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Standard Model corrections to Fermi transitions in light nuclei

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FRIB

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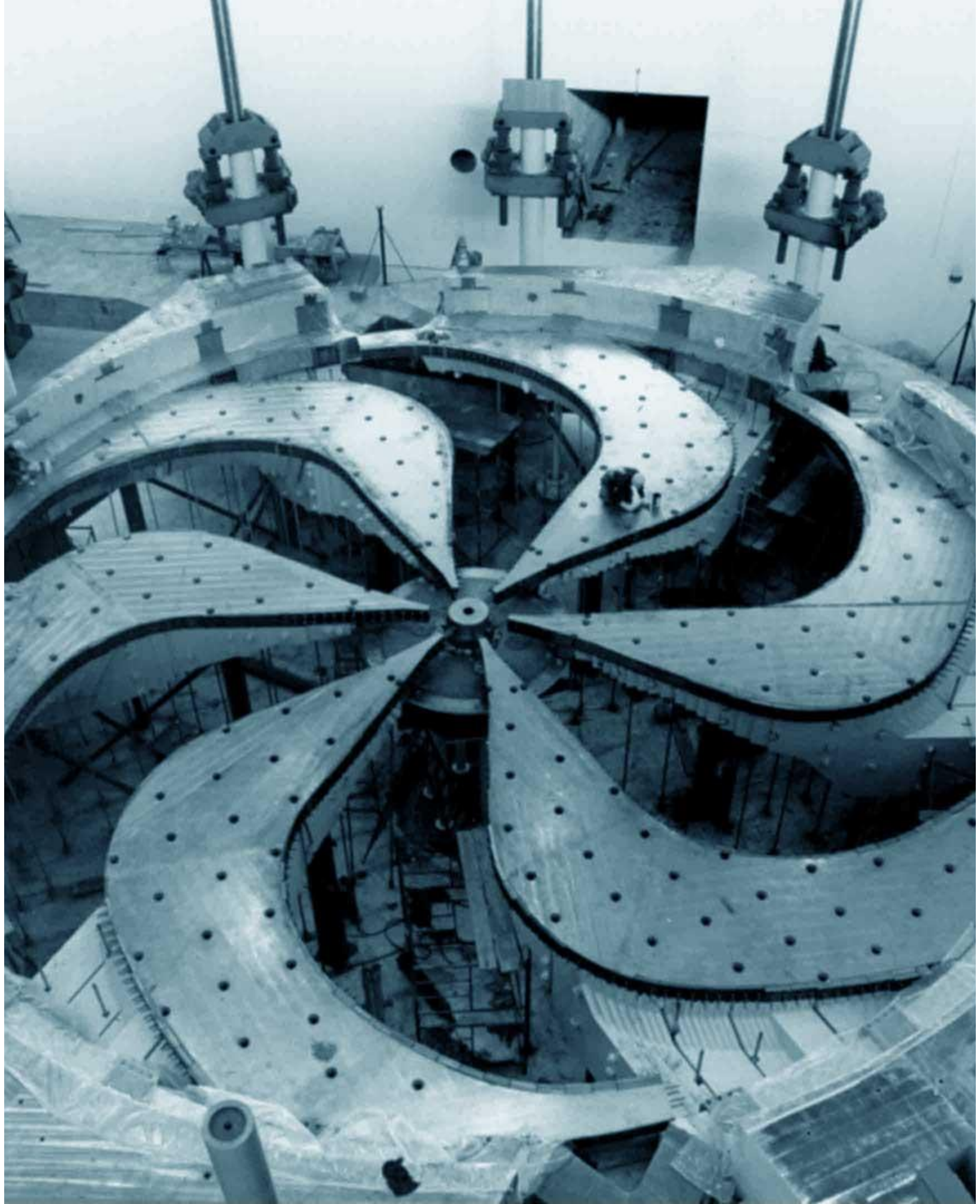
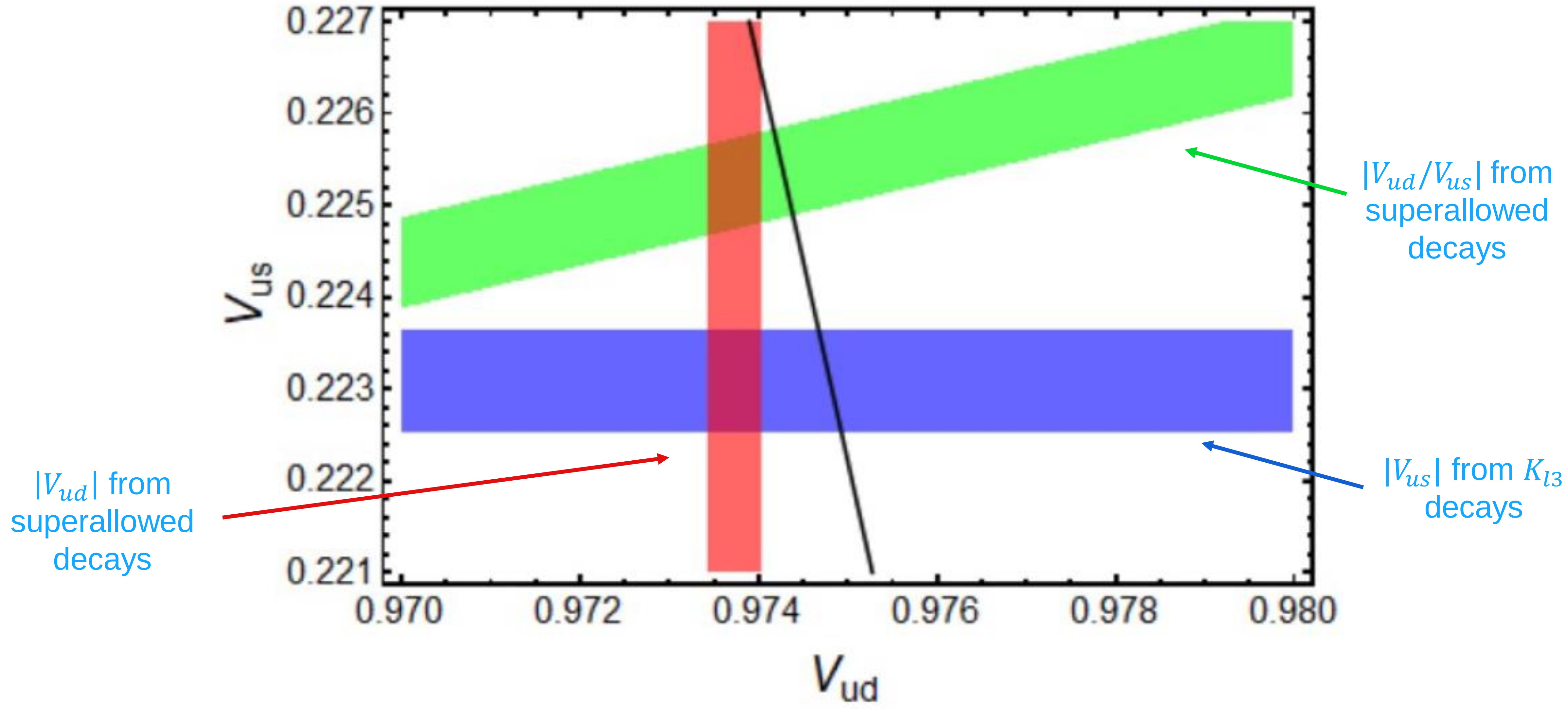


Discovery, accelerated



0.227
0.226
0.225
0.224
0.223
0.222
0.221
V_{us}
0.970
0.972
0.974
0.976
0.978
0.980
V_{ud}
$|V_{ud}/V_{us}|$ from superallowed decays
$|V_{us}|$ from K_{l3} decays
$|V_{ud}|$ from superallowed decays

[1] Seng (2022)

V_{ud} element of CKM matrix

	$\lvert V_{ud}\rvert$
superallowed	0.97373(31) [19]
n	0.97377(90) [20]
nuclear mirror	0.9739(10) [21]
π_{e3}	0.9740(28) [22]

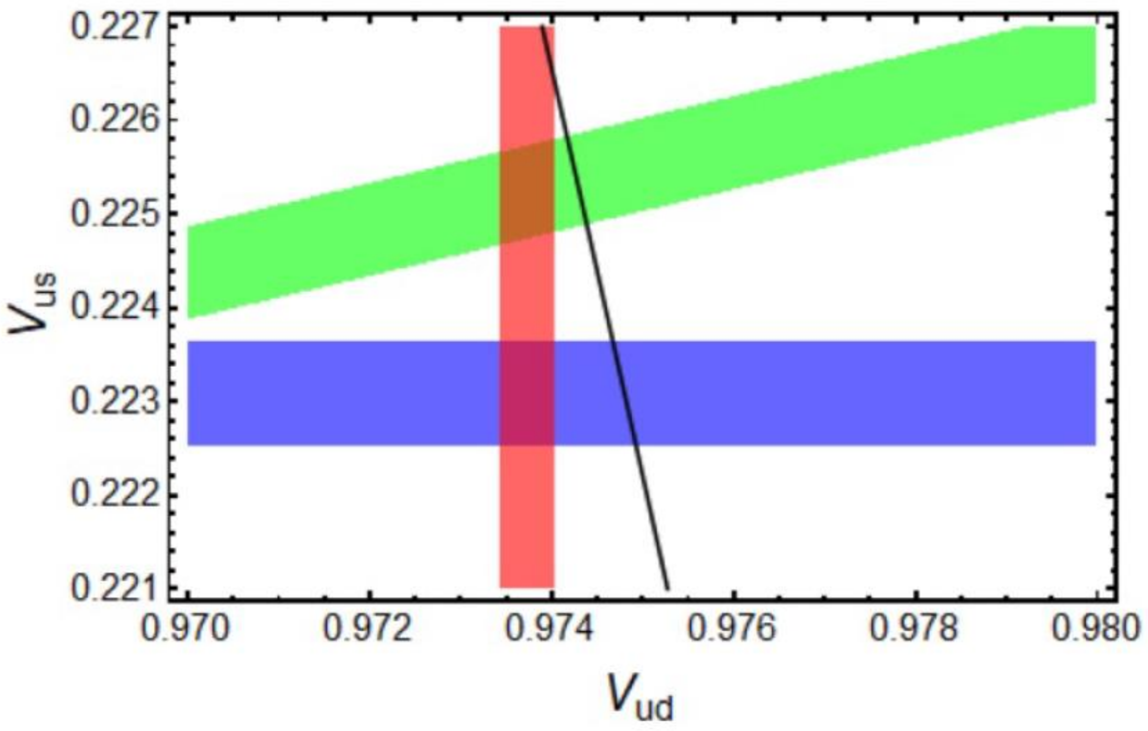
	$\lvert V_{us}\rvert$
$K_{\ell 3}$	0.22309(56) [23]
τ	0.2221(13) [24]
Hyperon	0.2250(27) [25]

	$\lvert V_{us}/V_{ud}\rvert$
$K_{\mu 2}/\pi_{\mu 2}$	0.23131(51) [23]
$K_{\ell 3}/\pi_{e3}$	0.22908(87) [23]

$$\lvert 0^+\rangle \to \lvert 0^+\rangle$$

$$n \to p e^- \bar{\nu}_e$$

$$\pi^+ \to \pi^0 e^+ \nu_e(\gamma)$$



V_{ud} element of CKM matrix

$$\mathcal{L}_{CC} = -\frac{g}{\sqrt{2}}(\bar{u}_L, \bar{c}_L, \bar{t}_L)\gamma^\mu W_\mu^+ V_{CKM} \begin{pmatrix} d_L \\ s_L \\ b_L \end{pmatrix} + h.c.$$

- Precise V_{ud} from superallowed Fermi transitions

$$|V_{ud}|^2 = \frac{\hbar^7}{G_F^2 m_e^5 c^4} \frac{\pi^3 \ln(2)}{\mathcal{F}t(1+\Delta_R^V)}$$

$G_F \equiv$ Fermi coupling constant
determined from muon β decay

  - hadronic matrix elements modified by nuclear environment
  - Fermi matrix element renormalized by isospin non–conserving forces

$$\mathcal{F}t = ft(1+\delta_R')\underline{(1-\delta_C+\delta_{NS})}$$

$$\mathcal{F}t = \frac{K}{G_V^2 |M_{F0}|^2 (1+\Delta_R^V)}$$

[2] Zyla et al. (2020)

Historical treatment

Last 30 years

- δ_{NS} from shell model and approximate single-nucleon currents
- δ_{C} from shell model with Woods-Saxon potential

Since 2018

- Data-driven dispersion integral approach for Δ_R^V **[3-4]**
- Reduced radiative correction uncertainty by factor of ~ 2
- Yields V_{ud} with $(2-3)\sigma$ deviation from unitarity

[3] Seng et al. (2018)
[4] Gorchtein et al. (2019)
[5] Hardy et al. (2020)

Historical treatment

Last 30 years

- δ_{NS} from shell model and approximate single-nucleon currents
- δ_{C}

Sinc

- Da
- Re
- Yields V_{ud} with $(2-3)\sigma$ deviation from unitarity

Evaluate corrections with *ab initio* NCSM

$^{10}C \rightarrow {}^{10}B$

[3] Seng et al. (2018)
[4] Gorchtein et al. (2019)
[5] Hardy et al. (2020)

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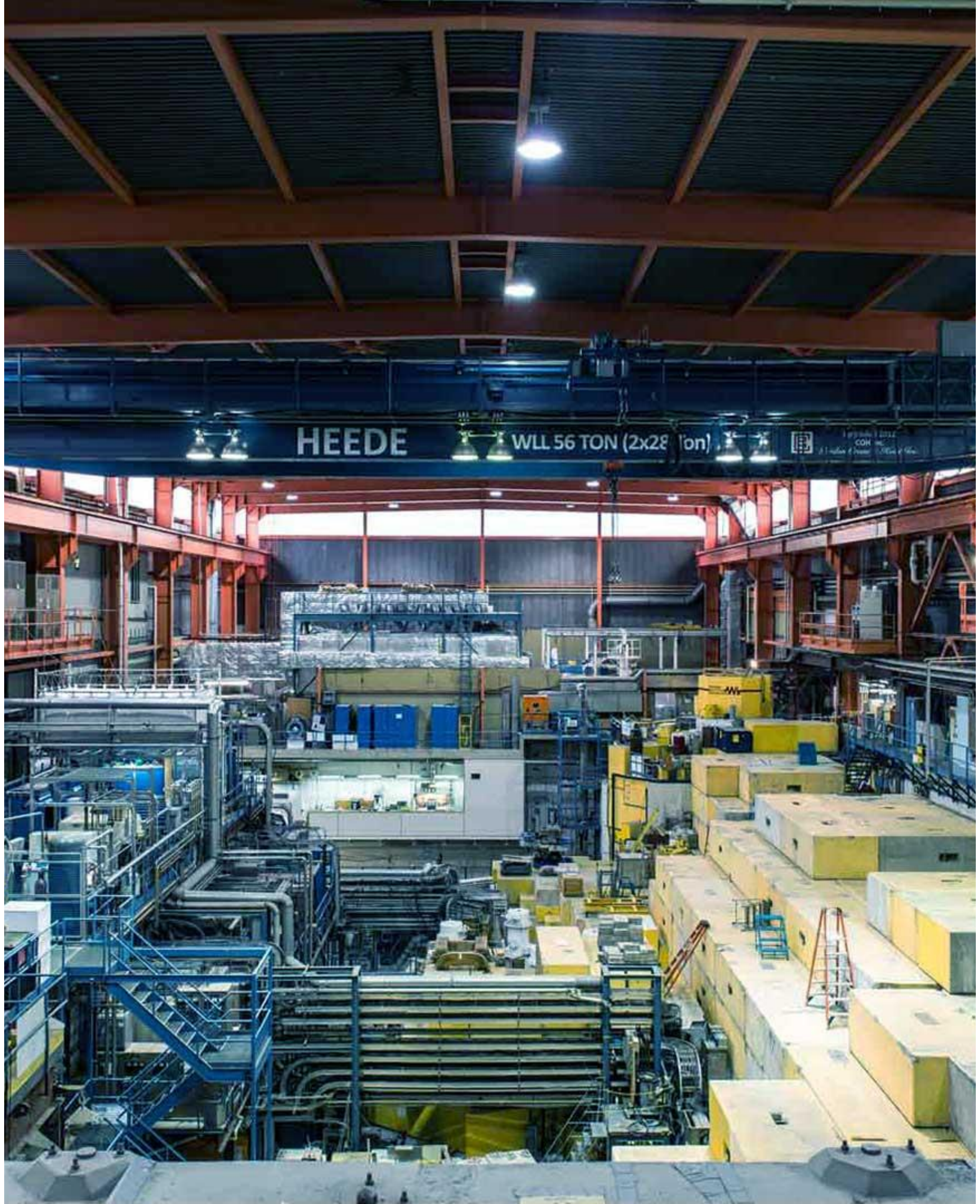
Electroweak radiative correction δ_{NS}



Discovery, accelerated

Δ_R^V and δ_{NS}

Leptonic current

NME of charged weak current

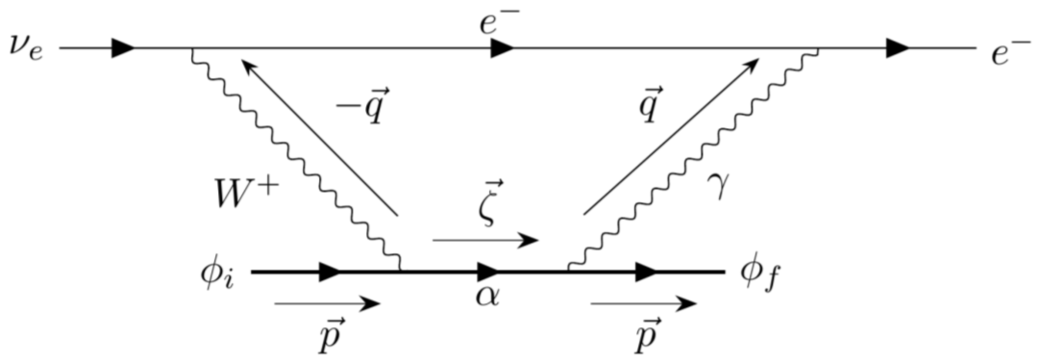
- Tree level beta decay amplitude

$$M_{tree} = -\frac{G_F}{\sqrt{2}} L_\lambda F^\lambda(p', p)$$

- Hadronic correction in forward scattering limit

$$\delta M = -i\sqrt{2} G_F e^2 L_\lambda \int \frac{d^4 q}{(2\pi)^4} \frac{M_W^2}{M_W^2 - q^2} \frac{\epsilon^{\mu\nu\alpha\lambda} q_\alpha}{[(p_e - q)^2 - m_e^2] q^2} \underline{T_{\mu\nu}(p', p, q)}$$

$$\boxed{\delta M = \Box_{\gamma W}(E_e) M_{tree}}$$



[6] Seng et al. (2023)

