Lattice QCD Determination of NNLO Valence PDF of the Pion at the Physical Point

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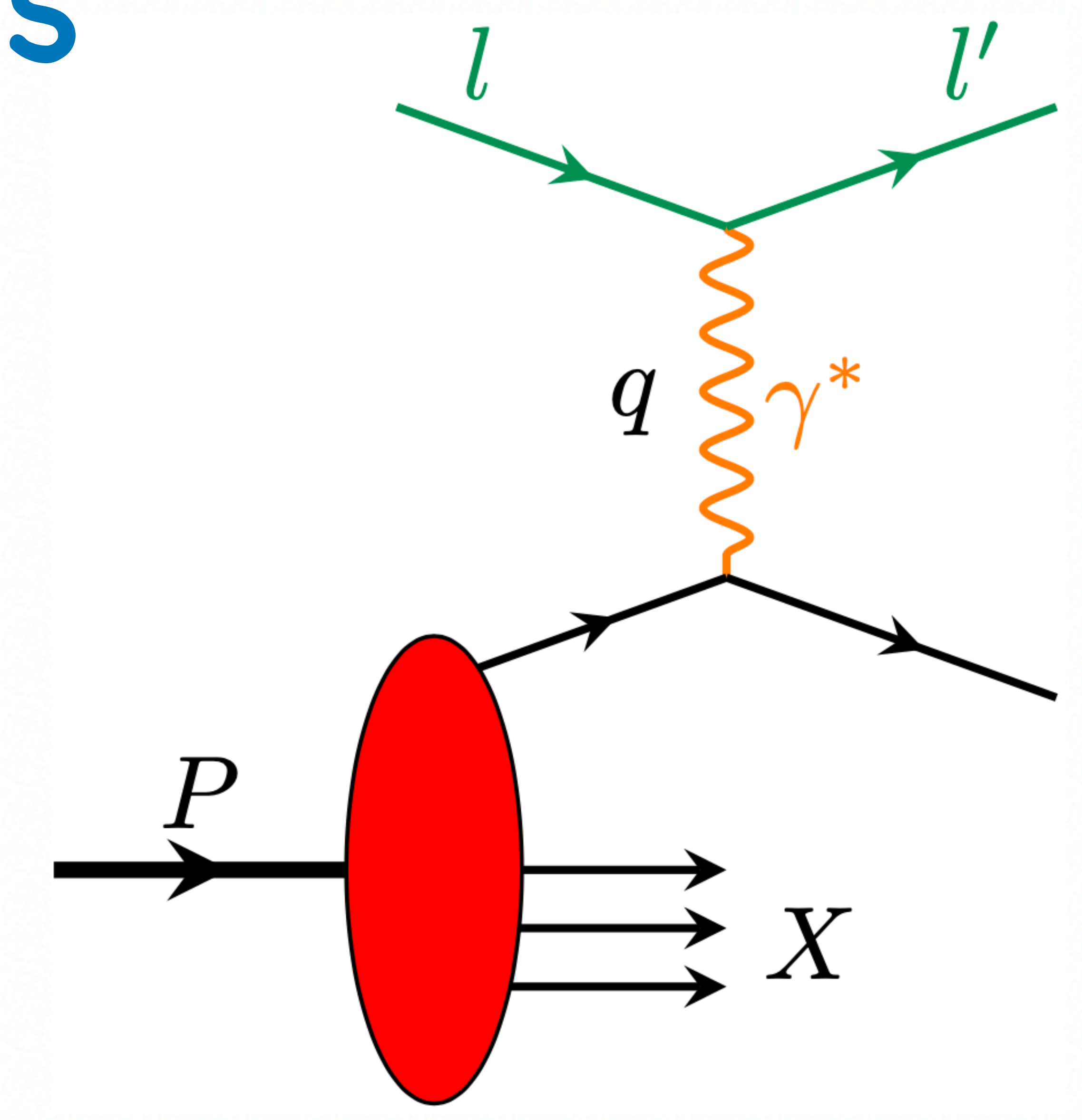
INT WORKSHOP INT-22-83W, Sep 12, 2022

Parton distribution functions

DIS



Non-perturbative PDFs

$$\sigma = \sum_i f_i(x, Q^2) \circledast \sigma\left\{eq_i(xP) \to eq_i(xP+q)\right\}$$

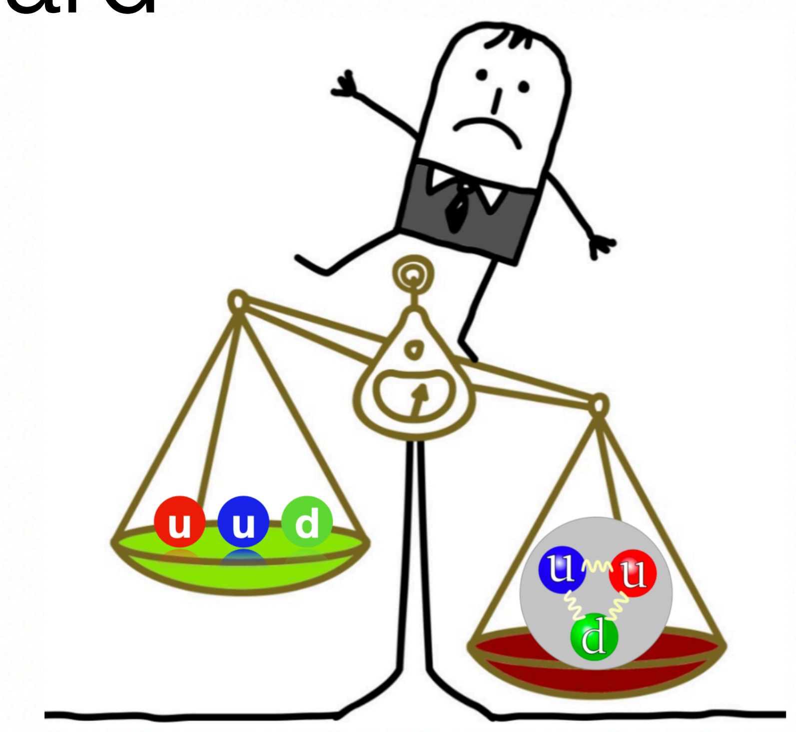
Perturbative parton process

Hadron Structure and Tomography:

- How hadrons are built.
- Mass and spin decomposition of hadron.

High-energy phenomenology:

- Standard Model backgrounds.
- Higgs physics and search for physics beyond the Standard Model.



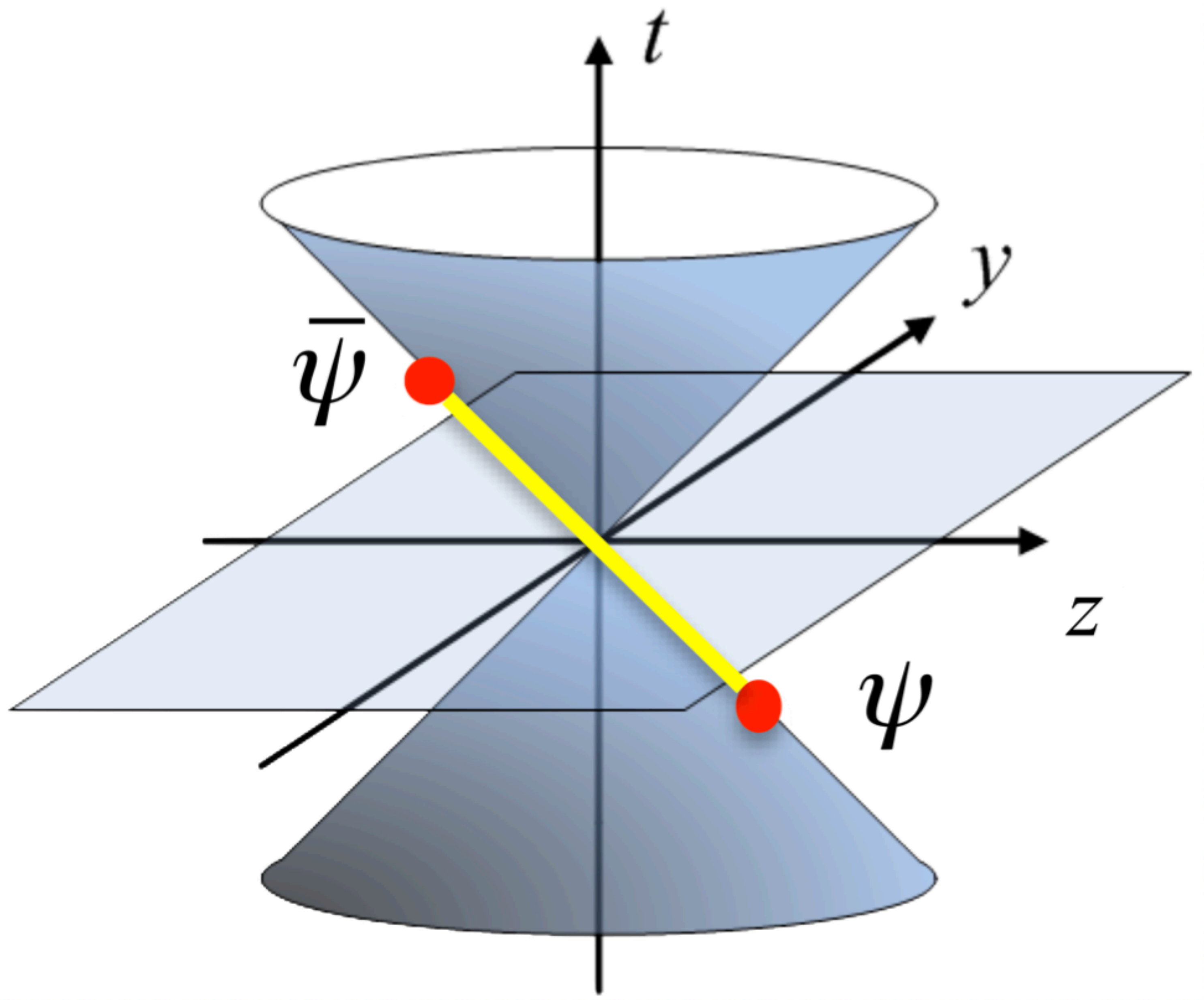
Parton distribution functions

Field theoretic Gauge-invariant and Lorentz invariant construction. (Soper ’77)

$$q(x) \equiv \int \frac{d\xi_-}{4\pi} e^{-ixP^+\xi_-} \langle P | O_\Gamma(\xi_-, \epsilon) | P \rangle, \; O_\Gamma(\xi_-, \epsilon) = \overline{\psi}(0) \Gamma W_-(0,\xi_-) \psi(\xi_-)$$

$$z + ct = 0, \;\; z - ct \neq 0$$

Not implementable in Monte-Carlo methodology!



Projecting to hadron state is easy on lattice, but presence of unequal time separation between $\psi(0)$ and $\psi(\xi_-)$ sandwiched between hadron states is a sign problem for Euclidean lattice.

Parton distribution functions from Lattice

Lattice computation of PDFs：

- Mellin or Gegenbauer Moments from leading-twist local operators.

  RQCD, PLB2017
  RQCD, JHEP2019

- Operator product expansion (OPE) of current-current matrix elements.

  W. Detmold and C. Lin, PRD 2006
  V. Braun and D. Müller, EPJC 2008
  A. J. Chambers, et al, PRL 2017

- Large-momentum effective theory: x-space matching of quasi-PDF.

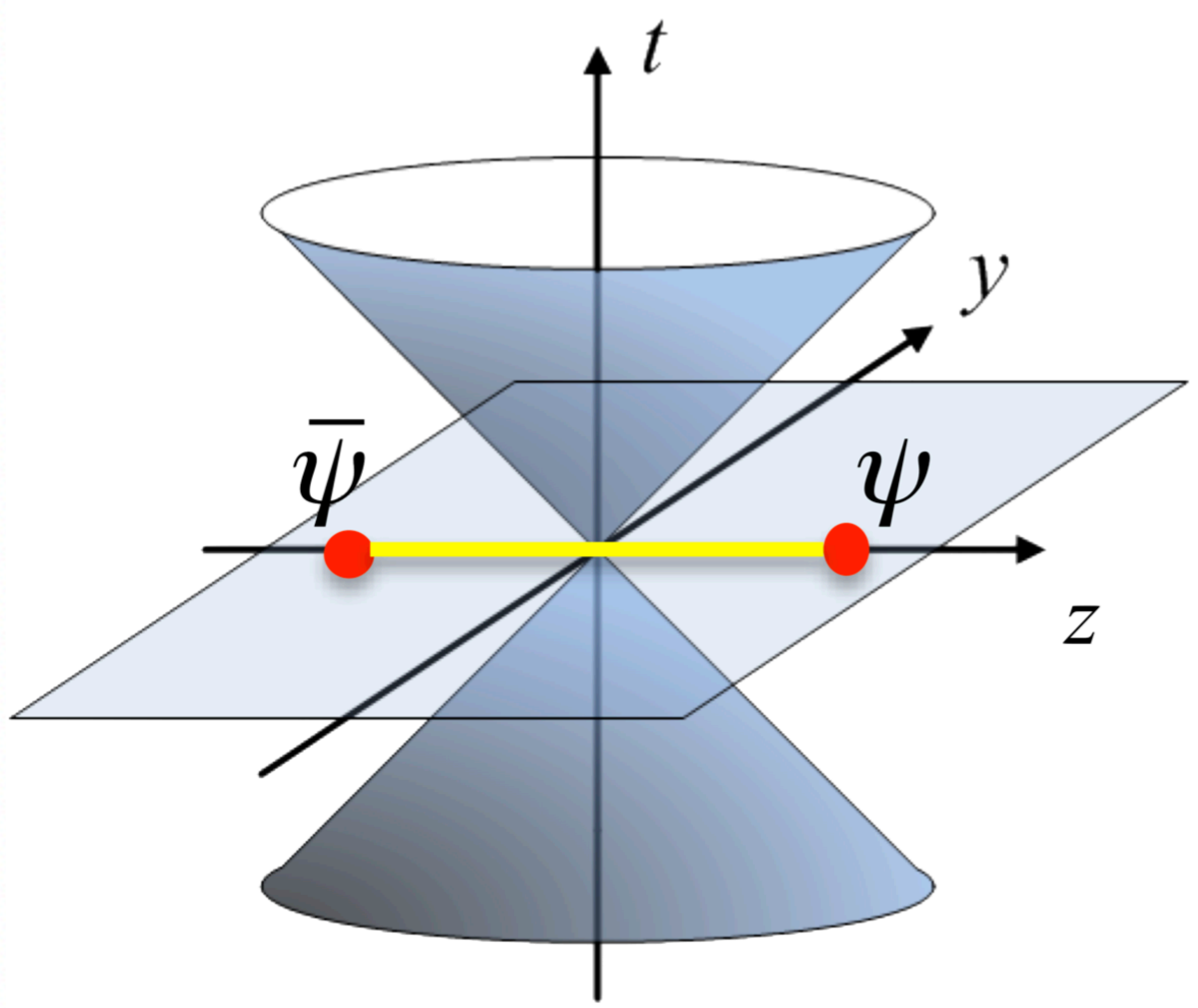
  X. Ji, PRL 2013
  X. Ji, et al, RevModPhys 2021

- Short distance factorization of the quasi-PDF matrix elements in position space or the pseudo-PDF approach.

  A. V. Radyushkin, PRD 2017
  A. V. Radyushkin, Int.J.Mod.Phys.A 2020

- …

$t = 0, \quad z \neq 0$



$$\tilde{q}(x) \equiv \int \frac{dz}{4\pi} e^{-ixP_z z} \langle P | \tilde{O}_\Gamma(z, \epsilon) | P \rangle,$$

$$\tilde{O}_\Gamma(z, \epsilon) = \overline{\psi}(0)\Gamma W_{\hat{z}}(0,z)\psi(z)$$

quasi-PDF matrix elements

Parton distribution functions from Lattice

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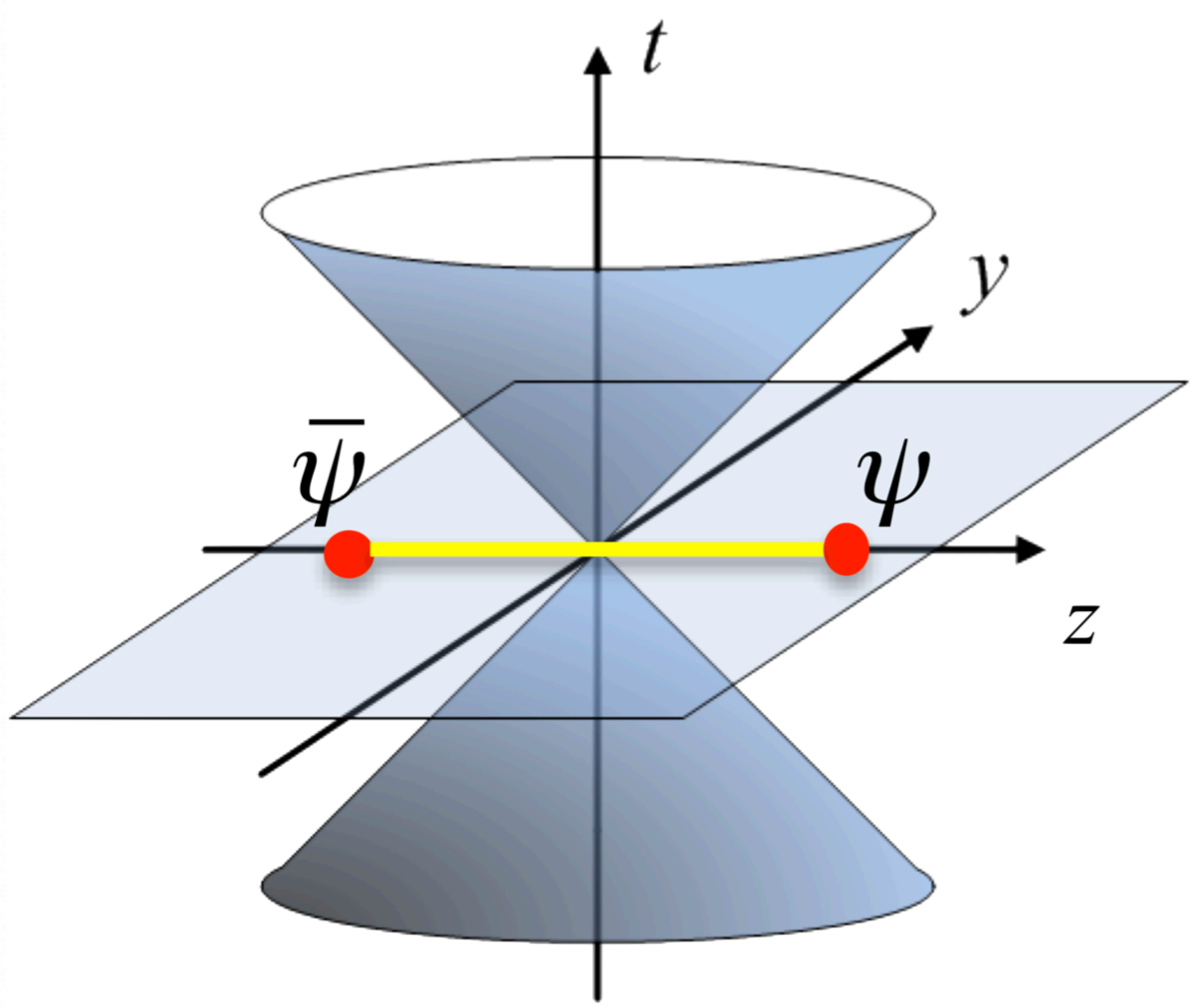

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quasi-PDF matrix elements

Large momentum effective theory

Quasi-PDFs Factorization

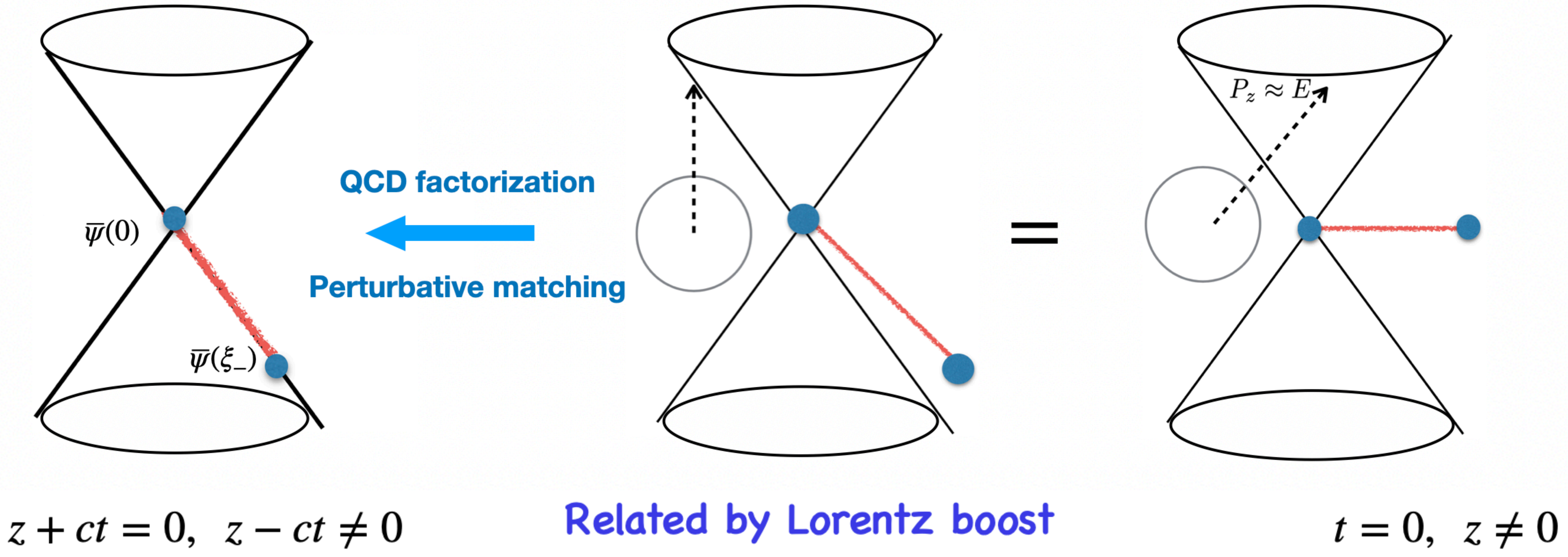
IR must cancel

$$\tilde{q}(x, P_z, \mu) = q(x, \mu) + \alpha_s(\mu)(\tilde{q}^{(1)}(x, P_z, \mu) - q^{(1)}(x, \mu))$$

$$= \int \frac{dy}{|y|} C(\frac{x}{y}, \frac{\mu}{yP_z}) q(y, \mu) + \mathcal{O}(\frac{\Lambda^2_{QCD}}{x^2 P_z^2}, \frac{\Lambda^2_{QCD}}{(1-x)^2 P_z^2})$$

