The Intrinsic Shape of ²³⁸U at the Relativistic Heavy Ion Collider

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OUTLINE

1 – Collective flow and nuclear structure in high-energy nuclear physics.

2 – The intrinsic shape of ²³⁸U in ultrarelativistic collisions.

3 – Prospects.

1 – Collective flow and nuclear structure in high-energy nuclear physics.

HIGH ENERGY NUCLEAR PHYSICS

Long Island (NY)



Huge program to explore emergent phenomena in strong-interaction matter.





How do we reconstruct the initial condition of the QGP?



Low-momentum particles follow the hydrodynamic expansion.

$$\frac{d^2 N}{dp_{\rm T} d\phi} = \frac{dN}{2\pi dp_{\rm T}} \left(1 + 2\sum_{n=1}^{\infty} v_n \cos n(\phi - \Phi_n) \right)$$
EXPLOSIVENESS
OF THE EXPANSION
EXPLOSIVENESS
ANISOTROPY OF
AZIMUTHAL DISTRIBUTION

[SEMINAR BY J-Y OLLITRAULT, WEEK 1]

Mapping initial-state geometry to final-state observables via pressure-gradient force.

$$F = -\nabla P$$

[SEMINAR BY J-Y OLLITRAULT, WEEK 1]



Shape and size of the QGP can be reconstructed from data!

Formation of QGP starts with an input from nuclear structure.



High-energy model

Scattering occurs mainly within nucleons.

"quantum measurement" of the nucleon positions.



[from Sandra Brandstetter (PI Heidelberg), Collapsed wave function of a gas of 10 ⁶Li atoms] **Origin of nucleon positions:** for "spherical" systems like 208Pb, independent sampling in common potential (<u>mean field</u>) is appropriate.



More realistic: Potential generated by effective nucleon-nucleon interaction (Gogny force, Skyrme force, etc.), in "Energy Density Functional" theory.

Goodness of mean field description justifies the Glauber Monte Carlo approach.



Interaction does not modify the global geometry of the overlap area.



Beyond ²⁰⁸Pb? Heavy-ion collisions require *a priori* knowledge of **all spatial correlations**.

Help from low-energy nuclear physics:

Spatial correlations encapsulated in "intrinsic shapes".

Instead of A-body correlation functions, use 1-body density with a deformed shape.

[SEMINAR BY W NAZAREWICZ, WEEK 1]



The bag of nucleons is now deformed and with a random orientation.

The collision selects one such orientation.

Intrinsic shapes are non-observable for direct measurements, but they leave their fingerprint on virtually all nuclear observables and phenomena Michael Bender – RBRC Workshop Jan 2022

$$\rho(r,\Theta,\Phi) \propto \frac{1}{1+\exp\left(\left[r-R(\Theta,\Phi)\right]/a\right)} , R(\Theta,\Phi) = R_0 \left[1+\underline{\beta_2}\left(\cos\gamma Y_{20}(\Theta)+\sin\underline{\gamma}Y_{22}(\Theta,\Phi)\right)+\underline{\beta_3}Y_{30}(\Theta)+\underline{\beta_4}Y_{40}(\Theta)\right] + \underline{\beta_4}Y_{40}(\Theta) + \underline{$$

Can we consistently understand the signatures of the intrinsic shape at high energy?

2 – The intrinsic shape of ²³⁸U in ultrarelativistic collisions.

Uranium-238 is the archetype of a well-deformed nucleus. $\beta_2 = 0.29$ from B(E2).

[Pritychenko et al., Atom.Data Nucl.Data Tabl. 107 (2016) 1-139]

From https://www.nndc.bnl.gov/nudat3/

8+

-518.1

.]=8

Ideal to study shape effects. U+U collisions are available at RHIC.

How far can we go?



[SEMINAR BY K WIMMER, WEEK 1]

First evidence of nuclear deformation in high-energy nuclear collisions.



How is elliptic flow enhanced? Simple model for collisions at b=0.

