The effects of light quarks on the glueball spectrum

INT WORKSHOP INT-20R-2C

Accessing and Understanding the QCD Spectra

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RESEARCH·TECHNOLOGY·INNOVATION

In collaboration with Jacob Finkenrath, Adam Lantos and Michael Teper based on arXiv:2212.02080

EURO NIKOS

Introduction

- Quantum ChromoDynamics – **a 50 year old theory**!
  - It is a theory of Quarks and Gluons
    - Very successful at high-energies (DIS, Jets,…)
    - Running coupling: No perturbation theory (PT) at low-energies
    - Lattice allows theory to be defined non-perturbatively at long distances

Gross and Wilczek; Politzer

- Glueballs:
  - Important footprint in literature (~ 1500)
  - Theoretical predictions mostly consistent
  - Experimental detection challenging: overlapped resonances
  - Essential for understanding of QCD

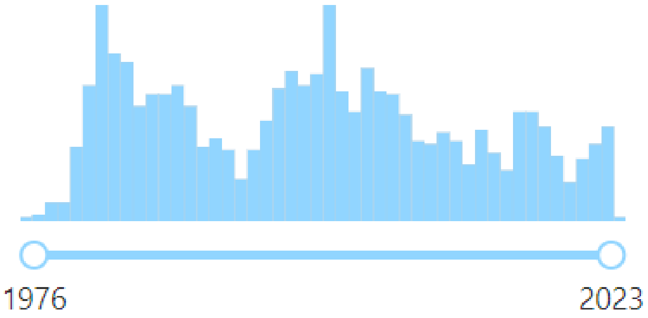
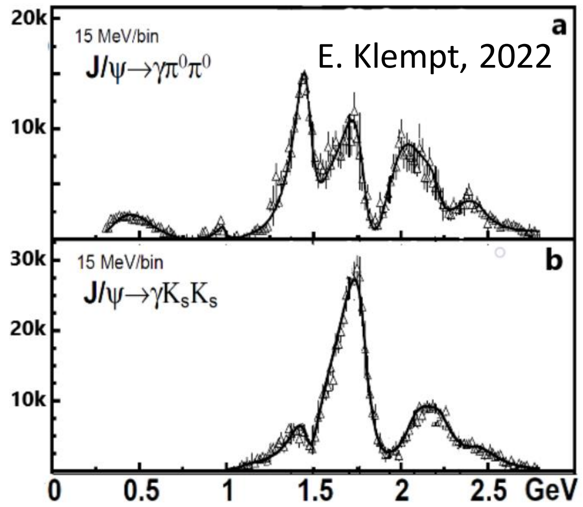


Figure: "find t glueball OR t glueballs"

- Not only Quantum ChromoDynamics…
  - Important for general non-Abelian gauge theory
  - Present in many scenarios on BSM, Dark Matter



General considerations 1

- Recently for Pure Gauge:
  - Extracted the masses of lightest glueballs for all 5 irreducible reps (A_1, A_2, E, T_1, T_2) of cubic rotation group and $P = \pm, C = \pm$: $N \in [2, 12]$, improving older works: *M. Teper, 1987; Morningstar and Peardon, 1997; Lucini, Rago, and Rinaldi, 2010*
  - We extrapolated to the continuum limit
  - For enough $N \in [2, 12]$, that we extrapolated to the $N = \infty$ limit
  - We have identified continuum spins J for the lightest states
  - Control systematic errors: multiglueball states; di-torelons; topological freezing
- Publications:
  - $SU(3)$: *AA and M. Teper, JHEP 11 (2020), 172, e-Print: 2007.06422 [hep-lat]*
  - $SU(\infty)$: *AA and M. Teper, JHEP 12 (2021), 082, e-Print: 2106.00364 [hep-lat]*
  - $SU(3) + 4\ flavors$: *AA, J. Finkenrath, A. Lantos, M. Teper, e-Print: 2212.02080 [hep-lat]*

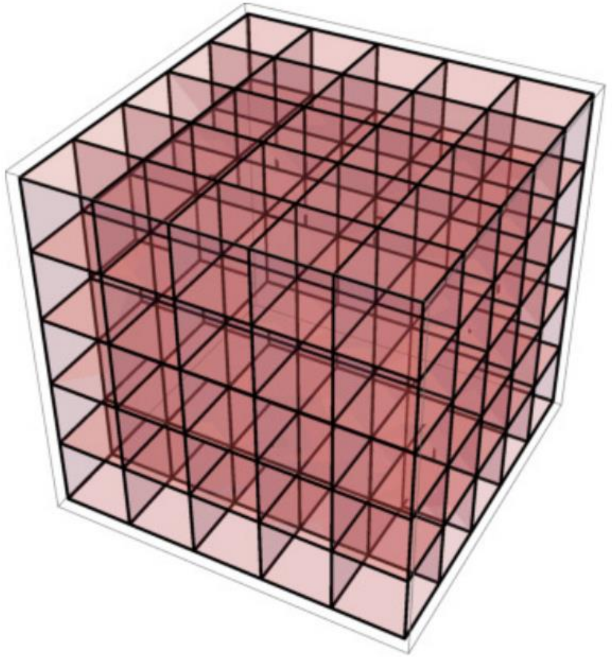
General considerations 2

- Glueball spectrum in $SU(3)$ has a phenomenological importance

- What is the effect of the light dynamical quarks?
  - Very noisy correlators – requires lots of statistics
  - Extract the spectrum with $N_f = 4$ light quarks with $m_\pi \approx 250$ MeV
    - Ensembles with 20k configurations
  - Investigate the low-lying spectrum of A_1^{++} with $N_f = 2+1+1$ light quarks
  - Ideally, perform an extended investigation for $N_f = 2+1+1$ light quarks

- What we know: *Chen, 2021 ($N_f = 2+1$, $m_\pi = 140$ MeV), Gregory, 2012 ($N_f = 2+1$, $m_\pi = 360$ MeV),*
- ***For a review see Vadacchino's plenary talk in Lattice2023:*** **A REVIEW ON GLUEBALL HUNTING**
  https://doi.org/10.5281/zenodo.7338133

Lattice Setup - fermions



- We use ensembles with Clover improved Twisted mass fermions produced with
  - $N_f = 4$ degenerate light flavors at three different lattice spacings
  - $N_f = 2 + 1 + 1$ (2 degenerate light flavors) + strange + charm
- We use the Iwasaki improved action

$$S_G = \frac{\beta}{3}\sum_x \left(c_0 \sum_{\substack{\mu,\nu=1\\ \mu<\nu}}^{4} \left[1 - \mathrm{Re}\,\mathrm{Tr}\left(U^{1\times1}_{x,\mu\nu}\right)\right] + c_1 \sum_{\substack{\mu,\nu=1\\ \mu\neq\nu}}^{4} \left[1 - \mathrm{Re}\,\mathrm{Tr}\left(U^{1\times2}_{x,\mu\nu}\right)\right] \right)$$

- The fermionic sector is implemented using the twisted mass formulation of lattice QCD, which for two mass degenerate quarks takes the form

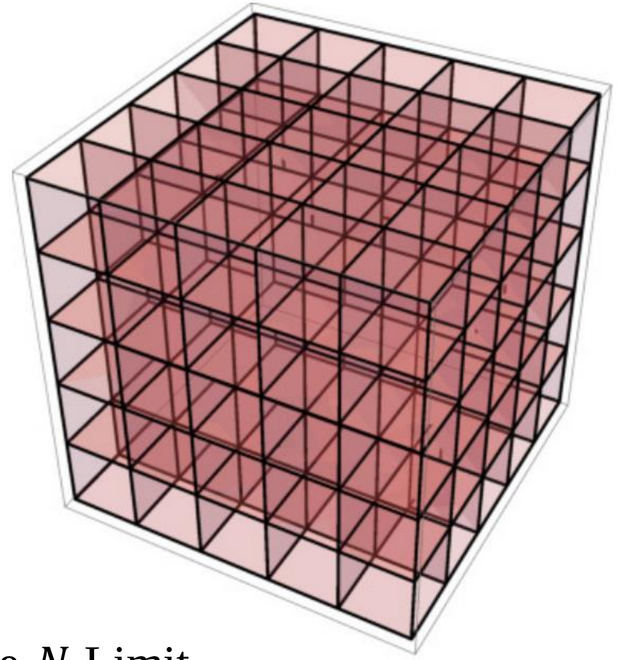
$$S_F^l = a^4 \sum_x \bar{\chi}^{(l)}(x) \left(D_W[U] + \frac{i}{4} c_{SW}\sigma^{\mu\nu}\mathcal{F}^{\mu\nu}[U] + m_{0,l} + i\mu_l\gamma_5\tau^3 \right) \chi^{(l)}(x)$$

	β	c_{sw}	μ	L	am_{PS}	t_0/a^2
$N_F = 4$						
cA4.60.16	1.726	1.74	0.006	16	0.2313(23)	3.565(39)
cB4.06.16	1.778	1.69	0.006	16	0.2652(53)	4.947(62)
cB4.06.24	1.778	1.69	0.006	24	0.1580(8)	4.667(17)
cC4.05.24	1.836	1.6452	0.005	24	0.1546(20)	6.422(48)
$N_F = 2+1+1$						
cA211.53.24	1.726	1.74	0.005	24	0.1661(4)	2.342(6)
cA211.25.32	1.726	1.74	0.003	32	0.1253(1)	2.392(4)

Extended Twisted Mass Collaboration

Lattice Setup – Pure Gauge

- We generate Pure Gauge ensembles
  - For several values of the lattice spacing (β)
  - For $N = 2$ to $N = 12$
- We use the Wilson action



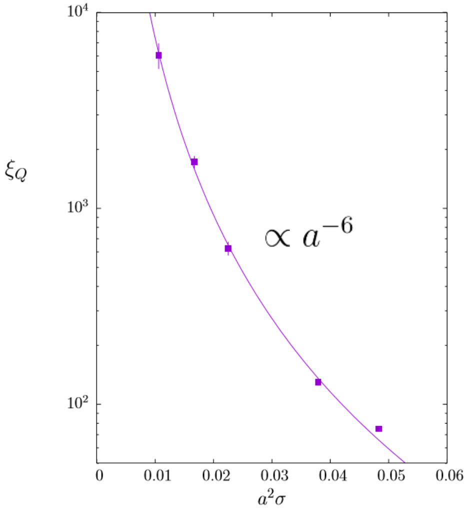
$$S_L = \beta \sum_p \{1 - \frac{1}{N} \mathrm{ReTr} U_p\}$$

$$\beta = \frac{2N}{g^2}$$

- Topological Freezing

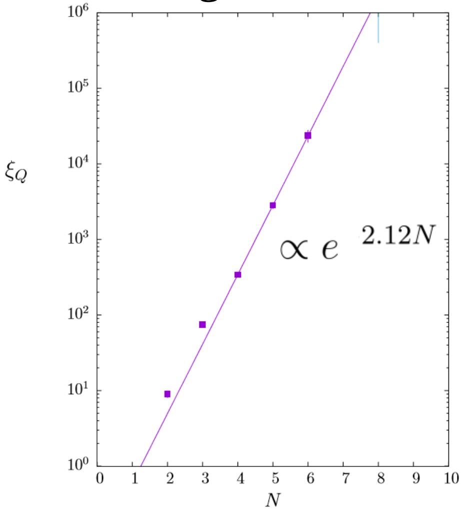
Correlation Length:
$\langle Q(is)Q(is+\xi_Q)\rangle / \langle Q^2 \rangle = e^{-1}$