Δ_R^V and δ_{NS}

Leptonic current

NME of charged weak current

- Tree level beta decay amplitude $M_{tree} = -\frac{G_F}{\sqrt{2}} L_\lambda F^\lambda(p', p)$
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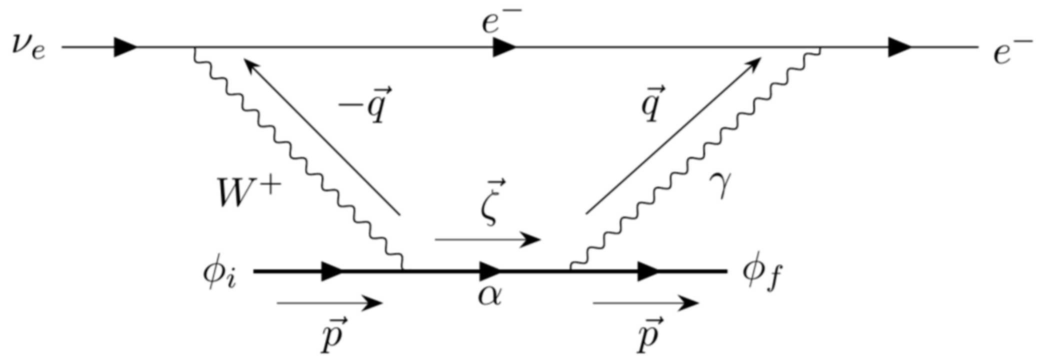
$$\delta M = \Box_{\gamma W}(E_e) M_{tree}$$



$$\boxed{\Box^b_{\gamma W}(E_e) = \frac{e^2}{M} \int \frac{d^4 q}{(2\pi)^4} \frac{M_W^2}{M_W^2 - q^2} \frac{1}{q^2 + i\epsilon} \frac{1}{(p_e - q)^2 + i\epsilon'} \frac{M\nu\left(\frac{p_e \cdot q}{p \cdot p_e}\right) - q^2}{\nu} \frac{T_3(\nu, |\vec{q}|)}{f_+(0)}}$$

[6] Seng et al. (2023)

Δ_R^V and δ_{NS}

Leptonic current

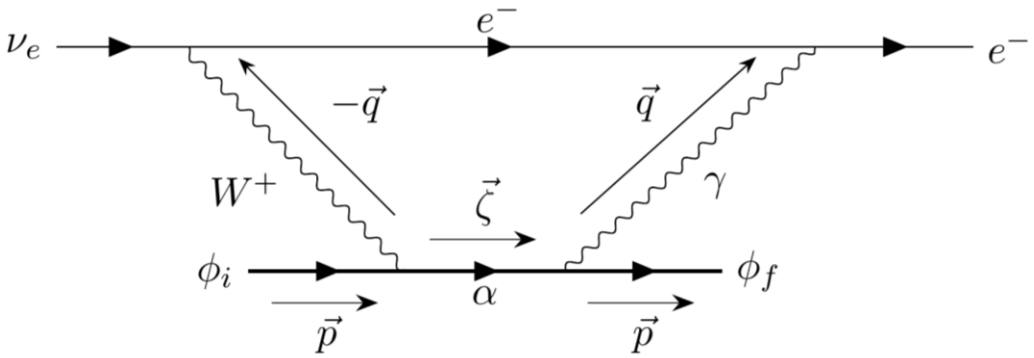
NME of charged weak current

- Tree level beta decay amplitude

$$M_{tree} = -\frac{G_F}{\sqrt{2}} L_\lambda F^\lambda(p', p)$$

- Hadronic correction in forward scattering limit

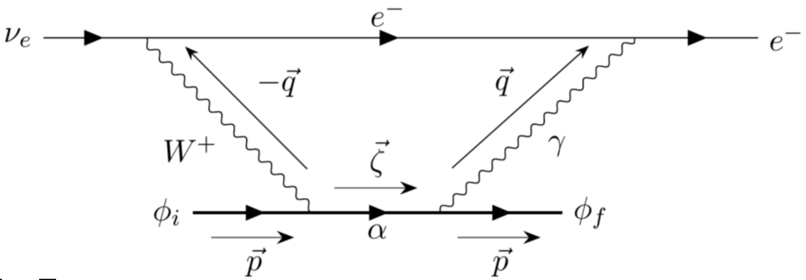
$$\delta_{NS} = 2[\Box_{\gamma W}^{\mathrm{b,nuc}} - \Box_{\gamma W}^{\mathrm{b,free\ n}}]$$



$$\boxed{T^{\mu\nu}(p,q) = \frac{1}{2}\int d^4x\ e^{iq\cdot x}\langle\phi_f(p)\big|T\big[J_{\mathrm{em}}^\mu(x)J_W^\nu(0)^\dagger\big]\big|\phi_i(p)\rangle}$$

[6] Seng et al. (2023)

Nonrelativistic Compton amplitude



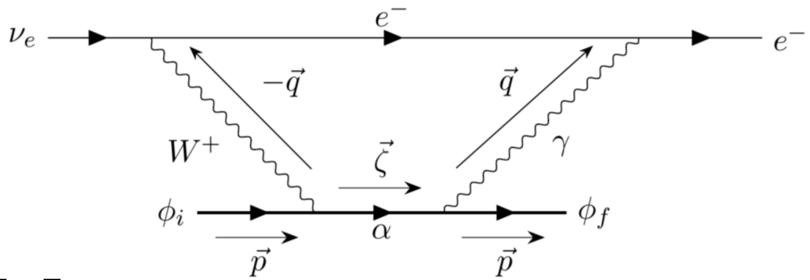
- **Goal**: Non-relativistic currents in momentum space **[7]**
- Rewrite currents with A-body propagators

$$J^\mu(t,\vec{x}) = e^{-iHt} J^\mu(0,\vec{x})\, e^{iHt} \longrightarrow \boxed{G(E) = \sum_n \frac{|n\rangle\langle n|}{E - E_n}}$$

$$\begin{aligned} T^{\mu\nu}(p,q) = &-\frac{i}{2}\int d^3x\, e^{-i\vec{q}\cdot\vec{x}} \langle \phi_f(p) \big| J^\mu_{em}(0,\vec{x})\, \underline{G(M_f+\nu+i\epsilon)}\, J^{\dagger\,\nu}_W(0,\vec{0}) \big| \phi_i(p)\rangle \\ &-\frac{i}{2}\int d^3x\, e^{-i\vec{q}\cdot\vec{x}} \langle \phi_f(p) \big| J^{\dagger\,\nu}_W(0,\vec{0})\, \underline{G(M_i-\nu+i\epsilon)}\, J^\mu_{em}(0,\vec{x}) \big| \phi_i(p)\rangle \end{aligned}$$

[7] Haxton et al. (2007)

Nonrelativistic Compton amplitude



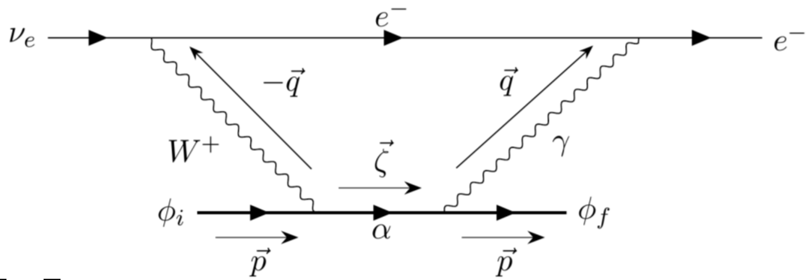
- **Goal**: Non-relativistic currents in momentum space **[7]**
- Rewrite currents with A-body propagators
- Fourier transform currents into momentum space

$$\boxed{J(\vec{r}) = \int \frac{d^3r}{(2\pi)^3}\; e^{i\vec{q}\cdot\vec{r}} J(\vec{q})}$$

$+$ Translation invariance

$$T^{\mu\nu}(p,q) = i\sqrt{M_i M_f}\,\langle \Phi_f | \underline{J^{\mu}_{em}(\vec{q})}\, G(M_f + \nu + i\epsilon)\, \underline{J^{\dagger\,\nu}_{W}(-\vec{q})} | \Phi_i \rangle$$
$$+ i\sqrt{M_i M_f}\,\langle \Phi_f | \underline{J^{\dagger\,\nu}_{W}(-\vec{q})}\, G(M_i - \nu + i\epsilon)\, \underline{J^{\mu}_{em}(\vec{q})} | \Phi_i \rangle$$

[7] Haxton et al. (2007)



Nonrelativistic Compton amplitude

- **Goal**: Non-relativistic currents in momentum space **[7]**
- Rewrite currents with A-body propagators
- Fourier transform currents into momentum space
- General multipole expansion of currents

$$M_{JM}(q) := \int d^3r\ \mathcal{M}_{JM}(q,\vec{r})\,\rho(\vec{r})$$

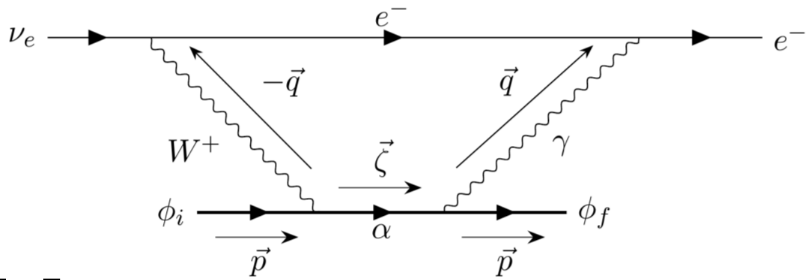
$$T^{\mathrm{el}}_{JM}(q) := \int d^3r\ \frac{1}{q}\left(\vec{\nabla}\times\vec{\mathcal{M}}^{M}_{JJ}(q,\vec{r})\right)\cdot\vec{J}(\vec{r})$$

$$L_{JM}(q) := \int d^3r\ \frac{i}{q}\left(\vec{\nabla}\mathcal{M}_{JM}(q,\vec{r})\right)\cdot\vec{J}(\vec{r})$$

$$T^{\mathrm{mag}}_{JM}(q) := \int d^3r\ \vec{\mathcal{M}}^{M}_{JJ}(q,\vec{r})\cdot\vec{J}(\vec{r})$$

[7] Haxton et al. (2007)

Nonrelativistic Compton amplitude



- **Goal**: Non-relativistic currents in momentum space **[7]**
- Rewrite currents with A-body propagators
- Fourier transform currents into momentum space
- General multipole expansion of currents

$$T_3(\nu,|\vec{q}\,|) = 4\pi i \frac{\nu}{|\vec{q}\,|}\sqrt{M_i M_f}\sum_{J=1}^{\infty}(2J+1)\,\langle\Psi_f|\Big\{T^{\mathrm{mag}}_{J0}\,G(\nu+M_f+i\epsilon)\,T^{5,\mathrm{el}}_{J0} + T^{\mathrm{el}}_{J0}\,G(\nu+M_f+i\epsilon)\,T^{5,\mathrm{mag}}_{J0}$$
$$+\,T^{5,\mathrm{mag}}_{J0}\,G(-\nu+M_i+i\epsilon)\,T^{\mathrm{el}}_{J0} + T^{5,\mathrm{el}}_{J0}\,G(-\nu+M_i+i\epsilon)T^{\mathrm{mag}}_{J0}\Big\}(|\vec{q}\,|)\,|\Psi_i\rangle$$

[7] Haxton et al. (2007)

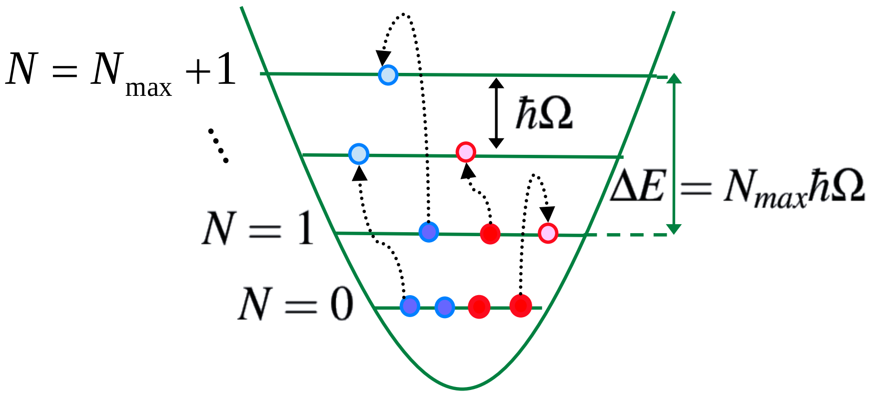
[8] Barrett et al. (2013)
[9] Weinberg (1991)
[10] Epelbaum (2009)
[11] Entem et al. (2017)
[12] Somà et al. (2020)

No-core shell model (NCSM)

- *Ab initio* approach to solving many-body Schrödinger equation **[8]**
- Sole input are nuclear interactions from chiral effective field theory
  - NN @ NN-N^{4}LO(500) **[11]**
  - 3N @ 3N(lnl)-N^{2}LO(650) **[12]**

$$H\left|\Psi_A^{J^\pi T}\right\rangle = E^{J^\pi T}\left|\Psi_A^{J^\pi T}\right\rangle$$

Anti-symmetrized products of many-body HO states



$$\left|\Psi_A^{J^\pi T}\right\rangle = \sum_{N=0}^{N_{max}} \sum_{\alpha} c_{N\alpha}^{J^\pi T}\left|\Phi_{N\alpha}^{J^\pi T}\right\rangle$$

$$\left|\Psi_A^{J^\pi T}\right\rangle_{\mathrm{SD}} = \sum_{N=0}^{N_{max}} \sum_{\alpha} [c^{(\mathrm{SD})}]_{N\alpha}^{J^\pi T}\left|\Phi_{N\alpha}^{J^\pi T}\right\rangle_{\mathrm{SD}} = \left|\Psi_A^{J^\pi T}\right\rangle \otimes \left|\Phi_{000}\right\rangle$$

[8] Barrett et al. (2013)
[9] Weinberg (1991)
[10] Epelbaum (2009)
[11] Entem et al. (2017)
[12] Somà et al. (2020)

No-core shell model (NCSM)

- *Ab initio* approach to solving many-body Schrödinger equation **[8]**
- Sole

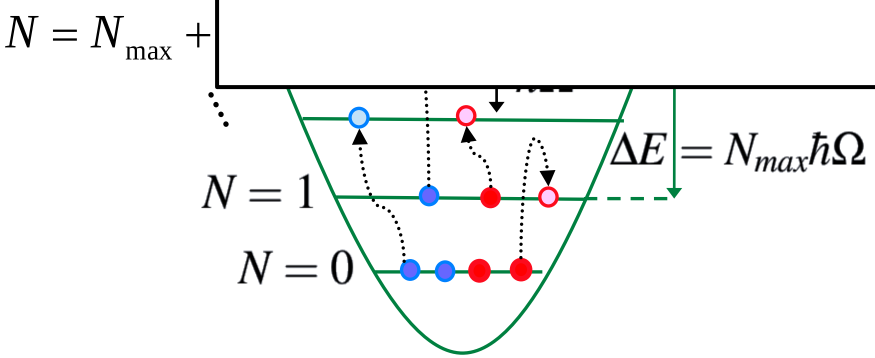
  – N

  – 3

Anti-
n

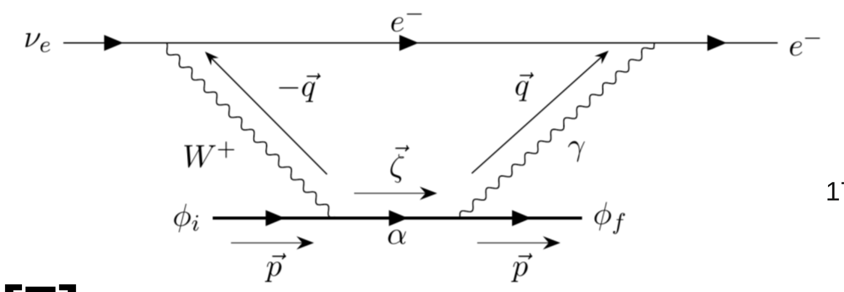
Accessible transitions

^{10}C → ^{10}B ^{14}O → ^{14}N ^{18}Ne → ^{18}F ^{22}Mg → ^{22}Na



$$\left|\Psi_A^{J^\pi T}\right\rangle_{\mathrm{SD}} = \sum_{N=0}^{N_{max}} \sum_{\alpha} [c^{(\mathrm{SD})}]_{N\alpha}^{J^\pi T} \left|\Phi_{N\alpha}^{J^\pi T}\right\rangle_{\mathrm{SD}} = \left|\Psi_A^{J^\pi T}\right\rangle \otimes \left|\Phi_{000}\right\rangle$$

Nonrelativistic Compton amplitude



- **Goal**: Non-relativistic currents in momentum space **[7]**
- Rewrite currents with A-body propagators
- Fourier transform currents into momentum space
- General multipole expansion of currents

Lanczos continued fraction method to compute nuclear Green's functions **[13-14]**

$$T_3(\nu, |\vec{q}\,|) = 4\pi i \frac{\nu}{|\vec{q}\,|} \sqrt{M_i M_f} \sum_{J=1}^{\infty} (2J+1) \langle \Psi_f | \Big\{ T_{J0}^{\mathrm{mag}} \boxed{G(\nu + M_f + i\epsilon)} T_{J0}^{5,\mathrm{el}} + T_{J0}^{\mathrm{el}} \boxed{G(\nu + M_f + i\epsilon)} T_{J0}^{5,\mathrm{mag}}$$

$$+ T_{J0}^{5,\mathrm{mag}} \boxed{G(-\nu + M_i + i\epsilon)} T_{J0}^{\mathrm{el}} + T_{J0}^{5,\mathrm{el}} \boxed{G(-\nu + M_i + i\epsilon)} T_{J0}^{\mathrm{mag}} \Big\} (|\vec{q}\,|) | \Psi_i \rangle$$

[7] Haxton et al. (2007)
[13] Hao et al. (2020)
[14] Froese et al. (2021)

Lanczos continued fraction method

- Reformulate as inhomogeneous Schrödinger equation

$$(H - E\mathbb{1})|\Phi\rangle = \hat{O}|\Psi\rangle$$

$$\boxed{\begin{aligned} H\mathbf{v}_1 &= \alpha_1\mathbf{v}_1 + \beta_1\mathbf{v}_2 \\ H\mathbf{v}_2 &= \beta_1\mathbf{v}_1 + \alpha_2\mathbf{v}_2 + \beta_2\mathbf{v}_3 \\ H\mathbf{v}_3 &= \beta_2\mathbf{v}_2 + \alpha_3\mathbf{v}_3 + \beta_3\mathbf{v}_4 \\ H\mathbf{v}_4 &= \beta_3\mathbf{v}_3 + \alpha_4\mathbf{v}_4 + \beta_4\mathbf{v}_5 \end{aligned}}$$

Select pivot as source term

$$|v_1\rangle = \frac{\hat{O}|\Psi\rangle}{\sqrt{\langle\Psi|\hat{O}^\dagger\hat{O}|\Psi\rangle}}$$

[15] Haydock (1974)
[16] Marchisio et al. (2003)

Lanczos continued fraction method

- Reformulate as inhomogeneous Schrödinger equation

$$(H - E\mathbb{1})|\Phi\rangle = \hat{O}|\Psi\rangle$$

$$\boxed{\begin{aligned}
H\mathbf{v}_1 &= \alpha_1\mathbf{v}_1 + \beta_1\mathbf{v}_2 \\
H\mathbf{v}_2 &= \beta_1\mathbf{v}_1 + \alpha_2\mathbf{v}_2 + \beta_2\mathbf{v}_3 \\
H\mathbf{v}_3 &= \qquad\qquad \beta_2\mathbf{v}_2 + \alpha_3\mathbf{v}_3 + \beta_3\mathbf{v}_4 \\
H\mathbf{v}_4 &= \qquad\qquad\qquad\qquad \beta_3\mathbf{v}_3 + \alpha_4\mathbf{v}_4 + \beta_4\mathbf{v}_5
\end{aligned}}$$