[Giacalone, Mehrabpour, in preparation] [see also Jia, PRC **105** (2022) 1, 014905]

Gaussian nuclei:
$$N_{A,B}(x,y,z) = e^{\frac{-(x^2+y^2+z^2)}{R_{A,B}^2}} \left(1 + \beta_{A,B} \left[\cos\gamma Y_2^0(x,y,z) + \frac{\sin\gamma}{\sqrt{2}} Re\{Y_2^2(x,y,z)\}\right]\right)$$

Thickness:
$$T_{A,B}(r,\phi,\Omega_{A,B}) = \int dz \ N_{A,B}\left(\frac{R_{zxz}(\Omega_{A,B})\vec{X}}{\text{Euler angles}}\right) \longrightarrow T_{A,B}(r,\phi) = \mathcal{F}_{A,B}(r) + \beta_{A,B} \ \mathcal{G}_{A,B}(r,\phi)$$

Eccentricity:
$$\epsilon(r,\phi,\Omega_A,\Omega_B) \propto \left(T_A(r,\phi,\Omega_A)T_B(r,\phi,\Omega_B)\right)^q \longrightarrow V_n \propto \mathcal{E}_n = -\frac{\int r dr d\phi \ r^n e^{in\phi} \epsilon(r,\phi,\Omega_A,\Omega_B)}{\int r dr d\phi \ r^n \epsilon(r,\phi,\Omega_A,\Omega_B)}$$

Mean squared anisotropy in many events (average over Euler angles and $\beta_A = \beta_B$):

$$\langle v_2^2 \rangle \propto \langle \mathcal{E}_2 \mathcal{E}_2^* \rangle = \underline{c(R_A, R_B, q)} \beta_2^2$$

Mean squared ellipticity receives a correction proportional to β^2 .

Very robust relation, works even away from central collisions.







Improper implementation of deformation in heavy-ion literature. Parameters of Woods-Saxon do not correspond to measured multipole moments. [SEMINAR BY W NAZAREWICZ, WEEK 1]

SHAPE IN INTRINSIC BODY FRAME

MEASURED MULTIPOLE MOMENT

$$\rho^{\mathrm{WS}}(\mathbf{r}) = \frac{\rho_0}{1 + \exp\left([r - R(\theta, \phi)]/a\right)} \text{ not the same thing!} \beta_{\ell} = \frac{4\pi}{(2\ell + 1)ZR_0^{\ell}} \sqrt{\frac{B(E\ell)\uparrow}{e^2}} R(\theta, \phi) = R_d \left[1 + \sum_{\ell=2}^{\ell_{\max}} \sum_{m=-\ell}^{\ell} \beta_{\ell m}^{\mathrm{WS}} Y_{\ell m}(\theta, \phi)\right]$$

[Ryssens, Giacalone, Schenke, Shen, in preparation]

IMPACT OF QUADRUPOLE-HEXADECAPOLE COUPLING

$$\beta_{2} = \frac{R_d^2}{R_0^2} \left[\beta_{20}^{\rm WS} + \frac{2}{7} \sqrt{\frac{5}{\pi}} (\beta_{20}^{\rm WS})^2 + \frac{12}{7\sqrt{\pi}} \frac{\beta_{20}^{\rm WS} \beta_{40}^{\rm WS}}{7\sqrt{\pi}} \right]$$

Uranium-238 has both a large β_2 and a sizable β_4 . Interplay plays a major role.

deformation on high-energy data! density with 21 different Skyrme functionals. 1.4 0.3Muonic x-rays $\blacktriangleright \beta_2^{\text{WS}} = 0.29$ Coulomb excitation 1.3ratio (U+U/Au+Au) β_2^{U} 0.25 B(E2) value 0.21.2 $\beta_{\ell}^{\rm WS}$ $\frac{1}{2}^{\text{WS}} = 0.25$ $eta_2^{
m Au}$ Cooperand BU **STAR data** 0.11.1BAu $v_2{2}$ IP-Glasma+MUSIC+UrQMD 0.00.10.20.30.0 Be 0.9 $\beta_{2 \text{ U}}^{\text{WS}} \in [0.24, 0.25]$ 4 % 3 2 We conclude:

Fitting Woods-Saxon profiles to one-body HFB

[Ryssens, Giacalone, Schenke, Shen, in preparation]

Impact of hexadecapole

MORE EXCLUSIVE OBSERVABLES

CAN WE DISCERN THE DIFFERENT ORIENTATIONS?