Continuum Limit



AA and M. Teper, arXiv:2007.06422

Large-N Limit



AA and M. Teper, arXiv:2106.00364

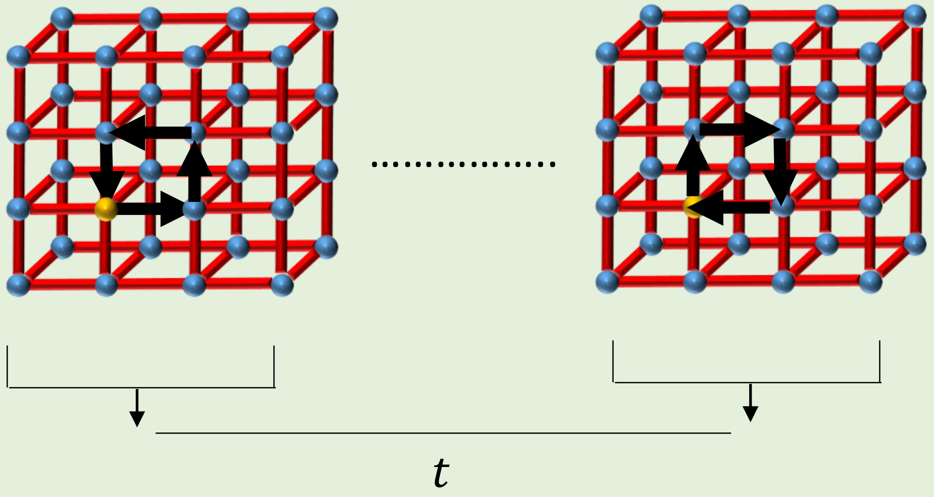
Quantum Numbers

- On the lattice, continuous rotational symmetry broken to the symmetry of the octahedral group
- Irreducible representations are A_1, A_2, E, T_1, T_2
- We use **gluonic** operators in irreducible representations of the octahedral group
- Near the continuum limit, full rotational symmetry is recovered
- Continuous spin obtained from the subduced representations of the rotation group $SO(3)$ restricted to the octahedral irreducible representations

J	A_1	A_2	E	T_1	T_2
0	1	0	0	0	0
1	0	0	0	1	0
2	0	0	1	0	1
3	0	1	0	1	1
4	1	0	1	1	1

- In addition, Parity and Charge Conjugation $P = \pm, C = \pm$

Correlation Function

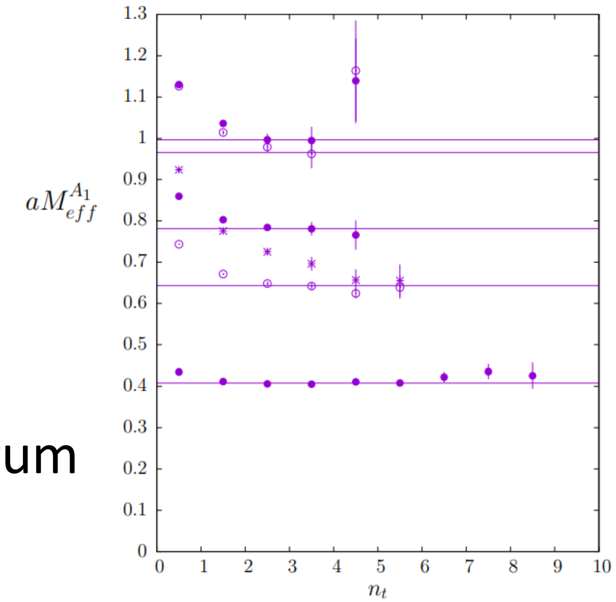


$$
\begin{aligned}
C(t) &= \langle \Phi^\dagger(t)\Phi(0)\rangle \\
&= \langle \Phi^\dagger(0)e^{-Ht}\Phi(0)\rangle \\
&= |\langle 0|\Phi(0)|vac\rangle|^2 e^{-E_0 t} \\
&\quad + \sum_{n=1} |\langle n|\Phi(0)|vac\rangle|^2 e^{-E_n t} \\
&\xrightarrow{t\to\infty} |\langle 0|\Phi(0)|vac\rangle|^2 e^{-E_0 t}
\end{aligned}
$$

- We calculate the effective energies

$$
\lim_{t\to\infty}\left[-\ln\left(\frac{C(t)}{C(t-a)}\right)\right] = aE_0
$$

Example of effective masses →



- We use the variational calculation to extract the excitation spectrum

Extraction of the Excitation Spectrum

- Construct a large basis of Operators $\Phi_i : i = 1, 2, \ldots$ with **RIGHT QUANTUM NUMBERS**
- Calculate the correlation function (Matrix) $C_{ij}(t) = \langle \Phi_i^\dagger(t) \Phi_j(0) \rangle$
- Diagonalize the matrix $C^{-1}(t=0)C(t=ma)$
- Extract the eigenvectors
- Extract the correlator for each state $(\sim \mathrm{e}^{-E_n t})$
- By fitting the results, we extract the mass (energy) for each state

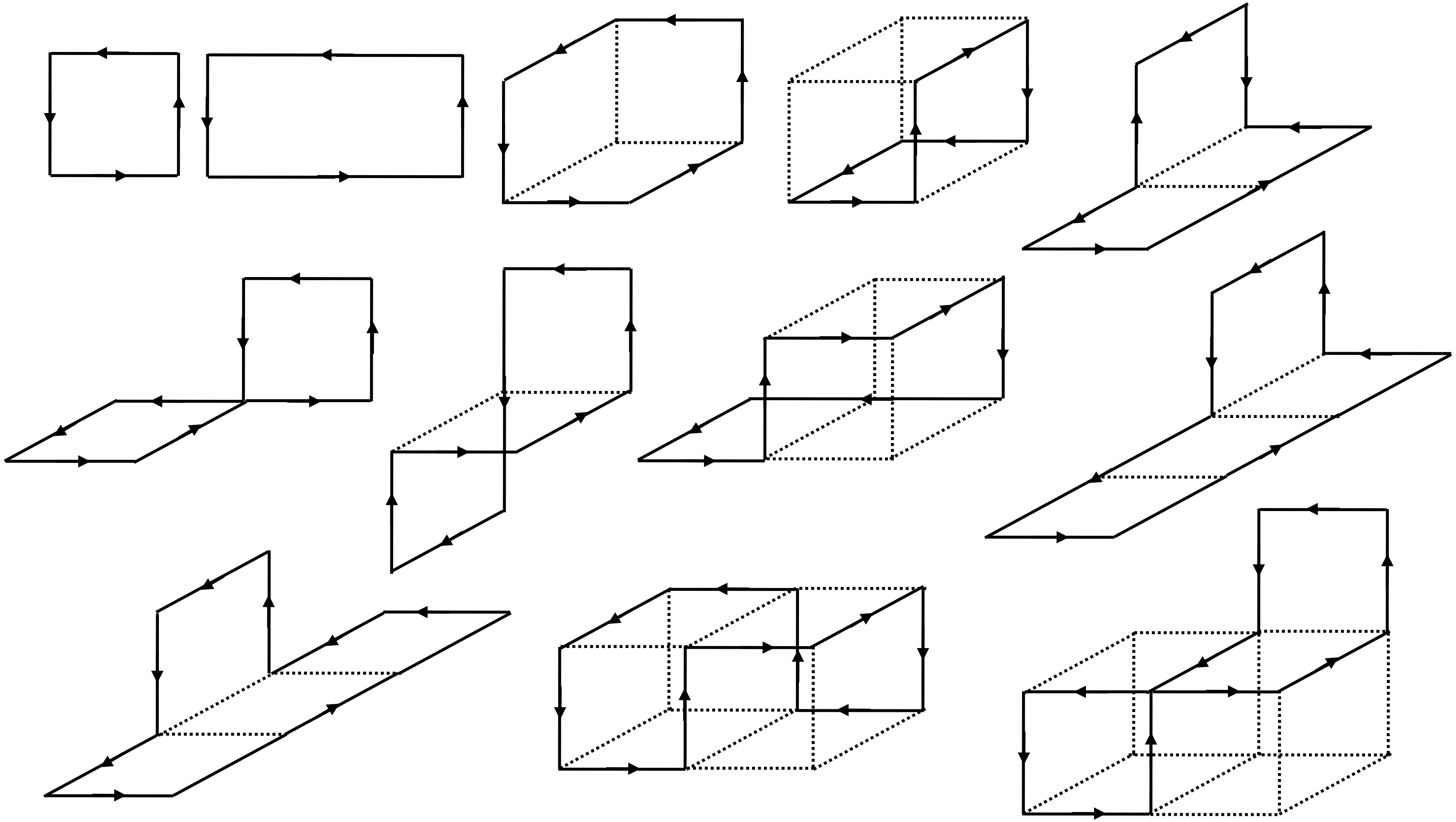
$$
\begin{bmatrix}
[A_1^{PC}] & 0 & 0 & 0 & 0 \\
0 & [A_2^{PC}] & 0 & 0 & 0 \\
0 & 0 & [E^{PC}] & 0 & 0 \\
0 & 0 & 0 & [T_1^{PC}] & 0 \\
0 & 0 & 0 & 0 & [T_2^{PC}]
\end{bmatrix}
$$

- This is the so called Generalized Eigenvalue Problem (GEVP)

M. Lüscher and U. Wolff, Nucl. Phys. B339 (1990) 222–252.

Lattice Operators

- Operators have been constructed by linear combinations of:



