Bose gases out of equilibrium: from Turbulence to Coarsening

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Compressible Turbulence:
From Cold Atoms to Neutron Star Mergers
24 Jun 2025

Science and Technology Facilities Council

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DesOEQ

People involved in the work

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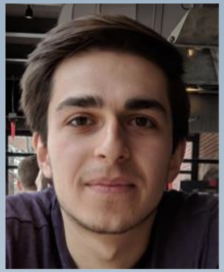
Andrey Karailiev



Gevorg Martirosyan

Jiří Etrych

Simon Fisher

Seb Morris

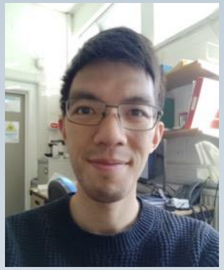
Maciej Gałka

Tanish Satoor

Christoph Eigen

Panos Christodoulou

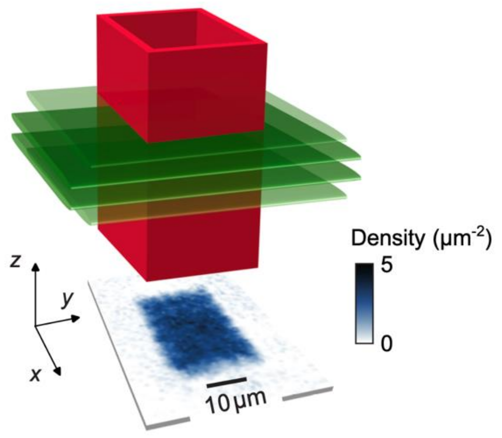
Julian Schmitt



Chris Ho

Experimental tools: ^{39}K Bose–Einstein Condensate in a box

2D Box trap (BEC 2)

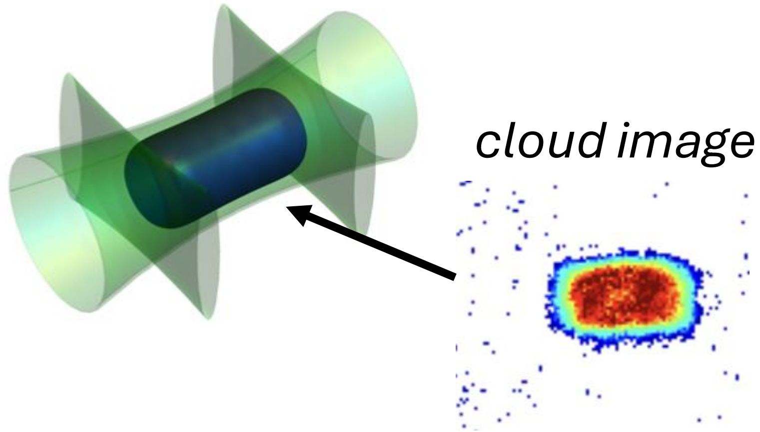


≈ 30–70k atoms

$\omega_z/(2\pi) \approx 1-3$ kHz

Trap depth: up to $k_B \times 800$ nK

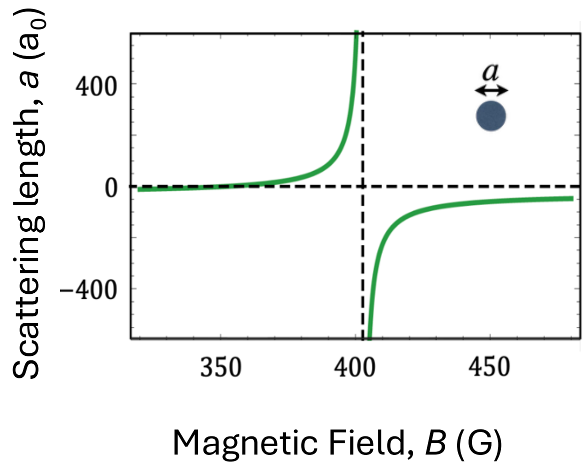
3D Box trap (BEC3)



up to ≈ 300k atoms

Trap depth: up to $k_B \times 250$ nK

Tunable interactions



Feshbach resonance
(with zero crossing)

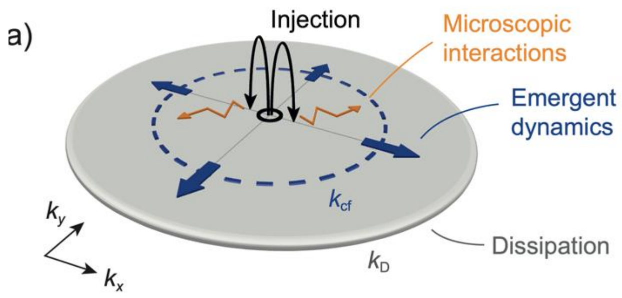
Turbulence in Bose gases

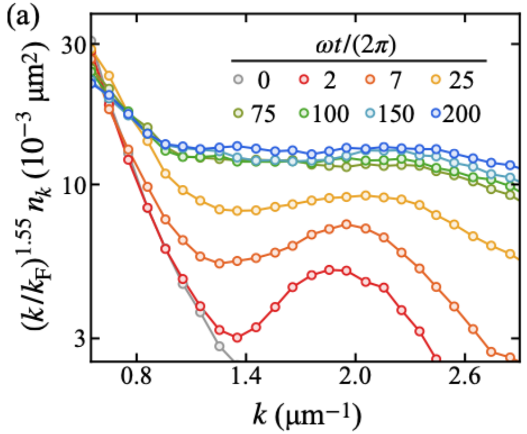
1. Driven steady-state turbulence

(2D gas)

Galka et.al., PRL **129**, 190402 (2022)

Karailiev et. al., PRL **133**, 243402 (2024)



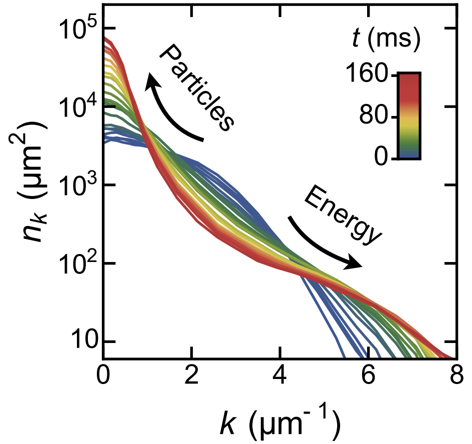


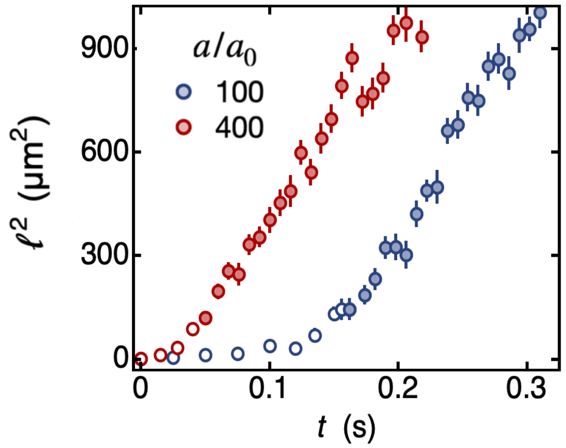
2. Free turbulence & universal coarsening

(2D & 3D gas)

Gazo et.al., arXiv:2312.09248 (2023)

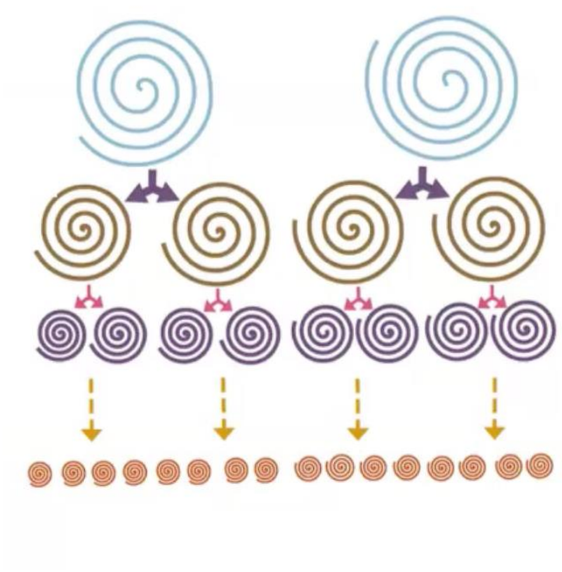
Martirosyan et. al., arXiv: 2411.19948 (2024)

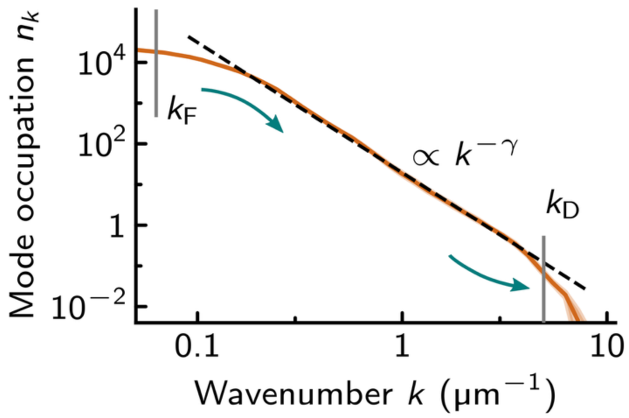




(Wave) turbulence

Direct cascade





Big whorls have little whorls
Which feed on their velocity,
And little whorls have lesser whorls
And so on to viscosity.

Lewis Richardson

Here:

whirls → (dominantly) waves

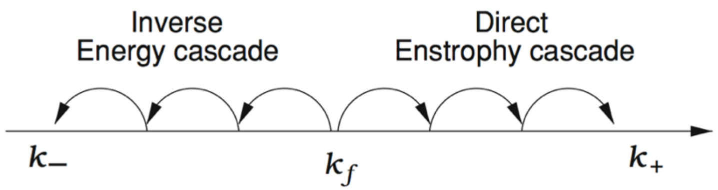
viscosity → trapdepth

Key signature:

Power law $n_k \sim k^{-\gamma}$

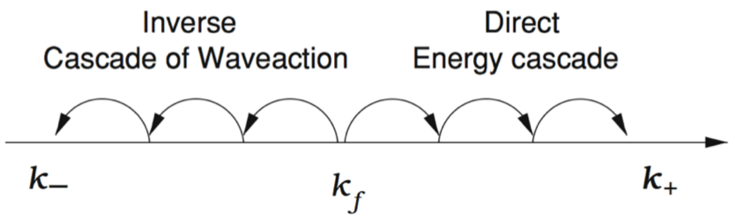
(Vortex) turbulence *elsewhere*

2D turbulence

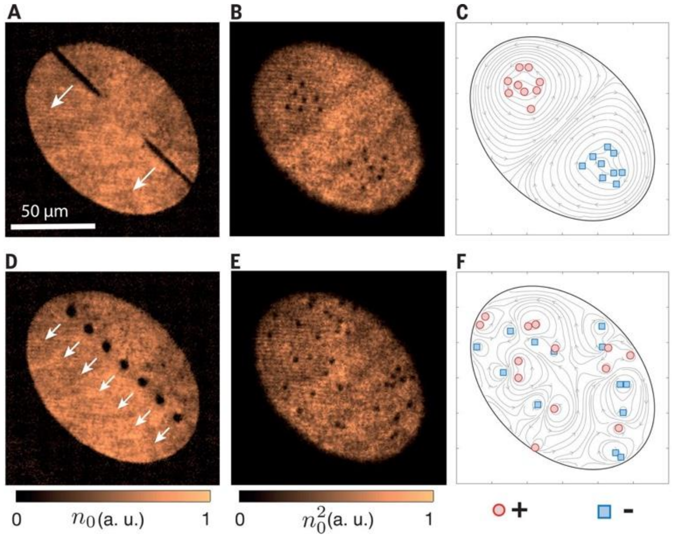


Drawing: Nazarenko

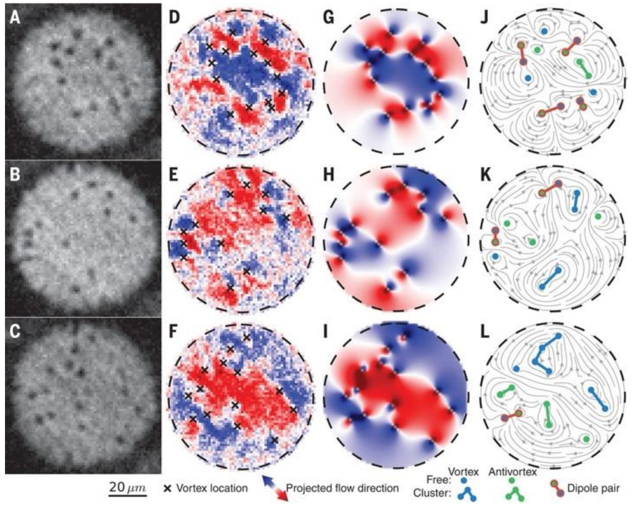
Weak wave turbulence



(Vortex turbulence)