- Resolvent reconstructed as linear combination of Lanczos vectors
- Avoids brute force calculation of intermediate states

[15] Haydock (1974)
[16] Marchisio et al. (2003)

Symmetry tests of T_3 amplitude

- Time reversal symmetry with exact isospin gives NME constraint
- Previously **assumed** nuclear T_3 matched nucleonic system

Nuclei	Nucleons	Pions
$\boxed{\begin{array}{c} T_3^{(0)}(-\nu,Q^2) = -T_3^{(0)}(\nu,Q^2) \\ T_3^{(1)}(-\nu,Q^2) = \cdots \end{array}}$	$\begin{array}{c} T_3^{(0)}(-\nu,Q^2) = -T_3^{(0)}(\nu,Q^2) \\ T_3^{(1)}(-\nu,Q^2) = T_3^{(1)}(\nu,Q^2) \end{array}$	$\begin{array}{c} T_3^{(0)}(-\nu,Q^2) = -T_3^{(0)}(\nu,Q^2) \\ T_3^{(1)}(\nu,Q^2) = 0 \end{array}$

$$\left\langle {}^{10}\mathrm{B}\middle| T_{J0}^{\mathrm{mag},(0)}(q)\, G(M+i\epsilon)\, T_{J0}^{5,\mathrm{el}}(q) \middle| {}^{10}\mathrm{C}\right\rangle = \left\langle {}^{10}\mathrm{C}\middle| T_{J0}^{\mathrm{mag},(0)}(q)\, G(M+i\epsilon)\, \tilde{T}_{J0}^{5,\mathrm{el}}(q) \middle| {}^{10}\mathrm{B}\right\rangle$$

$$\left\langle {}^{10}\mathrm{B}\middle| T_{J0}^{\mathrm{el},(0)}(q)\, G(M+i\epsilon)\, T_{J0}^{5,\mathrm{mag}}(q) \middle| {}^{10}\mathrm{C}\right\rangle = \left\langle {}^{10}\mathrm{C}\middle| T_{J0}^{\mathrm{el},(0)}(q)\, G(M+i\epsilon)\, \tilde{T}_{J0}^{5,\mathrm{mag}}(q) \middle| {}^{10}\mathrm{B}\right\rangle$$

[6] Seng et al. (2023)

About one year ago…

Poles of T_3

K.C. Greene

Poles

$$G(\nu + M_f + i\epsilon) = \sum_n \frac{\boxed{|n\rangle\langle n|}}{[\nu + M_f + i\epsilon] - \boxed{M_n}} \quad : \quad P_- = \{M_n - M_f - i\epsilon\}$$

$$G(-\nu + M_i + i\epsilon) = \sum_n \frac{\boxed{|n\rangle\langle n|}}{[-\nu + M_i + i\epsilon] - \boxed{M_n}} \quad : \quad P_+ = \{M_i - M_n + i\epsilon\}$$

- Numerical integration prone to instability
- Natural solution is Wick rotation

$$\nu = i\nu_E$$

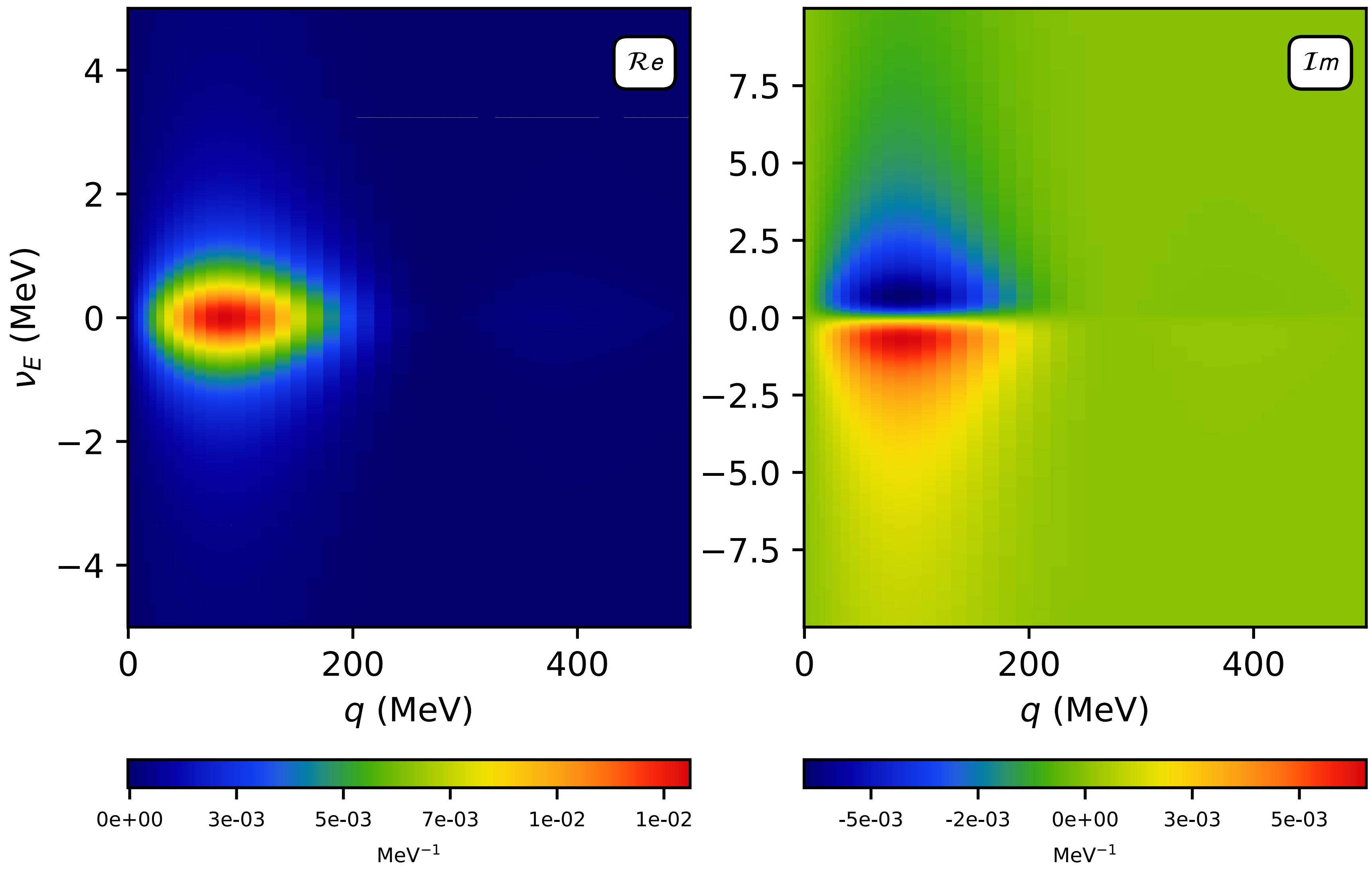
Initially thought NMEs and pole locations could not be extracted…

Wick rotated T_3

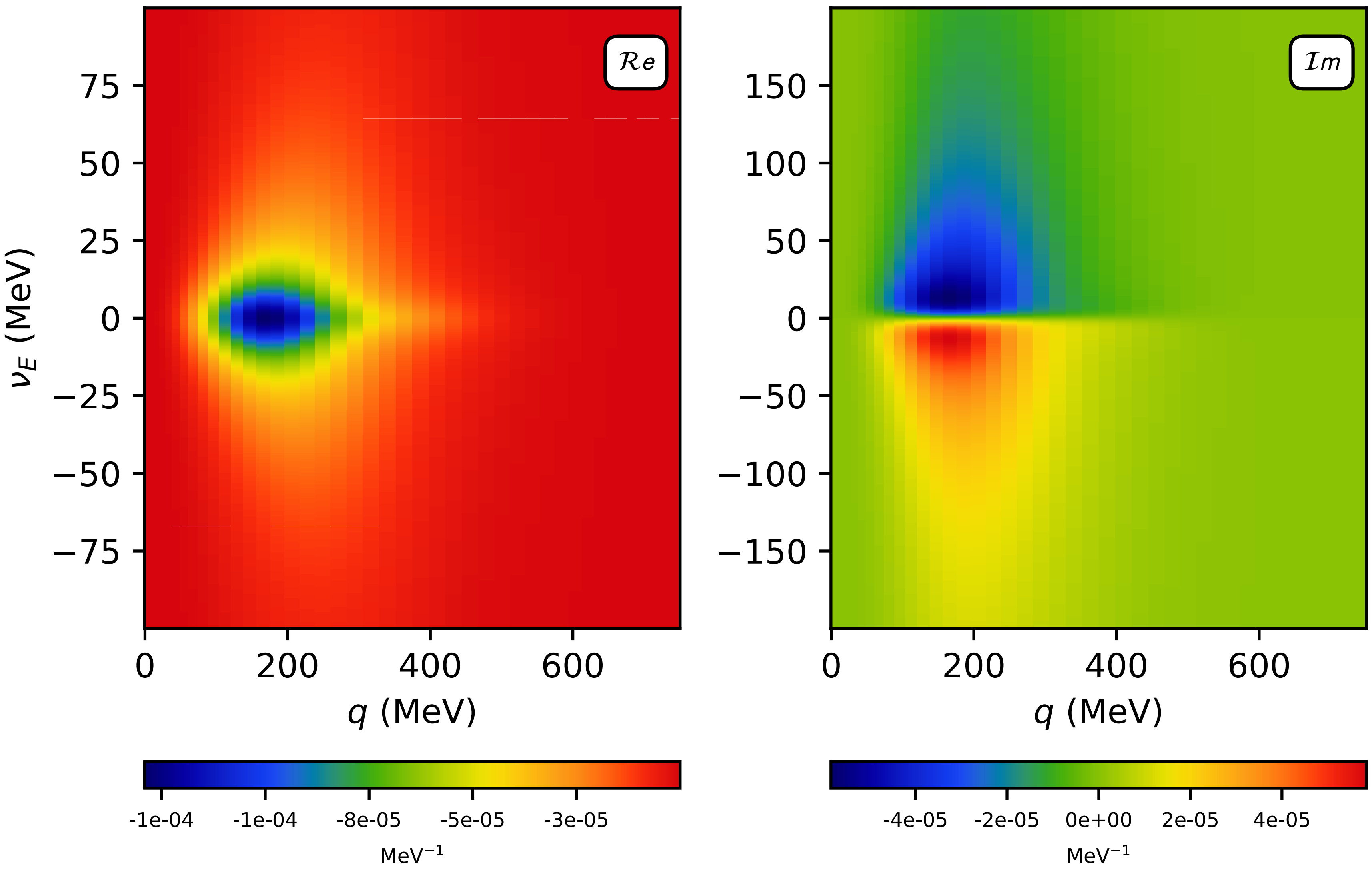
$$T_3(i\nu_E, |\vec{q}\,|) = -4\pi \frac{\nu_E}{|\vec{q}\,|} \sqrt{M_i M_f} \sum_{J=1}^{\infty} (2J+1) \left\langle \Psi_f \right| \left\{ T_{J0}^{\mathrm{mag}} \, \underline{G(M_f + i\nu_E)} \, T_{J0}^{5,\mathrm{el}} + T_{J0}^{\mathrm{el}} \, \underline{G(M_f + i\nu_E)} \, T_{J0}^{5,\mathrm{mag}} \right.$$

$$\left. + T_{J0}^{5,\mathrm{mag}} \, \underline{G(M_i - i\nu_E)} \, T_{J0}^{\mathrm{el}} + T_{J0}^{5,\mathrm{el}} \, \underline{G(M_i - i\nu_E)} T_{J0}^{\mathrm{mag}} \right\} (|\vec{q}\,|) \left| \Psi_i \right\rangle$$

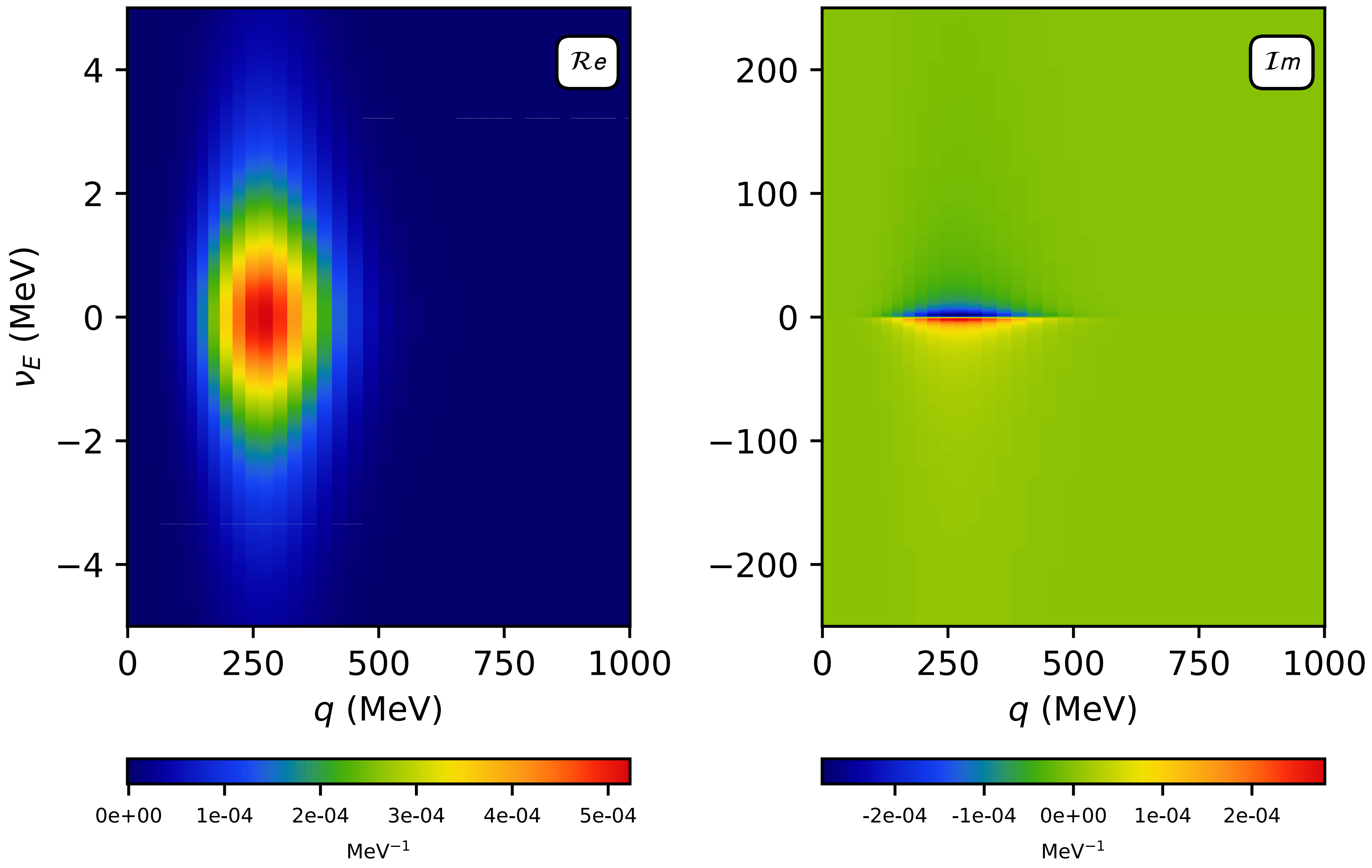
$$\left\langle {}^{10}\mathrm{B} \right| T^{\mathrm{mag}}_{J=1}(|\vec{q}\,|)\, G(M_f + i\nu_E)\, T^{5,\mathrm{el}}_{J=1}(|\vec{q}\,|) \left| {}^{10}\mathrm{C} \right\rangle$$



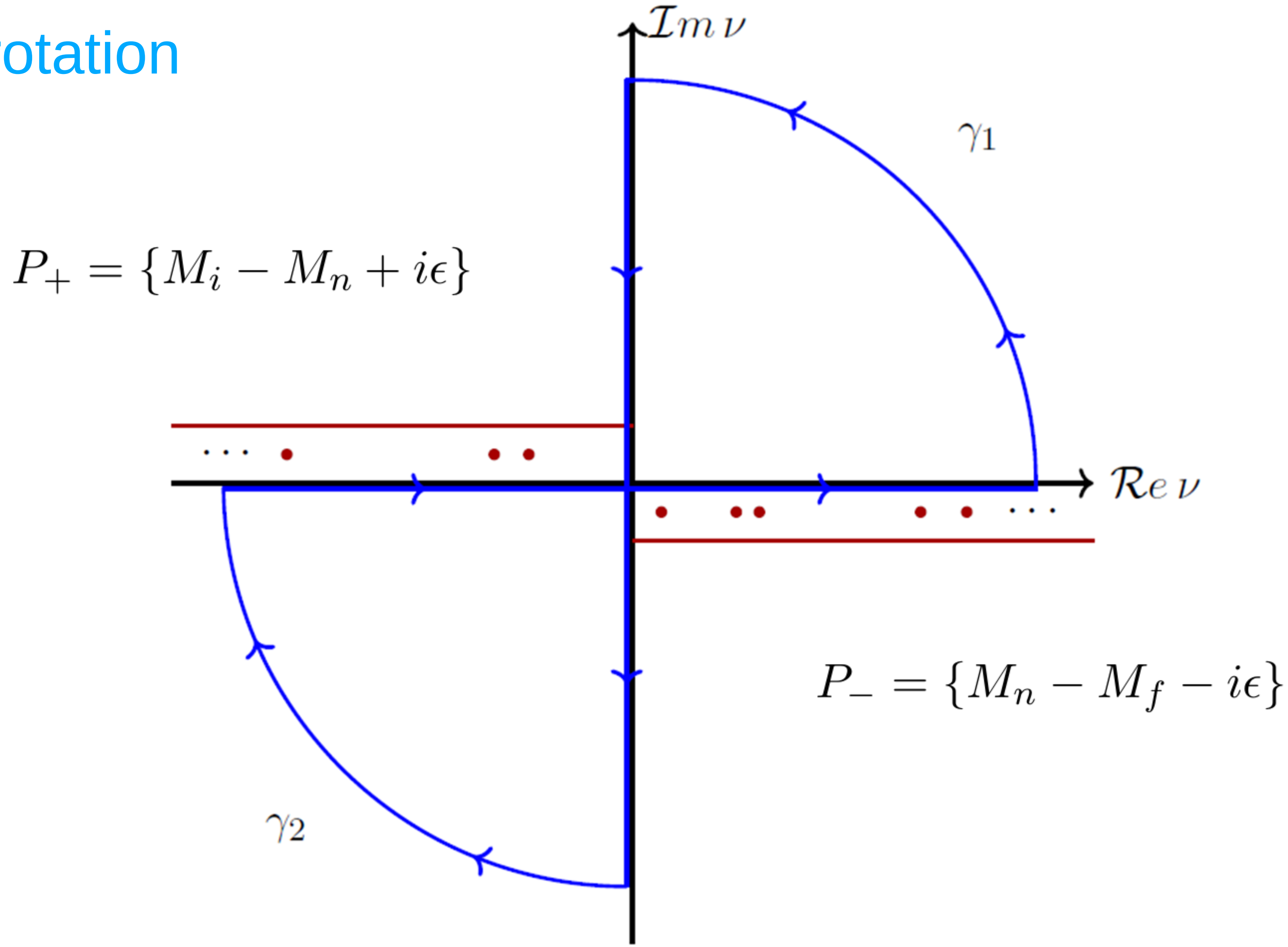
$$\left\langle {}^{10}\mathrm{B} \right| T^{\mathrm{mag}}_{J=2}(|\vec{q}\,|)\, G(M_f + i\nu_E)\, T^{5,\mathrm{el}}_{J=2}(|\vec{q}\,|) \left| {}^{10}\mathrm{C} \right\rangle$$



$$\langle {}^{10}\mathrm{B} |\, T^{\mathrm{mag}}_{J=3}(|\vec{q}\,|)\, G(M_f + i\nu_E)\, T^{5,\mathrm{el}}_{J=3}(|\vec{q}\,|)\, | {}^{10}\mathrm{C} \rangle$$

