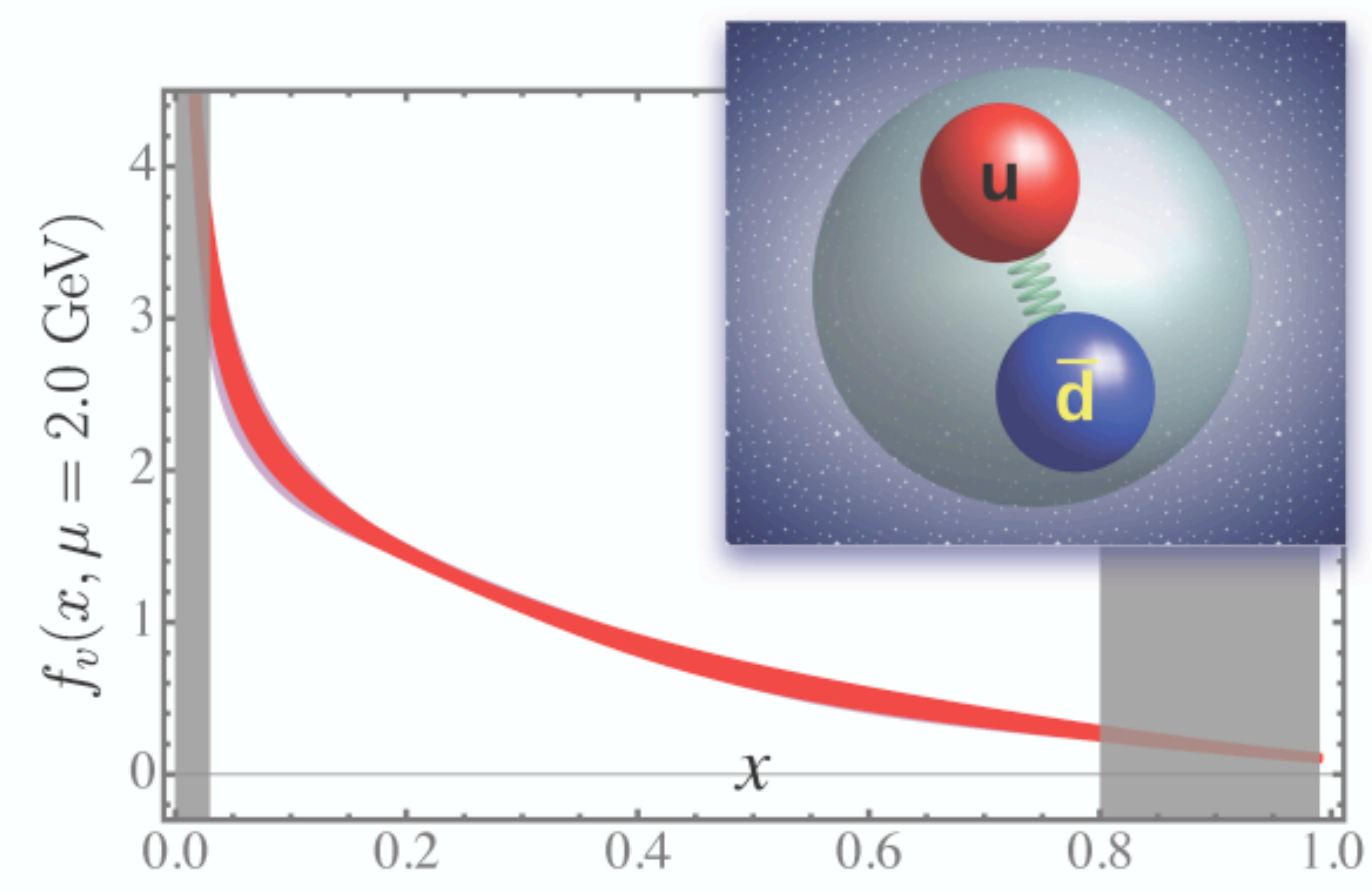
TMDs & factorization at sub-leading power "twist-3"