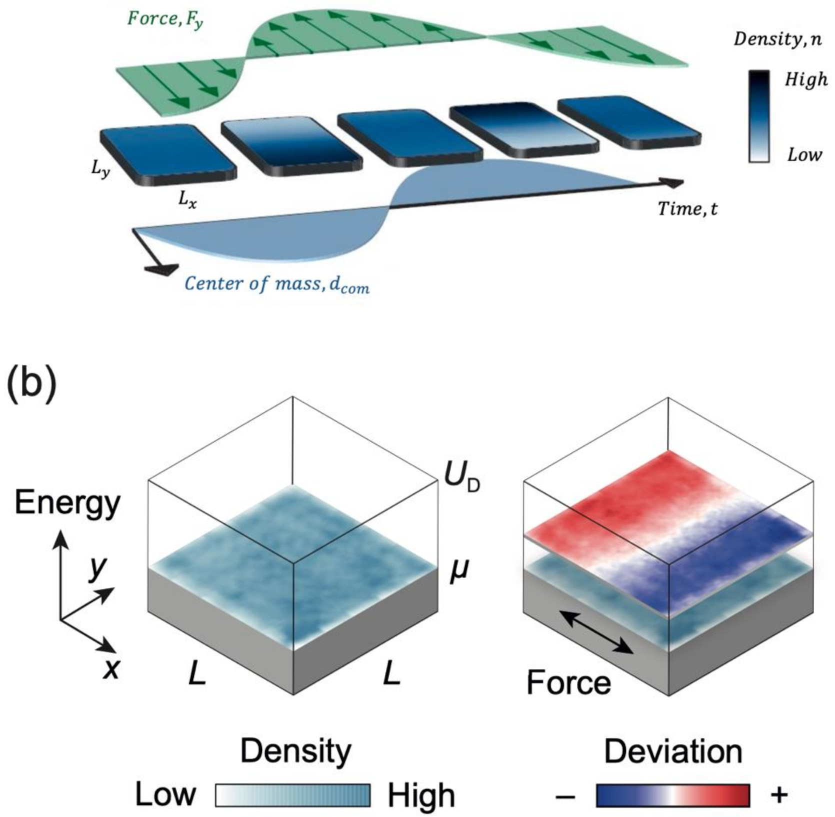


Queensland



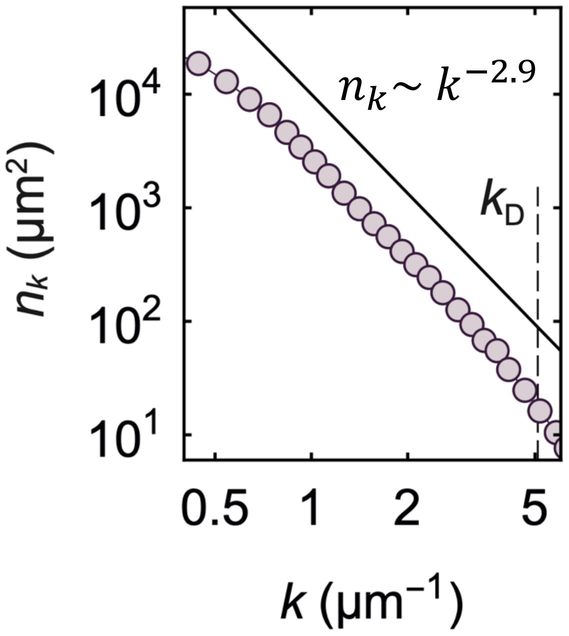
Monash

Steady-state turbulence in 2D gas



?

TOF, steady-state

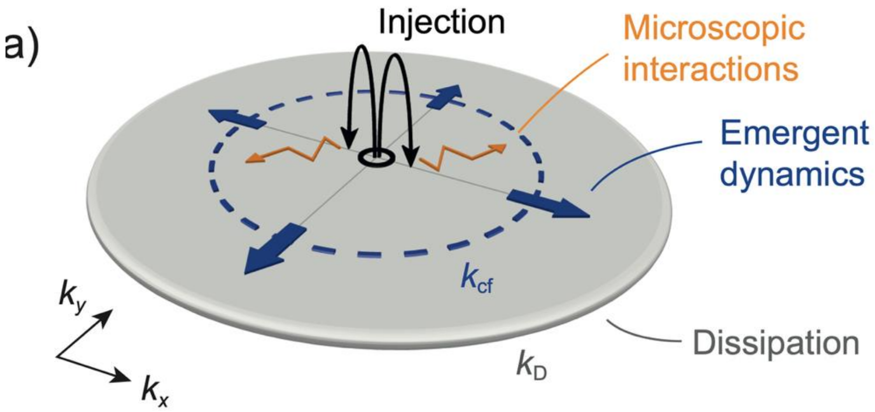


$$\gamma = 2.9 \neq d = 2$$

Usually $\gamma = d$, here finite-size effects

→ log-corrections

Emergence of the cascade



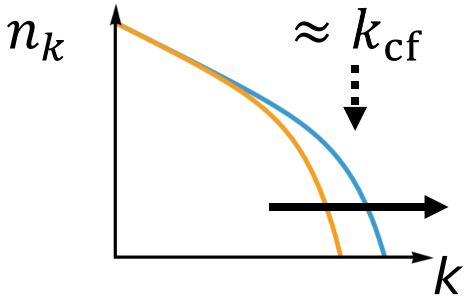
Key concepts:

1) Emergent isotropy

2) Dynamic scaling of momentum distributions

Cascade front: $k_{\mathrm{cf}} \sim t^{-\beta}$

In its wake: $n_k \sim k^{-\gamma}$



More generally:

$$n_k(k,t) = \left(\frac{t}{t_0}\right)^{\alpha} n_k\left[\left(\frac{t}{t_0}\right)^{\beta} k, t_0\right]$$

Here : $\alpha = \gamma\beta$

$\beta = 1/(\gamma - d - 2)$

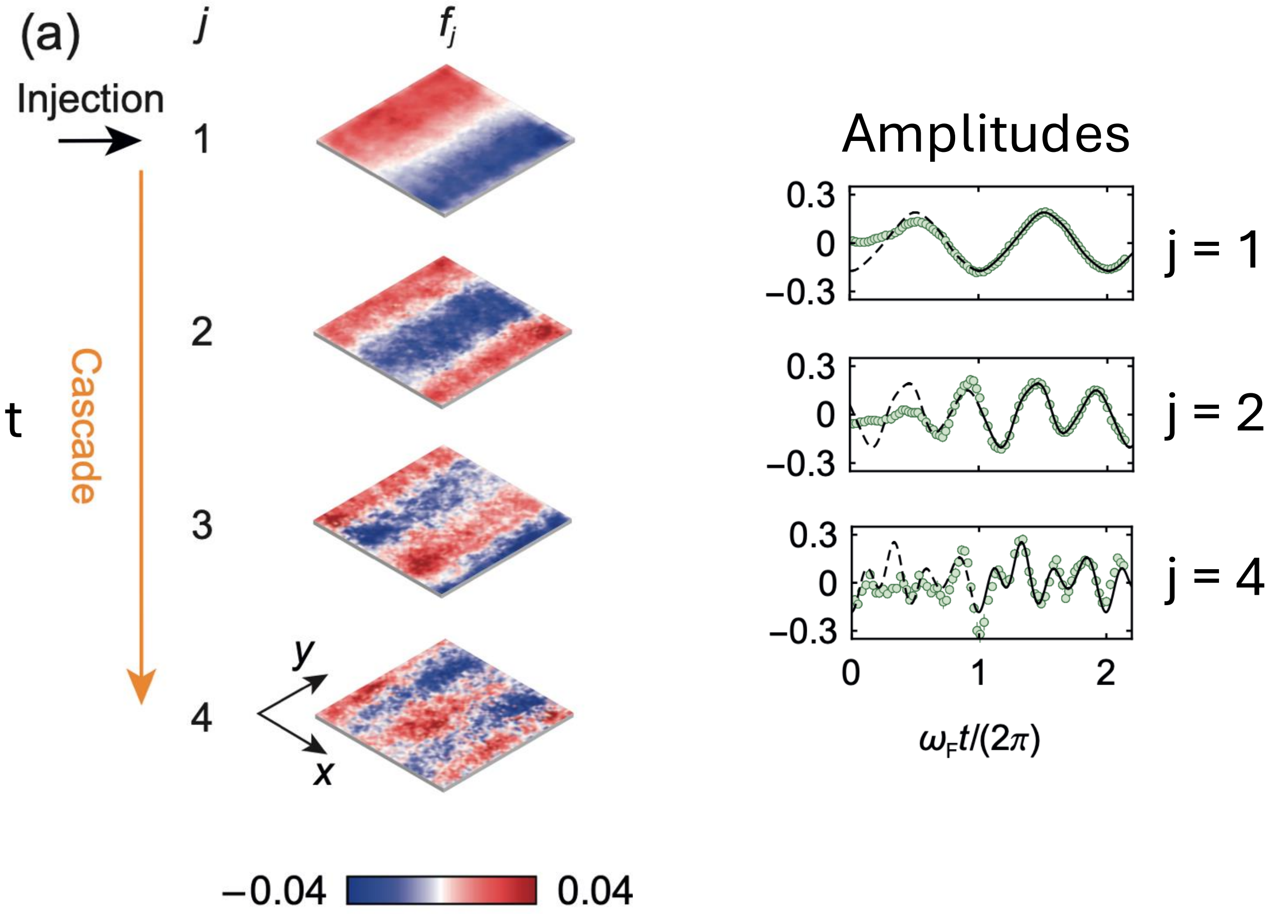
... also known in:
KPZ, NTFP,
phase-ordering

1) Isotropy of the cascade

Low k: PCA (in-situ)

Persistent anisotropy

Off resonant

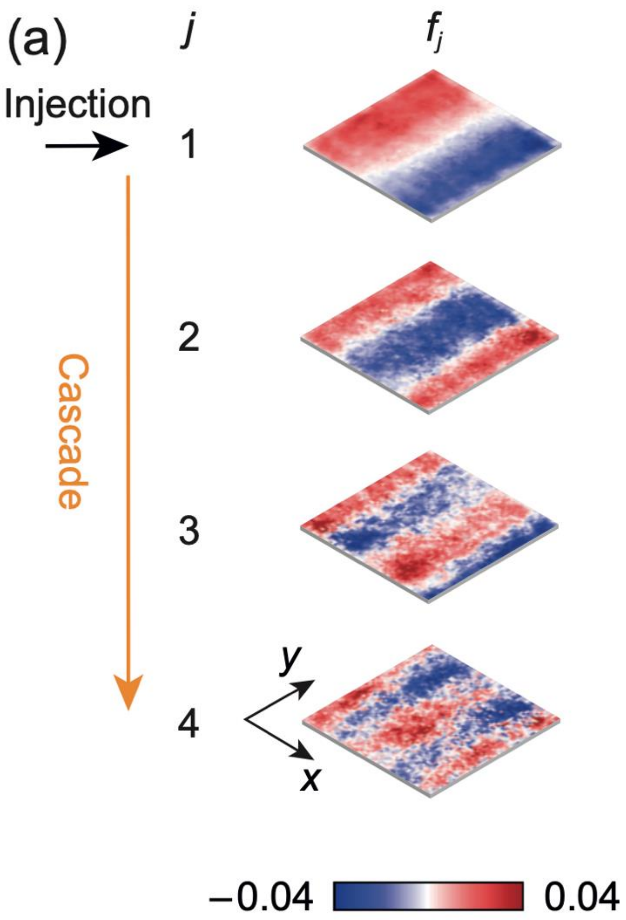


anisotropic acoustic cascade?