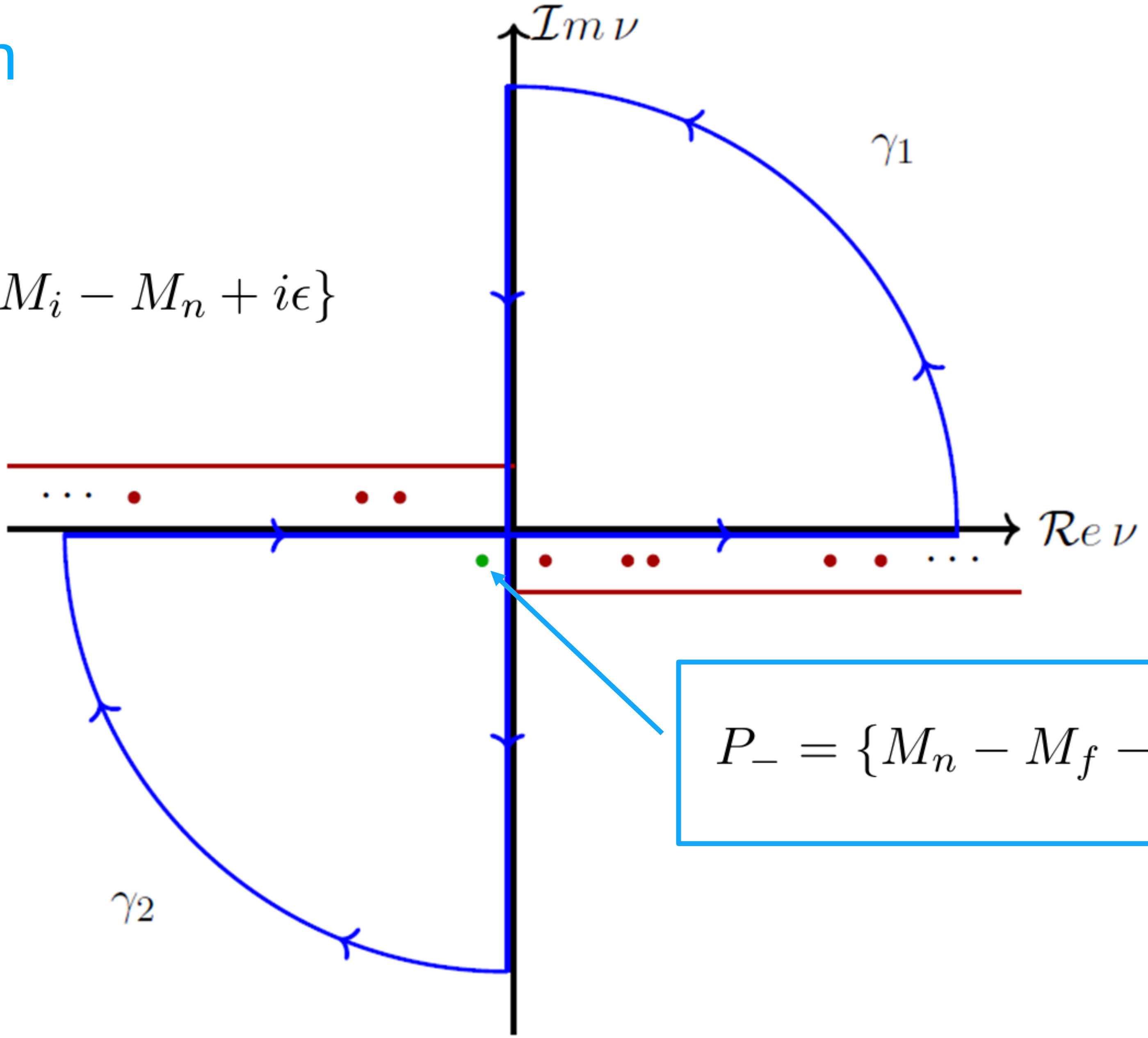


Wick rotation



Wick rotation

$$P_+ = \{M_i - M_n + i\epsilon\}$$



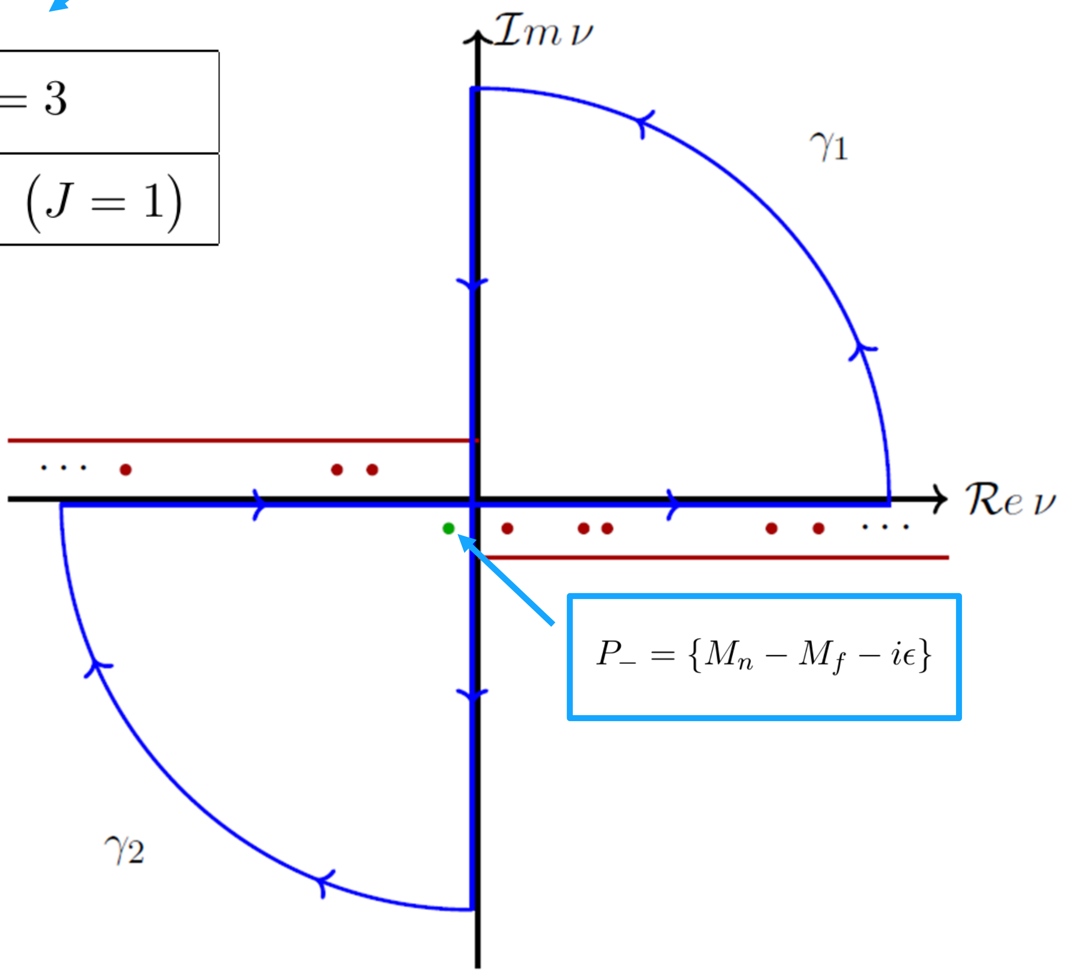
$$P_- = \{M_n - M_f - i\epsilon\}$$

Residues for $^{10}C \to {}^{10}B$ in NCSM

Second 1^+ below 0^+ sensitive to interaction and N_{max}

Poles	$n=1$	$n=2$	$n=3$
P_- [MeV]	$-1.6572\ (J=3)$	$-0.6974\ (J=1)$	$-0.1861\ (J=1)$

Table 1: Pole locations along ν axis corresponding to n–th excited state in T_3 for $^{10}\mathrm{C} \to {}^{10}\mathrm{B}$ transition at $N_{max} = 5$.



- Ground state 3^+ and low-lying 1^+ incur residues after Wick rotation
- Remaining pole in residue terms must also be treated

Electron energy expansion

$$\Box^b_{\gamma W}(E_e) = \underbrace{(\Box^b_{\gamma W})_{\mathrm{Wick}}(E_e) + (\Box^b_{\gamma W})_{\mathrm{Res},e}(E_e)} + \underline{(\Box^b_{\gamma W})_{\mathrm{Res},T_3}(E_e)}$$

- Wick rotated contour integral regular at $E_e = 0$
- Electron propagator residue regular at $E_e = 0$
- T_3 residue contribution **singular**

Expand in electron energy

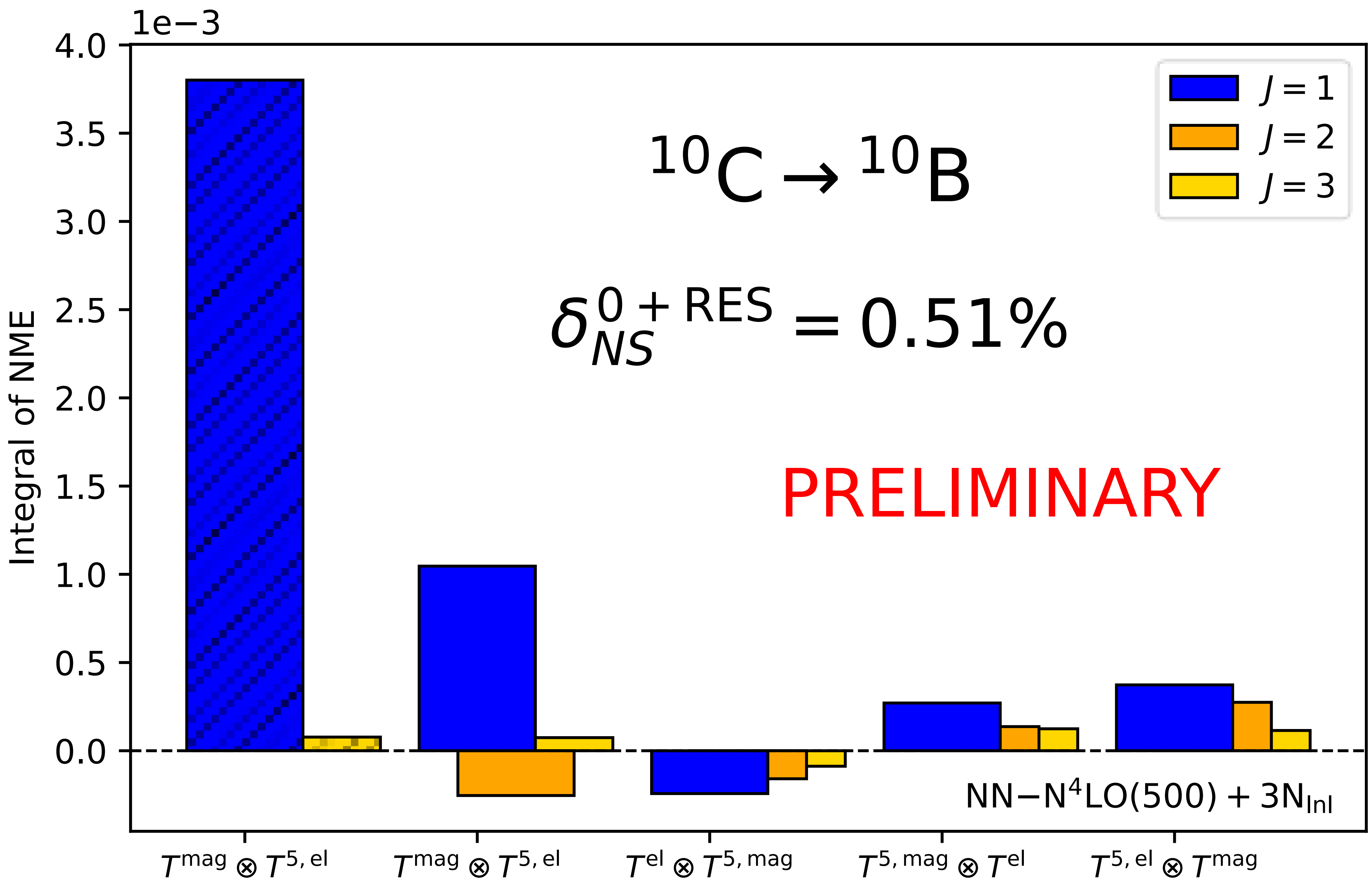
$$\boxed{\Box^b_{\gamma W}(E_e) = \boxminus_0 + E_e\,\boxminus_1 + (\Box^b_{\gamma W})_{\mathrm{Res},T_3}(E_e) + \mathcal{O}(E_e^2)}$$

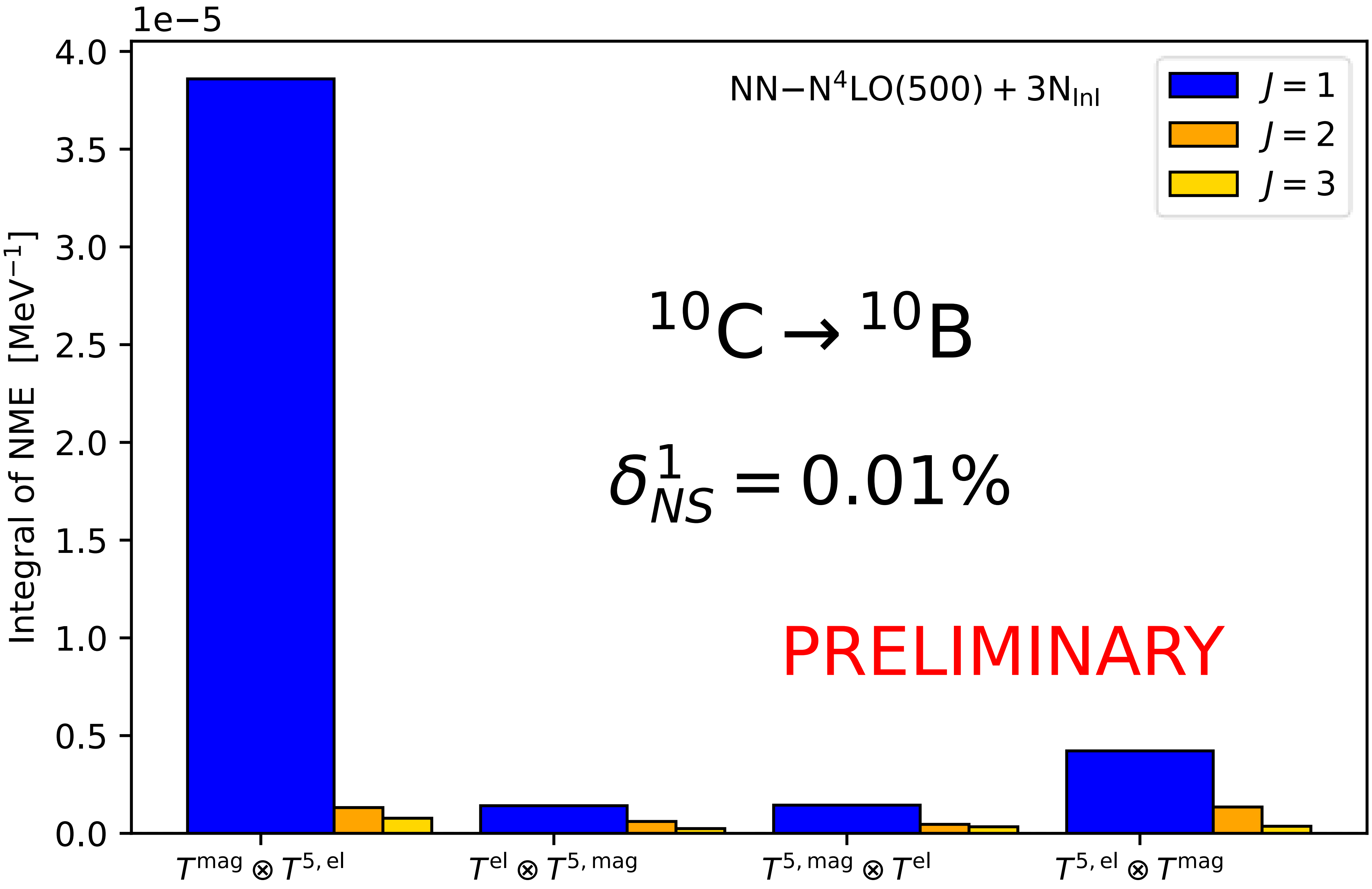
Comment on γW-box diagram subtraction for δ_{NS}

- **No resolution** for nuclear γW-box above pion threshold
- Compensate asymptotics with contributions from free nucleon box
- δ_{NS} extracted with only free nucleon Born contribution

$$\delta_{NS}^{0+\mathrm{RES}} = 2\left[\boxminus_0 - \Box_{\gamma W}^{\mathrm{n,\ Born}}\right] + 2\frac{\int_{m_e}^{E_m} dE_e\ |\vec{p}_e| E_e (E_e - E_m)^2 F(Z_f, E_e) \left(\Box_{\gamma W}^{b}\right)_{\mathrm{Res},T_3}(E_e)}{\int_{m_e}^{E_m} dE_e\ |\vec{p}_e| E_e (E_e - E_m)^2 F(Z_f, E_e)}$$

$$\delta_{NS}^{1} = 2\boxminus_1 \frac{\int_{m_e}^{E_m} dE_e\ |\vec{p}_e| E_e^2 (E_e - E_m)^2 F(Z_f, E_e)}{\int_{m_e}^{E_m} dE_e\ |\vec{p}_e| E_e (E_e - E_m)^2 F(Z_f, E_e)}$$


1e−3
4.0
3.5
3.0
2.5
2.0
1.5
1.0
0.5
0.0
Integral of NME
$J = 1$
$J = 2$
$J = 3$
$^{10}C \rightarrow {}^{10}B$
$\delta_{NS}^{0+\,RES} = 0.51\%$
PRELIMINARY
$NN{-}N^4LO(500) + 3N_{lnl}$
$T^{mag} \otimes T^{5,el}$
$T^{mag} \otimes T^{5,el}$
$T^{el} \otimes T^{5,mag}$
$T^{5,mag} \otimes T^{el}$
$T^{5,el} \otimes T^{mag}$

