Simultaneous SM & BSM calculations for allowed & forbidden β-decays

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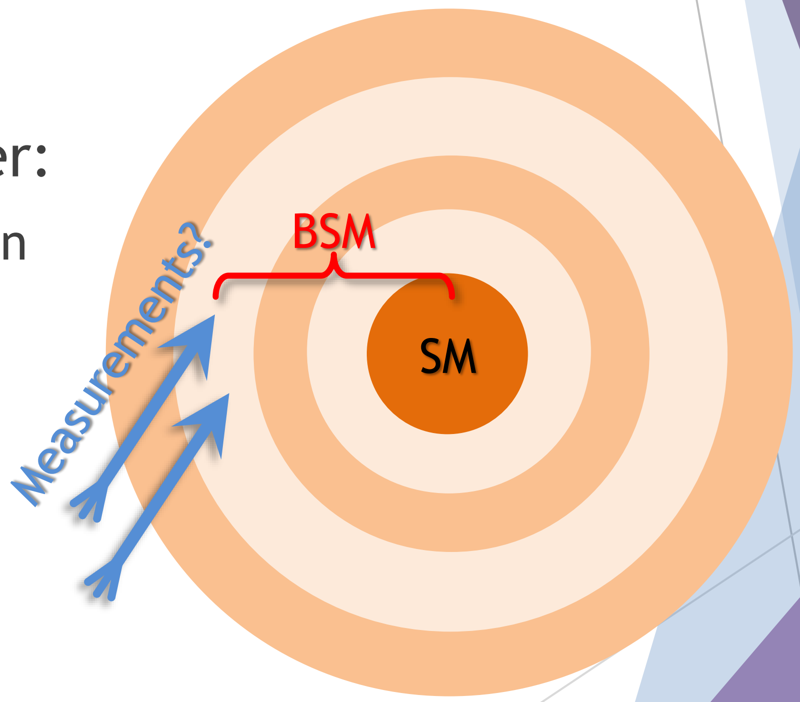
Why are we here?

- The Standard Model is incomplete (Dark matter, neutrino's mass)
- Experiments are searching for BSM signatures
- LHC → TeV energy frontier
- Nuclear phenomena are the precision frontier:
  - New experiments will have $\sim 0.1\%$ level precision
    - Sensitive to new physics at the TeV scale



- The theoretical goal: BSM predictions

  vs.

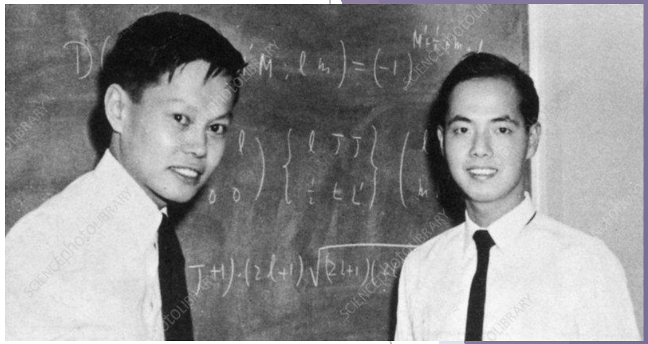
  SM corrections

Introduction: Weak interaction & β-decay BSM Searches

SM	Theory	Theory with Controlled accuracy
	Expt.	New Precise Bounds on BSM Interactions (^{23}Ne)
BSM	Theory	The Missing Theory for Forbidden Decays (Tensor+)
	Expt.	New Opportunities and Predictions

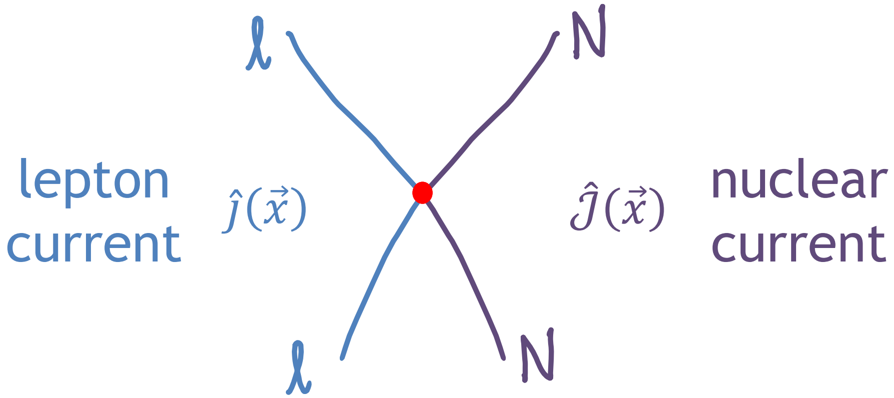
Summary: We Can Do It (and more)

Weak interaction



Theory: C.N. Yang and T.D. Lee

Low energy reaction of leptons with nucleons



$$\hat{\mathcal{H}}_W \sim C\, \hat{\jmath}(\vec{x}) \cdot \hat{J}(\vec{x})$$

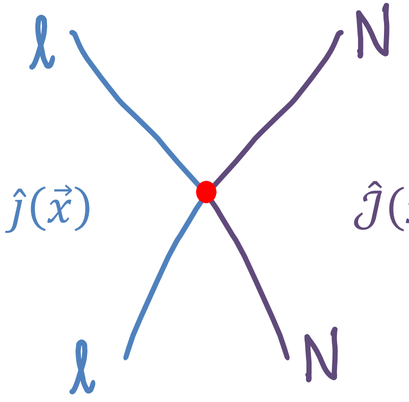
A-priori:

- Scalar (C_S)
- PseudoScalar (C_P)
- Vector (C_V)
- Axial vector (C_A)
- Tensor (C_T)

Weak interaction

Low energy reaction of
leptons with nucleons

lepton current $\hat{j}(\vec{x})$



$\hat{J}(\vec{x})$ nuclear current

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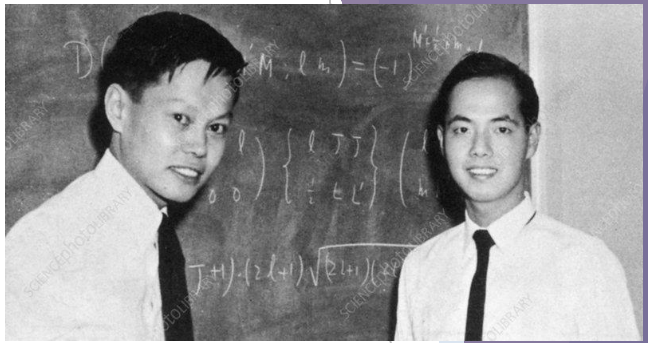

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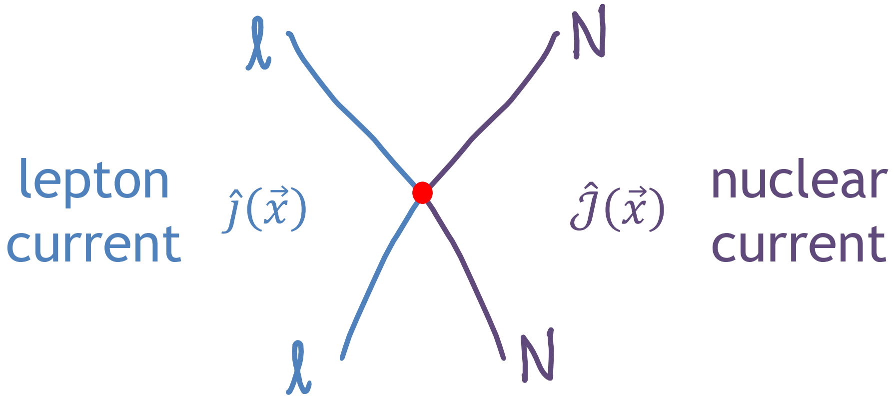
Experiment: C.S. Wu

Weak interaction

Low energy reaction of
leptons with nucleons



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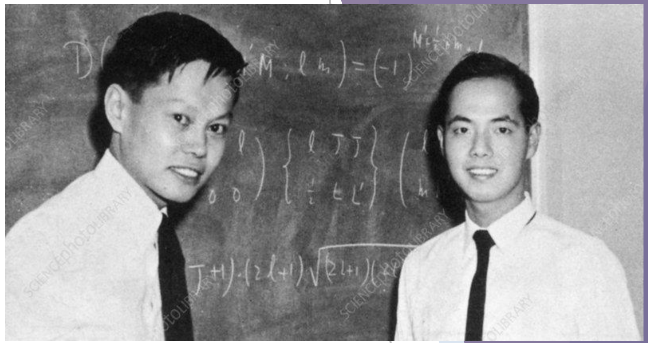

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The SM is incomplete

>> Ongoing searches for C_S, C_P, C_T
in precision ***nuclear β-decay*** experiments

Nuclear β-decay

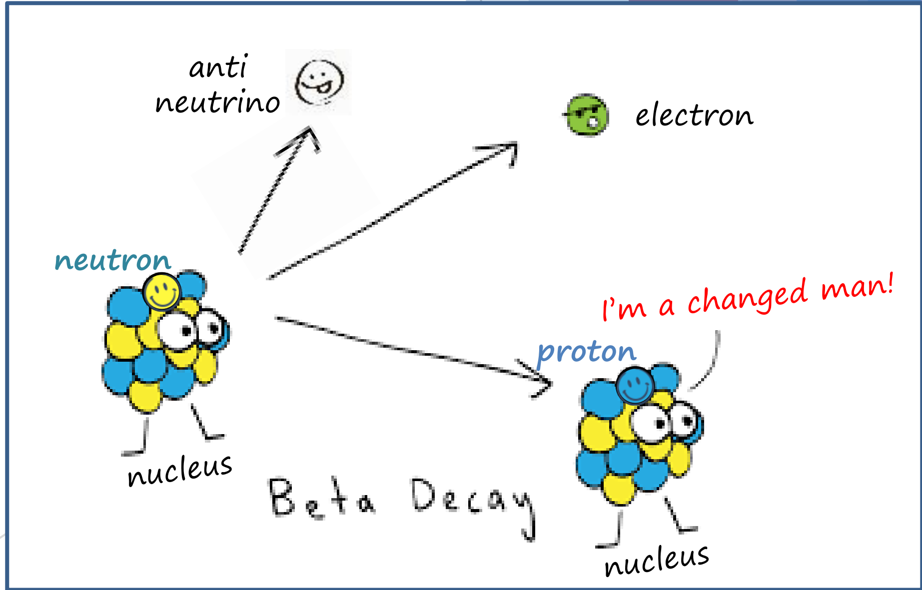
Low momentum transfer: $q \sim 0 - 10\ \mathrm{MeV/c}$

angular momentum

parity

Transitions $J^{\Delta\pi}$:

Beta decay, Khan Academy, cdn.kastatic.org/ka-perseus-images/8d978444f15f9bbc3bcadb0549816bc7e264b977.svg



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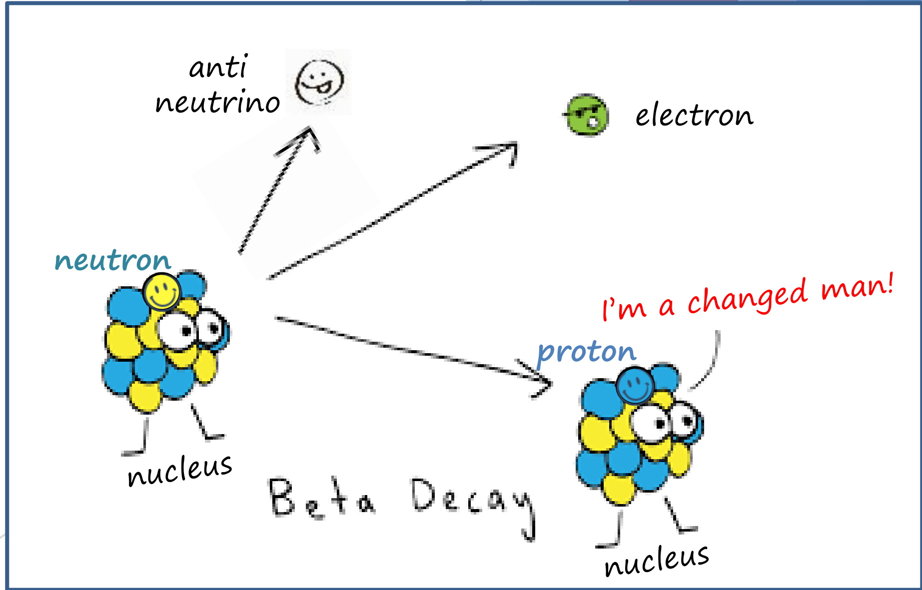

q – momentum transfer
$\vec{\beta} \equiv \frac{\vec{k}}{\epsilon}$ - electron's normalized momentum
$\hat{\nu} \equiv \frac{\vec{\nu}}{\nu}$ - neutrino's normalized momentum

Nuclear β-decay formalism

β-decay rate: $d\omega \propto kE(E_0 - E)^2 F_0 C_{\text{corr}}\, \Theta(q, \vec{\beta}\cdot\hat{\nu})$

k → electron's momentum

E → electron's energy

E_0 → maximal electron's energy

C_{corr} → $L_0 \cdot R \cdot X \cdot r \cdot S \cdot C_I \cdot Q \cdot U \cdot D_{\text{FS}} \cdot D_C \cdot R_N$

Item	Effect	Formula	Magnitude
1	Phase space factor [a]	$k\epsilon(\epsilon_0 - \epsilon)^2$	Unity or larger
2	Traditional Fermi function	F_0	
3	Finite size of the nucleus	L_0	10^{-1}-10^{-2}
4	Radiative corrections	R	
5	Shape factor	C	
6	Atomic exchange	X	
7	Atomic mismatch	r	
8	Atomic screening	S	10^{-3}-10^{-4}
9	Shake-up	See item 7	
10	Shake-off	See item 7	
11	Isovector correction	C_I	
12	Recoil Coulomb correction	Q	
13	Diffuse nuclear surface	U	
14	Nuclear deformation	D_{FS} & D_C	
15	Recoiling nucleus	R_N	
16	Molecular screening	ΔS_{Mol}	
17	Molecular exchange	Case by case	
18	Bound state β decay	Γ_b/Γ_c	Smaller than $1 \cdot 10^{-4}$
19	Neutrino mass	Negligible	

Hayen *et al.*, RMP 2018

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$$\Theta(q, \vec{\beta}\cdot\hat{\nu}) \propto \left|\langle\psi_f\|\hat{H}_W\|\psi_i\rangle\right|^2 \overset{\substack{\textit{allowed}\\ q\to 0}}{\propto} 1 + a_{\beta\nu}\vec{\beta}\cdot\hat{\nu} + b_F\frac{m_e}{E}$$

Observables

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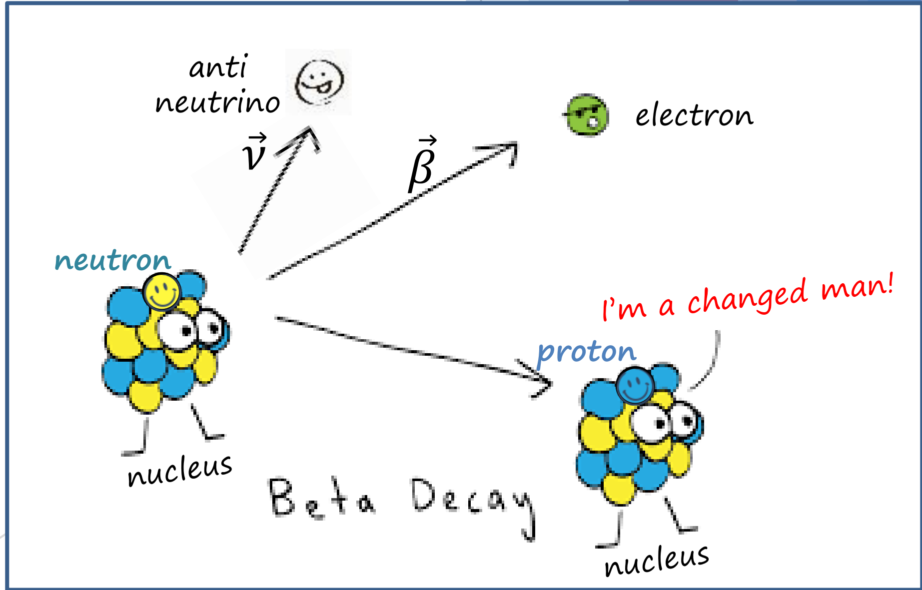

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Observables

Similar terms for Fermi ($T \to S$)

Angular correlation: $a_{\beta\nu} = -\frac{1}{3}\left(1 - \frac{|C_T|^2 + |C'_T|^2}{2|C_A|^2}\right)$

GT — BSM — $10^{-6} \sim$

- Quadratic in C_T, C'_T

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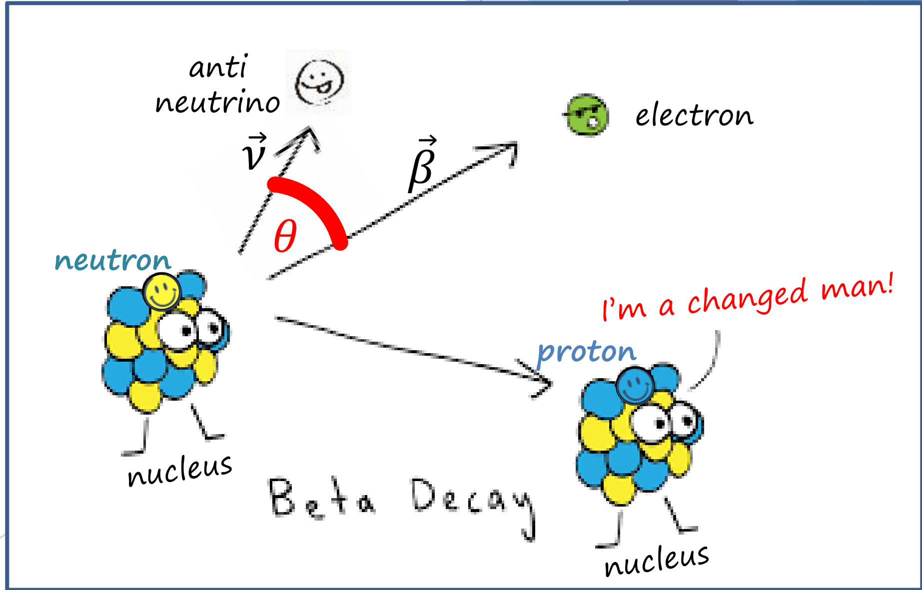

$C_A = 1.27$ Axial vector coupling constant (SM)
$C_T, C'_T \lesssim 10^{-3}$ Tensor coupling constants (BSM), unknown

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Nuclear β-decay formalism

Nuclear structure

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Angular correlation: $a_{\beta\nu} = -\frac{1}{3}\left(1 - \frac{|C_T|^2 + |C_T'|^2}{2|C_A|^2}\right)$ (GT; BSM term $\sim 10^{-6}$)

- Quadratic in C_T, C_T'

β Energy spectrum: Fierz term $b_{\text{F}}^{\beta^{\mp}} = \pm\frac{C_T + C_T'}{C_A}$ ($\sim 10^{-3}$)

- Vanishes for right-handed neutrinos ($C_T = -C_T'$)

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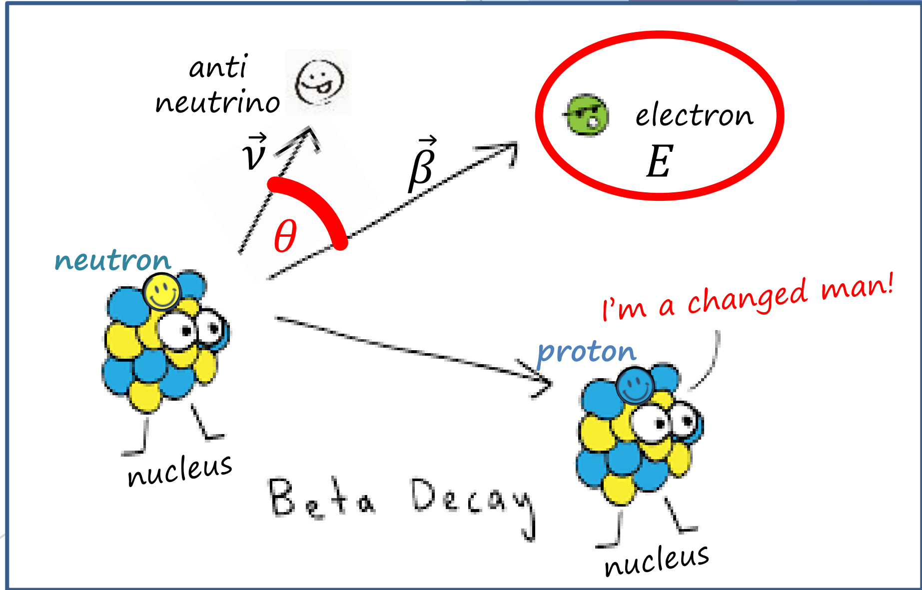