Old idea: Multiplicity should be higher in tip-tip collisions (larger Ncoll). Not supported by data.

[see e.g. Goldschmidt et al., PRC 92 (2015) 4, 044903]

HOW TO DO THAT? SHAPE-SIZE CORRELATION.

[Giacalone, PRL 124 (2020) 20, 202301]



CENTRAL COLLISIONS OF (PROLATE) DEFORMED IONS

The ellipticity of the quark-gluon plasma is positively correlated with its area. Deformation yields a negative correlation between v_2 and the $<p_t>$.



[Giacalone, PRC 102 (2020) 2, 024901]

SEEING BODY-BODY COLLISIONS "BY EYE"



Interplay of ellipticity and mean transverse momentum. Simple model for collisions at b=0.

Gaussian nuclei:
$$N_{A,B}(x,y,z) = e^{\frac{-(x^2+y^2+z^2)}{R_{A,B}^2}} \left(1 + \beta_{A,B} \left[\cos\gamma Y_2^0(x,y,z) + \frac{\sin\gamma}{\sqrt{2}} Re\{Y_2^2(x,y,z)\}\right]\right)$$

Thickness:
$$T_{A,B}(r,\phi,\Omega_{A,B}) = \int dz \ N_{A,B}\left(\frac{R_{zxz}(\Omega_{A,B})\vec{X}}{\text{Euler angles}}\right) \longrightarrow T_{A,B}(r,\phi) = \mathcal{F}_{A,B}(r) + \beta_{A,B} \ \mathcal{G}_{A,B}(r,\phi)$$

Eccentricity and total energy:

$$\epsilon(r,\phi,\Omega_A,\Omega_B) \propto \left(T_A(r,\phi,\Omega_A) T_B(r,\phi,\Omega_B) \right)^q \longrightarrow V_n \propto \mathcal{E}_n = -\frac{\int r dr d\phi \ r^n e^{in\phi} \epsilon(r,\phi,\Omega_A,\Omega_B)}{\int r dr d\phi \ r^n \epsilon(r,\phi,\Omega_A,\Omega_B)}$$
$$[p_t] \propto \underline{E} = \int r dr d\phi \ \epsilon(r,\phi,\Omega_A,\Omega_B)$$

Covariance of two quantities (average over Euler angles):

[Giacalone, Mehrabpour, in preparation] [see also Jia, PRC **105** (2022) 4, 044905]

$$\rho(v_2^2, \langle p_t \rangle) = \frac{\langle \delta v_2^2 \delta \langle p_t \rangle \rangle}{\sqrt{\langle \left(\delta v_2^2\right)^2 \rangle \langle \left(\delta \langle p_t \rangle\right)^2 \rangle}}$$

Covariance measured via Bozek's correlator:

[Bozek, PRC 93 (2016) 4, 044908]



Wrapping up: what have we learned from ²³⁸U+²³⁸U collisions?

1. Elliptic flow is enhanced by fluctuations in nuclear orientations. $\langle v_2^2 \rangle = \sigma_0^2 + c\beta_2^2 + v_{\rm RP}^2$

2. In ²³⁸U, interplay of quadrupole and hexadecapole deformations is crucial.

$$\beta_{20} = \frac{R_d^2}{R_0^2} \left[\beta_{20}^{\text{WS}} + \frac{2}{7} \sqrt{\frac{5}{\pi}} (\beta_{20}^{\text{WS}})^2 + \frac{12}{7\sqrt{\pi}} \beta_{20}^{\text{WS}} \beta_{40}^{\text{WS}} \right]$$

Optimal input from nuclear structure:

• Nuclear structure input: $\langle \Phi(\bar{\beta}, \bar{\gamma}) | a_r^{\dagger} a_r | \Phi(\bar{\beta}, \bar{\gamma}) \rangle \rightarrow \text{WS fit}$ Bally *et al.*, PRL 128, 082301 (2022) [SEMINAR BY B BALLY, WEEK 1]

3. We can use $<p_{T}>$ in conjunction with v_2 to discern body-body and tip-tip collisions.

3 – Prospects.

