Precise predictions for semi-inclusive DIS

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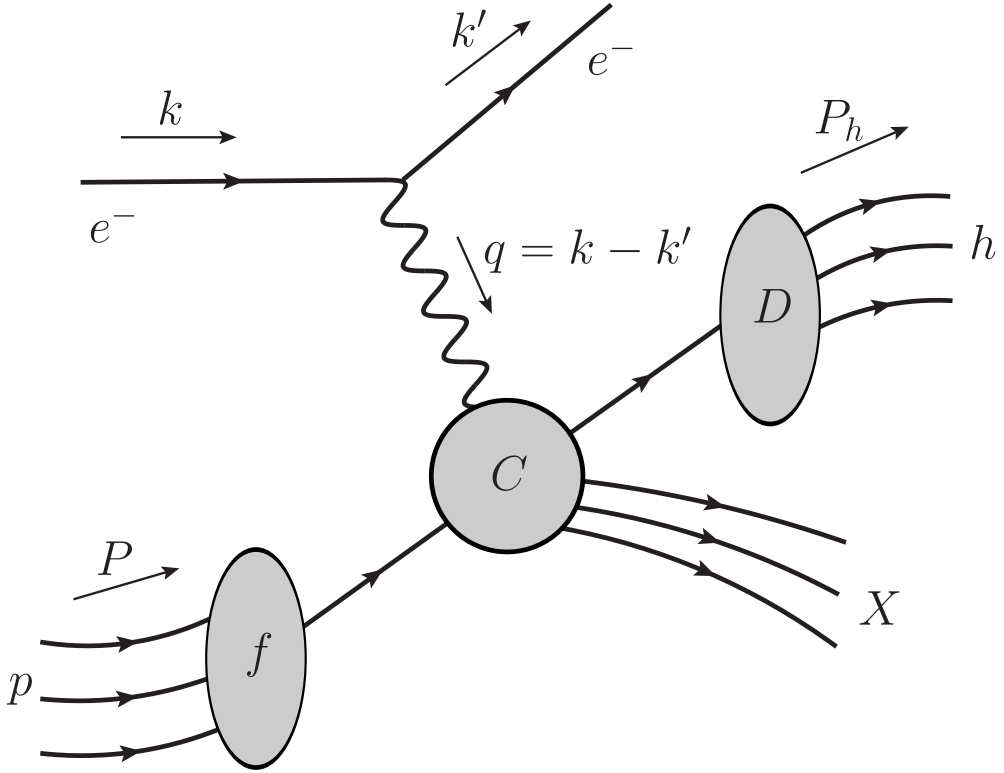
Semi-inclusive deep-inelastic scattering (SIDIS)

Production of identified hadrons in DIS

- multiple hadron species: π,K,D,p,n,Λ,...
- hadron structure:
  parton distribution function (PDF) $f(x,\mu)$
- parton-to-hadron fragmentation:
  fragmentation function (FF) $D(z,\mu)$
- probe flavour structure of PDFs and FFs
- flavour decomposition of sea quark PDFs
- constraints on polarized PDFs
- flavour separation of FFs for different hadrons



is becoming precision physics

State-of-the-art: fragmentation functions

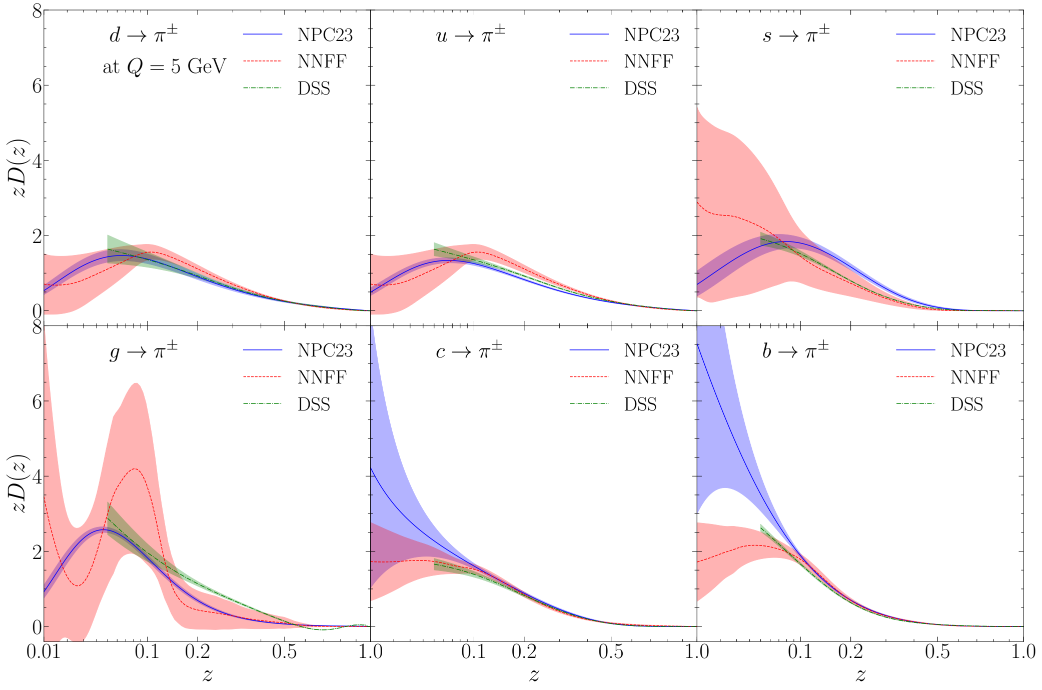
Global fits of collinear fragmentation functions

- data sets
  - Semi-inclusive annihilation (SIA) $e^+e^- \rightarrow h+X$
  - Semi-inclusive deep-inelastic scattering (SIDIS) $ep \rightarrow eh+X$
  - Single-inclusive hadro-production (SIH) $pp \rightarrow h+X$
- perturbative order until recently
  - NNLO for time-like Altarelli-Parisi splitting functions
    [A.Almasy, S.Moch, A.Vogt; M.X.Luo, T.Z.Yang, H.X.Zhu, Y.J.Zhu; M.Ebert, B.Mistlberger, G.Vita]
  - NNLO and N3LO for SIA coefficient functions [P.Rijken, W. van Neerven; C.Q.He, H.Xing, T.Z.Yang, H.X.Zhu]
  - NLO for SIDIS coefficient functions
    [G.Altarelli, K.Ellis, G.Martinelli, S.Pi; D.de Florian, M.Stratmann, W.Vogelsang]
  - NLO for SIH coefficient functions
    [F.Aversa, P.Chiappetta, M.Greco, J.P.Guillet; B.Jäger, A.Schäfer, M.Stratmann, W.Vogelsang]
  - approximate NNLO from threshold resummation for SIDIS [M.Abele, D.de Florian, W.Vogelsang]

State-of-the-art: fragmentation functions

Global fits of collinear fragmentation functions: $\pi^{\pm 0}$,$K^{\pm 0}$,Λ,p,...

- NLO global fits
  - BKK [J.Binnewies, B.Kniehl, G.Kramer]
  - DSS [R.Sassot, D.de Florian, M.Stratmann]
  - NNFF [V.Bertone, S.Carrazza, N.Hartland, E.Nocera, J.Rojo, L.Rottoli]
  - JAM [E.Moffat, W.Melnitchouk, T.C.Rogers, N.Sato]
  - NPC [J.Gao, C.Y.Liu, X.M.Shen, H.Xing, Y.Zhao, B.Zhou]
- NNLO SIA and aNNLO SIA+SIDIS+SIH fits
  - SIA-only [D.Anderle, F.Ringer, M.Stratmann]
  - BDSSV [I.Borsa, D.de Florian, R.Sassot, M.Stratmann, W.Vogelsang]
  - MAP [R.Abdul Khalek, V.Bertone, A.Khoudii, E.Nocera]
  - NNFF [V.Bertone, S.Carrazza, N.Hartland, E.Nocera, J.Rojo]
  - NPC [J.Gao, C.Y.Liu, X.M.Shen, H.Xing, Y.Zhao, B.Zhou]