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Nuclear β-decay formalism

Searches for deviations from the SM "V-A" structure

$$\Theta(q,\vec{\beta}\cdot\hat{\nu}) \propto \left|\langle\psi_f\|\widehat{H}_W\|\psi_i\rangle\right|^2 \overset{\substack{\text{allowed}\\ q\to 0}}{\propto} 1 + a_{\beta\nu}\vec{\beta}\cdot\hat{\nu} + b_F\frac{m_e}{E}$$

Observables ($a_{\beta\nu}$, b_F)

Angular correlation:
$$a_{\beta\nu} \overset{\text{GT}}{=} -\frac{1}{3}\left(1 - \overset{\text{BSM}}{\frac{|C_T|^2+|C_T'|^2}{2|C_A|^2}}\right)$$

$10^{-6}\sim$

- Quadratic in C_T, C_T'

$\sim 10^{-3}$

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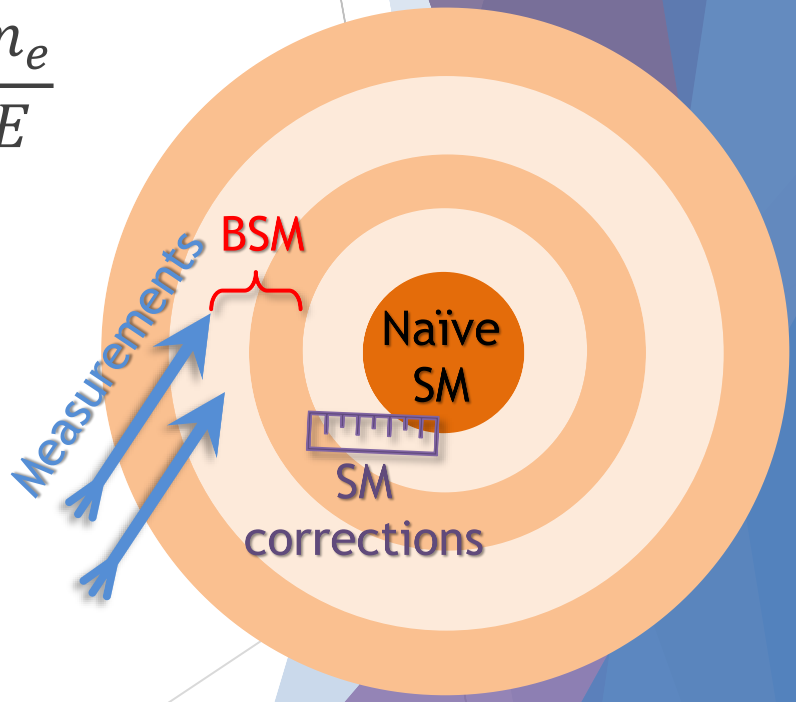

Required: BSM predictions vs. SM corrections

Standard Model: controlled accuracy

Identifying small parameters

q - momentum transfer
R - nucleus's radius
m_N - nucleon's mass
P_{fermi} - Fermi momentum
α - fine structure constant
Z - final nucleus's charge

- Kinematic parameters – β-decays have low momentum transfer:
  - $\epsilon_{qr} \sim qR \approx 0.01A^{1/3}$ *
  - $\epsilon_{\text{recoil}} \sim \frac{q}{m_N} \approx 0.002$ *

* For an endpoint of $\sim 2\ MeV$

- The nuclear model:
  - $\epsilon_{\text{NR}} \sim \frac{P_{\text{fermi}}}{m_N} \approx 0.2$
  - $\epsilon_{\text{EFT}} \sim 0.1 - 0.3$
- The Coulomb force:
  - $\epsilon_c \sim \alpha Z \approx 0.007Z$
- Numeric calculation:
  - ϵ_{solver}

AGM & Gazit, J.Phys.G 2022

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SM Multipole Expansion

Nuclear structure

β-decay rate: $d\omega \propto kE(E_0 - E)^2 F_0 C_{\text{corr}}\, \Theta(q, \vec{\beta}\cdot\hat{\nu})$

$$\Theta(q,\vec{\beta}\cdot\hat{\nu}) \propto \left|\langle\psi_f\|\hat{H}_W\|\psi_i\rangle\right|^2 \propto \sum_{J=0}^{\infty} f^V(\vec{\beta},\hat{\nu})\,\langle\psi_f\|\hat{O}_J^V\|\psi_i\rangle\langle\psi_f\|\hat{Q}_J^V\|\psi_i\rangle^*$$

$$\hat{H}_W \sim C_V \int d^3r\, \hat{j}^\mu(\vec{r})\,\hat{\mathcal{J}}_\mu^V(\vec{r})$$

Vector coupling constant

Lepton current

Vector nuclear current

Observables: lepton traces calculations (analytic)

Multipole operators:

$$\hat{C}_{JM}^V = \int d^3r\, j_J(qr) Y_{JM}(\hat{r}) \hat{\mathcal{J}}_0^V(\vec{r})$$

$$\hat{L}_{JM}^V = \frac{i}{q}\int d^3r \{\vec{\nabla}[j_J(qr)Y_{JM}(\hat{r})]\}\cdot\vec{\mathcal{J}}^V(\vec{r})$$

$$\hat{E}_{JM}^V = \frac{i}{q}\int d^3r \{\vec{\nabla}\times[j_J(qr)\,\vec{Y}_{JJ1}^M(\hat{r})]\}\cdot\vec{\mathcal{J}}^V(\vec{r})$$

$$\hat{M}_{JM}^V = \int d^3r [j_J(qr)\,\vec{Y}_{JJ1}^M(\hat{r})]\cdot\vec{\mathcal{J}}^V(\vec{r})$$

Vector nuclear current

and the same for the Axial (A) symmetry

AGM & Gazit, J.Phys.G 2022

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Lepton current | Vector nuclear current

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Multipole operators:

Axial (A) symmetry

"Allowed" (when $q \to 0$)	• 0^+: Fermi • 1^+: Gamow-Teller
"Forbidden" (vanish for $q \to 0$)	• All the rest ($J^{\Delta\pi}$)

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Vector nuclear current

AGM & Gazit, J.Phys.G 2022

Nuclear β-decay

For a general β-decay transition $J_i^{\pi_i} \to J_f^{\pi_f}$:

$$\Theta(q, \vec{\beta} \cdot \hat{\nu}) = \sum_{J=|J_i - J_f|}^{J_i + J_f} \Theta^{J^{\Delta\pi}}(q, \vec{\beta} \cdot \hat{\nu})$$

angular momentum — parity

$$J_i^{\pi_i} \to J_f^{\pi_f}$$

$$|J_i - J_f| \le J \le J_i + J_f$$

$$\Delta\pi = \pi_i \cdot \pi_f$$

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AGM & Gazit, J.Phys.G 2022

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angular momentum

parity

$$J_i^{\pi_i} \to J_f^{\pi_f}$$

$$|J_i - J_f| \leq J \leq J_i + J_f$$

$$\Delta\pi = \pi_i \cdot \pi_f$$

	$\Delta\pi$	Transition	Multipole operators
$J=0$	$+$	Fermi	$\hat{C}_0^V \sim 1$
	$-$	1^{st} forbidden	$\hat{L}_0^A \sim \epsilon_{qr} \sim q$ $\hat{C}_0^A \sim \epsilon_{NR} \sim q$
$J>0$	$(-)^J$	J^{th} forbidden	$\hat{C}_J^V \sim \epsilon_{qr}^J \sim q^J$ $\hat{M}_J^A \sim \epsilon_{qr}^J \sim q^J$
	$(-)^{J-1}$	Gamow Teller ($J=1$) unique $(J-1)^{\text{th}}$ forbidden	LO: $\hat{L}_J^A \sim \epsilon_{qr}^{J-1} \sim q^{J-1}$ NLO: $\hat{C}_J^A, \hat{M}_J^V \sim \epsilon_{\text{recoil}}\epsilon_{qr}^{J-1}$, $\epsilon_{NR}\epsilon_{qr}^J \sim q^{J+1}$

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SM corrections

- β-decay rate:

e.g., Gamow-Teller

$$d\omega \propto \left|\langle\psi_f\|\widehat{H}_W\|\psi_i\rangle\right|^2 \propto 1 + a_{\beta\nu}\vec{\beta}\cdot\hat{\nu} + b_F\frac{m_e}{E}$$

Spectrum shape — SM correction — SM $1+\delta_1$

Angular correlation — SM correction — SM $-\frac{1}{3}(1+\delta_a)$

Fierz term — SM correction — SM $0+\delta_b$

Multipole operator's matrix elements between the nuclear states

- $$\delta = f\left(\frac{\langle\psi_f\|\hat{C}_1^A\|\psi_i\rangle}{\langle\psi_f\|\hat{L}_1^A\|\psi_i\rangle}, \frac{\langle\psi_f\|\hat{M}_1^V\|\psi_i\rangle}{\langle\psi_f\|\hat{L}_1^A\|\psi_i\rangle}\right) + \mathcal{O}\left(\frac{\epsilon_{qr}^2}{15}, \epsilon_c^2\right)$$

$\sim \epsilon_{\text{NR}}\epsilon_{qr}, \epsilon_{\text{recoil}} \sim 10^{-2}$; $\sim 5\cdot 10^{-4}$

$\epsilon_{\text{NR}} \sim \frac{p_{\text{fermi}}}{m_N} \approx 2\cdot 10^{-1}$

$\epsilon_{\text{EFT}} \sim 1\cdot 10^{-1}$

$\epsilon_{qr} \sim qR \approx 5\cdot 10^{-2}$

$\epsilon_c \sim \alpha Z_f \approx 2\cdot 10^{-2}$

$\epsilon_{\text{recoil}} \sim \frac{q}{m_N} \approx 4\cdot 10^{-3}$

Multipole Expansion

General Theory – for any nucleus & transition

Controlled accuracy!

AGM & Gazit, J.Phys.G 2022

$^{23}\mathrm{Ne} \rightarrow {}^{23}\mathrm{Na}$

Soreq Nuclear Research Center

$\hat{H}$

Nuclear Hamiltonian

USDB (USD)

$\langle \psi_f \| \quad \| \psi_i \rangle$

Nuclear wave functions

$\hat{J}(\vec{x})$

Nuclear currents

LO (1-body currents)

$\hat{O}_J$

Multipole operators

$\langle \psi_f \| \hat{O}_J \| \psi_i \rangle$

Nuclear matrix elements

Shell Model $\sim\mathbf{10^{-1}}$ **uncertainty**

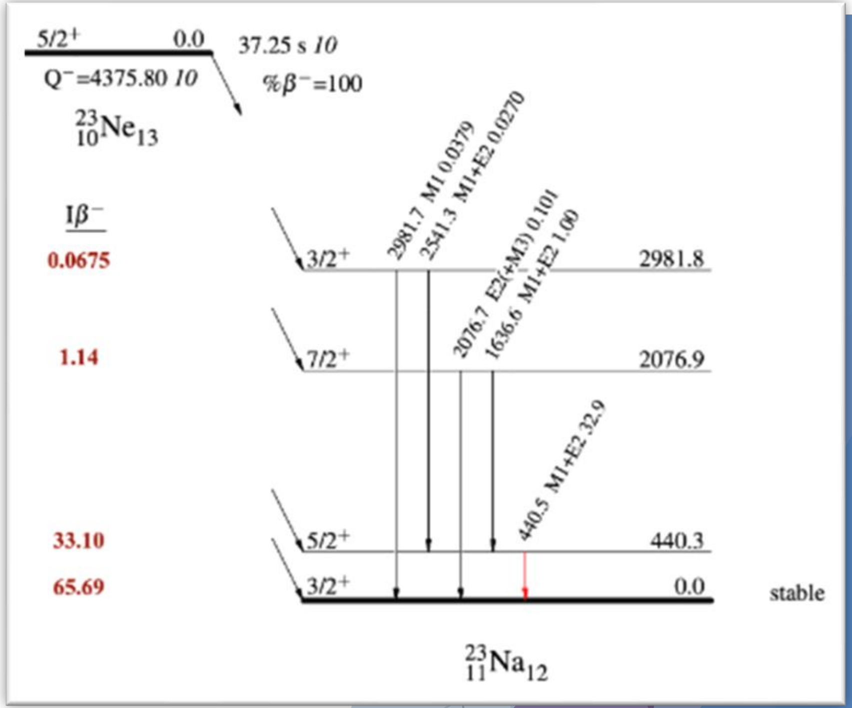
$\delta_1, \delta_a, \delta_b$

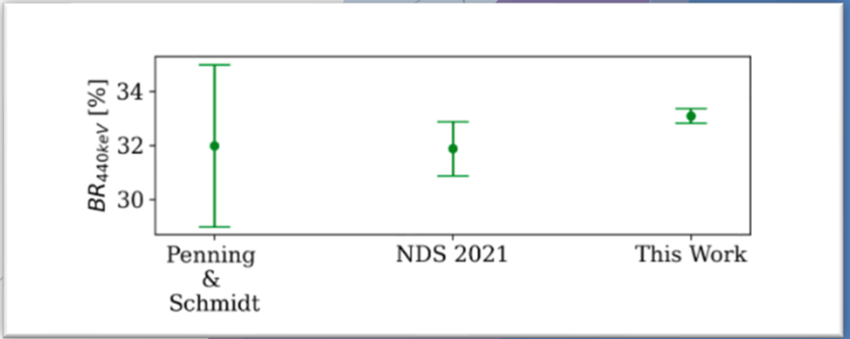
Observables' corrections

Numerical calculations of the nuclear structure

. . .

SARAF: measuring $^{23}\mathrm{Ne}$'s branching ratio with a $\sim\mathbf{5 \cdot 10^{-3}}$ **uncertainty**

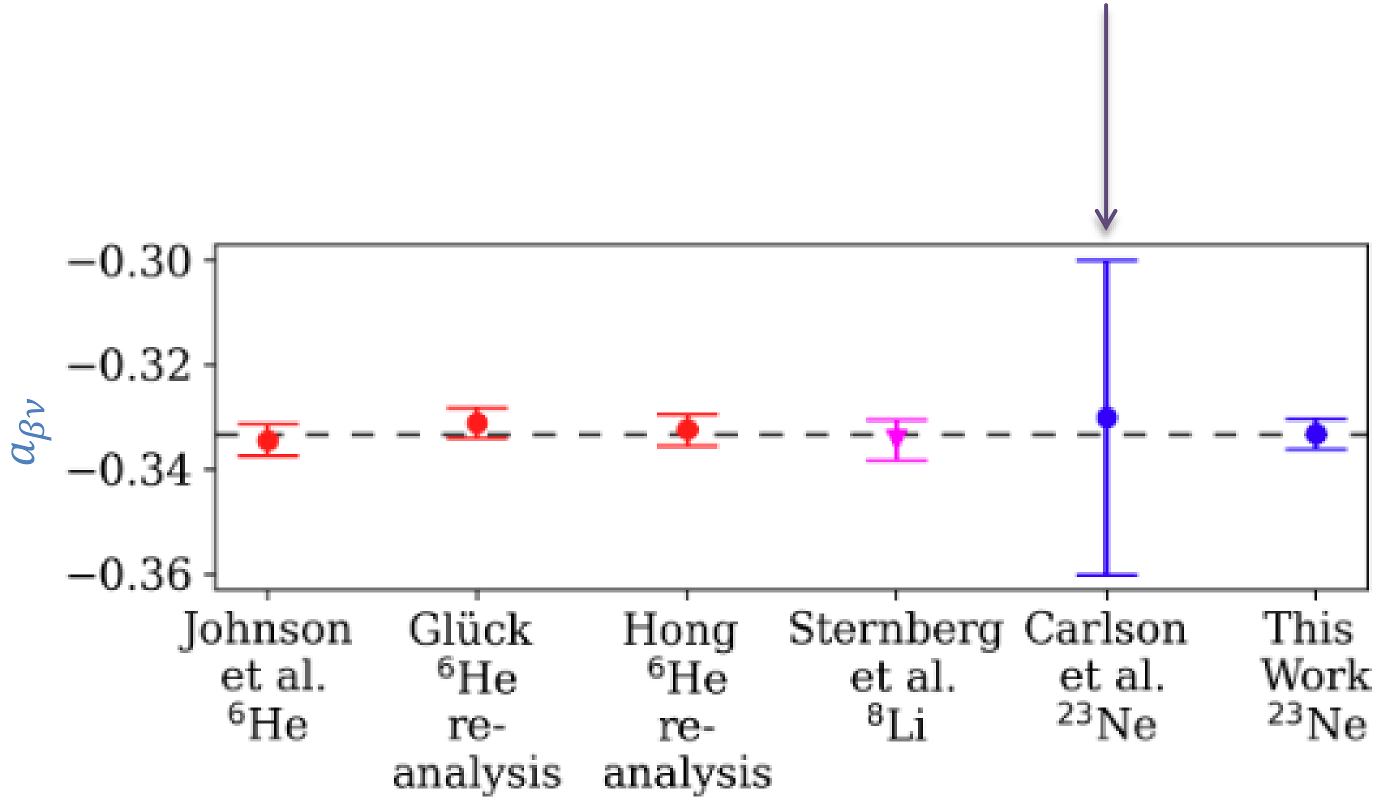




Mishnayot, AGM, *et al.*

$^{23}Ne \rightarrow {}^{23}Na$

Reanalyzing measurements of Carlson *et al.*, PhysRev132.2239 (1963)



Mishnayot, AGM, *et al.*

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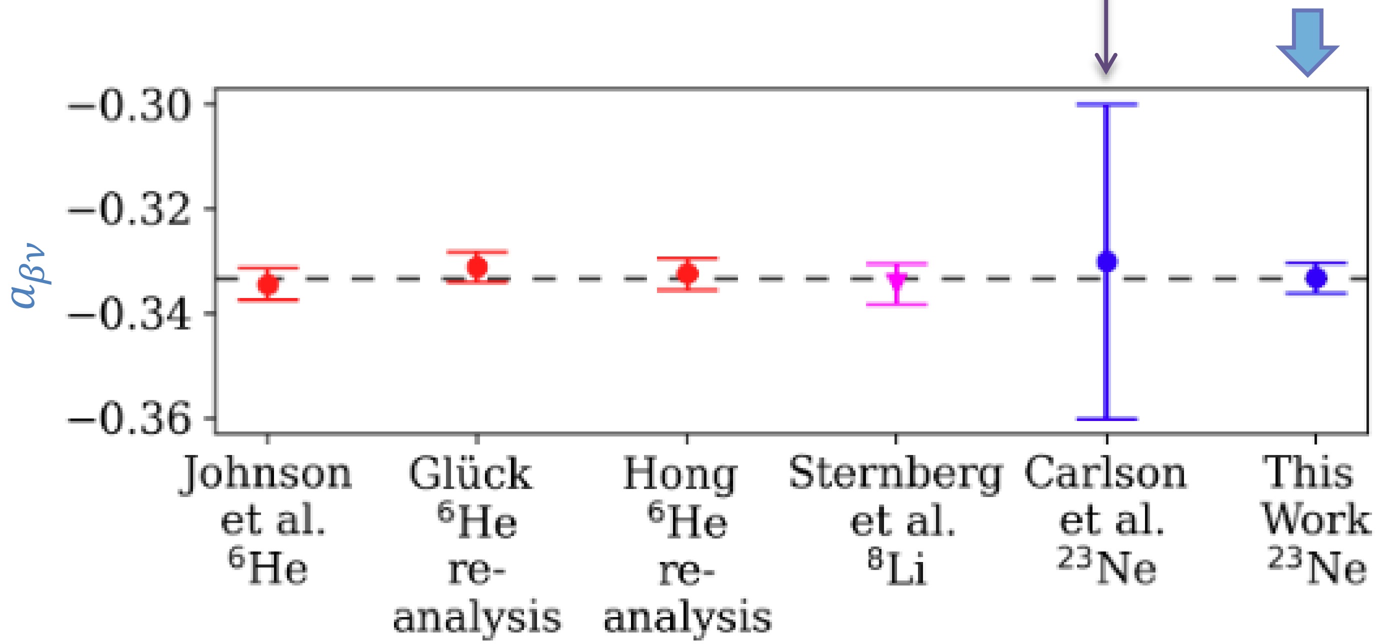
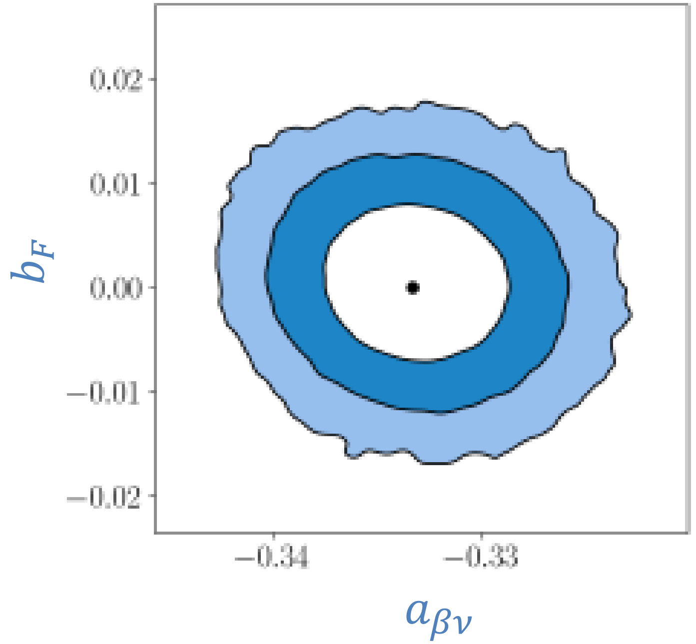