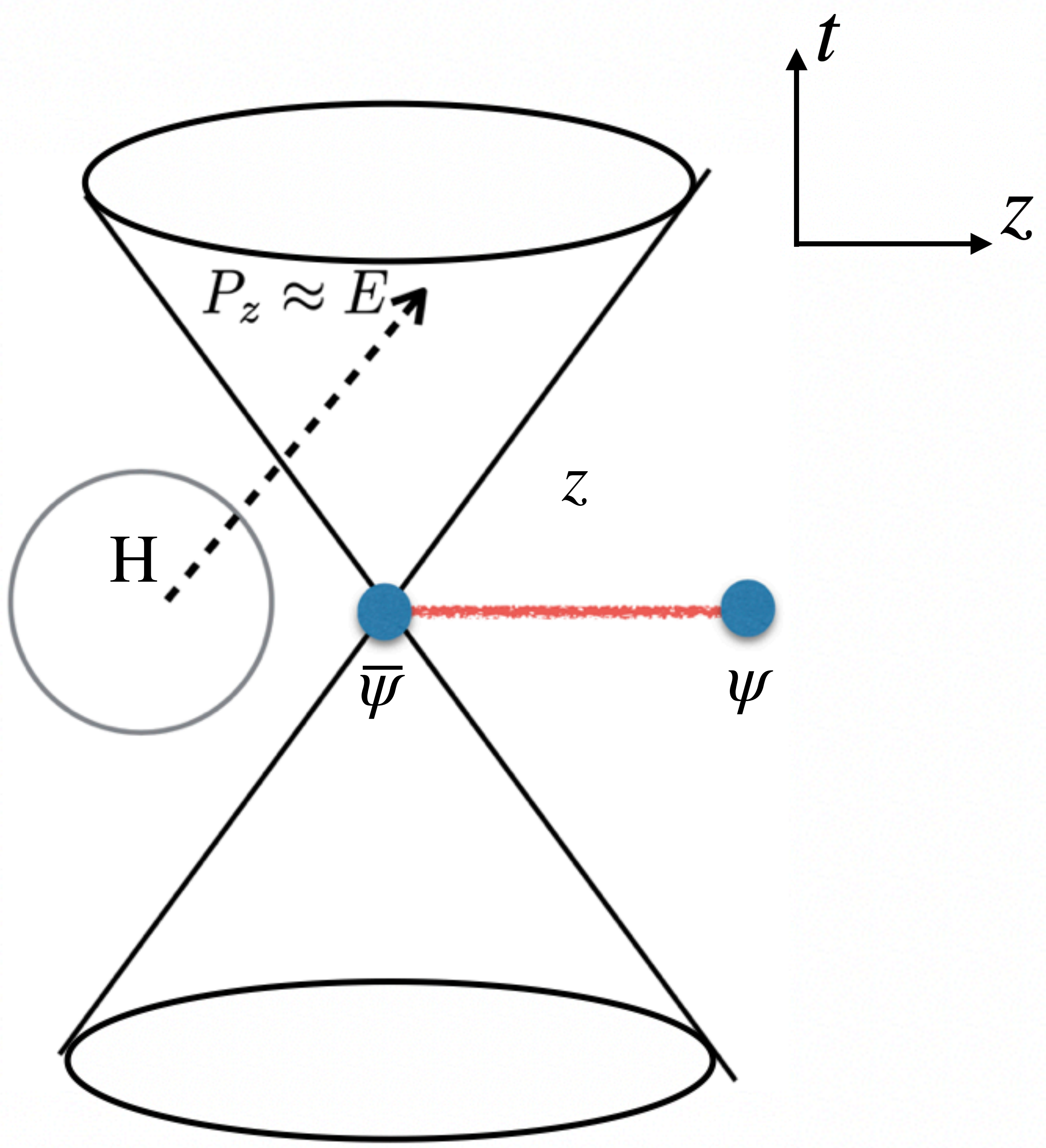
large P_z is essential

- X. Ji, PRL 110 (2013); SCPMA57 (2014);
- X. Xiong, X. Ji, et al, PRD 90 (2014);
- Y.-Q. Ma, et al, PRD98 (2018), PRL 120 (2018);
- T. Izubuchi, X. Ji, et al PRD98 (2018).
- X. Ji, Y. Zhao, et al, RMP 93 (2021).



Short distance factorization

$t = 0, \qquad z \neq 0$



quasi-PDF matrix elements

$\langle P | \tilde{O}_\Gamma(z, \epsilon) | P \rangle$

$\tilde{O}_\Gamma(z, \epsilon) = \overline{\psi}(0)\Gamma W_{\hat{z}}(0,z)\psi(z)$

Coordinate-space factorization:

- V. Braun et al., EPJC 55 (2008)
- A. V. Radyushkin et al., PRD 96 (2017)
- Y. Ma et al., PRL 120 (2018)
- T. Izubuchi et al., PRD 98 (2018)

$$h^R(z, P_z, \mu) = h^R(\lambda, z^2, \mu)$$

$$= \int_{-1}^{1} d\alpha \mathscr{C}(\alpha, \mu^2 z^2) Q(\alpha\lambda, \mu) + \mathcal{O}(z^2 \Lambda_{QCD}^2)$$

Perturbative kernel

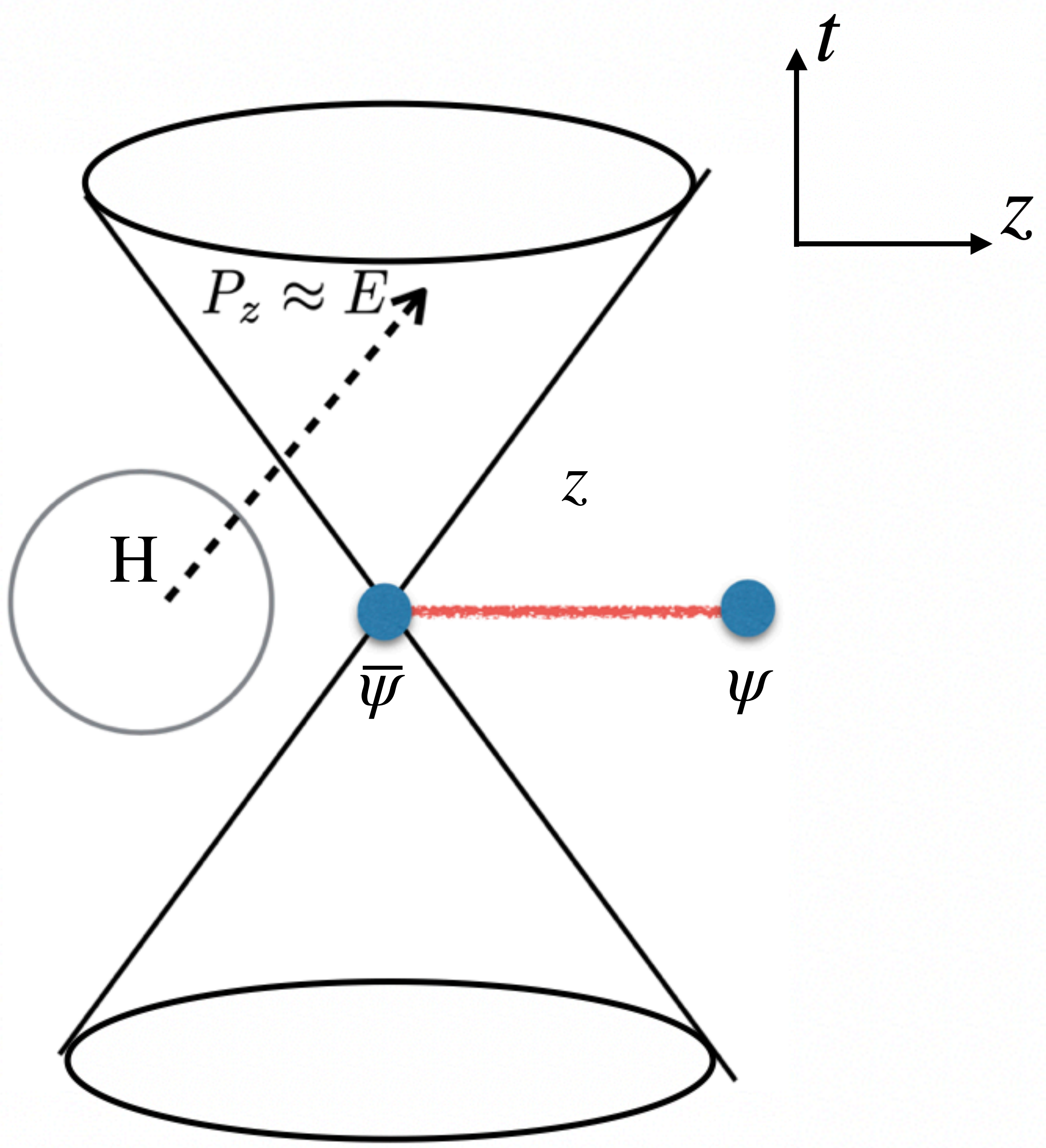
$$\lambda = z P_z$$

with light-cone Ioffe-time distribution:

$$Q(\lambda, \mu) = \int_{-1}^{1} dy e^{-iy\lambda} q(y, \mu)$$

Short distance factorization

$t = 0, \qquad z \neq 0$



quasi-PDF matrix elements

$$\langle P \,|\, \tilde{O}_\Gamma(z,\epsilon) \,|\, P \rangle$$

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$$h^R(z, P_z, \mu) = h^R(\lambda, z^2, \mu)$$

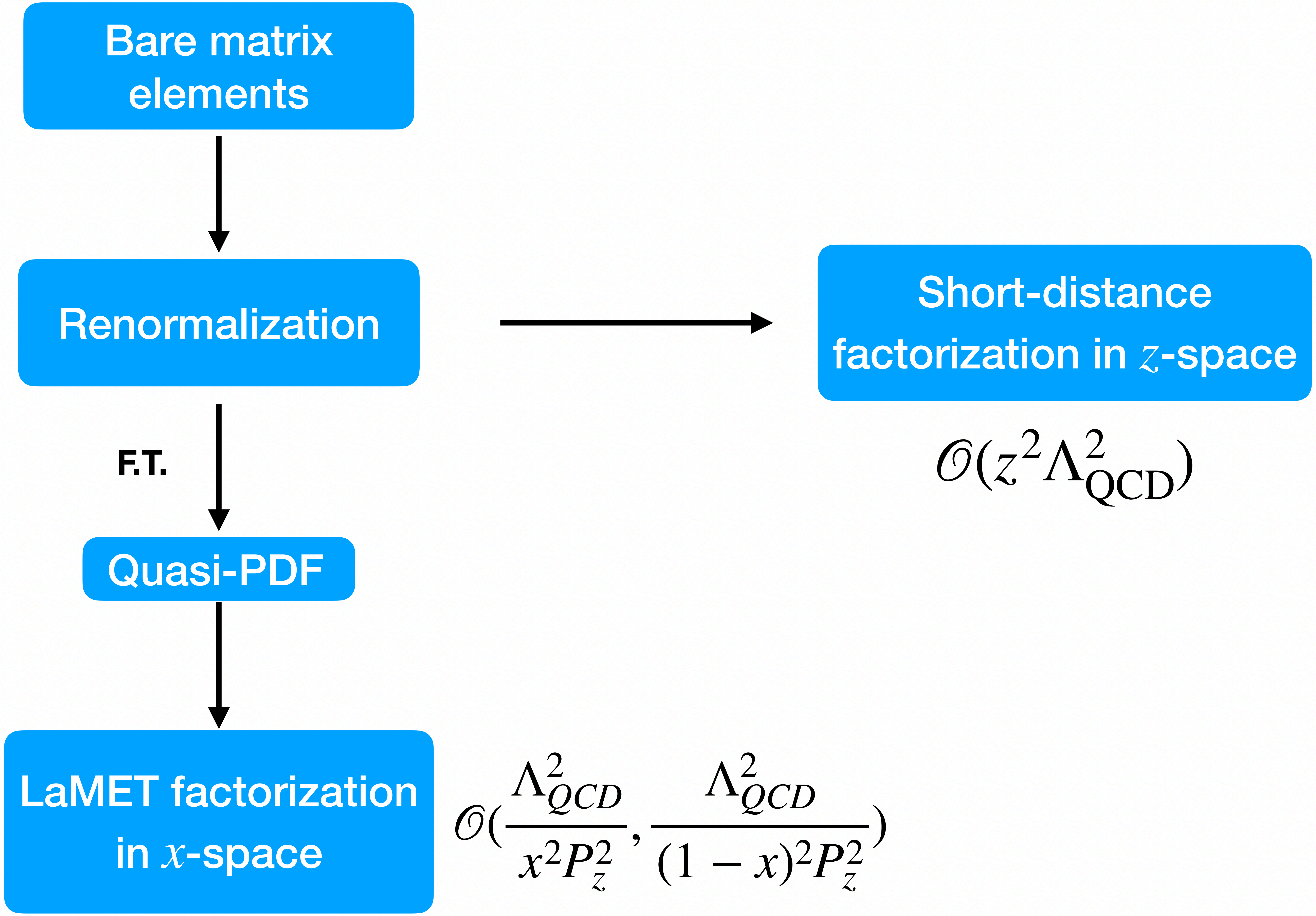
$$= \int_{-1}^{1} d\alpha \mathscr{C}(\alpha, \mu^2 z^2) Q(\alpha\lambda, \mu) + \mathcal{O}(z^2 \Lambda_{QCD}^2)$$

Perturbative kernel

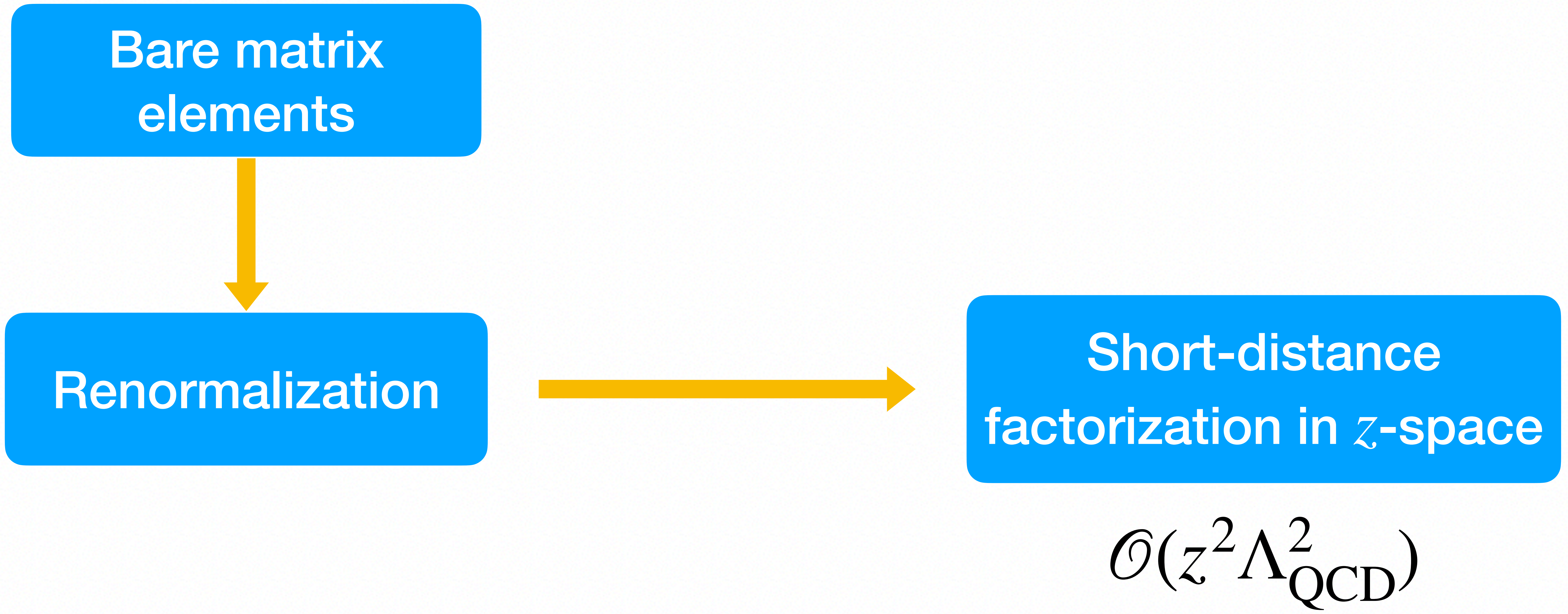
- The perturbative matching is valid in short range of z.
- The information that lattice data contains is limited by the range of finite $\lambda = zP_z$.

Large momentum P_z is still the key!

Parton distribution functions from Lattice



Parton distribution functions from Lattice

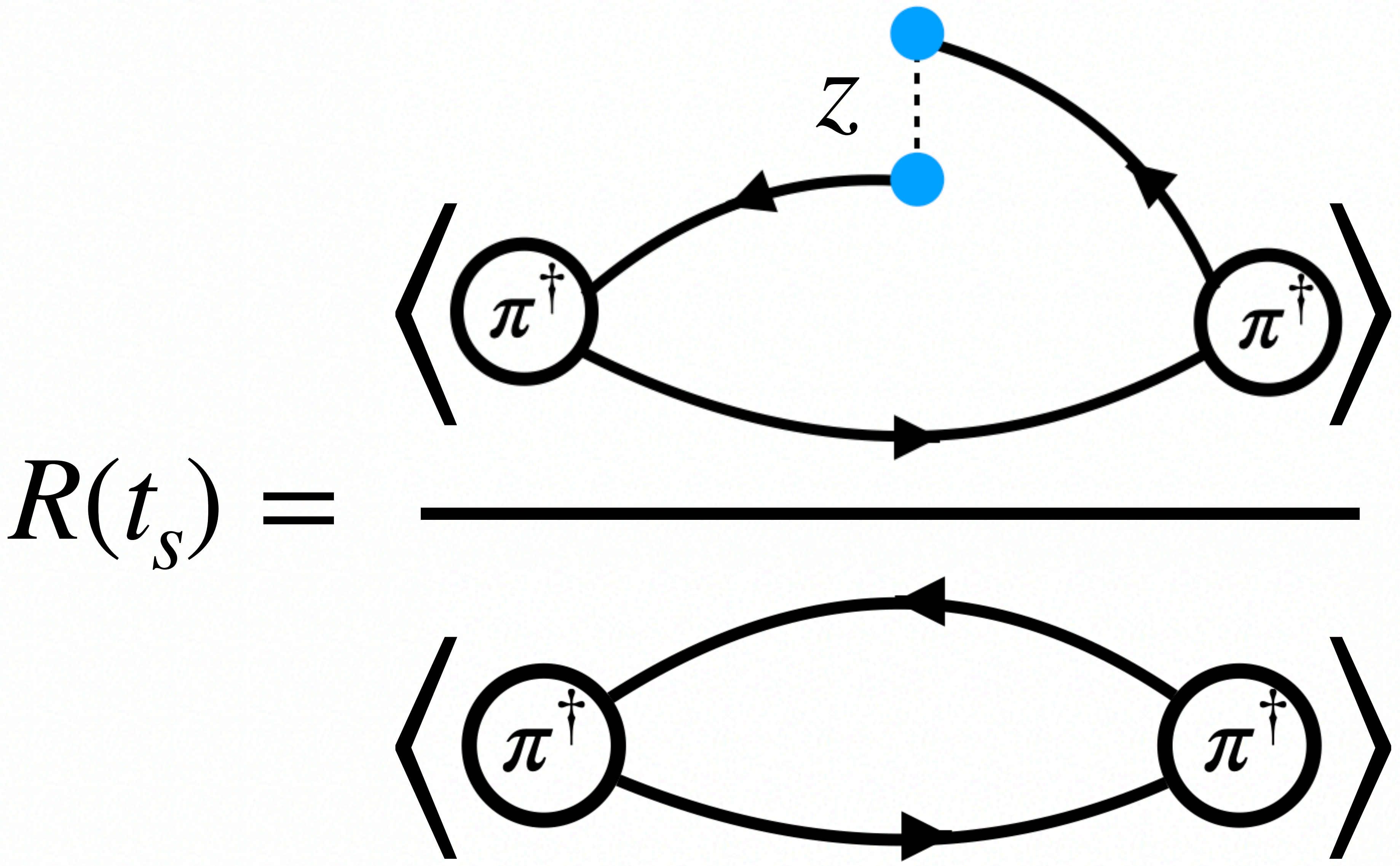


Lattice calculation

Lattice setup:

