS oscillations (Simplified)

- For $\lambda \neq 0$, the axion modes mix with the fluid, and the outgoing boundary condition becomes:

$$f(\bar{\omega}) \equiv (\bar{k}_a^2 - \bar{k}_-^2)\bar{k}_+ \tan(\bar{k}_+ \bar{x}_0) - (\bar{k}_a^2 - \bar{k}_+^2)\bar{k}_- \tan(\bar{k}_- \bar{x}_0) + i\bar{k}_{out}\sqrt{\Delta} = 0,$$



$$k_{\text{out}} = \sqrt{\omega^2 - {m_a}^2}$$

Two distinct families of modes appear:

- Fluid-dominated modes (near real axis, small imaginary parts)
- Axion modes (strongly damped)

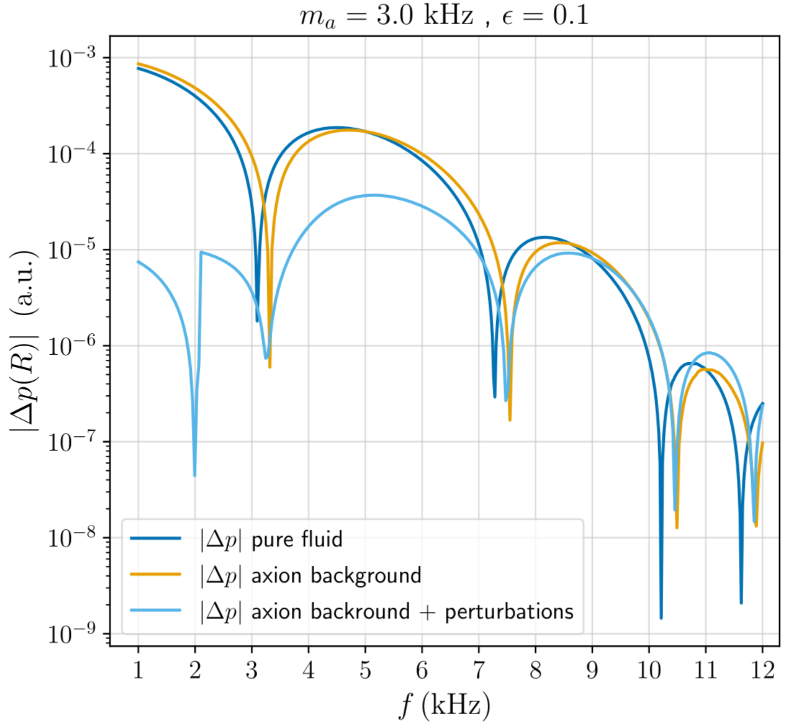
Axion-condensed NS oscillations

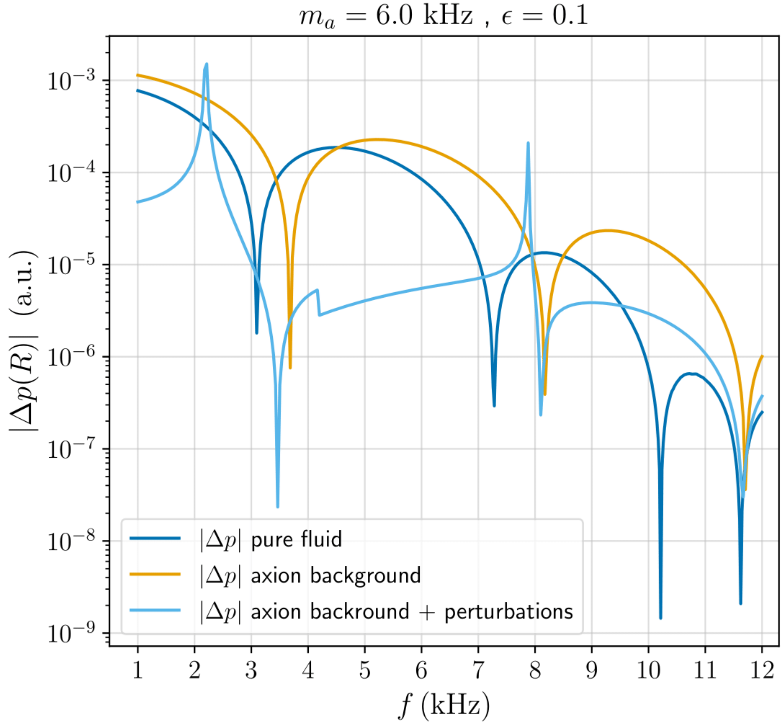
- In the **full system**, there are two effects at play:
  - Modification of background structure
  - Interaction with axion perturbations
- First, only include background structure effects due to the QCD axion condensate