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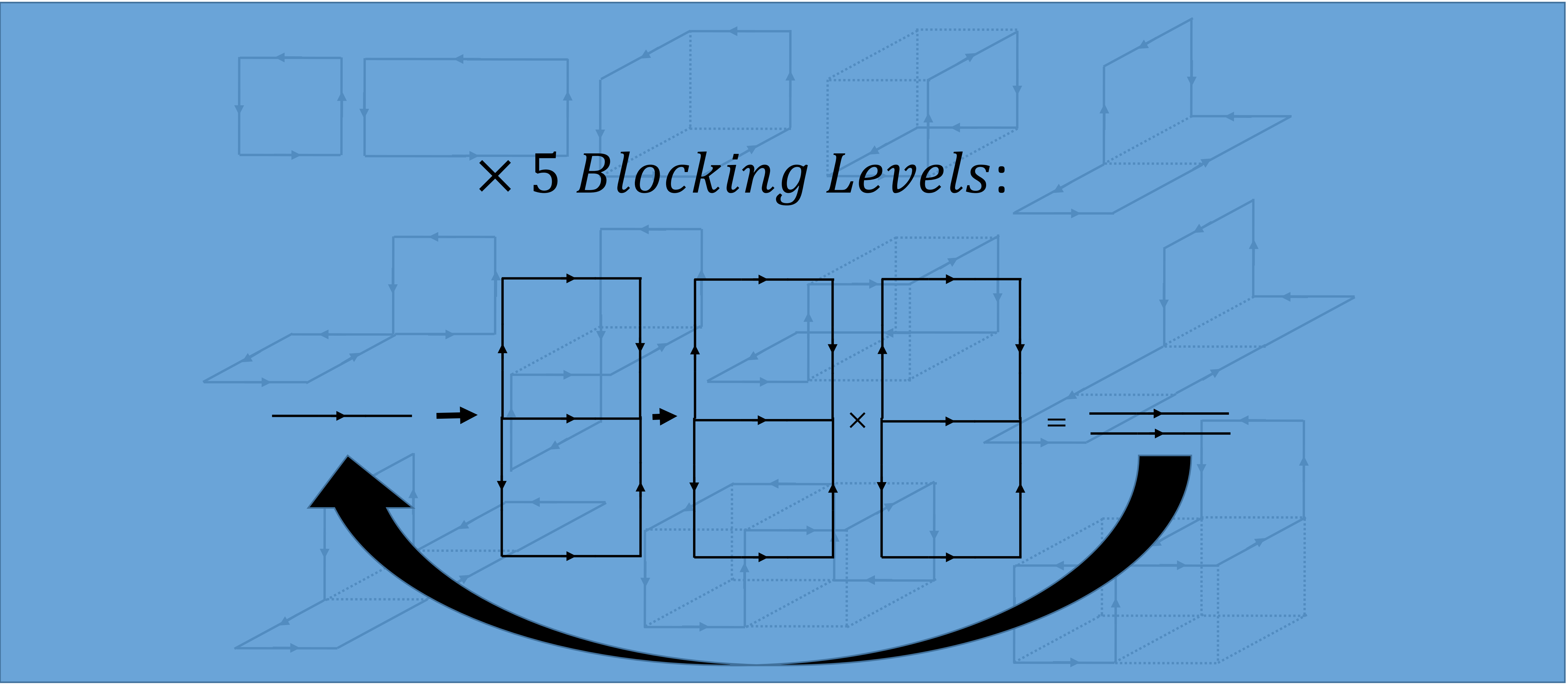
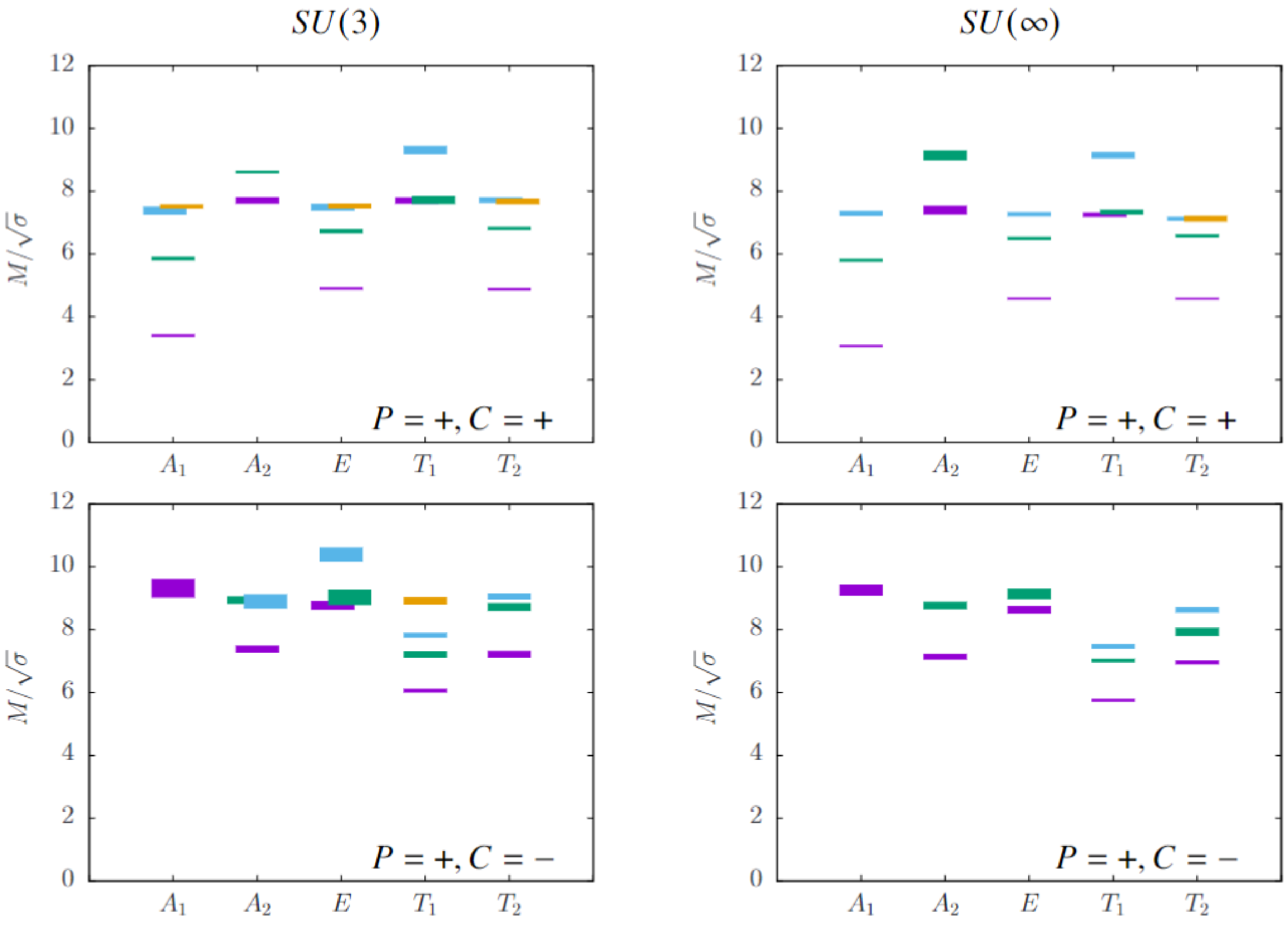


Recent results on Pure Gauge Theory



AA and M. Teper, JHEP 11 (2020) 172

AA and M. Teper, JHEP 12 (2021) 082

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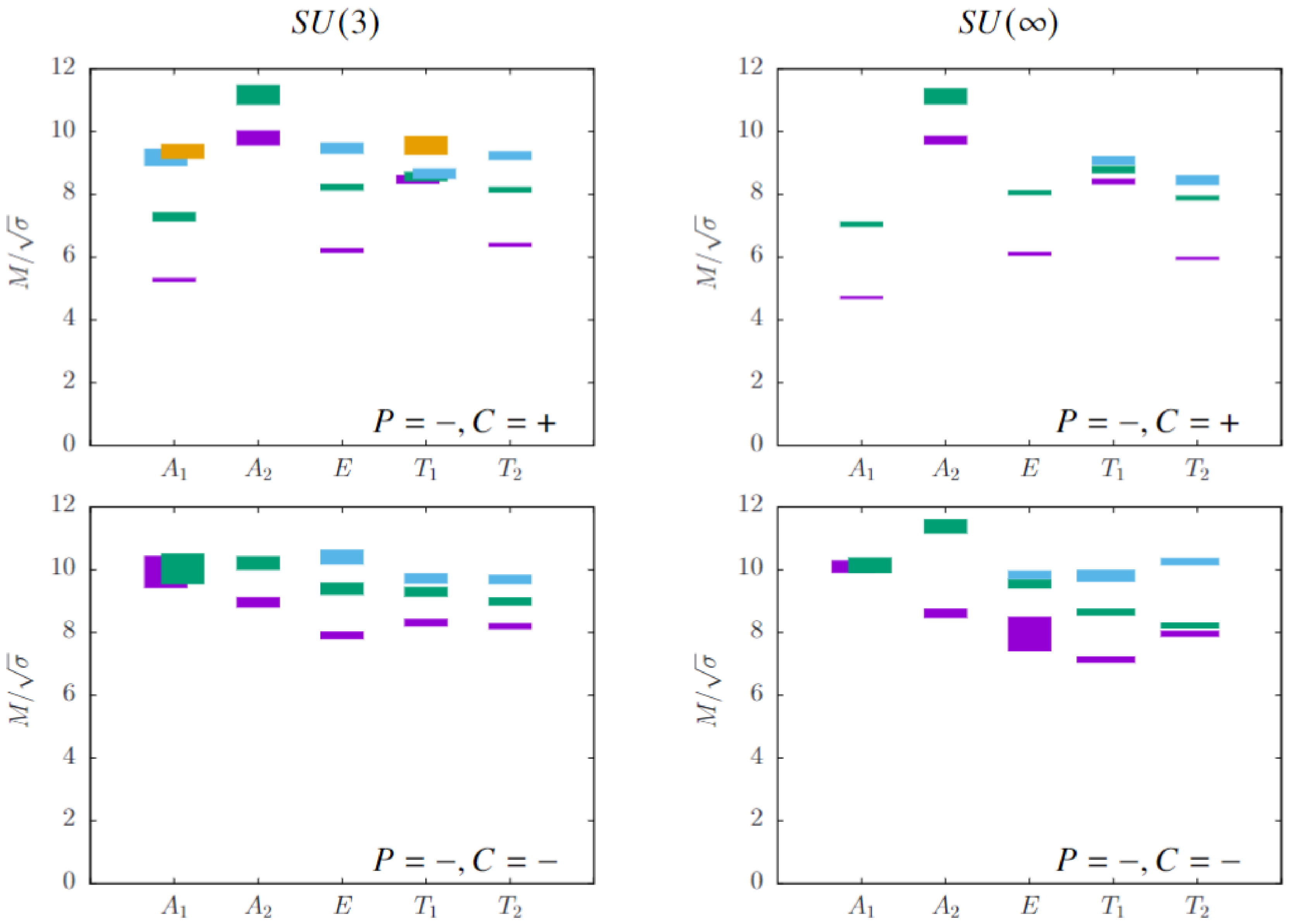