Leonard Gamberg

w/ Zhongbo Kang, John Terry, Ding-Yu Zhao, Fany Zhao

Parton Distributions and Nucleon Structure
September 15, 2022

PennState Berks



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TMD Collaboration

Motivation of my discussion/preamble

- We explore subheading power TMDs in the context of factorization theorem
  - Rely on "*TMD formalism*" —extension of CSS, Abyat Rogers, Boer Pijlman Mulders-framework
  - "Revisit matching" Consider consistency of matching onto collinear factorization
    see Bacchetta, Boer, Diehl, Mulders JHEP 2008 also in context of EOMs
  - Focus on Cahn effect & matching related to early picture of importance intrinsic $\mathbf{k}_T$
  - *INTRINSIC subleading twist TMDs—historical maybe not optimal*
  - See recent work:
    MIT group, Gao, Ebert, Stewart JHEP 2022
    Vladimirov & Rodini JHEP 2022

However, its an old subject in QCD … background

Important Literature (incomplete)

L. Gamberg, D Hwang, A Metz, M. Schlegel, Phys.Lett.B 639 (2006), hep-ph/0604022 [hep-ph]-demonstrate rapidity div. @tw3
A.P. Chen, J.P. Ma, Phys. Lett. B 768 (2017) 380, arXiv:1610 .08634.
I. Feige, D.W. Kolodrubetz, I. Moult, I.W. Stewart, J. High Energy Phys. 11 (2017) 142, arXiv:1703 .03411.
I. Balitsky, A. Tarasov, J. High Energy Phys. 07 (2017) 095, arXiv:1706 .01415.
I. Balitsky, A. Tarasov, J. High Energy Phys. 05 (2018) 150, arXiv:1712 .09389.
M.A. Ebert, I. Moult, I.W. Stewart, F.J. Tackmann, G. Vita, H.X. Zhu, J. High Energy Phys. 12 (2018) 084, arXiv:1807.10764.
M.A. Ebert, I. Moult, I.W. Stewart, F.J. Tackmann, G. Vita, H.X. Zhu, J. High Energy Phys. 04 (2019) 123, arXiv:1812 .08189
Moult, I.W. Stewart, G. Vita, arXiv:1905 .07411, 201
A. Bacchetta et al. / Physics Letters B 797 (2019) 134850
M. Ebert A. Gao I. Stewart JHEP 2022
Vladimirov & Rodini JHEP 2022

Why TMDs @ twist-3 → NLP

Some History-context

- **Georgi Politzer, PRL 1978**
  QCD analysis of hard gluon radiation in SIDIS to predict absolute value of $\boldsymbol{P_T}$
  & the angular distribution relative to lepton scattering plane
  "*Clean Tests of QCD*",
  "…angular correlations should be insensitive to nonperturbative effects"

- **Cahn, PLB 1978, also earlier Ravndal, PLB 1972**
  ***"Critique of the parton model calculation of azimuthal dependence in leptoproduction"*,**
  importance intrinsic k_T …
  "…The results can doubt on the utility of such azimuthal asymmetry as a clean test of quantum chromodynamics"

Clean tests of QCD

PHYSICAL REVIEW LETTERS

VOLUME 40 | 2 JANUARY 1978 | NUMBER 1

Clean Tests of Quantum Chromodynamics in μp Scattering

Howard Georgi
Lyman Laboratory of Physics, Harvard University, Cambridge, Massachusetts 02138

and

H. David Politzer
California Institute of Technology, Pasadena, California 91125
(Received 25 October 1977)

Hard gluon bremsstrahlung in μp scattering produces final-state hadrons with a large component of momentum transverse to the virtual-photon direction. Quantum chromodynamics can be used to predict not only the absolute value of the transverse momentum, but also its angular distribution relative to the muon scattering plane. The angular correlations should be insensitive to nonperturbative effects.