Mishnayot, AGM, *et al.*

$^{23}Ne \rightarrow {}^{23}Na$

Reanalyzing measurements of Carlson *et al.*, PhysRev132.2239 (1963)

Constraining $a_{\beta\nu}$ & b_F *simultaneously*



statistics experiment theory

$$a_{\beta\nu} = -0.3331 \pm 0.0028 \pm 0.0004 \pm 0.0002$$
$$b_F = 0.0007 \pm 0.0049 \pm 0.0003 \pm 0.0001$$

Mishnayot, AGM, *et al.*

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Reanalyzing measurements of Carlson *et al.*, PhysRev132.2239 (1963)

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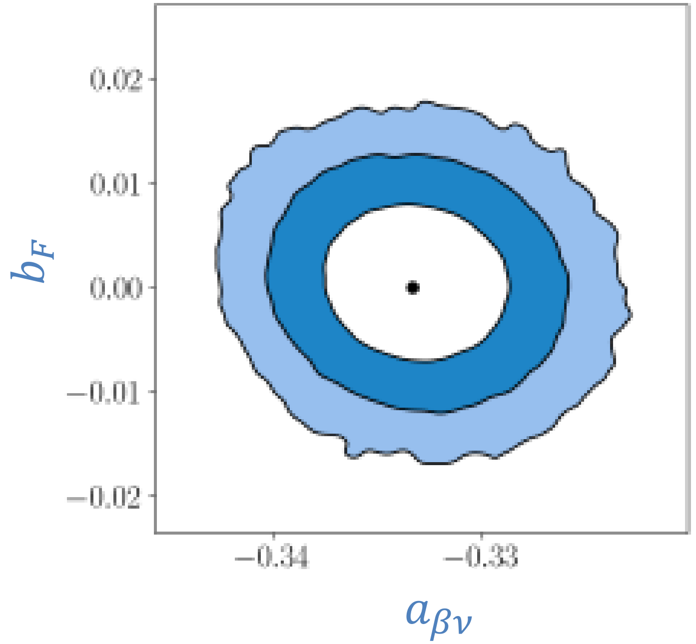

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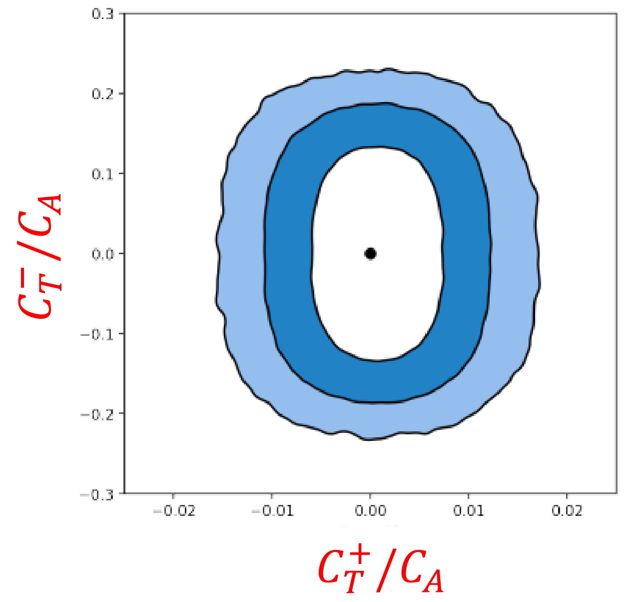
Mishnayot, AGM, *et al.*

$^{23}Ne \rightarrow {}^{23}Na$

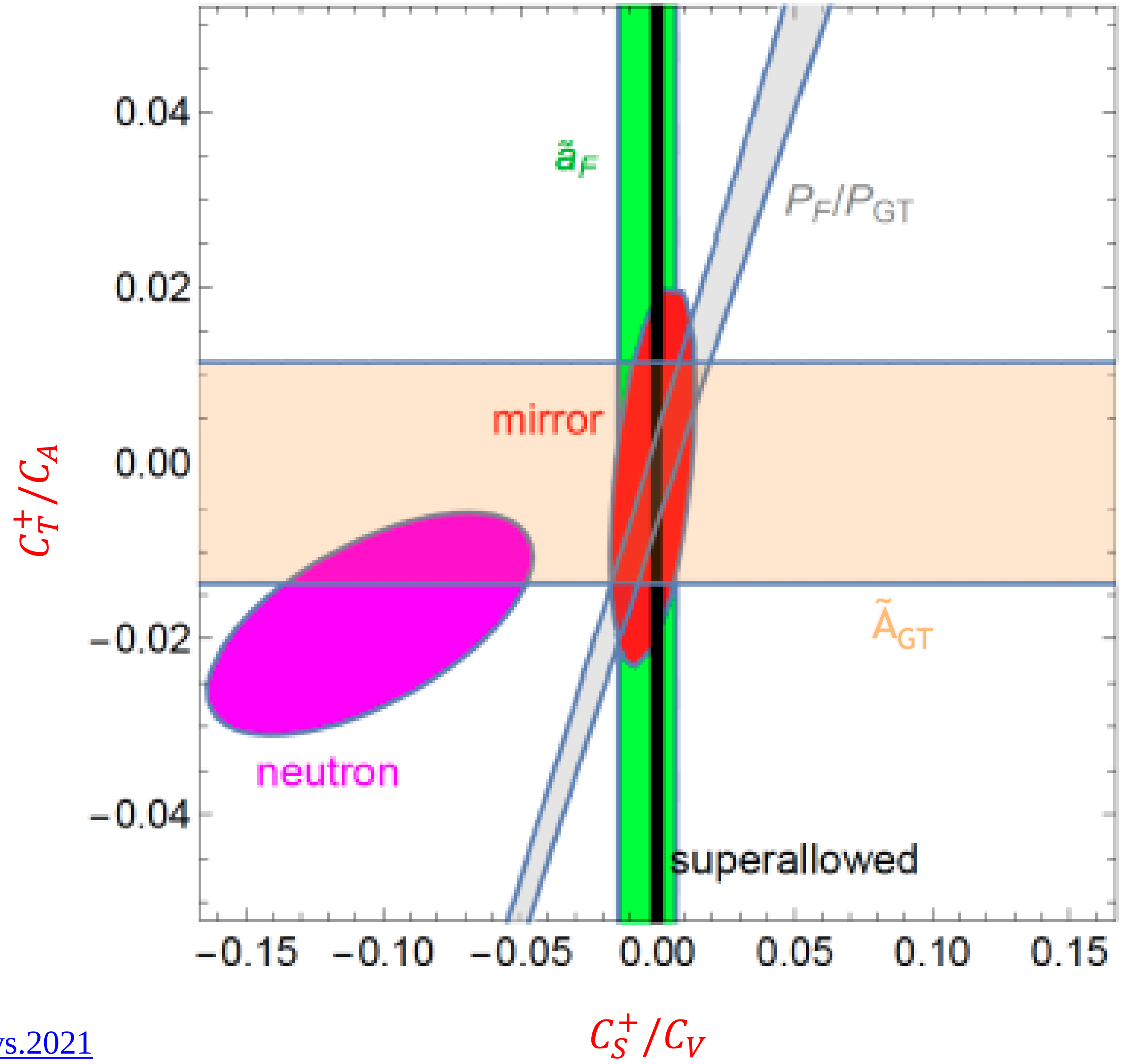
Reanalyzing measurements of Carlson *et al.*, PhysRev132.2239 (1963)

New constraints on the existence of exotic Tensor interactions

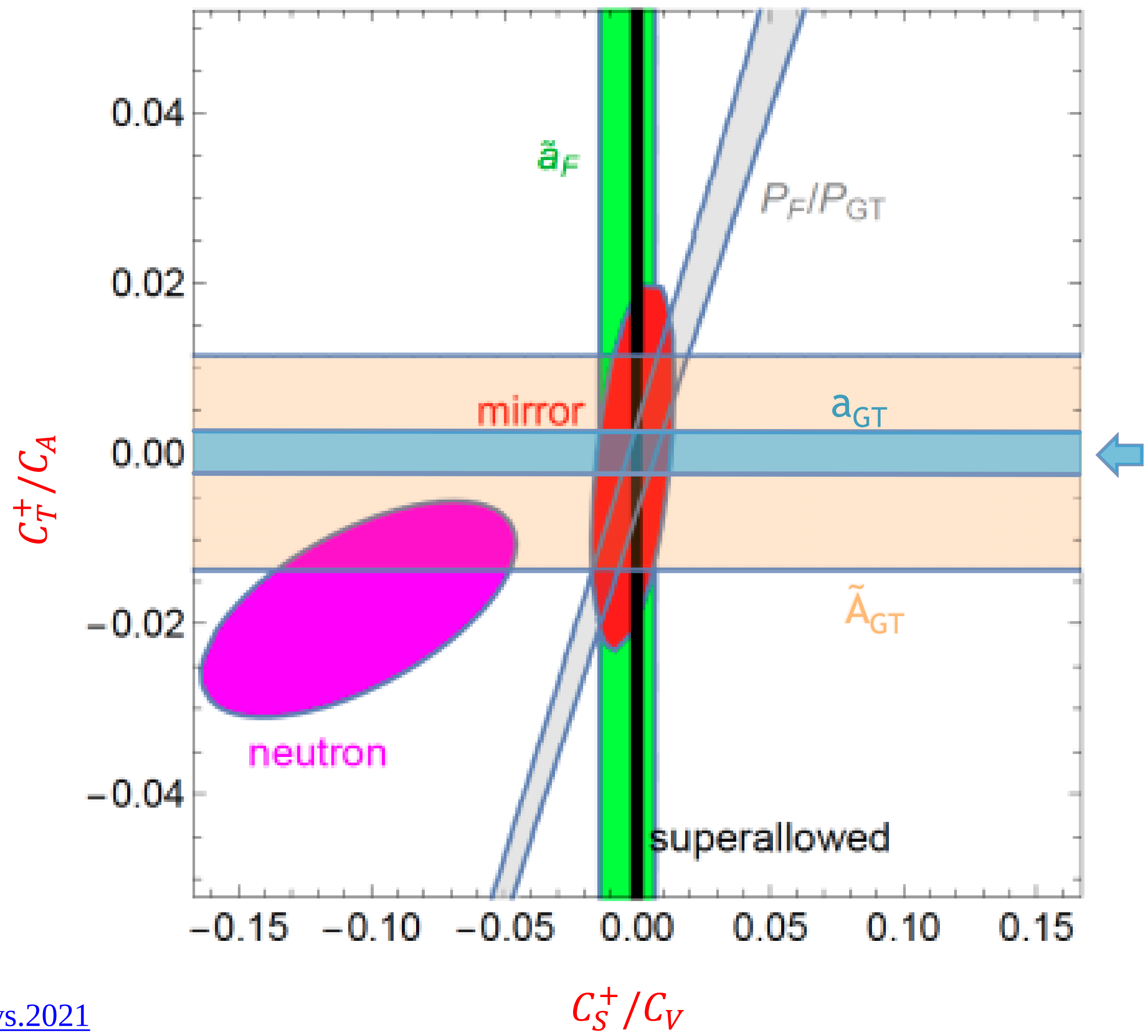


$$\frac{C_T^+}{C_A} = 0.0007 \pm 0.0049 \qquad \frac{C_T^-}{C_A} = 0.0001 \pm 0.0823$$

Mishnayot, AGM, *et al.*



Falkowski *et al.*, J.High Energ.Phys.2021



[Mishnayot, AGM, *et al.*](#)

[Falkowski *et al.*, J.High Energ.Phys.2021](#)

BSM missing theory: forbidden decays (tensor+)

SM Multipole Expansion

q - momentum transfer
$\vec{\beta} \equiv \frac{\vec{k}}{\epsilon}$ - electron's normalized momentum
$\hat{\nu} \equiv \frac{\vec{\nu}}{\nu}$ - neutrino's normalized momentum

Nuclear structure

β-decay rate: $d\omega \propto kE(E_0 - E)^2 F_0 C_{corr}\, \Theta(q, \vec{\beta}\cdot\hat{\nu})$

$$\Theta(q,\vec{\beta}\cdot\hat{\nu}) \propto \left|\langle\psi_f\|\hat{H}_W\|\psi_i\rangle\right|^2 \propto \sum_{J=0}^{\infty} f^V(\vec{\beta},\hat{\nu})\, \langle\psi_f\|\hat{O}_J^V\|\psi_i\rangle\langle\psi_f\|\hat{Q}_J^V\|\psi_i\rangle^*$$

$$\hat{H}_W \sim C_V \int d^3r\, \hat{j}^\mu(\vec{r})\, \hat{J}_\mu^V(\vec{r})$$

Vector coupling constant

Vector Lepton current

Vector nuclear current

Observables: lepton traces calculations (analytic)

Multipole operators:

$$\hat{C}_{JM}^V = \int d^3r\, j_J(qr) Y_{JM}(\hat{r}) \hat{J}_0^V(\vec{r})$$

$$\hat{L}_{JM}^V = \frac{i}{q}\int d^3r \{\vec{\nabla}[j_J(qr)Y_{JM}(\hat{r})]\}\cdot \vec{J}^V(\vec{r})$$

$$\hat{E}_{JM}^V = \frac{i}{q}\int d^3r\{\vec{\nabla}\times[j_J(qr)\, \vec{Y}_{JJ1}^M(\hat{r})]\}\cdot\vec{J}^V(\vec{r})$$

$$\hat{M}_{JM}^V = \int d^3r [j_J(qr)\, \vec{Y}_{JJ1}^M(\hat{r})]\cdot \vec{J}^V(\vec{r})$$

Vector nuclear current

and the same for the Axial (A) symmetry

q - momentum transfer
$\vec{\beta} \equiv \frac{\vec{k}}{\epsilon}$ - electron's normalized momentum
$\hat{\nu} \equiv \frac{\vec{\nu}}{\nu}$ - neutrino's normalized momentum

BSM Multipole Expansion

Nuclear structure

β-decay rate: $d\omega \propto kE(E_0 - E)^2 F_0 C_{\text{corr}}\, \Theta(q, \vec{\beta}\cdot\hat{\nu})$

$$\Theta(q,\vec{\beta}\cdot\hat{\nu}) \propto \left|\langle\psi_f\|\hat{H}_W\|\psi_i\rangle\right|^2 \propto \sum_{J=0}^{\infty} f^T(\vec{\beta},\hat{\nu})\,\langle\psi_f\|\hat{O}_J^T\|\psi_i\rangle\langle\psi_f\|\hat{Q}_J^T\|\psi_i\rangle^*$$

$$\hat{H}_W \sim C_T \int d^3r\, \hat{j}^{\mu\nu}(\vec{r})\,\hat{\mathcal{J}}^T_{\mu\nu}(\vec{r})$$

Tensor coupling constant

Tensor Lepton current

Tensor nuclear current

Observables: lepton traces calculations (analytic)

Multipole operators:

$$\hat{C}^V_{JM} = \int d^3r\, j_J(qr) Y_{JM}(\hat{r})\, \hat{\mathcal{J}}^V_0(\vec{r})$$

$$\hat{L}^V_{JM} = \frac{i}{q}\int d^3r \{\vec{\nabla}[j_J(qr) Y_{JM}(\hat{r})]\}\cdot \vec{\mathcal{J}}^V(\vec{r})$$

$$\hat{E}^V_{JM} = \frac{i}{q}\int d^3r \{\vec{\nabla}\times[j_J(qr)\,\vec{Y}^M_{JJ1}(\hat{r})]\}\cdot \vec{\mathcal{J}}^V(\vec{r})$$

$$\hat{M}^V_{JM} = \int d^3r [j_J(qr)\,\vec{Y}^M_{JJ1}(\hat{r})]\cdot \vec{\mathcal{J}}^V(\vec{r})$$

Vector nuclear current

We want to have the same for the Tensor coupling

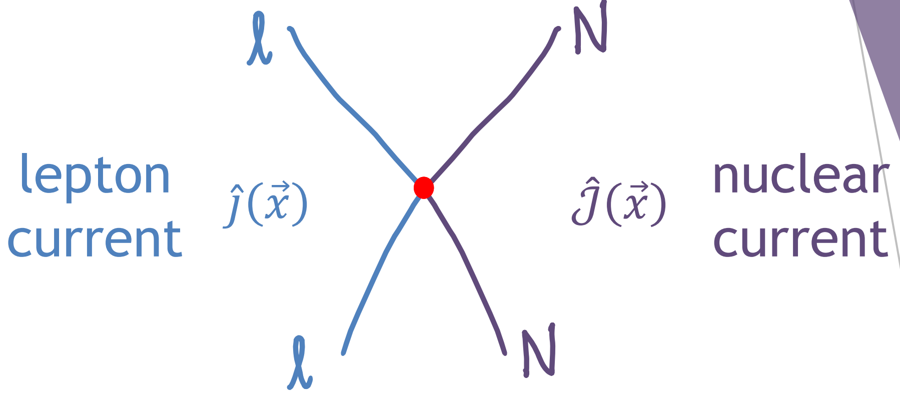
- The currents are tensors: $\hat{j}^{\mu\nu}(\vec{x})\hat{\mathcal{J}}_{\mu\nu}(\vec{x})$

The SM multipoles are defined for vector (axial) currents

- with vector spherical harmonics $\vec{Y}^M_{JJ1}(\hat{r})$

AGM & Gazit, PRD 2023

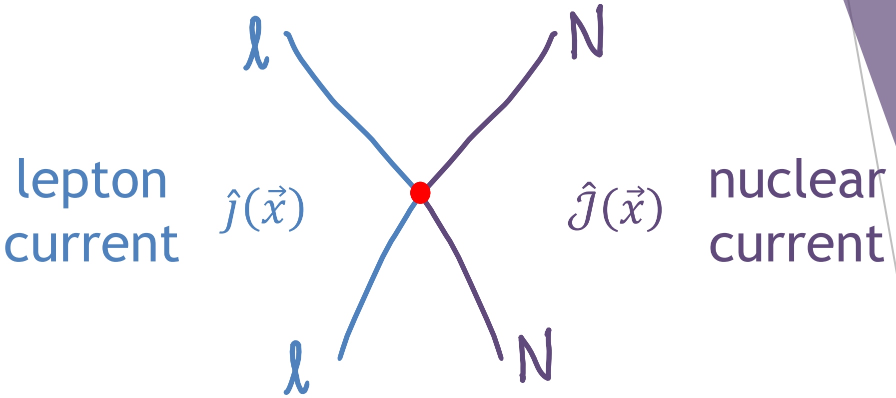
Tensor



$$\hat{\mathcal{H}}_W \sim C_\mathrm{T}\ \hat{j}^{\mu\nu}(\vec{x})\ \hat{J}_{\mu\nu}(\vec{x})$$

AGM & Gazit, PRD 2023

Tensor

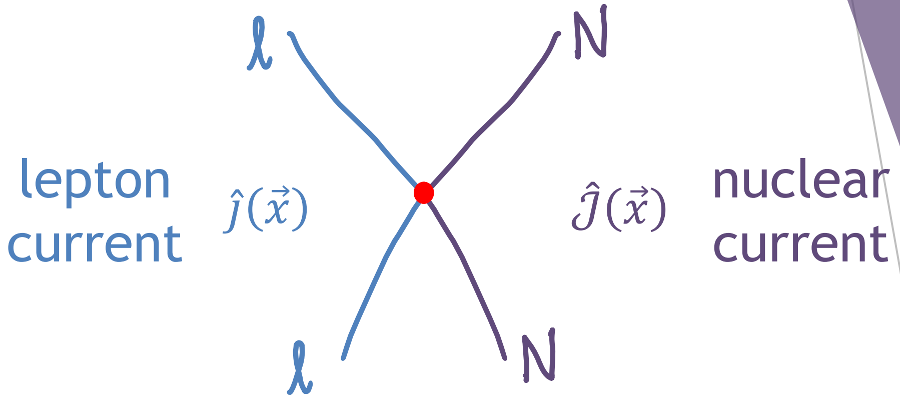


Tensor interactions $\hat{\mathcal{H}}_W \sim C_{\mathrm{T}}\ \hat{j}^{\mu\nu}(\vec{x})\, \hat{J}_{\mu\nu}(\vec{x})$