1) Isotropy of the cascade

Low k: PCA (in-situ)

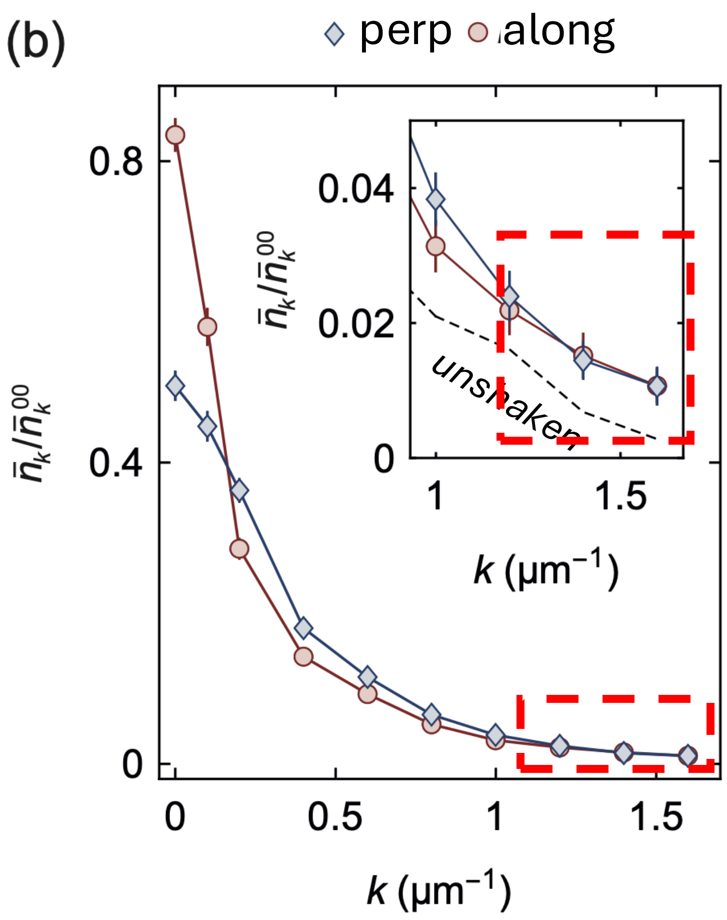
Persistent anisotropy



Intermediate k: Bragg spectroscopy ($@\omega t = 1$)

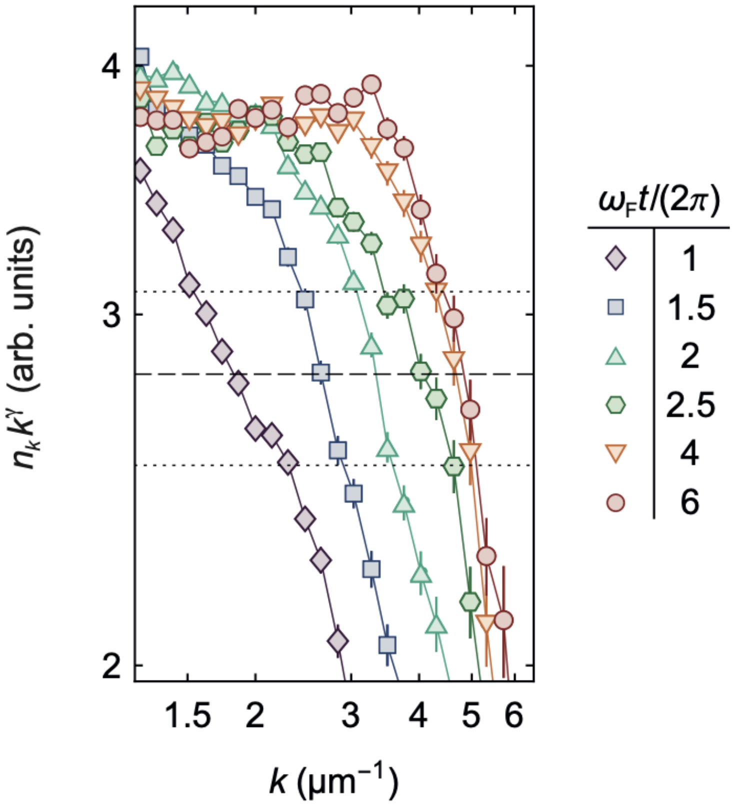
Emergent isotropy

$(k_\xi \approx 1.0\ \mu\mathrm{m}^{-1})$

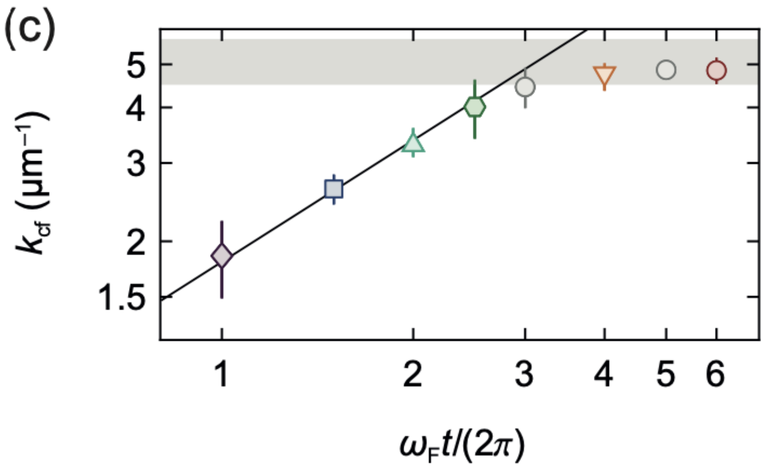


2) Dynamic scaling – large k

Plot $k^{\gamma} n_k$ $(\gamma = 2.9)$

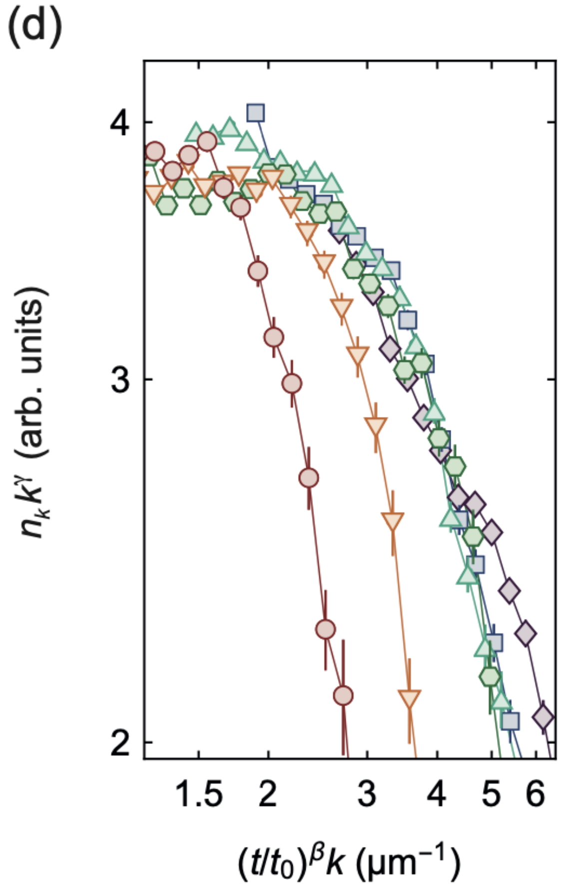


Predict $\beta = \frac{1}{\gamma - d - 2} \approx -0.91$

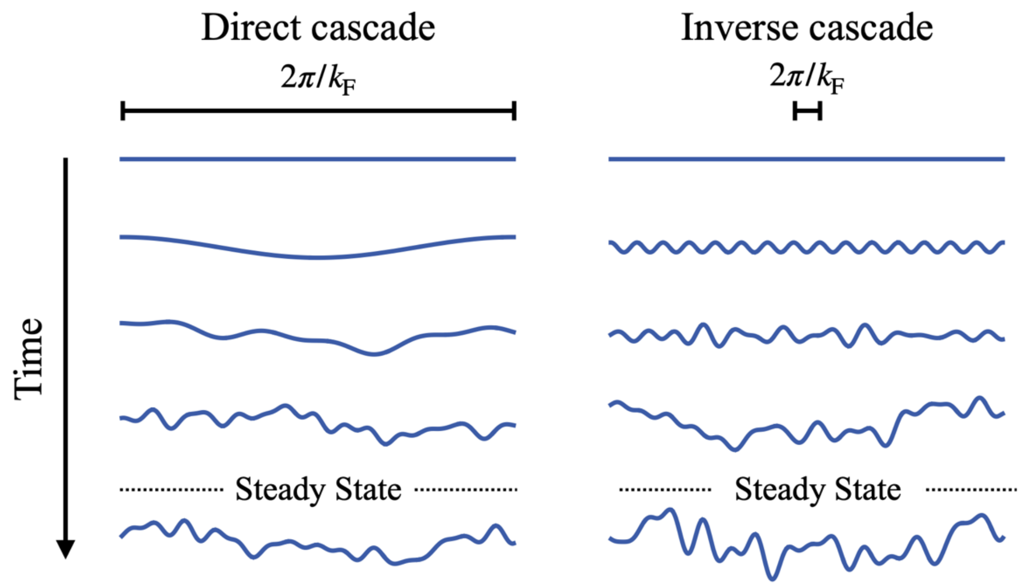


Or fit: $d = \gamma - 2 - \frac{1}{\beta} = 2.1(2)$

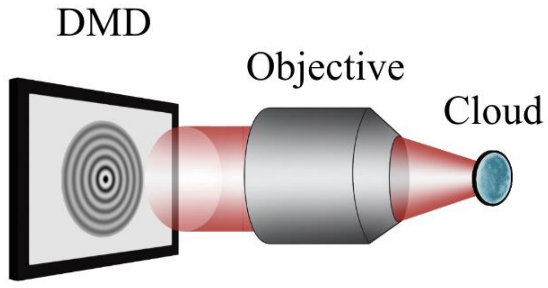
Scaling + breakdown

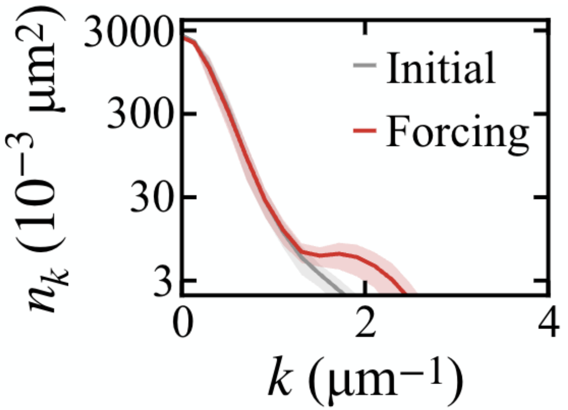


Inverse wave cascade in 2D gas



Isotropic excitation $k_F \approx 2\ \mu\text{m}^{-1} > k_\xi = 0.8\ \mu\text{m}^{-1}$