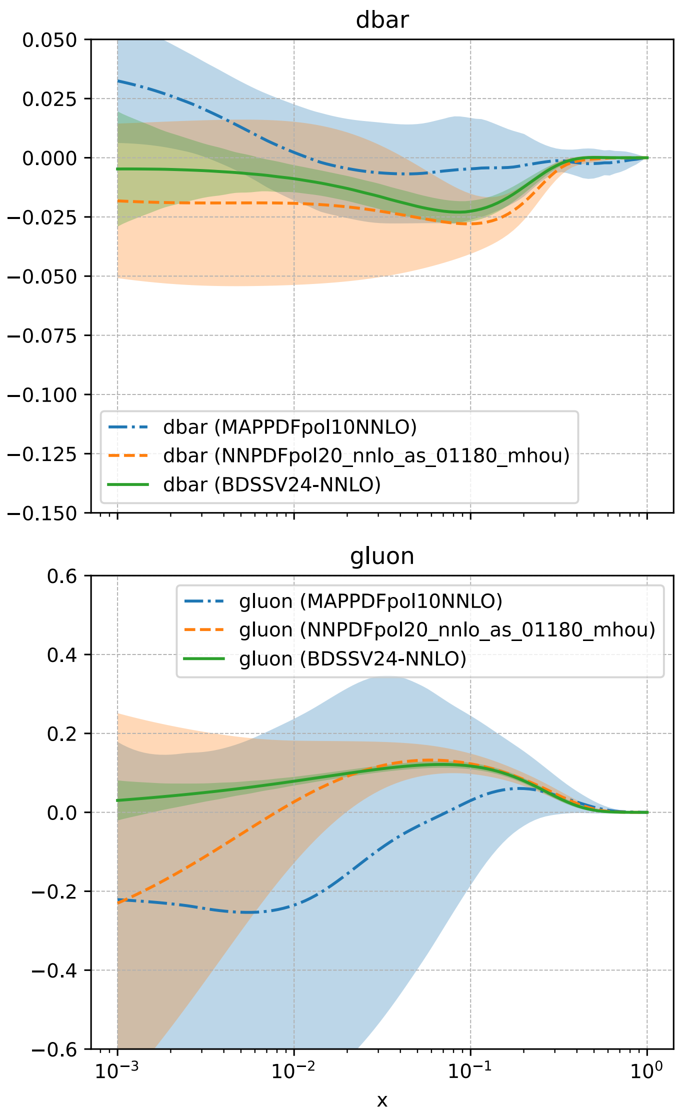


- Fits differ in data selection and detailed methodology

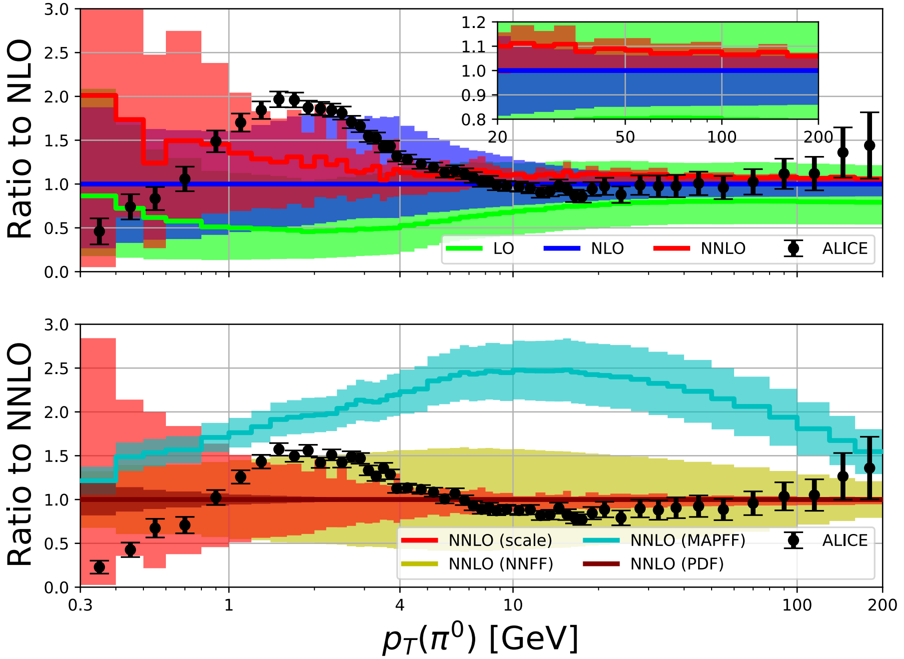
State-of-the-art: polarized PDFs

Global fits of polarized parton distributions

- data sets
  - inclusive polarized DIS
  - polarized SIDIS
  - polarized proton-proton (RHIC)
- perturbative order
  - NNLO splitting functions
    [S.Moch, J.Vermaseren, A.Vogt; J. Blümlein, C.Schneider, K.Schönwald]
  - NNLO inclusive DIS and Drell-Yan
    [E.Zijstra, W.van Neerven; R.Boughezal, H.Li, F.Petriello]
  - aNNLO SIDIS [M.Abele, D.de Florian, W.Vogelsang]
- most recently first NNLO fits
  - MAP24 [V.Bertone, E.Chiefa, E.Nocera]
  - BDSSV24 [I.Borsa, D.de Florian, R.Sassot, M.Stratmann, W.Vogelsang]
  - NNPDFpol2.0 [J.Cruz-Martinez et al.]



New NNLO results for identified hadrons



Why compute higher orders?

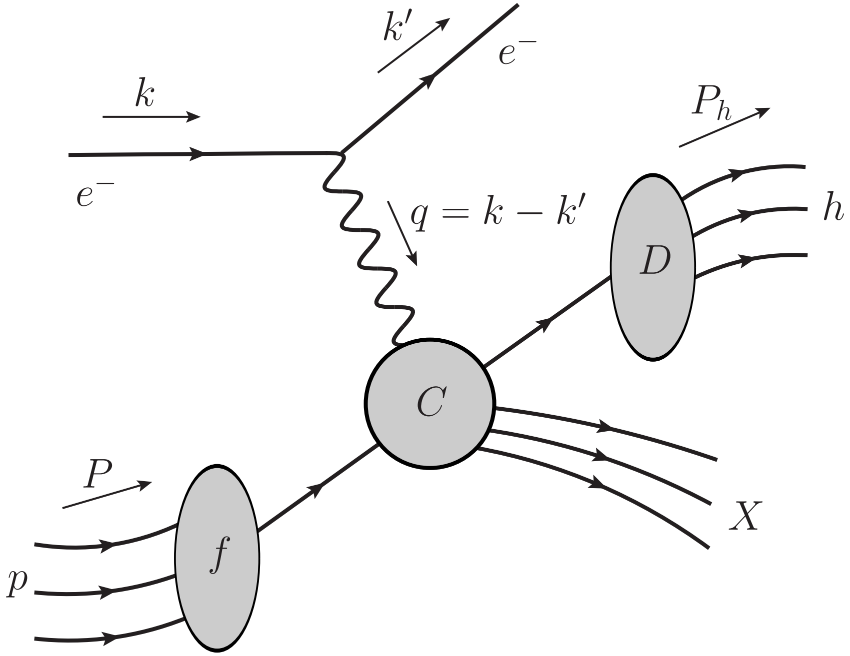
- improve perturbative QCD description
- reliable quantification of uncertainties
- new kinematical features and partonic channels
- match quality of precision data
- strong coupling typically not small: α_s(10 GeV) ~ 0.2

Recent results

- NNLO unpolarized SIDIS [S.Goyal, S.Moch, V.Pathak, N.Rana, V.Ravindran; L.Bonino, G.Stagnitto, TG]
- NNLO longitudinally polarized SIDIS [S.Goyal, R.Lee, S.Moch, V.Pathak, N.Rana, V.Ravindran; L.Bonino, M.Löchner, K.Schönwald, G.Stagnitto, TG]
- NNLO unpolarized SIH [M.Czakon, T.Generet, A.Mitov, R.Poncelet]

SIDIS kinematics

$$l(k) + p(P) \to l(k') + h(P_h) + X$$



variables

$$Q^2 = -q^2 \qquad x = \frac{Q^2}{2P \cdot q} \qquad z = \frac{P \cdot P_h}{P \cdot q}$$

cross sections

$$\frac{\mathrm{d}^3\sigma^h}{\mathrm{d}x\mathrm{d}y\mathrm{d}z} = \frac{4\pi\alpha^2}{Q^2}\left[\frac{1+(1-y)^2}{2y} F_T^h(x,z,Q^2) + \frac{1-y}{y} F_L^h(x,z,Q^2)\right]$$

$$\frac{\mathrm{d}^3\Delta\sigma^h}{\mathrm{d}x\mathrm{d}y\mathrm{d}z} = \frac{4\pi\alpha^2}{Q^2}(2-y)\, g_1^h(x,z,Q^2)$$