More features of the intrinsic shape?



BEYOND THE QUADRUPOLE



BEYOND THE MEAN FIELD? OCTUPOLE DEFORMATION

EDF CALCULATIONS WITH THE GOGNY FUNCTIONAL

[SEMINAR BY T RODRIGUEZ, WEEK 2]





ENHANCEMENT OF TRIANGULAR FLOW?



$$\langle v_3^2 \rangle = \sigma^2 + c\beta_3^2$$

From this model setup and assuming:



RHIC has observed pear-shaped ²³⁸U!



Off-central collisions: Centrality dependence of v₂ in U+U is not captured.



34

θ (π)

INDEPENDENT PROBES (AND ISSUES!) OF THE SKIN



A STAR: Signal $\pi^+\pi^-$ Pairs Would Get I 4×10^{-2} Model I 10^{-2} Model I 10^{-3} R=7.47 fm, a=0.54 fm 10^{-3} 10^{-3

[STAR Collaboration, Sci.Adv. 9 (2023) 1, eabq3903] [SEMINAR BY D BRANDENBURG, WEEK 5]

 $\Delta r_{np}[\text{STAR}] = 0.17 \pm 0.03 \text{ (stat.)} \pm 0.08 \text{ (syst.) fm}$

Fully consistent with state-of-the-art density functional results:

 $\Delta r_{np}[\text{MREDF}] = 0.17 \text{ fm}$

[Bally, Giacalone, Bender, arXiv:2301.02420]

INDEPENDENT PROBES (AND ISSUES!) OF THE SKIN





[STAR Collaboration, Sci.Adv. 9 (2023) 1, eabq3903] [SEMINAR BY D BRANDENBURG, WEEK 5]

Measured skin of uranium-238:

$$\Delta r_{np} \, [\text{STAR}] = 0.44 \pm 0.05 \, (\text{stat.}) \pm 0.08 \, (\text{syst.})$$

Inconsistent with low-energy physics. Why?

MORE THOUGHTS ...



Triaxiality

Beyond mean field calculations predict $\gamma \approx 5-10^{\circ}$. [Bertsch *et al.*, PRL **99** (2007) 032502] [Delaroche *et al.*, PRC **81** (2010) 014303]

Visible? Observables sensitive to $(\beta_2)^3 \cos(3\gamma)$.



Same analysis at different energy?

- Nuclear deformation with full 3+1D dynamics, @sqrt(s)~1 GeV. Role of shapes has to be addressed.

- Impact of small-x evolution?

[SEMINAR BY P SINGH, WEEK 2]



Ceterum autem censeo ...





[SEMINAR BY J-P EBRAN, WEEK 4]

[SEMINAR BY W VAN DER SCHEE, WEEK 4]



PRECISE INPUTS FROM AB INITIO NUCLEAR THEORY [WEEK 4]

Play same games in a <u>small system</u>, comparing to "spherical baseline" of O+O collisions **Unique insights on origin of collectivity and the response to the initial geometry.**

[ab initio PGCM + Trajectum frameworks, in preparation]

SUMMARY



- The initial geometry and the collective flow of high-energy nuclear collisions are strongly impacted by nuclear deformations.
- Understanding the effect of the ellipsoidal shape of ²³⁸U boosts confidence in understanding of the collision process. Much progress made.
- Prospects: more detailed features (octupole, skin...). Impact of collision runs at lower energies and with smaller systems?

THANK YOU

Intersection of nuclear structure and high-energy nuclear collisions: a new research direction.



Next Initial Stages conference (Copenhagen, 2023) will have a track related to nuclear structure.

Input to Nuclear Physics LRP in the US, both hot QCD (e.g. arXiv link) and nuclear theory.

Contributed input to NUPECC LRP 2024 [with Y. Zhou (NBI Copenhagen)] (link)

Just started a Topical Issue on EPJA on the intersection of the two areas (~20 papers in 2023) [T. Duguet, G. Giacalone, V. Somà, Y. Zhou]

IN CONCLUSION – NEW METHOD TO IMAGE NUCLEI

The ultimate nuclear shape experiment? Exploration of rare earth nuclei.