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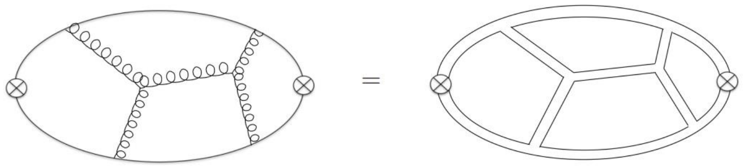
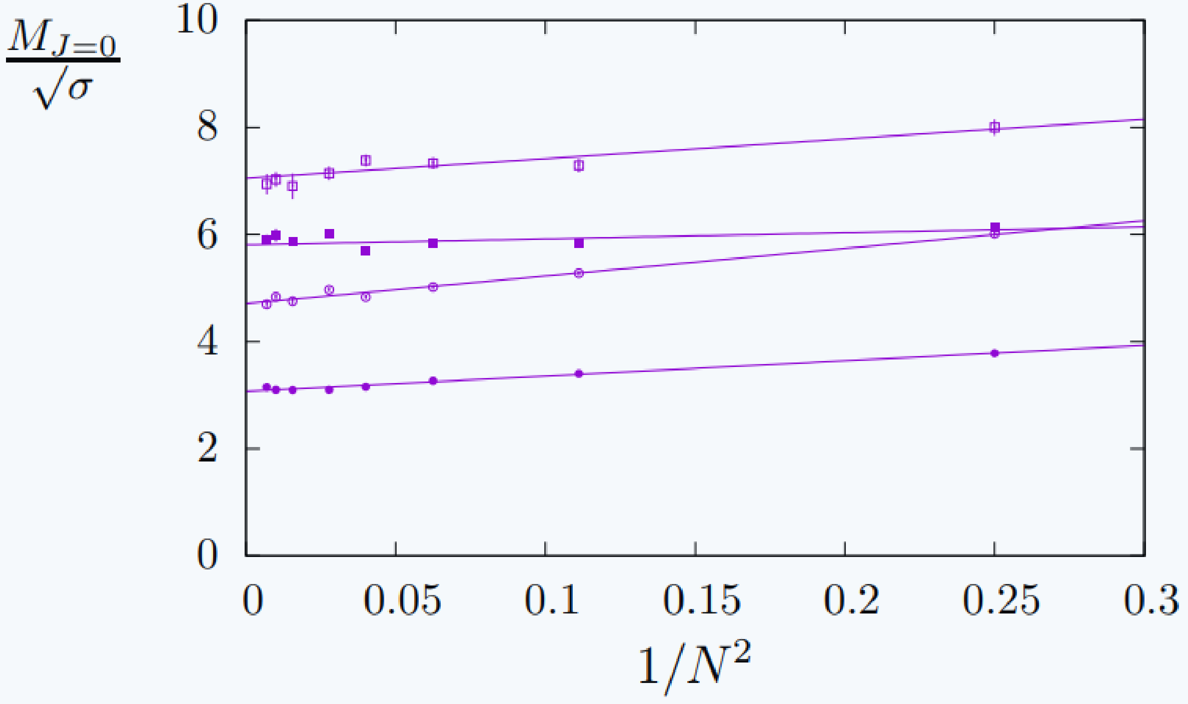
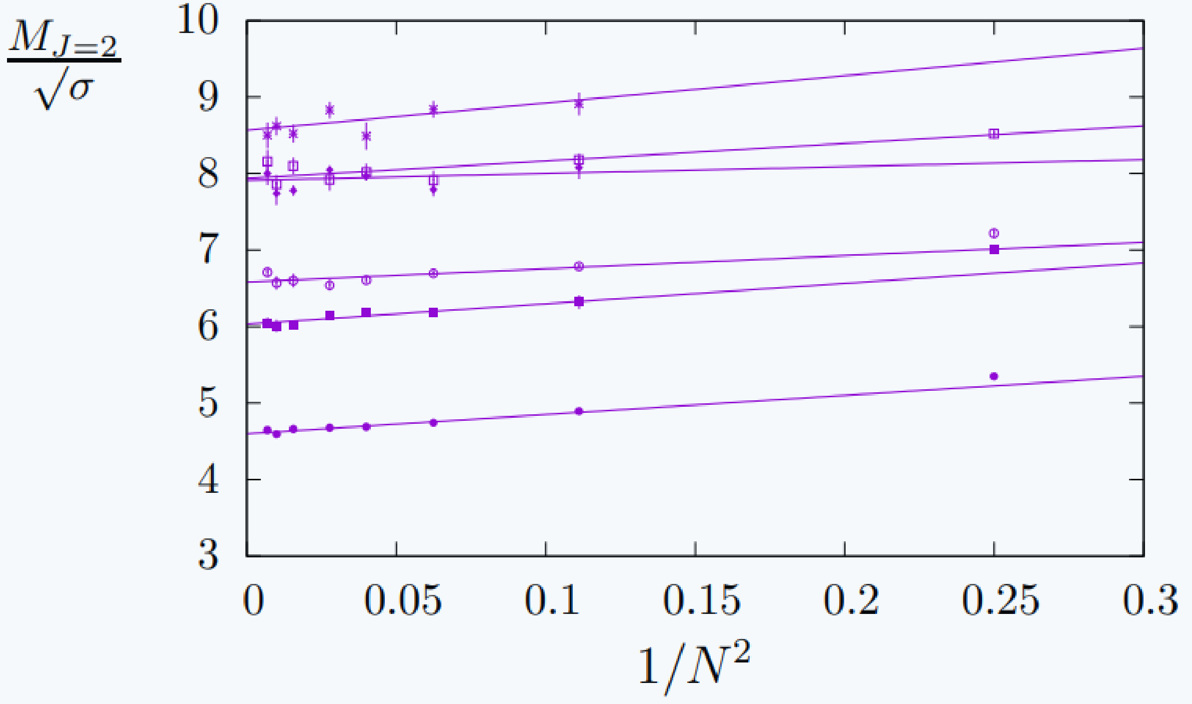
Results for Planar Limit

Extrapolations to the Large-N limit



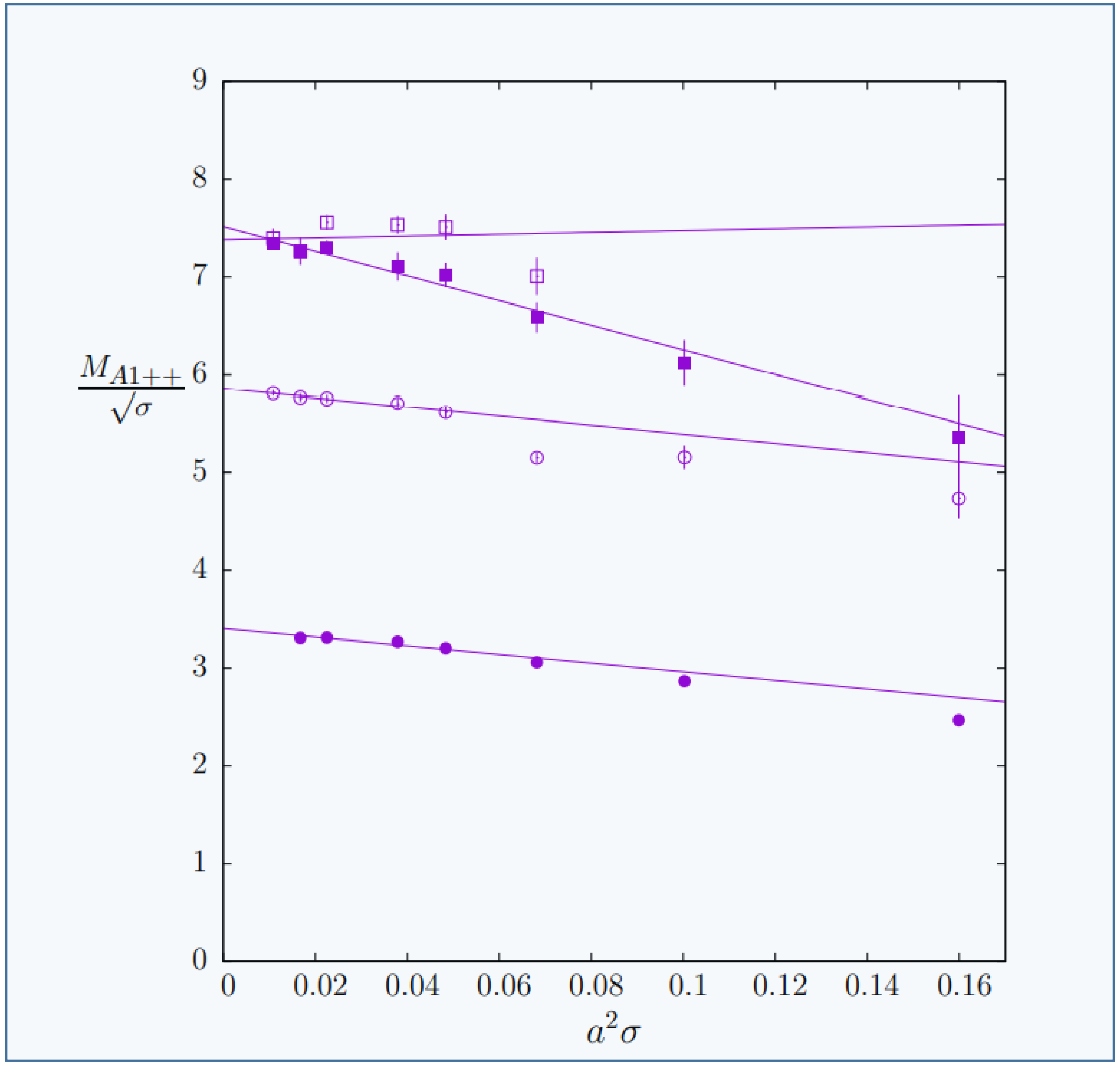


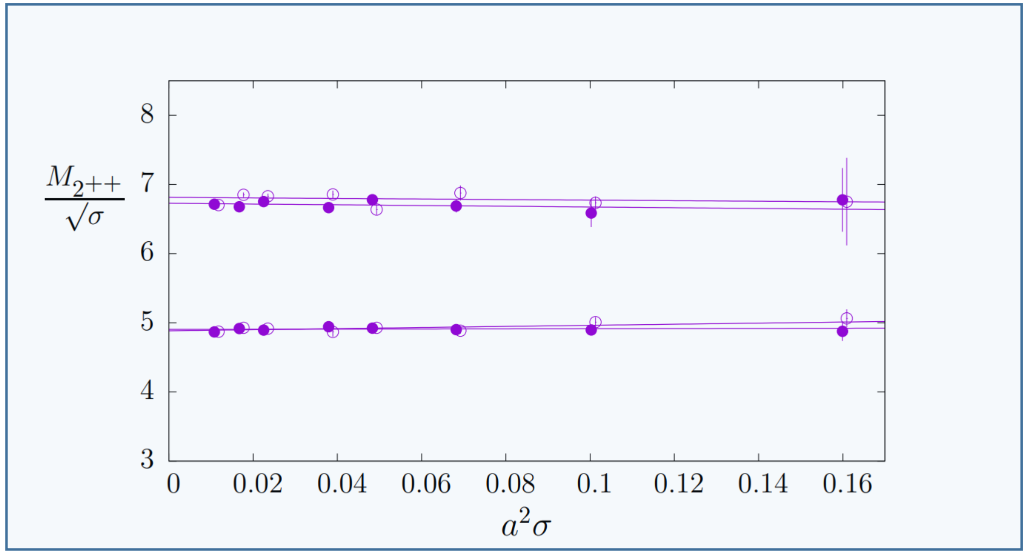
J^{PC}= 0^{++} ground (•) and first excited (■);
0^{-+} ground (◦) and first excited (□).
With extrapolations to $N = \infty$ from $N = 2 - 12$.

