Anomalous transport phenomena on the lattice

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INT Workshop Chirality and Criticality, Seattle, 25-08-2023

DFG

CRC-TR 211
Strong-interaction matter under extreme conditions

UNIVERSITÄT BIELEFELD

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Introduction

- Quantum anomalies + $\begin{matrix}\text{EM fields}\\ \text{Vorticity}\end{matrix}$ $\rightarrow$ non-dissipative transport effects:

  Anomalous transport phenomena

- Examples:
  - Chiral Magnetic Effect (CME)
  - Chiral Separation Effect (CSE)
  - Chiral Electric Separation Effect (CESE)
  - Chiral Vortical Effect (CVE)
  - ...

  For a review see Kharzeev, Liao, Voloshin, Wang '16

- Event-by-event CP-violation $\rightarrow$ non-trivial topology of QCD vacuum

Conductivities

- We focus on:
  - CME: Finite chiral density + Magnetic field → Vector current
  - CSE: Finite quark density + Magnetic field → Axial current
- Currents linear in B and μ/μ_5 to first order:

$$J^V_{\mathsf{CME}} = C_{\mathsf{CME}}\, eB\mu_5 + \mathcal{O}(\mu_5^3)$$
$$J^A_{\mathsf{CSE}} = C_{\mathsf{CSE}}\, eB\mu + \mathcal{O}(\mu^3)$$

Baryon chemical potential: $\mu\bar{\psi}\gamma_4\psi$, Chiral "chemical potential": $\mu_5\bar{\psi}\gamma_4\gamma_5\psi$

Conductivities

- Analytical predictions for **free fermions**
- CME Fukushima, Kharzeev, Warringa '08:

$$C_{\mathsf{CME}} = \begin{cases} \dfrac{1}{2\pi^2} \text{ out-of-equilibrium} \\ 0 \left(\text{or } \dfrac{1}{2\pi^2}?\right) \text{ in-equilibrium} \end{cases}$$

out-of-equilibrium: Son, Surowka '09; Kharzeev et al '16

in-equilibrium: Buividovich '14; Sheng et al '17

Importance of regularization! (more to this later)

Conductivities

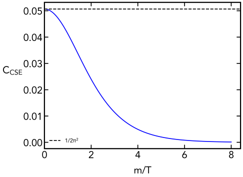
- Analytical predictions for **free fermions**
- CME Fukushima, Kharzeev, Warringa '08:

$$C_{\text{CME}} = \begin{cases} \dfrac{1}{2\pi^2} \text{ out-of-equilibrium} & \text{Son, Surowka '09} \quad \text{Kharzeev et al '16} \\ 0 \left(\text{or } \dfrac{1}{2\pi^2}?\right) \text{ in-equilibrium} & \text{Buividovich '14} \quad \text{Sheng et al '17} \end{cases}$$

Importance of regularization! (more to this later)

- CSE Son, Zhitnitsky '04 Metlitski, Zhitnitsky '05:

$$C_{\text{CSE}} = C_{\text{CSE}}(m/T) \xrightarrow{m \to 0} \frac{1}{2\pi^2}$$

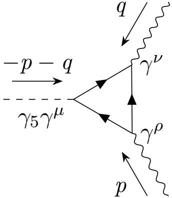


- Goal: Use gauge invariant lattice regularization and check corrections due to QCD!

Regulator sensitivity

- Well known example: triangle anomaly (with massive fermions)

$$\Gamma^{\mu\nu\rho}_{AVV}(p+q,p,q) =$$



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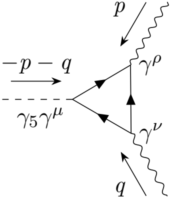

- No/Wrong regularization

$$(p+q)_\mu \Gamma^{\mu\nu\rho}_{AVV}(p+q,p,q) = mP_5(p,q)$$

Regulator sensitivity

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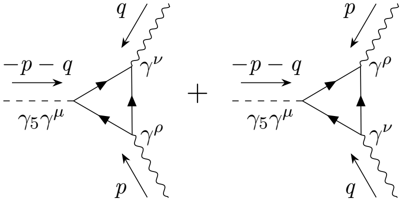