- ~~Symmetric:~~
  - ~~A space-time-metric and the~~ stress-energy tensor
- Antisymmetric
  - Fermionic probes

AGM & Gazit, PRD 2023

Tensor



$$\widehat{\mathcal{H}}_W \sim C_T \; \hat{j}^{\mu\nu}(\vec{x}) \, \hat{J}_{\mu\nu}(\vec{x})$$

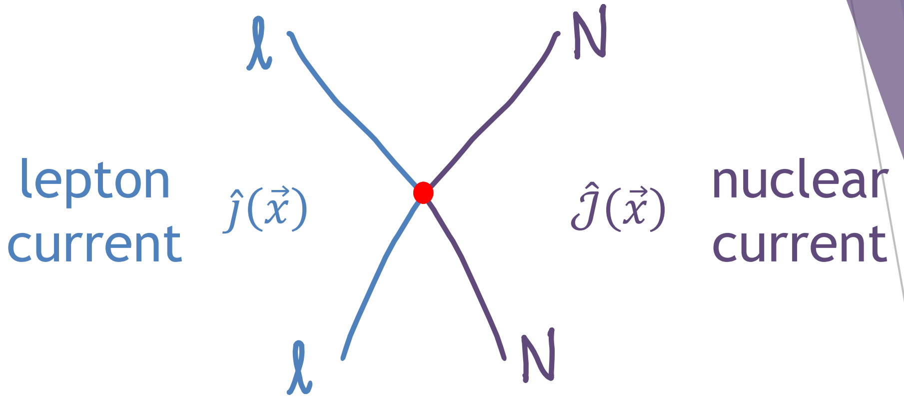
Tensor interactions

- ~~Symmetric:~~
  - ~~A space-time-metric and the stress-energy tensor~~
- Antisymmetric
  - Fermionic probes

$\Longrightarrow l_{00} = 0$

$$l_{\mu\nu} = \begin{pmatrix} \cancel{l_{00}} & \begin{pmatrix} \leftarrow & \vec{l}_{0\cdot} & \rightarrow \end{pmatrix} \\ \begin{pmatrix} \uparrow \\ \vec{l}_{\cdot 0} \\ \downarrow \end{pmatrix} & \begin{pmatrix} & & \\ & l_{ij} & \\ & & \end{pmatrix} \end{pmatrix}$$

Tensor



Tensor interactions

$$\hat{\mathcal{H}}_W \sim C_T \, \hat{j}^{\mu\nu}(\vec{x}) \, \hat{J}_{\mu\nu}(\vec{x})$$

- ~~Symmetric:~~
  - ~~A space-time-metric and the stress-energy tensor~~
- Antisymmetric
  - Fermionic probes

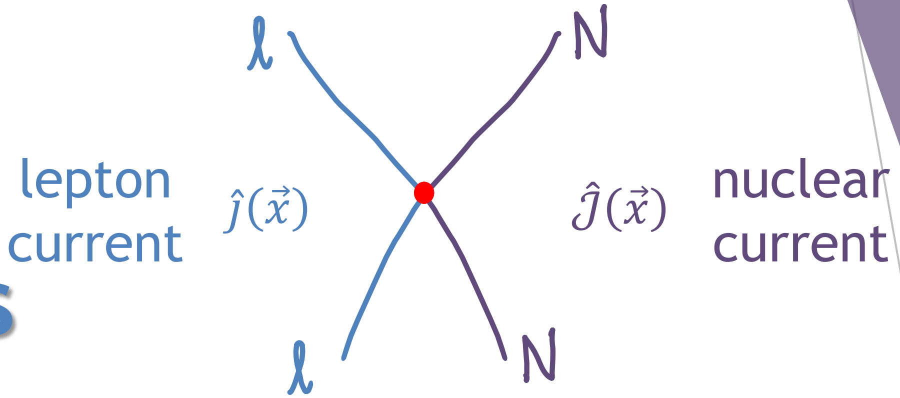
$$\Longrightarrow l_{00} = 0$$

$$\Longrightarrow l_{\cdot 0} = -l_{0\cdot}$$

$$l_{\mu\nu} = \begin{pmatrix} \cancel{l_{00}} & \begin{pmatrix} \leftarrow & \vec{l}_{0\cdot} & \rightarrow \end{pmatrix} \\ \begin{pmatrix} \uparrow \\ -\vec{l}_{0\cdot} \\ \downarrow \end{pmatrix} & \begin{pmatrix} \\ l_{ij} \\ \\ \end{pmatrix} \end{pmatrix}$$

AGM & Gazit, PRD 2023

Tensor
→ vector-like objects



$$\hat{\mathcal{H}}_W \sim C_T\ \hat{j}^{\mu\nu}(\vec{x})\ \hat{J}_{\mu\nu}(\vec{x})$$

Tensor interactions

- ~~Symmetric:~~
  - ~~A space-time-metric and the stress-energy tensor~~
- Antisymmetric
  - Fermionic probes

$$\Rightarrow l_{00} = 0$$
$$\Rightarrow l_{\cdot 0} = -l_{0\cdot}$$
$$\Rightarrow l_{ij} \longrightarrow [l_{ij}]^{(1)}$$

$$l_{\mu\nu} = \begin{pmatrix} \cancel{l_{00}} & \begin{pmatrix} \leftarrow & \vec{l}_{0\cdot} & \rightarrow \end{pmatrix} \\ \begin{pmatrix} \uparrow \\ -\vec{l}_{0\cdot} \\ \downarrow \end{pmatrix} & \begin{pmatrix} \vec{l}^{(1)} \end{pmatrix} \end{pmatrix}$$

AGM & Gazit, PRD 2023

Tensor → vector-like objects

- Tensor "vector-like" multipole operators with an **identified parity**
  - $\Delta\pi = (-)^{J-1}$: "Axial (vector)-like" tensor operators:

$$\hat{L}_J^T = -\frac{i}{\sqrt{2}}\frac{g_T}{g_A}\hat{L}_J^A + \mathcal{O}\left(\epsilon_{\mathrm{NR}}^2 \sim \frac{P_{\mathrm{fermi}}^2}{m_N^2} \sim 0.04\right)$$

BSM operator

Well known SM operator

  - $\Delta\pi = (-)^J$: "Vector" like tensor operators:

$$\hat{L}_J^{T'} \propto \epsilon_{qr} \sim \frac{q}{m_N} \sim 0.002 \text{ (for an end point of } \sim 2\ \mathrm{MeV})$$

- Predictions & Observables for forbidden decays for the first time

and the same **exact** relations for $\hat{E}_J^T, \hat{M}_J^T$

$\frac{g_S}{g_V}, \frac{g_T}{g_A} \sim 1$ nuclear charges

AGM & Gazit, PRD 2023

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BSM operator

Well known SM operator

  - $\Delta\pi = (-)^J$: "Vector" like tensor operators:

$\hat{L}_J^{T'} \propto \epsilon_{qr} \sim \frac{q}{m_N} \sim 0.002$ (for an end point of ~ 2 MeV)

and the same **exact** relations for $\hat{E}_J^T, \hat{M}_J^T$

- Predictions & Observables for forbidden decays for the first time

- No Coulomb multipole $\hat{C}_J^T$ (associated with the charge $l_{00} = 0$)

$$\hat{C}_J^S = -\frac{i}{\sqrt{2}}\frac{g_S}{g_V}\hat{C}_J^V + \mathcal{O}\left(\epsilon_{\text{NR}}^2 \sim \frac{P_{\text{fermi}}^2}{m_N^2} \sim 0.04\right)$$

$\frac{g_S}{g_V}, \frac{g_T}{g_A} \sim 1$ nuclear charges

AGM & Gazit, PRD 2023

Tensor → vector-like objects

- Tensor "vector-like" multipole operators with an **identified parity**

$$\hat{L}_J^T \approx -\frac{i}{\sqrt{2}}\frac{g_T}{g_A}\hat{L}_J^A$$

$$\hat{C}_J^S \approx -\frac{i}{\sqrt{2}}\frac{g_S}{g_V}\hat{C}_J^V$$

BSM operators

Well known SM operators

and the same **exact** relations for the other tensor operators $(\hat{E}_J^T, \hat{M}_J^T)$

- Predictions & Observables for forbidden decays for the first time

$\frac{g_S}{g_V}, \frac{g_T}{g_A} \sim 1$ nuclear charges

AGM & Gazit, PRD 2023

Tensor → vector-like objects

- Tensor "vector-like" multipole operators with an **identified parity**

and the same **exact** relations for the other tensor operators ($\hat{E}_J^T, \hat{M}_J^T$)

$$\hat{L}_J^T \approx -\frac{i}{\sqrt{2}}\frac{g_T}{g_A}\hat{L}_J^A$$

BSM operators

Well known SM operators

$$\hat{C}_J^S \approx -\frac{i}{\sqrt{2}}\frac{g_S}{g_V}\hat{C}_J^V$$

$$\hat{C}_J^P \approx \frac{q}{2m_N}\frac{g_P}{g_A}\hat{L}_J^A \quad \text{(Accidental relation)}$$

- Predictions & Observables for forbidden decays for the first time

Charge	Value
g_A	1.278(33)
g_T	0.987(55)
g_S	1.02(11)
g_P	349(9)

M. Gonzalez-Alonso *et al.*, PPNP 2019

AGM & Gazit, PRD 2023

$\frac{g_S}{g_V}, \frac{g_T}{g_A} \sim 1$ nuclear charges

Nuclear β-decay

For a general β-decay transition $J_i^{\pi_i} \to J_f^{\pi_f}$:

$$\Theta(q, \vec{\beta}\cdot\hat{\nu}) = \sum_{J=|J_i-J_f|}^{J_i+J_f} \Theta^{J^{\Delta\pi}}(q, \vec{\beta}\cdot\hat{\nu})$$

angular momentum — parity

$$J_i^{\pi_i} \to J_f^{\pi_f}$$
$$|J_i - J_f| \leq J \leq J_i + J_f$$
$$\Delta\pi = \pi_i \cdot \pi_f$$

	$\Delta\pi$	transition	multipoles	BSM
$J=0$	$+$	Fermi	$\hat{C}_0^V \sim 1$	$\hat{C}_0^S \approx \frac{g_S}{g_V}\hat{C}_0^V$
	$-$	1^{st} forbidden	$\hat{L}_0^A \sim \epsilon_{qr} \sim q$ $\hat{C}_0^A \sim \epsilon_{NR} \sim q$	$\hat{L}_0^T \approx -\frac{i}{\sqrt{2}}\frac{g_T}{g_A}\hat{L}_0^A$ $\hat{C}_0^P \approx \frac{q}{2m_N}\frac{g_P}{g_A}\hat{L}_0^A$
$J>0$	$(-)^J$	J^{th} forbidden	$\hat{C}_J^V \sim \epsilon_{qr}^J \sim q^J$ $\hat{M}_J^A \sim \epsilon_{qr}^J \sim q^J$	$\hat{C}_J^S \approx \frac{g_S}{g_V}\hat{C}_J^V$ $\hat{M}_J^T \approx -\frac{i}{\sqrt{2}}\frac{g_T}{g_A}\hat{M}_J^A$
	$(-)^{J-1}$	Gamow Teller ($J=1$) unique $(J-1)^{\text{th}}$ forbidden	LO: $\hat{L}_J^A \sim \epsilon_{qr}^{J-1} \sim q^{J-1}$ NLO: $\hat{C}_J^A, \hat{M}_J^V \sim \epsilon_{\text{recoil}}\epsilon_{qr}^{J-1}, \epsilon_{NR}\epsilon_{qr}^J \sim q^{J+1}$	$\hat{L}_J^T \approx -\frac{i}{\sqrt{2}}\frac{g_T}{g_A}\hat{L}_J^A$ $\hat{C}_J^P \approx \frac{q}{2m_N}\frac{g_P}{g_A}\hat{L}_J^A$

AGM & Gazit, PRD 2023

Multipole operator's matrix elements

$\hat{H}$

Nuclear Hamiltonian

↓

$\langle\psi_f\| \quad \|\psi_i\rangle$

Nuclear wave functions

$\hat{\mathcal{J}}_\mu(\vec{x})$

SM Nuclear currents

↓

$\hat{O}_J$

SM Multipole operators

$\langle\psi_f\|\hat{O}_J\|\psi_i\rangle$

SM Nuclear matrix elements

↓

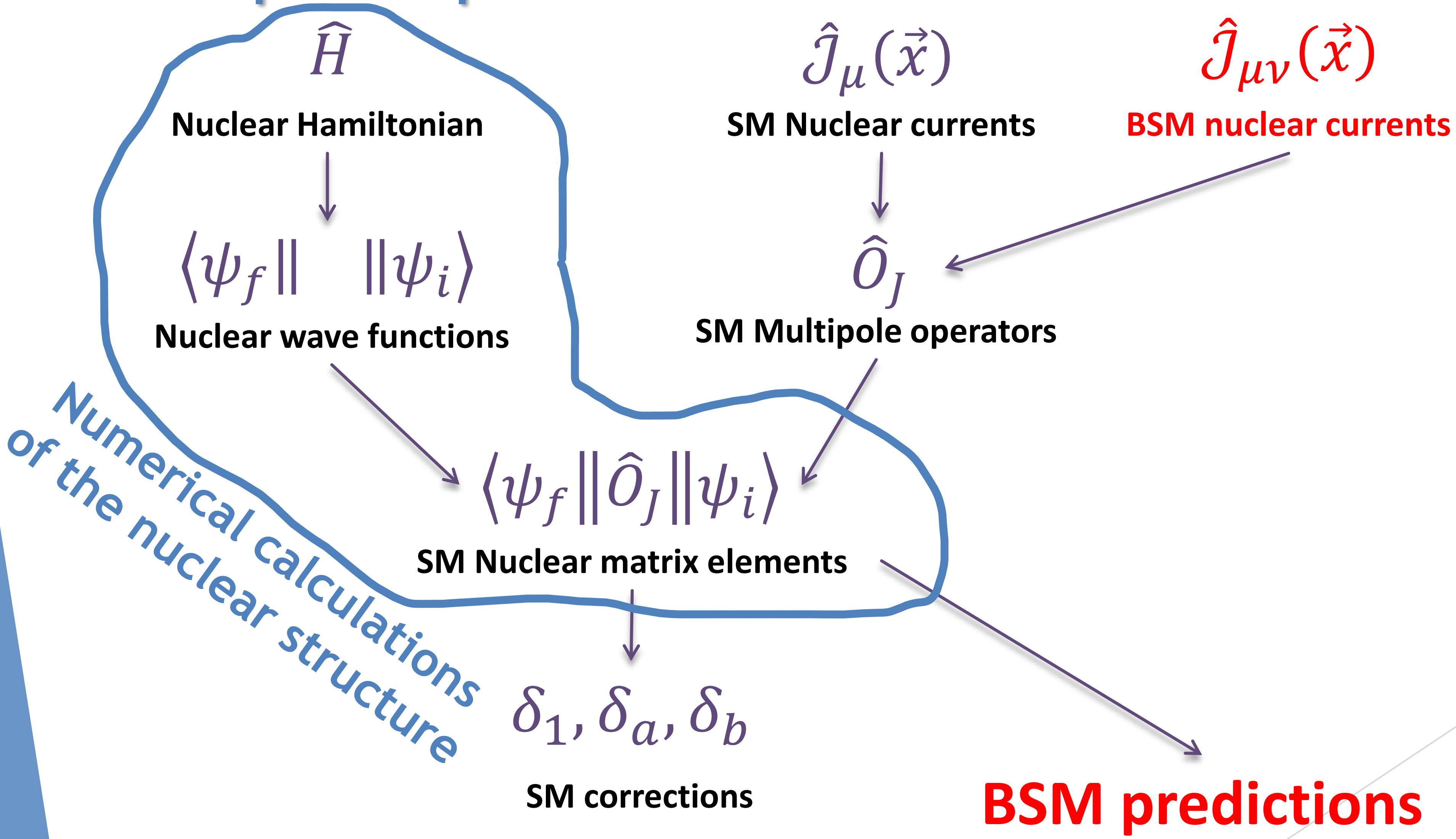
$\delta_1, \delta_a, \delta_b$

SM corrections

Numerical calculations of the nuclear structure

AGM & Gazit, PRD 2023

Multipole operator's matrix elements



AGM & Gazit, PRD 2023

Experimental status over the world

Energy spectrum - b_F

TABLE III. List of nuclear β-decay spectral measurements in search for non-SM physics [a]

Measurement	Transition Type	Nucleus	Institution/Collaboration	Goal
β spectrum	GT	^{114}In	MiniBETA-Krakow-Leuven	0.1 %
β spectrum	GT	^{6}He	LPC-Caen	0.1 %
β spectrum	GT	^{6}He, ^{20}F	NSCL-MSU	0.1 %
β spectrum	GT, F, Mixed	^{6}He, ^{14}O, ^{19}Ne	He6-CRES	0.1 %

Angular correlation - $a_{\beta\nu}$

TABLE I. List of nuclear β-decay correlation experiments in search for non-SM physics [a]

Measurement	Transition Type	Nucleus	Institution/Collaboration	Goal
$\beta - \nu$	F	^{32}Ar	Isolde-CERN	0.1 %
$\beta - \nu$	F	^{38}K	TRINAT-TRIUMF	0.1 %
$\beta - \nu$	GT, Mixed	^{6}He, ^{23}Ne	SARAF	0.1 %
$\beta - \nu$	GT	^{8}B, ^{8}Li	ANL	0.1 %
$\beta - \nu$	F	^{20}Mg, ^{24}Si, ^{28}S, ^{32}Ar, ...	TAMUTRAP-Texas A&M	0.1 %
$\beta - \nu$	Mixed	^{11}C, ^{13}N, ^{15}O, ^{17}F	Notre Dame	0.5 %
β & recoil asymmetry	Mixed	^{37}K	TRINAT-TRIUMF	0.1 %

Cirigliano *et al.*, 2019

Ab initio calculations of $^{6}\mathrm{He} \xrightarrow{\beta^-} {}^{6}\mathrm{Li}$

$\hat{H}$

Nuclear Hamiltonian

χEFT @ $N^2LO_{opt\,(sat)}$

↓

$\langle \psi_f \| \quad \| \psi_i \rangle$

Nuclear wave functions

$\hat{J}(\vec{x})$

Nuclear currents

LO (1-body currents)

↓

$\hat{O}_J$

Multipole operators

$\langle \psi_f \| \hat{O}_J \| \psi_i \rangle$

Nuclear matrix elements

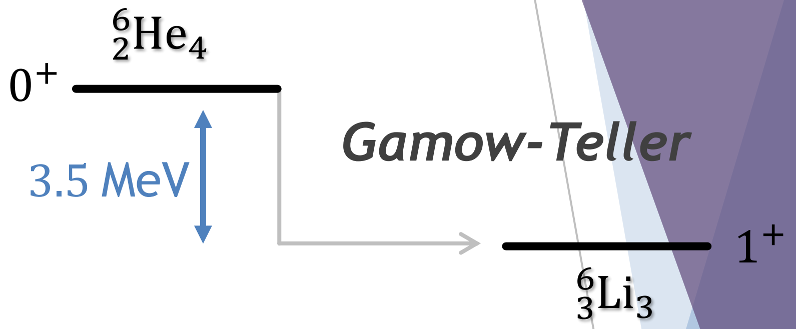
Ab initio No-Core Shell Model (NCSM)