m_a (kHz)	f_0 (kHz)	f_1 (kHz)	f_2 (kHz)	f_3 (kHz)
no axion	3.10	7.27	10.22	11.63
2.8	3.14	7.32	10.26	11.67
3.0	3.32	7.54	10.46	11.86
4.0	3.63	8.03	11.25	12.84
5.0	3.67	8.14	11.56	13.45
6.0	3.68	8.18	11.71	13.87
8.0	3.69	8.22	11.88	14.45
12.0	3.72	8.27	12.07	15.16

- Oscillation frequencies without including axion perturbations ($\epsilon = 0{,}1$ and $p_0 = 100\ \mathrm{MeV/fm^3}$)
- $\mathrm{M} = 1{,}53\ \mathrm{M}_{\odot}, \mathrm{R} = 11{,}7\ \mathrm{km}$

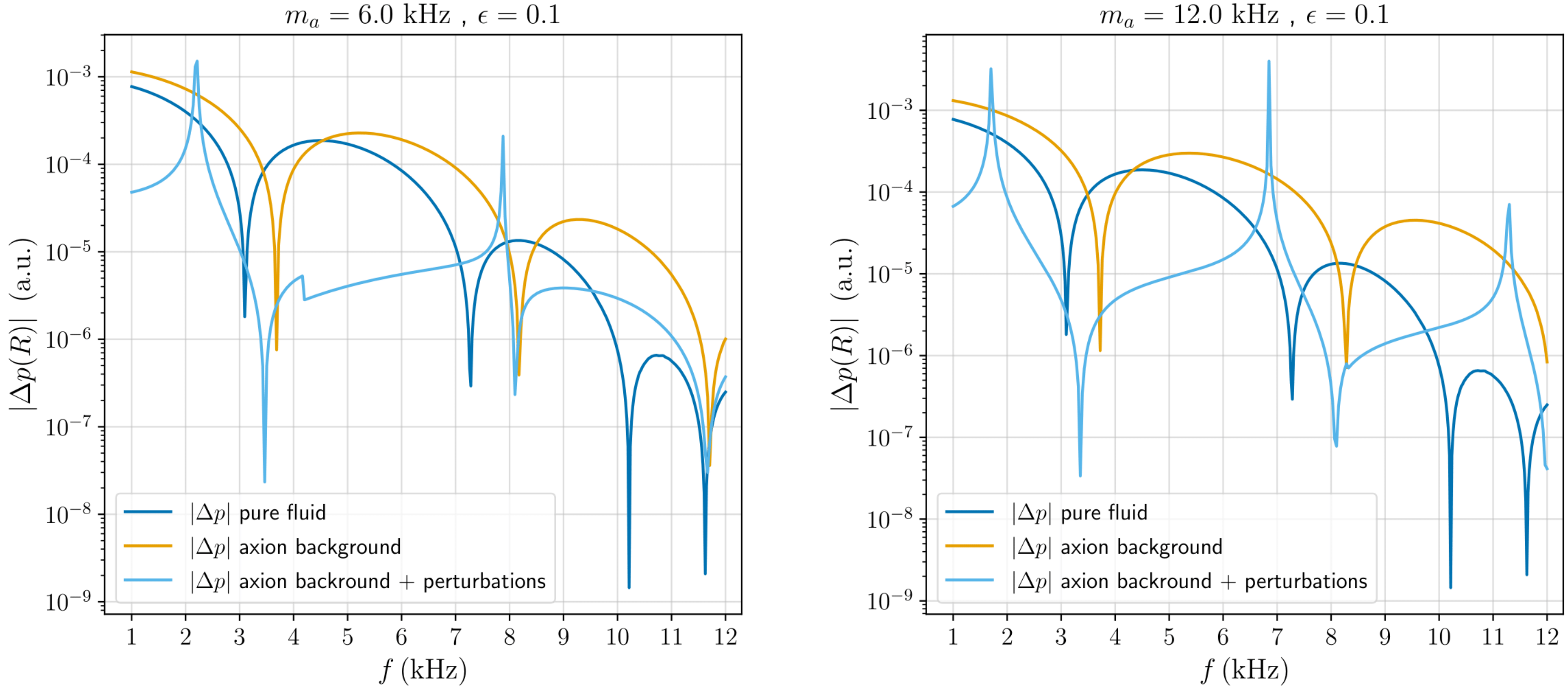
Axion-condensed NS oscillations





- The axion background changes the NS structure, increasing the compactness of the star.
- Fluid mode frequencies are shifted to larger values.