- No/Wrong regularization

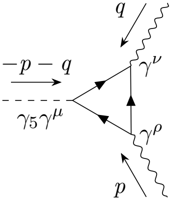
$$(p+q)_\mu \Gamma^{\mu\nu\rho}_{AVV}(p+q,p,q) = mP_5(p,q)$$

- Pauli-Villars regularization: new particles, with coeffs c_s ($s = 0, 1, 2, 3$, $s = 0$ physical fermion $m_0 \equiv m$) and masses $m_{s>0} \to \infty$

$$\begin{aligned}(p+q)_\mu \Gamma^{\mu\nu\rho}_{AVV}(p+q,p,q) =& mP_5(p,q) + \sum_{s=1} c_s m_s P^{\nu\rho}_{5,s}(p,q) \\ \to& mP_5(p,q) + \frac{\epsilon^{\alpha\beta\nu\rho} q_\alpha p_\beta}{4\pi^2}\end{aligned}$$

- $C_{\mathsf{CME}}/C_{\mathsf{CSE}}$ can also be written with the triangle diagram

$\Gamma^{\mu\nu\rho}_{AVV}(p+q,p,q) =$



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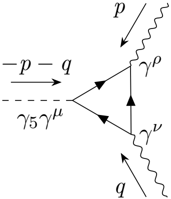
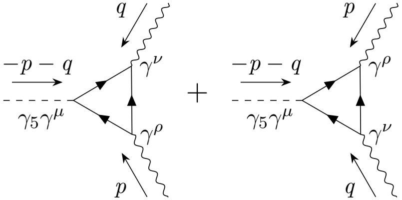


with $J_3 \sim A_3, \quad J_3^5 \sim A_3^5, \quad \boldsymbol{B}_3 \sim q_1 A_2, \quad \mu = A_0, \quad \mu_5 = A_0^5$

Regulator sensitivity

- $C_{\text{CME}}/C_{\text{CSE}}$ can also be written with the triangle diagram

$$\Gamma^{\mu\nu\rho}_{AVV}(p+q,p,q) =$$



with $J_3 \sim A_3, \quad J_3^5 \sim A_3^5, \quad \boldsymbol{B}_3 \sim q_1 A_2, \quad \mu = A_0, \quad \mu_5 = A_0^5$

- Kubo formulas

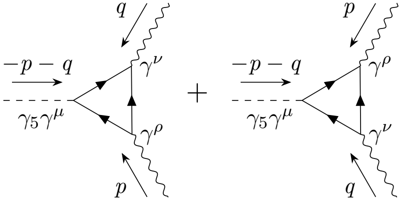
$$C_{\text{CME}} = \lim_{p,q,p+q\to 0} \frac{1}{q_1}\Gamma^{023}_{AVV}(p+q,p,q) = \frac{1}{2\pi^2} + \sum_{s=1} \frac{c_s}{2\pi^2} = 0$$

$$C_{\text{CSE}} = \lim_{p,q,p+q\to 0} \frac{1}{q_1}\Gamma^{320}_{AVV}(p+q,p,q) = -\frac{1}{\pi^2}\int \mathrm{d}k\, \frac{\partial n_F(E_k)}{\partial E_k}$$

Regulator sensitivity

- $C_{\text{CME}}/C_{\text{CSE}}$ can also be written with the triangle diagram

$$\Gamma^{\mu\nu\rho}_{AVV}(p+q,p,q) =$$



with $J_3 \sim A_3, \quad J^5_3 \sim A^5_3, \quad \boldsymbol{B}_3 \sim q_1 A_2, \quad \mu = A_0, \quad \mu_5 = A^5_0$

- Kubo formulas

$$C_{\text{CME}} = \lim_{p,q,p+q\to 0} \frac{1}{q_1} \Gamma^{023}_{AVV}(p+q,p,q) = \frac{1}{2\pi^2} + \sum_{s=1} \frac{c_s}{2\pi^2} = 0$$

$$C_{\text{CSE}} = \lim_{p,q,p+q\to 0} \frac{1}{q_1} \Gamma^{320}_{AVV}(p+q,p,q) = -\frac{1}{\pi^2} \int \mathrm{d}k \, \frac{\partial n_F(E_k)}{\partial E_k}$$

- C_{CME} is zero due to anomalous contribution!
- C_{CSE} agrees with known results Metlitski, Zhitnitsky '05