Observables: multiplicity

$$\frac{\mathrm{d}M_h}{\mathrm{d}z} = \frac{\mathrm{d}^3\sigma^h}{\mathrm{d}x\mathrm{d}y\mathrm{d}z} \Big/ \frac{\mathrm{d}^2\sigma^{\mathrm{DIS}}}{\mathrm{d}x\mathrm{d}y}$$

spin asymmetry

$$A_1^h(x,z,Q^2) = \frac{g_1^h(x,z,Q^2)}{F_1^h(x,z,Q^2)}$$

SIDIS coefficient functions

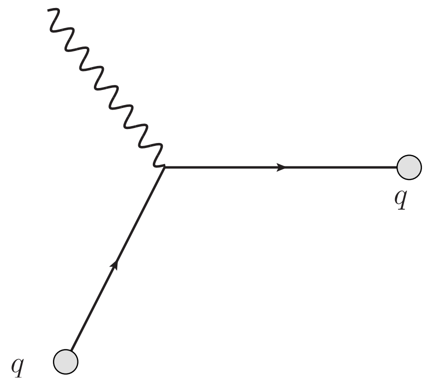
Parton model

$$F_i^h(x,z,Q^2) = \sum_{p,p'} \int_x^1 \frac{\mathrm{d}\hat{x}}{\hat{x}} \int_z^1 \frac{\mathrm{d}\hat{z}}{\hat{z}} f_p\left(\frac{x}{\hat{x}},\mu_F\right) D_{p'}^h\left(\frac{z}{\hat{z}},\mu_A\right) C_{p'p}^i\left(\hat{x},\hat{z},Q^2,\mu_R,\mu_F,\mu_A\right)$$

$$C_{p'p}^i\left(\hat{x},\hat{z},Q^2,\mu_R,\mu_F,\mu_A\right) = C_{p'p}^{i,(0)}\left(\hat{x},\hat{z}\right) + \frac{\alpha_s(\mu_R)}{2\pi} C_{p'p}^{i,(1)}\left(\hat{x},\hat{z},Q^2,\mu_R,\mu_F,\mu_A\right) + \left(\frac{\alpha_s(\mu_R)}{2\pi}\right)^2 C_{p'p}^{i,(2)}\left(\hat{x},\hat{z},Q^2,\mu_R,\mu_F,\mu_A\right) + \ldots$$

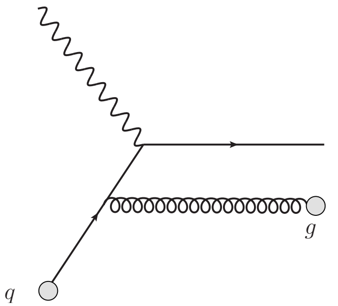
Leading order

$$C_{qq}^{T,(0)}\left(\hat{x},\hat{z}\right) = e_q^2\delta(1-\hat{x})\delta(1-\hat{z}) \qquad C_{qq}^{L,(0)}\left(\hat{x},\hat{z}\right) = 0$$

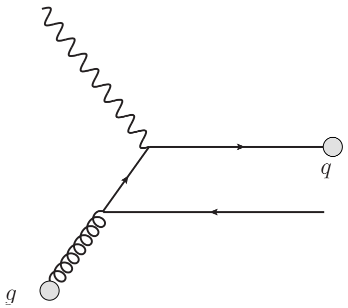


SIDIS coefficient functions

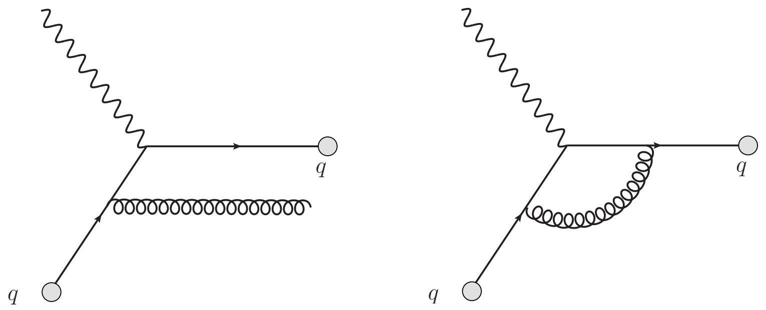
Next-to-leading order [G.Altarelli, K.Ellis, G.Martinelli, S.Pi; D.de Florian, M.Stratmann, W.Vogelsang]



$C_{gq}^{i,(1)}$



$C_{qg}^{i,(1)}$



$C_{qq}^{i,(1)}$

- real radiation at NLO: $q + p_i \to k_p^{\mathrm{id}} + k_j$

$$C_{\mathrm{R}}^{(1)} \sim \int \mathrm{d}\phi_2(k_p, k_j; q, k_i)\, |\mathcal{M}|^2_{\mathrm{R}}(s_{ip}, s_{ij}, s_{jp})\, \delta\left(z - \frac{s_{ip}}{s_{ip}+s_{ij}}\right)$$

$$(1-x)^{-1-2\epsilon} = -\frac{1}{2\epsilon}\delta(1-x) + \sum_n \frac{(-2\epsilon)^n}{n!}\mathcal{D}_n(1-x)$$

$$s_{ip} = -Q^2\frac{\hat{z}}{\hat{x}}$$

$$s_{ij} = -Q^2\frac{1-\hat{z}}{\hat{x}}$$

$$s_{jp} = Q^2\frac{1-\hat{x}}{\hat{x}}$$

SIDIS coefficient functions at NNLO

Partonic channels

- $$C_{qq}^{i,(2)} = C_{\bar{q}\bar{q}}^{i,(2)} = e_q^2 C_{qq}^{i,\mathrm{NS}} + \left(\sum_j e_{q_j}^2\right) C_{qq}^{i,\mathrm{PS}}\,,$$

$$C_{\bar{q}q}^{i,(2)} = C_{q\bar{q}}^{i,(2)} = e_q^2 C_{\bar{q}q}^{i}\,,$$

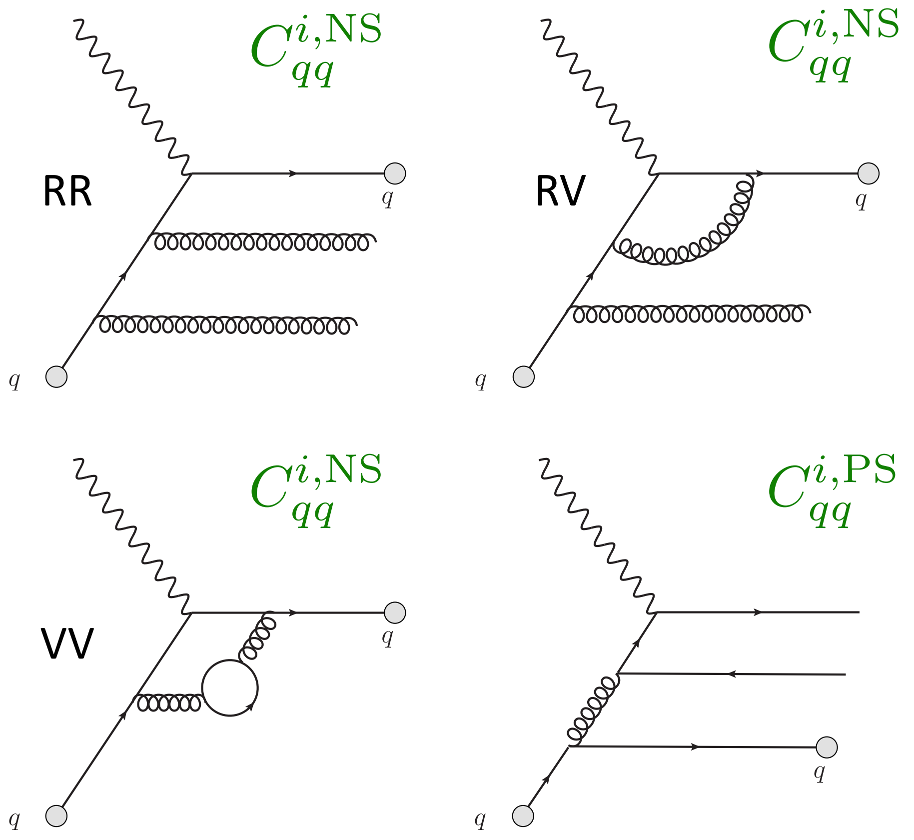
$$C_{q'q}^{i,(2)} = C_{\bar{q}'\bar{q}}^{i,(2)} = e_q^2 C_{q'q}^{i,1} + e_{q'}^2 C_{q'q}^{i,2} + e_q e_{q'} C_{q'q}^{i,3}\,,$$

$$C_{\bar{q}'q}^{i,(2)} = C_{q'\bar{q}}^{i,(2)} = e_q^2 C_{q'q}^{i,1} + e_{q'}^2 C_{q'q}^{i,2} - e_q e_{q'} C_{q'q}^{i,3}\,,$$

$$C_{gq}^{i,(2)} = C_{g\bar{q}}^{i,(2)} = e_q^2 C_{gq}^{i}\,,$$

$$C_{qg}^{i,(2)} = C_{\bar{q}g}^{i,(2)} = e_q^2 C_{qg}^{i}\,,$$

$$C_{gg}^{i,(2)} = \left(\sum_j e_{q_j}^2\right) C_{gg}^{i}\,,$$



not a forward scattering amplitude:
evaluate all contributions separately

NNLO corrections

VV: known massless two-loop form factors [T.Matsuura, W.van Neerven]