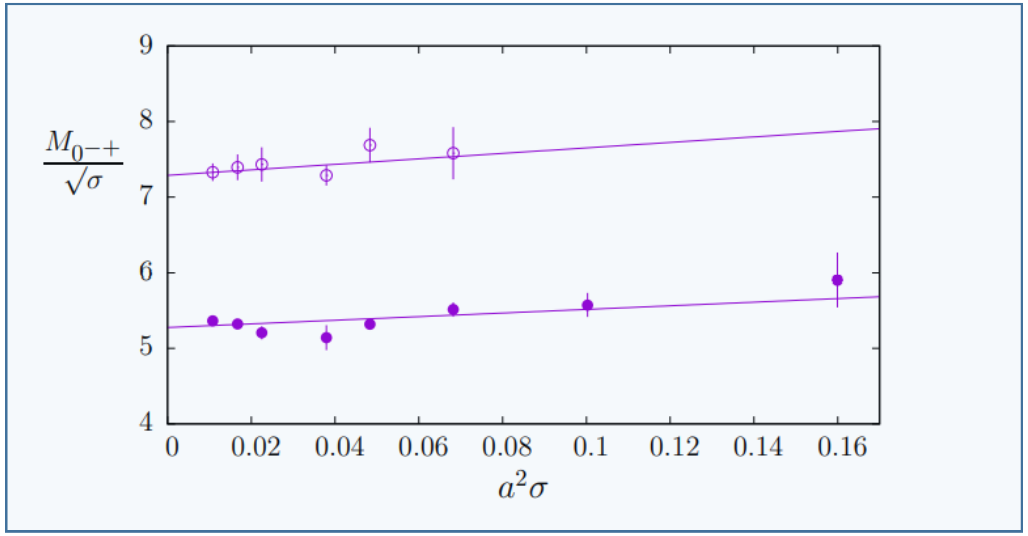


J^{PC}= 2^{++}ground (•) and first excited (◦) tensors; 2^{-+} ground (■) and first excited (□) pseudotensors; lightest 2^{+-} ($*$) and the lightest 2^{--} (♦).

Results for SU(3) Pure Gauge, continuum extrapolations







Asymptotic behaviour: $\sim a^2 \left[\frac{1}{-\log(a\Lambda)}\right]^{\hat{\gamma}_1}$, *Husung et al, Eur.Phys.J.C 80 (2020) 3, 200*

Topological Charge and scale setting

- We calculate the topological charge:

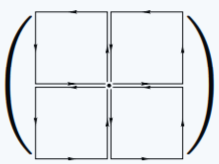
$$\mathcal{Q} = \int d^4 x q(x)$$

- With topological charge density:

$$q(x) = \frac{1}{32\pi^2} \epsilon_{\mu\nu\rho\sigma} \mathrm{Tr}\left\{ G_{\mu\nu} G_{\rho\sigma} \right\}$$

- We use the Clover definition:

$$C_{\mu\nu}\ (x) = \frac{1}{4} \mathrm{Im} \left(\quad \right)$$



- We smooth the UV fluctuations using the Wilson Flow.

- We solve the evolution equations:

$$\begin{aligned} \dot{V}_\mu(x,\tau) &= -g_0^2 \left[\partial_{x,\mu} S_G(V(\tau)) \right] V_\mu(x,\tau) \\ V_\mu(x,0) &= U_\mu(x) \,, \end{aligned}$$

- With link derivative defined as:

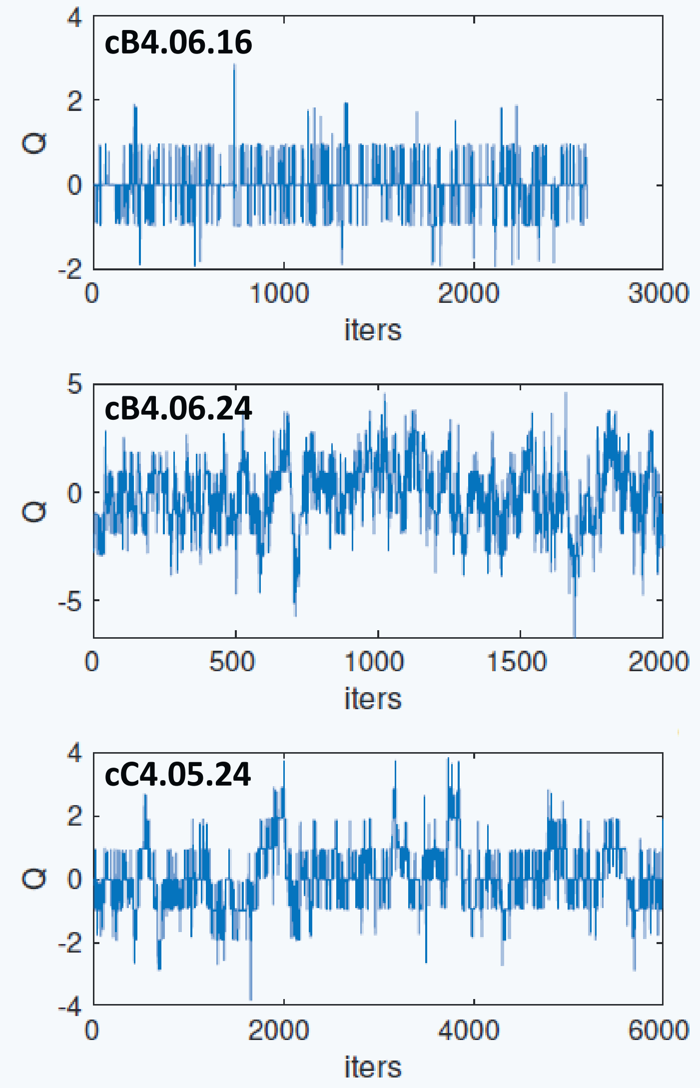
$$\begin{aligned} \partial_{x,\mu} S_G(U) &= i \sum_a T^a \frac{\mathrm{d}}{\mathrm{d}s} S_G\left(e^{isY^a} U \right) \Big|_{s=0} \\ &\equiv i \sum_a T^a \partial_{x,\mu}^{(a)} S_G(U)\,, \end{aligned}$$

- We can define a scale parameter t_0

$$F(t) = t^2 \langle E(t) \rangle \ \text{ where } E(t) = \frac{1}{4} B_{\mu\nu}^2(t)$$

$$F(t)|_{t=t_0(c)} = c$$

- With $c = 0.3$.