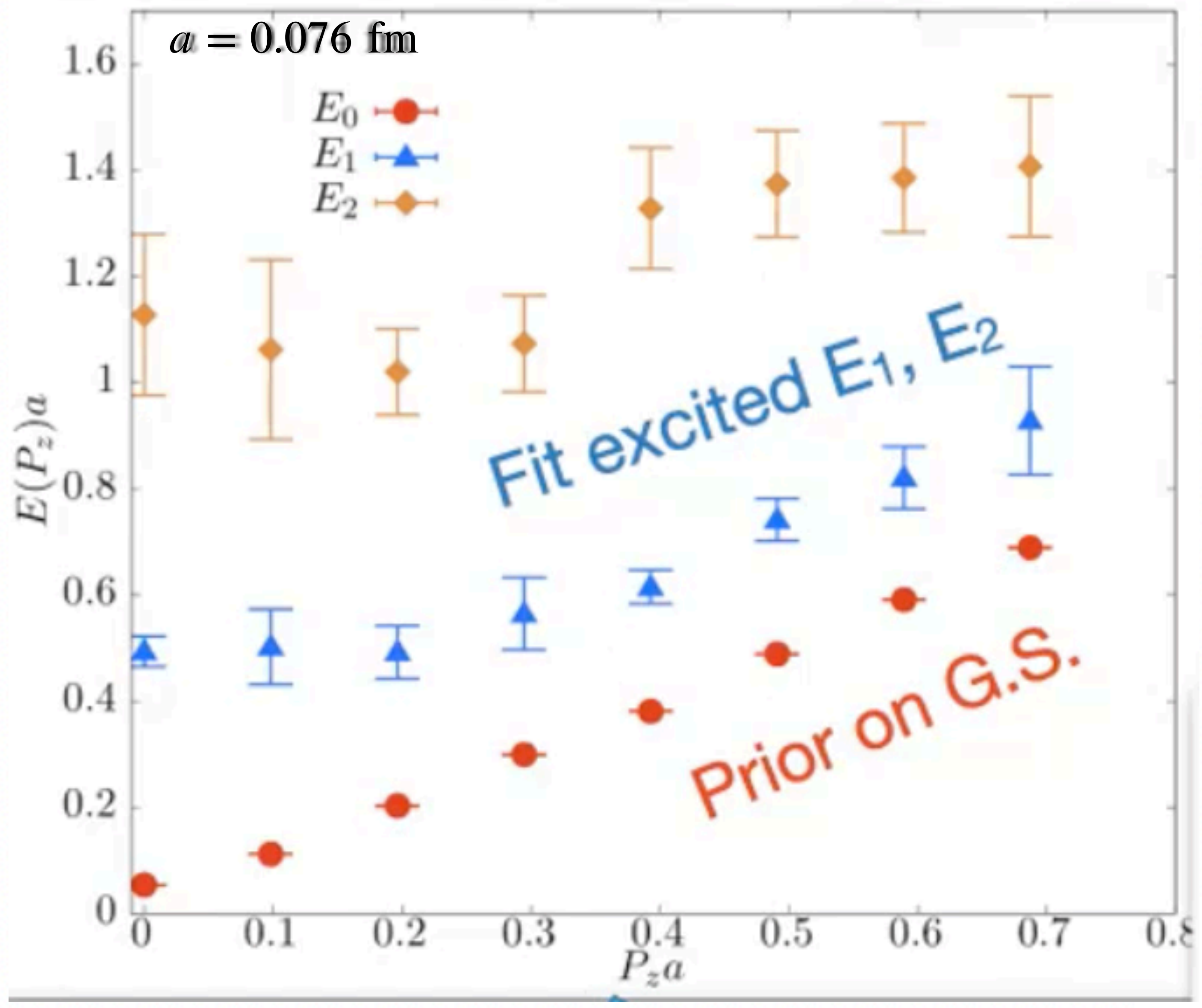
Lattice	a [fm]	m_π	P_z
$64^3 \times 64$	0.04	300 MeV	0 ~ 2.42 GeV
$48^3 \times 64$	0.06	300 MeV	0 ~ 2.15 GeV
$64^3 \times 64$	0.076	140 MeV	0 ~ 1.78 GeV

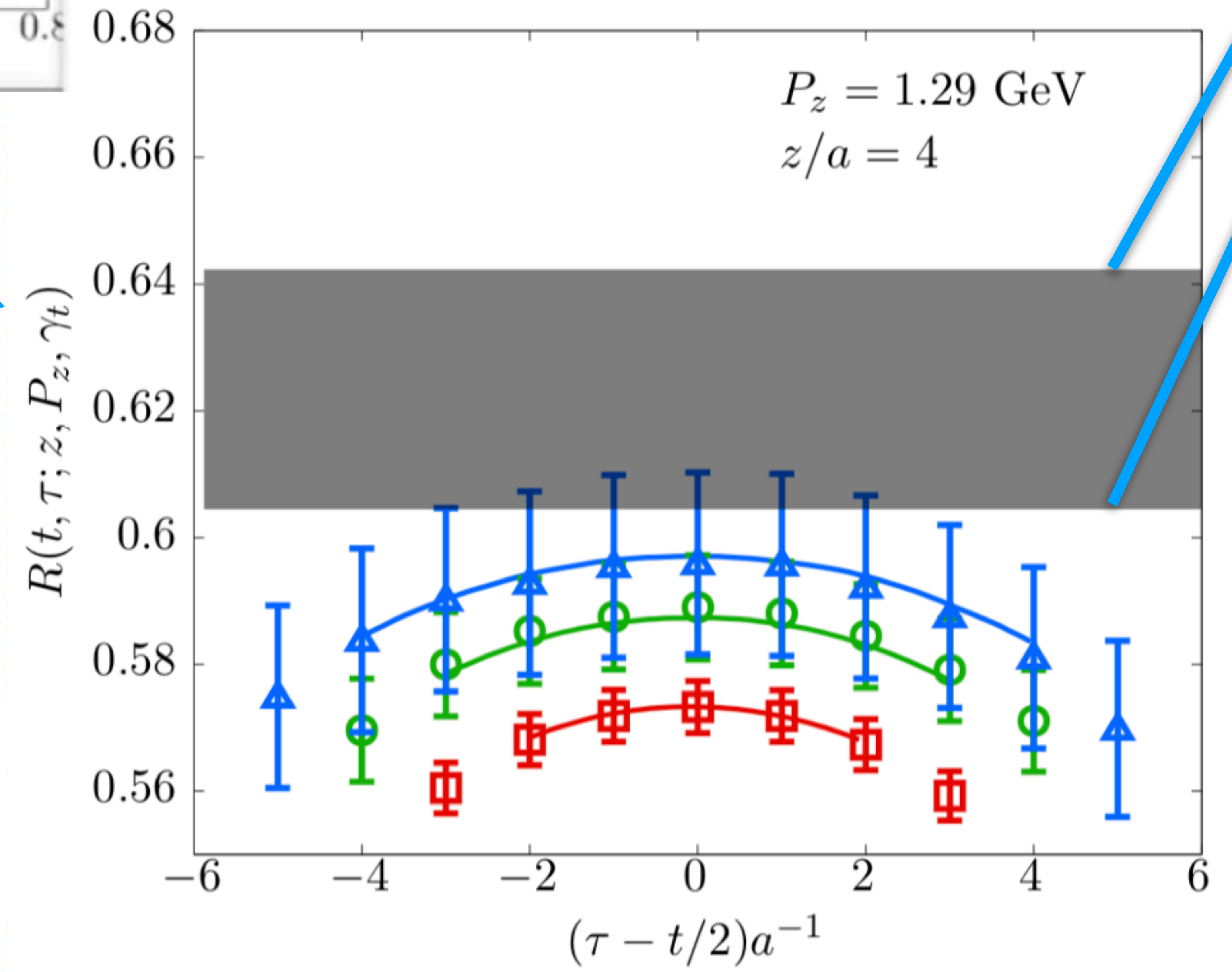
- ➡ Wilson-clover fermion on 2+1 flavor HISQ configurations.
- ➡ Boosted smearing
- ➡ 1-HYP smearing for Wilson line
- ➡ All modes averaging (AMA)

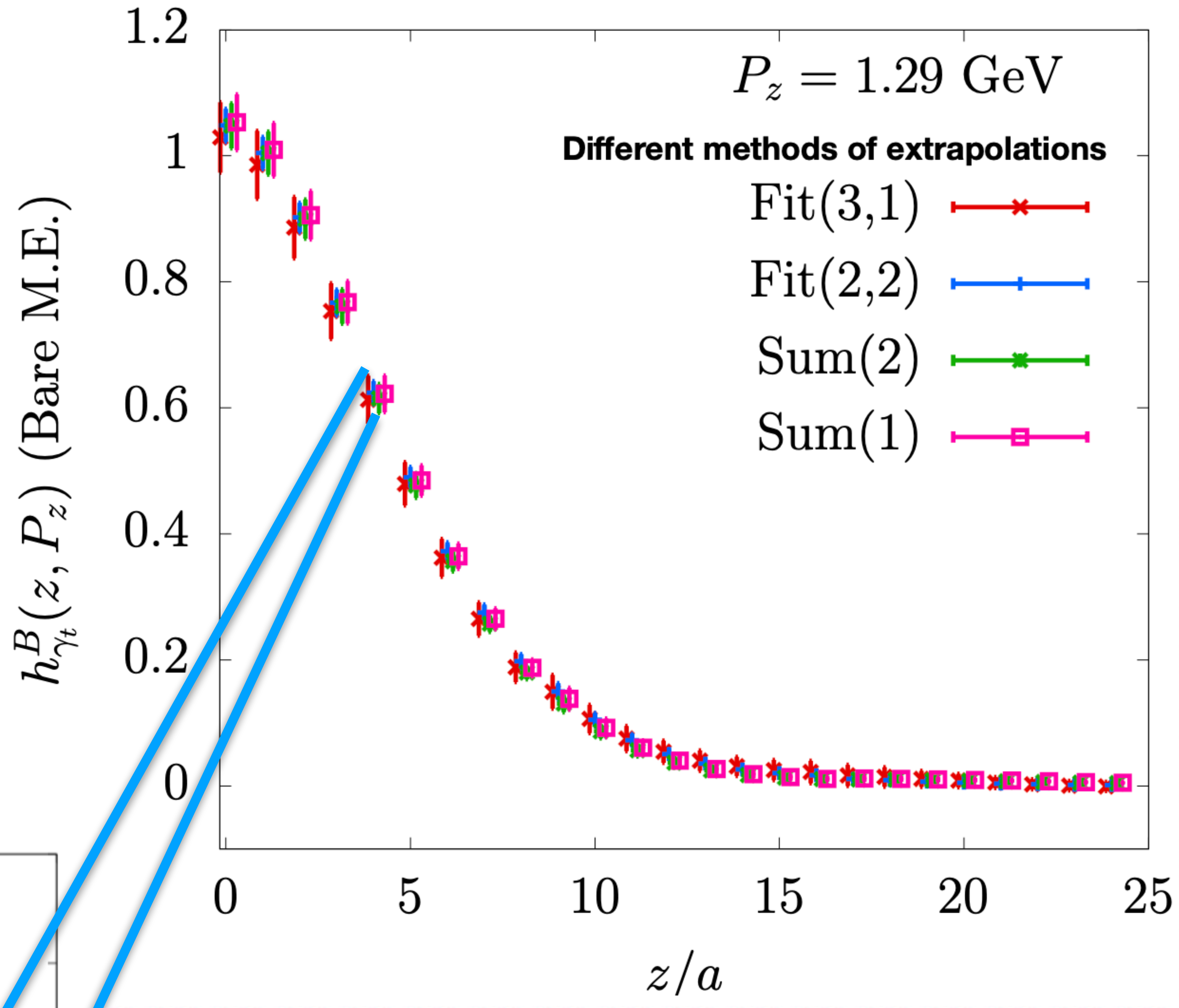


$$h^B(z, P_z) = \lim_{t\to\infty} \frac{\langle \pi(t; P_z) O_\Gamma(z; \tau) \pi^\dagger(0; P_z)\rangle}{\langle \pi(t; P_z)\pi^\dagger(0; P_z)\rangle}$$

Bare quasi-PDF matrix elements







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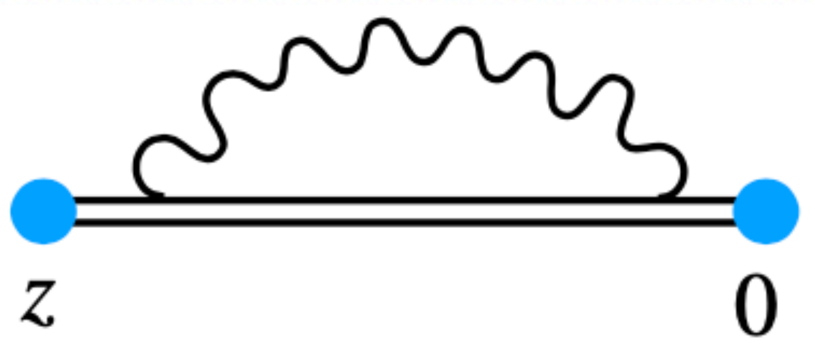
Bare matrix elements and Renormalization

The operator can be multiplicatively renormalized:

- X. Ji, J. H. Zhang and Y. Zhao, PRL120 (2018)
- J. Green, K. Jansen and F. Steffens, PRL121 (2018)
- T. Ishikawa, et al, PRD 96 (2017)



$$= \delta m(a)|z| \propto \frac{|z|}{a}$$

$$[\overline{\psi}(0)\Gamma W_{\hat{z}}(0,z)\psi(z)]_B$$

$$= e^{-\delta m(a)|z|} Z(a) [\overline{\psi}(0)\Gamma W_{\hat{z}}(0,z)\psi(z)]_R$$

$$\delta m = m_{-1}/a + m_0$$

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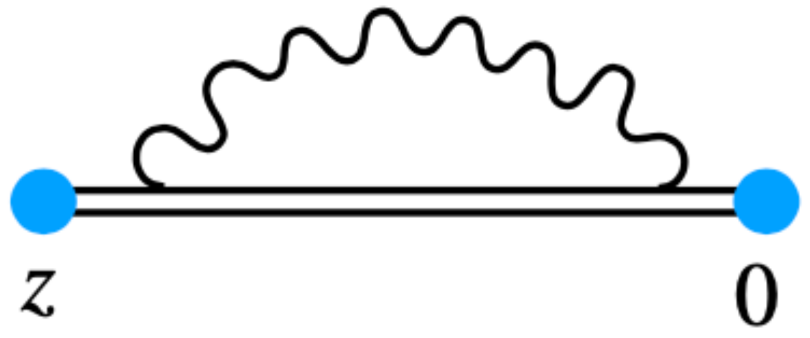

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$$\delta m = m_{-1}/a + m_0$$

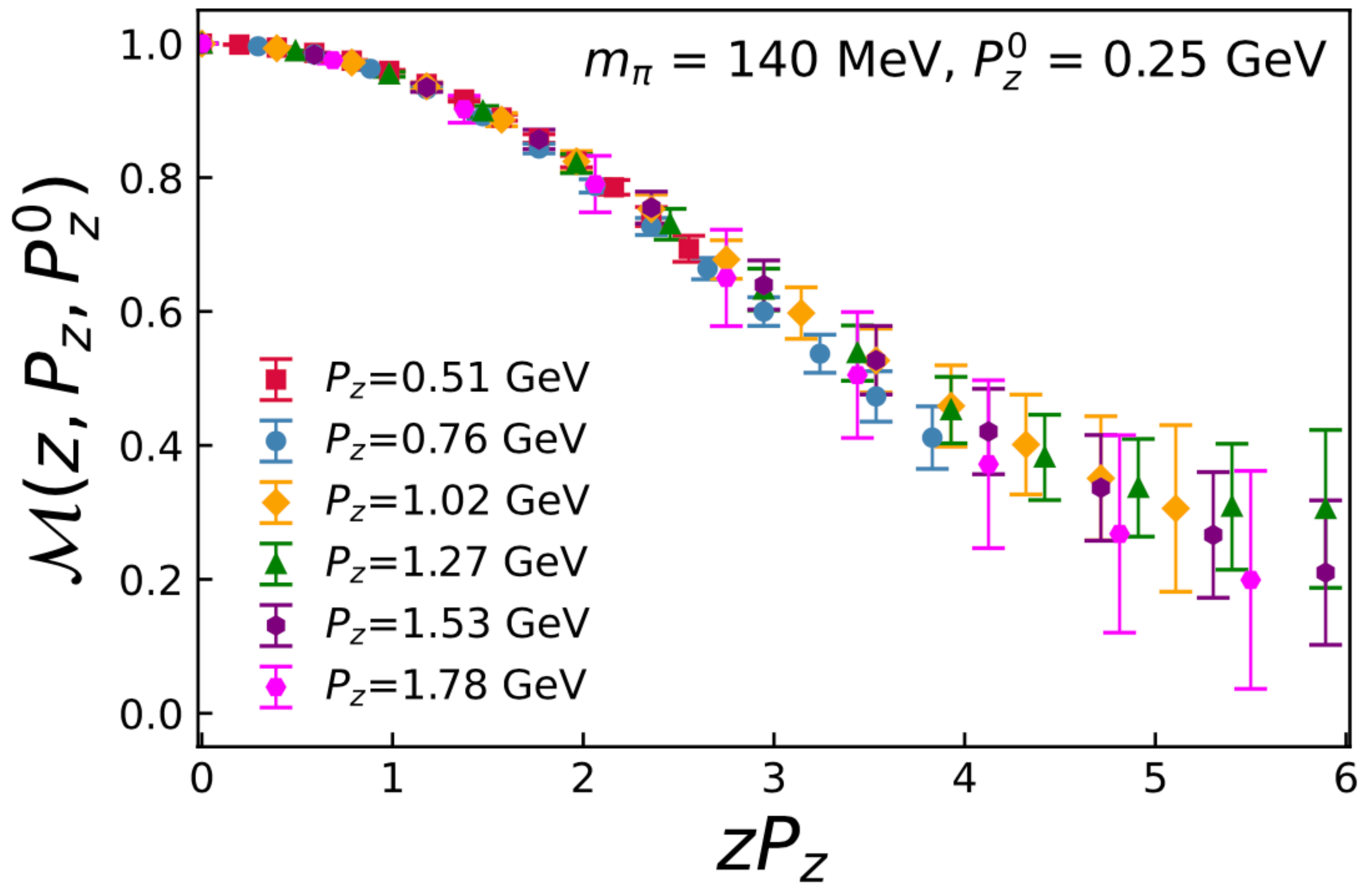
Ratio scheme renormalization

A. V. Radyushkin, PRD 2017
K. Orginos, et al, PRD 96, 2017
Bálint Joó, et al, PRL125, 2020
X. Gao, et al, PRD 102, 2020
Z. Fan, et al, PRD 102, 2020

$$M(z, P_z, P_z^0) = \frac{h^B(z, P_z, a)}{h^B(z, P_z^0, a)} = \frac{h^R(z, P_z, \mu)}{h^R(z, P_z^0, \mu)}$$

- Construct the RG-invariant ratio.

Bare matrix elements and Renormalization



$$M(z, P_z, P_z^0) = \frac{h^B(z, P_z, a)}{h^B(z, P_z^0, a)} = \frac{h^R(z, P_z, \mu)}{h^R(z, P_z^0, \mu)}$$

Insert the SDF formula

$$h^R(z, P_z, \mu) = h^R(\lambda, z^2, \mu)$$

$$= \int_{-1}^{1} d\alpha \mathscr{C}(\alpha, \mu^2 z^2) Q(\alpha\lambda, \mu) + \mathcal{O}(z^2 \Lambda_{QCD}^2)$$

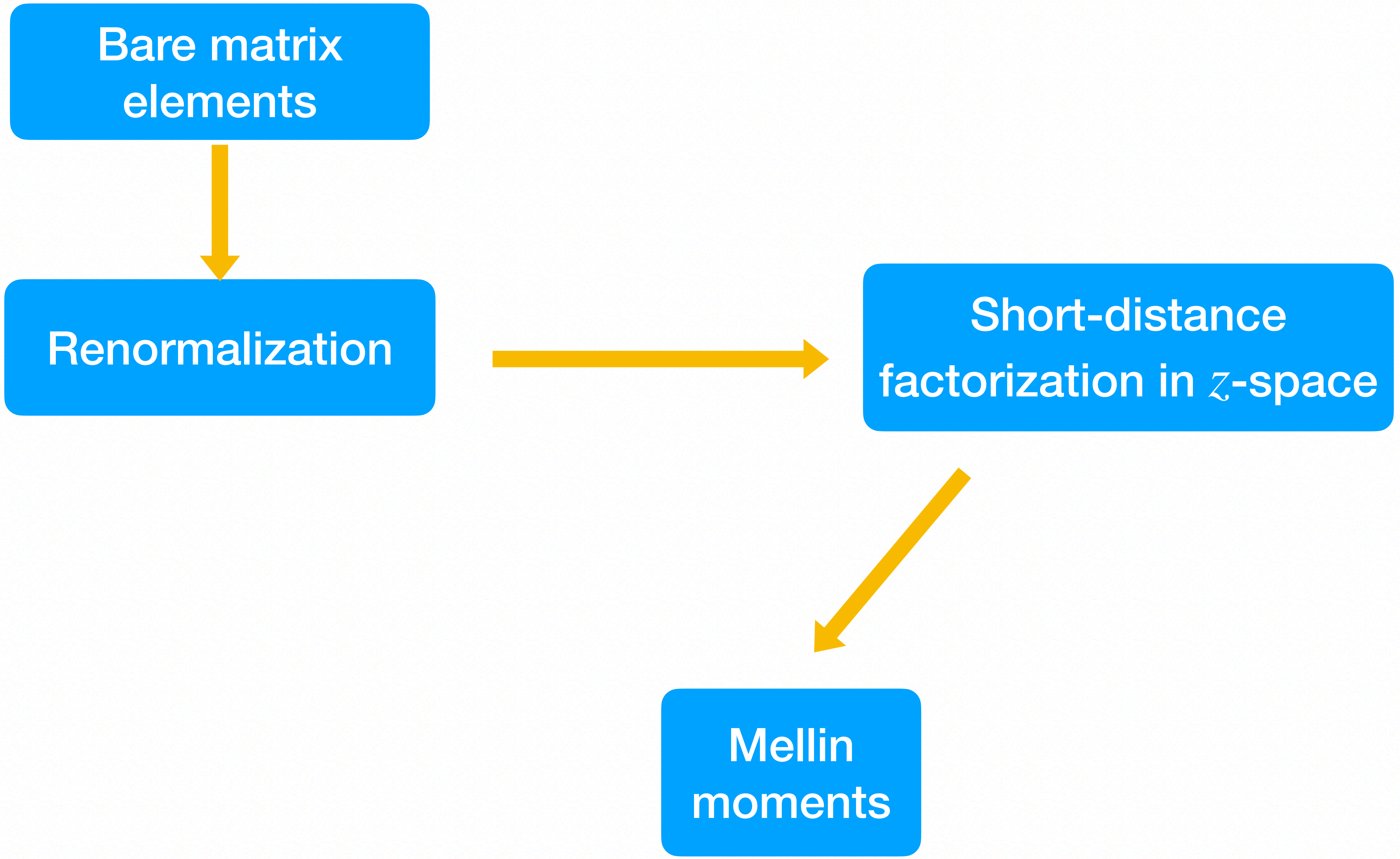
Light-cone ITD

$$= \sum_{n=0}^{\infty} \frac{(-i\lambda)^n}{n!} C_n(z^2\mu^2) \langle x^n \rangle (\mu) + \mathcal{O}(z^2 \Lambda_{\rm QCD}^2)$$

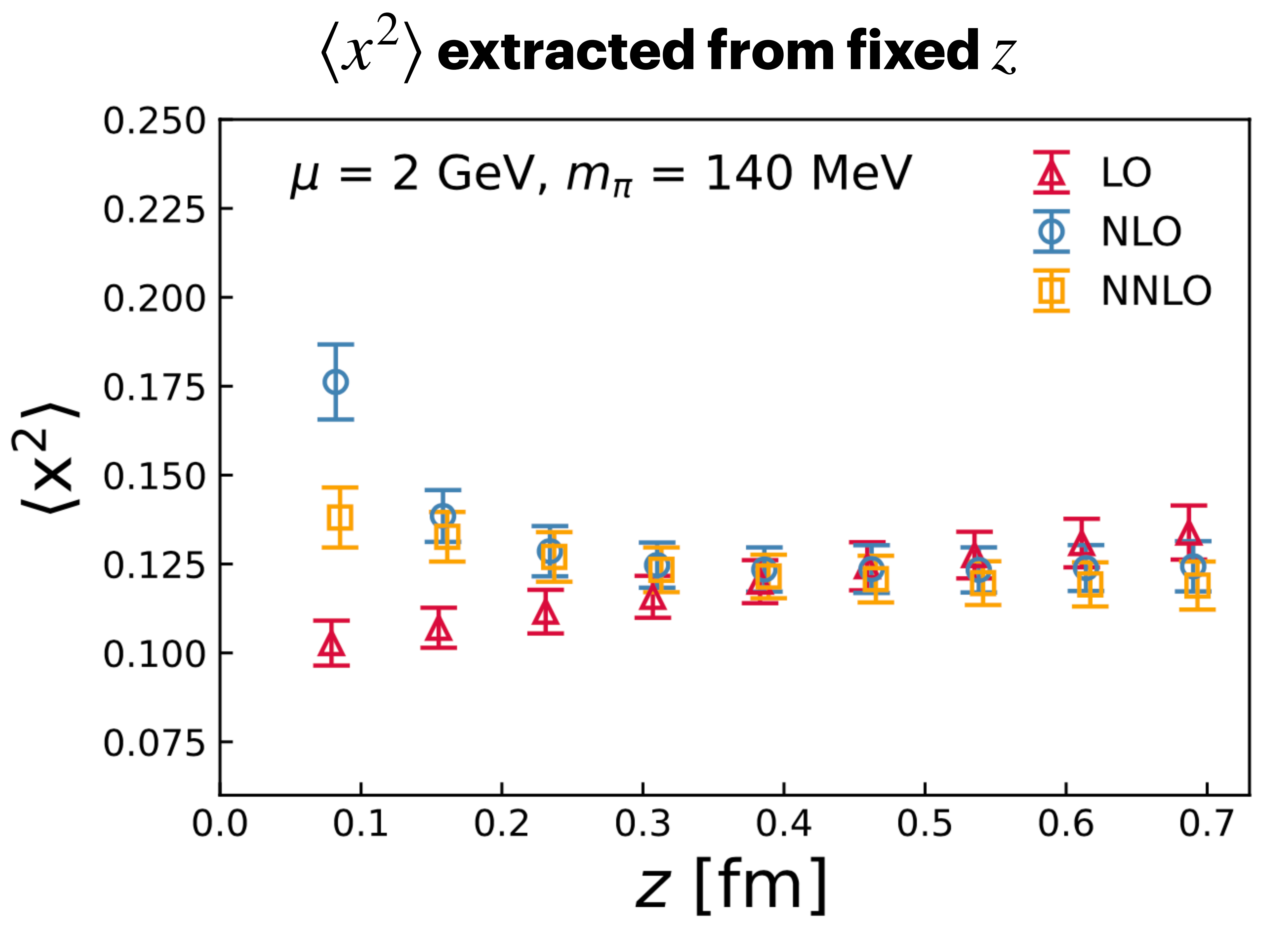
Mellin moments

- Extract the light-cone ITD up to $\lambda = z_{\rm max} P^z_{\rm max}$ or Mellin moments by truncating the OPE up to $n \leq N$.
- The discretization effect and higher-twist effect are supposed to be reduced by the ratio.