RV: one-loop single-real matrix elements

$$C_{\mathrm{RV}}^{(2)} \sim \int \mathrm{d}\phi_2(k_p, k_j; q, k_i)\, |\mathcal{M}|^2_{\mathrm{RV}}(s_{ip}, s_{ij}, s_{jp})\, \delta\left(\hat{z} - \frac{s_{ip}}{s_{ip} + s_{ij}}\right)$$

- phase space integral fully constrained, expand in distributions [R.Schürmann, TG]

RR: tree-level double-real matrix elements

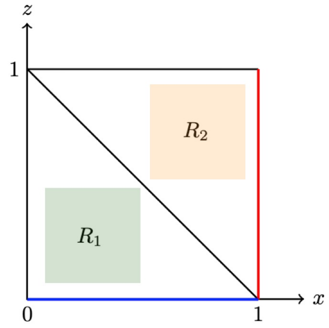
$$C_{\mathrm{RR}}^{(2)} \sim \int \mathrm{d}\phi_3(k_p, k_j, k_k; q, k_i)\, |\mathcal{M}|^2_{\mathrm{RR}}(\{s_{ab}\})\, \delta\left(\hat{z} - \frac{s_{ip}}{s_{ip} + s_{ij} + s_{ik}}\right)$$

- phase space integrals with kinematical constraint
- reduce to phase-space master integrals, computed through differential equations [L.Bonino, M.Marcoli, R.Schürmann, G.Stagnitto, TG]

NNLO corrections: RV

RV: one-loop single-real matrix elements: massless bubble and box functions

$$C^{(2)}_{\mathrm{RV}} \sim \int \mathrm{d}\phi_2(k_p, k_j; q, k_i)\, |\mathcal{M}|^2_{\mathrm{RV}}(s_{ip}, s_{ij}, s_{jp})\, \delta\left(\hat{z} - \frac{s_{ip}}{s_{ip}+s_{ij}}\right)$$



- requires careful analytic continuation of one-loop functions (4 segments)
- can combine into single-valued functions [J.Haug, F.Wunder]
- example: Box(s_{12}=s_{ij} , s_{23}=s_{pj})

$$\begin{aligned}
&\mathrm{Box}(s_{ij}, s_{ik}) \\
&= \frac{2(1-2\epsilon)}{\epsilon} A_{2,LO} \frac{1}{s_{ij}s_{ik}} \\
&\quad \times \left[\left(\frac{s_{ij}s_{ik}}{s_{ij}-s_{ijk}}\right)^{-\epsilon} {}_2F_1\left(-\epsilon, -\epsilon; 1-\epsilon; \frac{s_{ijk}-s_{ij}-s_{ik}}{s_{ijk}-s_{ij}}\right) \right. \\
&\quad + \left(\frac{s_{ij}s_{ik}}{s_{ik}-s_{ijk}}\right)^{-\epsilon} {}_2F_1\left(-\epsilon, -\epsilon; 1-\epsilon; \frac{s_{ijk}-s_{ij}-s_{ik}}{s_{ijk}-s_{ik}}\right) \\
&\quad \left. - \left(\frac{-s_{ijk}s_{ij}s_{ik}}{(s_{ij}-s_{ijk})(s_{ik}-s_{ijk})}\right)^{-\epsilon} {}_2F_1\left(-\epsilon, -\epsilon; 1-\epsilon; \frac{s_{ijk}(s_{ijk}-s_{ij}-s_{ik})}{(s_{ijk}-s_{ij})(s_{ijk}-s_{ik})}\right) \right]
\end{aligned}$$

$$\begin{aligned}
a_1(s_{12}, s_{23}) &= \frac{s_{123}-s_{12}-s_{23}}{s_{123}-s_{12}} = -\frac{z}{1-x-z}\,, \\
a_2(s_{12}, s_{23}) &= \frac{s_{123}-s_{12}-s_{23}}{s_{123}-s_{23}} = z\,, \\
a_3(s_{12}, s_{23}) &= \frac{s_{123}\, s_{13}}{(s_{13}+s_{23})(s_{12}+s_{13})} = -\frac{x\,z}{1-x-z}
\end{aligned}$$

R_1

$$\begin{aligned}
\tilde{a}_1(s_{12}, s_{23}) &= 1 - \frac{1}{a_1(s_{12}, s_{23})} = \frac{1-x}{z}\,, \\
\tilde{a}_3(s_{12}, s_{23}) &= 1 - \frac{1}{a_3(s_{12}, s_{23})} = \frac{(1-x)(1-z)}{xz}
\end{aligned}$$

R_2

NNLO corrections: RR

RR: tree-level double-real matrix elements

$$C_{\rm RR}^{(2)} \sim \int {\rm d}\phi_3(k_p, k_j, k_k; q, k_i)\, |\mathcal{M}|^2_{\rm RR}(\{s_{ab}\})\, \delta\left(\hat{z} - \frac{s_{ip}}{s_{ip}+s_{ij}+s_{ik}}\right)$$

- correspond to cut two-loop integrals

$$C_{\rm RR}^{(2)} \sim \int {\rm d}^{4-2\epsilon}k_p {\rm d}^{4-2\epsilon}k_j\, |\mathcal{M}|^2_{\rm RR}(\{s_{ab}\})\, \delta^+(k_p^2)\, \delta^+(k_j^2)\, \delta^+((q+k_i-k_p-k_j)^2)\, \delta\left(\hat{z} - \frac{s_{ip}}{s_{ip}+s_{ij}+s_{ik}}\right)$$

- use Cutkosky rule to arrive at Standard integral form

$$\delta^+(p_i^2) = \frac{1}{2\pi i}\left(\frac{1}{p_i^2+i0} - \frac{1}{p_i^2-i0}\right)$$

$$I_{t,r,s}(p_1,\ldots,p_n) = \int \frac{d^dk}{(2\pi)^d}\frac{d^dl}{(2\pi)^d}\frac{1}{D_1^{m_1}\ldots D_t^{m_t}} S_1^{n_1}\ldots S_q^{n_q}$$

$$S_i = \{k\cdot p_j, l\cdot p_j\}$$
$$D_i = \{(k-p_j)^2, (l-p_j)^2, (k-l-p_j)^2\}$$

NNLO corrections: RR

Reduction to master integrals: integration-by-part (IBP) equations [K. Chetyrkin, F. Tkachov]

$$\int \frac{d^d k}{(2\pi)^d} \frac{d^d l}{(2\pi)^d} \frac{\partial}{\partial a^\mu} \left[b^\mu f(k,l,p_i) \right] = 0 \qquad \text{with } a^\mu = k^\mu, l^\mu; b^\mu = k^\mu, l^\mu, p_i^\mu$$

- yield large system of linear relations among integrals
- solved using lexicographic ordering (Laporta algorithm): Reduze2 [A. von Manteuffel, C.Studerus]

Computation of master integrals: differential equations [E.Remiddi, TG]

- differential equations in x and z derived at integrand level
- generic solution by direct integration
- specific solution (matching to boundary condition) by integration over z and comparison with inclusive RR integrals (DIS coefficient functions)

NNLO corrections: RR

Integrals [L.Bonino, M.Marcoli, R.Schürmann, G.Stagnitto, TG]