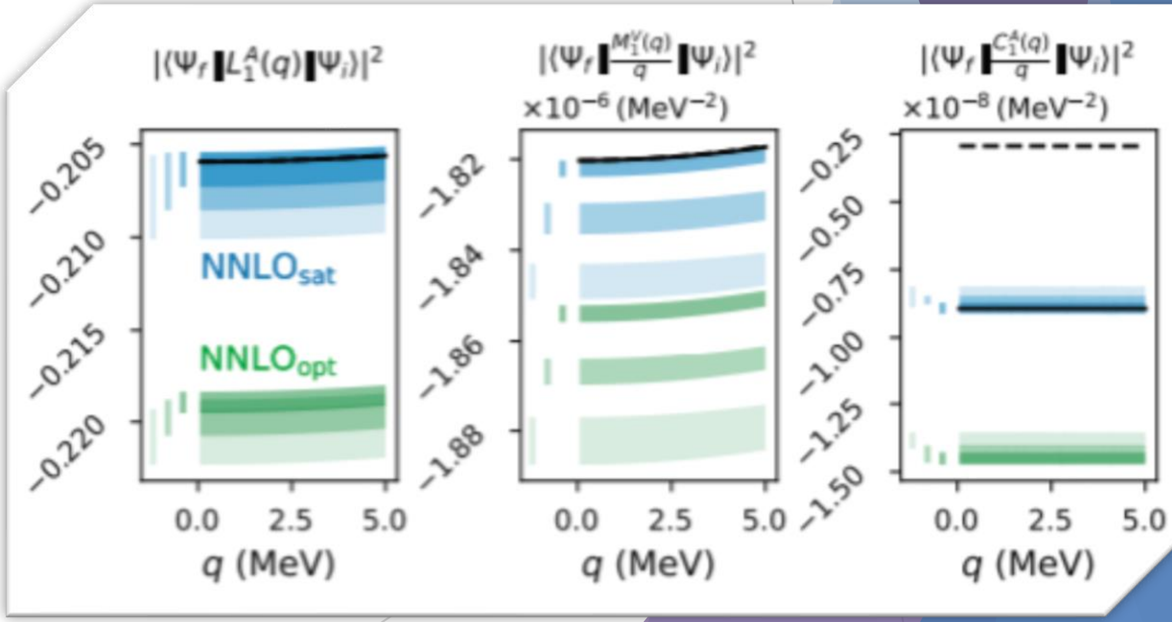
↓

$\delta_1, \delta_a, \delta_b$

Observables' corrections

Numerical calculations of the nuclear structure

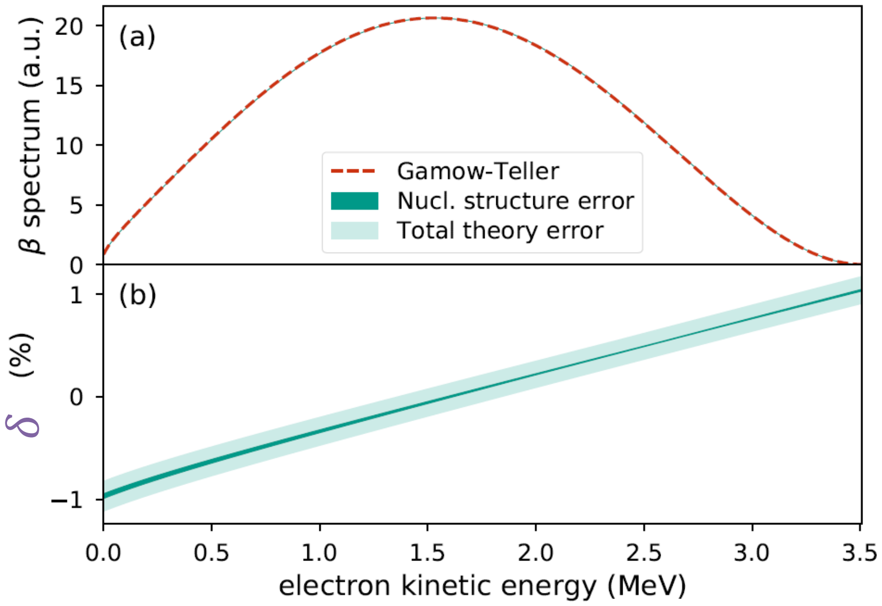




AGM, Forssén, Gazda, Gazit, Gysbers & Navrátil, PLB 2022

➢ Petr Navrátil's talk

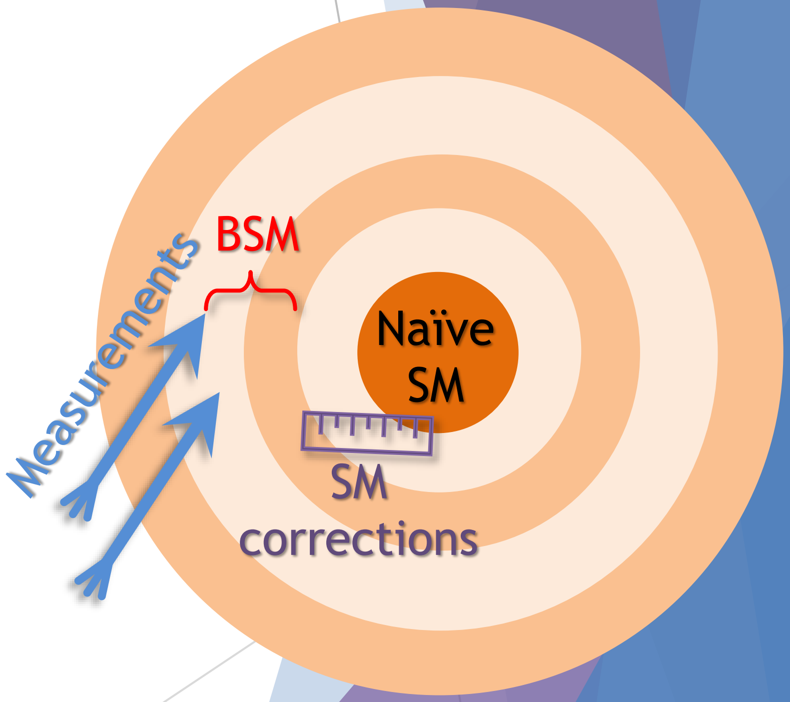
$^{6}\mathrm{He} \rightarrow {}^{6}\mathrm{Li}$ β-energy spectrum



- Experiments are aiming a 10^{-3} accuracy
- The spectrum is used to find Fierz term:

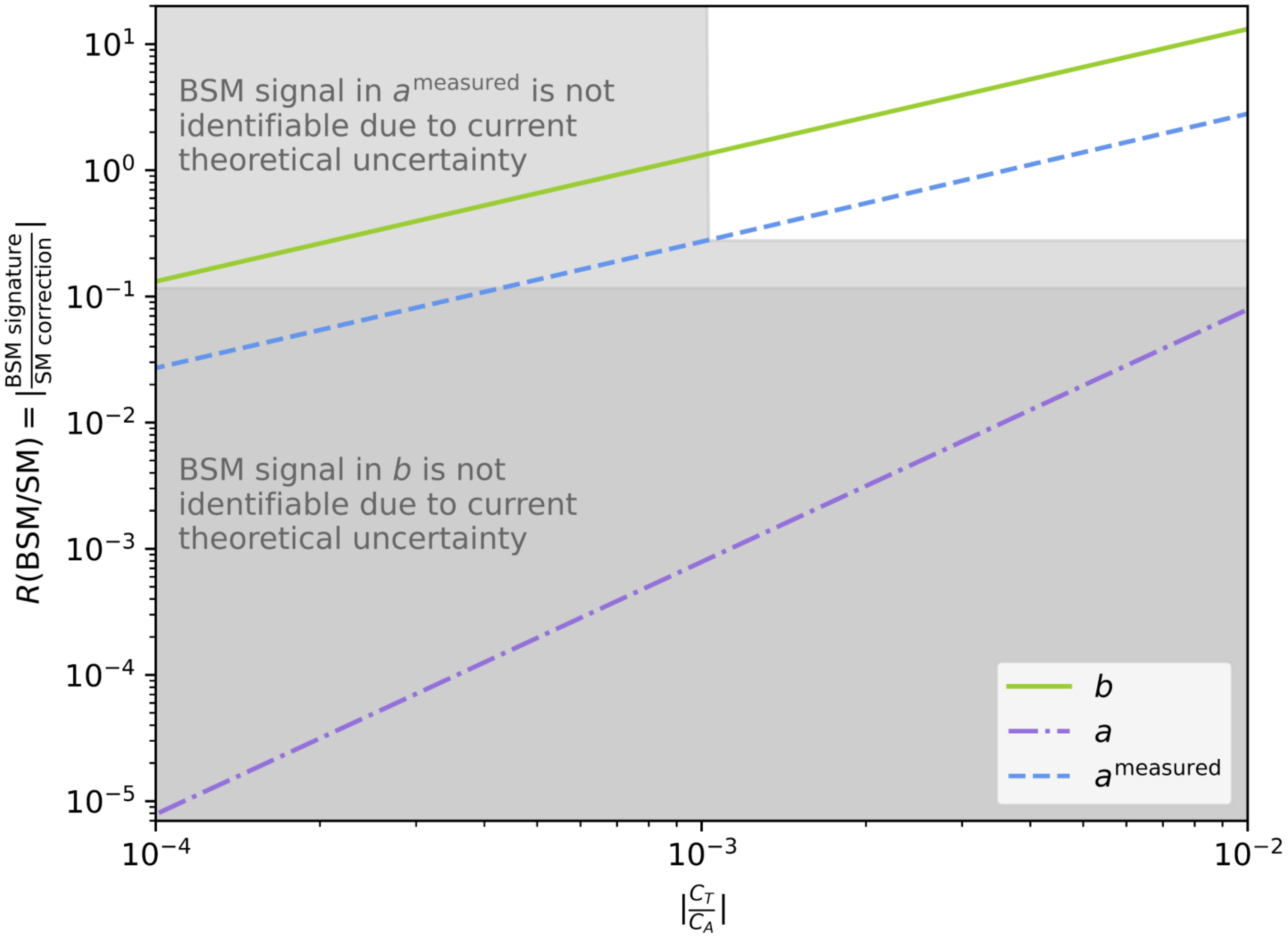
SM, SM correction, BSM

- $b_{\mathrm{F}} = 0 + \delta_b + \frac{C_T^+}{C_A}$
- Looking for $\frac{C_T^+}{C_A} \sim 10^{-3}$
- $\delta_b = -1.52(18) \cdot 10^{-3}$
- Uncertainty $< 2 \cdot 10^{-4}$



AGM, Forssén, Gazda, Gazit, Gysbers & Navrátil, PLB 2022

➢ Doron Gazit's talk



$^{6}He \rightarrow {}^{6}Li$

AGM & Gazit, PRD 2023

BSM predictions: unique 1st-forbidden decay

$$d\omega \propto 1 + a_{\beta\nu}\left[1 - \left(\hat{\beta}\cdot\hat{\nu}\right)^2\right] + b_F\frac{m_e}{\epsilon}$$

- Predictions & Observables for forbidden decays for the first time

AGM *et al*, PLB 2017

BSM predictions: unique 1st-forbidden decay

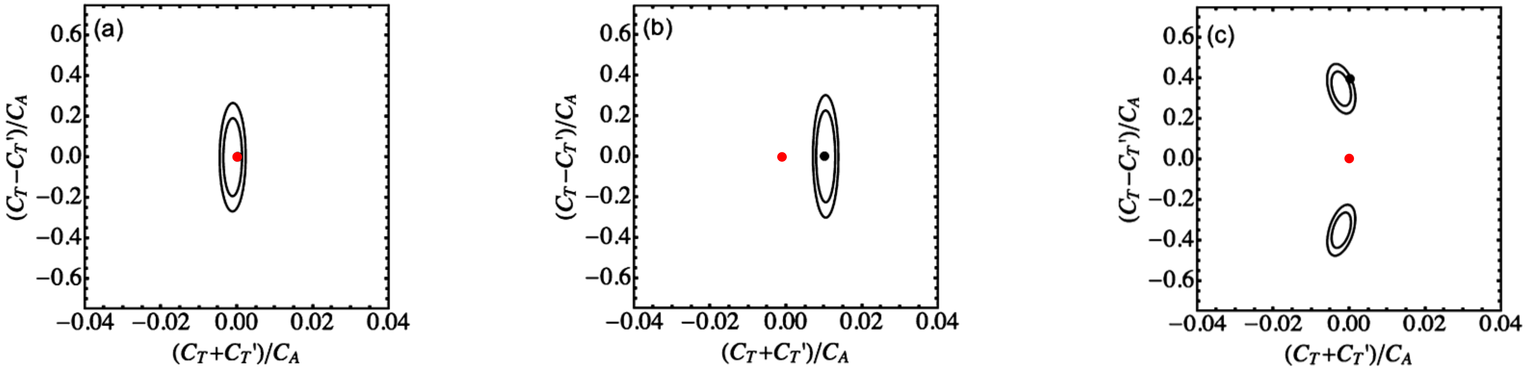
$$d\omega \propto 1 + a_{\beta\nu}\left[1 - (\hat{\beta}\cdot\hat{\nu})^2\right] + b_F \frac{m_e}{\epsilon}$$

The β-energy spectrum is sensitive to both $a_{\beta\nu}$ & b_F

- Allows simultaneous extraction of C_T and C_T'
- Increases the accuracy level

- Predictions & Observables for forbidden decays for the first time

No BSM coupling | Left-handed coupling | Right-handed coupling

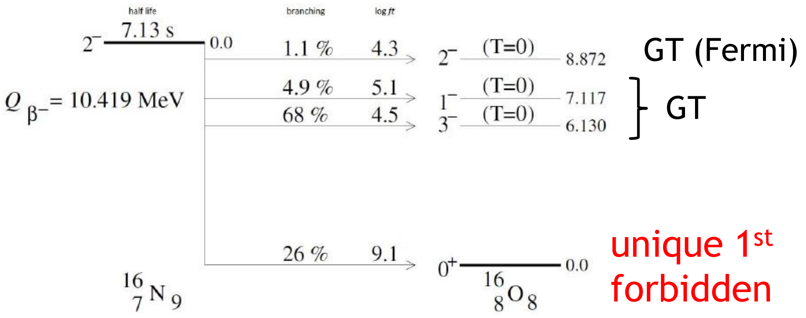


Formalism is nice, but applications are nicer…

AGM *et al*, PLB 2017

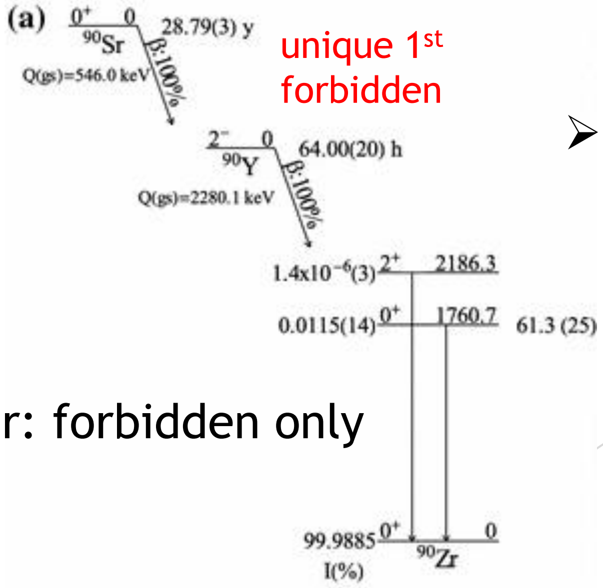
Unique 1st-forbidden experiments

- Petr Navrátil's talk



^{16}N: Large energy separation between the forbidden and allowed branches

Ohayon, Chocron, Hirsh, AGM, *et al.*, Hyp.Int.2018



^{90}Sr: forbidden only

- Doron Gazit's talk

Fig.: Morozov et al. J.Rad.Nuc.Chem.2010

Unique 1st-forbidden experiments

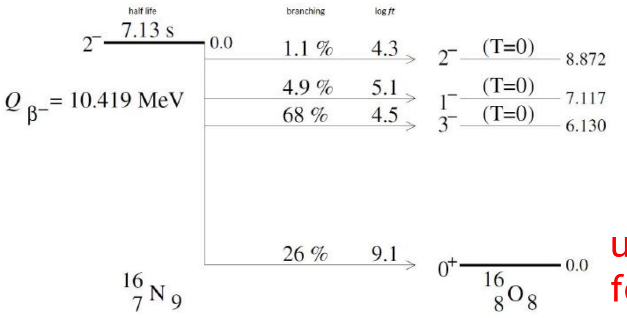
PHYSICAL REVIEW C **105**, 054312 (2022)

Determination of β-decay feeding patterns of ^{88}Rb and ^{88}Kr using the Modular Total Absorption Spectrometer at ORNL HRIBF

P. Shuai,[1,2,3,4] B. C. Rasco,[1,2,3,*] K. P. Rykaczewski,[2] A. Fijałkowska,[5,3] M. Karny,[5,2,1] M. Wolińska-Cichocka,[6,2,1] R. K. Grzywacz,[3,2,1] C. J. Gross,[2] D. W. Stracener,[2] E. F. Zganjar,[7] J. C. Batchelder,[8,1] J. C. Blackmon,[7] N. T. Brewer,[1,2,3] S. Go,[3] M. Cooper,[3] K. C. Goetz,[9,3] J. W. Johnson,[2] C. U. Jost,[2] T. T. King,[2] J. T. Matta,[2] J. H. Hamilton,[10] A. Laminack,[2] K. Miernik,[5] M. Madurga,[3] D. Miller,[3,11] C. D. Nesaraja,[2] S. Padgett,[3] S. V. Paulauskas,[3] M. M. Rajabali,[12] T. Ruland,[7] M. Stepaniuk,[5] E. H. Wang,[10] and J. A. Winger[13]

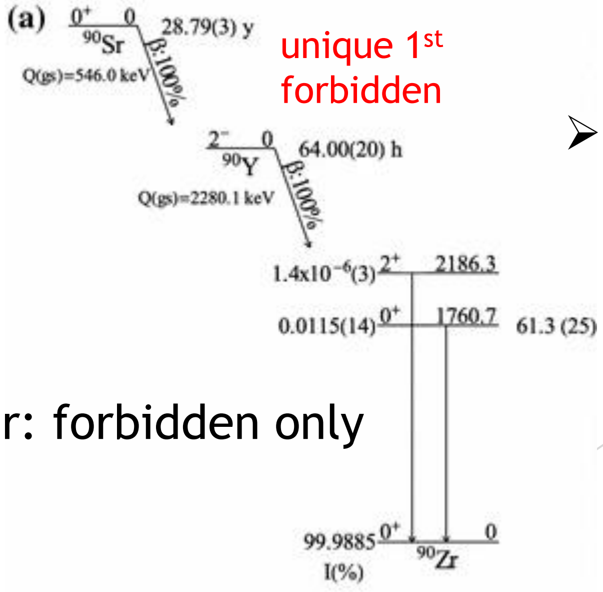
^{88}Rb decay spectra suggests that MTAS can distinguish an allowed β spectral shape from a first forbidden unique β spectral shape.

- Charlie Rasco's talk
- Petr Navrátil's talk



^{16}N: Large energy separation between the forbidden and allowed branches

Ohayon, Chocron, Hirsh, AGM, *et al.*, Hyp.Int.2018



- Doron Gazit's talk

^{90}Sr: forbidden only

Fig.: Morozov et al. J.Rad.Nuc.Chem.2010

Summary

- β-decay rate: $d\omega \propto kE(E_0 - E)^2 F_0 C_{\mathrm{corr}}\, \Theta(q, \vec{\beta}\cdot\hat{\nu})$

$$\Theta(q, \vec{\beta}\cdot\hat{\nu}) = \sum_{J=|J_i - J_f|}^{J_i + J_f} \Theta^{J^{\Delta\pi}}(q, \vec{\beta}\cdot\hat{\nu})$$

$$J_i^{\pi_i} \rightarrow J_f^{\pi_f}$$
$$|J_i - J_f| \le J \le J_i + J_f$$
$$\Delta\pi = \pi_i \cdot \pi_f$$

SM: controlled accuracy

- Identifying small parameters
- Corrections to observables
- Theory with controlled level of accuracy
  - ^{6}He: corrections with 10^{-4} uncertainty
  - ^{23}Ne: new bounds on BSM Tensor interactions

BSM: new opportunities

- Tensor forbidden's observables for the first time
- Uses the already-known SM matrix elements
  - No need for new matrix elements calculations
- Forbidden decays – BSM sensitivity
  - Experiments @SARAF, @ORNL

Gives significant constraints even for the naivest nuclear calculations

Can be done for any nucleus & decay (allowed/forbidden)

Paving the way for new, even higher precision experiments and discoveries

Required: BSM predictions vs. SM corrections

Thanks!