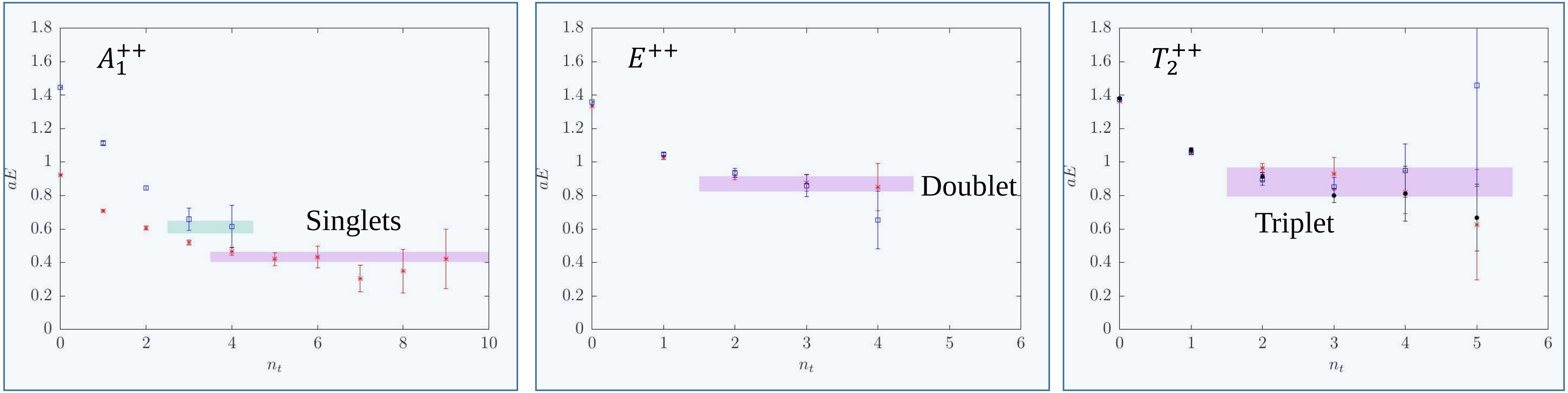
Effective Masses

- Correlation functions of specific operators used for extracting the spectrum

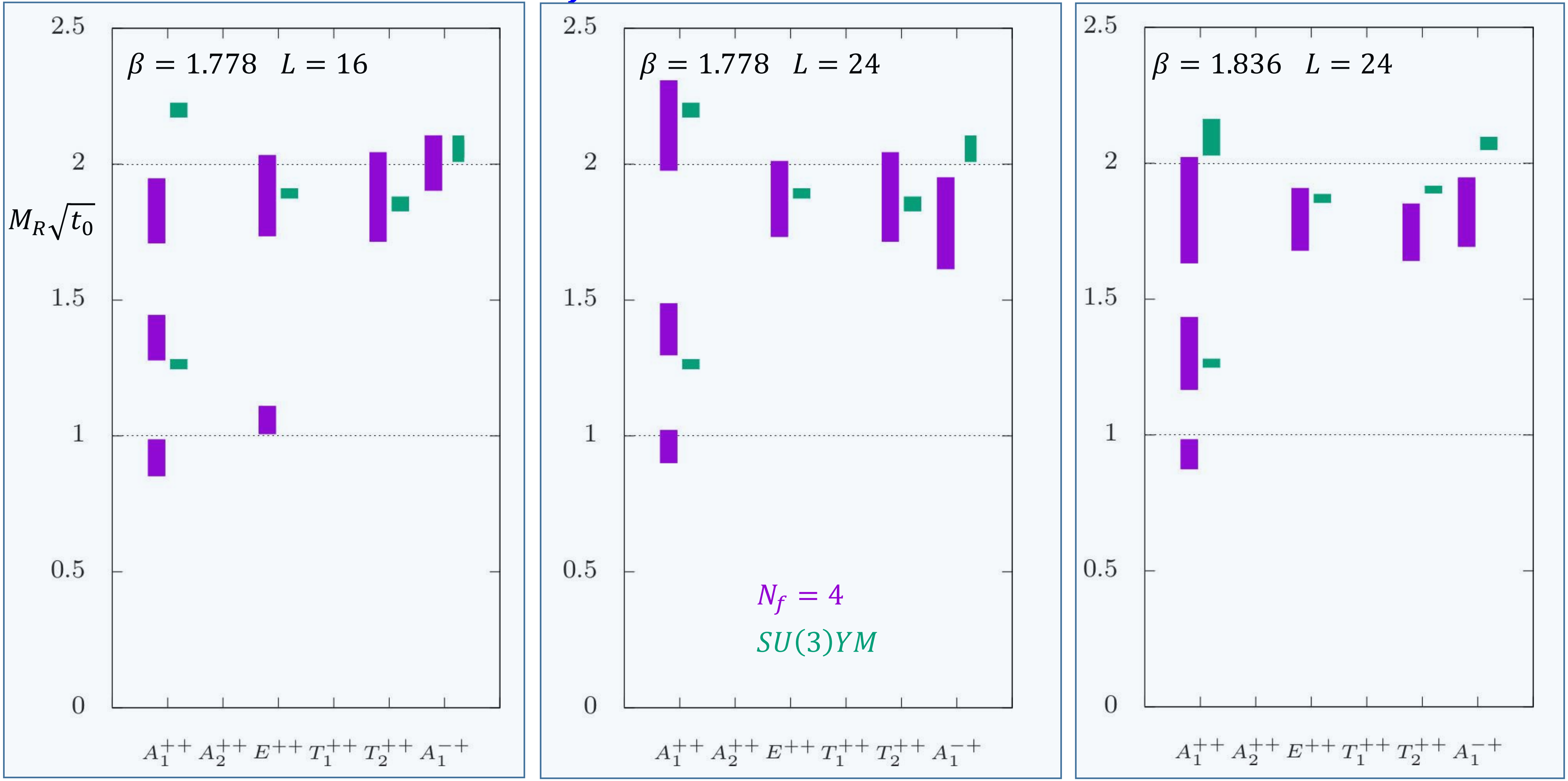
$$
\begin{aligned}
C(t) &= \langle \Phi^\dagger(t)\Phi(0)\rangle = \langle \Phi^\dagger(0)e^{-Ht}\Phi(0)\rangle \\
&= |\langle 0|\Phi(0)|vac\rangle|^2 e^{-E_0 t} + \sum_{n=1} |\langle n|\Phi(0)|vac\rangle|^2 e^{-E_n t} \stackrel{t\to\infty}{\longrightarrow} |\langle 0|\Phi(0)|vac\rangle|^2 e^{-E_0 t}