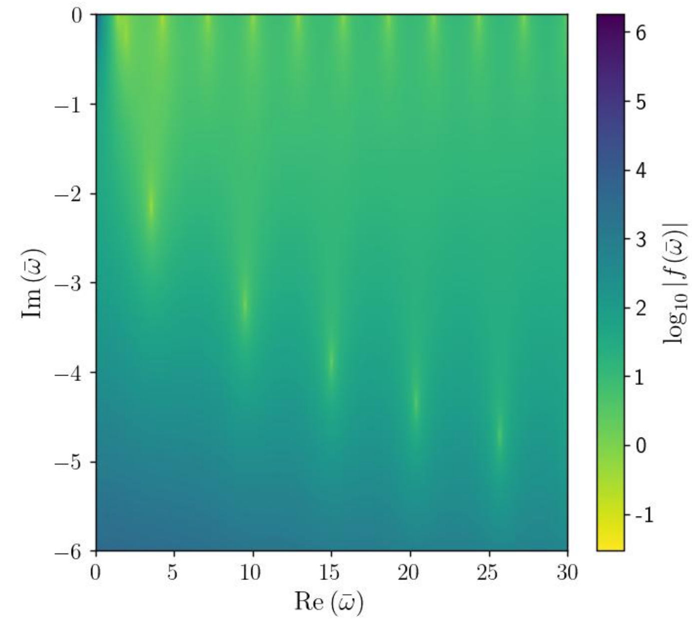
Axion-condensed NS oscillations

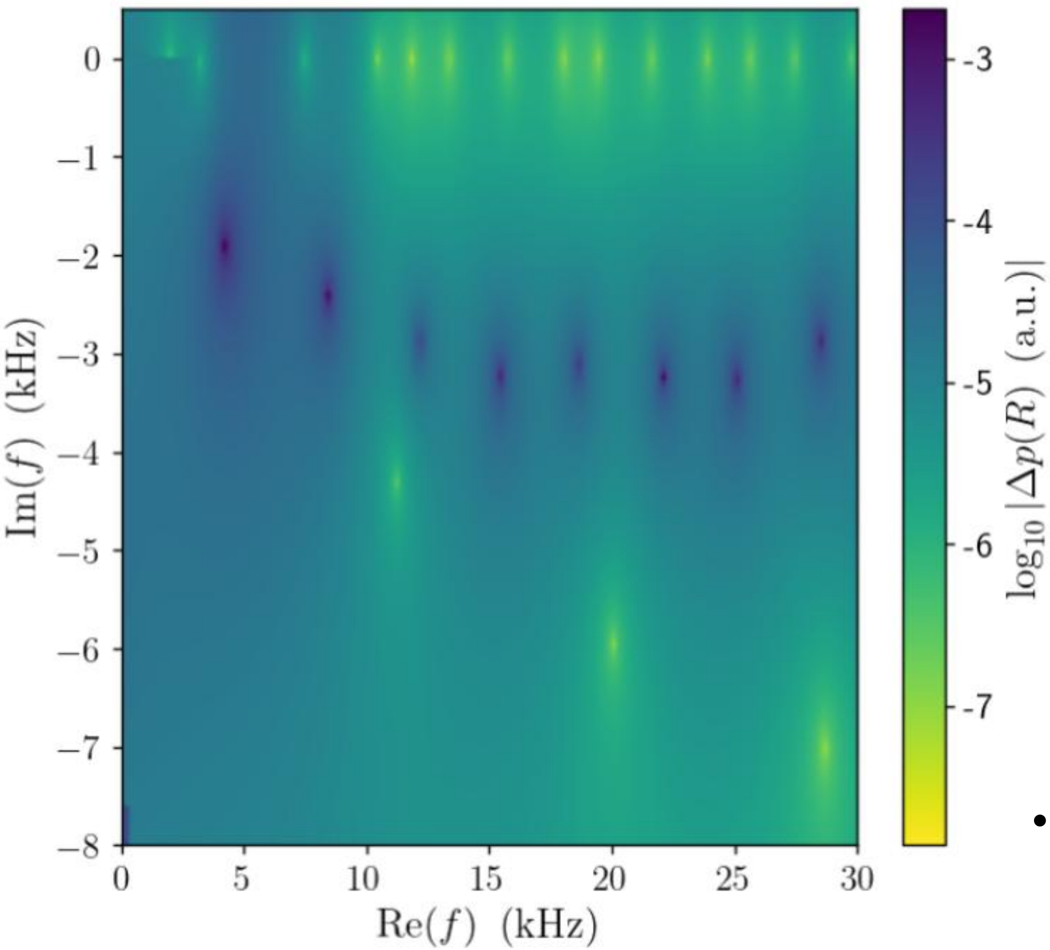


- Including $\delta\theta$ reduces frequency relative to background-changed model.
- Fluid modes develop imaginary parts.