Hebrew University
Doron Gazit
Guy Ron
Hitesh Rahangdale
Vishal Srivastava

TRIUMF
Petr Navrátil
Peter Gysbers
Lotta Jokiniemi

Chalmers University
Christian Forssén

ÚJF rez
Daniel Gazda

University of Barcelona
Javier Menéndez

NCSU
Leendert Hayen

LLNL
Nicholas Scielzo
Yonatan Mishnayot
Jason Harke
Aaron Gallant
Richard Hughes

SARAF (SOREQ)
Sergey Vaintraub
Tsviki Hirsh
Leonid Waisman
Arik Kreisel
Boaz Kaizer
Hodaya Dafna
Maayan Buzaglo

ETH Zurich
Ben Ohayon

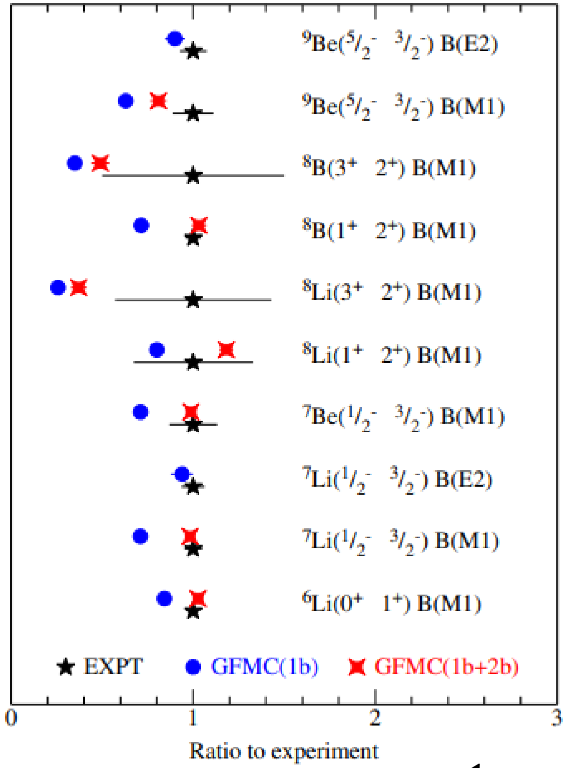
Weizamnn Institute
Michael Hass

Ministry of Science and Technology, Israel
Israeli Science Foundation (ISF)
European Research Council (ERC)

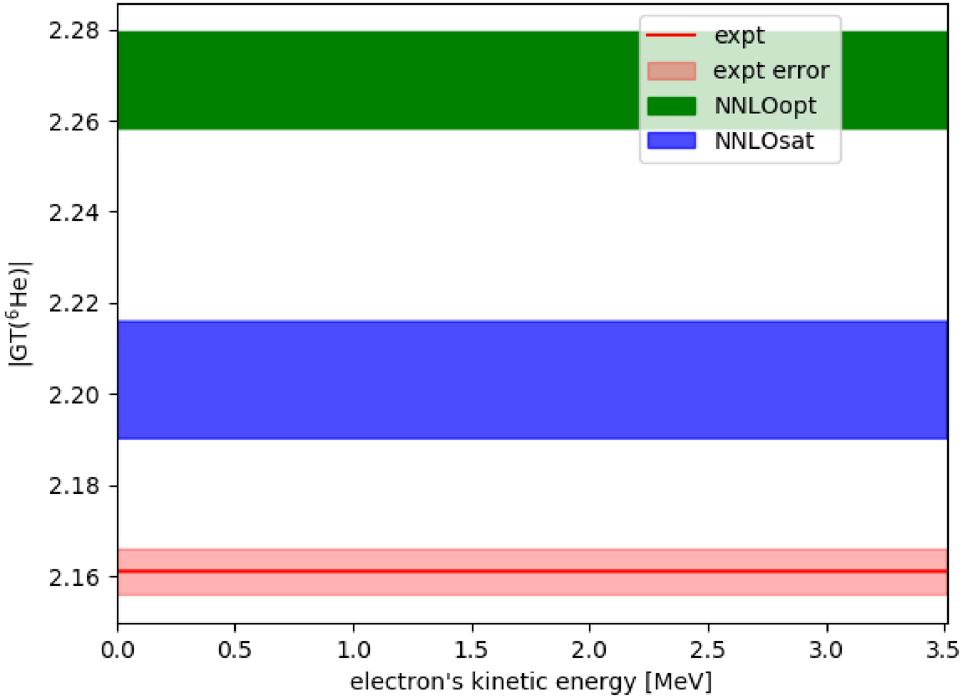
Some Details

Estimating the multipoles' EFT error (2b currents)

Electro-magnetic transitions



6He beta decay GT half life operator



$$^{6}\text{Li}(0^{+} \rightarrow 1^{+})B(M1) = \frac{1}{3}\left|\langle\|\hat{M}_1^V\|\rangle\right|^2$$

2b: $\langle\|\hat{M}_1^V\|\rangle \sim 10\% \sim \mathcal{O}(\epsilon_{EFT})$

$$\left|GT({}^{6}\text{He})\right| = \frac{\sqrt{12\pi}}{g_A}\left|\langle\|\hat{L}_1^A\|\rangle\right|^2$$

2b: $\langle\|\hat{L}_1^A\|\rangle, \langle\|\hat{C}_1^A\|\rangle \sim 1.6\% \sim \mathcal{O}(\epsilon_{EFT}^2)$

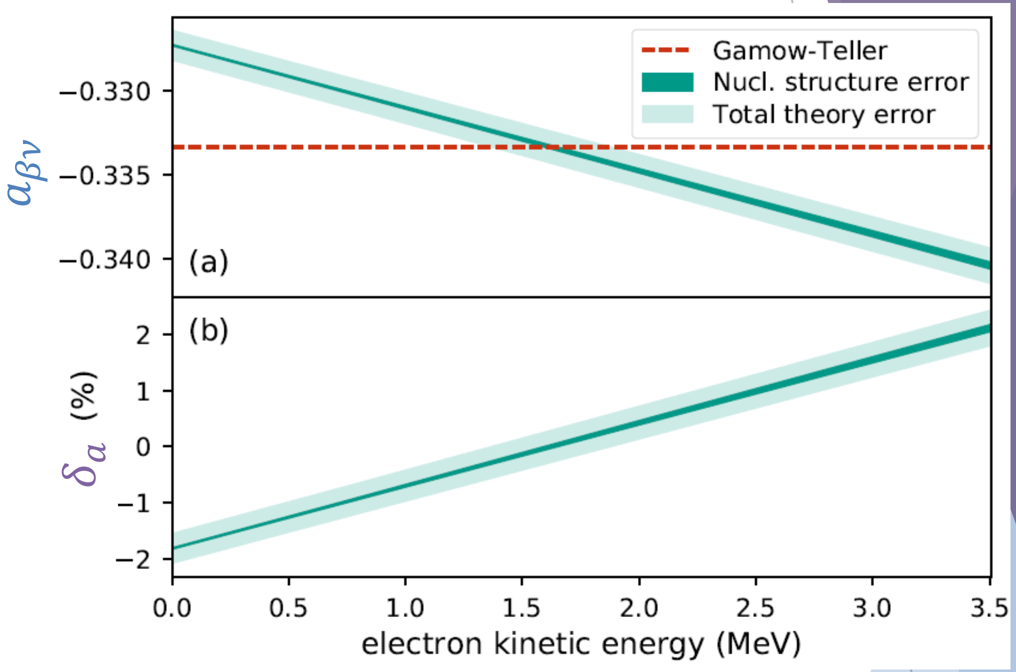
Pastore *et al.*, PRC87 035503 (2013)
Friman-Gayer *et al.*, PRL126 102501 (2021)

$^{6}He \rightarrow {}^{6}Li$ angular correlation

- Experiments are aiming a 10^{-3} accuracy

GT / SM correction / BSM

- $$a_{\beta\nu} = -\frac{1}{3}\left(1 + \tilde{\delta}_a + \frac{|C_T^+|^2 + |C_T^-|^2}{4|C_A|^2}\right)$$



$$\tilde{\delta}_a = -2.54(68) \cdot 10^{-3}$$

$$C_T^+(C_T^-) \sim 10^{-3}$$

AGM, Forssén, Gazda, Gazit, Gysbers & Navrátil, PLB 2022

$^{6}He \rightarrow {}^{6}Li$ angular correlation

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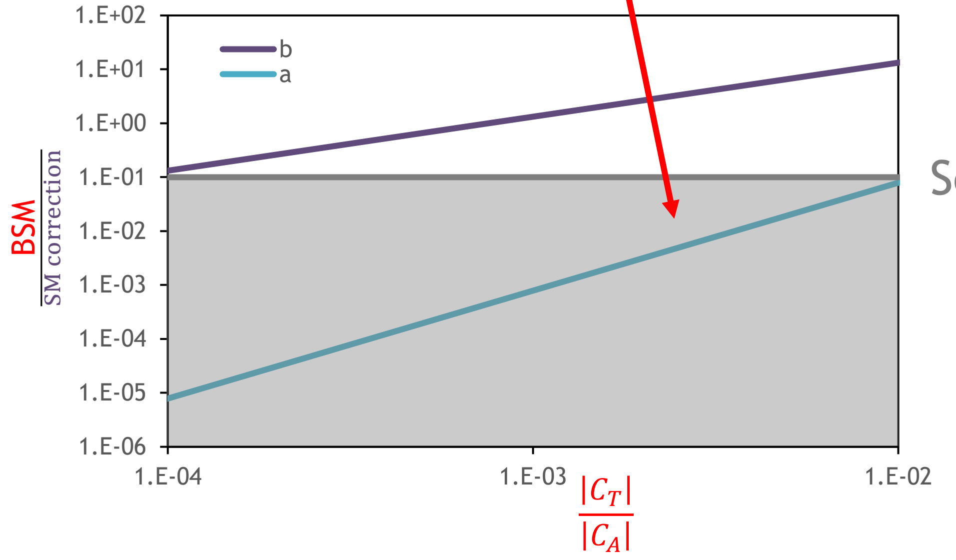
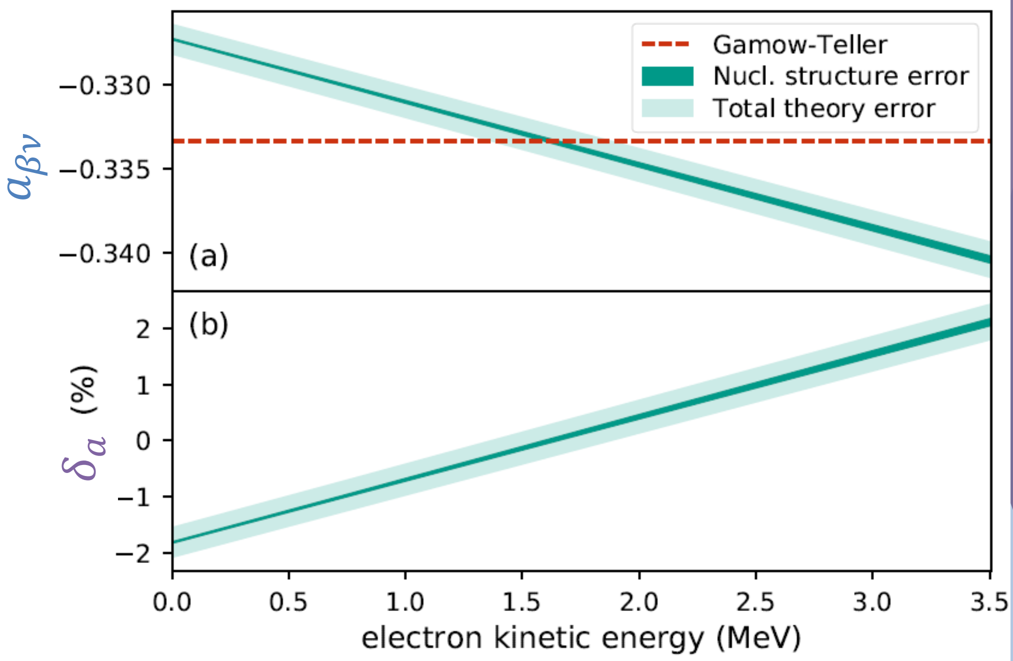
GT, SM correction, BSM

- $a_{\beta\nu} = -\frac{1}{3}\left(1 + \tilde{\delta}_a + \frac{|C_T^+|^2 + |C_T^-|^2}{4|C_A|^2}\right)$

- Looking for $\frac{|C_T^+|^2 + |C_T^-|^2}{4|C_A|^2} \sim 10^{-6}$???





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AGM, Forssén, Gazda, Gazit, Gysbers & Navrátil, PLB 2022

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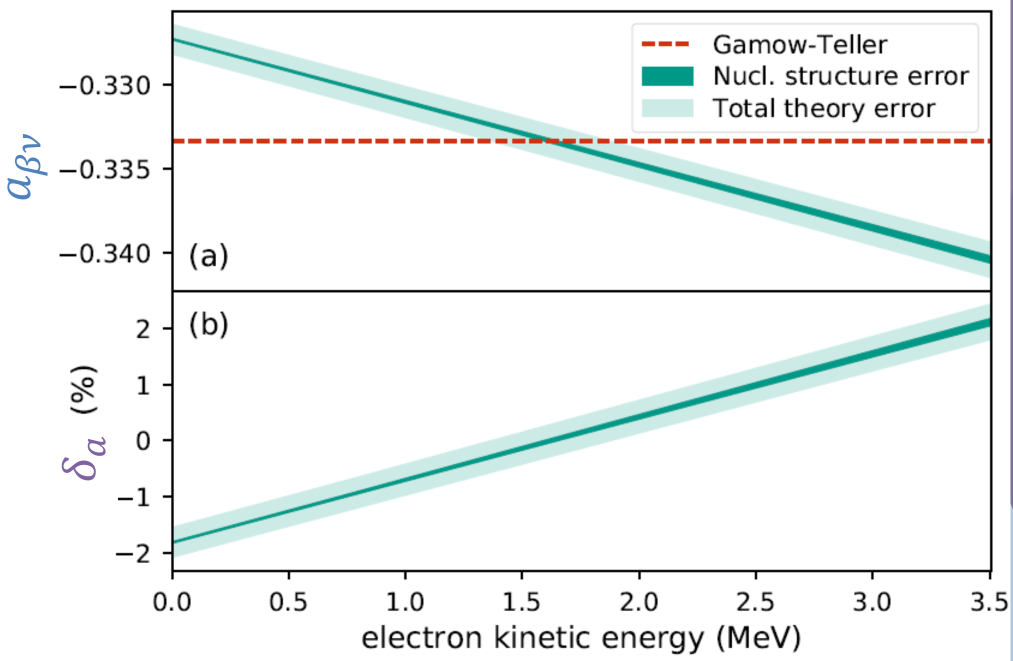
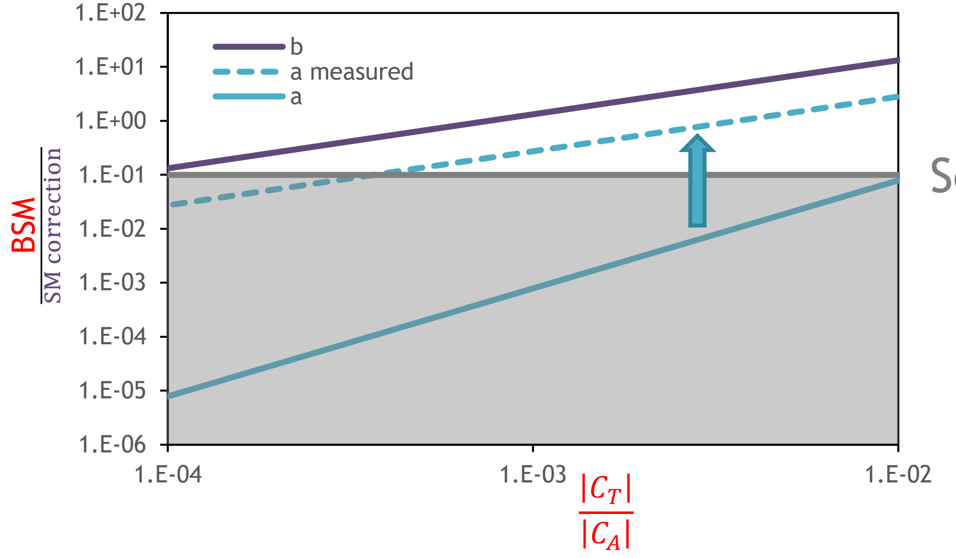




Sensitivity

$\tilde{\delta}_a = -2.54(68) \cdot 10^{-3}$

$C_T^+(C_T^-) \sim 10^{-3}$

$a_{\beta\nu}^{measured} \propto \cdots - 0.1 b_F \sim 0.1 \frac{C_T^+}{C_A} \sim 10^{-4}$ **!!!**

Future experiments aim at < 10^{-3}

AGM, Forssén, Gazda, Gazit, Gysbers & Navrátil, PLB 2022

PHYSICAL REVIEW LETTERS **129,** 182502 (2022)

β-Nuclear-Recoil Correlation from ^{6}He Decay in a Laser Trap

P. Müller,[1] Y. Bagdasarova,[2] R. Hong,[2] A. Leredde,[1] K. G. Bailey,[1] X. Fléchard,[3] A. García,[2] B. Graner,[2] A. Knecht,[2,4] O. Naviliat-Cuncic,[3,5] T. P. O'Connor,[1] M. G. Sternberg,[2] D. W. Storm,[2] H. E. Swanson,[2] F. Wauters,[2,6] and D. W. Zumwalt[2]

$$\hat{a} = -0.3268(46)_{\text{stat}}(41)_{\text{syst}}. \tag{4}$$

Assuming tensor contributions with right-handed neutrinos ($b = 0$ or $\tilde{C}_T = -\tilde{C}'_T$) the result above implies $|\tilde{C}_T|^2 \leq 0.022$ (90% C.L.) On the other hand, assuming purely left-handed neutrinos ($\tilde{C}_T = +\tilde{C}'_T$) yields

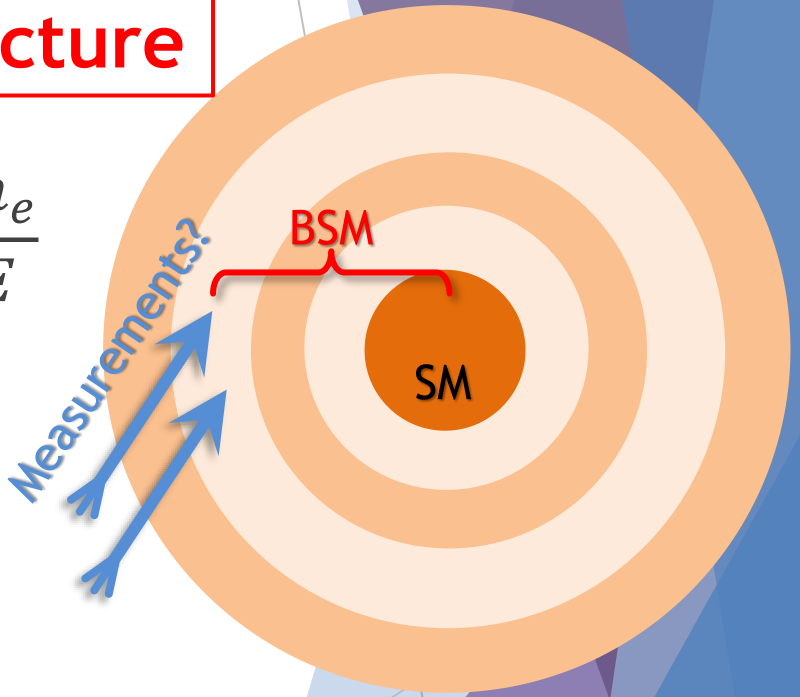
$$0.007 < \tilde{C}_T < 0.111 \text{ (90\% C.L.)}. \tag{5}$$

Nuclear β-decay formalism

Searches for deviations from the SM "V-A" structure

$$\Theta(q, \vec{\beta}\cdot\hat{\nu}) \propto \left|\langle\psi_f \| \hat{H}_W \| \psi_i\rangle\right|^2 \overset{\textit{allowed}}{\propto} 1 + a_{\beta\nu}\vec{\beta}\cdot\hat{\nu} + b_F \frac{m_e}{E}$$

Observables



Angular correlation: $a_{\beta\nu} = -\frac{1}{3}\left(1 - \frac{|C_T|^2 + |C'_T|^2}{2|C_A|^2}\right)$ (GT; BSM term $10^{-6}\sim$)

- Quadratic in C_T, C'_T

β Energy spectrum: Fierz term $b_F^{\beta^\mp} = \pm\frac{C_T + C'_T}{C_A}$ ($\sim 10^{-3}$)

- Vanishes for right-handed neutrinos ($C_T = -C'_T$)

Required: BSM predictions

$C_A = 1.27$ Axial vector coupling constant (SM)
$C_T, C'_T \lesssim 10^{-3}$ Tensor coupling constants (BSM), unknown

q - momentum transfer
$\vec{\beta} \equiv \frac{\vec{k}}{\epsilon}$ - electron's normalized momentum
$\hat{\nu} \equiv \frac{\vec{\nu}}{\nu}$ - neutrino's normalized momentum