Parton distribution functions from Lattice



Mellin moments from model independent fit

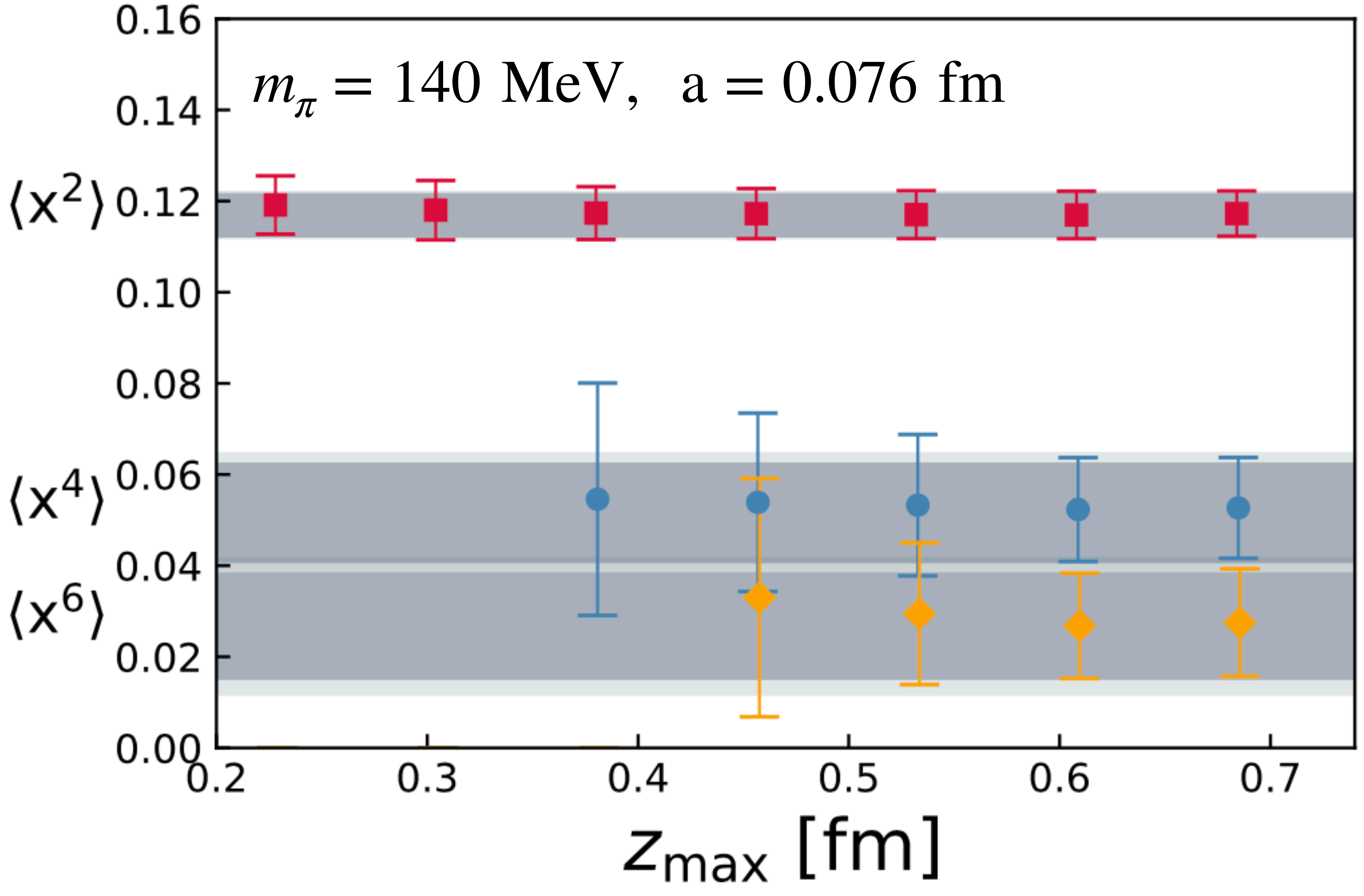


- The tree-level ($\alpha_s = 0$) result show clear z dependence.
- Beyond LO, the perturbative kernels are supposed to compensate the z dependence.
- NNLO produce similar results with NLO but works better when $1/z$ is far from μ.

$$\mathscr{M}(z, P_z, P_z^0) = \frac{\sum_n C_n(\mu^2 z^2) \frac{(-izP_z)^n}{n!} \langle x^n \rangle(\mu) + \mathcal{O}(z^2 \Lambda_{\mathrm{QCD}}^2)}{\sum_n C_n(\mu^2 z^2) \frac{(-izP_z^0)^n}{n!} \langle x^n \rangle(\mu) + \mathcal{O}(z^2 \Lambda_{\mathrm{QCD}}^2)}$$

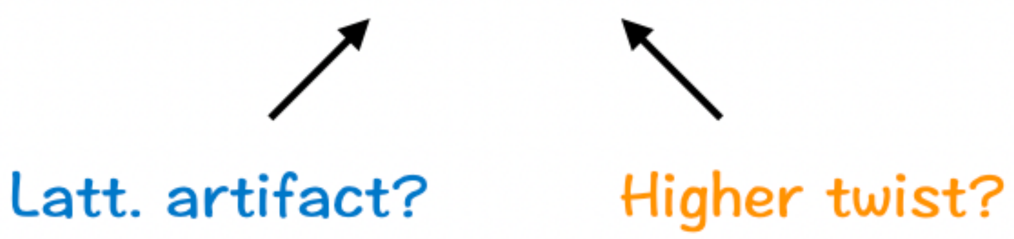
Mellin moments from model independent fit

$\langle x^n \rangle$ extracted from combined z



Combine z fit to stabilize the fit using

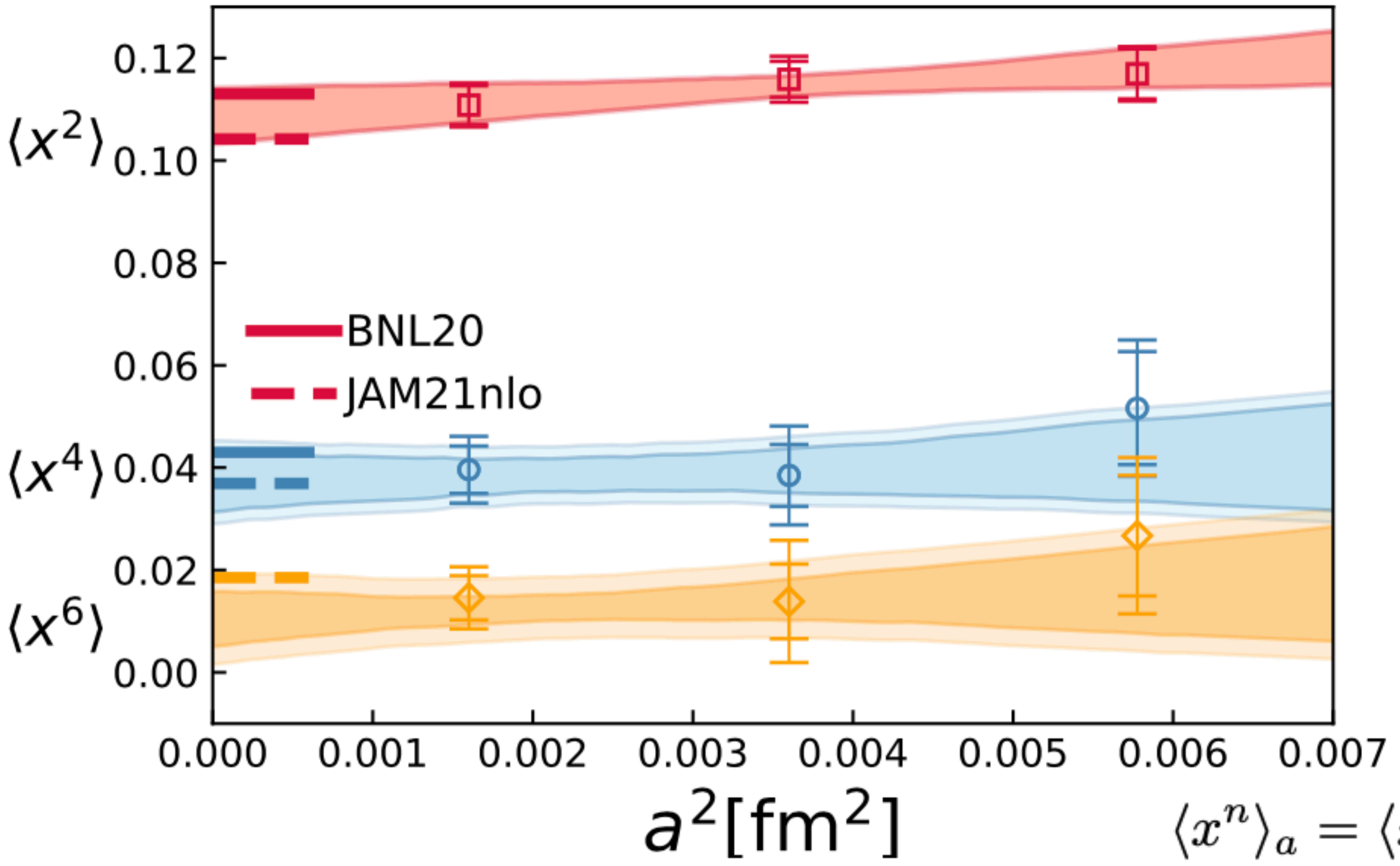
$$z \in [z_{min}, z_{max}]$$



- We vary z_{min} ($2a$ and $3a$) and z_{max} (0.48 fm to 0.72 fm) to estimate the systematic errors.

Mellin moments from model independent fit

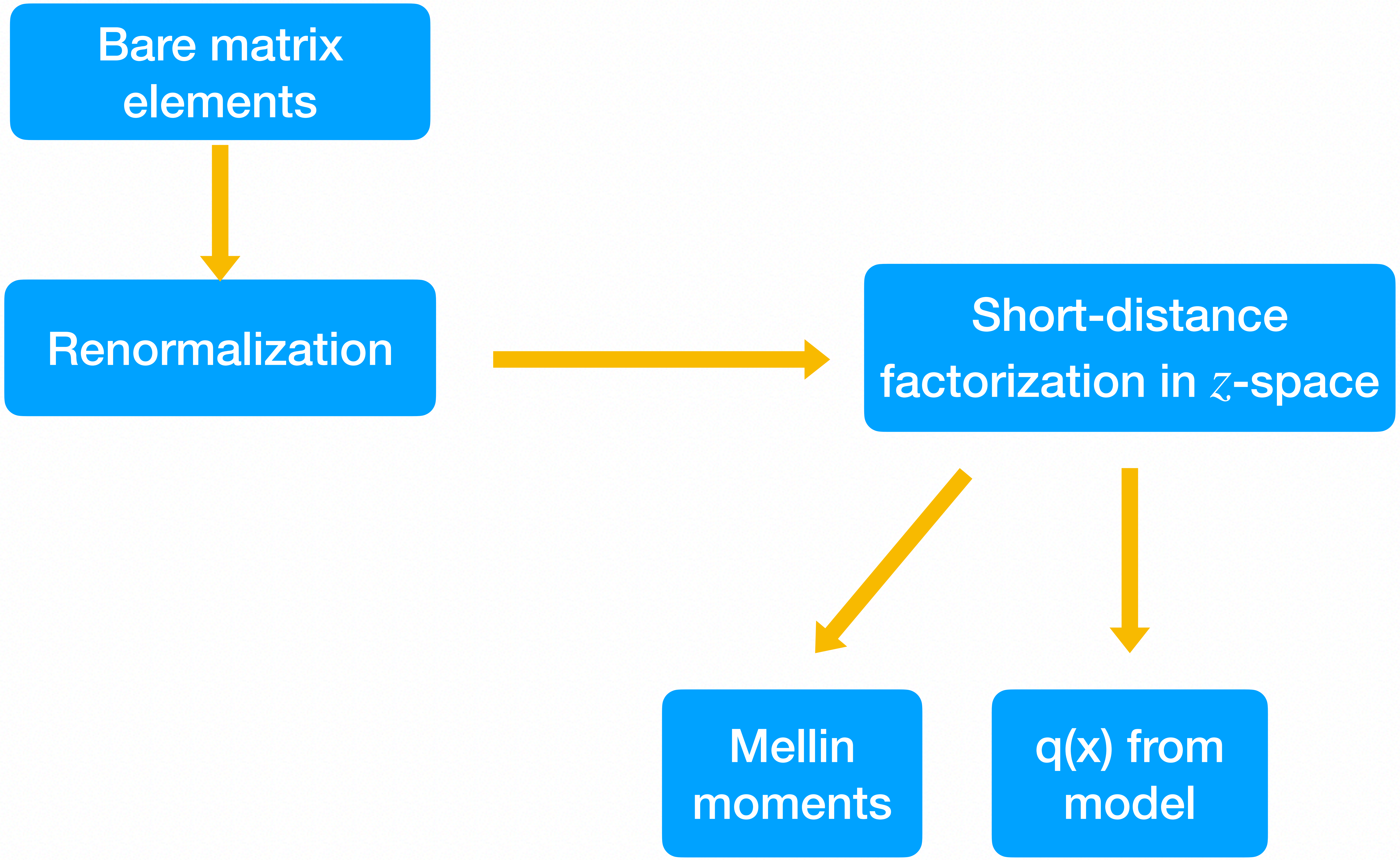
Continuum extrapolation



$$\langle x^n \rangle_a = \langle x^n \rangle_{a^2 \to 0} + d_n a^2$$

- Mild mass and lattice spacing dependence.
- Good agreements with previous NLO determination (BNL20) and JAM21 global analysis of experimental data.

Parton distribution functions from Lattice



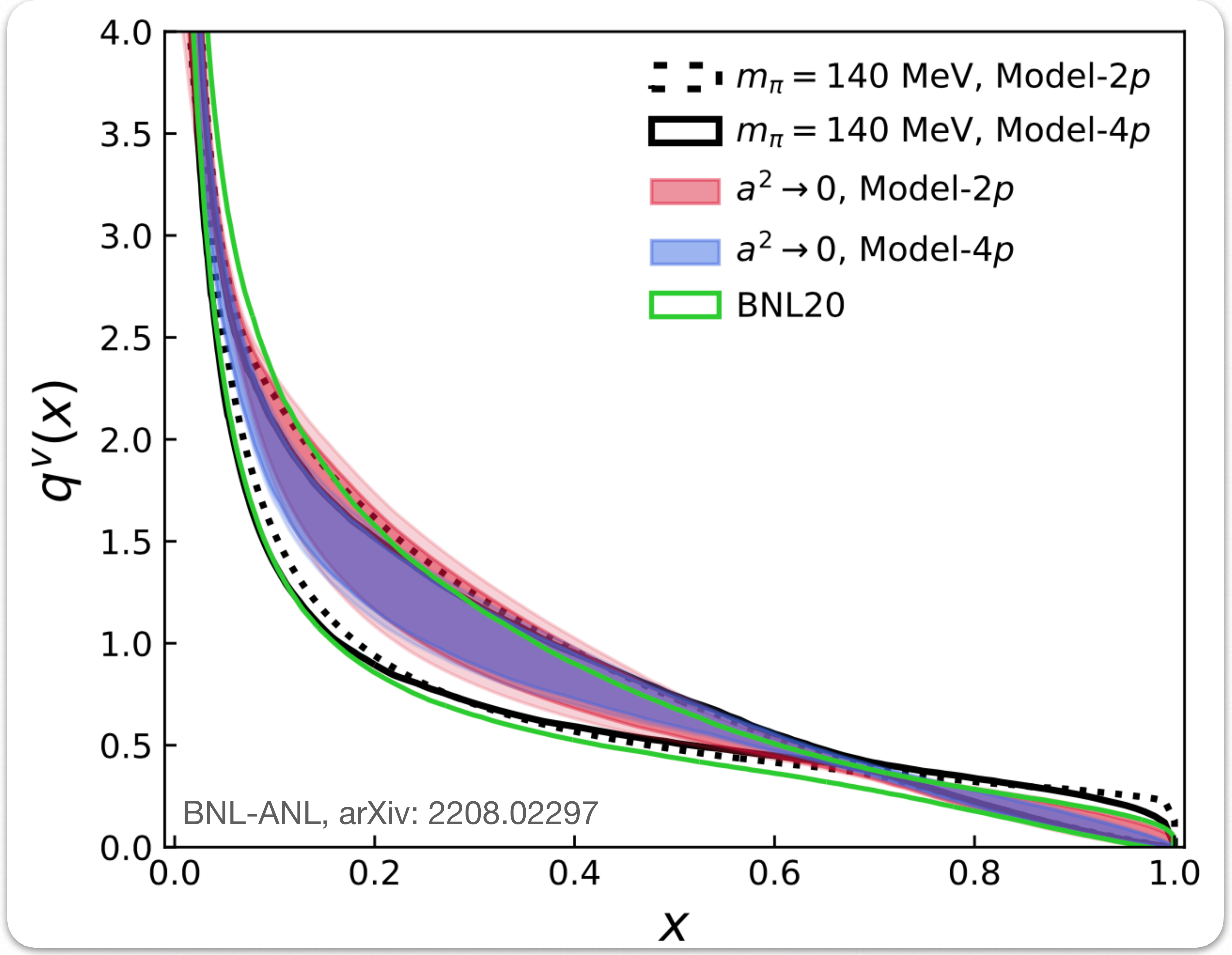
Pion valence PDF from model reconstruction