\end{aligned}
$$

- We calculate the effective energy $\lim_{t\to\infty}\left[-\ln\left(\frac{C(t)}{C(t-a)}\right)\right] = aE_0$

- Effective Energies for **cB4.06.24**



The $N_f = 4$ Glueball Spectrum



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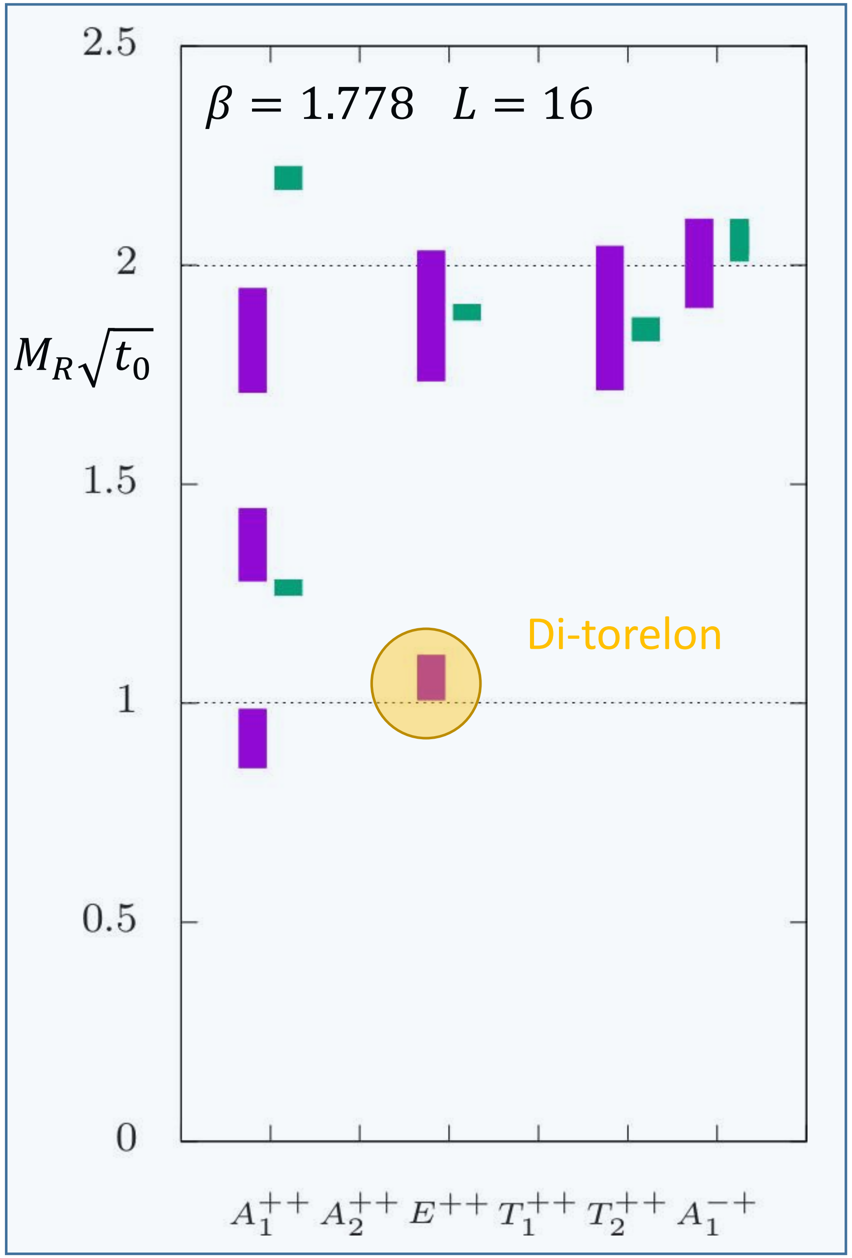
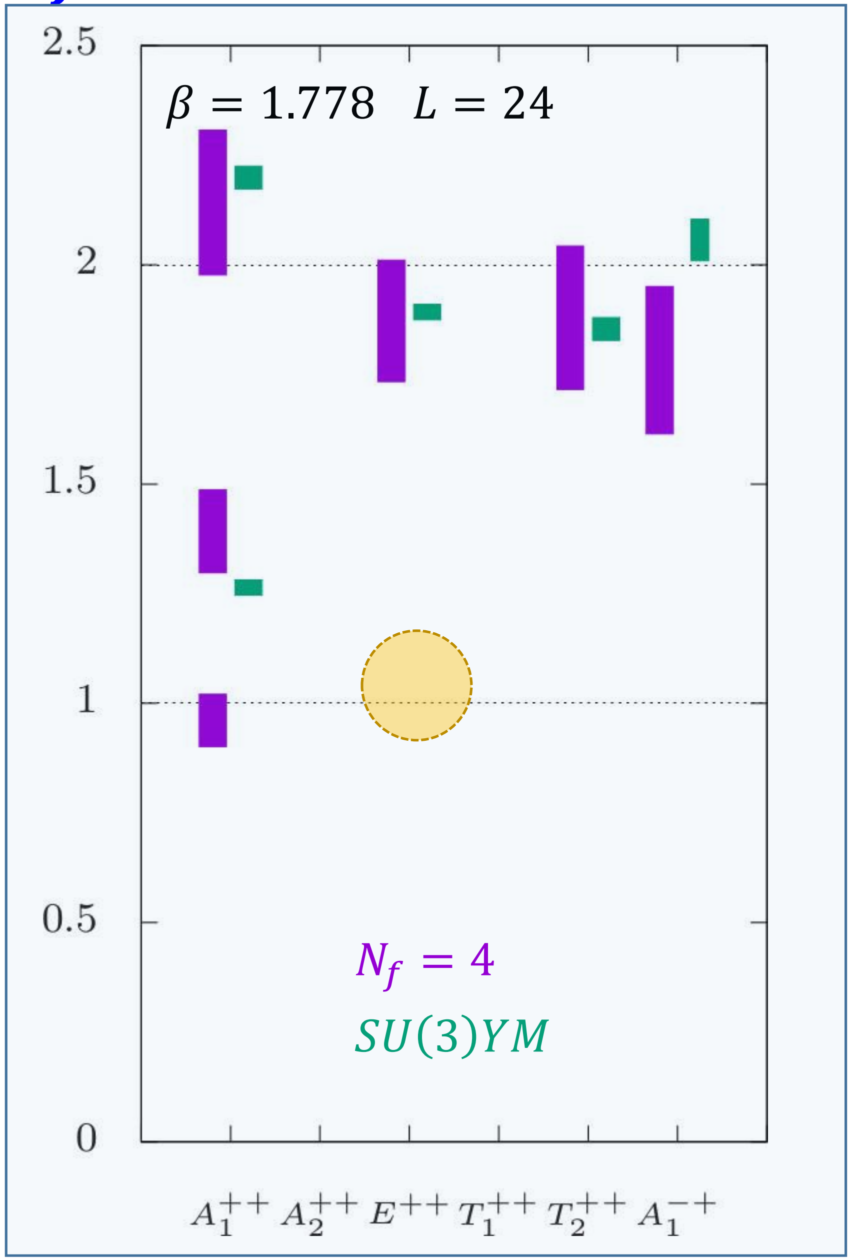
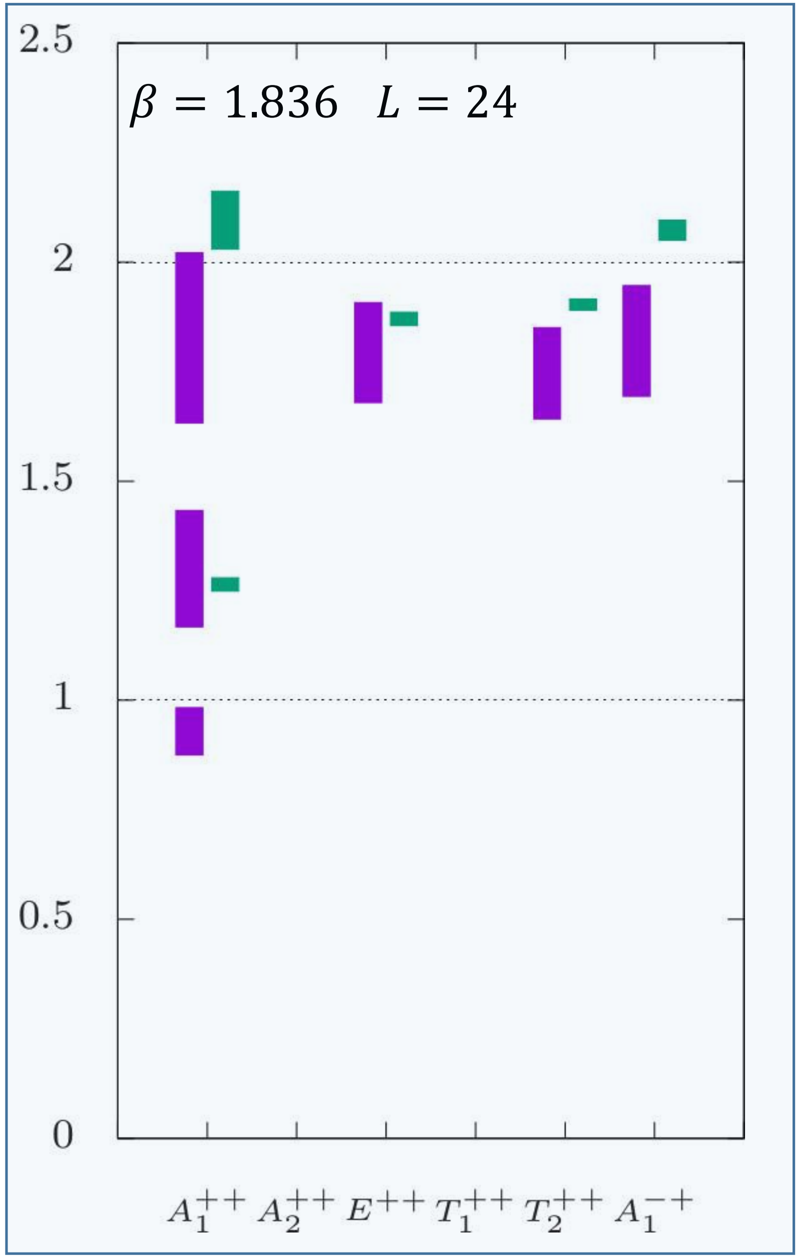
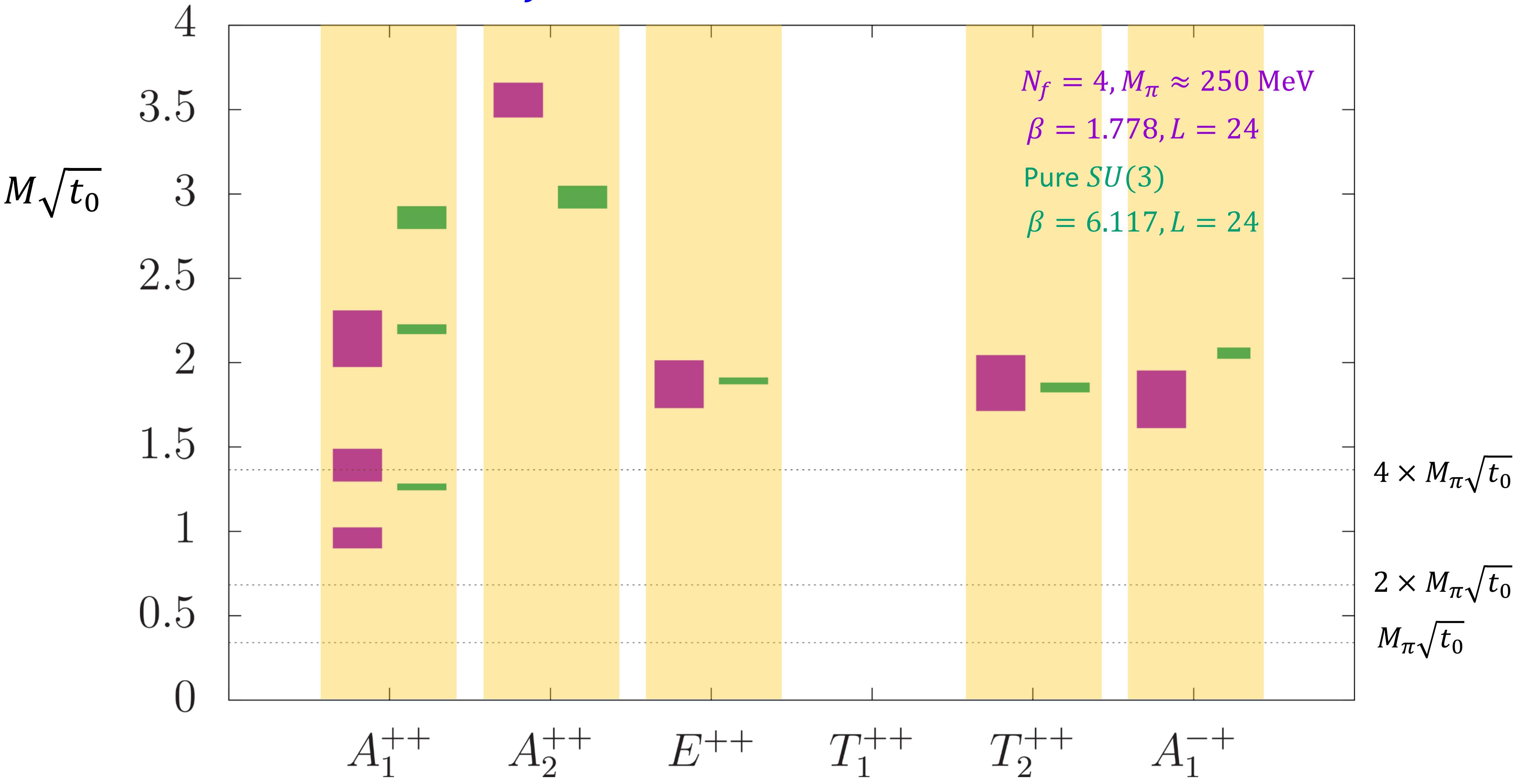