Fluid Modes

- Simplified model



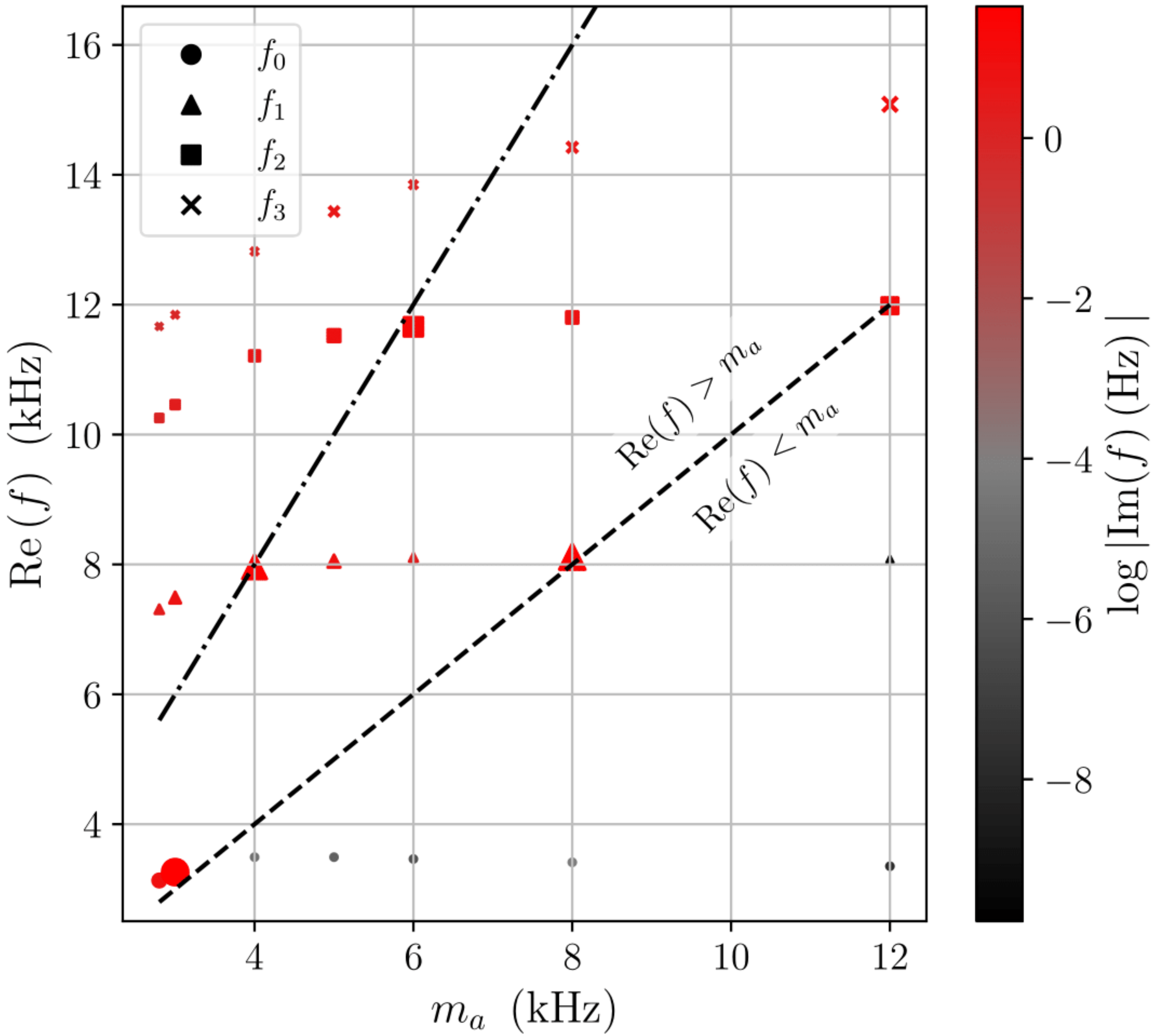


- Full system QNM spectrum for $m_a = 3$ kHz

Fluid Modes

- Fluid modes acquire damping of the order of Hz.
- Damping larger than other radial oscillation viscous mechanisms.
- Fluid modes with $f < m_a$ remain undamped.

m_a (kHz)	f_0		f_1		f_2		f_3	
	Ref (kHz)	Imf (Hz)	Ref (kHz)	Imf (Hz)	Ref (kHz)	Imf (Hz)	Ref (kHz)	Imf (Hz)
no axion	3.10		7.27		10.22		11.63	
2.8	3.13	−8.80	7.31	−2.60	10.25	−1.04	11.66	−0.394
3.0	3.26	−43.7	7.49	−6.10	10.46	−2.15	11.84	−0.854