- 12 propagators (4 cut),
  7 of them linearly independent
- 13 integral families with
  total 21 master integrals
- analytical results throughout

$$\mathrm{Ti}_2(y) = \int_0^y \frac{\arctan x}{x} \mathrm{d}x$$

- family A,B,C previously computed for photon fragmentation
  [R.Schürmann, TG]

$$\begin{aligned}
D_1 &= (q - k_p)^2\,, \\
D_2 &= (p_i + q - k_p)^2\,, \\
D_3 &= (p_i - k_l)^2\,, \\
D_4 &= (q - k_l)^2\,, \\
D_5 &= (p_i + q - k_l)^2\,, \\
D_6 &= (q - k_p - k_l)^2\,, \\
D_7 &= (p_i - k_p - k_l)^2\,, \\
D_8 &= (k_p + k_l)^2\,, \\
D_9 &= k_p^2\,, \\
D_{10} &= k_l^2\,, \\
D_{11} &= (q + p_i - k_p - k_l)^2\,, \\
D_{12} &= (p_i - k_p)^2 + Q^2 \frac{z}{x}\,,
\end{aligned}$$

family	master	deepest pole	at $x = 1$	at $z = 1$
	$I[0]$	ϵ^0	$(1-x)^{1-2\epsilon}$	$(1-z)^{1-2\epsilon}$
A	$I[5]$	ϵ^{-1}	$(1-x)^{-2\epsilon}$	$(1-z)^{1-2\epsilon}$
	$I[2,3,5]$	ϵ^{-2}	$(1-x)^{-1-2\epsilon}$	$(1-z)^{-1-2\epsilon}$
B	$I[7]$	ϵ^0	$(1-x)^{1-2\epsilon}$	$(1-z)^{1-2\epsilon}$
	$I[-2,7]$	ϵ^0	$(1-x)^{1-2\epsilon}$	$(1-z)^{1-2\epsilon}$
	$I[-3,7]$	ϵ^0	$(1-x)^{1-2\epsilon}$	$(1-z)^{1-2\epsilon}$
	$I[2,3,7]$	ϵ^{-2}	$(1-x)^{-2\epsilon}$	$(1-z)^{-1-2\epsilon}$
C	$I[5,7]$	ϵ^{-1}	$(1-x)^{-2\epsilon}$	$(1-z)^{1-2\epsilon}$
	$I[3,5,7]$	ϵ^{-2}	$(1-x)^{-2\epsilon}$	$(1-z)^{-2\epsilon}$
D	$I[1]$	ϵ^0	$(1-x)^{-2\epsilon}$	$(1-z)^{-2\epsilon}$
	$I[1,4]$	ϵ^0	$(1-x)^{-2\epsilon}$	$(1-z)^{-2\epsilon}$
	$I[1,3,4]$	ϵ^{-1}	$(1-x)^{-2\epsilon}$	$(1-z)^{-1-2\epsilon}$
E	$I[1,3,5]$	ϵ^{-2}	$(1-x)^{-2\epsilon}$	$(1-z)^{-1-2\epsilon}$
G	$I[1,3,8]$	ϵ^{-2}	$(1-x)^{-2\epsilon}$	$(1-z)^{-1-2\epsilon}$
H	$I[1,4,5]$	ϵ^{-1}	$(1-x)^{-1-2\epsilon}$	$(1-z)^{-2\epsilon}$
I	$I[2,4,5]$	ϵ^{-2}	$(1-x)^{-1-2\epsilon}$	$(1-z)^{-2\epsilon}$
J	$I[4,7]$	ϵ^0	$(1-x)^{-2\epsilon}$	$(1-z)^{-2\epsilon}$
	$I[3,4,7]$	ϵ^{-1}	$(1-x)^{-2\epsilon}$	$(1-z)^{-2\epsilon}$
K	$I[3,5,8]$	ϵ^{-2}	$(1-x)^{-1-2\epsilon}$	$(1-z)^{-2\epsilon}$
L	$I[4,5,7]$	ϵ^{-1}	$(1-x)^{-1-2\epsilon}$	$(1-z)^{-2\epsilon}$
M	$I[4,5,8]$	ϵ^{-1}	$(1-x)^{-1-2\epsilon}$	$(1-z)^{-2\epsilon}$

Table 1. Summary of the double real radiation master integrals.

NNLO corrections: RR

Example: I[3,5,8]

$$\frac{\partial I[358](Q^2,x,z)}{\partial Q^2} = -\frac{2(1+\epsilon)}{Q^2} I[358](Q^2,x,z)\,,$$
$$\frac{\partial I[358](Q^2,x,z)}{\partial x} = \left(\frac{1+2\epsilon}{1-x} + \frac{2+2\epsilon}{x}\right) I[358](Q^2,x,z)\,,$$
$$\frac{\partial I[358](Q^2,x,z)}{\partial z} = -\frac{1+2\epsilon}{z} I[358](Q^2,x,z) - \frac{2x^3(1-2\epsilon)^2(1+z)}{(Q^2)^3(1-x)^2\epsilon z^2(1-z)^2} I[0](Q^2,x,z)$$
$$+\frac{2x^2\epsilon}{(Q^2)^2(1-x)z^2} I[5](Q^2,x,z)\,.$$

$$I_{\rm inc}[358](Q^2,x) = \frac{3(1-2\epsilon)(4-6\epsilon)(2-6\epsilon)}{\epsilon^3}\frac{x^3}{(Q^2)^3(1-x)^2} I[0](Q^2,x)\,,$$

$$I_{\rm inc}[0](Q^2,x) = N_\Gamma (Q^2)^{1-2\epsilon}(1-x)^{1-2\epsilon}x^{-1+2\epsilon}\frac{\Gamma(2-2\epsilon)\Gamma(1-\epsilon)}{\Gamma(3-3\epsilon)}\,.$$

$$I[358](Q^2,x,z) = N_\Gamma \left(\frac{1-2\epsilon}{\epsilon}\right)^2 (Q^2)^{-2-2\epsilon}(1-x)^{-1-2\epsilon}x^{2+2\epsilon}z^{-1-2\epsilon}$$
$$\times\Bigg(-2(1-z)^{-2\epsilon}z^{\epsilon} + 2z^{\epsilon}\,{}_3F_2(\epsilon,\epsilon,2\epsilon,1+\epsilon,1+\epsilon,z)$$
$$-\frac{2\epsilon\Gamma(1-2\epsilon)\Gamma(1+\epsilon)}{\Gamma(1-\epsilon)}(\pi\cot(\pi\epsilon)+\ln(z))\Bigg)\,.$$

Numerical results

$$\frac{d^3\sigma^{\pi^+}}{dx dy dz} = \frac{4\pi\alpha^2}{Q^2}\left[\frac{1+(1-y)^2}{2y} F_T^{\pi^+}(x,z,Q^2) + \frac{1-y}{y} F_L^{\pi^+}(x,z,Q^2)\right]$$

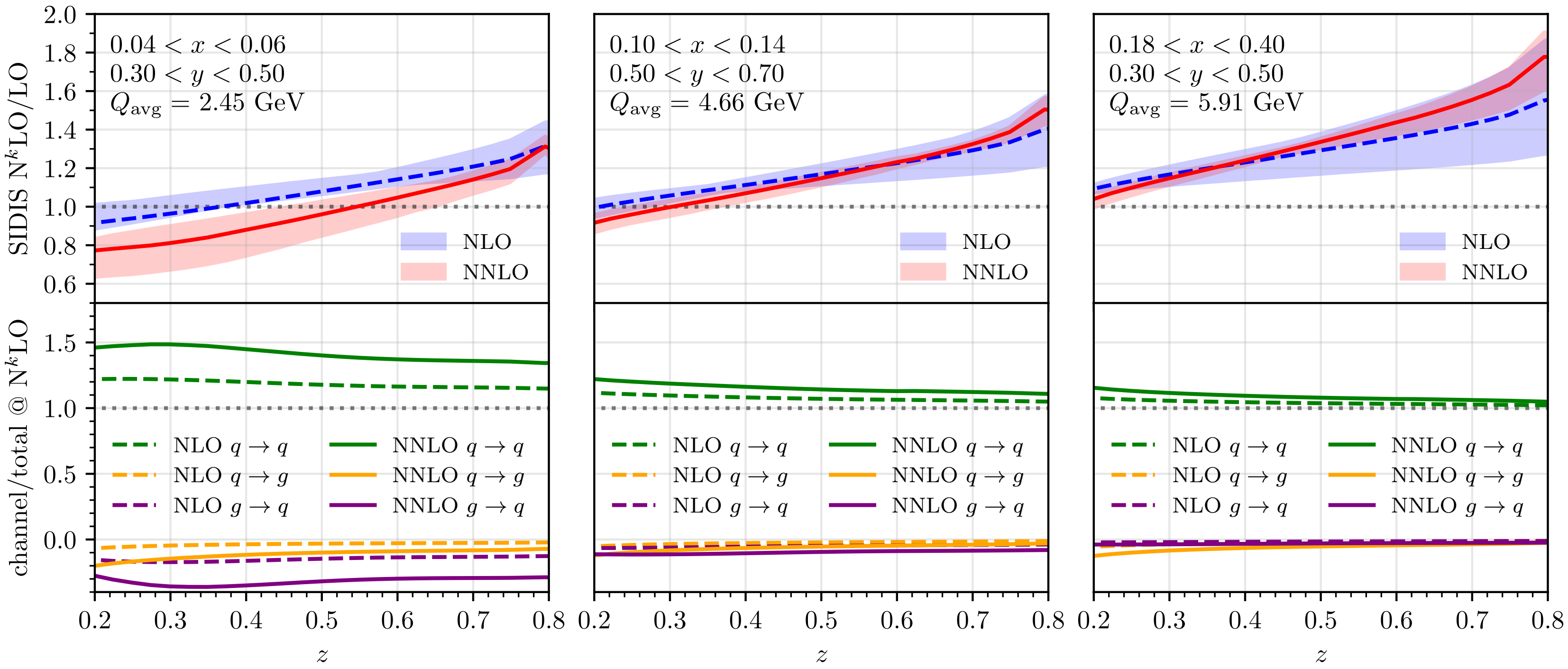
Unpolarized SIDIS

[L.Bonino, G.Stagnitto, TG]