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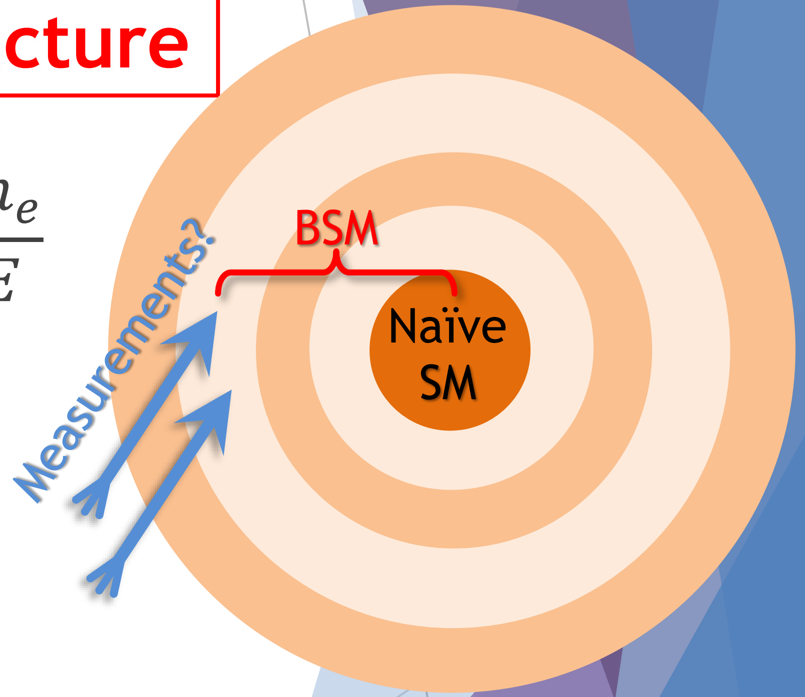
$$\Theta(q, \vec{\beta} \cdot \hat{\nu}) \propto \left|\langle \psi_f \| \hat{H}_W \| \psi_i \rangle\right|^2 \underset{q \to 0}{\overset{allowed}{\propto}} 1 + a_{\beta\nu} \vec{\beta} \cdot \hat{\nu} + b_F \frac{m_e}{E}$$

Observables



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Observables

Angular correlation: $a_{\beta\nu} = -\frac{1}{3}\left(1 - \frac{|C_T|^2+|C_T'|^2}{2|C_A|^2}\right)$ (GT; BSM term $10^{-6}\sim$)

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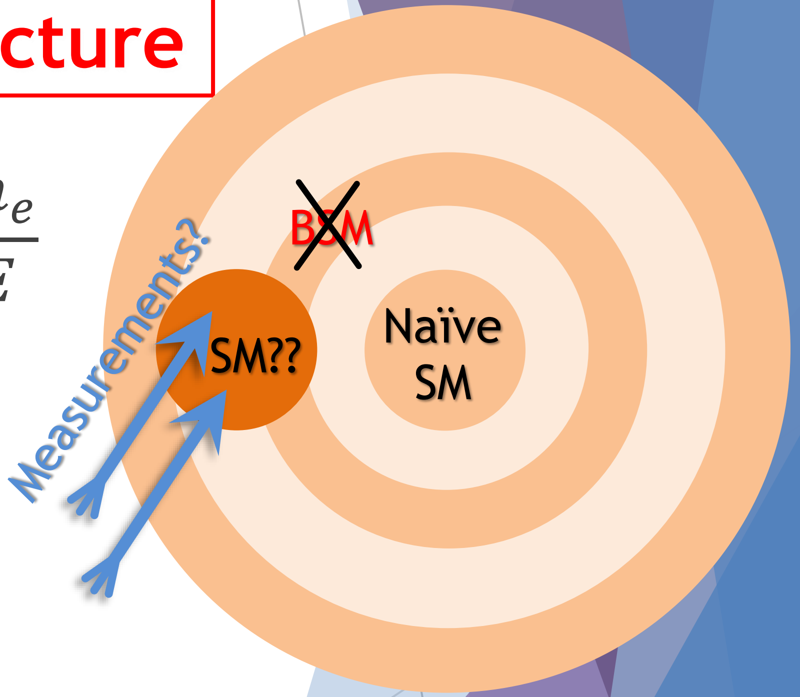
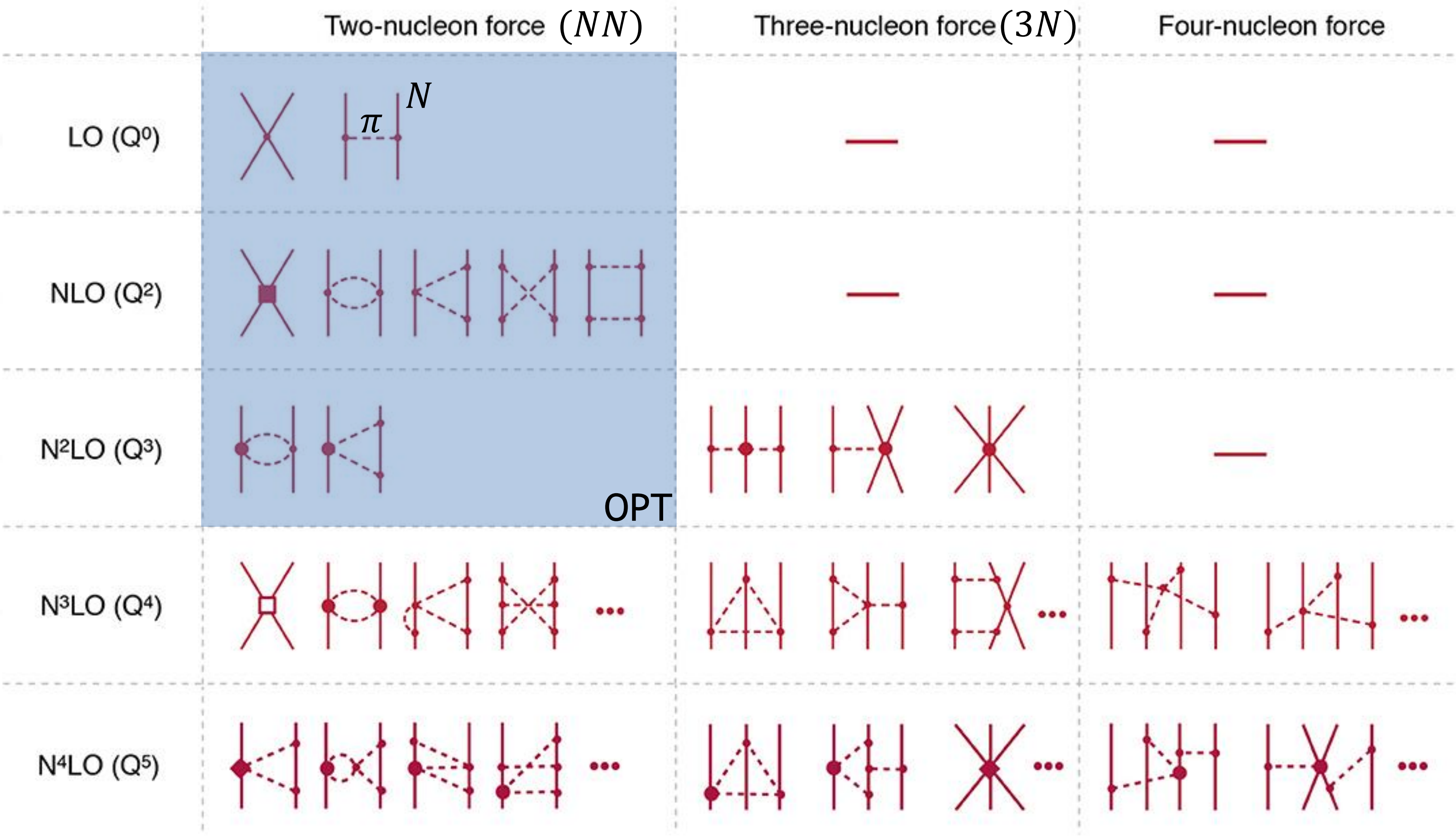


Required: BSM predictions vs. SM corrections

χEFT

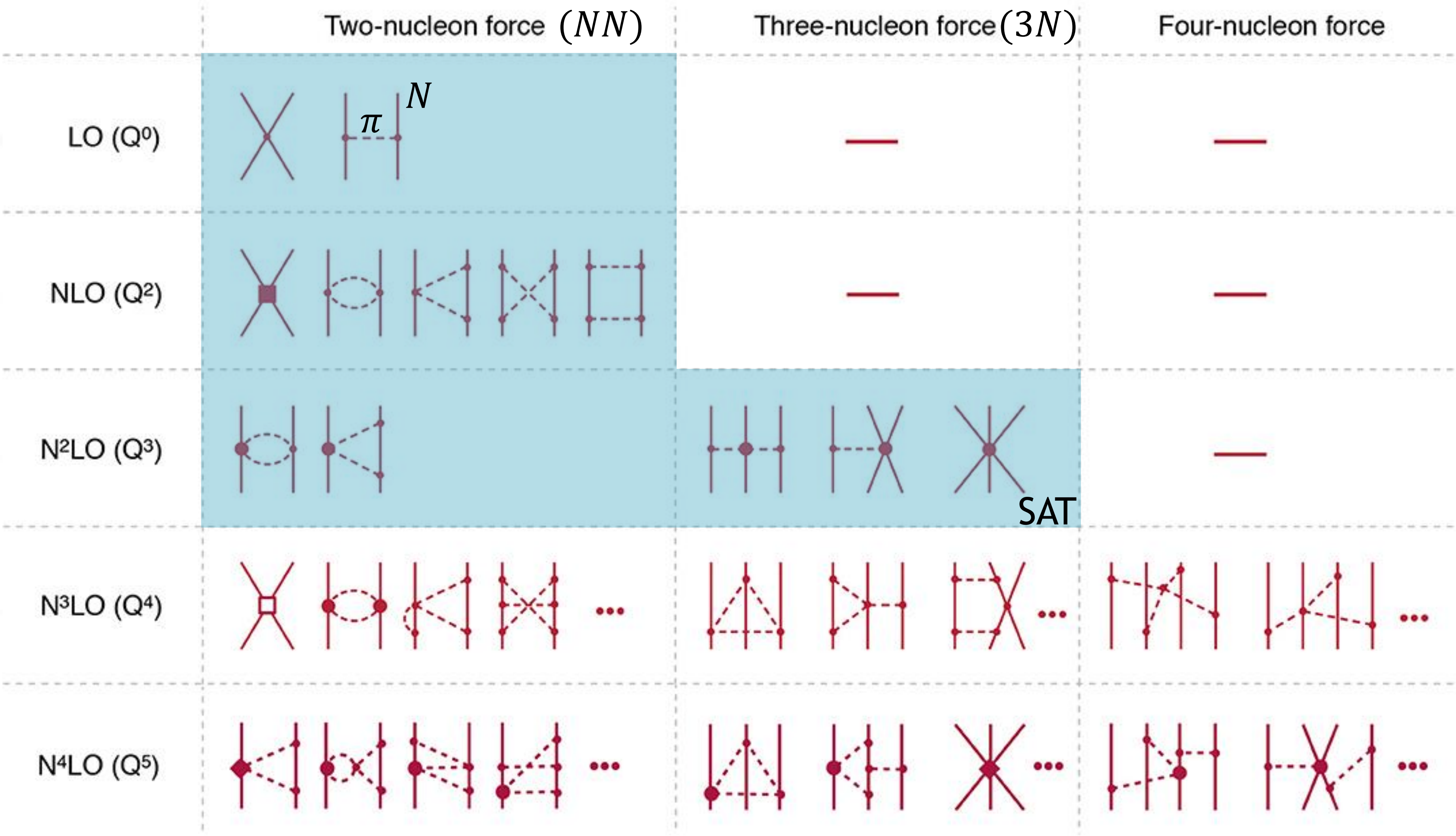


$$\hat{H} = \frac{1}{A}\sum_{i<j=1}^{A}\frac{(\vec{p}_i-\vec{p}_j)^2}{2m_N} + \sum_{i<j=1}^{A} V_{ij}^{NN}$$

χEFT @ N^{2}LO interactions

Required: SM high accuracy predictions

χEFT



$$\hat{H} = \frac{1}{A}\sum_{i<j=1}^{A}\frac{(\vec{p}_i-\vec{p}_j)^2}{2m_N} + \sum_{i<j=1}^{A} V_{ij}^{NN} + \sum_{i<j<k=1}^{A} V_{ijk}^{3N}$$

χEFT @ N^2LO interactions

Required: SM high accuracy predictions

Ab initio No-Core Shell Model

$$\hat{H} = \frac{1}{A}\sum_{i<j=1}^{A}\frac{(\vec{p}_i-\vec{p}_j)^2}{2m_N} + \sum_{i<j=1}^{A} V_{ij}^{NN} + \sum_{i<j<k=1}^{A} V_{ijk}^{3N}$$

χEFT @ N^{2}LO interactions

Schrodinger equation: $\hat{H}|\alpha\rangle = E_{\lambda T_z}^{I^\pi T}|\alpha\rangle$

single-particle harmonic-oscillator base states (depend on single-particle coordinates $\vec{r}$)

Required: SM high accuracy predictions

Ab initio No-Core Shell Model

$$\hat{H} = \frac{1}{A}\sum_{i<j=1}^{A}\frac{(\vec{p}_i-\vec{p}_j)^2}{2m_N} + \sum_{i<j=1}^{A} V_{ij}^{NN} + \sum_{i<j<k=1}^{A} V_{ijk}^{3N}$$

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single-particle harmonic-oscillator base states (depend on single-particle coordinates $\vec{r}$)

$$\langle\psi_f\|\sum_{j=1}^{A}\hat{O}_J(\vec{r}_j)\|\psi_i\rangle = \frac{-1}{\sqrt{2J+1}}\sum_{|\alpha|,|\beta|}\langle|\alpha|\|\hat{O}_J(\vec{r})\||\beta|\rangle\langle\psi_f\|(a^{+}_{|\alpha|}\tilde{a}_{|\beta|})_J\|\psi_i\rangle$$

Required: SM high accuracy predictions

Translational-invariant 1-body density matrices

$$\hat{H} = \frac{1}{A}\sum_{i<j=1}^{A}\frac{(\vec{p}_i-\vec{p}_j)^2}{2m_N} + \sum_{i<j=1}^{A} V_{ij}^{NN} + \sum_{i<j<k=1}^{A} V_{ijk}^{3N}$$

χEFT @ $\mathrm{N^2LO}$ interactions

Schrodinger equation: $\hat{H}|\alpha\rangle = E_{\lambda T_z}^{I^\pi T}|\alpha\rangle$

single-particle harmonic-oscillator base states (depend on single-particle coordinates $\vec{r}$)

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transformation matrix $|a||b| \to |\alpha||\beta|$

$$\langle\psi_f\left\|\sum_{j=1}^{A}\hat{O}_J(\vec{\xi}_j)\right\|\psi_i\rangle = \frac{-1}{\sqrt{2J+1}}\sum_{\substack{|\alpha|,|\beta|\\|a|,|b|}}\langle|a|\|\hat{O}_J(\vec{\xi})\||b|\rangle\left(M^J\right)^{-1}_{|a||b||\alpha||\beta|}\langle\psi_f\left\|\left(a^+_{|\alpha|}\tilde{a}_{|\beta|}\right)_J\right\|\psi_i\rangle$$

depend on single-particle Jacobi coordinates $\vec{\xi} \propto \vec{r} - \vec{R}_{\mathrm{CM}}$

Required: SM high accuracy predictions

Navrátil, Phys. Rev. C104, 064322 (2021)

Calculations

$\hat{H}$

Nuclear Hamiltonian

χEFT @ $N^2LO_{opt\ (sat)}$

↓

$\langle \psi_f \| \quad \| \psi_i \rangle$

Nuclear wave functions

$\hat{\mathcal{J}}(\vec{x})$

Nuclear currents

LO (1-body currents)

↓

$\hat{O}_J$

Multipole operators

↘ ↙

$\langle \psi_f \| \hat{O}_J \| \psi_i \rangle$

Nuclear matrix elements

Ab initio No-Core Shell Model (NCSM)

↓

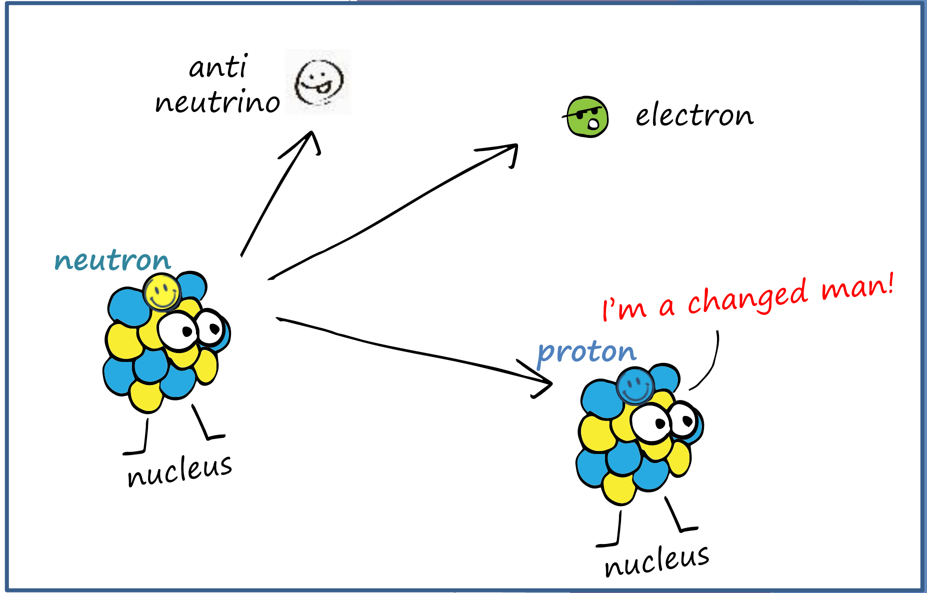
$\delta_1, \delta_a, \delta_b$

Observables' corrections

Required: SM high accuracy predictions

Nuclear β-decay

Beta decay, Khan Academy: cdn.kastatic.org/ka-perseus-images/8d978444f15f9bbc3bcadb0549816bc7e264b977.svg



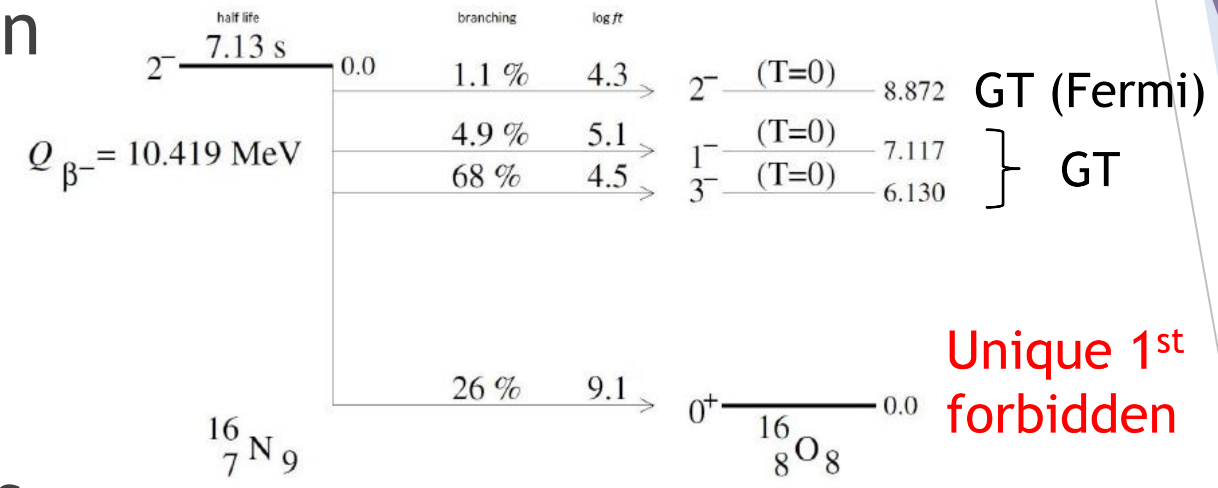
- Low momentum transfer: $q \sim 0 - 10\ \mathrm{MeV/c}$

Total angular momentum — parity

- Transitions (different ΔJ^{π}):
  - Allowed:
    - Fermi (F, $\Delta J^{\pi} = 0^{+}$)
    - Gamow-Teller (GT, $\Delta J^{\pi} = 1^{+}$)
  - Forbidden - all the rest
    - Vanish for $q \to 0$
- Recent experiments: Deviations from the V-A structure (Scalar & Tensor)
  - Missing precise theory for forbidden Tensor interactions