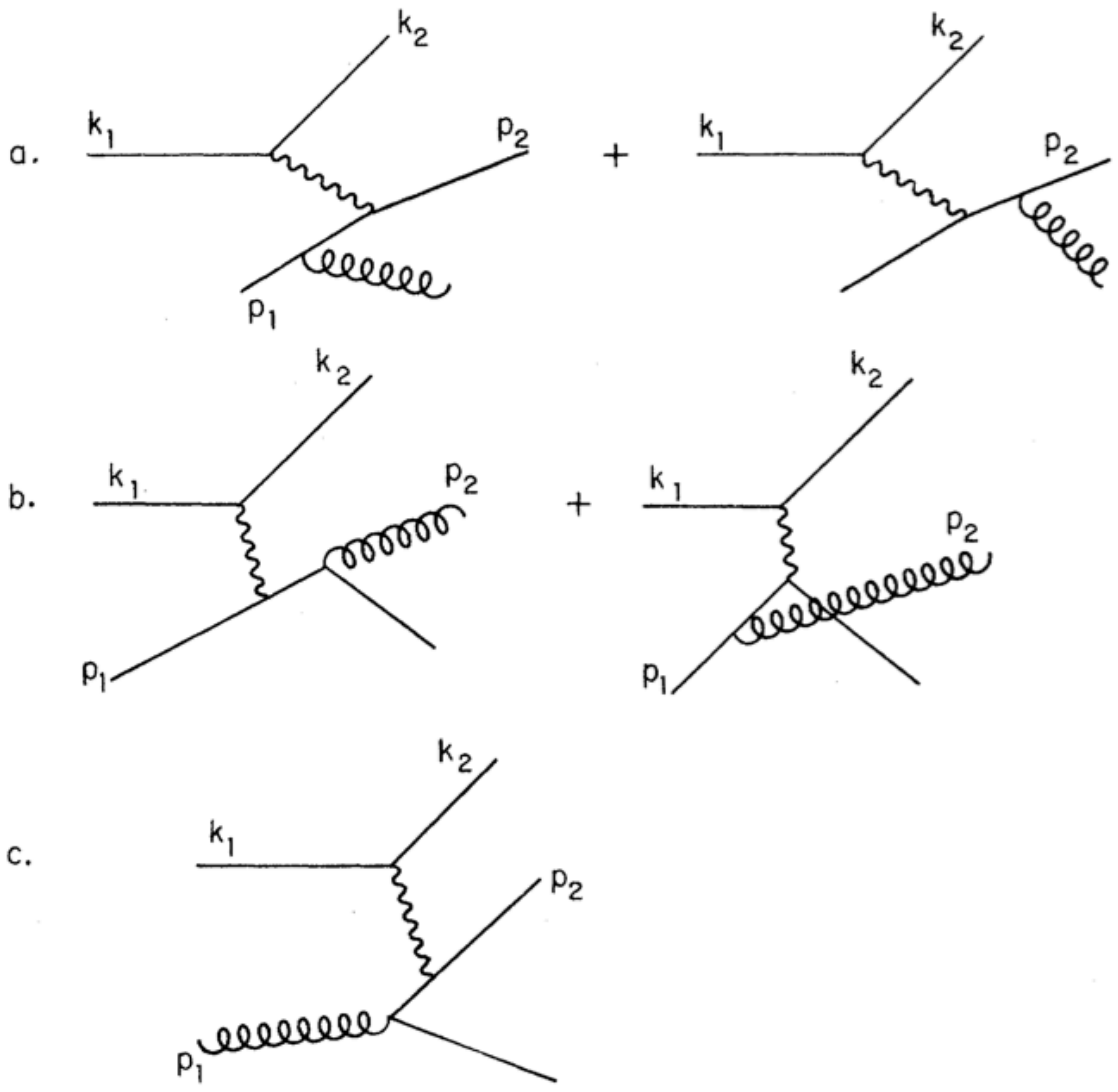


FIG. 1. Diagrams contributing to semi-inclusive μ-parton scattering to first order in α_s. k (p) denotes muon (parton) momentum. The wavy line is a virtual photon. The curly line is a gluon.