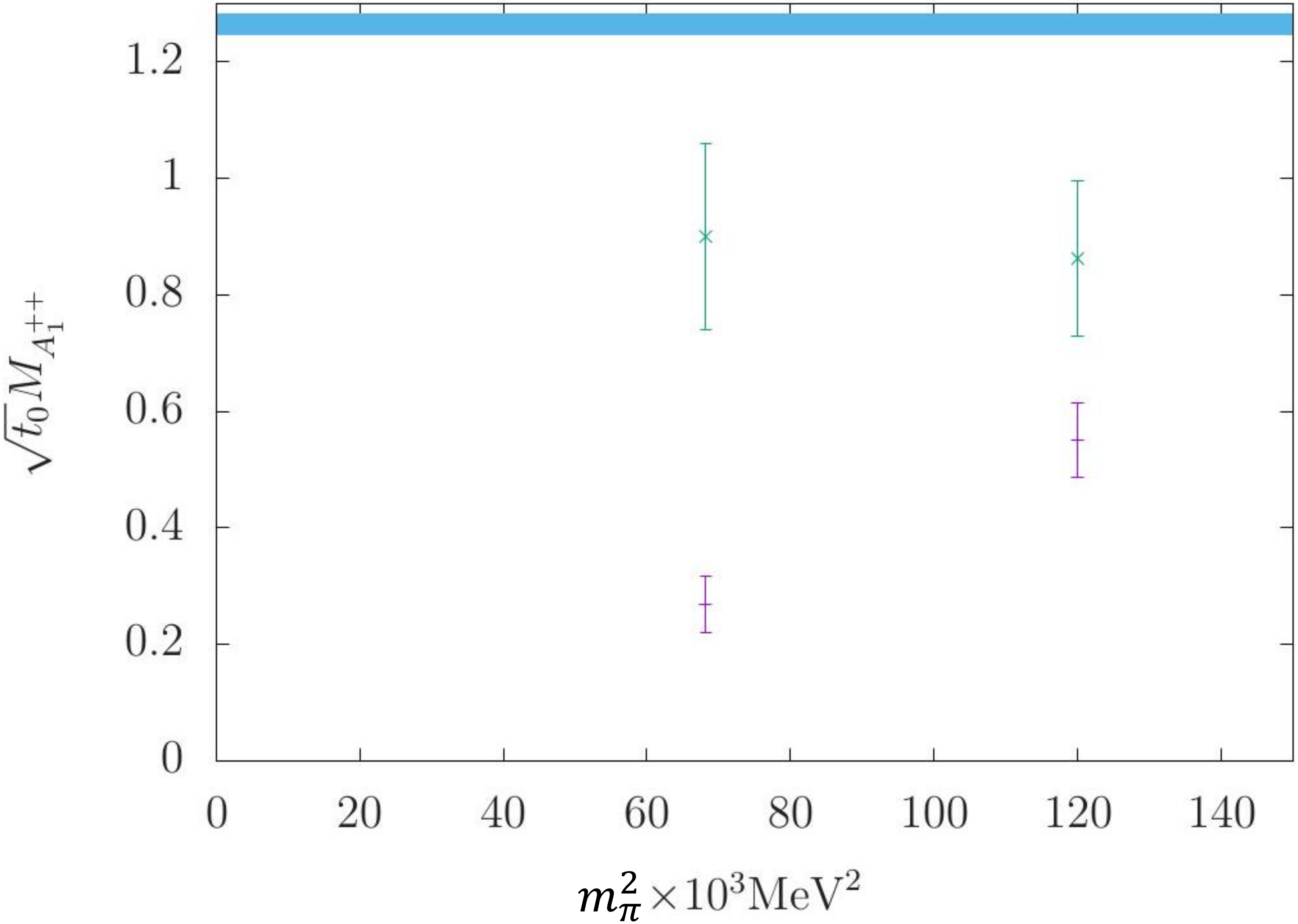




The $N_f = 4$ Glueball Spectrum

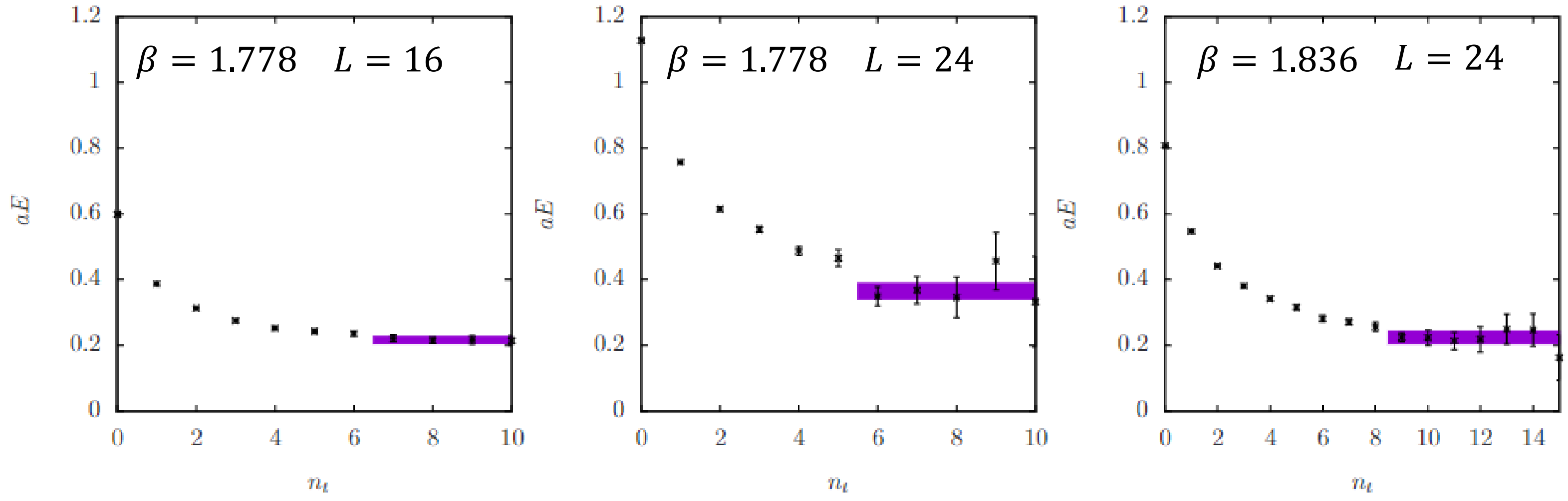


The A_1^{++} channel with $N_f = 2 + 1 + 1$ fermions



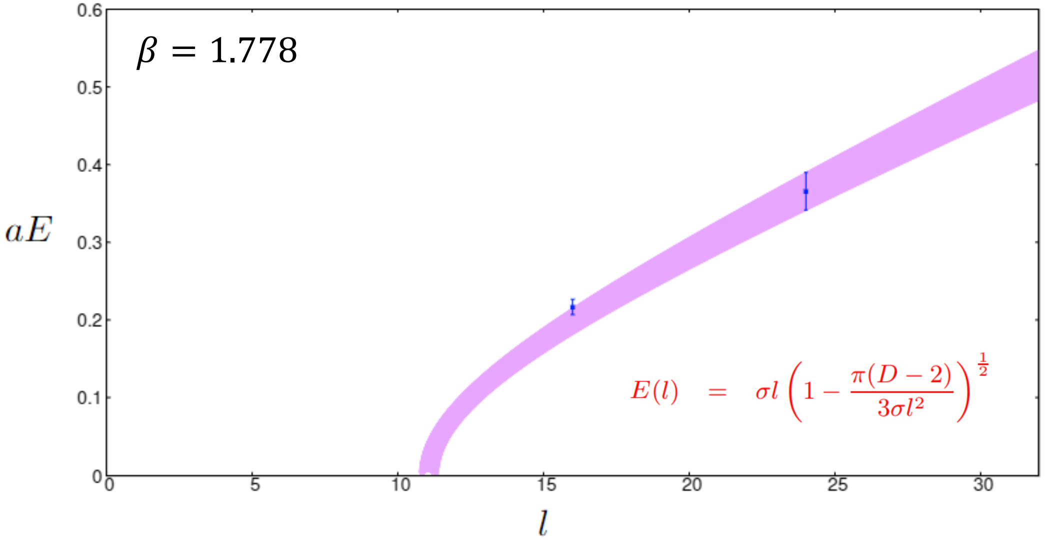
String tension

Extract the string tension from torelon masses:



Extract string tension from Nambu-Goto:

$$E_0(l) = \sigma l \left(1 - \frac{\pi(D-2)}{3\sigma l^2}\right)^{\frac{1}{2}}$$

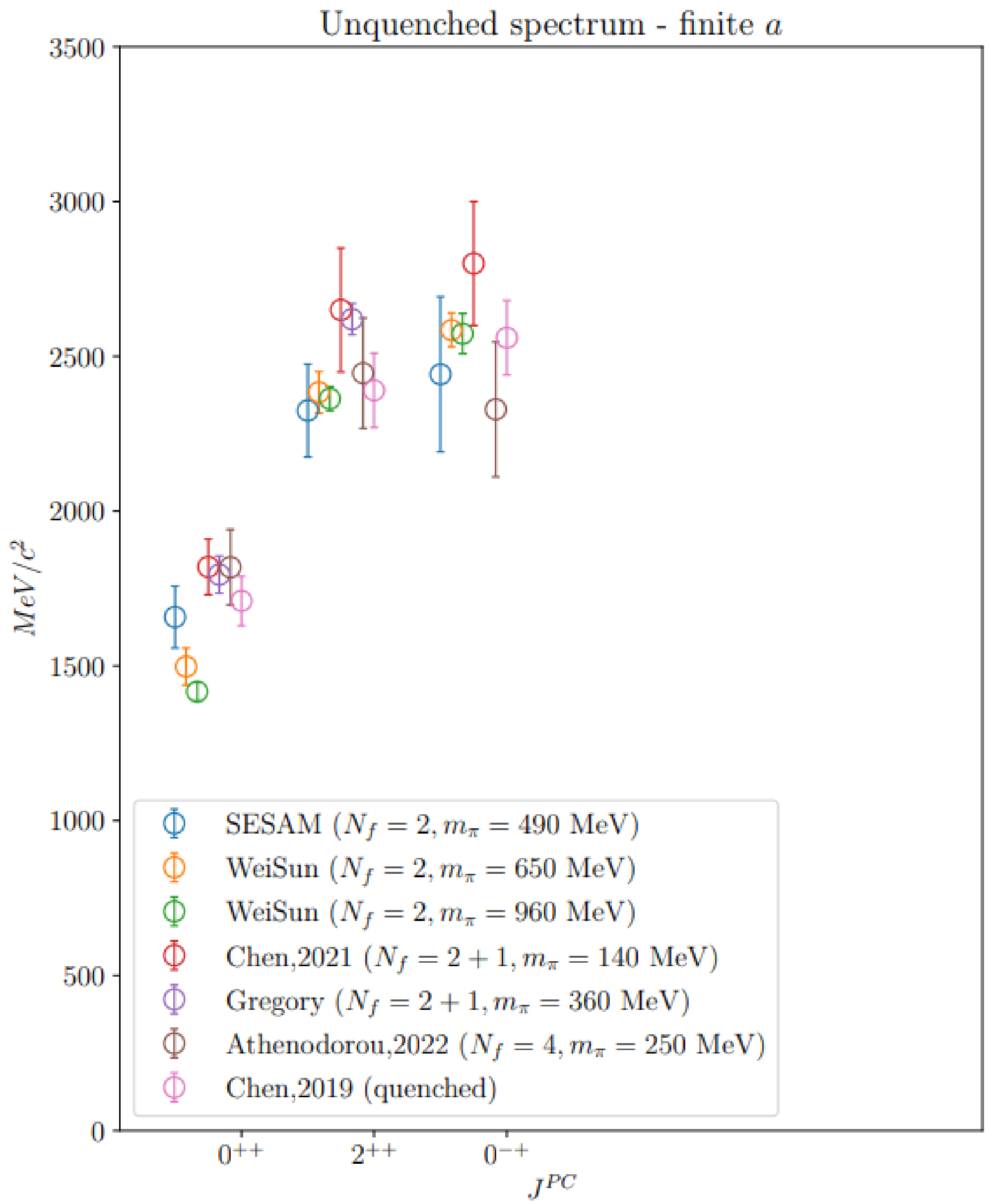


Comparison of string tensions

Ensemble	ground state	$a^2\sigma$	$a^2\sigma_{SU(3)}$
cB4.06.16	0.2167(100)	0.01824(60)	0.03063(32)
cB4.06.24	0.3655(241)	0.01715(100)	0.03172(43)
cC4.05.24	0.2247(183)	0.011365(735)	0.02350(18)

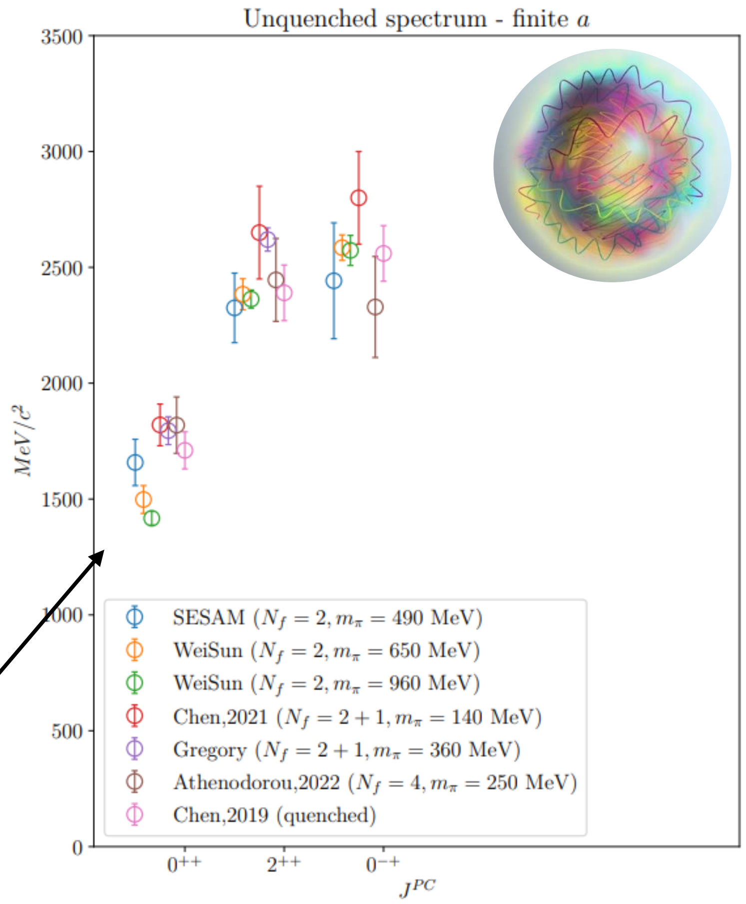
Unquenched spectrum

- Comparison from other calculations
  - Scalar ground state $m(0^{++}) \sim 1700\ \mathrm{MeV/c^2}$
  - Tensor ground state $m(2^{++}) \sim 2400\ \mathrm{MeV/c^2}$
  - Pseudo-scalar ground state $m(0^{-+}) \sim 2400\ \mathrm{MeV/c^2}$
- No continuum extrapolation



Observations

- Observations for Pure Gauge:
  - $N = 3$ 'close to' $N = \infty$: modest $O(1/N^2)$ correction suffices
  - $J^{PC} = 0^{++}$ scalar is the lightest glueball
  - $J^{PC} = 2^{++}$ next with mass $\sim 1.5 \times 0^{++}$
  - $J^{PC} = 0^{-+}$ mass is next, very close to 2^{++}
  - $J^{PC} = 1^{+-}$ is next, very close to first excited 0^{++}
  - Other $C = -$ states are much heavier
- Observations for QCD with light dynamical quarks
  - A_1^{++} includes an additional state
  - A_1^{++} ground state depends strongly on m_π
  - $J^{PC} = 2^{++}$ ground state is consistent with Pure Gauge $SU(3)$
  - $J^{PC} = 0^{-+}$ ground state is very close to 2^{++}
- The glueball masses are affected negligibly by dynamical quarks



From Vadacchino's Lattice2023 plenary

Thanks for your attention!!!