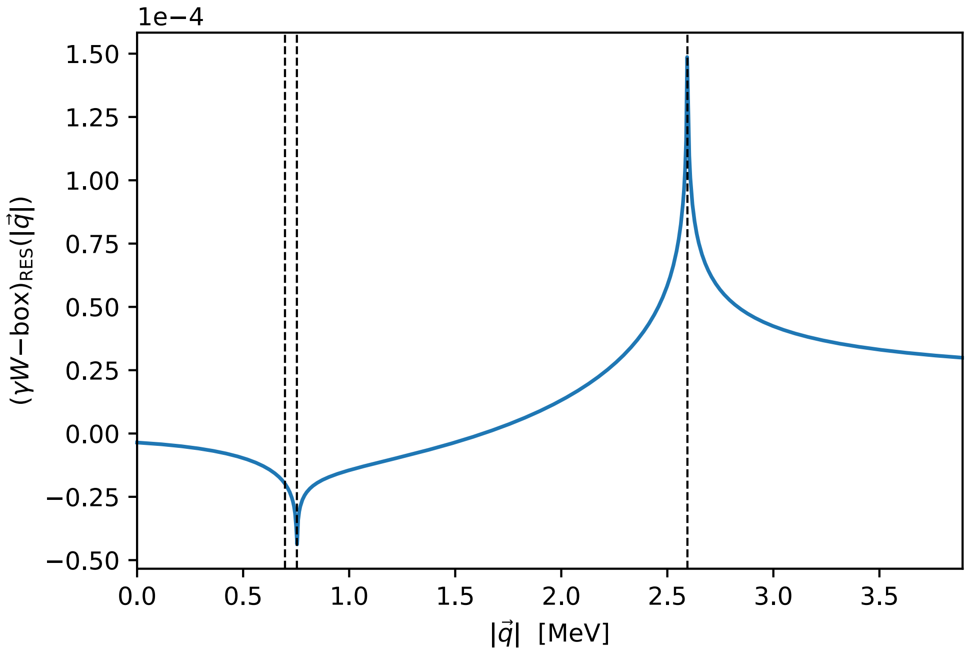

1e−5
4.0
3.5
3.0
2.5
2.0
1.5
1.0
0.5
0.0
Integral of NME [MeV^{-1}]
NN−N^{4}LO(500) + 3N$_{\mathrm{lnl}}$
$J = 1$
$J = 2$
$J = 3$
$^{10}\mathrm{C} \rightarrow {}^{10}\mathrm{B}$
$\delta^1_{NS} = 0.01\%$
PRELIMINARY
$T^{\mathrm{mag}} \otimes T^{5,\mathrm{el}}$
$T^{\mathrm{el}} \otimes T^{5,\mathrm{mag}}$
$T^{5,\mathrm{mag}} \otimes T^{\mathrm{el}}$
$T^{5,\mathrm{el}} \otimes T^{\mathrm{mag}}$

T_3 residue contribution

$$\left(\Box^b_{\gamma W}\right)_{\mathrm{Res},T_3} = \frac{e^2}{M}\mathcal{R}e\int_0^\infty \frac{d|\vec{q}\,|}{(2\pi)^2}|\vec{q}\,|^2 \sum_k \frac{M_W^2}{M_W^2 - q_k^2}\frac{\mathcal{A}(E_e,\nu_k,|\vec{q}\,|)}{q_k^2}\frac{\left[i\mathrm{Res}\,T_3(\nu_k,|\vec{q}\,|)\right]}{f_+(0)}$$

$$\mathrm{Res}\,T_3(\nu_k,|\vec{q}\,|) = \lim_{\nu\to\nu_k} T_3(\nu,|\vec{q}\,|)\,(\nu-\nu_k)$$

- NME residues are transition matrix elements to low-lying eigenstates
- Residue integral contains additional pole in photon propagator
- Numerical techniques for safe integration

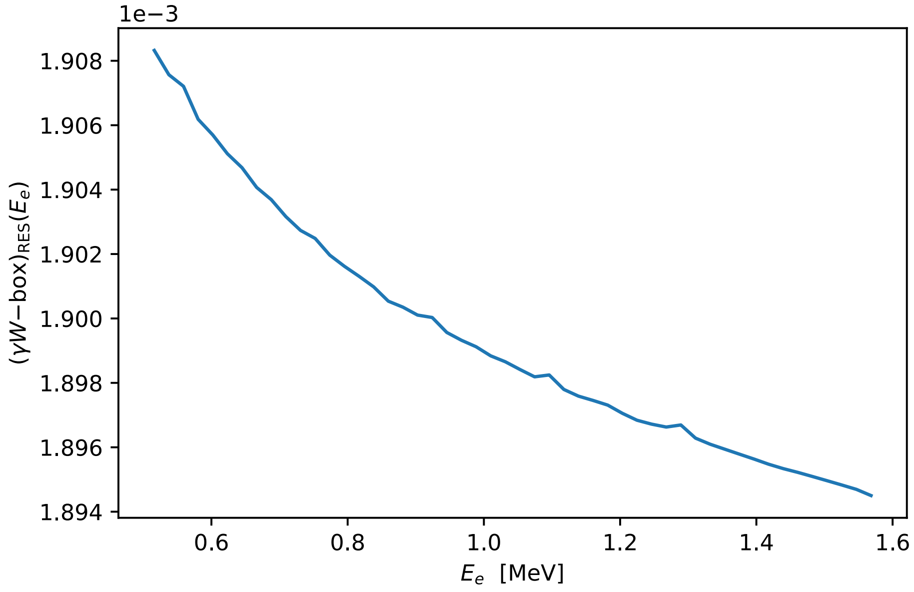


T_3 residue contribution

$$\left(\Box^b_{\gamma W}\right)_{\mathrm{Res},T_3} = \frac{e^2}{M}\mathcal{R}e\int_0^\infty \frac{d|\vec{q}\,|}{(2\pi)^2}|\vec{q}\,|^2 \sum_k \frac{M_W^2}{M_W^2 - q_k^2}\,\frac{\mathcal{A}(E_e,\nu_k,|\vec{q}\,|)}{q_k^2}\,\frac{\left[i\mathrm{Res}\,T_3(\nu_k,|\vec{q}\,|)\right]}{f_+(0)}$$

$$\mathrm{Res}\,T_3(\nu_k,|\vec{q}\,|) = \lim_{\nu\to\nu_k} T_3(\nu,|\vec{q}\,|)\,(\nu-\nu_k)$$

- NME residues are transition matrix elements to low-lying eigenstates
- Residue integral contains additional pole in photon propagator
- Numerical techniques for safe integration



Benchmarking δ_{NS} results

- Structure function F_3 instead of Compton amplitude T_3
- Analytic results for integral over boson energy ν

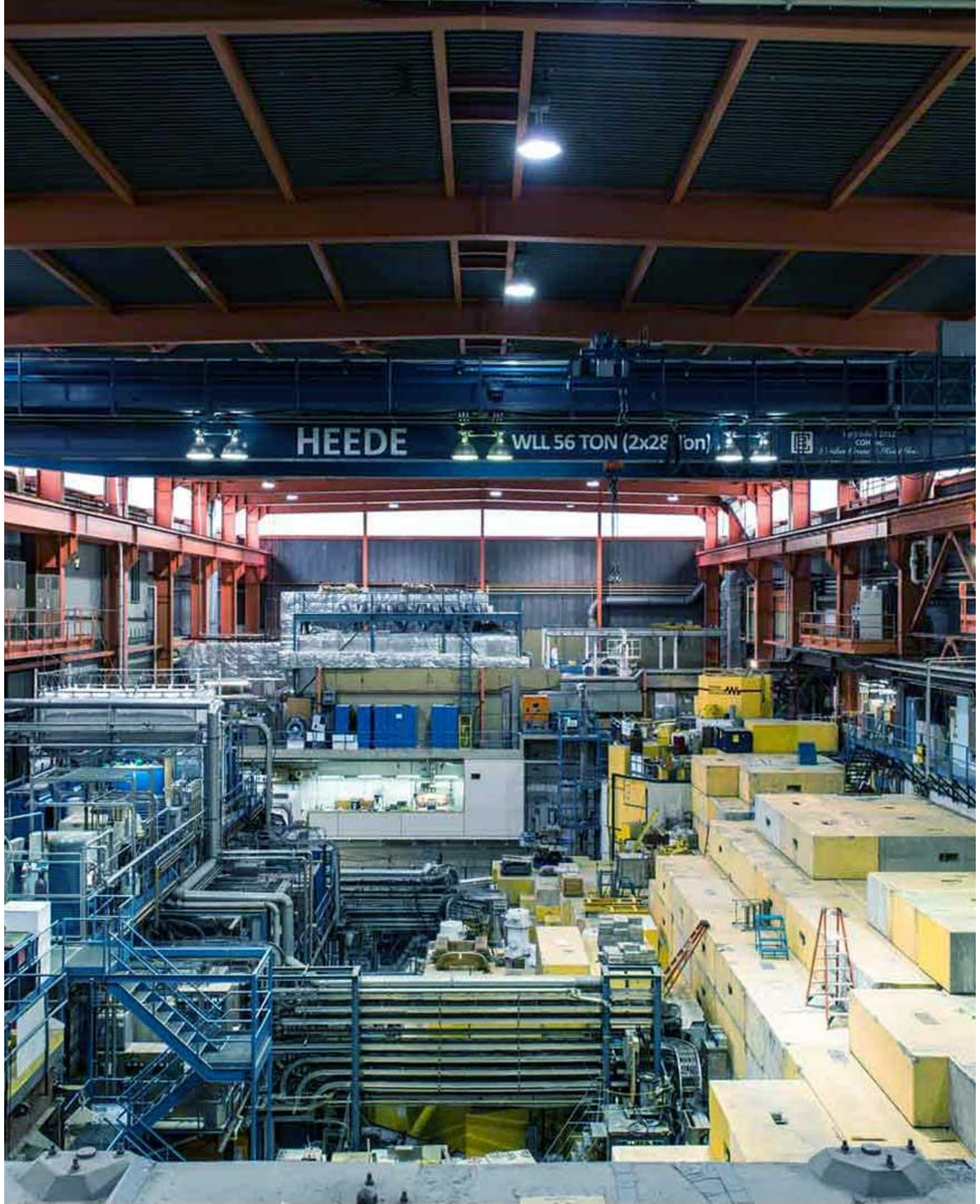
$$\boxminus_0 = -2e^2 \int \frac{d^3q}{(2\pi)^3} \sum_k \frac{M_W^2}{M_W^2 - q_k^2} \frac{|\vec{q}|}{q_k^4} \frac{\mathrm{Res}\, T_3(\nu_k, |\vec{q}|)}{f_+(0)}$$

$\mathrm{Res}\, T_3(\nu_k, |\vec{q}|)$

i. Need all residue positions in nuclear spectra

ii. Need transition matrix elements with all excited states in spectra

Isospin symmetry breaking correction δ_C



Discovery, accelerated

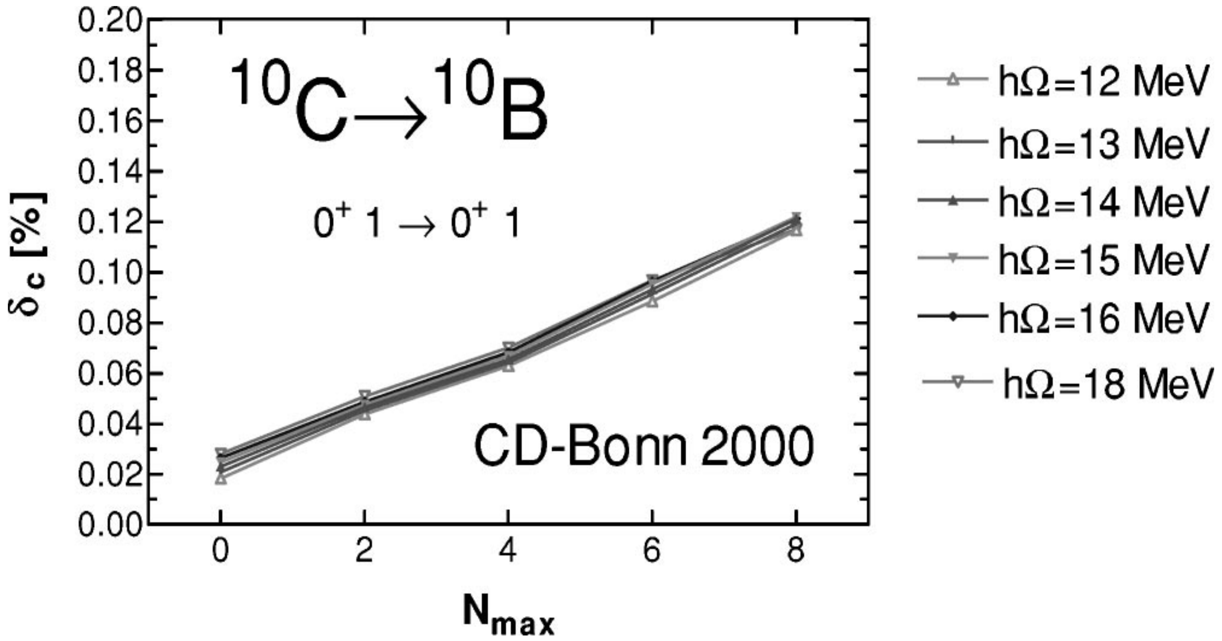
The pathway to δ_C

- δ_C in *ab initio* NCSM over 20 years ago

PHYSICAL REVIEW C **66**, 024314 (2002)

***Ab initio* shell model for $A=10$ nuclei**

E. Caurier,[1] P. Navrátil,[2] W. E. Ormand,[2] and J. P. Vary[3]
[1]*Institut de Recherches Subatomiques (IN2P3-CNRS-Université Louis Pasteur), Batiment 27/1, 67037 Strasbourg Cedex 2, France*
[2]*Lawrence Livermore National Laboratory, L-414, P.O. Box 808, Livermore, California 94551*
[3]*Department of Physics and Astronomy, Iowa State University, Ames, Iowa 50011*
(Received 10 May 2002; published 13 August 2002)



HO expansion incompatible with reaction theory

i. imprecise asymptotics
ii. missing correlations in excited states
iii. description of scattering not feasible

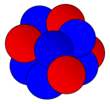
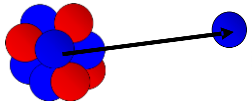
Combine NCSM with resonating group method (RGM) **[17]**

[17] Navrátil et al. (2009)

No–core shell model with continuum (NCSMC)

- Generalized basis with NCSM states and microscopic cluster states

$$\left|\Psi_A^{J^\pi T}\right\rangle = \sum_\alpha c_\alpha \left|\psi_A^{J^\pi T};\alpha\right\rangle + \sum_\nu \int d\vec{r}\,\gamma_\nu(\vec{r})\,\hat{\mathcal{A}}_\nu \left|\Phi_{\nu r}^{J^\pi T}\right\rangle$$

$\Big|$  $,\alpha\Big\rangle_{\mathrm{NCSM}} \quad + \quad \Big[\Big|$  $,\nu\Big\rangle^{(s)} Y_l(\hat{r}_{12})\Big]^{(J^\pi)}$

Static solutions to Schrödinger equation

RGM cluster states describing long-range relative motion of projectile and target

[18] Baroni et al. (2013)

δ_C in NCSMC

- Compute Fermi matrix element in NCSMC **[19]**

$$M_F = \left\langle \Psi^{J^\pi T_f M_{T_f}} \middle| T_+ \middle| \Psi^{J^\pi T_i M_{T_i}} \right\rangle \longrightarrow |M_F|^2 = |M_{F0}|^2(1-\delta_C)$$

- Total isospin operator $T_+ = T_+^{(1)} + T_+^{(2)}$ for partitioned system

$$M_F \sim \boxed{\langle A\lambda_f J_f T_f M_{T_f}|T_+|A\lambda J_i T_i M_{T_i}\rangle} + \boxed{\langle A\lambda J_f T_f M_{T_f}|T_+\mathcal{A}_{\nu i}|\Phi_{\nu r}^{J_i T_i M_{T_i}}\rangle}$$
$$+ \boxed{\langle \Phi_{\nu r}^{J_f T_f M_{T_f}}|\mathcal{A}_{\nu f} T_+|A\lambda_i J_i T_i M_{T_i}\rangle} + \boxed{\langle \Phi_{\nu r}^{J_f T_f M_{T_f}}|\mathcal{A}_{\nu f} T_+ \mathcal{A}_{\nu i}|\Phi_{\nu r}^{J_i T_i M_{T_i}}\rangle}$$

NCSM matrix element

NCSM-Cluster matrix elements

Continuum (cluster) matrix element

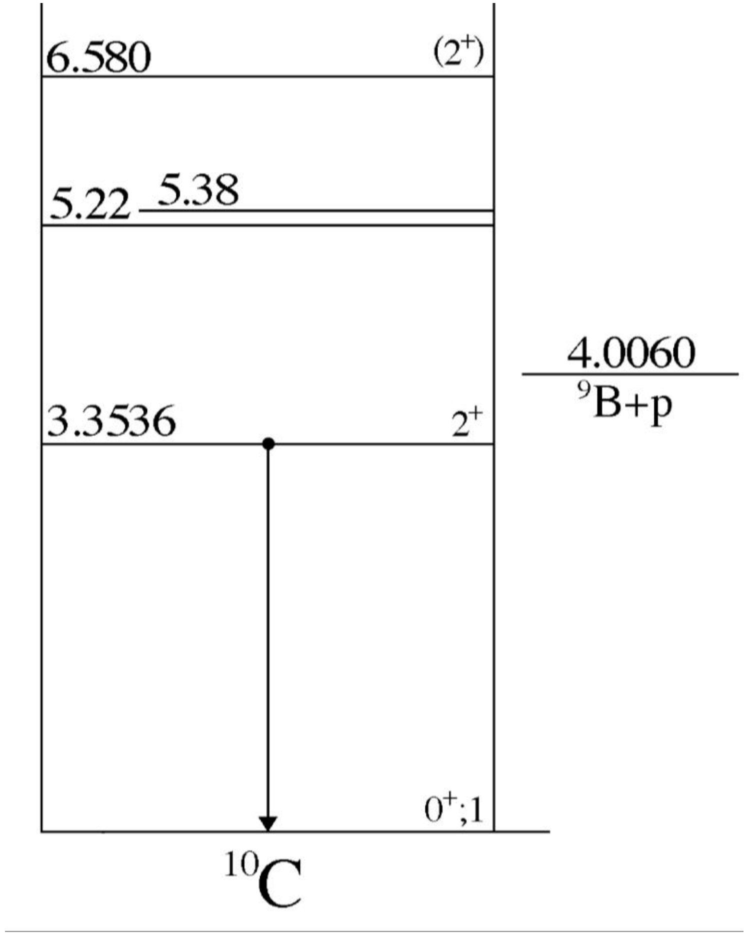
[19] Atkinson et al. (2022)