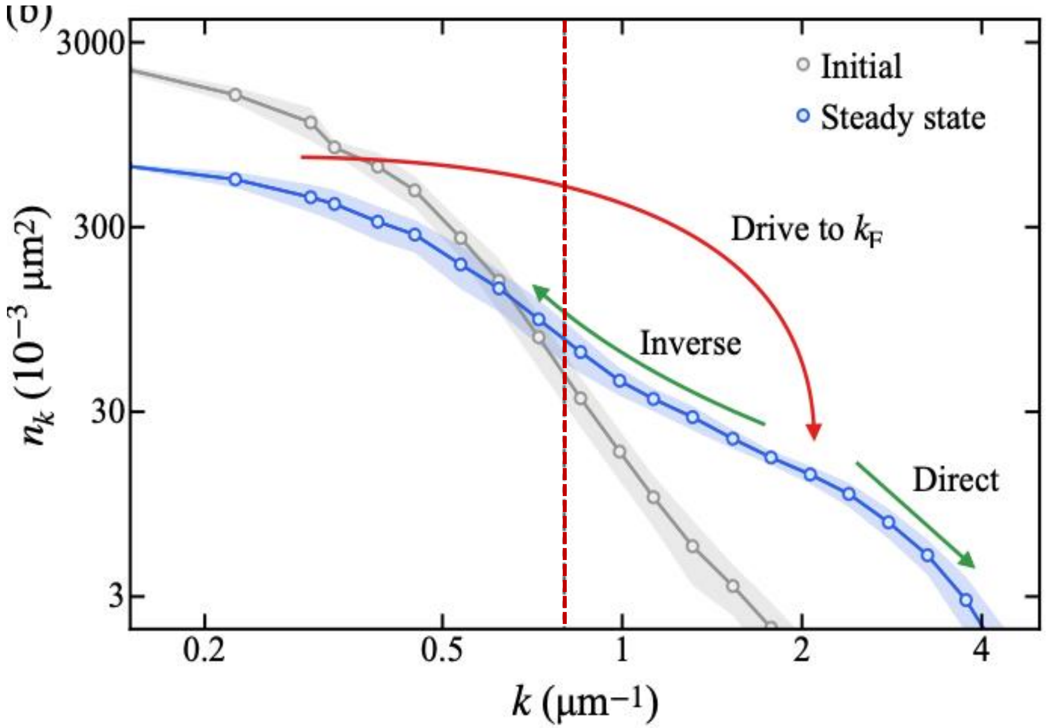


Dynamic potential

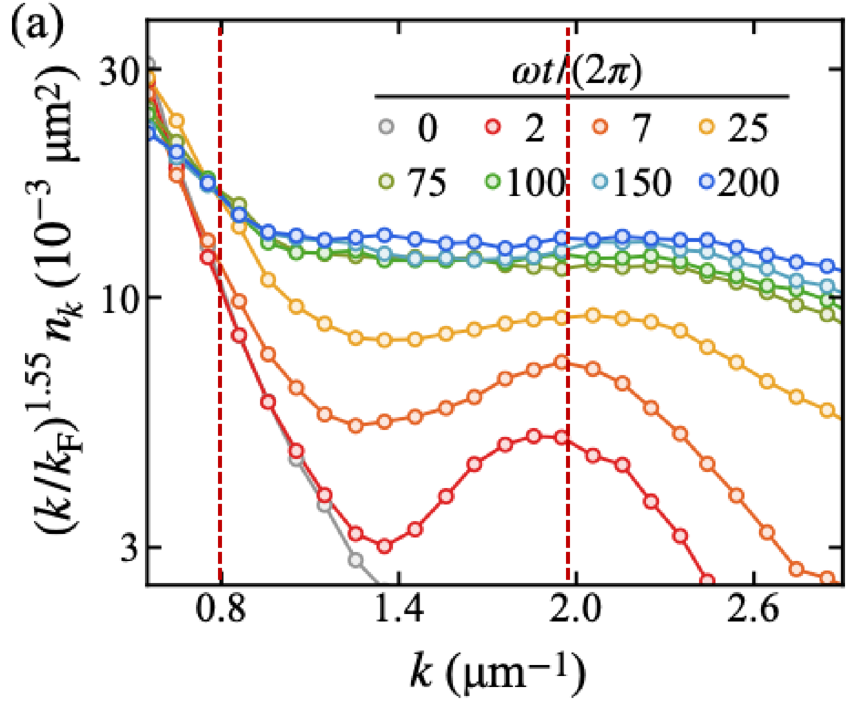
$\omega/(2\pi) \approx 700$ Hz

Inverse wave cascade in 2D gas

Long-time steady state ($\omega t > 75$)



Look at: $k_\xi < k < k_F$



Dual cascade: Inverse & Direct

Dissipation k_D finite → direct cascade with Π_N

Injection k_F finite → inverse cascade with Π_E

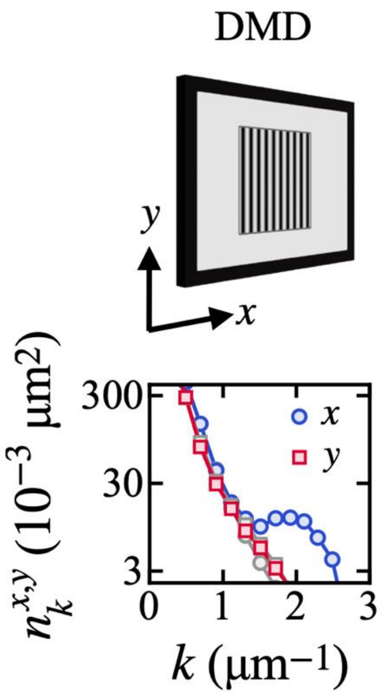
Experiment: $\gamma = 1.55(15)$

Theory: $\gamma = 4/3$

(ignoring all problems)

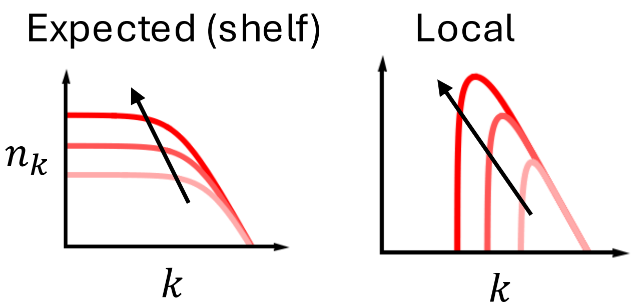
(Weakly) nonlocal cascade, reflected fluxes

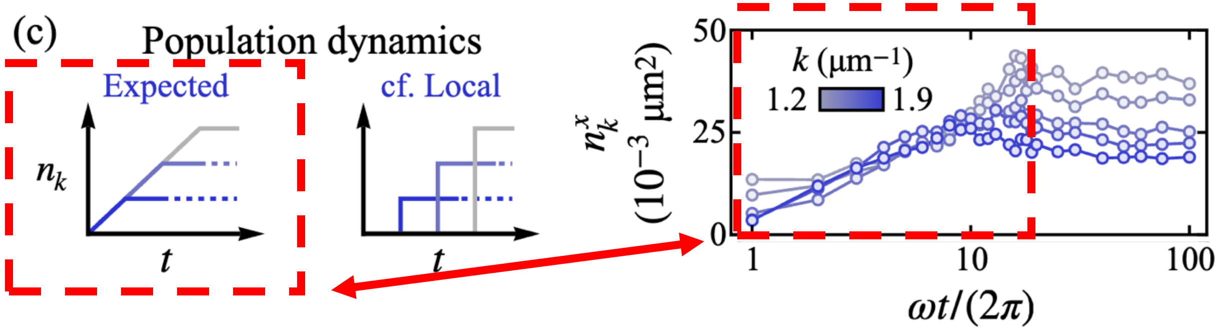
Anisotropic drive



Get still $\sim k^{-1.5}$ along the drive

Nonlocality of the cascade





Qualitatively similar

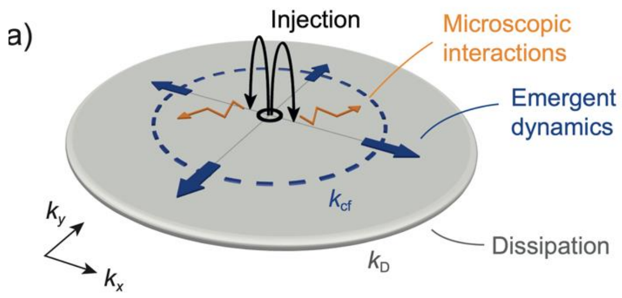
Turbulence in Bose gases

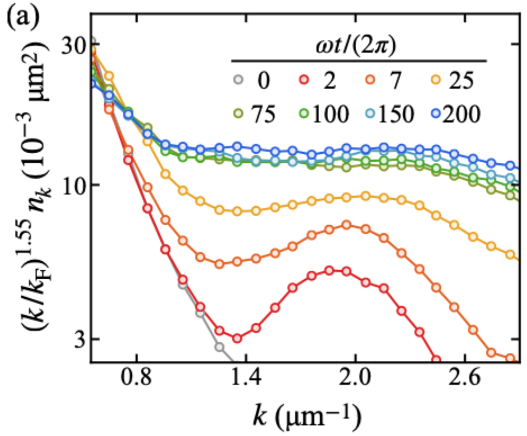
1. Driven steady-state turbulence

(2D gas)

Galka et.al., PRL **129**, 190402 (2022)

Karailiev et. al., PRL **133**, 243402 (2024)





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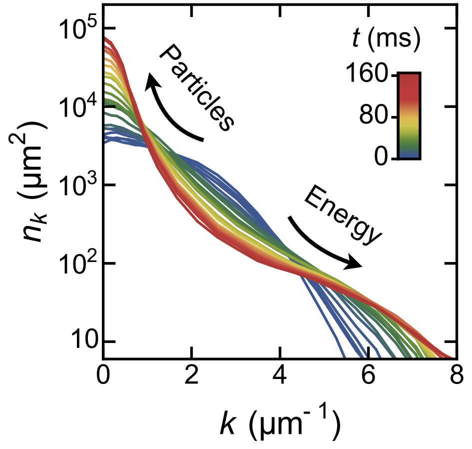
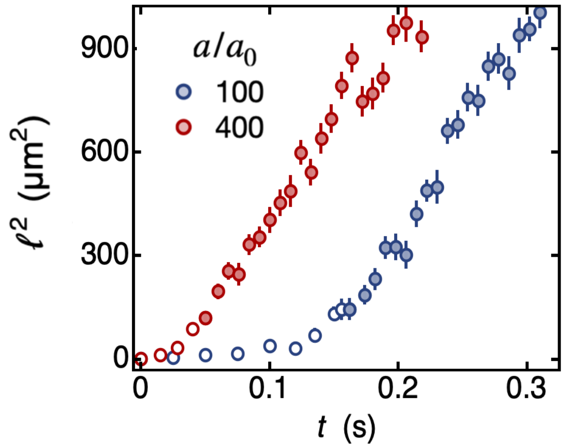
(2D & 3D gas)

Gazo et.al., arXiv:2312.09248 (2023)

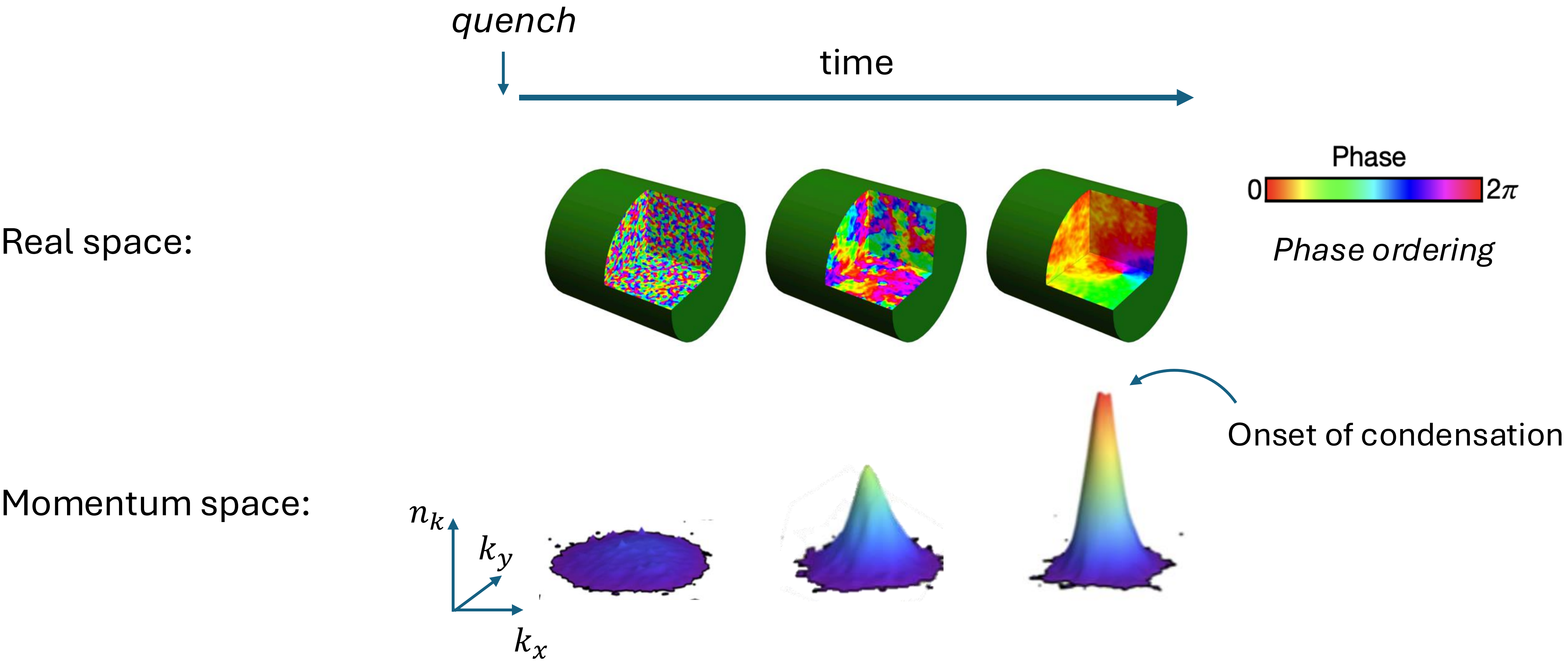
Martirosyan et. al., arXiv: 2411.19948 (2024)



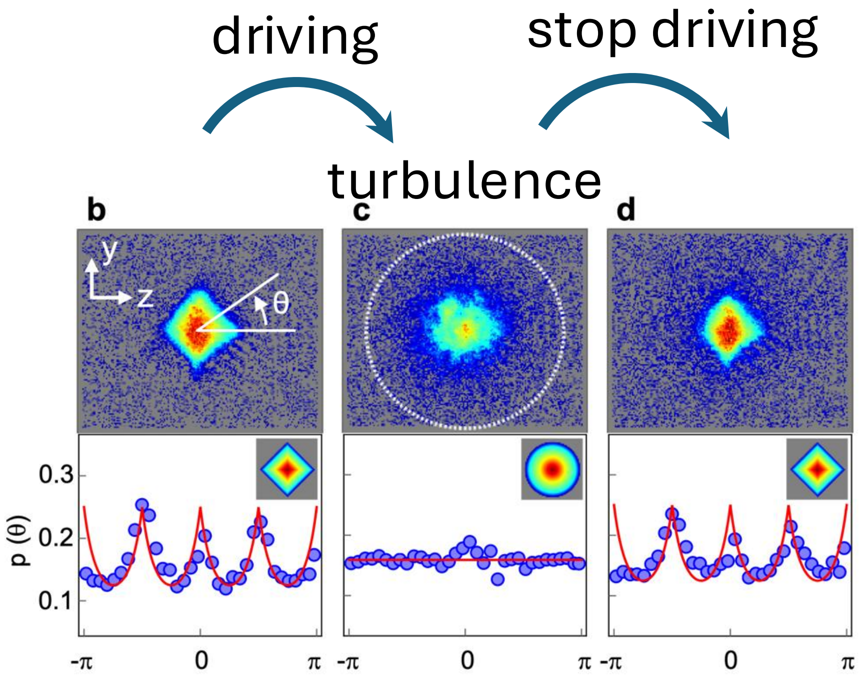


How does a Bose–Einstein condensate form?