We tried two phenomenology inspired model for the PDF,

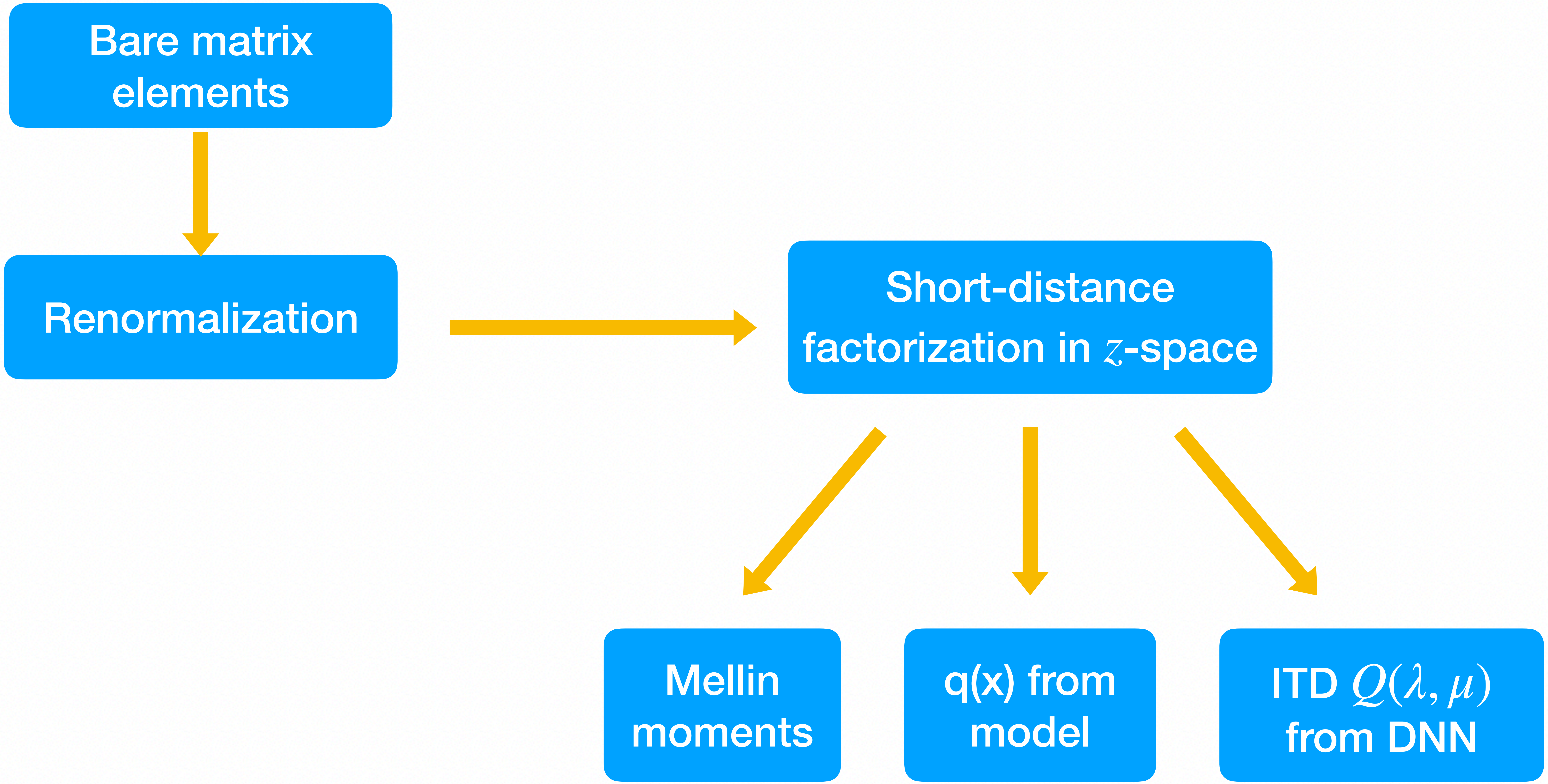
$$q(x;\alpha,\beta) = \mathcal{N} x^{\alpha}(1-x)^{\beta},$$
$$q(x;\alpha,\beta,s,t) = \mathcal{N}' x^{\alpha}(1-x)^{\beta}(1+s\sqrt{x}+tx)$$

then substitute the moments by the model parameters and fit,

$$\mathscr{M}_{\mathrm{model}}(z,P_z,P_z^0) = \frac{\sum_n C_n(\mu^2 z^2)\frac{(-izP_z)^n}{n!}\langle x^n\rangle_{q(x;\alpha,\ldots)} + \mathcal{O}(z^2\Lambda_{\mathrm{QCD}}^2)}{\sum_n C_n(\mu^2 z^2)\frac{(-izP_z^0)^n}{n!}\langle x^n\rangle_{q(x;\alpha,\ldots)} + \mathcal{O}(z^2\Lambda_{\mathrm{QCD}}^2)}$$



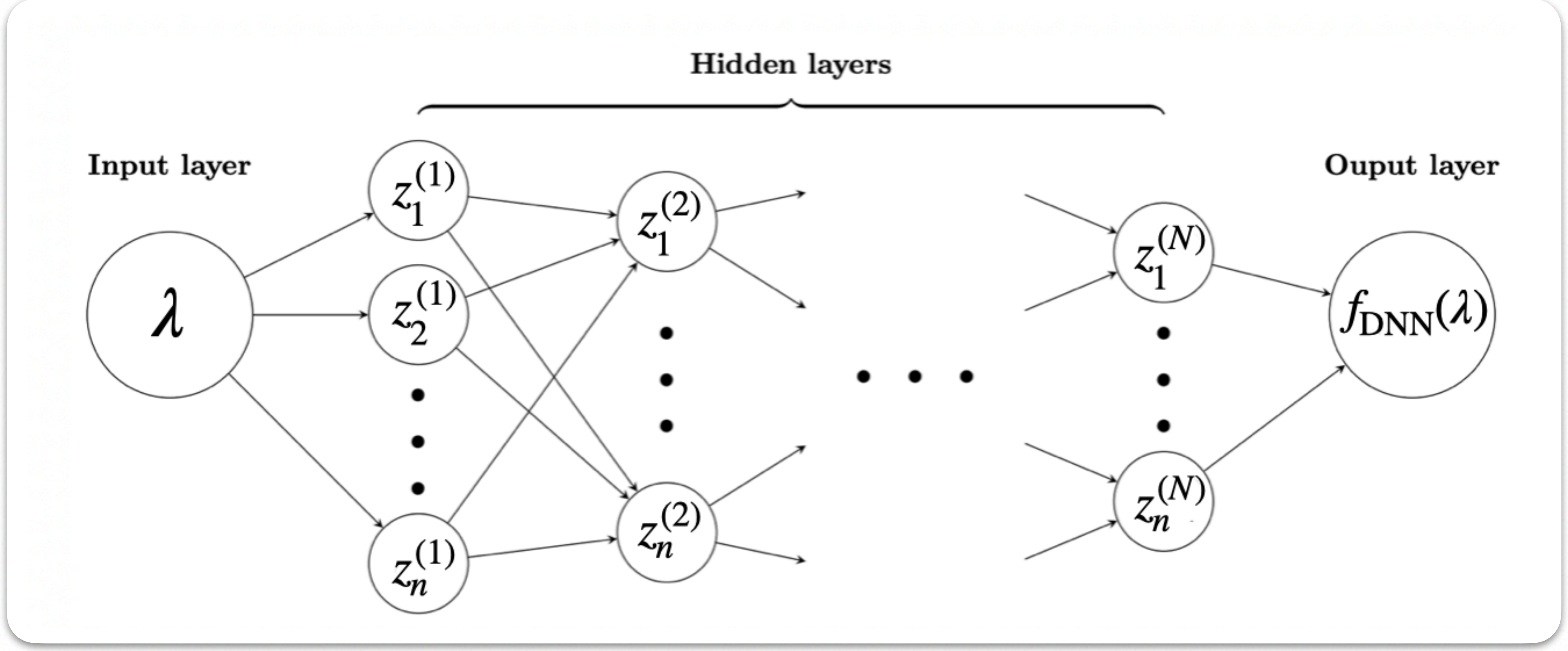
Parton distribution functions from Lattice



DNN representation of Ioffe-time distribution

We express the ITD as

$$Q_{\rm DNN}(\lambda,\mu) \equiv \frac{f_{\rm DNN}(\boldsymbol{\theta};\lambda)}{f_{\rm DNN}(\boldsymbol{\theta};0)}$$



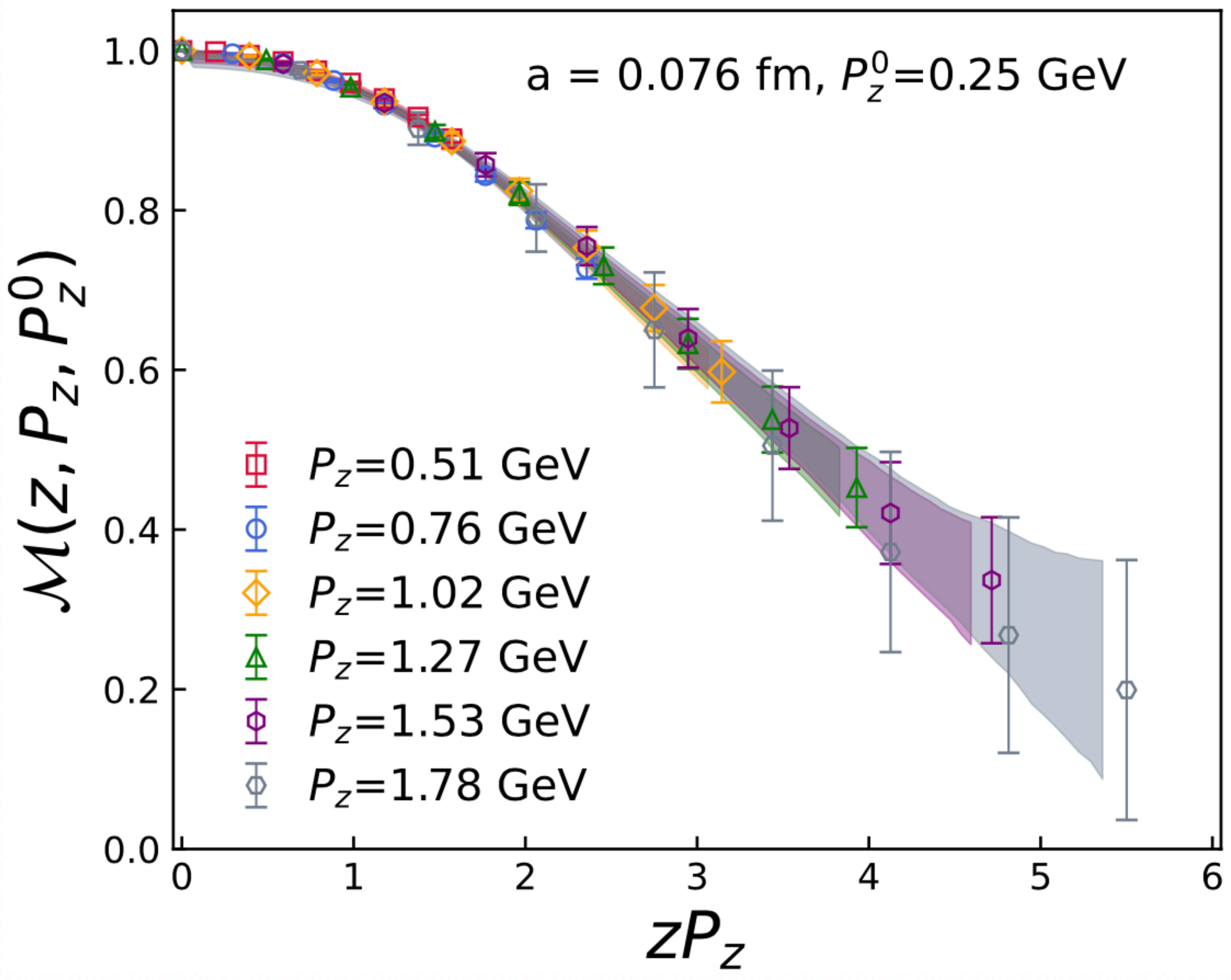
Short distance factorization

$$\mathscr{M}(z,P_z,P_z^0) = \frac{\int_{-1}^{1} d\alpha\, \mathscr{C}(\alpha,\mu^2 z^2)\, Q(\alpha\lambda,\mu) + \mathcal{O}(z^2\Lambda_{QCD}^2)}{\int_{-1}^{1} d\alpha\, \mathscr{C}(\alpha,\mu^2 z^2)\, Q(\alpha\lambda^0,\mu) + \mathcal{O}(z^2\Lambda_{QCD}^2)},$$

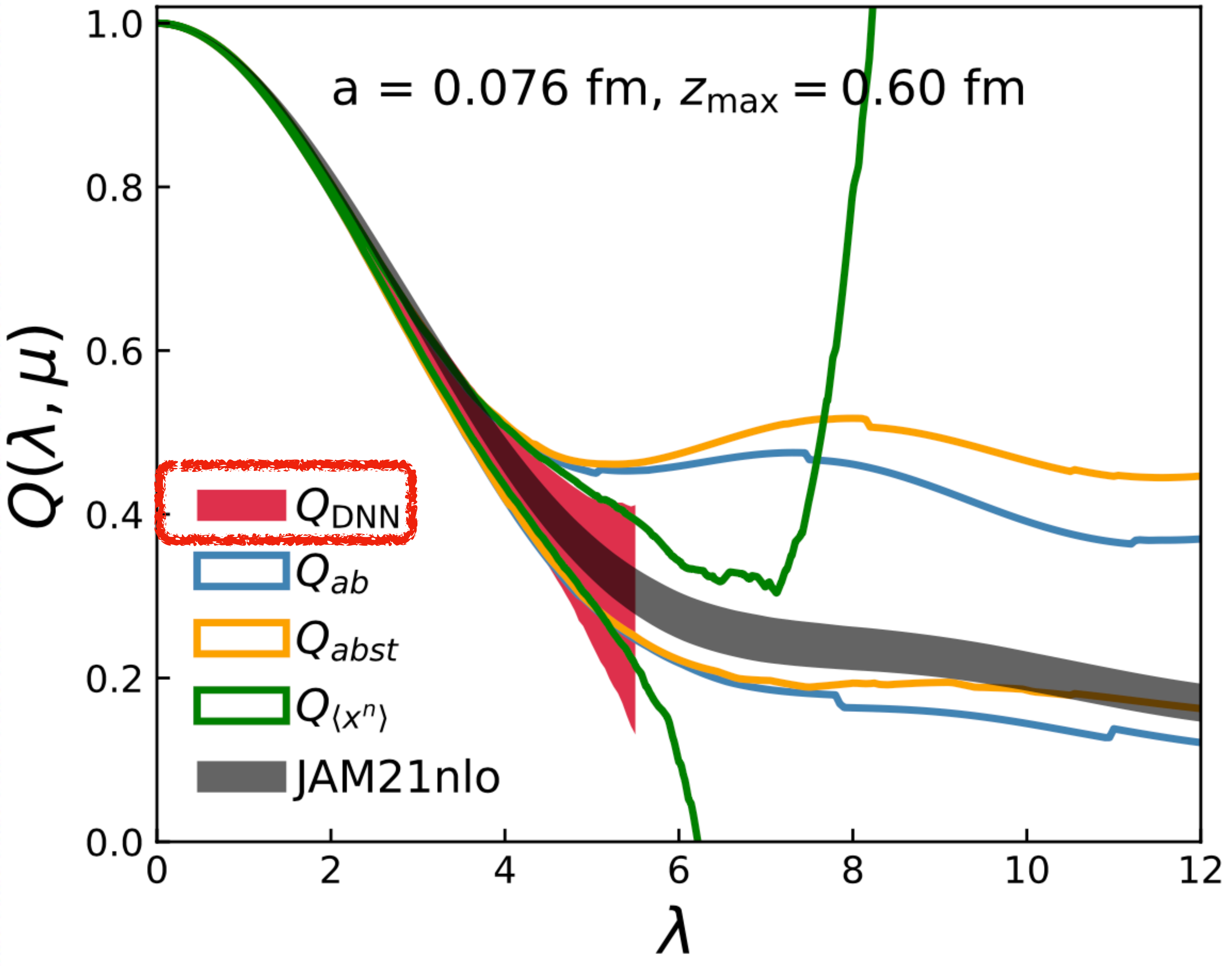
- $Q(\lambda,\mu)$ is the most straight-forward observable that the lattice data sensitive to and is free of truncation of Mellin OPE.
- Inverse problem.

DNN representation of Ioffe-time distribution

Ratio-scheme renormalized matrix elements

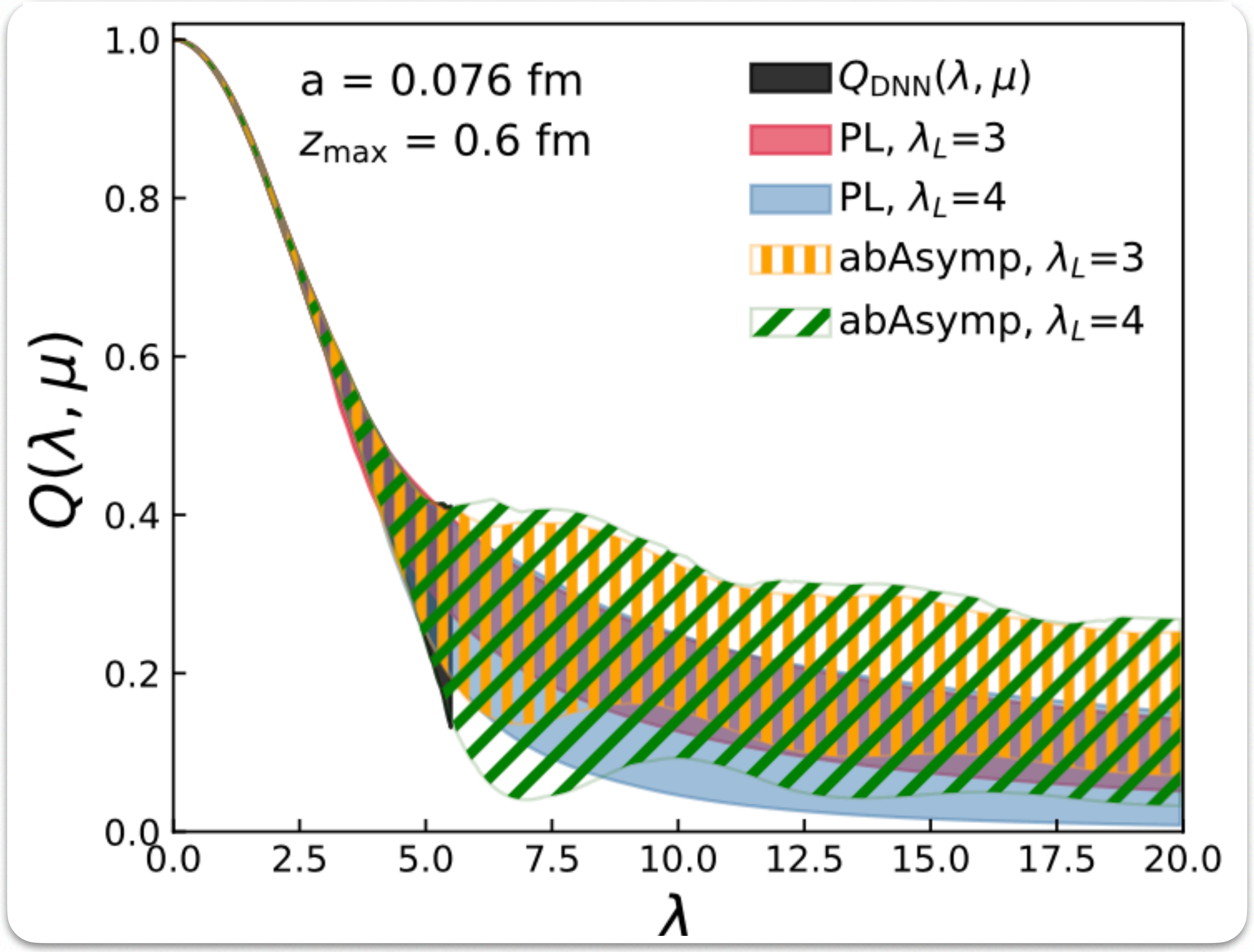


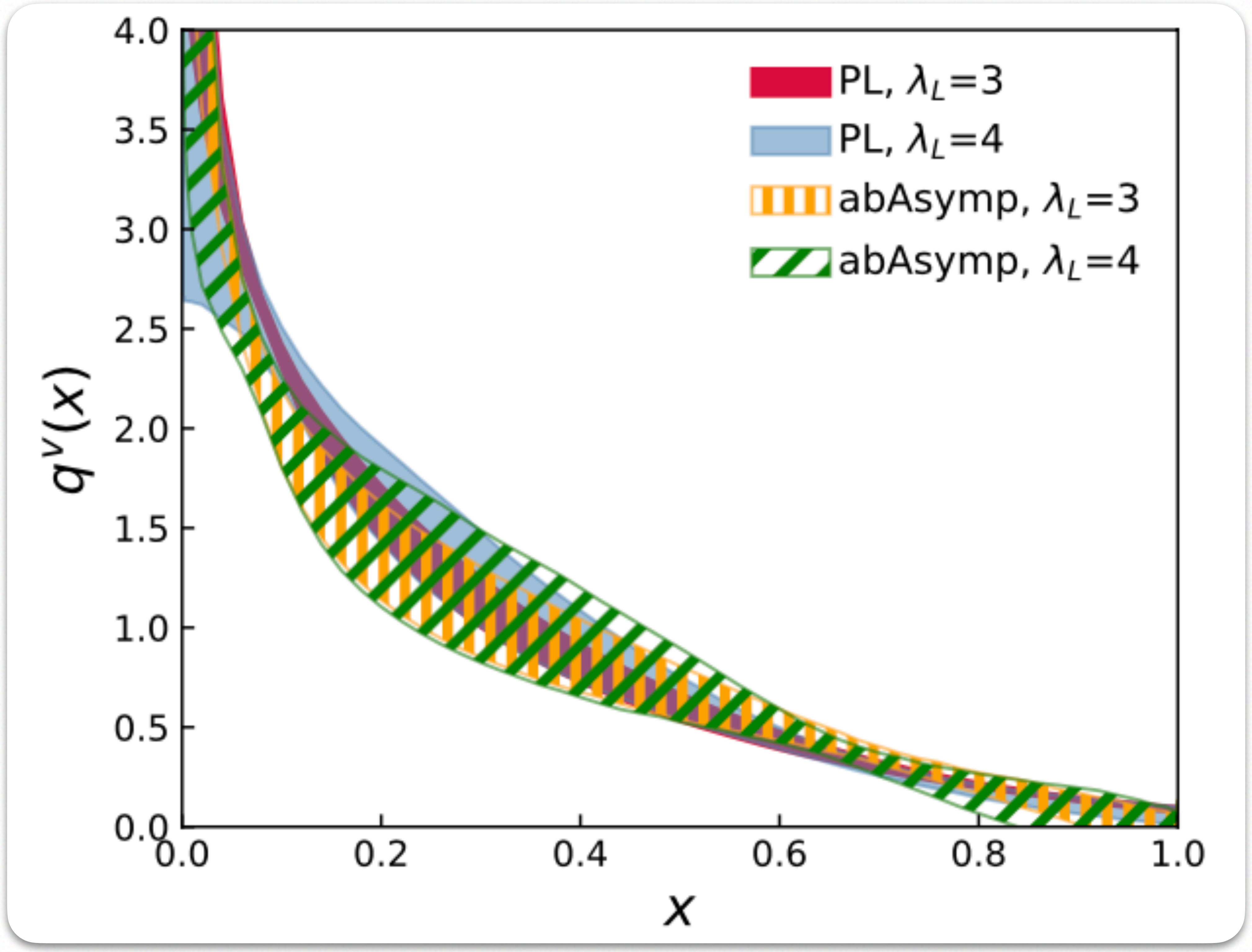
Ioffe-time distribution



- The lattice data has sensitivity up to $\lambda_{\max} = z_{\max} P^z_{\max}$.