^{10}C structure at $N_{max} = 9$

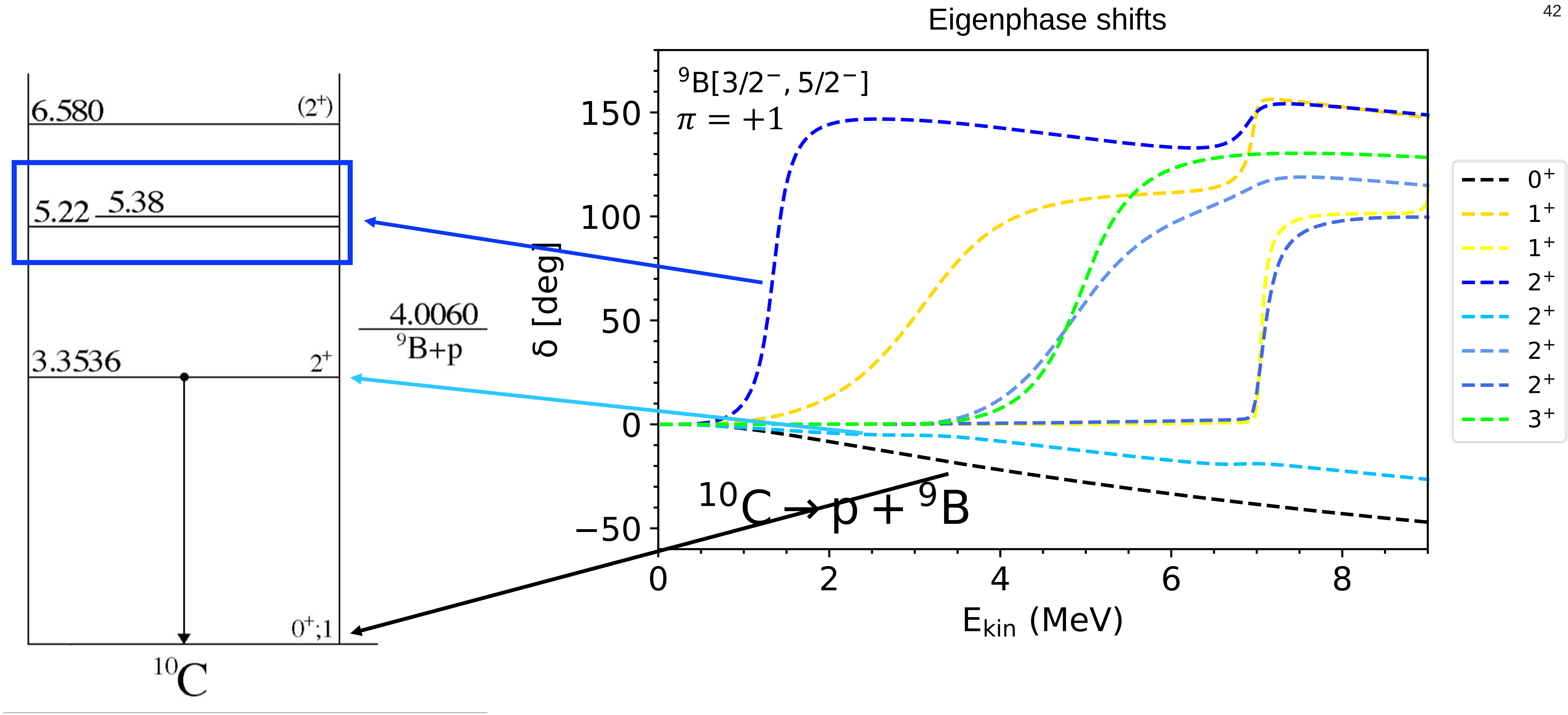
$$|{}^{10}\mathrm{C}\rangle = \sum_{\alpha} c_{\alpha} |{}^{10}\mathrm{C}, \alpha\rangle_{\mathrm{NCSM}} + \sum_{\nu} \int dr\ \gamma_{\nu}^{J^{\pi}T}(r) \mathcal{A}_{\nu} |{}^{9}\mathrm{B} + \mathrm{p}, \nu\rangle$$

- Treat as mass partition of proton plus ^{9}B
- Use $3/2^{-}$ and $5/2^{-}$ states of ^{9}B
- Known bound states captured by NCSMC

State	E_{NCSM} (MeV)	E (MeV)	E_{exp} (MeV)
0^{+}	-3.09	-3.46	-4.006
2^{+}	$+0.40$	-0.03	-0.652

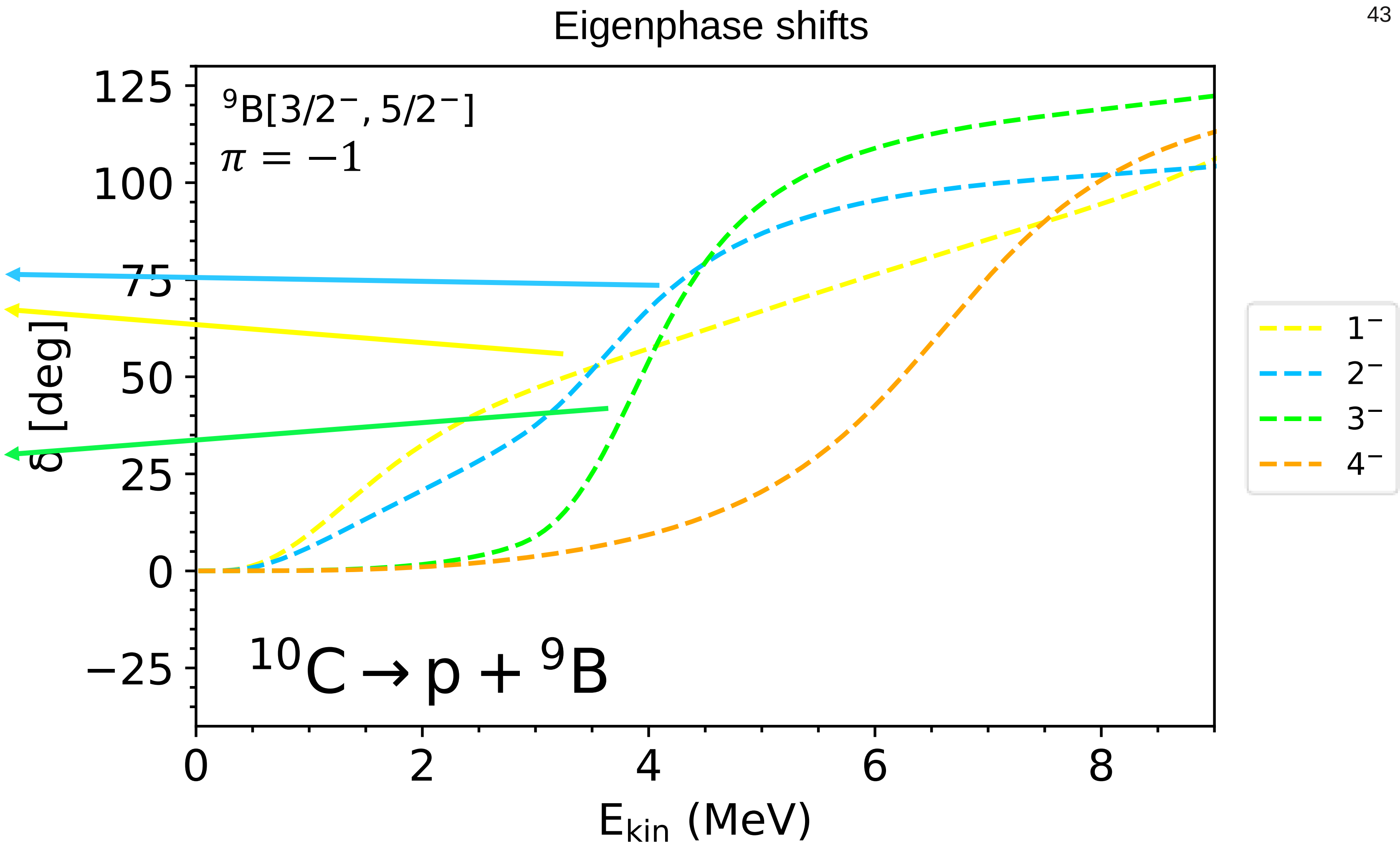


^{10}C structure at $N_{max} = 9$

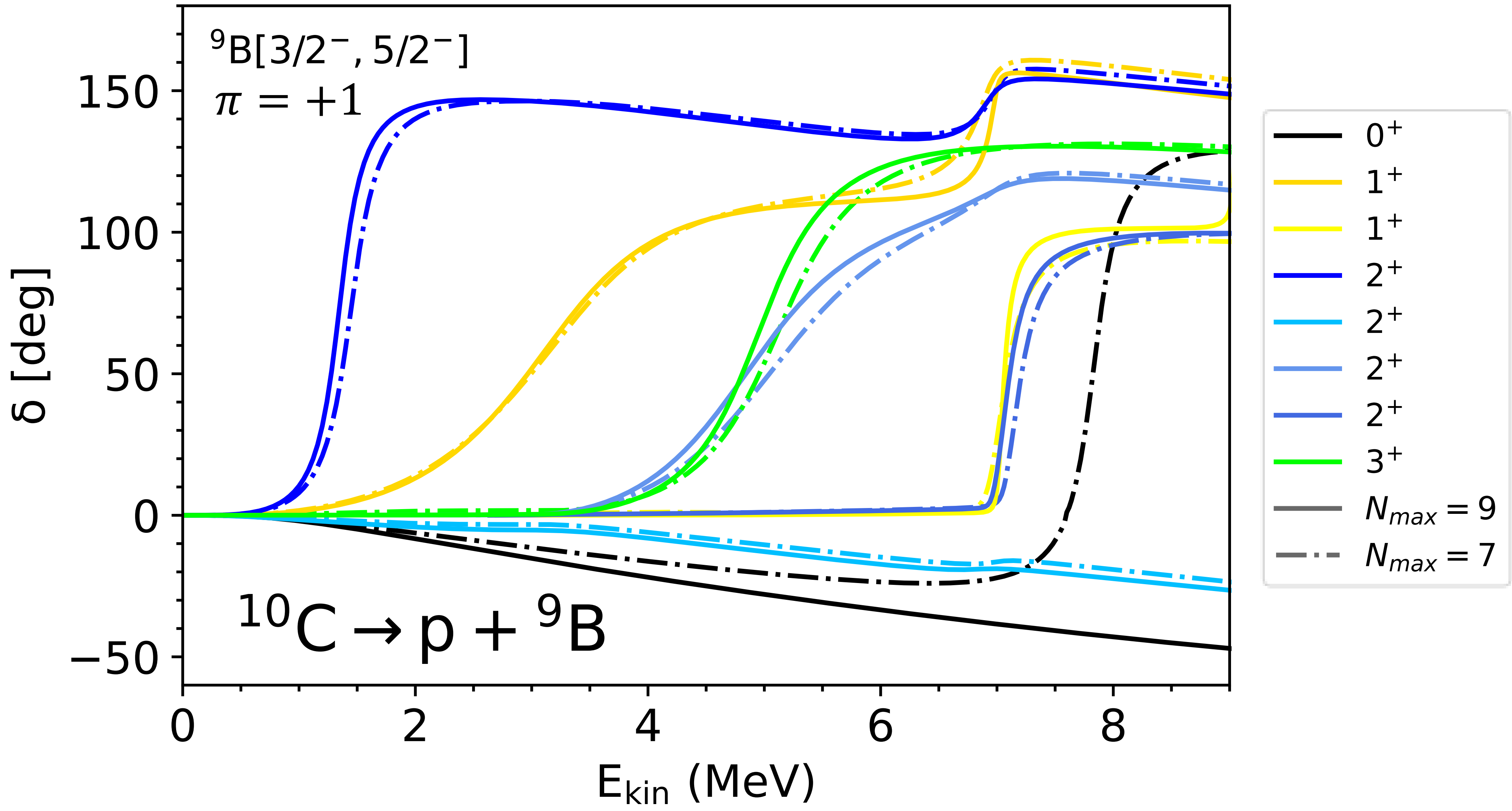


^{10}C structure at $N_{max} = 9$

- 1^- and 2^- resonances in ^{10}C analogs of ^{10}Be halo states
- 3^- resonance present

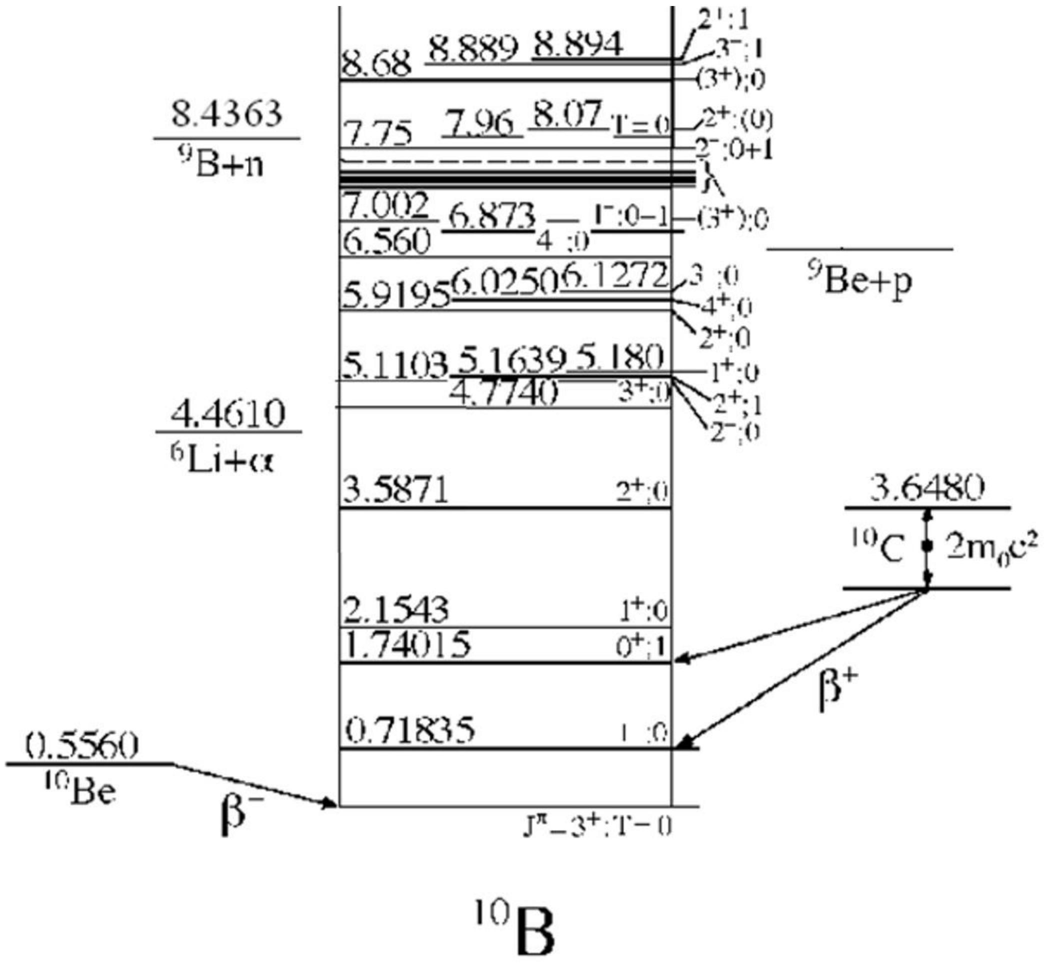


^{10}C eigenphase shifts $N_{max} = 7 - 9$ comparison



^{10}B structure result at $N_{max} = 9$

$$|^{10}\mathrm{B}\rangle = \sum_{\alpha} c_\alpha \, |^{10}\mathrm{B}, \alpha\rangle_{\mathrm{NCSM}} + \sum_{\nu} \int dr \, \gamma_\nu(r) \mathcal{A}_\nu \, |^{9}\mathrm{Be} + p, \nu\rangle + \sum_{\mu} \int dr \, \gamma_\mu(r) \mathcal{A}_\mu \, |^{9}\mathrm{B} + n, \mu\rangle$$



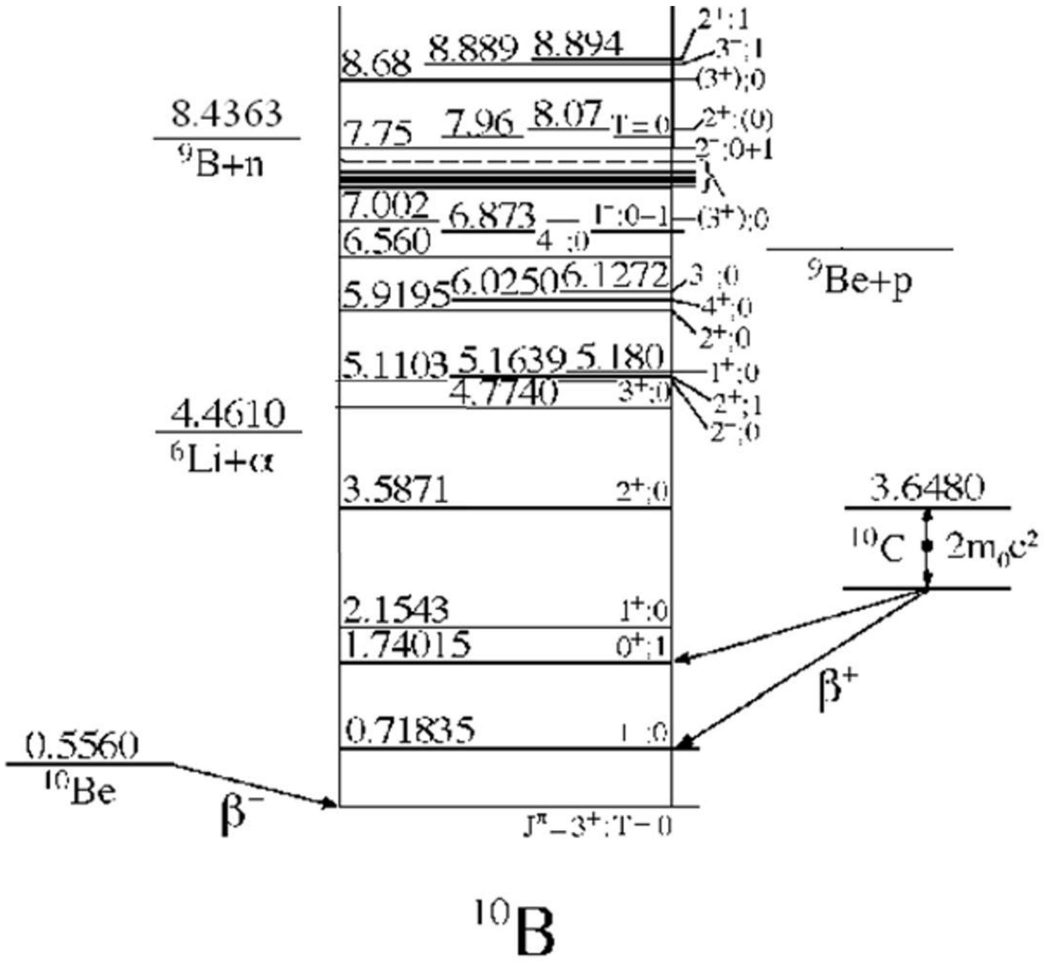
- Use $3/2^-$ and $5/2^-$ states of ^{9}B and ^{9}Be
- Eight of twelve bound states predicted

State	E (MeV)	E_{exp} (MeV)
3^+	-5.75	-6.5859
1^+	-5.33	-5.8676
0^+	-4.30	-4.8458
1^+	-4.26	-4.4316
2^+	-2.69	-2.9988
2^+	-0.93	-1.4220
2^+	-0.70	-0.6664
4^+	-0.19	-0.5609

[20] Caurier et al. (2002)
[21] Navrátil et al. (2004)

^{10}B structure result at $N_{max} = 9$

$$|^{10}\mathrm{B}\rangle = \sum_{\alpha} c_\alpha \,|^{10}\mathrm{B}, \alpha\rangle_{\mathrm{NCSM}} + \sum_{\nu} \int dr\, \gamma_\nu(r) \mathcal{A}_\nu \,|^{9}\mathrm{Be} + p, \nu\rangle + \sum_{\mu} \int dr\, \gamma_\mu(r) \mathcal{A}_\mu \,|^{9}\mathrm{B} + n, \mu\rangle$$



- Correct ordering of 3^+ and excited 1^+
- Sensitive to 3N part of Hamiltonian **[20-21]**