Typical situation: a rapid <u>quench</u> through a phase transition into the <u>ordered phase</u>



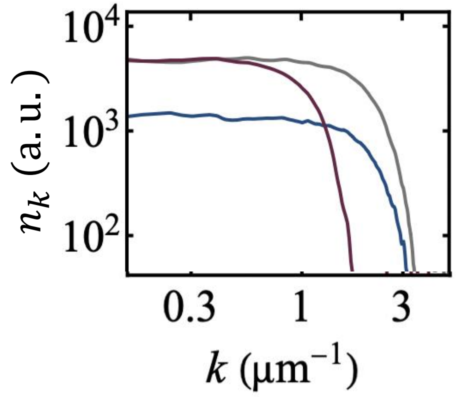
Preparing far-from-equilibrium states



Navon et. al,
Nature (2016)

Chaotic driving
@ zero interactions, but disorder important

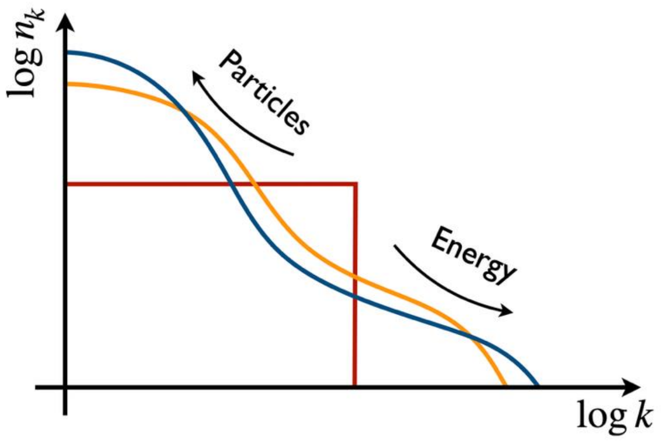
Destroys condensate faster



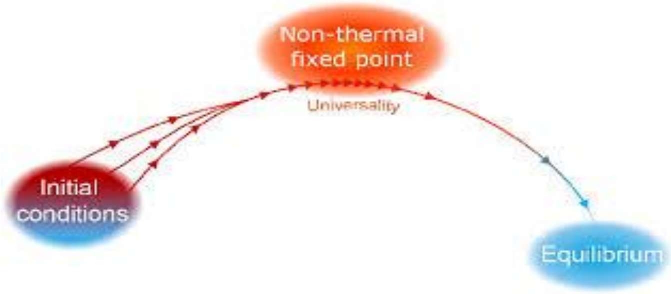
Experiment: Martirosyan et.al., PRL, 132, 113401 2024

Theory: Zhang et.al., 2024

Bidirectional relaxation



Nonthermal fixed points (NTFPs)

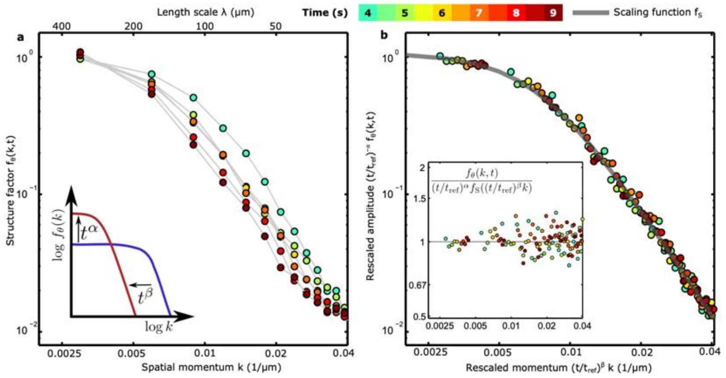


Dynamic self-similar scaling (IR and UV):

$$n_k(k,t) = \left(\frac{t}{t_0}\right)^{\alpha} n_k\left[\left(\frac{t}{t_0}\right)^{\beta} k, t_0\right]$$

Berges et.al., 101, 041603 (2008).
Schmied et.al., 2019

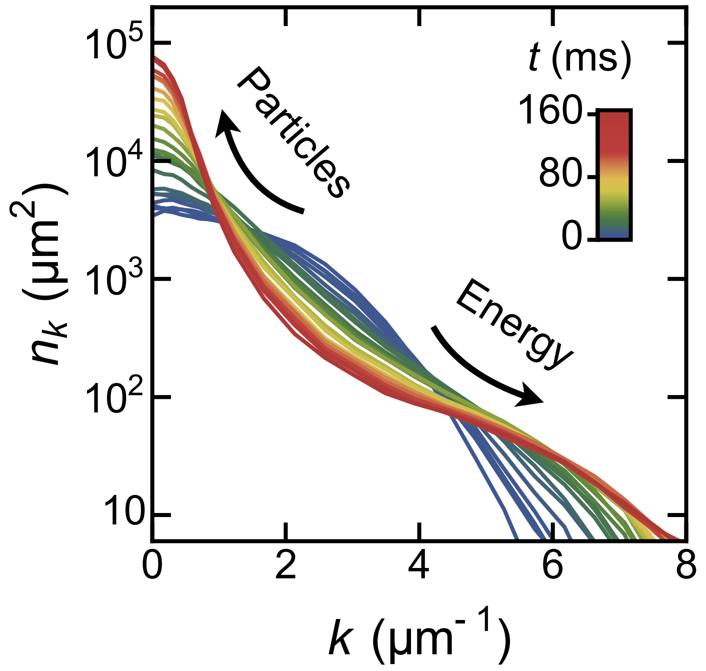
Past experiments?



Prufer et al., Nature **563**, 217–220 (2018)
Erne et al., Nature **563**, 225 (2018)
Glidden et al., Nat. Phys. **17**, 457 (2021)
(+ more recent ones...)

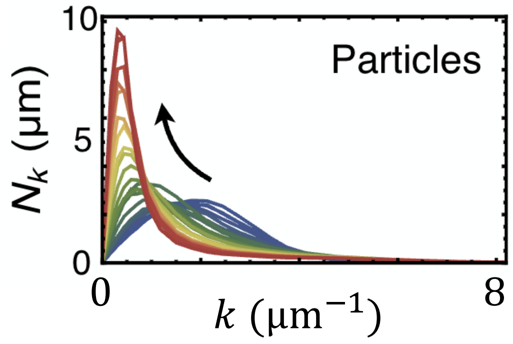
Dynamic scaling observed in experiments, but with exponents not fully understood

Experiment (2D)



Bidirectional relaxation

(particles to IR, energy to UV)

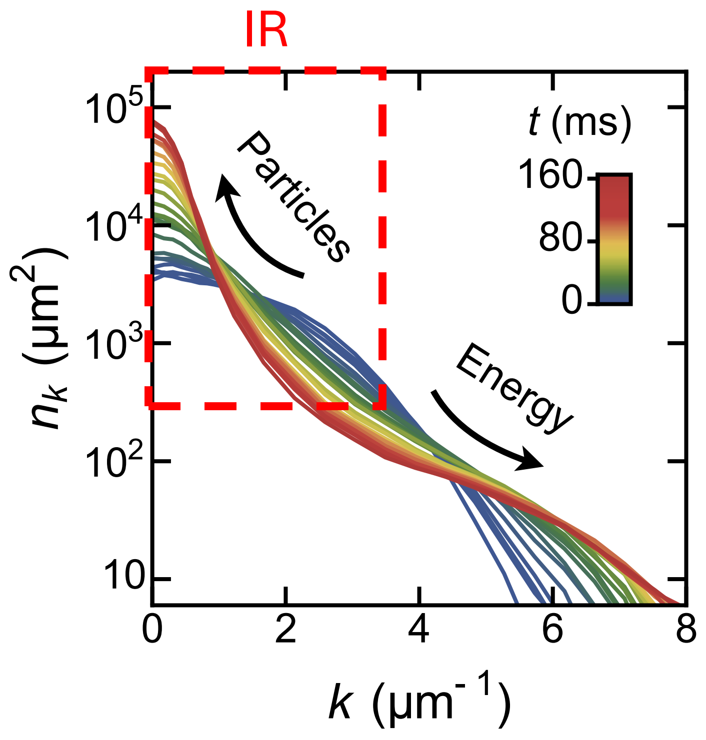


Spectral particle density

$$N_k \propto k^{d-1}\, n_k$$

Gazo et. al. 2023

Experiment (2D)



Bidirectional relaxation

(particles to IR, energy to UV)

First, focus on the IR:

$$n_k(k,t) = \left(\frac{t}{t_0}\right)^{\alpha} n_k\left[\left(\frac{t}{t_0}\right)^{\beta} k, t_0\right]$$

$$\alpha = d\beta = d/z = d/2 \quad \Rightarrow \alpha = 1$$

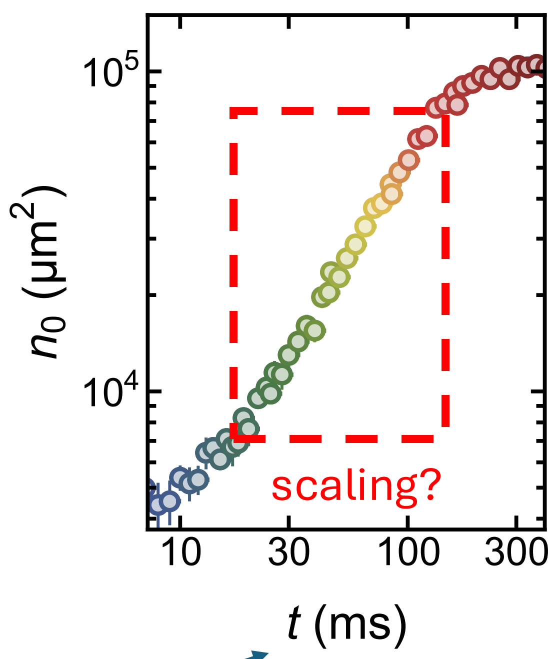
"decreasing level of confidence"

For condensate population n_0:

$$n_0 = n_k(k=0) \propto t^{\alpha}$$

Condensate growth

$n_0(t)$



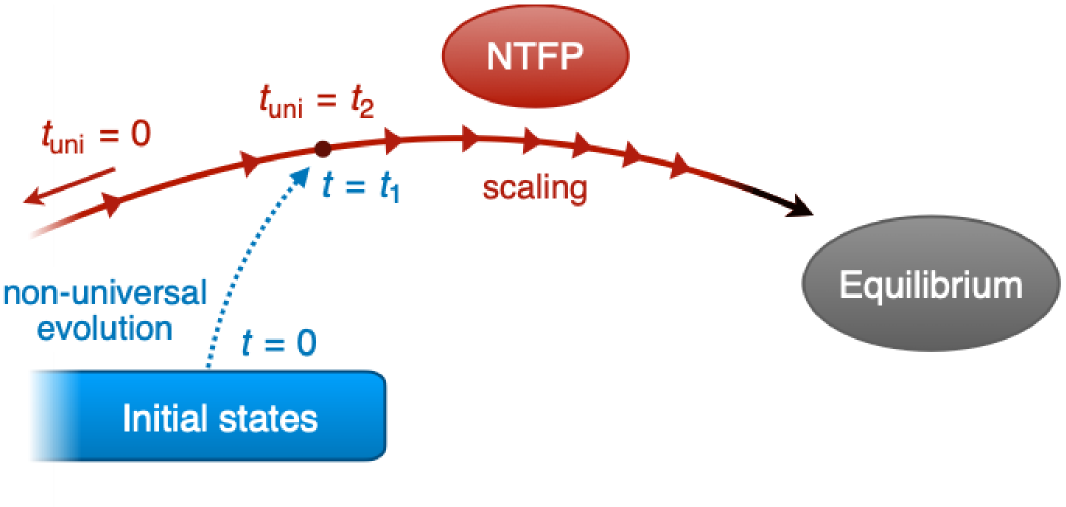
These two times are actually **not** the same!