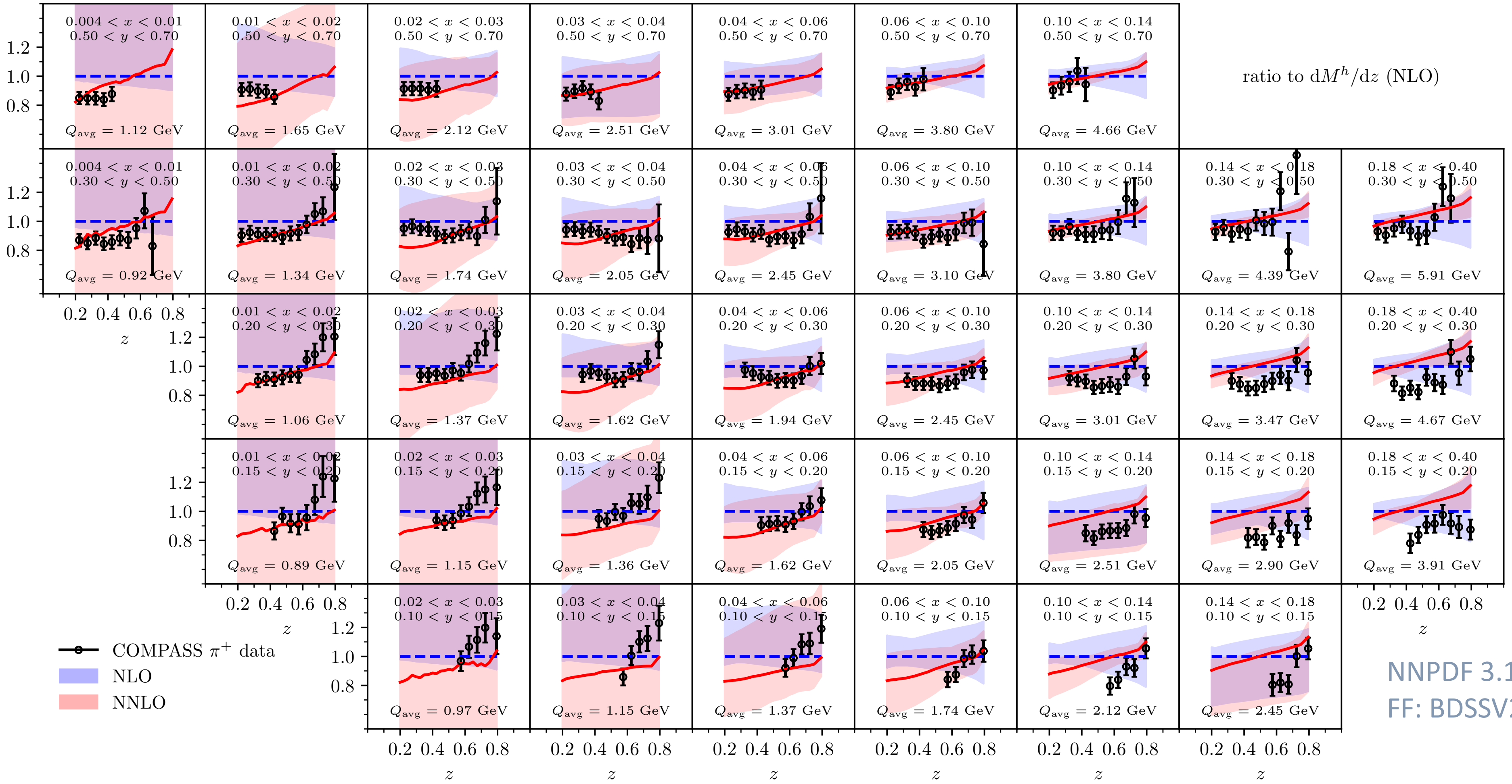
- COMPASS kinematics

$$\sqrt{s} = 17.35 \text{ GeV}$$
$$Q^2 > 1 \text{ GeV}^2$$
$$W > 5 \text{ GeV}$$

- PDF: NNPDF3.1
- FF: BDSSV22 [I.Borsa, D.de Florian, R.Sassot, M.Stratmann, W.Vogelsang]
- 7-point scale variation



Numerical results: π^+ multiplicity



Experiment	# data	NLO	Approx. NNLO	NNLO
SIA	288	1.05	0.96	0.85
COMPASS	510	0.98	1.14	0.96
HERMES	224	2.24	2.27	2.52
TOTAL	1022	1.27	1.33	1.26

[I.Borsa: BDSSV22 revisited]

NNPDF 3.1
FF: BDSSV22

Numerical results

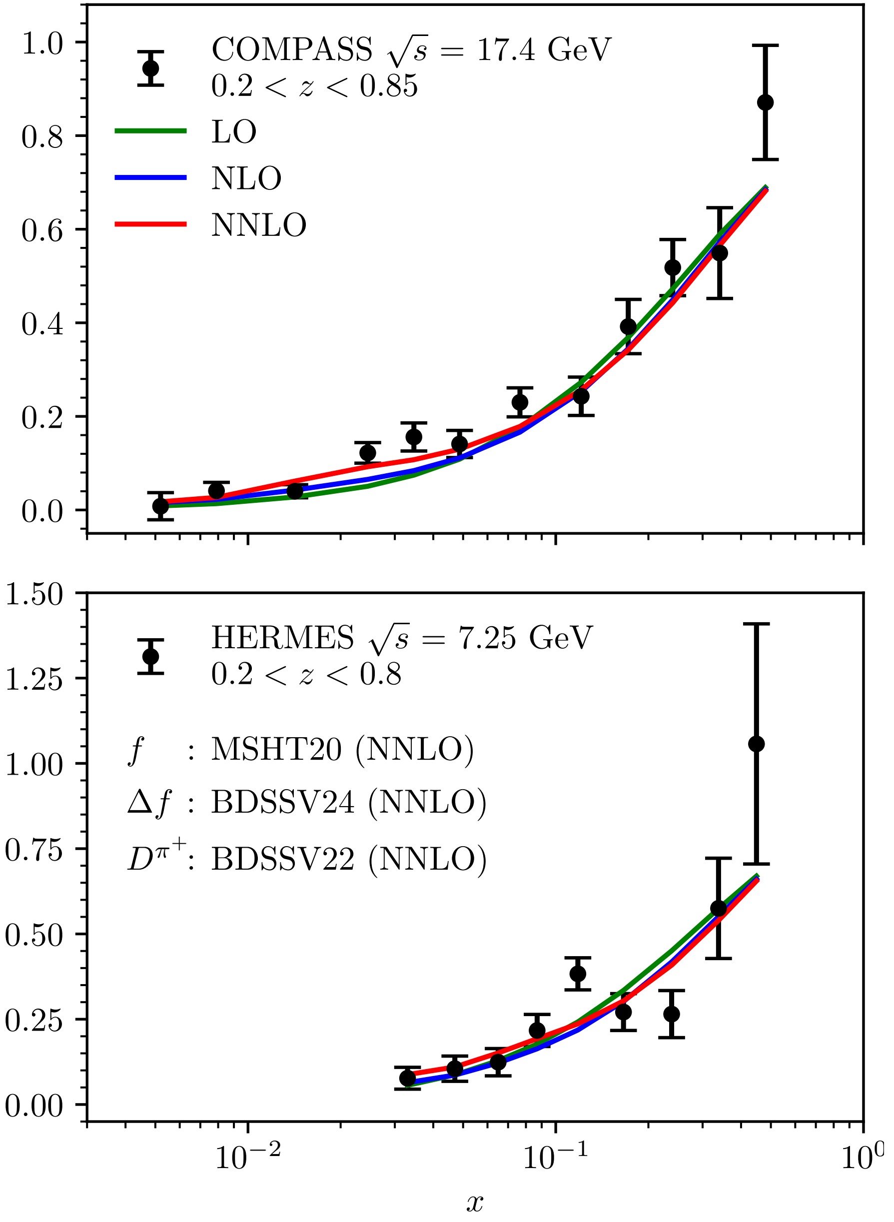
Polarized SIDIS [L.Bonino, M.Löchner, K.Schönwald, G.Stagnitto, TG]