4.0	3.49	0	7.96	−38.9	11.21	−4.55	12.82	−1.06
5.0	3.50	0	8.05	−8.38	11.52	−7.74	13.44	−4.02
6.0	3.47	0	8.11	−2.05	11.66	−22.5	13.85	−2.01
8.0	3.42	0	8.12	−36.2	11.80	−6.71	14.42	−5.22
12.0	3.36	0	8.09	0	11.98	−12	15.08	−11.4



f_0
f_1
f_2
f_3
$\mathrm{Re}(f) > m_a$
$\mathrm{Re}(f) < m_a$
$\mathrm{Re}(f)$ (kHz)
m_a (kHz)
$\log|\mathrm{Im}(f)\ (\mathrm{Hz})|$

Axion Modes

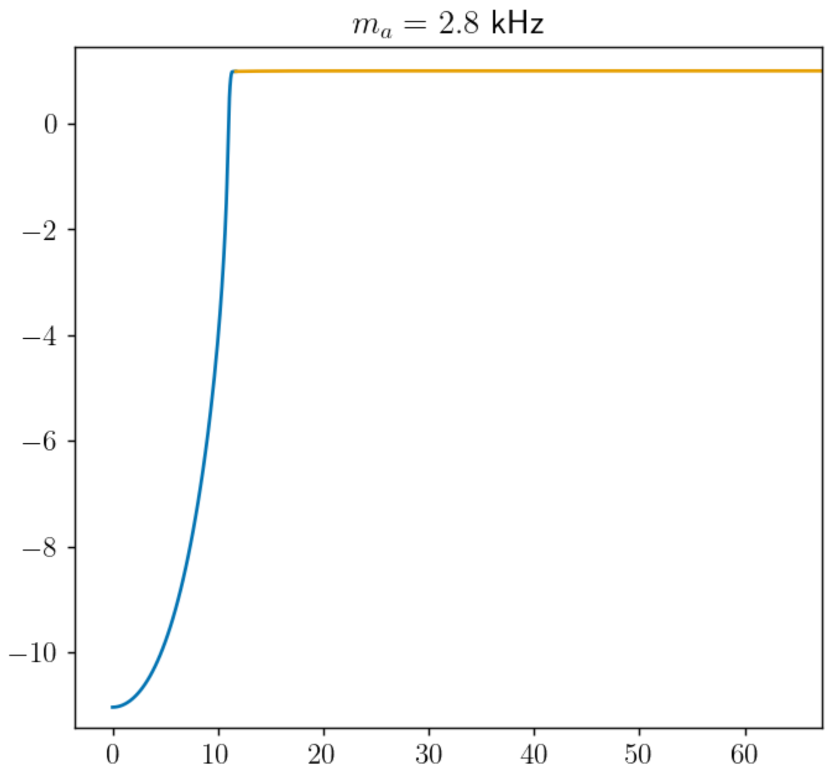
- Secondary, strongly damped family of modes.
- These are associated to oscillations of the QCD axion field.
- Supported by the variation of ${m_a}^2$ inside the NS.