Gazo et. al. 2023

Story of two clocks

$$n_k(k,t) = \left(\frac{t}{t_0}\right)^{\alpha} n_k\left[\left(\frac{t}{t_0}\right)^{\beta} k, t_0\right]$$

Universal scaling



the universal highway
has its one clock

$$t_1 \neq t_2$$

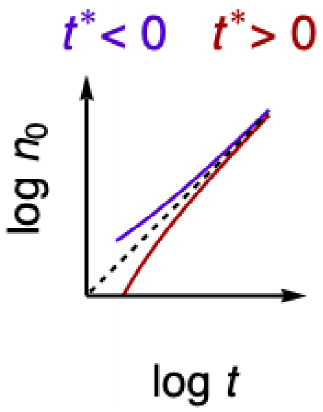
$$t^* = t_1 - t_2$$

$$t_{uni} = t - t^*$$

$$n_0 \propto (t - t^*)^{\alpha}$$

Ignoring t^*:

$$\frac{d \ln n_0}{d \ln t} = \frac{\alpha}{1 - t^*/t}$$



Theorist: prescaling
(... and just waits)

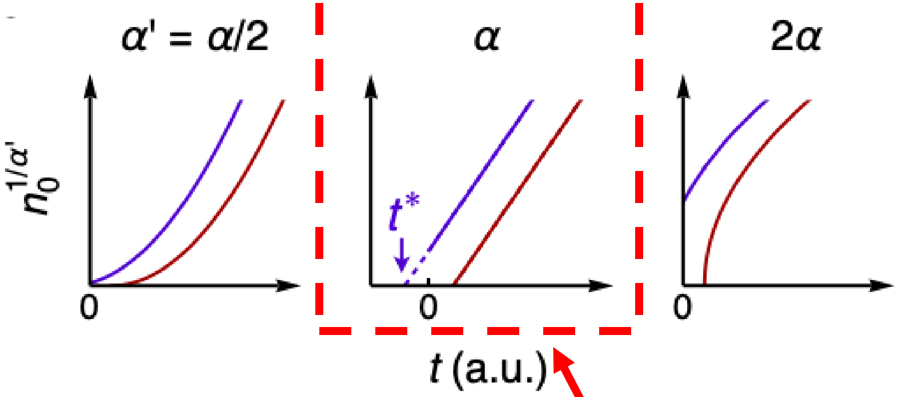
Mazeliauskas et. al., **122**, 122301 (2019)
Schmied et. al., **122**, 170404 (2019)
Heller et. al., **132**, 071602 (2024)

Experimentalist: gets wrong exponents
(... and cannot wait)

Gazo et. al. 2023

A solution

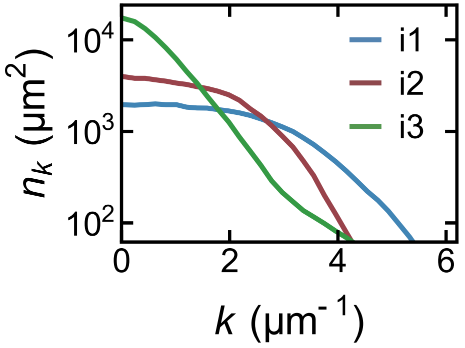
Linear ⇒ Correct!



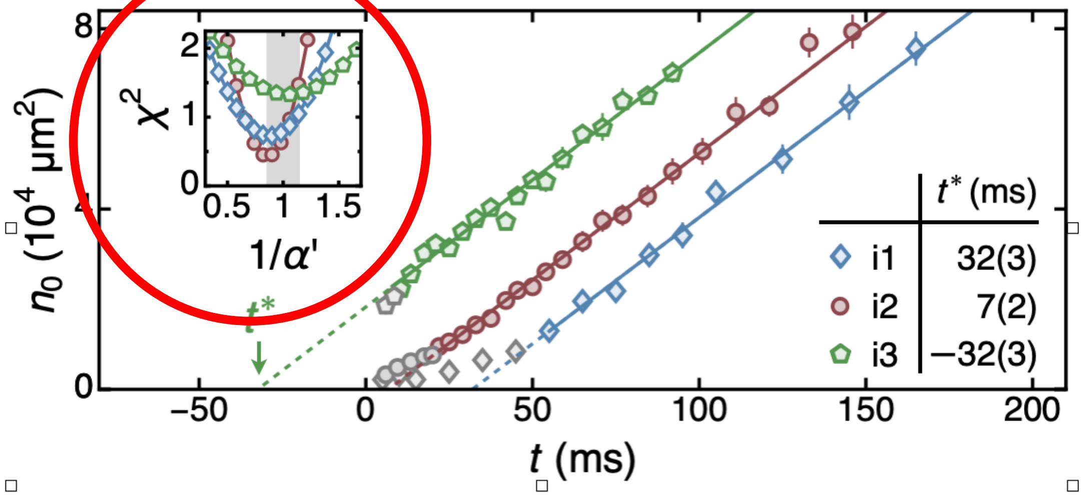
With unknown α: $n_0(t) \propto (t - t^*)^\alpha$

Plot $n_0^{1/\alpha'}$, get straight lines only for correct $\alpha' = \alpha$

3 initial conditions



Straight lines for $\alpha' \approx 1$

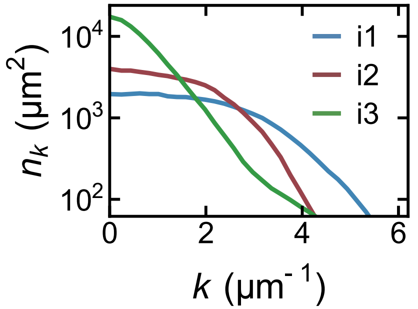


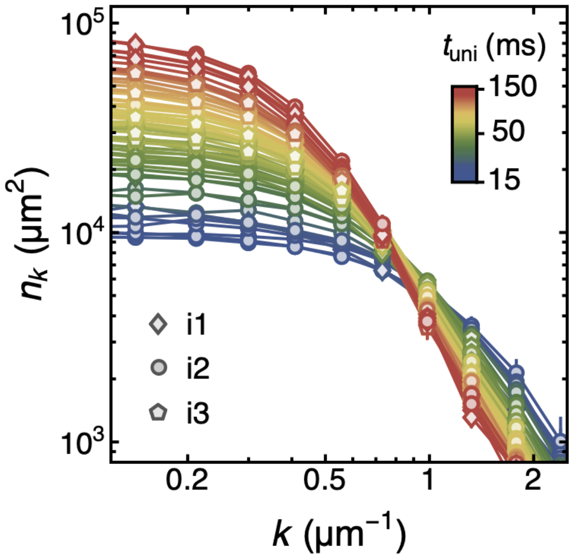
Moreover, this reveals values of t^*

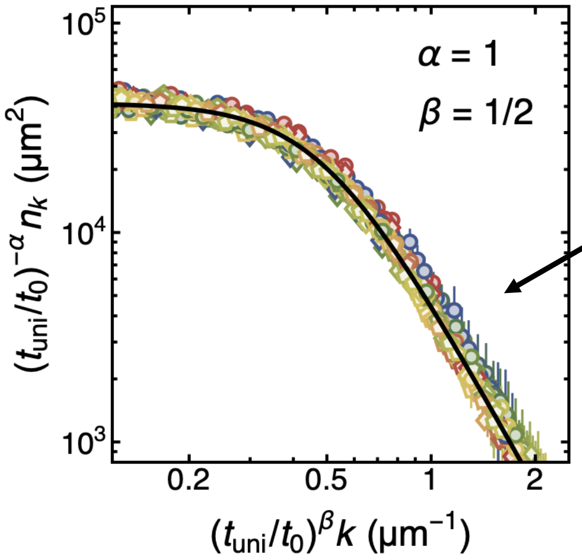
Gazo et. al. 2023

Collapse full IR n_k curves with theoretical exponents:

3 initial conditions







Shape also theoretically understood!

$$n_k(k, t_0) = \frac{A}{1 + (k/k_0)^{\kappa}}$$

Exponent κ in the shape:

Vortices: $\kappa = 4$ Porod's law, phase-ordering kinetics *e.g.* Bray, Adv. Phys. 51, 481 (2002)

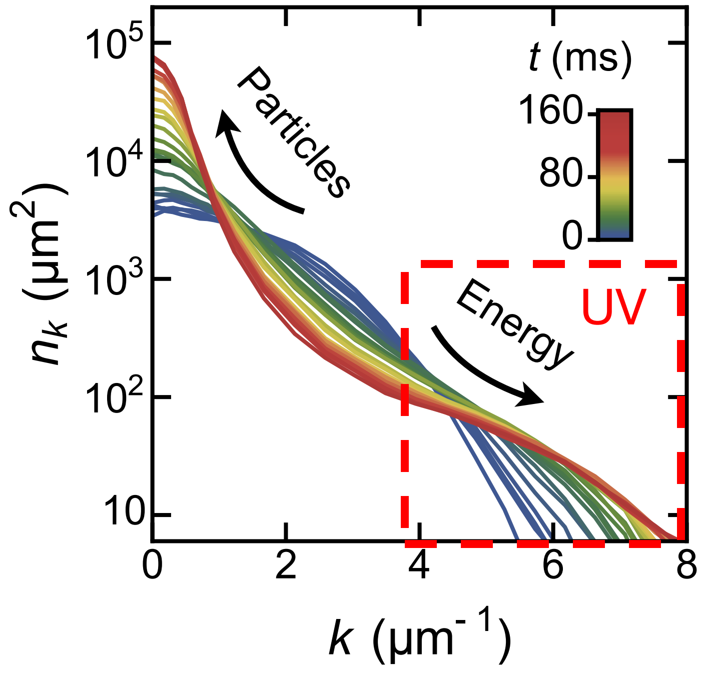
Waves: $\kappa = 3$ Resummed kinetic theory Chantesana et.al., PRA 99, 043620 (2019)

Rosenhaus, Falkovich, Arxiv: (2024)

Experiment: $\kappa = 2.9(2)$ → wave-dominated dynamics!