$^{16}N \rightarrow {}^{16}O$ experiment @ SARAF

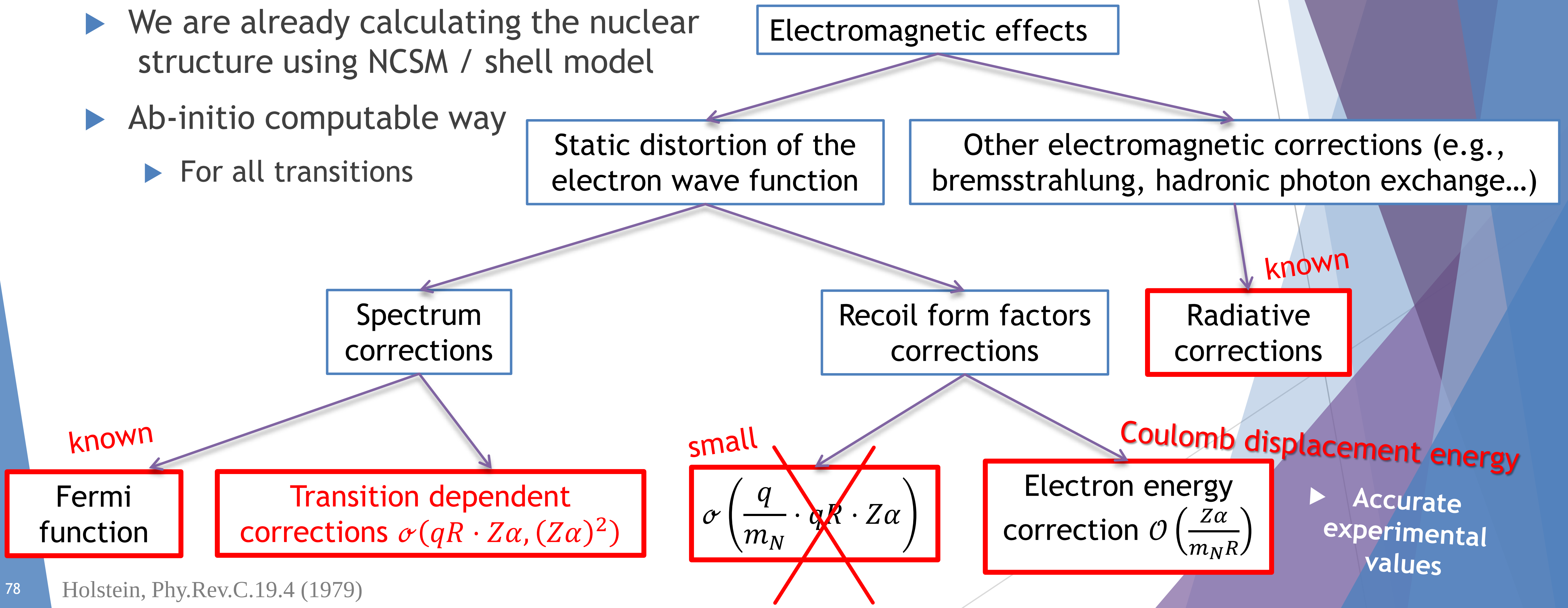
- Unique 1st forbidden sensitivity to BSM signatures
- Energy separation
  - Ideal case study
- BSM & SM predictions
- Ab initio NCSM & eigenvector continuation emulators with Christian Forssén (Chalmers)
- Theoretical & Experimental constrains on the measured observables



AGM, Mishnayot, *et al.*, PLB767 285-288 (2017)
Ohayon, Chocron, Hirsh, AGM, et al., Hyp.Int.239,57 (2018)
Djärv, Ekström, Forssén, Johansson, arXiv:2108.13313 (2021)

Coulomb corrections

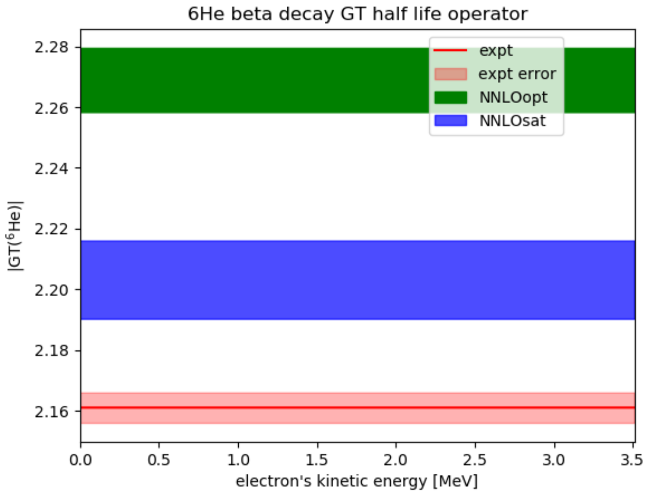
- Usually considered by approximations treating the nucleus with simple models
  - Only for specific transitions
- We are already calculating the nuclear structure using NCSM / shell model
- Ab-initio computable way
  - For all transitions



Holstein, Phy.Rev.C.19.4 (1979)

Calculating 2b currents

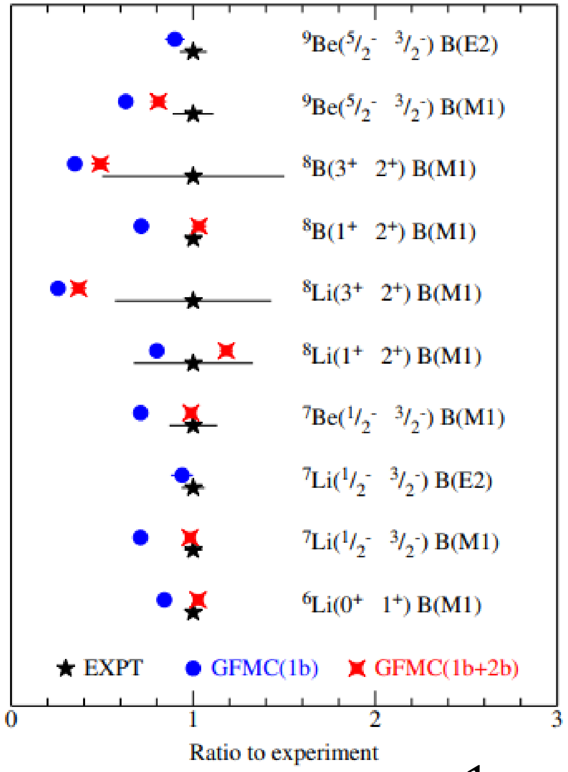
- Significant improvement in accuracy:



$$|GT(\,^6\mathrm{He})| = \frac{\sqrt{12\pi}}{g_A} |\langle \| \hat{L}_1^A \| \rangle|^2$$

2b: $\langle \| \hat{L}_1^A \| \rangle, \langle \| \hat{C}_1^A \| \rangle \sim 1.57\% \sim \mathcal{O}(\epsilon_{EFT}^2)$

Electro-magnetic transitions



$$^6\mathrm{Li}(0^+ \rightarrow 1^+) B(M1) = \frac{1}{3} |\langle \| \hat{M}_1^V \| \rangle|^2$$

2b: $\langle \| \hat{M}_1^V \| \rangle \sim 10\% \sim \mathcal{O}(\epsilon_{EFT})$

$\hat{J}(\vec{x})$

Nuclear currents

↓

$\hat{O}_J$

Multipole operators

↓

$\langle \psi_f \| \hat{O}_J \| \psi_i \rangle$

Nuclear matrix elements

↓

$\delta_1, \delta_a, \delta_b$

Observables' corrections

Pastore *et al.*, PRC87 035503 (2013)
Friman-Gayer *et al.*, PRL126 102501 (2021)

Recoil ion spectrum

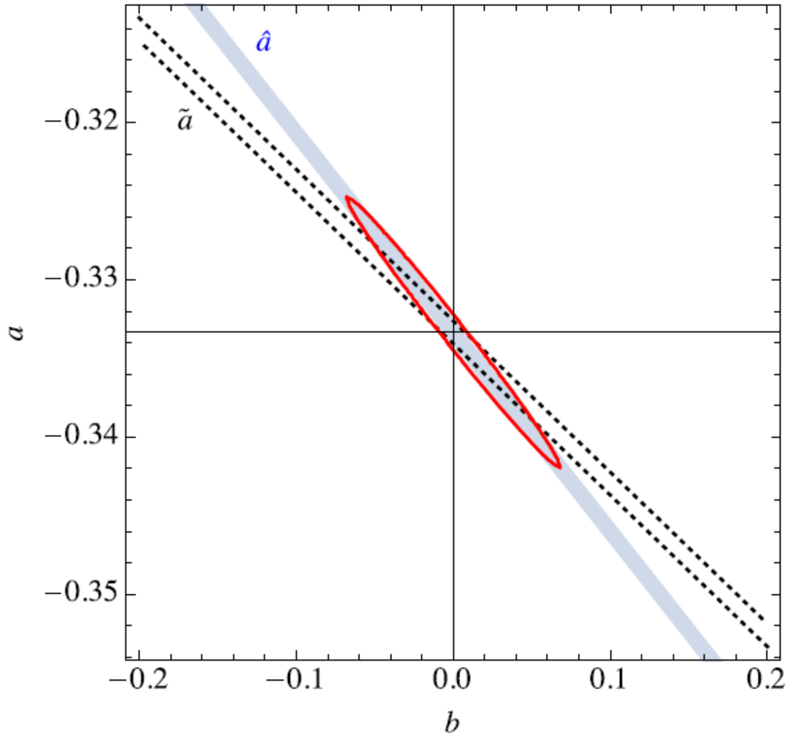


FIG. 5. The solid red ellipse shows the 1σ region obtained from a fit of simulated recoil momentum spectra with 10^7 events, for the ^{6}He decay, where both a and b were left as free parameters. The blue shaded band shows the 1σ bound on the combination $\hat{a} = a + 0.127b$, whereas the black dotted lines represent the 1σ bound obtained using the $\tilde{a}$ prescription.

- β spectrum: $a_{\beta\nu} = \frac{a_{\beta\nu}^{\text{measured}}}{1+\left\langle\frac{m_e}{\epsilon}\right\rangle b_F}$
- Recoil ion spectrum: $a_{\beta\nu}$ has a non-trivial dependence on b_F
- Case study: ^{23}Na ion measurements @ SARAF
- Coupled-Cluster calculations with Sonia Bacca (Mainz)

Gonzalez-Alonso & Naviliat-Cuncic, Phys.Rev.C94

E.g., GT & unique $(J-1)^{\text{th}}$ forbidden

$$\Theta^{J^{(-)^{J-1}}}(q,\vec{\beta}\cdot\hat{v}) = \frac{2J+1}{J}\left(1+\delta_1^{J^{(-)^{J-1}}}\right)\left\{1 + a_{\beta\nu}\vec{\beta}\cdot\hat{v} + b_{\mathrm{F}}\frac{m_e}{\epsilon} + c_{\mathrm{squared}}\left[\vec{\beta}^2 - \left(\vec{\beta}\cdot\hat{v}\right)^2\right]\right\}\left|\langle\psi_f\|\hat{L}_J\|\psi_i\rangle\right|^2$$

- $a_{\beta\nu} = -\frac{1}{2J+1}\left(1+\tilde{\delta}_a^{J^{(-)^{J-1}}}\right)$
- $b_{\mathrm{F}} = \delta_b^{J^{(-)^{J-1}}}$
- $c_{\mathrm{squared}} = \frac{1}{2J+1}\frac{\epsilon(\epsilon_0-\epsilon)}{q^2}\left(1-\delta_1^{J^{(-)^{J-1}}}\right)$
- $\delta_1 = \frac{2}{2J+1}\Re\mathrm{e}\left[-J\epsilon_0\frac{\langle\|\hat{C}_J^A/q\|\rangle}{\langle\|\hat{L}_J^A\|\rangle} \pm \sqrt{J(J+1)}(\epsilon_0-2\epsilon)\frac{\langle\|\hat{M}_J^V/q\|\rangle}{\langle\|\hat{L}_J^A\|\rangle}\right] + \mathcal{O}\left(\frac{\epsilon_{qr}^2}{15}, \epsilon_c^2\right)$
- $\tilde{\delta}_a = \frac{4}{2J+1}\Re\mathrm{e}\left[(J+1)\epsilon_0\frac{\langle\|\hat{C}_J^A/q\|\rangle}{\langle\|\hat{L}_J^A\|\rangle} \pm \sqrt{J(J+1)}(\epsilon_0-2\epsilon)\frac{\langle\|\hat{M}_J^V/q\|\rangle}{\langle\|\hat{L}_J^A\|\rangle}\right] + \mathcal{O}\left(\frac{\epsilon_{qr}^2}{15}, \epsilon_c^2\right)$
- $\delta_b = \frac{2}{2J+1}m_e\Re\mathrm{e}\left[J\frac{\langle\|\hat{C}_J^A/q\|\rangle}{\langle\|\hat{L}_J^A\|\rangle} \pm \sqrt{J(J+1)}\frac{\langle\|\hat{M}_J^V/q\|\rangle}{\langle\|\hat{L}_J^A\|\rangle}\right] + \mathcal{O}\left(\frac{\epsilon}{m_e}\frac{\epsilon_{qr}^2}{15}, \frac{\epsilon}{m_e}\epsilon_c^2\right)$

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Nuclear Currents

Vector: $$\langle p(p_p)|\bar{u}\gamma_\mu d|n(p_n)\rangle = \bar{u}_p(p_p)\left[g_V(q^2)\gamma_\mu + \frac{\tilde{g}_{T(V)}(q^2)}{2m_N}\sigma_{\mu\nu}q^\nu + \frac{\tilde{g}_S(q^2)}{2m_N}q_\mu\right]u_n(p_n)$$

Axial: $$\langle p(p_p)|\bar{u}\gamma_\mu\gamma_5 d|n(p_n)\rangle = \bar{u}_p(p_p)\left[g_A(q^2)\gamma_\mu + \frac{\tilde{g}_{T(A)}(q^2)}{2m_N}\sigma_{\mu\nu}q^\nu + \frac{\tilde{g}_P(q^2)}{2m_N}q_\mu\right]\gamma_5 u_n(p_n)$$

$$\begin{aligned}
\langle p(p_p)|\bar{u}d|n(p_n)\rangle &= g_S(0)\,\bar{u}_p(p_p)\,u_n(p_n) + \mathcal{O}\left(q^2/m_N^2\right)\\
\langle p(p_p)|\bar{u}\gamma_5 d|n(p_n)\rangle &= g_P(0)\,\bar{u}_p(p_p)\,\gamma_5 u_n(p_n) + \mathcal{O}\left(q^2/m_N^2\right)\\
\langle p(p_p)|\bar{u}\sigma_{\mu\nu}d|n(p_n)\rangle &= g_T(0)\,\bar{u}_p(p_p)\,\sigma_{\mu\nu}u_n(p_n) + \mathcal{O}\left(q/m_N\right)
\end{aligned}$$

$$\hat{C}_{JM} = \int d^3x\, j_J(qx) Y_{JM}(\hat{x}) \hat{\mathcal{J}}_0(\vec{x})$$

$$\hat{L}_{JM} = \frac{i}{q}\int d^3x \{\vec{\nabla}[j_J(qx) Y_{JM}(\hat{x})]\} \cdot \vec{\mathcal{J}}(\vec{x})$$

$$\hat{E}_{JM} = \frac{i}{q}\int d^3x \{\vec{\nabla} \times [j_J(qx)\, \vec{Y}^M_{JJ1}(\hat{x})]\} \cdot \vec{\mathcal{J}}(\vec{x})$$

$$\hat{M}_{JM} = \frac{i}{q}\int d^3x [j_J(qx)\, \vec{Y}^M_{JJ1}(\hat{x})] \cdot \vec{\mathcal{J}}(\vec{x})$$

$$\hat{C}_J^A(q) = -\frac{iq}{m_N}\sum_{j=1}^{A}\tau_j^{\pm}\left[g_A\Omega_J(q\vec{r}_j) + \frac{1}{2}\left(g_A + \tilde{g}_{T(A)} - \frac{\omega}{2m_N}\tilde{g}_P\right)\Sigma_J''(q\vec{r}_j)\right] + O\left(\frac{r_j^J q^{J+3}}{m_N^3}\right)$$

$$\hat{L}_J^A(q) = i\left(g_A - \frac{\omega}{2m_N}\tilde{g}_{T(A)}\right)\sum_{j=1}^{A}\tau_j^{\pm}\Sigma_J''(q\vec{r}_j) + O\left(\frac{r^{J-1}q^{J+1}}{m_N^2}\right)$$

$$\hat{E}_J^A(q) = i\left(g_A - \frac{\omega}{2m_N}\tilde{g}_{T(A)}\right)\sum_{j=1}^{A}\tau_j^{\pm}\Sigma_J'(q\vec{r}_j) + O\left(\frac{r^J q^{J+2}}{m_N^2}\right)$$

$$\hat{M}_J^A(q) = \left(g_A - \frac{\omega}{2m_N}\tilde{g}_{T(A)}\right)\sum_{j=1}^{A}\tau_j^{\pm}\Sigma_J(q\vec{r}_j) + O\left(\frac{r^J q^{J+2}}{m_N^2}\right).$$

$$J_0^V(\vec{r}) = \sum_{j=1}^{A}\left[g_V + \frac{\omega}{2m_N}\tilde{g}_S\right]\tau_j^{\pm}\delta^{(3)}(\vec{r}-\vec{r}_j) + O\left(\frac{q^2}{m_N^2}\right)$$

$$\vec{J}^V(\vec{r}) = \frac{1}{2m_N}\sum_{j=1}^{A}\left[g_V\left\{\vec{p}_j, \delta^{(3)}(\vec{r}-\vec{r}_j)\right\} + \left(g_V + \tilde{g}_{T(V)}\right)\vec{\nabla}\times\vec{\sigma}_j\delta^{(3)}(\vec{r}-\vec{r}_j) + \right.$$
$$\left. - i\tilde{g}_S\vec{\nabla}\delta^{(3)}(\vec{r}-\vec{r}_j)\right]\tau_j^{\pm} + O\left(\frac{q^2}{m_N^2}\right)$$

$$J_0^A(\vec{r}) = \frac{1}{2m_N}\sum_{j=1}^{A}\left[g_A\left\{\vec{p}_j, \delta^{(3)}(\vec{r}-\vec{r}_j)\right\} + i\left(\tilde{g}_{T(A)} - \frac{\omega}{2m_N}\tilde{g}_P\right)\vec{\nabla}\delta^{(3)}(\vec{r}-\vec{r}_j)\right]\cdot\vec{\sigma}_j\tau_j^{\pm} + O\left(\frac{q^3}{m_N^3}\right)$$

$$\vec{J}^A(\vec{r}) = \sum_{j=1}^{A}\left[g_A - \frac{\omega}{2m_N}\tilde{g}_{T(A)}\right]\vec{\sigma}_j\tau_j^{\pm}\delta^{(3)}(\vec{r}-\vec{r}_j) + O\left(\frac{q^2}{m_N^2}\right), \qquad (4)$$

$$\hat{C}_J^V(q) = \sum_{j=1}^{A}\tau_j^{\pm}\left(g_V + \frac{\omega}{2m_N}\tilde{g}_S\right)M_J(q\vec{r}_j) + O\left(\frac{r_j^J q^{J+2}}{m_N^2}\right)$$

$$\hat{L}_J^V(q) = -\frac{q}{\omega}\hat{C}_J^V(q)$$

$$\hat{E}_J^V(q) = \frac{q}{m_N}\sum_{j=1}^{A}\tau_j^{\pm}\left[g_V\Delta_J'(q\vec{r}_j) + \frac{g_V + \tilde{g}_{T(V)}}{2}\Sigma_J(q\vec{r}_j)\right] + O\left(\frac{r_j^J q^{J+2}}{m_N^2}\right)$$

$$\hat{M}_J^V(q) = -\frac{iq}{m_N}\sum_{j=1}^{A}\tau_j^{\pm}\left[g_V\Delta_J(q\vec{r}_j) - \frac{g_V + \tilde{g}_{T(V)}}{2}\Sigma_J'(q\vec{r}_j)\right] + O\left(\frac{r_j^J q^{J+2}}{m_N^2}\right)$$

$$\Delta_J(q\vec{r}) \equiv \vec{M}_{JJ1}(q\vec{r})\cdot\frac{1}{q}\vec{\nabla}$$

$$\Delta_J'(q\vec{r}) \equiv -i\left[\frac{1}{q}\vec{\nabla}\times\vec{M}_{JJ1}(q\vec{r})\right]\cdot\frac{1}{q}\vec{\nabla}$$

$$\Sigma_J(q\vec{r}) \equiv \vec{M}_{JJ1}(q\vec{r})\cdot\vec{\sigma}$$

$$\Sigma_J'(q\vec{r}) \equiv -i\left[\frac{1}{q}\vec{\nabla}\times\vec{M}_{JJ1}(q\vec{r})\right]\cdot\vec{\sigma}$$

$$\Sigma_J''(q\vec{r}) \equiv \left[\frac{1}{q}\vec{\nabla}M_J(q\vec{r})\right]\cdot\vec{\sigma}$$

$$\Omega_J(q\vec{r}) \equiv M_J(q\vec{r})\,\vec{\sigma}\cdot\frac{1}{q}\vec{\nabla}$$

$$\Omega_J'(q\vec{r}) \equiv \Omega_J(q\vec{r}) + \frac{1}{2}\Sigma_J''(q\vec{r}),$$