DNN representation of Ioffe-time distribution





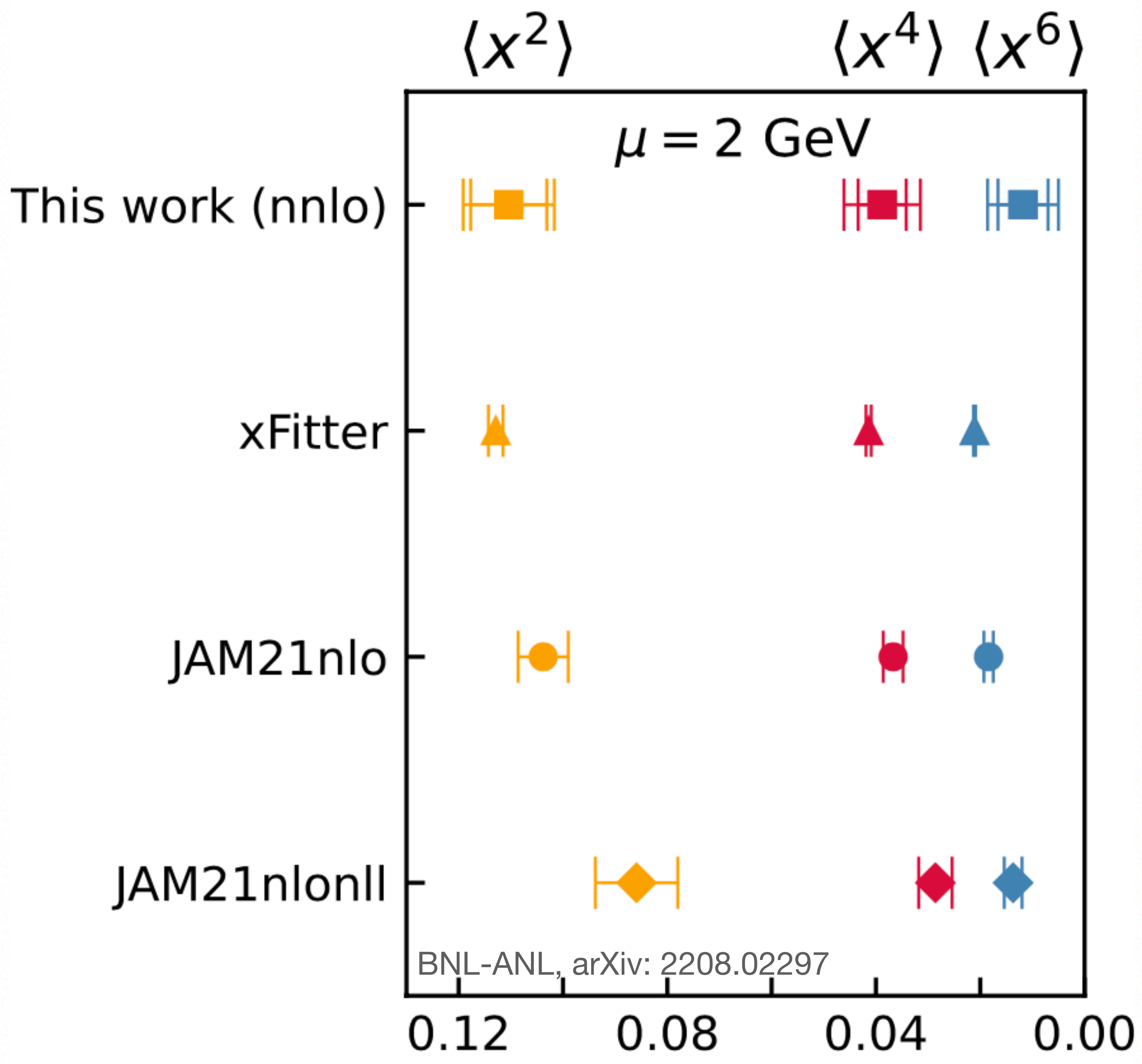
Extrapolation

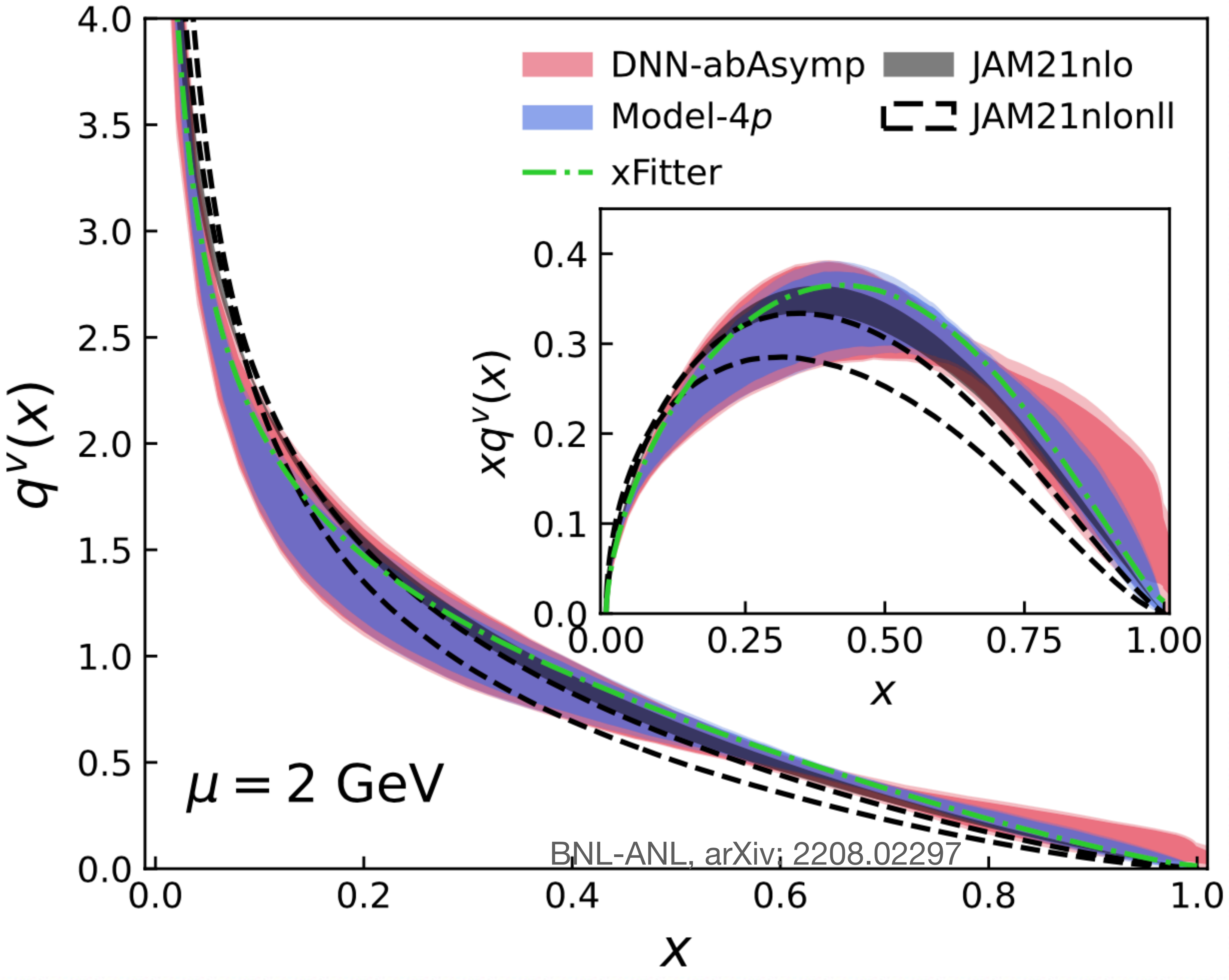
$$Q_{\rm PL}(\lambda, \mu) = \frac{A}{|\lambda|^{\alpha+1}}$$

$$\lambda \in [\lambda_L, \lambda_{\rm max}]$$

$$Q_{\rm abAsymp}(\lambda, \mu) = \Re\left[A\left(\frac{\Gamma(1+\alpha)}{(-i|\lambda|)^{\alpha+1}} + e^{i\lambda}\frac{\Gamma(1+\beta)}{(i|\lambda|)^{\beta+1}}\right)\right]$$

PDFs from Short distance factorization





All our results are in broad agreement with the results of global fits to the experimental data carried out by the xFitter and JAM collaborations.

Parton distribution functions from Lattice

Bare matrix elements

↓

Renormalization

↓ F.T.

Quasi-PDF

↓

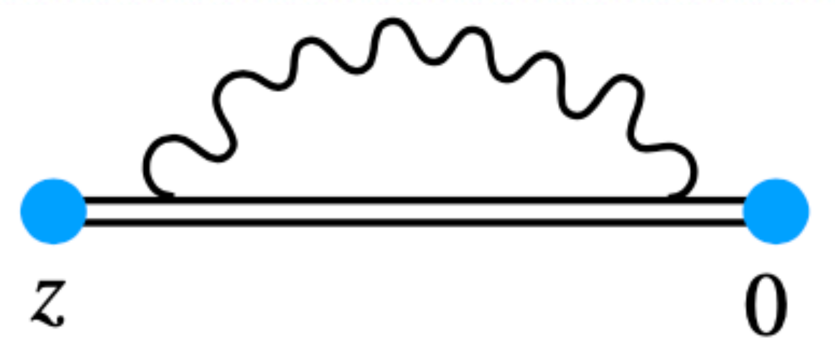
LaMET factorization in x-space

$$\mathcal{O}\left(\frac{\Lambda_{QCD}^2}{x^2P_z^2}, \frac{\Lambda_{QCD}^2}{(1-x)^2P_z^2}\right)$$

Bare matrix elements and renormalization

The operator can be multiplicatively renormalized:

- X. Ji, J. H. Zhang and Y. Zhao, PRL120 (2018)
- J. Green, K. Jansen and F. Steffens, PRL121 (2018)
- T. Ishikawa, et al, PRD 96 (2017)

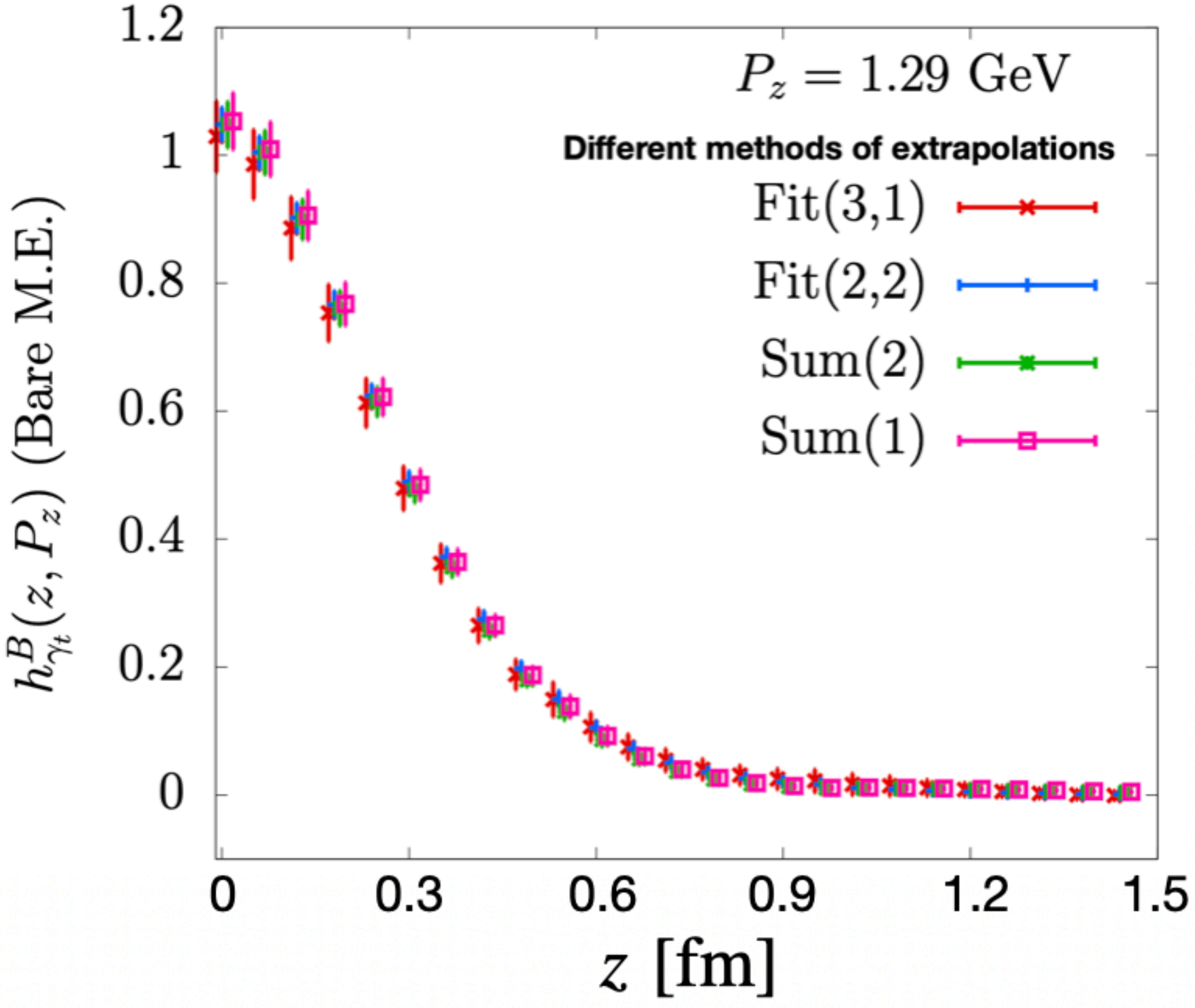


$$= \delta m(a)|z| \propto \frac{|z|}{a}$$

$$[\overline{\psi}(0)\Gamma W_{\hat{z}}(0,z)\psi(z)]_B$$

$$= e^{-\delta m|z|} Z(a) [\overline{\psi}(0)\Gamma W_{\hat{z}}(0,z)\psi(z)]_R$$

Bare matrix elements of **boosted** pion state



$$\delta m(a) = m_{-1}(a)/a + m_0$$

Wilson-line self energy + renormalon ambiguity

Hybrid renormalization

Hybrid renormalization:

- X. Ji, et al., NPB 964 (2021).

- Short distance $z \in [0, z_s],\ z_s \ll \Lambda_{QCD}$:

$$h^R = \frac{\tilde{h}(z, P_z, a)}{\tilde{h}(z, 0, a)}$$

Ratio scheme

- Long distance $z \in [z_s, +\infty]$:

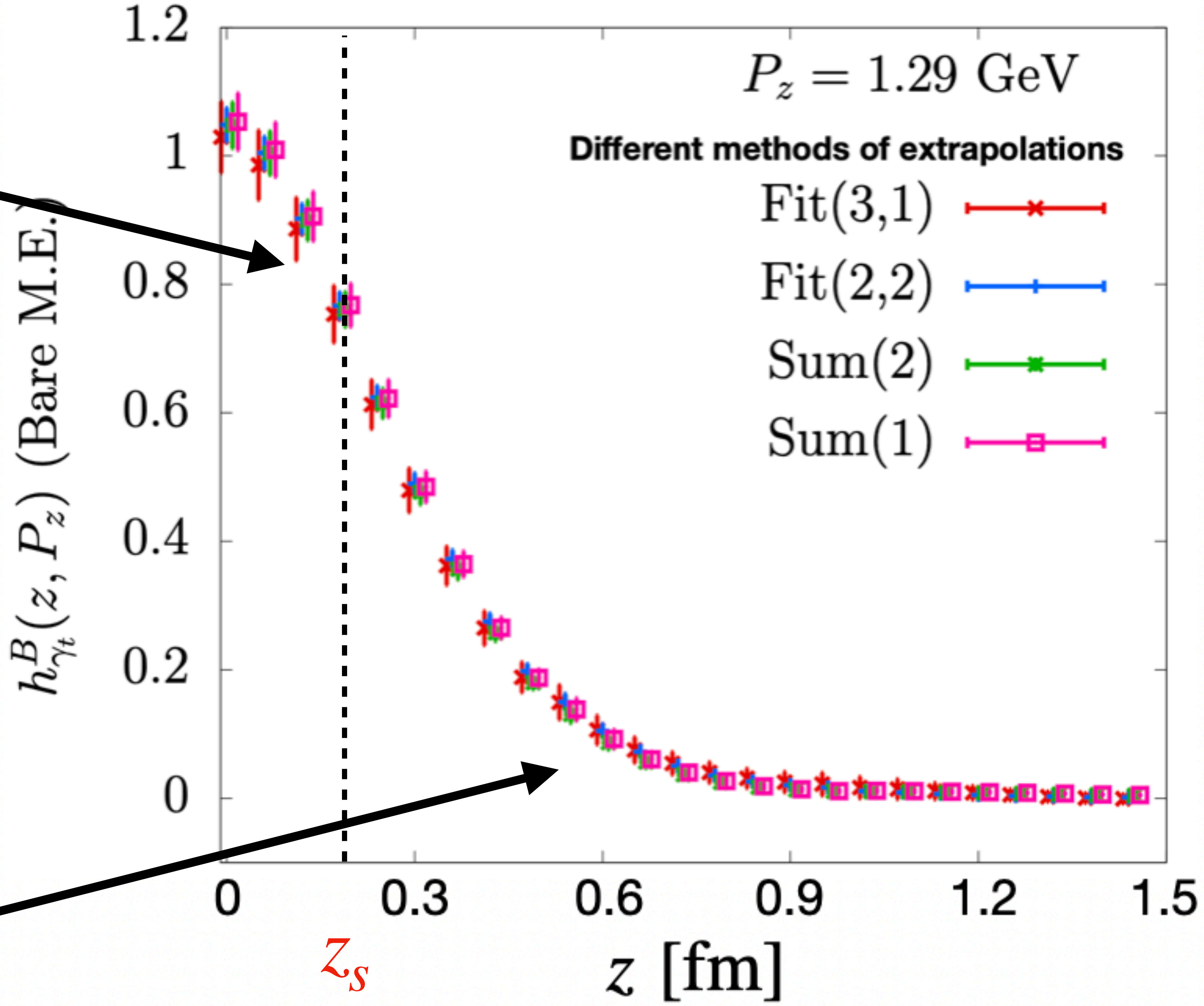
A "minimal" subtraction

$$h^R = e^{\delta m|z - z_s|} \frac{\tilde{h}(z, P_z, a)}{\tilde{h}(z_s, 0, a)}$$

$$\delta m = m_{-1}/a + m_0$$

$$[\bar{\psi}(0)\Gamma W_{\hat{z}}(0,z)\psi(z)]_B = e^{-\delta m|z|} Z(a) [\bar{\psi}(0)\Gamma W_{\hat{z}}(0,z)\psi(z)]_R$$

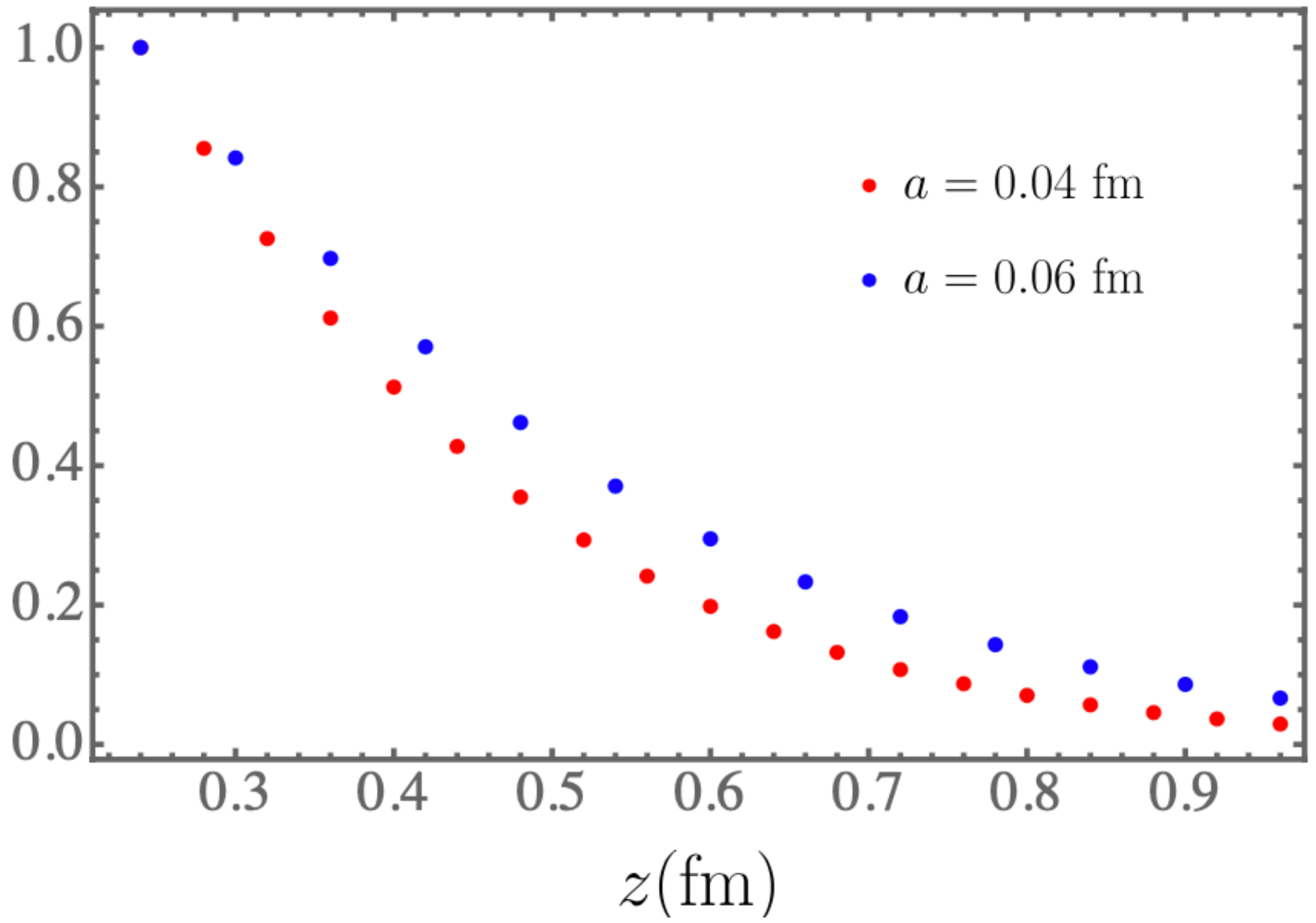
Bare matrix elements of **boosted** pion state



Hybrid renormalization: Wilson-line mass

matrix elements before mass subtraction

$$\frac{\tilde{h}(z, P_z = 0, a)}{\tilde{h}(z_s, P_z = 0, a)}$$



- Clear lattice dependence before subtract $\delta m(a)$

$$[\bar{\psi}(0)\Gamma W_{\hat{z}}(0,z)\psi(z)]_B = e^{-\delta m|z|} Z(a) [\bar{\psi}(0)\Gamma W_{\hat{z}}(0,z)\psi(z)]_R$$