State	E (MeV)	E_{exp} (MeV)
3^+	-5.75	-6.5859
1^+	-5.33	-5.8676
0^+	-4.30	-4.8458
1^+	-4.26	-4.4316
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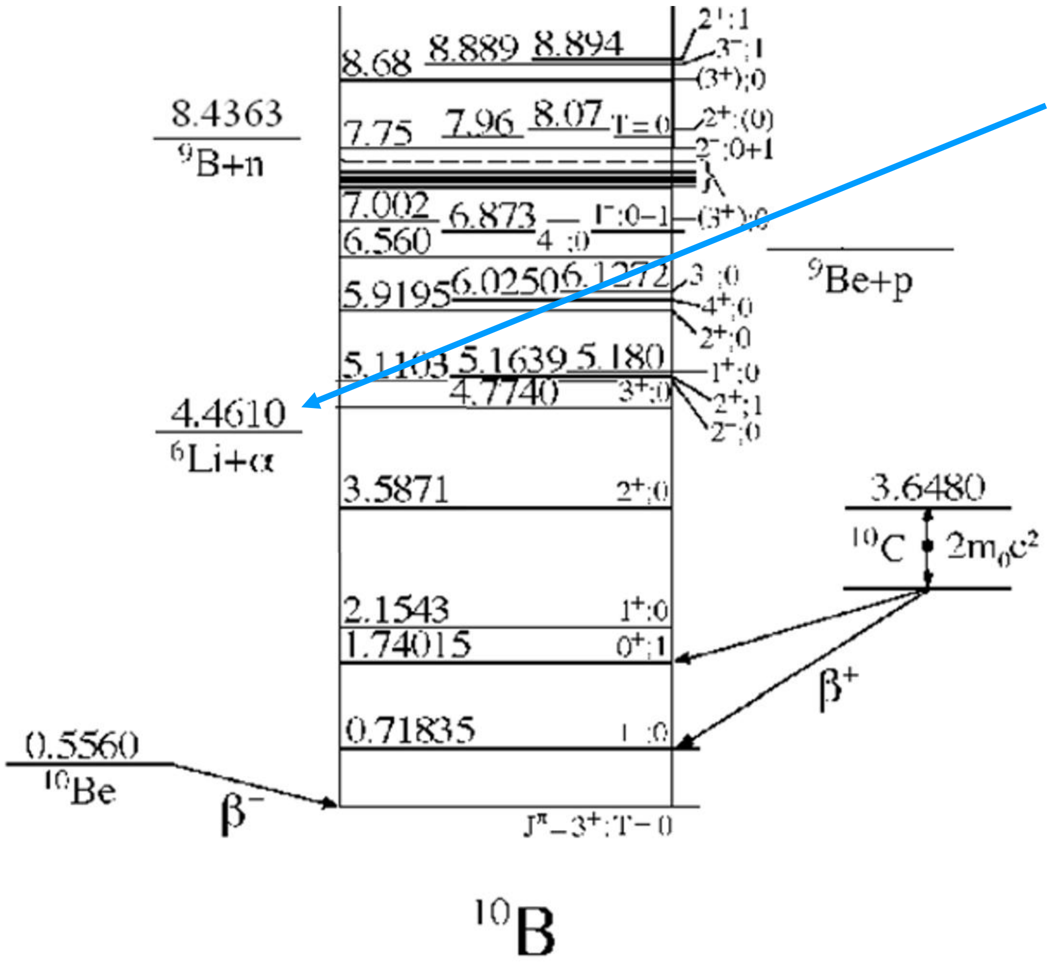
^{10}B structure result at $N_{max} = 9$

$$|^{10}\mathrm{B}\rangle = \sum_{\alpha} c_{\alpha}\, |^{10}\mathrm{B}, \alpha\rangle_{\mathrm{NCSM}} + \sum_{\nu} \int dr\, \gamma_{\nu}(r) \mathcal{A}_{\nu}\, |^{9}\mathrm{Be} + p, \nu\rangle + \sum_{\mu} \int dr\, \gamma_{\mu}(r) \mathcal{A}_{\mu}\, |^{9}\mathrm{B} + n, \mu\rangle$$



- $\alpha + {}^{6}\mathrm{Li}$ impacts structure of resonances and bound states above threshold

State	E (MeV)	E_{exp} (MeV)
3^+	-5.75	-6.5859
1^+	-5.33	-5.8676
0^+	-4.30	-4.8458
1^+	-4.26	-4.4316
2^+	-2.69	-2.9988
2^+	-0.93	-1.4220
2^+	-0.70	-0.6664
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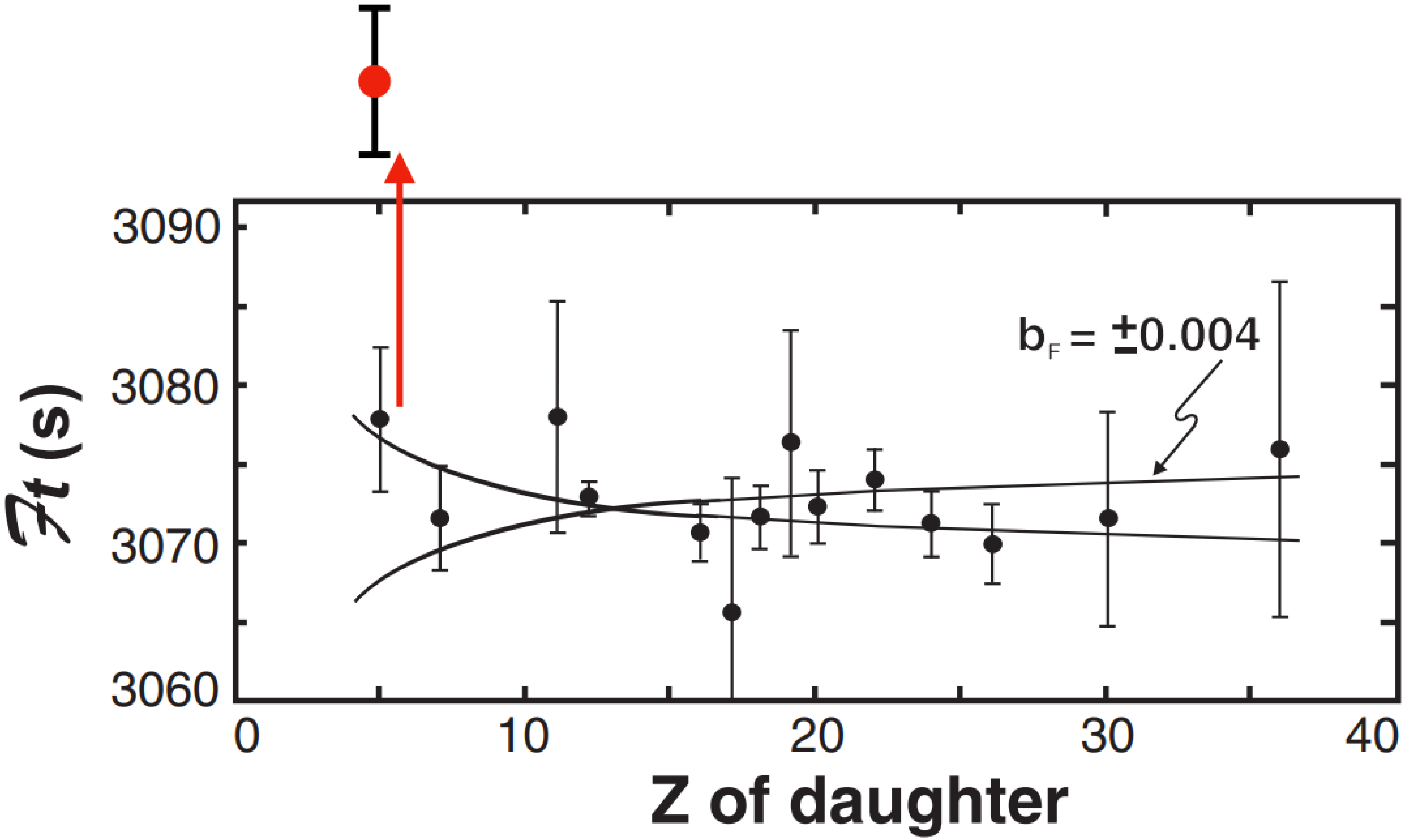

3090
3080
3070
3060
0
10
20
30
40
$\mathcal{F}t$ (s)
Z of daughter
$b_F = \pm 0.004$

- **Goal:** consistent nuclear theory corrections to Fermi transitions
- Larger basis NCSM calculations of δ_{NS}
  - first fully consistent NCSM calculation
  - residue could be dominant feature
- NCSMC calculations for δ_{C} ongoing with Mack Atkinson

Outlook

- Benchmarking δ_{NS} via Lanczos strength function approach
- Tackle large number of many-body calculations with realistic N_{max}
  - seperate inhomogeneous Schrödinger equation at each $|\vec{q}|$
  - $N_{|\vec{q}|} \times N_{terms} \times J_{max} = 50 \times 4 \times 3 = 600$ many body calculations
- Improve limited uncertainty quantification
- Heavier transitions, e.g., $^{14}\mathrm{O} \rightarrow {}^{14}\mathrm{N}$

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Thank you
Merci

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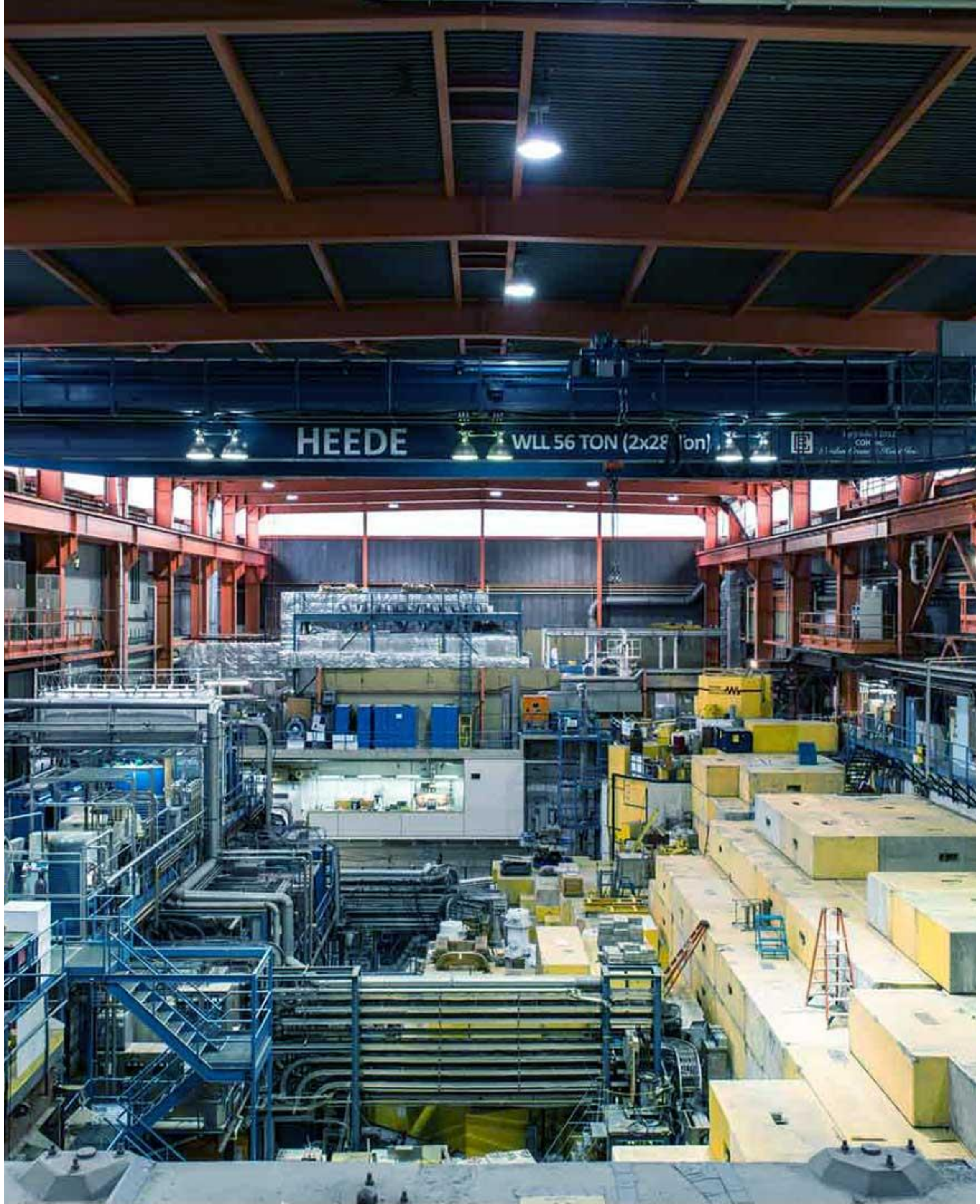
Backup slides for multipole expansion and δ_{NS}



Electron energy expansion

$$\boxed{\Box^b_{\gamma W}(E_e) = \boxminus_0 + E_e \, \boxminus_1 + \cdots + \left(\Box^b_{\gamma W}\right)_{\mathrm{Res},T_3}(E_e)}$$

$$\boxminus_0 = \frac{e^2}{M} \int \frac{d^3 q}{(2\pi)^3} \int \frac{d\nu_E}{2\pi} \frac{M_W^2}{M_W^2 - q^2} \frac{|\vec{q}\,|^2}{\nu_E (q^2 + i\epsilon_1)^2} \frac{T_3\left(i\nu_E, |\vec{q}\,|\right)}{f_+(0)}$$

$$\boxminus_1 = \frac{8}{3} \frac{e^2}{M} \int \frac{d^3 q}{(2\pi)^3} \int \frac{d\nu_E}{2\pi} \frac{M_W^2}{M_W^2 - q^2} \frac{|\vec{q}\,|^2}{(q^2 + i\epsilon_1)^3} \frac{i T_3\left(i\nu_E, |\vec{q}\,|\right)}{f_+(0)}$$

Multipole expansion of amplitude

$$J^\mu(q) = \Big(\rho(\vec{q}), \vec{J}(\vec{q})\Big) \longrightarrow \vec{J}(\vec{q}) = \sum_\lambda J(\vec{q},\lambda)\,\vec{\epsilon}^{\,*}_\lambda$$

$$e^{-i\vec{q}\cdot\vec{r}} = 4\pi \sum_{J=0}^{\infty} \sum_{M_J} (-i)^J j_J(qr)\, Y_{JM_J}(\hat{q}) Y^*_{JM_J}(\hat{q})$$

$$\mathcal{M}_{JM}(q,\vec{r}) = j_J(qr)\, Y_{JM}(\hat{r}) \qquad \vec{\mathcal{M}}^M_{JL}(q,\vec{r}) = j_L(qr)\, \vec{Y}^M_{JL1}(\hat{r})$$

[7] Walecka (2004)

Multipole expansion of amplitude

$$\rho(\vec{q}) = \sqrt{4\pi} \sum_{J=0}^{\infty} (-i)^J \sqrt{2J+1}\, M_{J0}(q)$$

$$J(\vec{q}, \lambda = 0) = \sqrt{4\pi} \sum_{J=0}^{\infty} (-i)^J \sqrt{2J+1}\, L_{J0}(q)$$

$$J(\vec{q}, \lambda = \pm 1) = -\sqrt{2\pi} \sum_{J=1}^{\infty} (-i)^J \sqrt{2J+1} \left(\lambda T^{\text{mag}}_{J\lambda}(q) - T^{\text{el}}_{J\lambda}(q) \right)$$

Nuclear matrix elements of multipole operators

$$\langle N(p_f s_f m_{T_f}) | V^{\mu}_{TM_T}(0) | N(p_i s_i m_{T_i}) \rangle = \bar{u}_{s_f}(p_f) \left[\boxed{F_1^{(T)}} \gamma^\mu + \frac{i \boxed{F_2^{(T)}}}{2m_N} \sigma^{\mu\nu}(p_f - p_i)_\nu \right] u_{s_i}(p_i) \langle m_{T_f} | \Gamma_{TM_T} | m_{T_i} \rangle$$

$$\langle N(p_f s_f m_{T_f}) | A^{\mu}_{TM_T}(0) | N(p_i s_i m_{T_i}) \rangle = \bar{u}_{s_f}(p_f) \left[\boxed{G_A^{(T)}} \gamma^\mu \gamma_5 - \frac{\boxed{G_P^{(T)}}}{2m_N} \gamma_5 (p_f - p_i)^\mu \right] u_{s_i}(p_i) \langle m_{T_f} | \Gamma_{TM_T} | m_{T_i} \rangle$$

$$\mathcal{M}_{JM}(q, \vec{r}) = j_J(qr)\, Y_{JM}(\hat{r})$$

$$\Delta_{JM}(q, \vec{r}) := \vec{\mathcal{M}}^M_{JJ}(q, \vec{r}) \cdot \frac{1}{q} \vec{\nabla}$$

$$\Delta'_{JM}(q, \vec{r}) := -i \left(\frac{1}{q} \vec{\nabla} \times \vec{\mathcal{M}}^M_{JJ}(q, \vec{r}) \right) \cdot \frac{1}{q} \vec{\nabla}$$

$$\Sigma_{JM}(q, \vec{r}) := \vec{\mathcal{M}}^M_{JJ}(q, \vec{r}) \cdot \vec{\sigma}$$

$$\Sigma'_{JM}(q, \vec{r}) := -i \left(\frac{1}{q} \vec{\nabla} \times \vec{\mathcal{M}}^M_{JJ}(q, \vec{r}) \right) \cdot \vec{\sigma}$$

$$\Sigma''_{JM}(q, \vec{r}) := \left(\frac{1}{q} \vec{\nabla} \mathcal{M}_{JM}(q, \vec{r}) \right) \cdot \vec{\sigma}$$

$$\Omega_{JM}(q, \vec{r}) := \left(\mathcal{M}_{JM}(q, \vec{r})\ \vec{\sigma} \right) \cdot \frac{1}{q} \vec{\nabla}$$