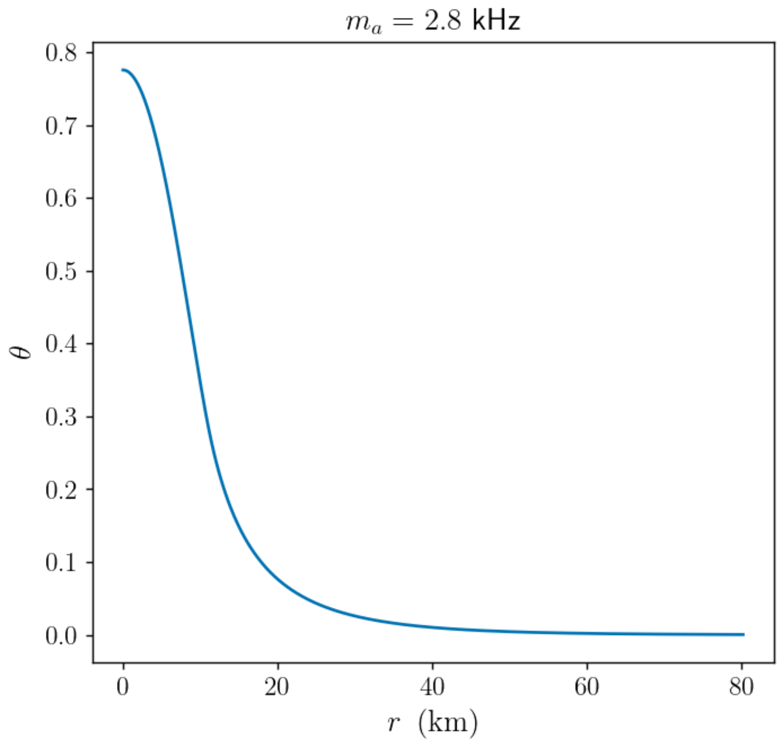
m_a (kHz)	$f_0^{\rm a}$		$f_1^{\rm a}$		$f_2^{\rm a}$	
	Ref (kHz)	Imf (kHz)	Ref (kHz)	Imf (kHz)	Ref (kHz)	Imf (kHz)
2.8	0.749	0	10.50	−4.30	19.29	−5.82
3.0	2.00	0	11.23	−4.31	20.06	−5.94
4.0	2	−1.4	16.36	−3.45	22.99	−5.37
5.0	1.8	−2	20.30	−2.22	25.54	−4.70
6.0	2	−2	23.51	−1.50	28.22	−3.81
8.0	3	−2	29.86	−0.98	33.55	−2.50
12.0	4	−3	42.97	−1.08	45.19	−1.50