Cahn intrinsic k_T

Volume 78B, number 2,3 PHYSICS LETTERS 25 September 1978

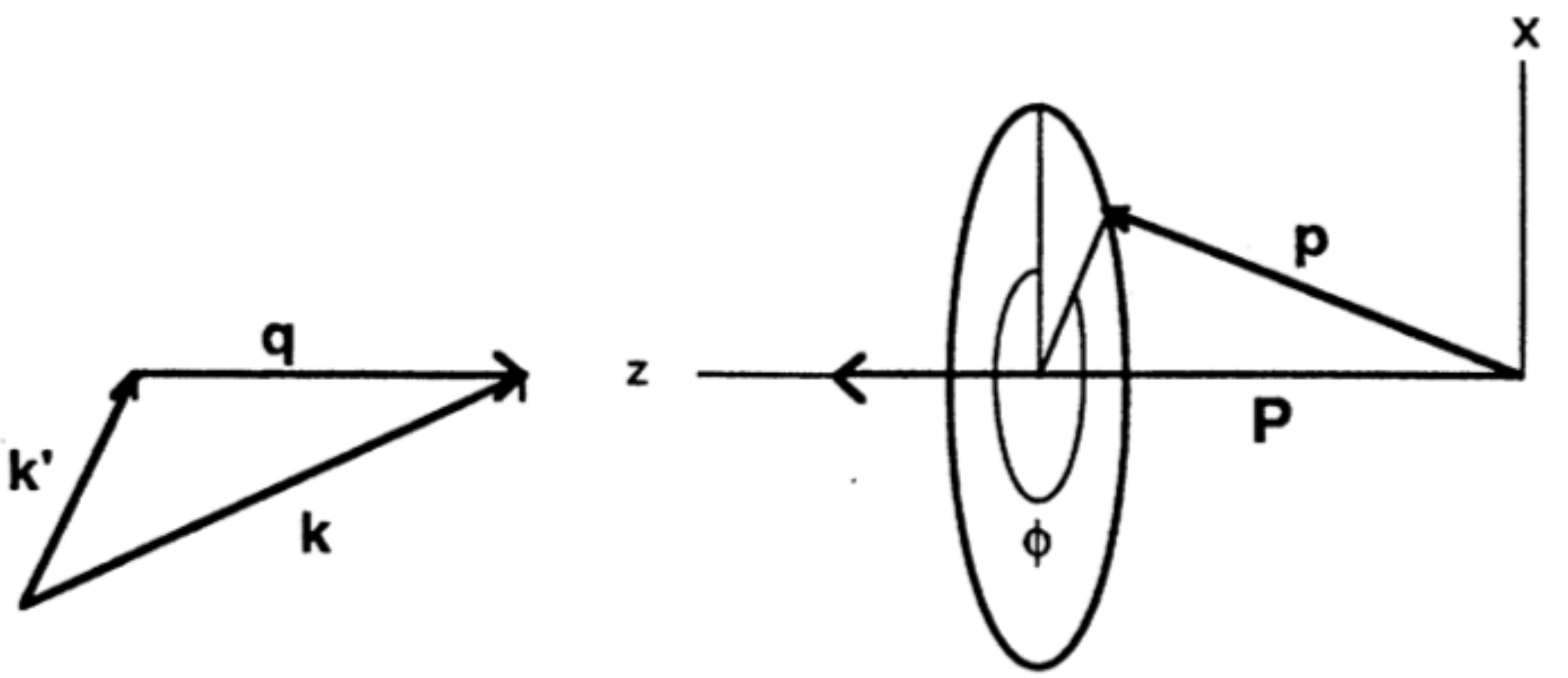
AZIMUTHAL DEPENDENCE IN LEPTOPRODUCTION: A SIMPLE PARTON MODEL CALCULATION☆

Robert N. CAHN
Department of Physics, University of Michigan, Ann Arbor, MI 48109, USA

Received 5 June 1978

Semi-inclusive leptoproduction, $\ell + p \to \ell' + h + X$, is considered in the naive parton model. The scattered parton shows an azimuthal asymmetry about the momentum transfer direction. Simple derivations for the effects in ep, νp and $\bar{\nu}$p scattering are given. Reduction of the asymmetry due to fragmentation of partons into hadrons is estimated. The results cast doubt on the utility of such azimuthal asymmetry as a clean test of quantum chromodynamics.