Gazo et. al. 2023

UV dynamics: characteristic lengthscale



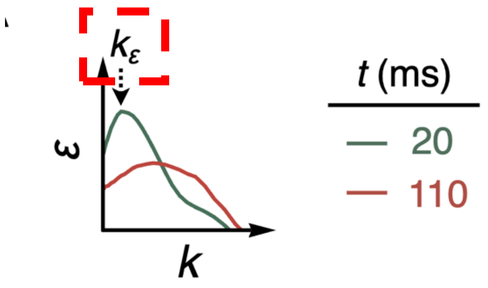
$$n_k(k,t) = \left(\frac{t}{t_0}\right)^{\alpha} n_k\left[\left(\frac{t}{t_0}\right)^{\beta} k, t_0\right]$$

weak-wave free turbulence

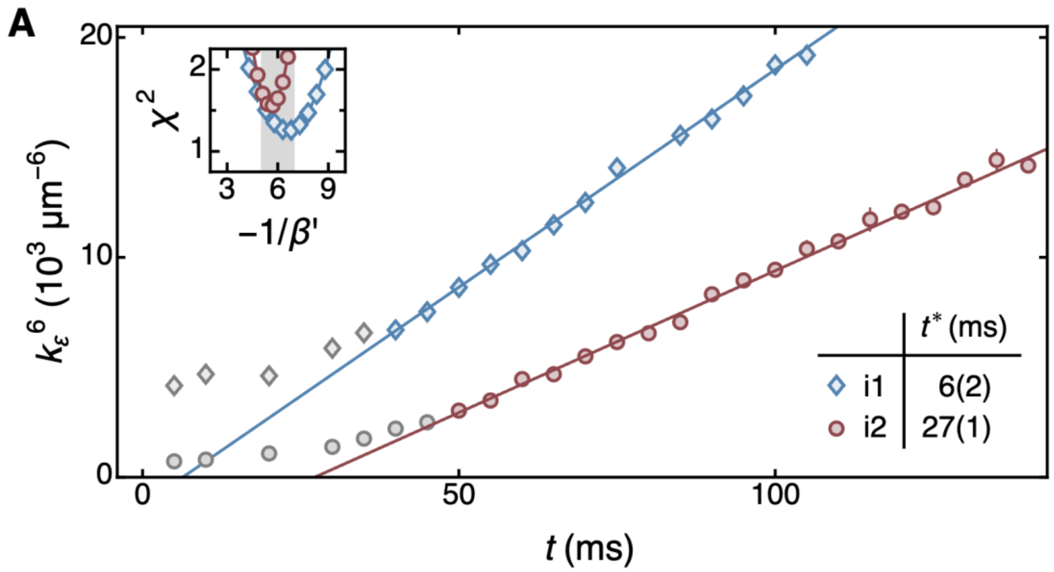
$$\beta = -1/6$$
$$\alpha = (d+2)\beta$$

But no special k givin a simple power law...

Peak of $\epsilon \sim k^3 n_k$

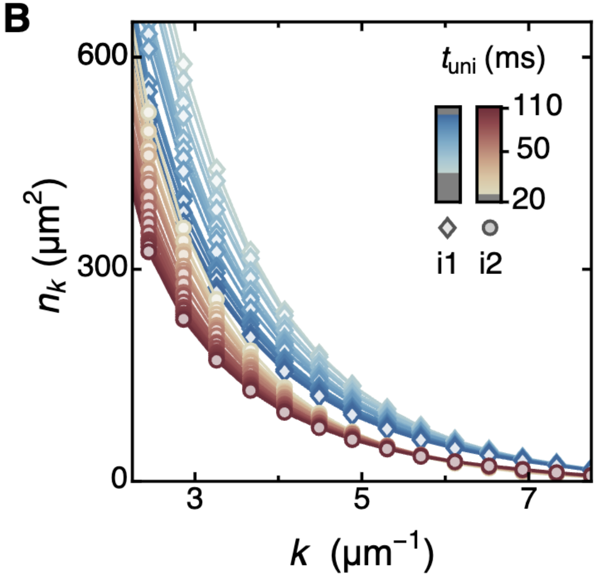


$k_\varepsilon \sim (t - t^*)^{1/6}$

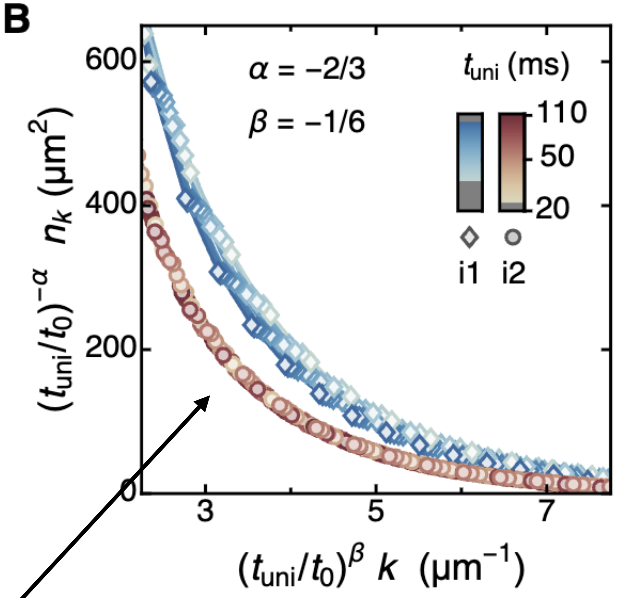


UV dynamics: dynamic scaling

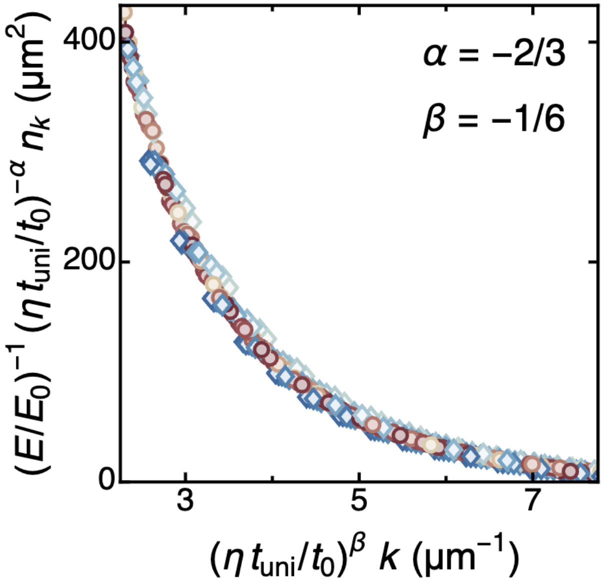
Unscaled



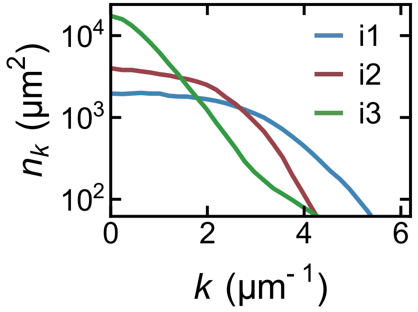
Dynamic scaling



Rescale by E and speeds



Different E cannot collapse



3 initial conditions

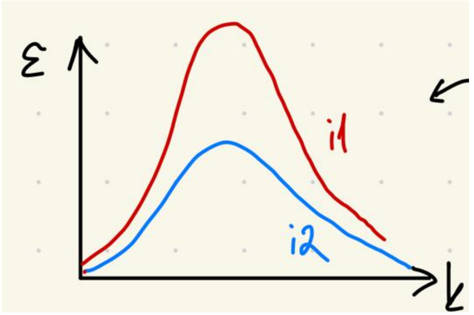
Can we understand different E?

UV dynamics: relation to fluxes

$$n_k(k,t) = \left(\frac{t}{t_0}\right)^{\alpha} n_k\left[\left(\frac{t}{t_0}\right)^{\beta} k, t_0\right]$$

Energy spectral density $\epsilon_k \sim k^3 n_k$

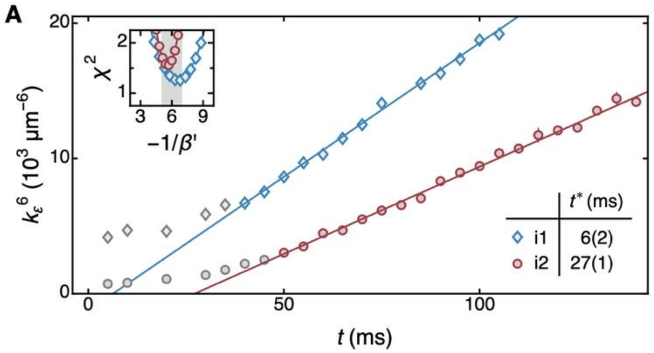
Energy flux Π_ε : $\nabla\, \Pi_\varepsilon = \dot{\epsilon}_k$ (k-dependent energy flux)

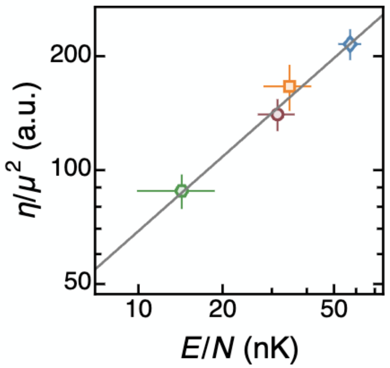


$$\boxed{\Pi_\varepsilon = -\frac{\beta}{t_{\mathrm{uni}}} k\, \varepsilon_k\,.}$$

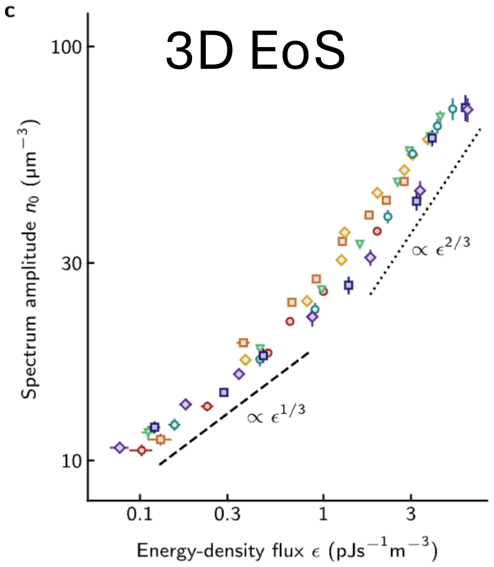
n_k reveals fluxes

$\Pi_\epsilon \sim \epsilon_k \sim n_k$? EoS: $n_0 \sim \Pi_\varepsilon^\kappa$ with $\kappa < 1$ t_{uni} has to be different!





Estimate EoS exponent:
$\kappa \approx 0.6$



3D Bose gas

Key universal features also **work** in 3D:

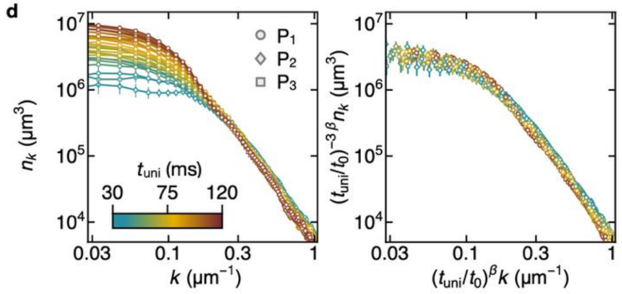
1) *Coarsening:*

$$\alpha = d\beta = d/z = 3/2$$

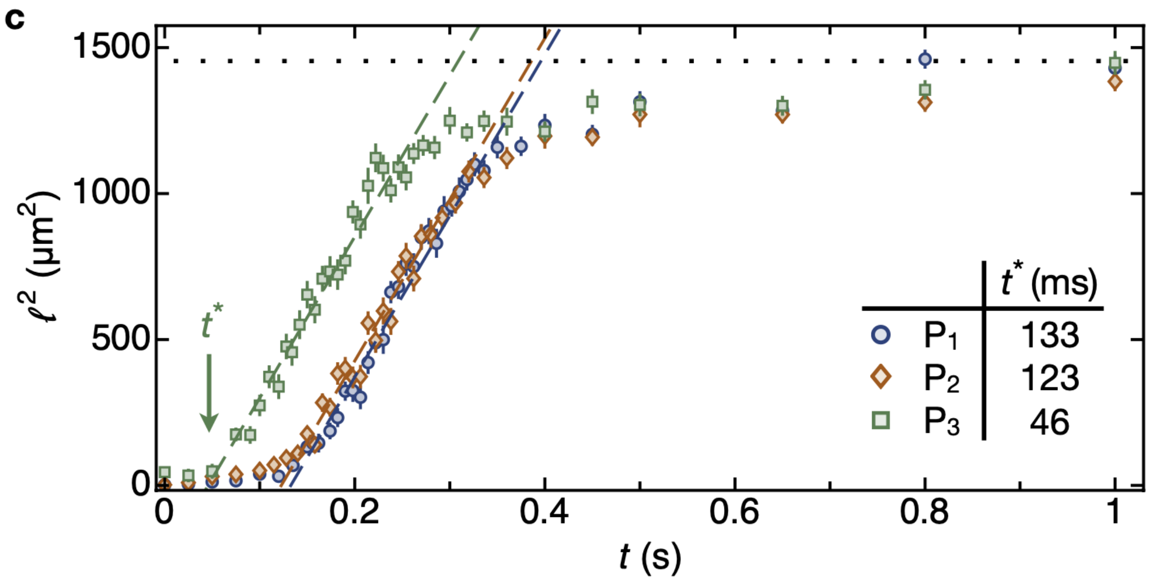
$\Rightarrow n_0^{2/3}$ is linear in time ✓