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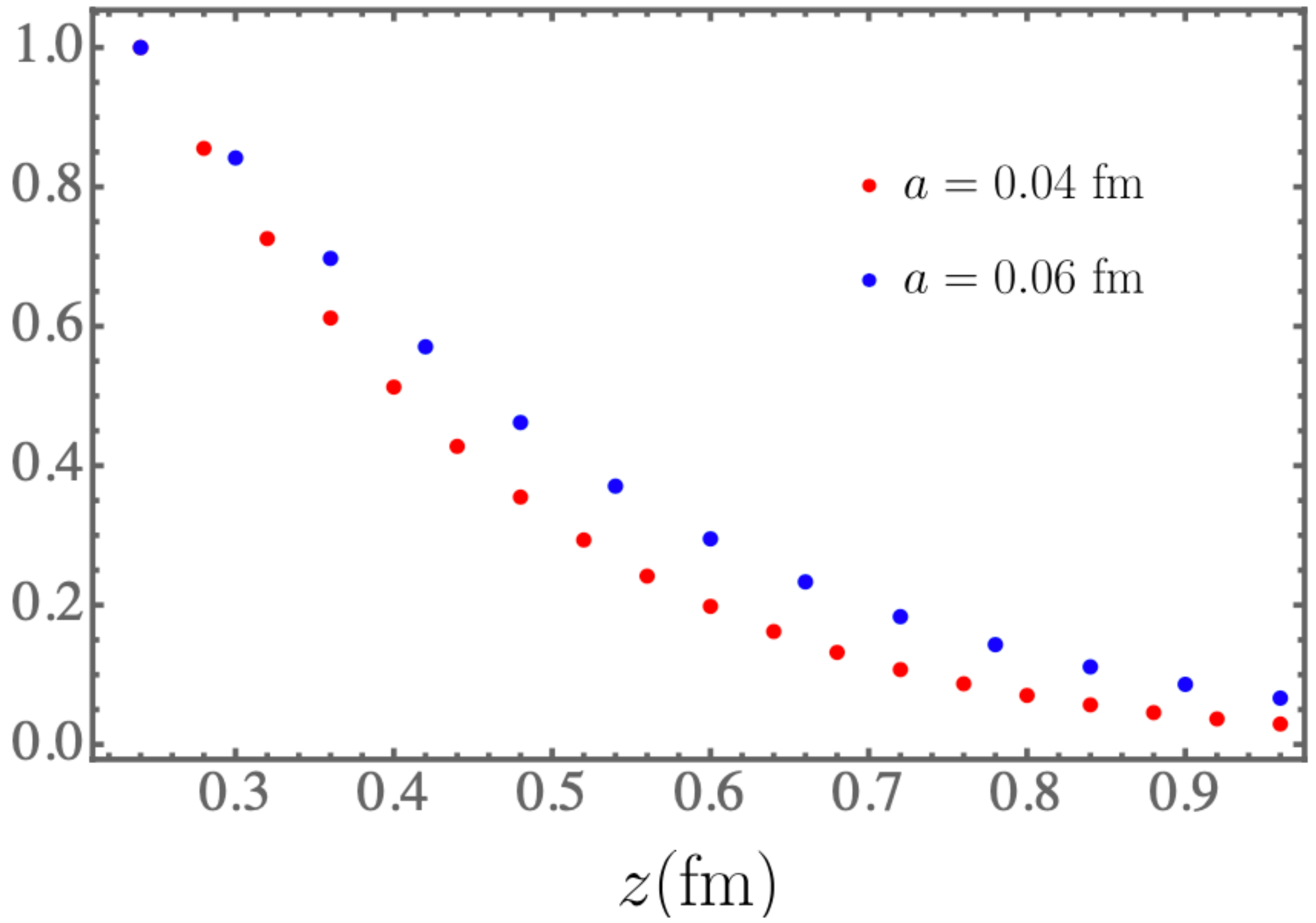


- Clear lattice dependence before subtract $\delta m(a)$

$$[\bar{\psi}(0)\Gamma W_{\hat{z}}(0,z)\psi(z)]_B = e^{-\delta m|z|} Z(a) [\bar{\psi}(0)\Gamma W_{\hat{z}}(0,z)\psi(z)]_R$$

- $\delta m(a)$ from static quark-antiquark potential with renormalization condition

$$V^{\text{lat}}(r,a)\Big|_{r=r_0} + 2\delta m(a) = 0.95/r_0$$

- C. Bauer, G. Bali and A. Pineda, PRL108 (2012).
- A. Bazavov et al., TUMQCD, PRD98 (2018).

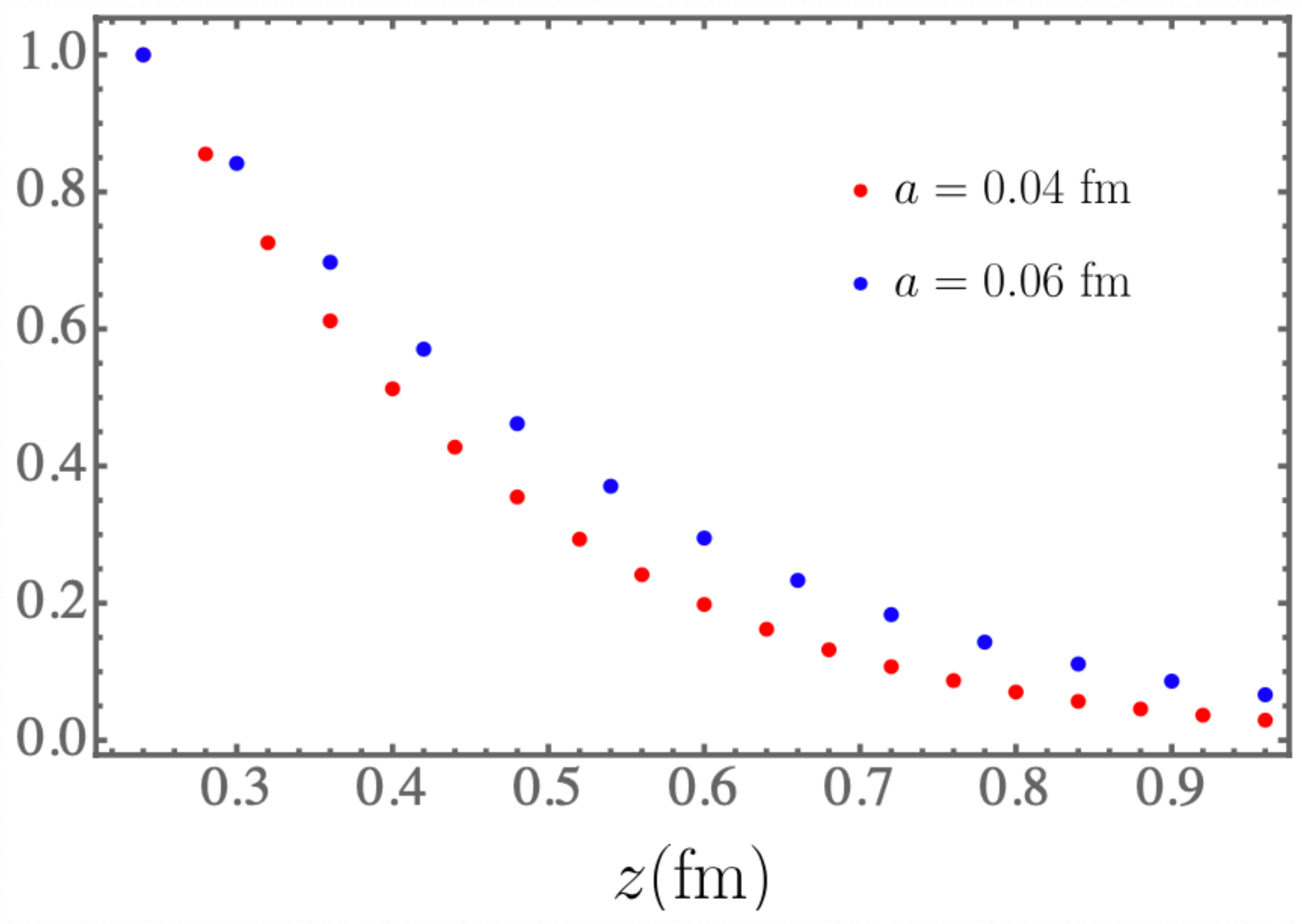
$$a\delta m(a = 0.04 \text{ fm}) = 0.1508(12)$$

$$a\delta m(a = 0.06 \text{ fm}) = 0.1586(8)$$

Hybrid renormalization: Wilson-line mass

matrix elements before mass subtraction

$$\frac{\tilde{h}(z, P_z = 0, a)}{\tilde{h}(z_s, P_z = 0, a)}$$

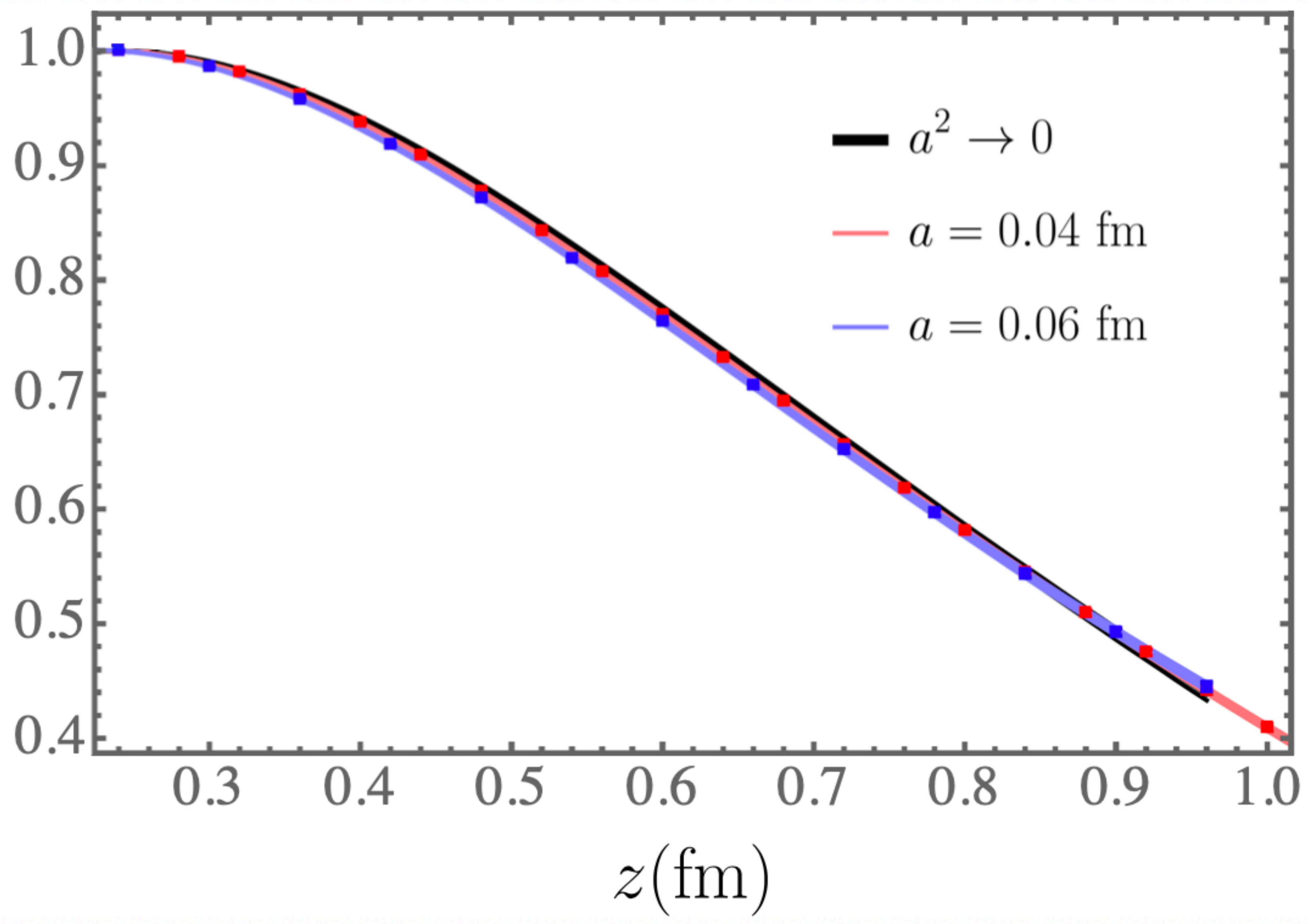


- Clear lattice dependence before subtract $\delta m(a)$

$$[\bar{\psi}(0)\Gamma W_{\hat{z}}(0,z)\psi(z)]_B = e^{-\delta m|z|} Z(a) [\bar{\psi}(0)\Gamma W_{\hat{z}}(0,z)\psi(z)]_R$$

$\delta m(a)$ subtracted matrix elements :

$$e^{\delta m(a)(z-z_s)} \frac{\tilde{h}(z, P_z = 0, a)}{\tilde{h}(z_s, P_z = 0, a)}$$



- Good continuum condition
- The linear divergences have been sufficiently subtracted by $\delta m(a)$

Hybrid renormalization: UV renormalon

OPE of $\overline{\text{MS}}$ matrix elements

- L.-B. Chen, R.-L. Zhu and W. Wang, PRL126 (2021);
- Z.-Y. Li, Y.-Q. Ma and J.-W. Qiu, PRL126 (2021);
- V. Braun and K. G. Chetyrkin, JHEP 07 (2020).

Perturbative:

Known to NNLO

$$\tilde{h}^{\overline{\text{MS}}}(z, P^z = 0, \mu) = e^{-m_0^{\overline{\text{MS}}}(z-z_0)} \left[C_0(\alpha_s(\mu), z^2\mu^2) + \mathcal{O}(z^2\Lambda^2_{\text{QCD}}) \right]$$

UV renormalon,

- M. Beneke and V. Braun, NPB 426 (1994)

IR renormalon:

Leading IR renormalon

- V. Braun, A. Vladimirov and J.-H. Zhang, PRD99 (2019)

Matching the mass-subtracted ratio to the $\overline{\text{MS}}$ OPE ratio

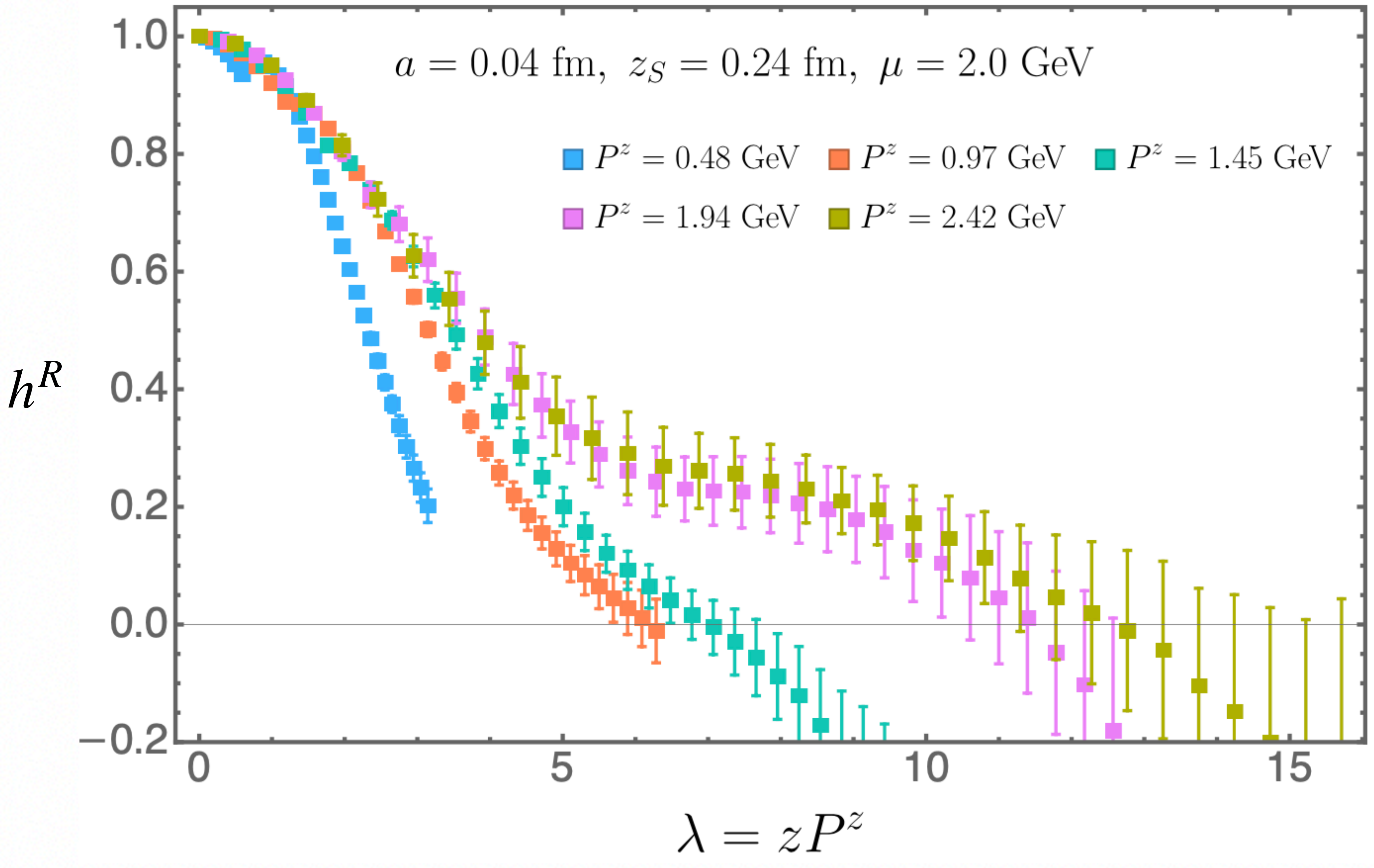
$$\lim_{a\to 0} e^{\delta m(a)(z-z_s)} \frac{\tilde{h}(z, P_z = 0, a)}{\tilde{h}(z_s, P_z = 0, a)} = e^{-\bar{m}_0(z-z_s)} \frac{C_0(\alpha_s(\mu), z^2\mu^2) + \Lambda z^2}{C_0(\alpha_s(\mu), z_s^2\mu^2) + \Lambda z_s^2}$$

$$[\overline{\psi}(0)\Gamma W_{\hat{z}}(0,z)\psi(z)]_B = e^{-\delta m|z|} Z(a) [\overline{\psi}(0)\Gamma W_{\hat{z}}(0,z)\psi(z)]_R$$

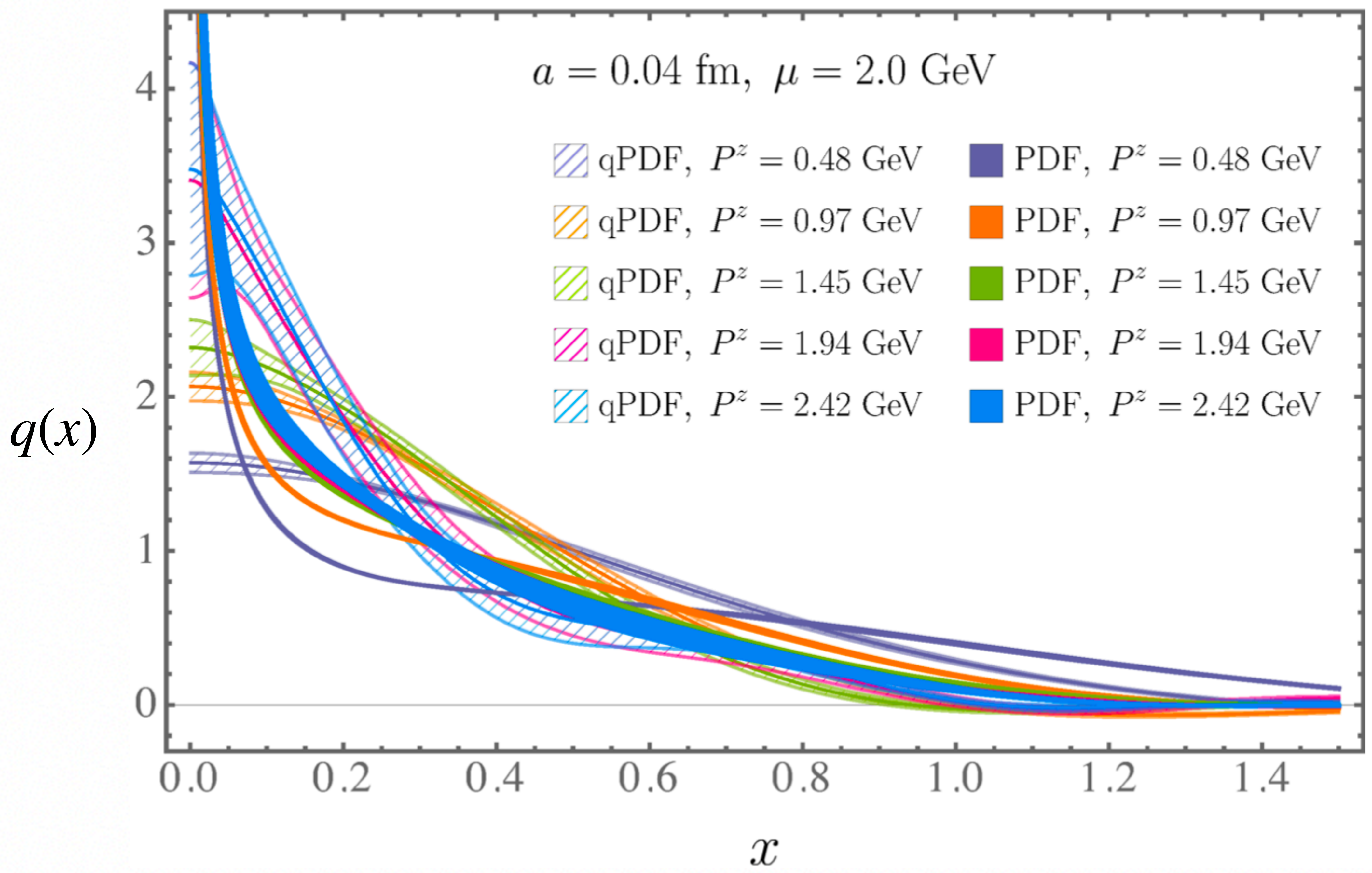
$$\bar{m}_0 = m_0^{\overline{\text{MS}}} + m_0^{\text{Lat}/\overline{\text{MS}}}$$