Cahn intrinsic k_T



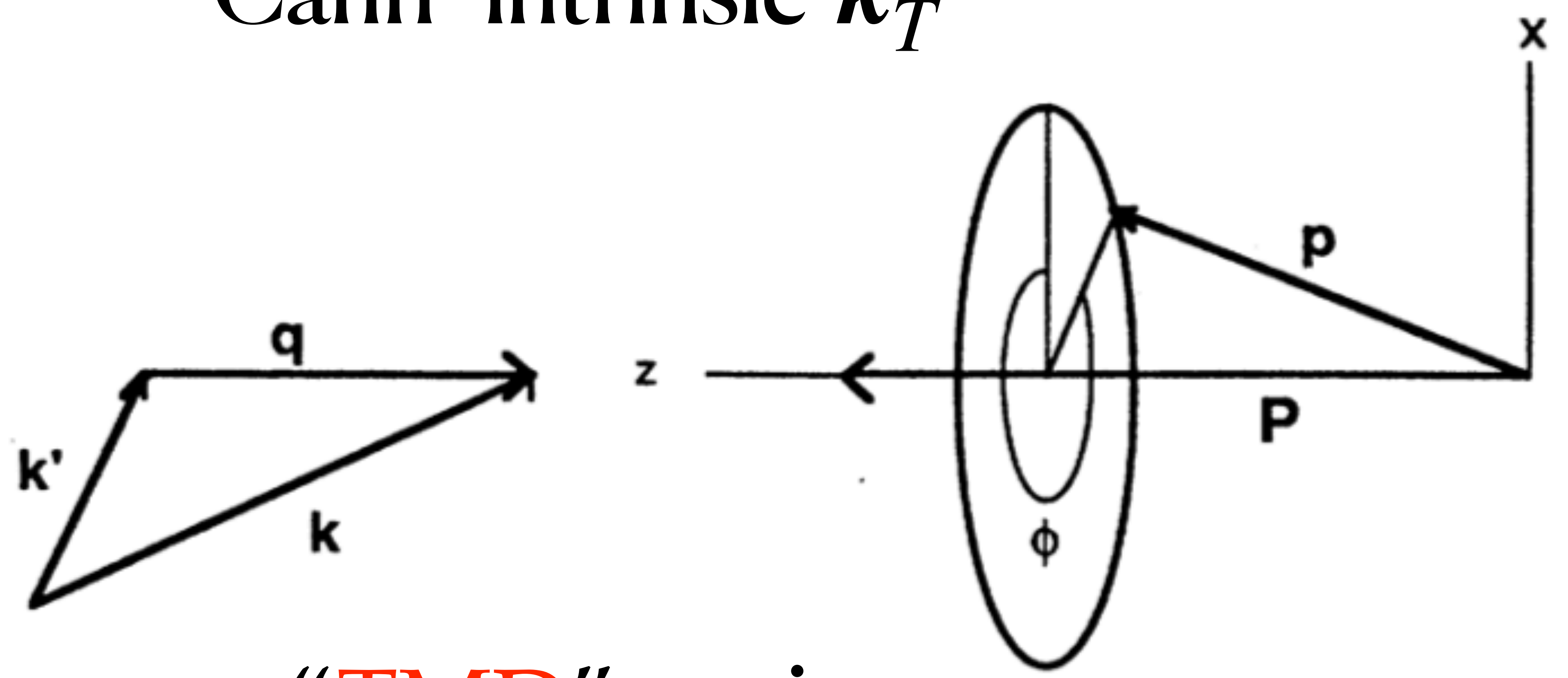
Simple parton model argument

$$\sigma_{ep} \propto \hat{s}^{\,2} + \hat{u}^{\,2} \propto \left[1 - \frac{2p_\perp}{Q}\sqrt{1-y}\ \cos\phi\right]^2 + (1-y)^2\left[1 - \frac{2p_\perp}{Q\sqrt{1-y}}\cos\phi\right]^2$$

$$\langle \cos\phi \rangle_{ep} = -\left[\frac{2p_\perp}{Q}\right]\frac{(2-y)\sqrt{1-y}}{1+(1-y)^2}$$