Lattice QCD in a nutshell

- We use **Lattice QCD**
- Partition function

$$\mathcal{Z} = \int \mathcal{D}\mathcal{U}\mathcal{D}\bar{\psi}\mathcal{D}\psi\, e^{-S_E} = \int \mathcal{D}\mathcal{U}\ \det M\ e^{-S_G}$$

with S_E the Wick-rotated, finite temperature QCD action

$$S_E = S_G + S_F = \int_0^{1/T} \mathrm{d}\tau \int \mathrm{d}^3x \left[\frac{\mathrm{Tr}\, F^2}{2g^2} + \sum_f \bar{\psi}_f \underbrace{\left(\not{D} + m_f\right)}_{\equiv M} \psi_f \right]$$

- Inherently in **equilibrium**
- Observables

$$\langle \mathcal{O} \rangle = \int \mathcal{D}\mathcal{U}\ \det M\ e^{-S_G}\, \mathcal{O}$$

Lattice QCD in a nutshell

- Discretize the action, with **lattice spacing** a, and use **importance sampling Monte Carlo** integration to evaluate the path integral

- Some useful jargon:
  - Lattice $N_s^3 \times N_t,\ T = 1/N_t a,\ V = a^4 N_s^3 N_t$
  - **Continuum limit**: $a \to 0$ (equiv. $N_t \to \infty$) while V, T fix
  - **Quenched** approximation: $\det M = 1$ (exact when $m_q \to \infty$)

- Different fermion discretizations (doublers appear!):
  - Wilson: doublers are given a **cutoff dependant mass** to decouple in the continuum limit ($\leftrightarrow$PV)
  - Staggered: Dirac and flavor structure is **mixed with coordinate dependence** to reduce doubling problem
  - Overlap, Domain Wall, ...

Lattice setup

- Some previous results:
  CME
  - Wilson: Quenched and full QCD Yamamoto '11
    Full QCD: $C_{\text{CME}} = 0.013$ at high T $(1/2\pi^2 \approx 0.05)$
    Quenched: $C_{\text{CME}} = 0.02 - 0.03$ at high T
  - Results are neither 0 nor $1/2\pi^2$!

  CSE
  - Overlap: Quenched QCD Puhr, Buividovich '17
    No significant corrections found to the free fermions result
  - Wilson/Domain Wall: SU(2) Buividovich, Smith, von Smekal '21
    CSE suppressed at low T

  No results for full QCD with physical masses!

- Our setup:
  - Dynamical staggered fermions, $2+1$ flavors, physical quark masses
  - Quenched staggered and Wilson
  - Background B field ($z \equiv 3$ direction)

Lattice setup

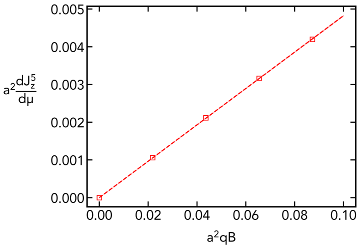
- Measure derivatives of the currents:

$$C_{\mathsf{CME}}\, eB_3 = \frac{\mathrm{d}\langle J_3^V \rangle}{\mathrm{d}\mu_5}\bigg|_{\mu_5=0} \sim \left\langle J_3^V J_4^A \right\rangle_{\mu_5=0}$$

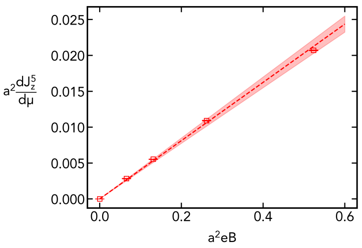
$$C_{\mathsf{CSE}}\, eB_3 = \frac{\mathrm{d}\langle J_3^A \rangle}{\mathrm{d}\mu}\bigg|_{\mu=0} \sim \left\langle J_4^V J_3^A \right\rangle_{\mu=0}$$