- use Larin γ_5
- finite scheme transformation of polarized PDFs to $\overline{\text{MS}}$ scheme

$$2g_1^h(x,z,Q^2) = \sum_{p,p'} \int_x^1 \frac{d\hat{x}}{\hat{x}} \int_z^1 \frac{d\hat{z}}{\hat{z}} \Delta f_p\left(\frac{x}{\hat{x}},\mu_F\right) D_{p'}^h\left(\frac{z}{\hat{z}},\mu_A\right) \times \Delta C_{p'p}\left(\hat{x},\hat{z},Q^2,\mu_R,\mu_F,\mu_A\right)$$

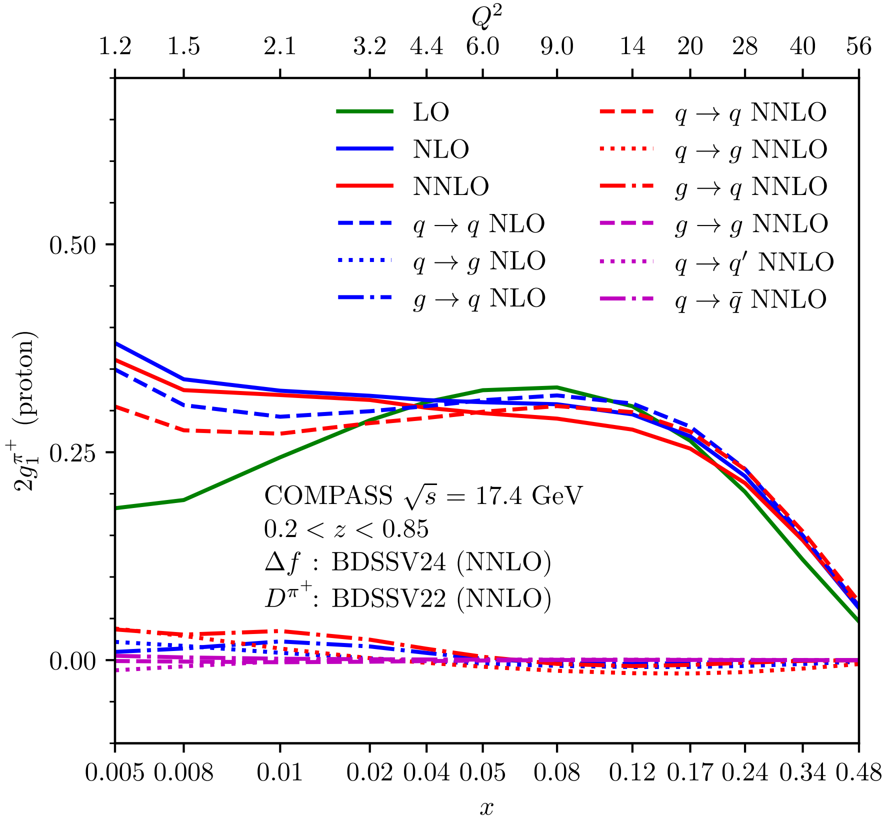
- PDF: BDSSV24 (pol) / MSHT20 (unpol)
  [I.Borsa, D.de Florian, R.Sassot, M.Stratmann, W.Vogelsang; S.Bailey, T.Cridge, L.Harland-Lang, A.Martin, R.Thorne]
- FF: BDSSV21

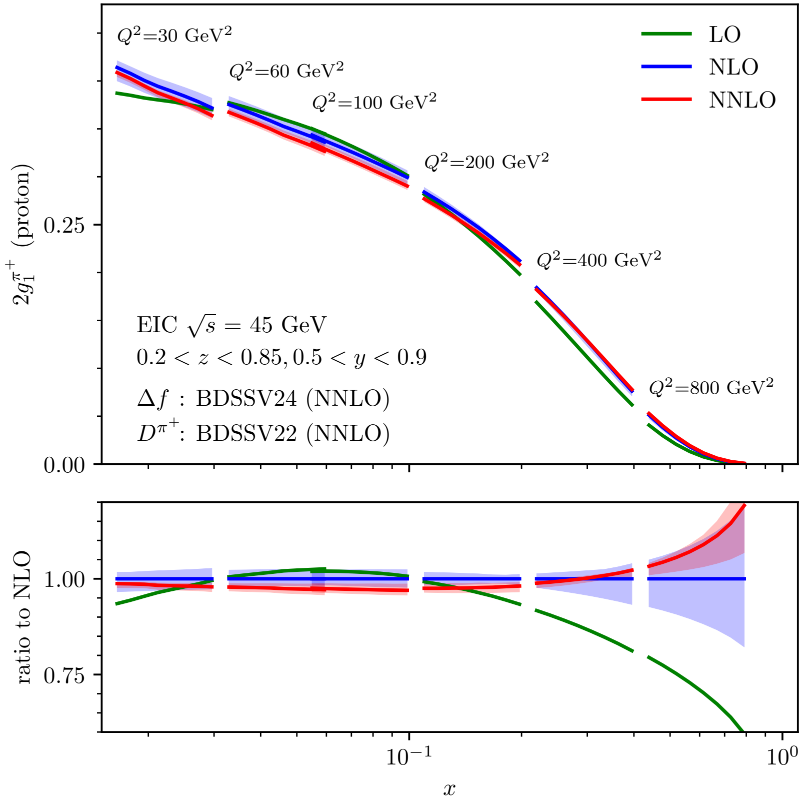
$$A_1^h(x,z,Q^2) = \frac{g_1^h(x,z,Q^2)}{F_1^h(x,z,Q^2)}$$



Numerical results

Polarized SIDIS [L.Bonino, M.Löchner, K.Schönwald, G.Stagnitto, TG]





- analytical agreement with competing group [S.Goyal, R.Lee, S.Moch, V.Pathak, N.Rana, V.Ravindran]

Neutrino-induced SIDIS

Charged-current DIS reaction

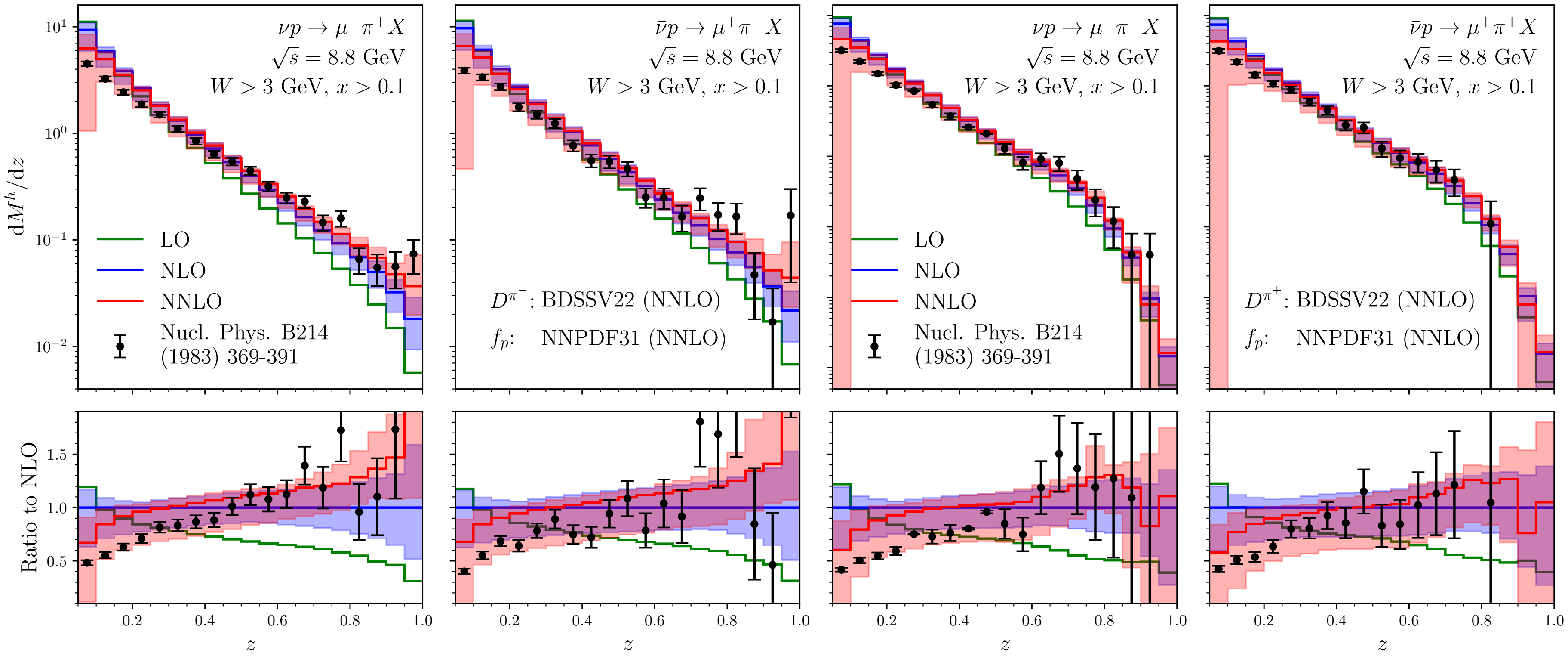
- legacy data from bubble chamber experiments at CERN and Fermilab $\nu p \to l^{\pm} h^{\pm} X$
- prepare for charged-current processes at EIC: $e^{\pm} p \to \nu h^{\pm} X$
- four charge combinations measured
  - $\nu p \to l^{-} \pi^{+} X$
  - $\nu p \to l^{+} \pi^{-} X$
  - $\nu p \to l^{-} \pi^{-} X$
  - $\nu p \to l^{+} \pi^{+} X$
- charged current selects quark charge
- charge hierarchy of fragmentation functions