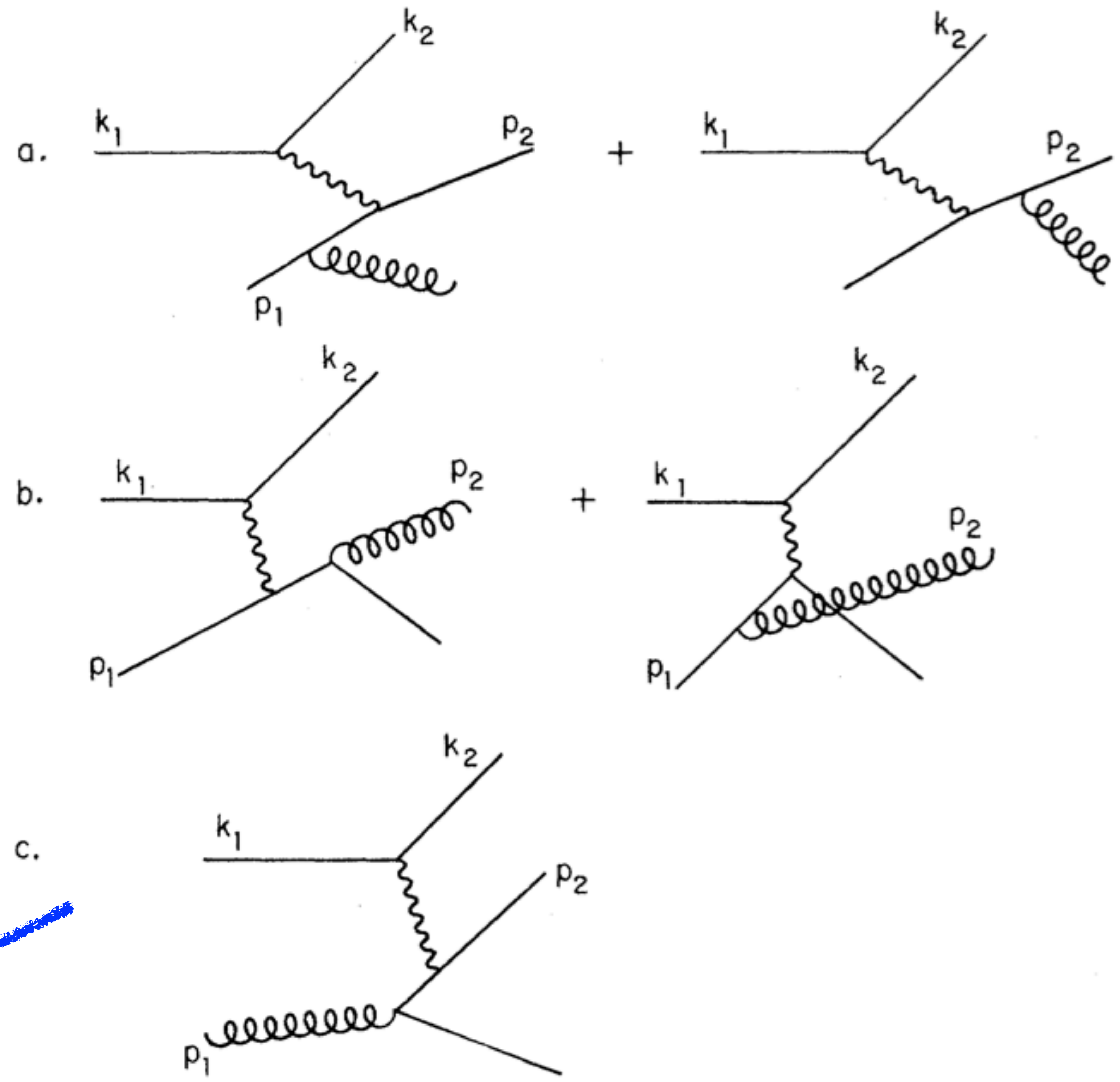
Two mechanisms ?