Hybrid renormalization and quasi-PDF

Renormalized matrix elements



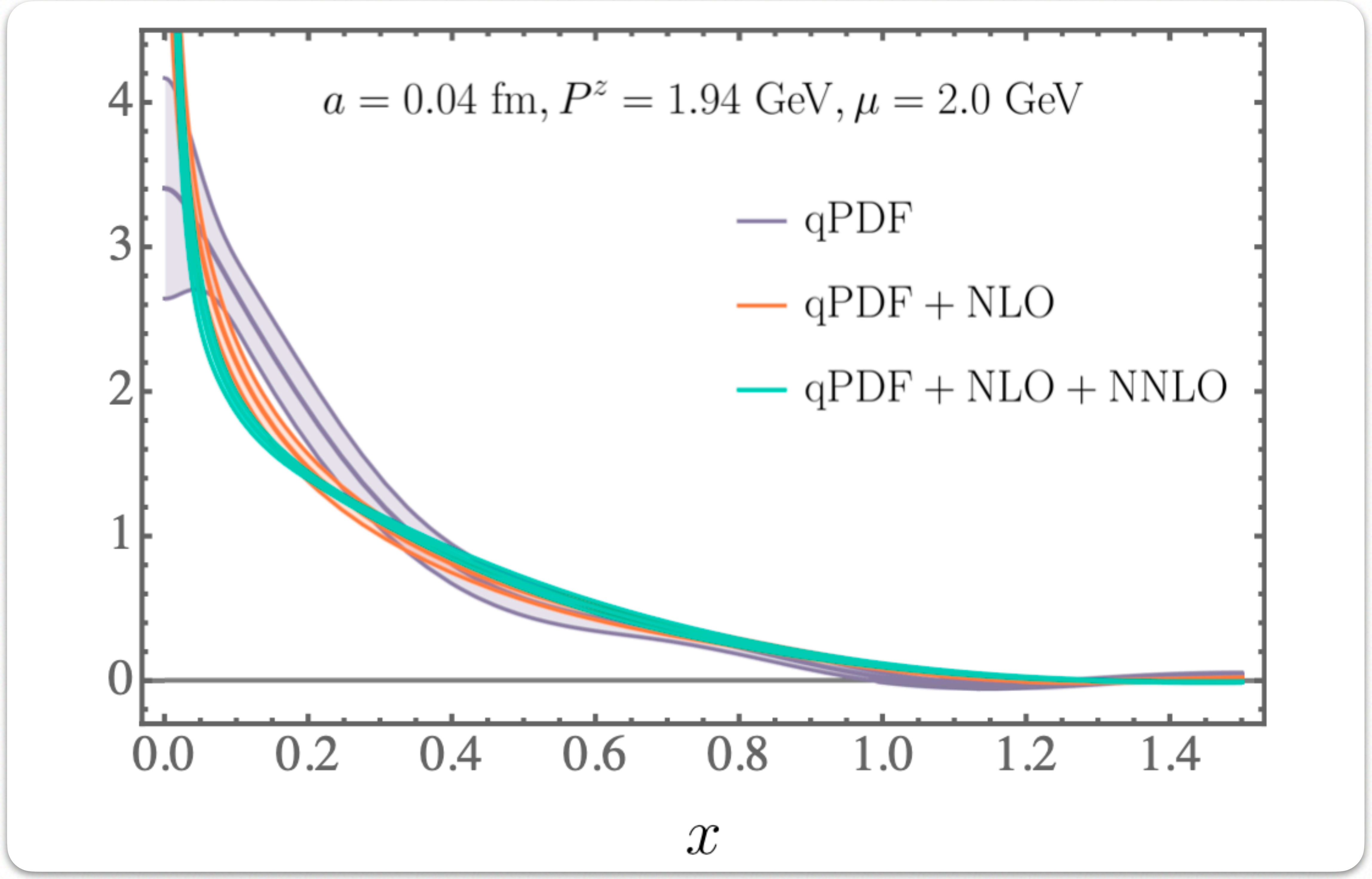
qPDF and PDF after matching



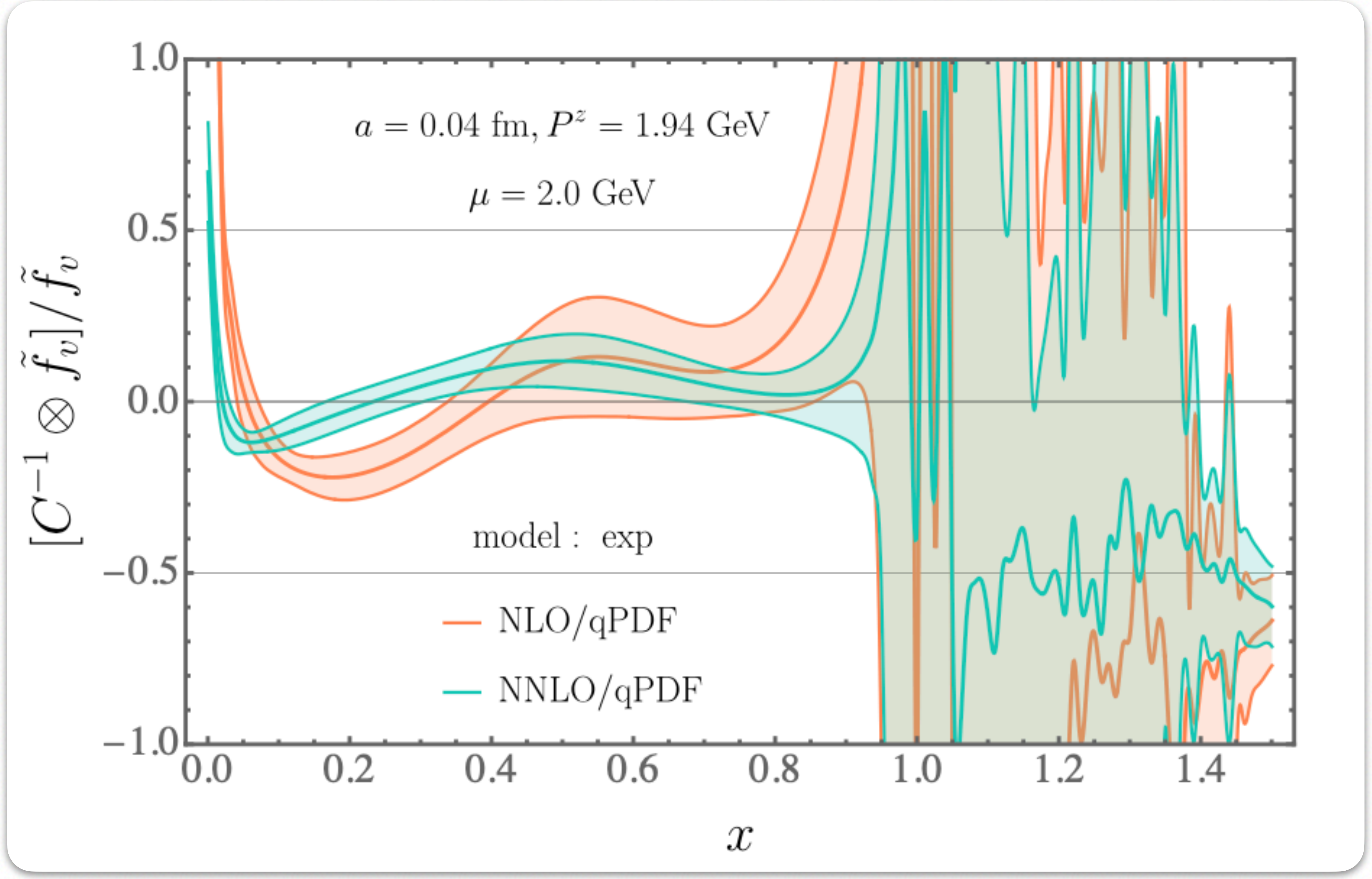
- Matrix elements and qPDFs start to converge for large P_z (perturbation region).
- Matching makes the convergence faster and drives the quasi-PDF to smaller x.

Systematics

LO → NLO → NNLO



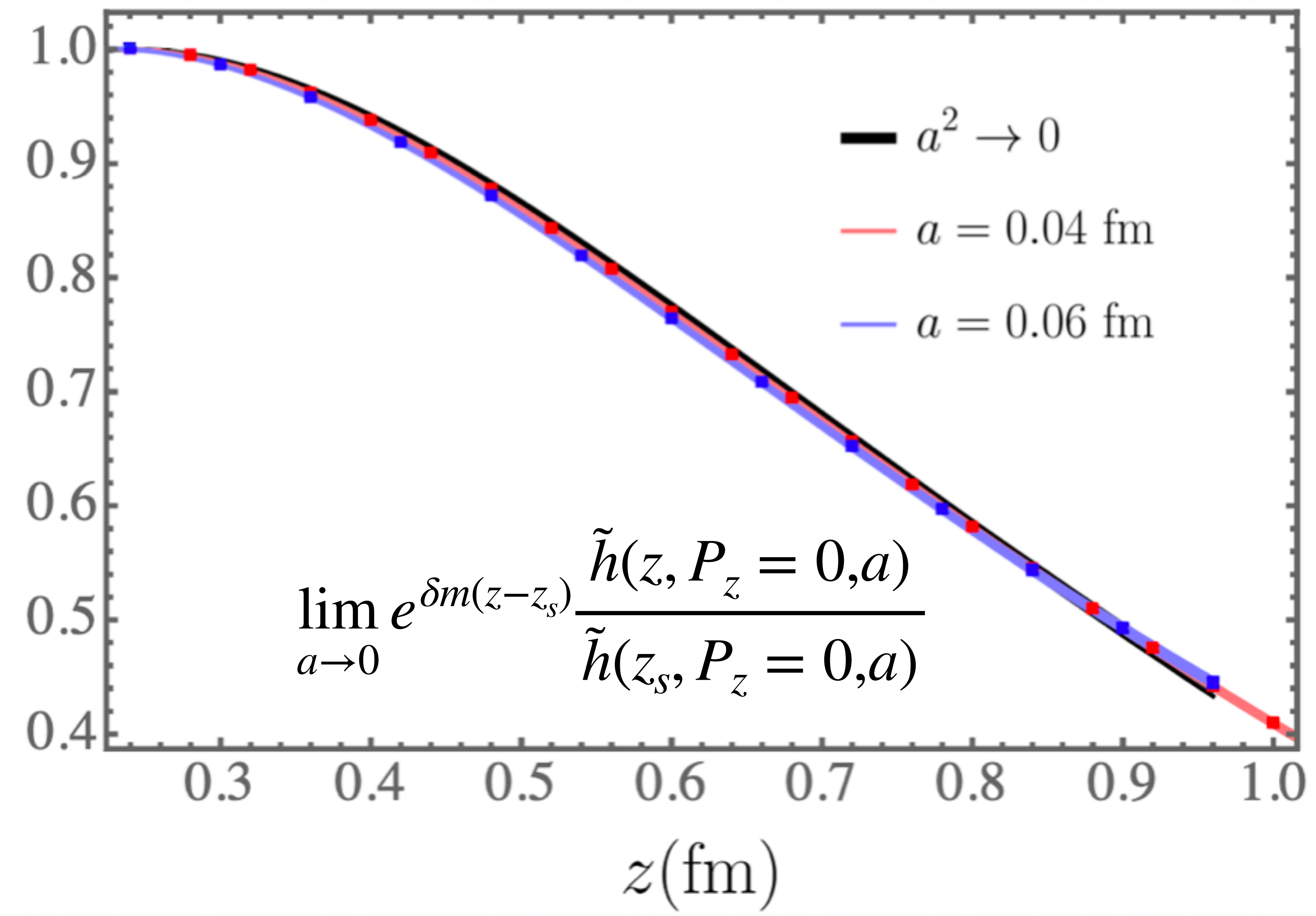
pQCD correction to the qPDF



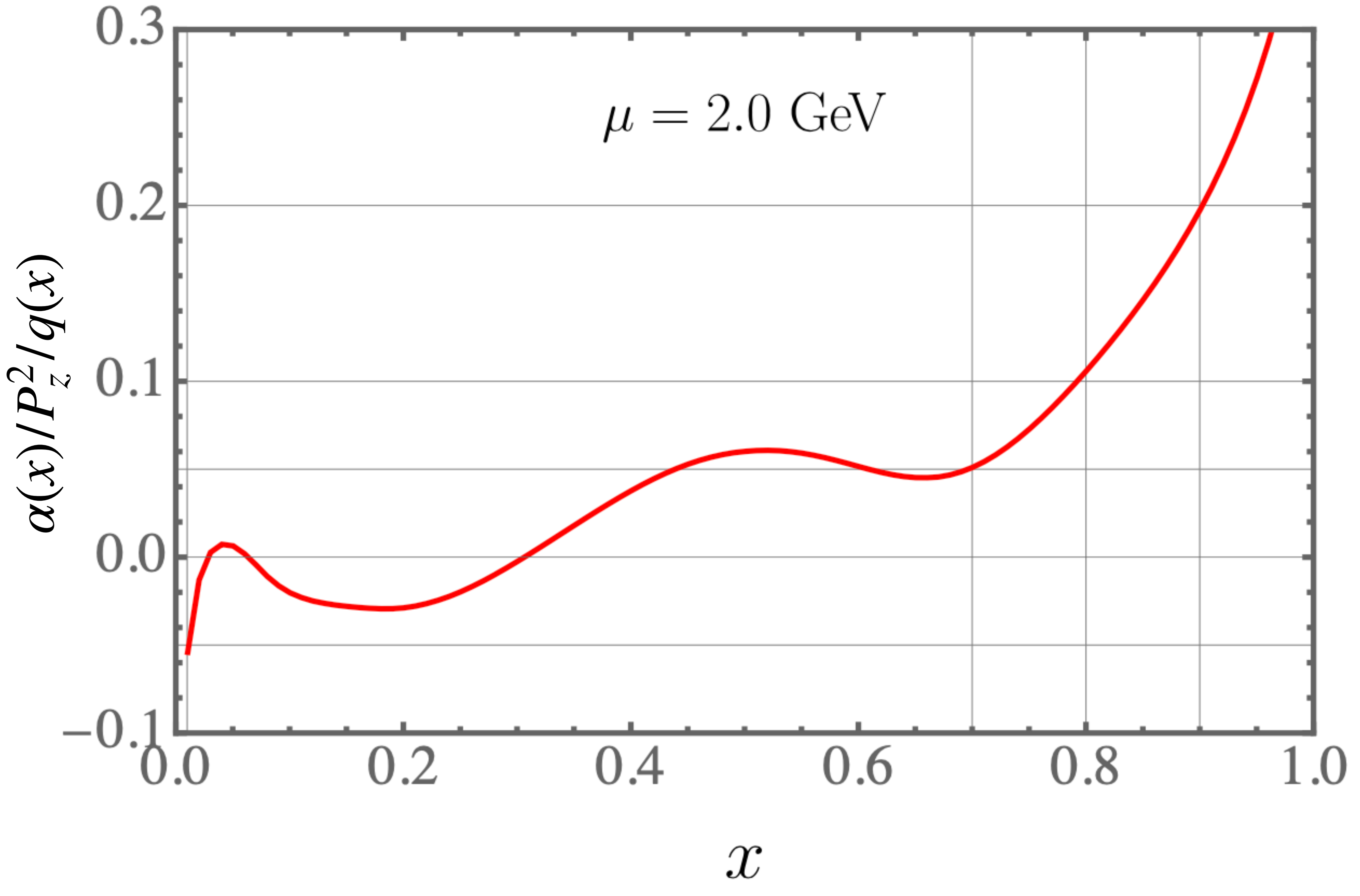
- Good convergence at moderate x.
- Large corrections in end-point regions, need resummation.

Systematics

Wilson-line mass subtraction



Power correction $q(x, P_z) = q(x) + \alpha(x)/P_z^2$

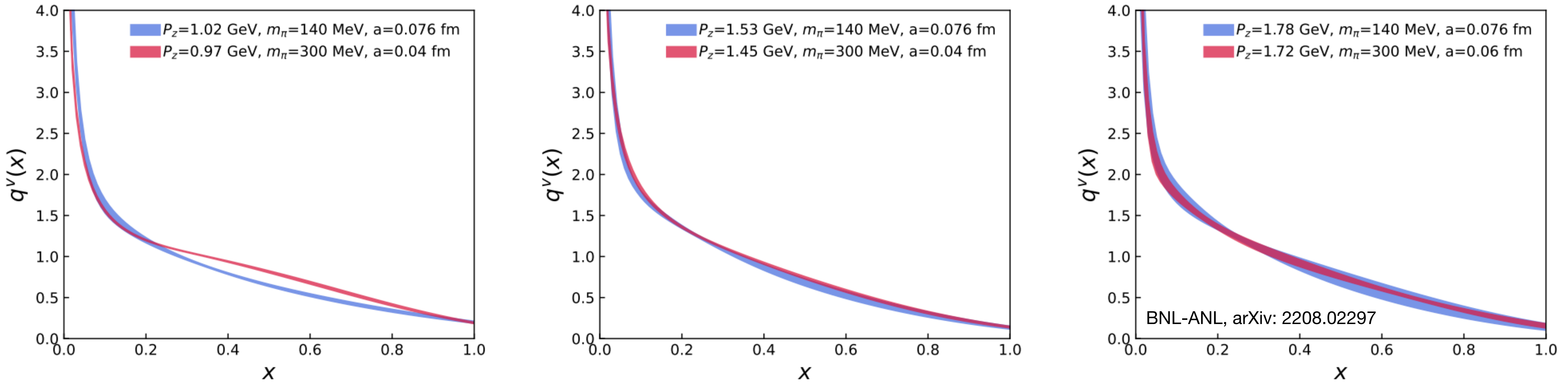


- Mild lattice spacing dependence

- Small P_z dependence in middle x

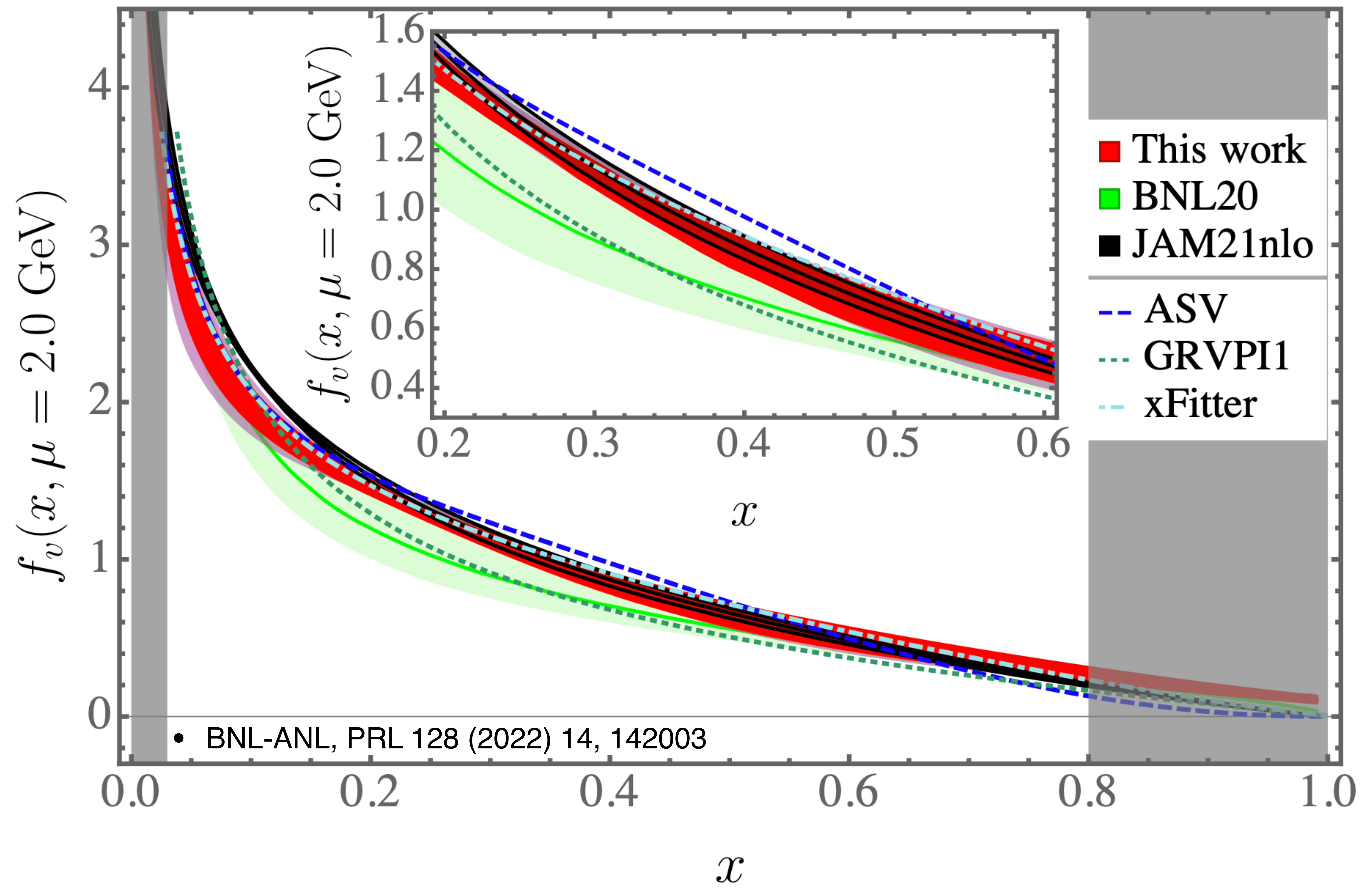
Systematics

qPDF from different lattice spacing and pion mass



- Pion mass dependence absent in large P_z.
- Lattice spacing dependence is small.

x dependent PDFs from LaMET



The shaded regions x < 0.03 and x > 0.8 are excluded by requiring that estimates of $O(\alpha_s^3)$ and power corrections be smaller than 5% and 10%, respectively.

- Lattice prediction show good agreement with Global analysis from JAM, xFitter in moderate-x region.
- Reduced uncertainty and model dependence compared to the short-distance factorization (BNL20).

x	Statistical	Scale	$\mathcal{O}(\alpha_s^3)$	Power corrections	$\mathcal{O}(a^2 P_z^2)$
0.03	0.10	0.04	< 0.05	< 0.01	< 0.01
0.40	0.07	< 0.01	< 0.05	0.04	< 0.01
0.80	0.15	0.03	< 0.05	0.10	< 0.01

Summary

- We carried out lattice calculation of the quasi-PDF matrix elements of pion with large momentum and renormalized them using ratio scheme and hybrid scheme.
- We performed analysis in coordinate-space based on NNLO short distance factorization and extract the Mellon moments, model-based PDF and the light-cone ITD.
- We performed analysis in x space based on NNLO LaMET matching and predict x-dependence of pion valence PDF.
- All our results are in broad agreement with the results of global fits to the experimental data carried out by the xFitter and JAM collaborations.

Thanks for your attention