$$D_u^{\pi^+}(z) > D_d^{\pi^+}(z)$$

- same-charge combinations probe subdominant FF (sea content of final hadron)

Neutrino-induced SIDIS

Impact of NNLO corrections [L.Bonino, M.Löchner, K.Schönwald, G.Stagnitto, TG]

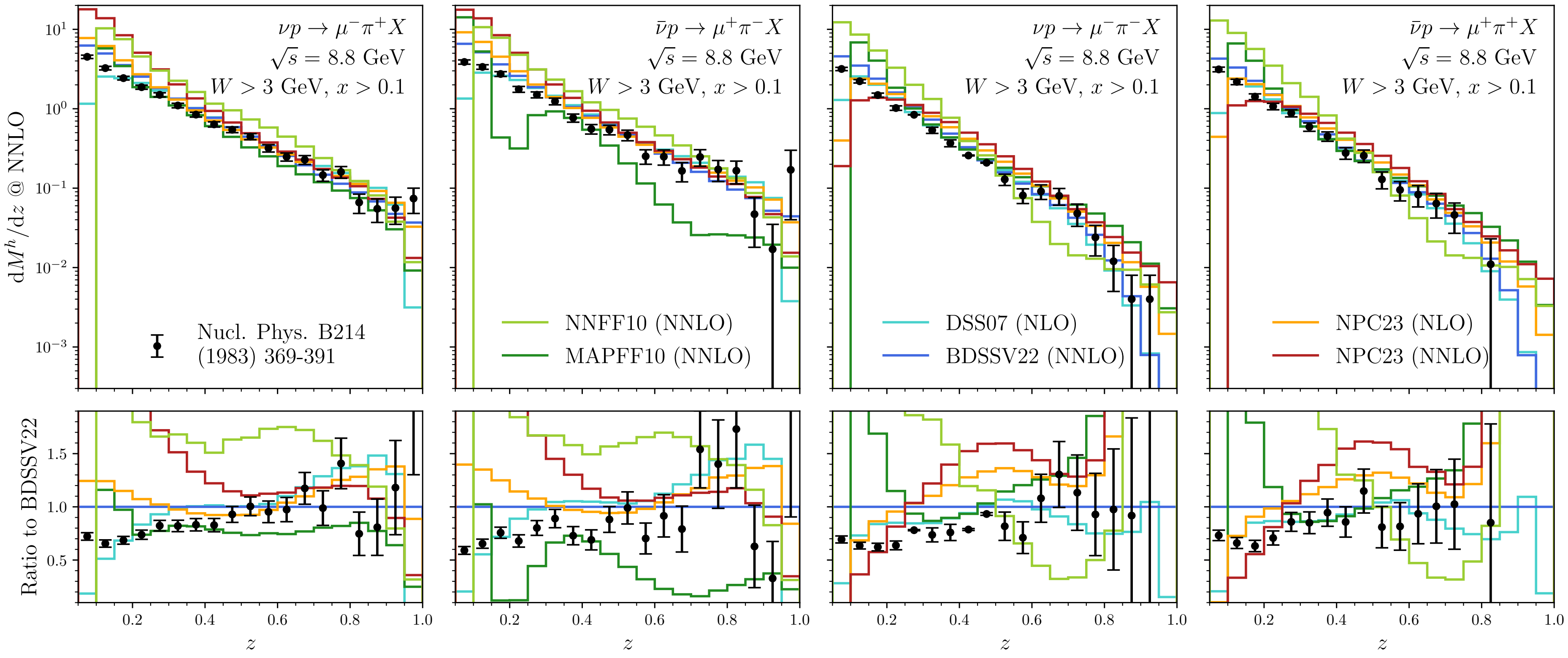


flavour-favoured

flavour-unfavoured

Neutrino-induced SIDIS

Sensitivity on fragmentation functions

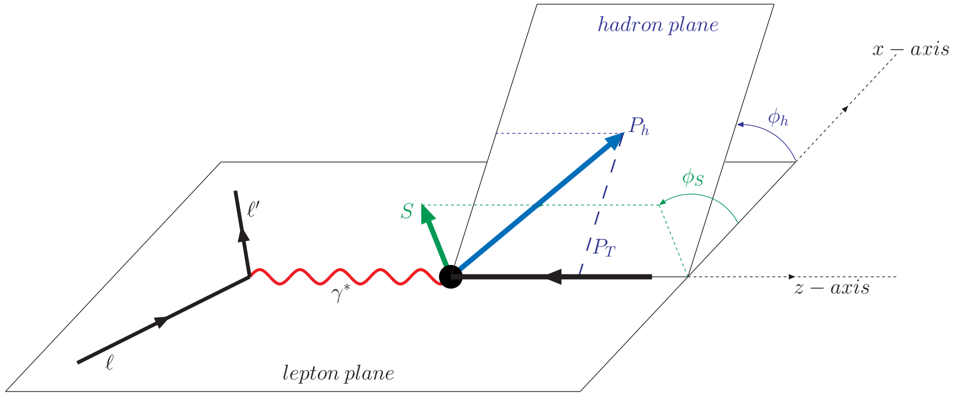


d → u → π+ u→ d → π- d → u → π- u→ d → π+

Outlook

Fully differential SIDIS kinematics

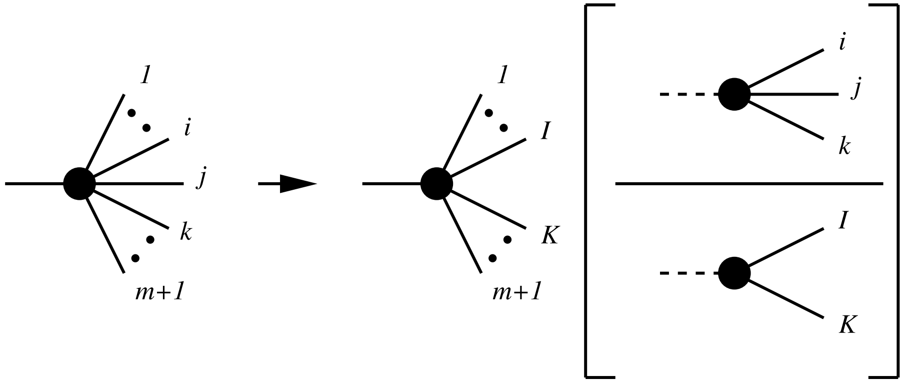
$$(x, Q^2, z, p_T^h, \phi_h)$$



- requires recoil at parton level
- Born processes eq → eqg and eg → eqq are $O(\alpha_s)$
- analogous to two-jet production in DIS

NNLO corrections to fully differential SIDIS

- NNLO parton-level event generator: separate numerical evaluation of RR, RV, VV
- requires infrared subtraction scheme at NNLO
- including initial-state polarization
- antenna subtraction for hadron fragmentation processes up to NNLO
  [L.Bonino, M.Marcoli, R.Schürmann, G.Stagnitto, TG]



Summary

NNLO corrections to SIDIS coefficient functions now available

- fully analytical expressions
- enable precision phenomenology with SIDIS data
- combine with numerical NNLO results on $pp \longrightarrow h+X$
- allow consistent NNLO fits of fragmentation functions and polarized PDFs

Fully differential SIDIS

- kinematics of DIS 2j production
- precision calculations: numerical parton-level event generation
- in principle feasible at NNLO with present-day methods