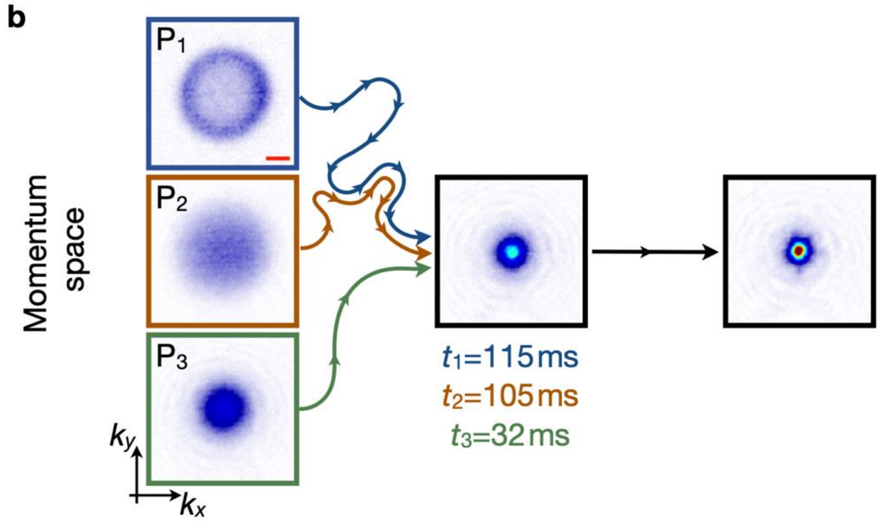
2) *Universal shape:*

memory loss of the initial state



✓





Martirosyan et. al., arXiv: 2411.19948

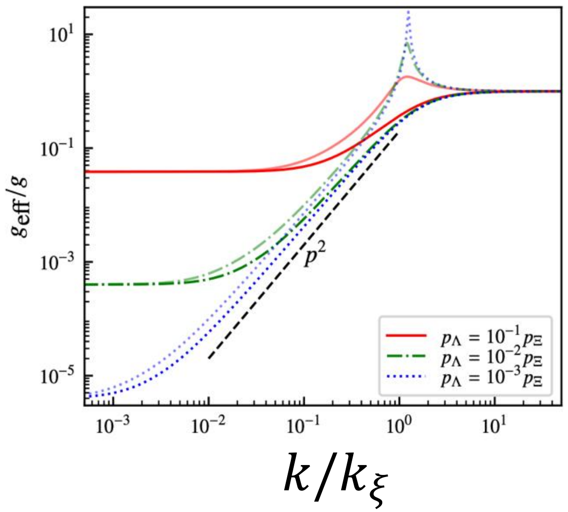
Interaction strength dependence

In WWT: timescales $\propto g^2$

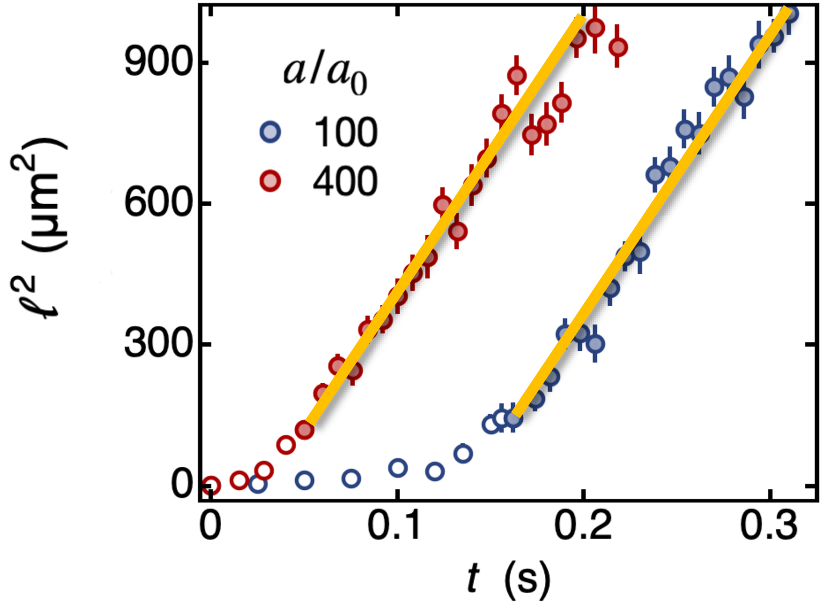
Coarsening: ?

$g_{\text{eff}} \sim p^2$ independent on bare g

Effective interaction strength



Chantesana et.al.,
PRA 99, 043620 (2019)
Rosenhaus, Falkovich,
arXiv:2501.12451 (2025)

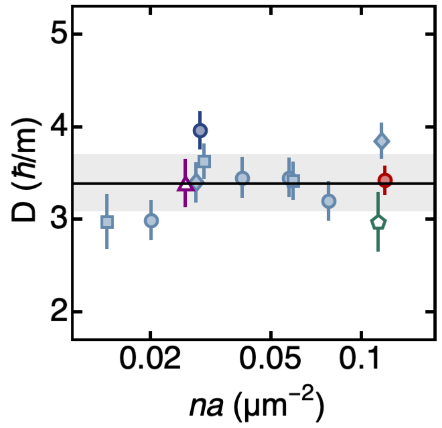


Martirosyan et. al., arXiv: 2411.19948

1. Non-scaling part (open symbols):
   ...depends on g

2. Universal NTFP part (solid symbols):
   ...independent on bare g!

Universal slope:



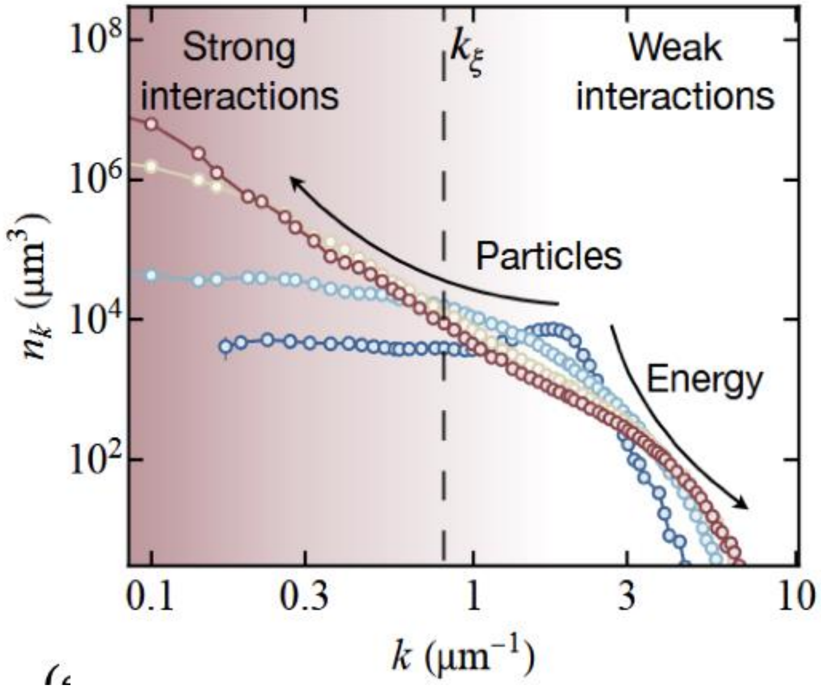
Require $k < k_\xi \sim \sqrt{na}$

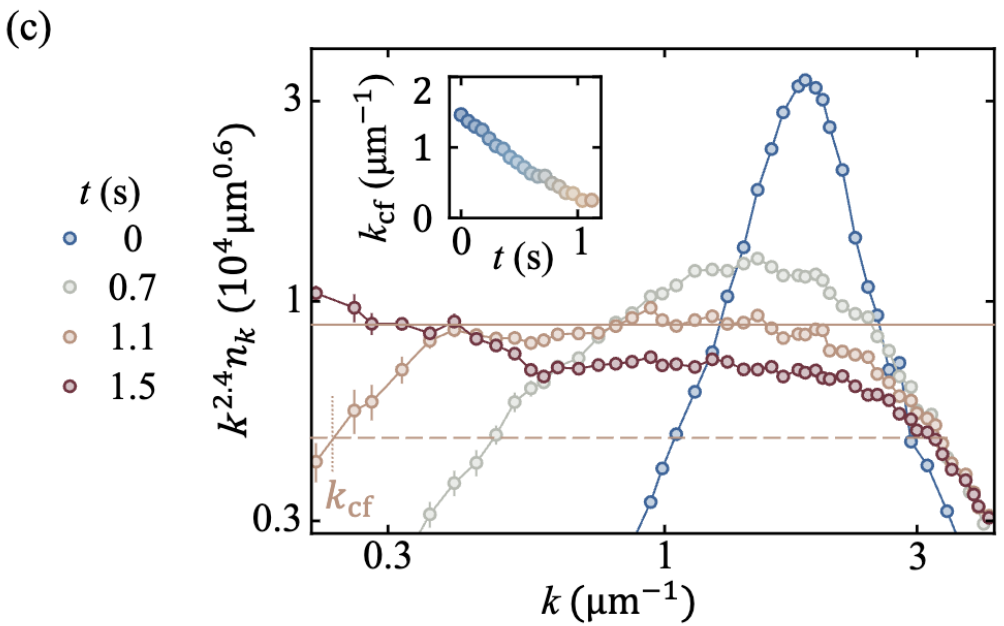
Transition from the weakly interacting (perturbative) regime

So far: IR dynamics $k \lesssim 1/\xi$ (strongly-interacting regime)

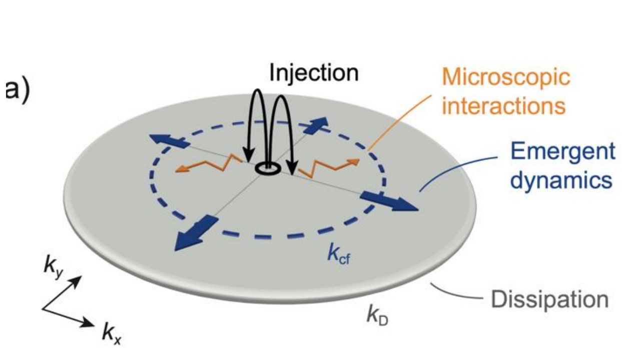
Opposite regime also possible $k > 1/\xi$: ~~coarsening~~ → weak-wave turbulence

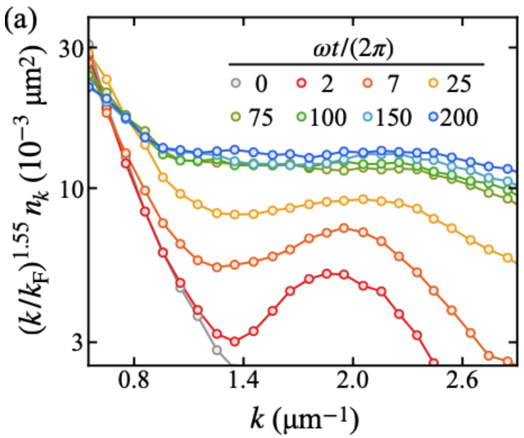
Reduce $k_\xi \rightarrow 0.4\ \mu\mathrm{m}^{-1}$

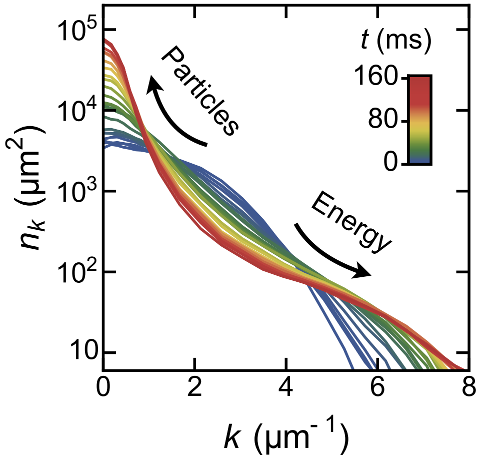


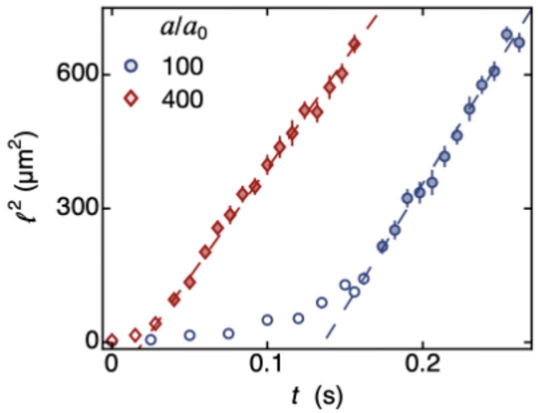


Predictions for inverse cascade: ≈ 2.5 (not 7/3)









Thank you for you attention!

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