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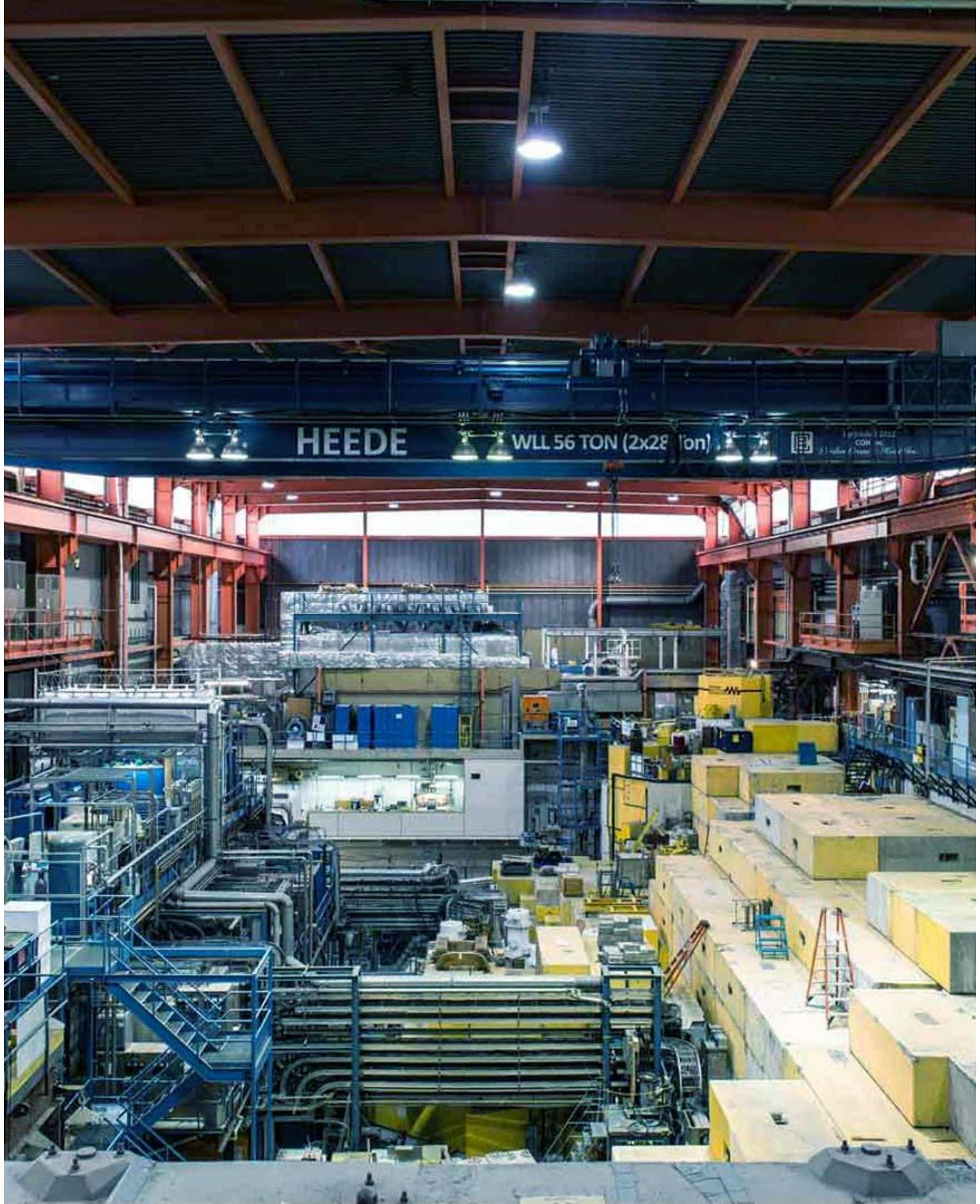
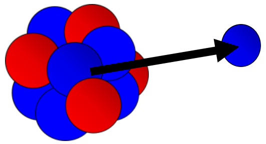
Backup slides for NCSM/RGM



Discovery, accelerated

NCSM/RGM



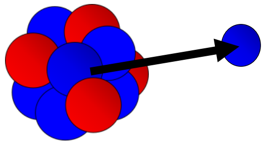
- Combine NCSM with resonating group method (RGM) **[15]**
  - $(A-a)$-target and a-nucleon projectile in ${}^{2s+1}l_J$ relative motion waves
  - $\hat{r}_{A-a,a}$ connects c.m. of each cluster

$$\left|\Phi^{J^\pi T}_{\nu r}\right\rangle = \left[\left(\left|A-a\;\alpha_1 I_1^{\pi_1} T_1\right\rangle \otimes \left|a\;\alpha_2 I_2^{\pi_2} T_2\right\rangle\right)^{(sT)} Y_l(\hat{r}_{A-a,a})\right]^{(J^\pi T)} \frac{\delta(r-r_{A-a,a})}{r\,r_{A-a,a}}$$

$$H^{(A-a)}\left|\Psi^{I_1^{\pi_1}T_1}_{A-a}\right\rangle = E^{I_1^{\pi_1}T_1}\left|\Psi^{I_1^{\pi_1}T_1}_{A-a}\right\rangle \qquad H^{(a)}\left|\Psi^{I_2^{\pi_2}T_2}_{a}\right\rangle = E^{I_2^{\pi_2}T_2}\left|\Psi^{I_2^{\pi_2}T_2}_{a}\right\rangle$$

[15] Navrátril et al. (2009)

NCSM/RGM



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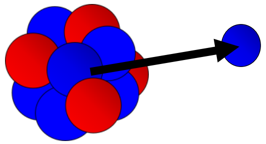
- Require anti-symmetrization to preserve Pauli principle

$$\hat{\mathcal{A}}_\nu = \sqrt{\frac{(A-a)!\,a!}{A!}}\left(1 + \sum_{P \neq \mathbb{1}} (-1)^p P_\nu\right) \longrightarrow$$

Anti-symmetrize between clusters

[15] Navrátril et al. (2009)

NCSM/RGM



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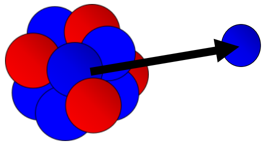
- Require anti-symmetrization to preserve Pauli principle
- Use anti-symmetrized channel states as continuous basis ansatz

$$\left|\Psi^{J^\pi T}\right\rangle = \sum_\nu \int dr\; r^2 \mathcal{A}_\nu \left|\Phi^{J^\pi T}_{\nu r}\right\rangle \frac{\left[\mathcal{N}^{-\frac{1}{2}}\cdot\chi\right]^{J^\pi T}_\nu(r)}{r}$$

Linear variational amplitudes

[15] Navrátril et al. (2009)

Solving RGM equations



- Solve orthogonalized RGM equations

$$\sum_{\nu'} \int dr' \, r'^2 \, \left[\mathcal{N}^{-\frac{1}{2}} \mathcal{H} \mathcal{N}^{-\frac{1}{2}}\right]^{J^\pi T}_{\nu\nu'} (r, r') \, \frac{\chi^{J^\pi T}_{\nu'}(r')}{r'} = E \frac{\chi^{J^\pi T}_{\nu}(r)}{r}$$

- Norm and Hamiltonian kernels primary computational challenge

$$\mathcal{H}^{J^\pi T}_{\nu'\nu}(r', r) = \left\langle \Phi^{J^\pi T}_{\nu' r'} \right| \hat{\mathcal{A}}_{\nu'} \mathcal{H} \hat{\mathcal{A}}_{\nu} \left| \Phi^{J^\pi T}_{\nu r} \right\rangle$$

Hamiltonian kernels

$$\mathcal{N}^{J^\pi T}_{\nu'\nu}(r', r) = \left\langle \Phi^{J^\pi T}_{\nu' r'} \right| \hat{\mathcal{A}}_{\nu'} \hat{\mathcal{A}}_{\nu} \left| \Phi^{J^\pi T}_{\nu r} \right\rangle$$

Norm kernels

Well established solutions of multi-channel
Schrödinger equations

[20] Lane et al. (1958)
[21] Hesse et al. (1998)
[22] Descouvemont et al. (2010)

Solving RGM equations

$$\sum_{\nu'} \int dr'\, r'^2 \left[\mathcal{N}^{-\frac{1}{2}} \mathcal{H} \mathcal{N}^{-\frac{1}{2}}\right]^{J^\pi T}_{\nu\nu'} (r, r') \frac{\chi^{J^\pi T}_{\nu'}(r')}{r'} = E \frac{\chi^{J^\pi T}_{\nu}(r)}{r}$$

- Solve coupled channel nonlocal integro–differential equations **[20-22]**
  - split configuration space by large matching radius r_0
  - require continuity of wave function and derivative

Internal region

$$\chi^{J^\pi T}_{\nu}(r) = \frac{i}{2v_\nu}\left[\delta_{\nu i} H^-_l(\kappa_\nu r) - S^{J^\pi T}_{\nu i} H^+_l(\kappa_\nu r)\right]$$

- Coulomb functions
- Expand over square integrable Lagrange functions

External region

$$\chi^{J^\pi T}_{\nu}(r) = C^{J^\pi T}_{\nu} W_l(\kappa_\nu r)$$

- Whittaker function asymptotics
- Normalization constant $C^{J^\pi T}_\nu$

[20] Lane et al. (1958)
[21] Hesse et al. (1998)
[22] Descouvemont et al. (2010)

Solving RGM equations

$$\sum_{\nu'} \int dr'\, r'^2 \left[\mathcal{N}^{-\frac{1}{2}}\, \mathcal{H}\, \mathcal{N}^{-\frac{1}{2}}\right]^{J^\pi T}_{\nu\nu'}(r, r')\, \frac{\chi^{J^\pi T}_{\nu'}(r')}{r'} = E\, \frac{\chi^{J^\pi T}_{\nu}(r)}{r}$$

- Solve coupled channel nonlocal integro–differential equations **[20-22]**
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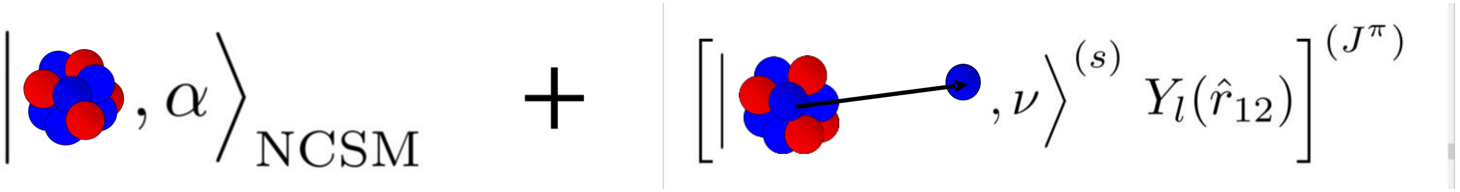
- Eigenstates and eigenenergies for bound states
- Scattering matrix and eigenstates for unbound states
- *Ab initio* description of scattering off light–nuclei

NCSMC

- Generalize NCSM/RGM expansion with discrete NCSM eigenstates **[16]**

$$\left|\Psi^{J^\pi T}\right\rangle = \sum_\alpha c_\alpha^{J^\pi T}\left|A\alpha J^\pi T\right\rangle + \sum_\nu \int dr\ r^2 \mathcal{A}_\nu \left|\Phi_{\nu r}^{J^\pi T}\right\rangle \frac{\left[\mathcal{N}^{-\frac{1}{2}}\cdot\chi\right]_\nu^{J^\pi T}(r)}{r}$$



$$\begin{pmatrix} \mathbb{E} & \bar{h} \\ \bar{h} & \mathcal{H} \end{pmatrix} \begin{pmatrix} c \\ \chi \end{pmatrix} = E \begin{pmatrix} \mathbb{1} & \bar{g} \\ \bar{g} & \mathcal{I} \end{pmatrix} \begin{pmatrix} c \\ \chi \end{pmatrix}$$

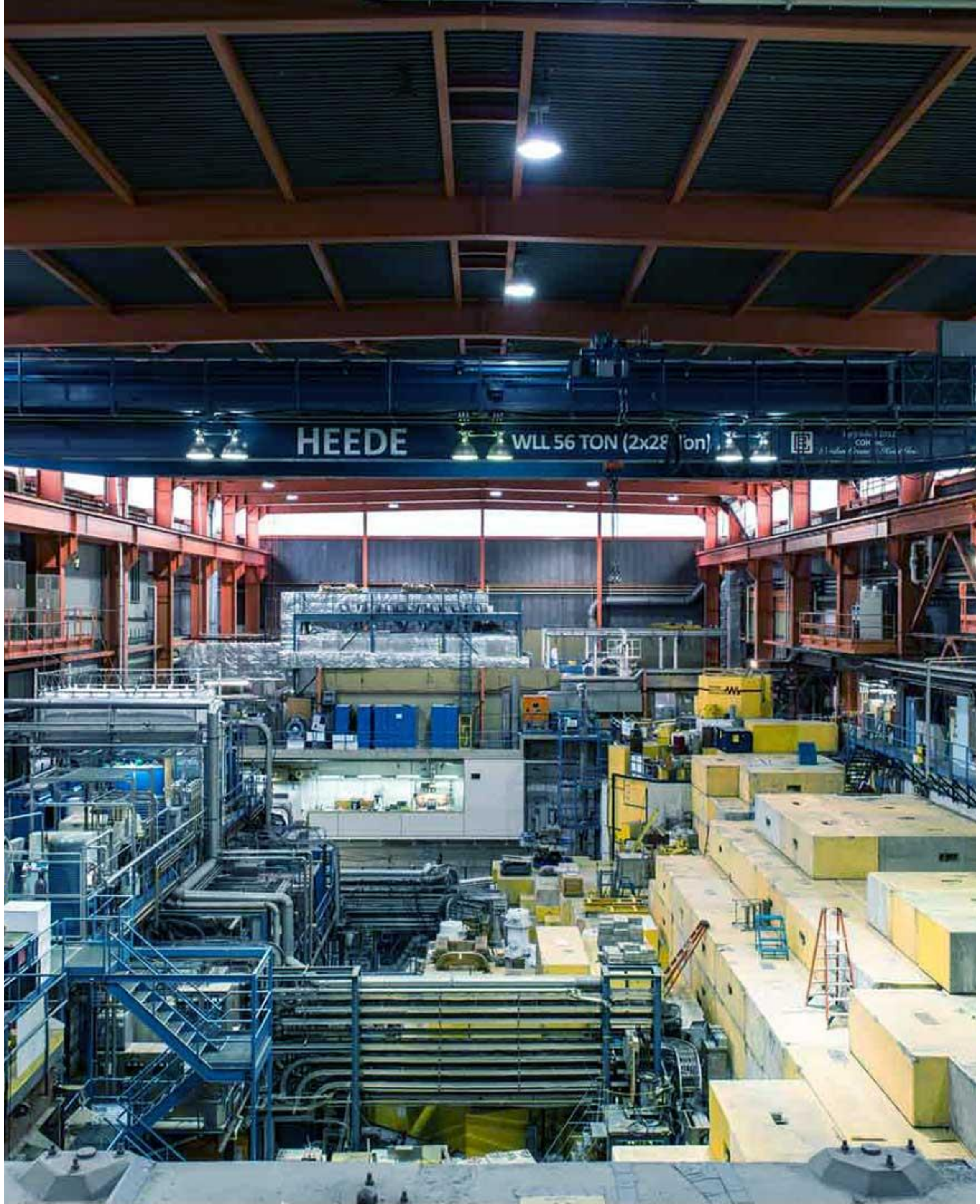
c_α and $\gamma_\nu(r)$ from solving coupled equations

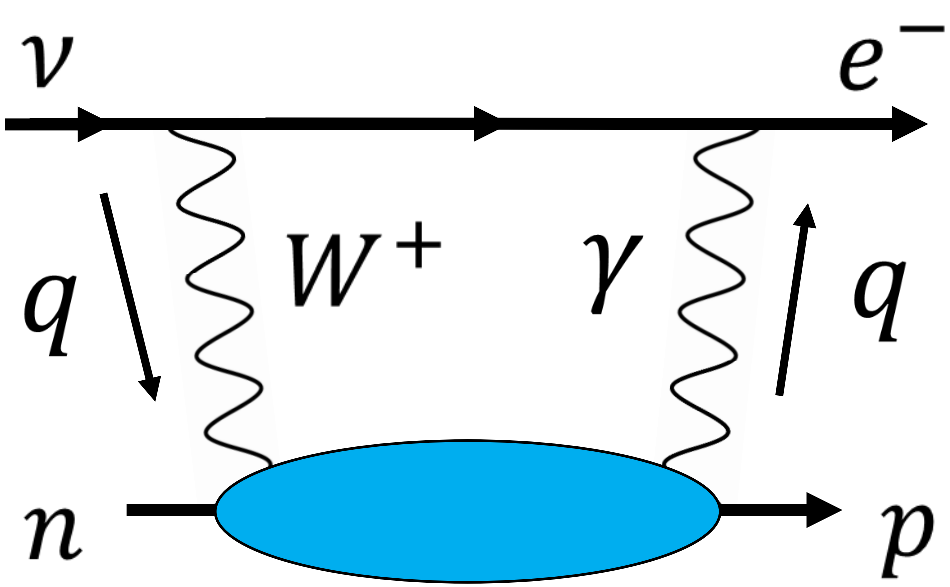
[16] Baroni et al. (2013)

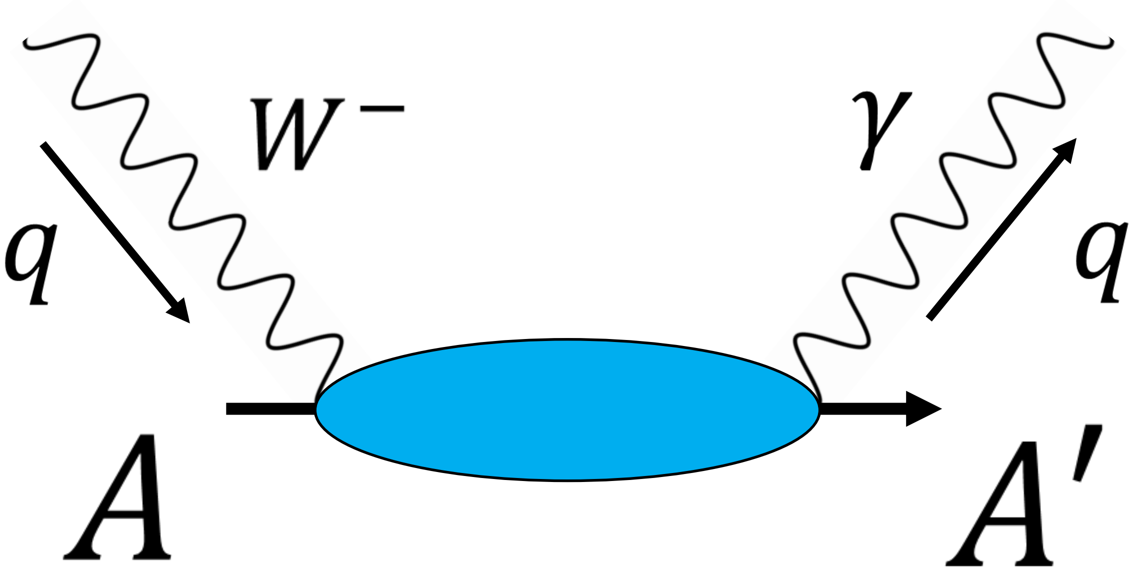
References

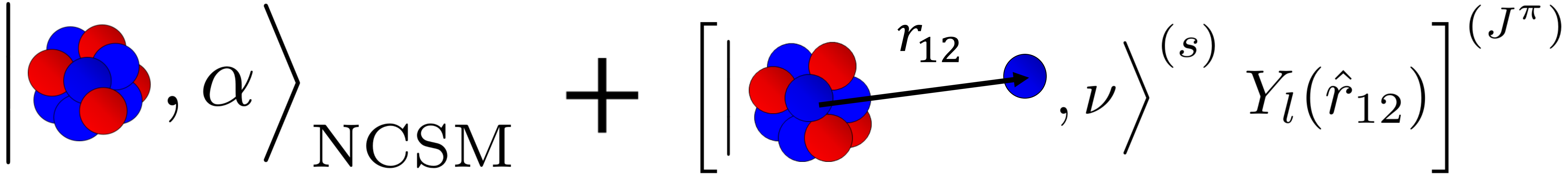
20. A.M. Lane & R.G. Thomas. Rev. Mod. Phys. **30**, 257 (1958)
21. M. Hesse, J.-M. Sparenberg, F. Van Raemdonck, & D. Baye. Nuc. Phys. A **640**, pp. 37-51 (1998)
22. P. Descouvemont & D. Baye. Rep. Prog. Phys. **73**, 036301 (2010)

The real end




ν
e^-
q
W^+
γ
q
n
p


q
W^-
γ
q
A
A'


$, \alpha \rangle_{\mathrm{NCSM}}$
$+$
r_{12}
$, \nu \rangle^{(s)}$
$Y_l(\hat{r}_{12})$
$]^{(J^\pi)}$