- Simulations at $\mu = 0 \rightarrow$ no sign problem
- Numerical derivative (linear fit) w.r.t. B to obtain $C_{\mathsf{CME/CSE}}$:

free fermions



full QCD



Currents in staggered

- Staggered "gammas" (free fermions and quark chemical potential):

$$\Gamma_\nu(n,m) = \frac{1}{2}\eta_\nu(n)[e^{a\mu\delta_{\nu,4}}\delta_{n+\hat{\nu},m} + e^{-a\mu\delta_{\nu,4}}\delta_{n-\hat{\nu},m}]$$

$$\Gamma_5(n,m) = \frac{1}{4!}\sum_{i,j,k,l}\epsilon_{ijkl}\Gamma_i\Gamma_j\Gamma_k\Gamma_l$$

$$\Gamma_{\nu 5}(n,m) = \frac{1}{3!}\sum_{i,j,k}\epsilon_{ijk}\Gamma_i\Gamma_j\Gamma_k \quad i,j,k \neq \nu$$

- Conserved vector current and anomalous axial current:

$$j_\nu^V = \bar{\chi}\Gamma_\nu\chi$$

$$j_\nu^A = \bar{\chi}\Gamma_{\nu 5}\chi$$

- Staggered observable has a tadpole term, for example CSE

$$\left.\frac{\mathrm{d}\left\langle J_3^A\right\rangle}{\mathrm{d}\mu}\right|_{\mu=0} \sim \left\langle J_4^V J_3^A\right\rangle_{\mu=0} + \left\langle \frac{\partial J_3^A}{\partial\mu}\right\rangle_{\mu=0}$$

Currents in Wilson

- Local currents (don't fulfill a WI/AWI)

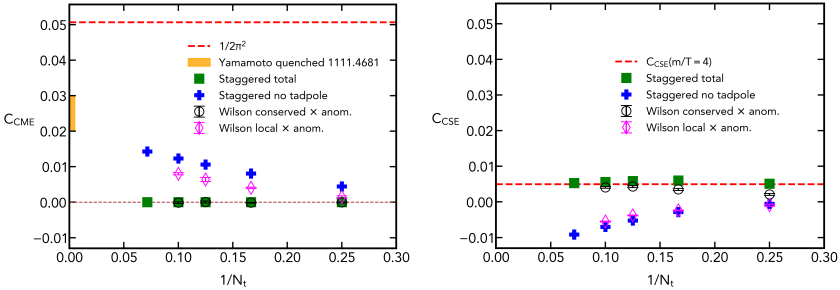
$$j_\mu^{VL} = \bar{\psi}\gamma_\mu\psi$$
$$j_\mu^{AL} = \bar{\psi}\gamma_\mu\gamma_5\psi$$

- Conserved vector current and anomalous axial current:

$$j_\mu^{VC}(n) = \frac{1}{2}\left[\bar{\psi}(n)(\gamma_\mu - r)\psi(n+\hat{\mu})) + \bar{\psi}(n)(\gamma_\mu + r)\psi(n-\hat{\mu}))\right]$$
$$j_\mu^{AA}(n) = \frac{1}{2}\left[\bar{\psi}(n)\gamma_\mu\gamma_5\psi(n+\hat{\mu})) + \bar{\psi}(n)\gamma_\mu\gamma_5\psi(n-\hat{\mu})\right]$$

- For correlators like $\left\langle J_4^V J_3^A \right\rangle$ we can use different combinations, for example $\left\langle J_4^{VC} J_3^{AA} \right\rangle$, $\left\langle J_4^{VL} J_3^{AA} \right\rangle$, ...

- Consistency check in the free case
- For $m/T = 4$ (similar behavior for other m/T's)



- Using the correct currents is **crucial**

Chirality

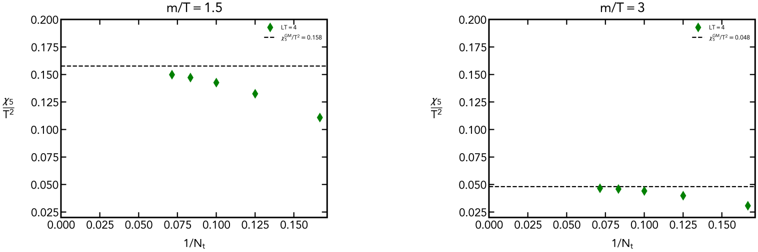
- Naturally

$$n5\Big|_{\mu_5=0} = \frac{T}{V}\frac{\mathrm{d}\log\mathcal{Z}}{\mathrm{d}\mu_5} = 0$$