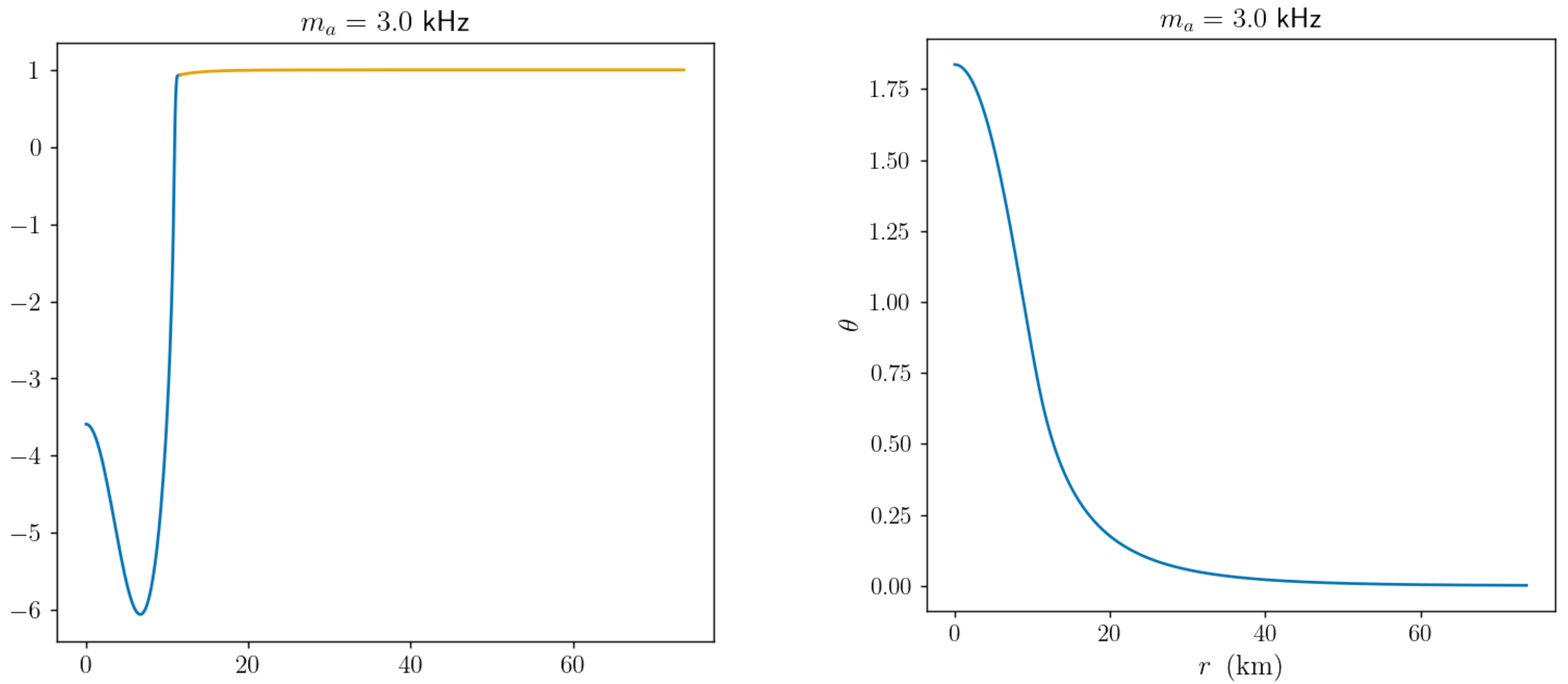
Conclusions

- As one would expect from a massive scalar field, axion emission occurs when $\mathrm{Re}(f) > m_a$
- Damped and undamped families of modes, possible resonances around m_a (expected to remain in the non-radial case)
- Presence or absence of damping in an oscillation spectrum could provide information about the axion mass.
- This would mean that future GW detectors could probe axion physics (if it couples to nuclear physics…)
- We **expect** that **extending this work** provides a pathway to *axion asteroseismology*.

Thanks for your attention!


$m_a = 2.8$ kHz
0
−2
−4
−6
−8
−10
0
10
20
30
40
50
60


$m_a = 2.8$ kHz
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0.0
θ
0
20
40
60
80
r (km)


$m_a = 3.0$ kHz
1
0
−1
−2
−3
−4
−5
−6
0
20
40
60
$m_a = 3.0$ kHz
1.75
1.50
1.25
1.00
0.75
0.50
0.25
0.00
θ
0
20
40
60
r (km)