- But

$$n_5(\mu_5) = \frac{T}{V}\frac{\mathrm{d}^2\log\mathcal{Z}}{\mathrm{d}\mu_5^2}\Big|_{\mu_5=0}\mu_5 + \mathcal{O}(\mu_5^2) = \chi_5\mu_5 + \mathcal{O}(\mu_5^2)$$

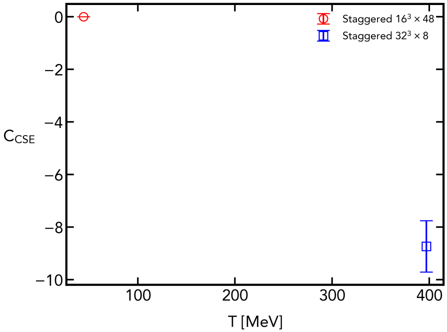
- χ_5 can be calculated with PV and compared to lattice



- μ_5 does induce chirality in our system

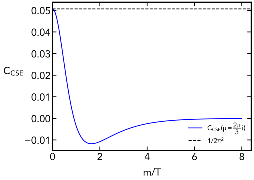
Quenched results: CSE

- Quenched staggered results for CSE



- Large negative result?

- There is an explanation!



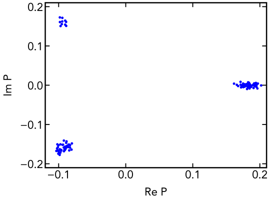
Imaginary Polyakov loop sectors

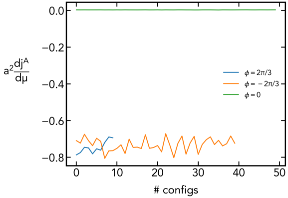
$\updownarrow$

Imaginary chemical potential

$\mu = \pm i\, 2\pi/3$

- Contribution to CSE (not CME), at $T = 400$ MeV in a $32^3 \times 8$ lattice:

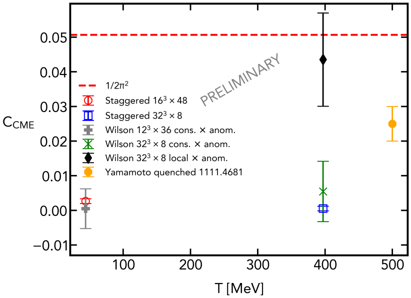


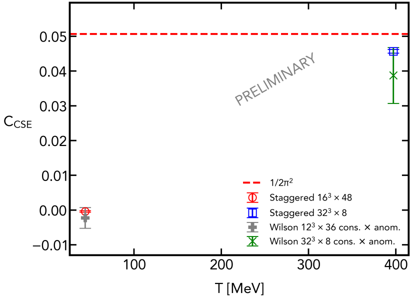


- In full QCD, quark masses break the Z_3 symmetry $\rightarrow$ we consider only the real sector

Quenched results: CSE and CME

- Quenched results: Staggered $m_\pi = 415$ MeV, Wilson $m_\pi = 620$ MeV

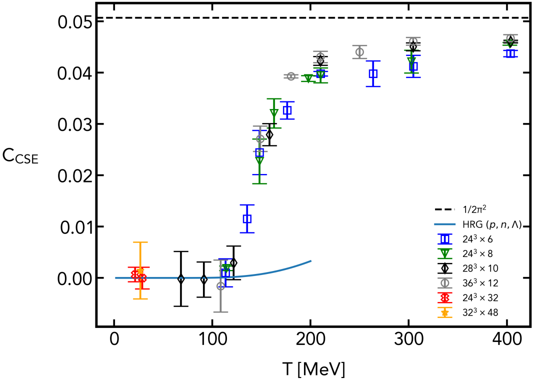




- Vanishing CME for correct currents
- CSE suppressed at low T, free result for high T

Results for CSE: full QCD

- 2+1 flavors, physical masses



Results for CSE: full QCD

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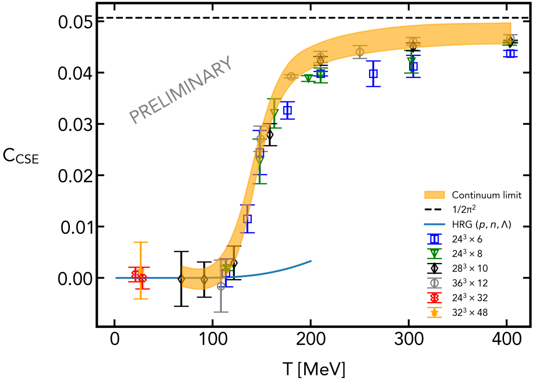
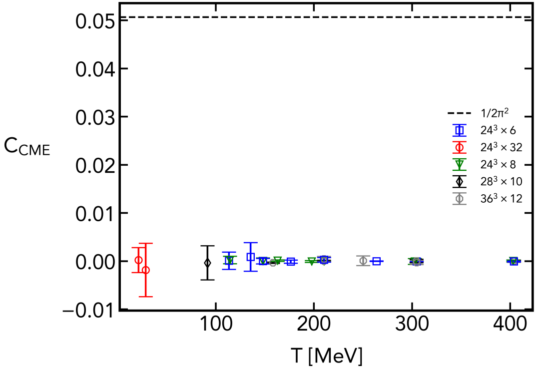


- High T ($T > T_c$): approaches free case value
- Low T ($T < T_c$): CSE suppressed Buividovich, Smith, von Smekal '21
  Chiral effective theories Avdoshkin, Sadofyev, Zakharov '18

Results for CME: full QCD

- 2+1 flavors, physical masses



- **CME vanishes** in our setup for free fermions and in QCD for physical and higher than physical pion masses, for all temperatures
- Chiral density is finite and non-zero for $\mu_5 \neq 0$ in our setup

Summary

- First study of CME and CSE with staggered fermions, 2+1 flavors, physical masses
- Free case consistent with analytical prediction $\rightarrow$ Importance of the currents used
- Quenched:
  - Imaginary Polyakov loop sectors for quenched CSE give an unphysical contribution
  - Cross check with Wilson fermions
- Full QCD:

  CME

  - CME current is zero in equilibrium in a gauge invariant regularization with conserved vector and anomalous axial currents

  CSE

  - Suppression at low T, approach free case value at high T
  - Example of how interactions can modify the free case results
  - Implications for experimental searches of the chiral magnetic wave

Backup slides

- Transport effects:

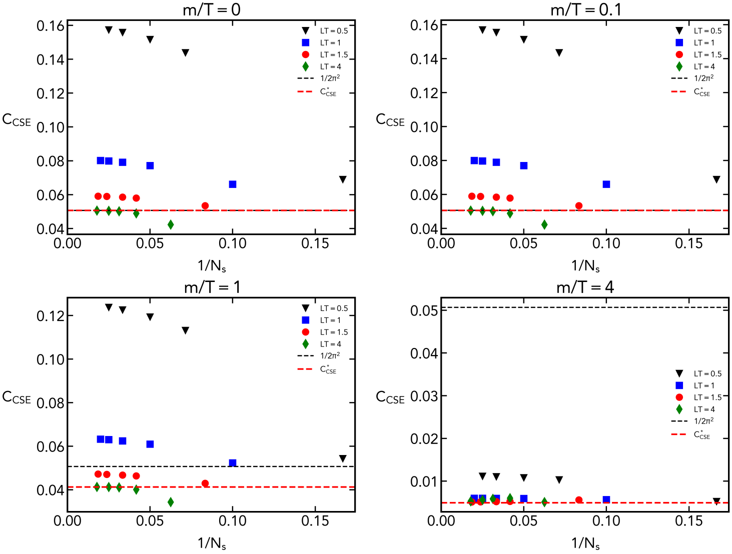
$$\begin{pmatrix} \vec{J} \\ \vec{J}_5 \end{pmatrix} = \begin{pmatrix} \sigma_{\text{Ohm}} & \sigma_{\text{CME}} \\ \sigma_{\text{CESE}} & \sigma_{\text{CSE}} \end{pmatrix} \begin{pmatrix} \vec{E} \\ \vec{B} \end{pmatrix}$$

- Chiral Vortical Effect: vector/axial current generated by rotation + $\mu + \mu_5$:

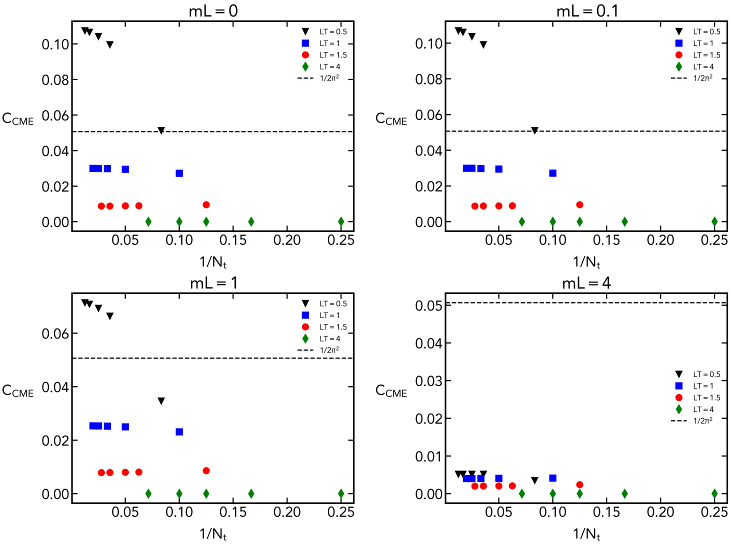
$$\vec{J} = \frac{1}{\pi^2}\mu_5\mu\vec{\omega}$$

$$\vec{J}_5 = \left[\frac{1}{6}T^2 + \frac{1}{2\pi^2}(\mu_5^2 + \mu^2)\right]\vec{\omega}$$

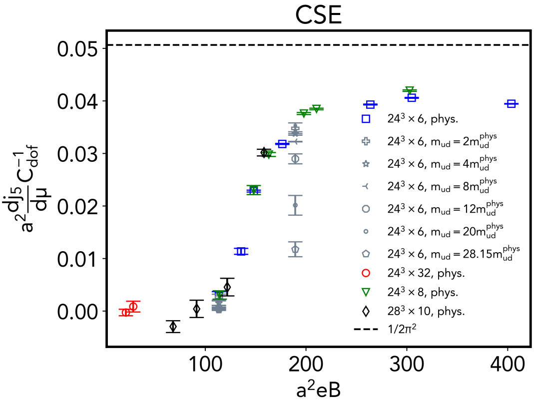
CSE free case



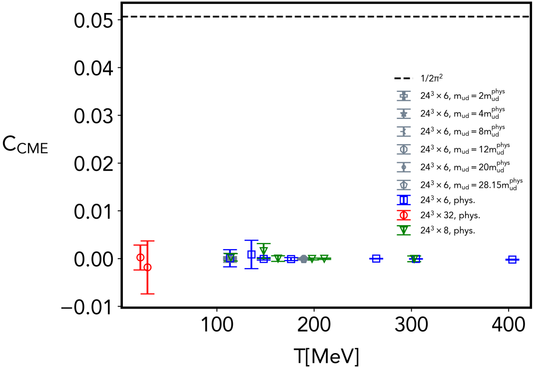
CME free case



CSE masses


CSE
0.05
0.04
0.03
0.02
0.01
0.00
$a^2 \frac{dj_5}{d\mu} C_{dof}^{-1}$
100
200
300
400
a^2eB
$24^3 \times 6$, phys.
$24^3 \times 6$, $m_{ud} = 2m_{ud}^{phys}$
$24^3 \times 6$, $m_{ud} = 4m_{ud}^{phys}$
$24^3 \times 6$, $m_{ud} = 8m_{ud}^{phys}$
$24^3 \times 6$, $m_{ud} = 12m_{ud}^{phys}$
$24^3 \times 6$, $m_{ud} = 20m_{ud}^{phys}$
$24^3 \times 6$, $m_{ud} = 28.15m_{ud}^{phys}$
$24^3 \times 32$, phys.
$24^3 \times 8$, phys.
$28^3 \times 10$, phys.
$1/2\pi^2$

CME masses



0.05
0.04
0.03
0.02
0.01
0.00
−0.01
C_CME
100
200
300
400
T[MeV]
1/2π²
24³ × 6, m_ud = 2m_ud^phys
24³ × 6, m_ud = 4m_ud^phys
24³ × 6, m_ud = 8m_ud^phys
24³ × 6, m_ud = 12m_ud^phys
24³ × 6, m_ud = 20m_ud^phys
24³ × 6, m_ud = 28.15m_ud^phys
24³ × 6, phys.
24³ × 32, phys.
24³ × 8, phys.