Cahn intrinsic $\boldsymbol{k}_T$



- "TMD" region

$(p_T \sim k_T) \sim q_T \ll Q$



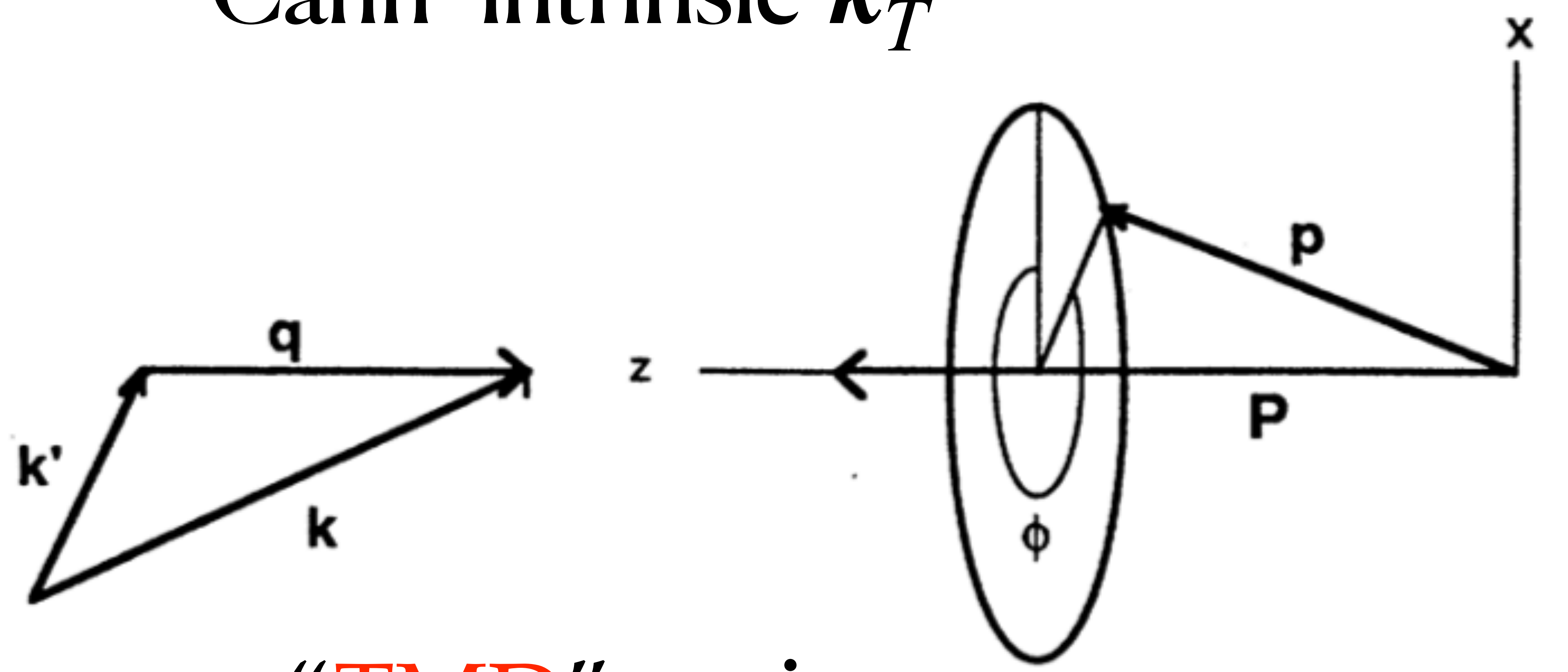
$$\frac{d^5\sigma}{dx_{bj}\,dQ^2\,dz_f\,dq_T^2\,d\phi} = \frac{\alpha_e^2\alpha_s}{8\pi x_{bj}^2 S_{ep}^2 Q^2}\sum_k \mathcal{A}_k \int_{x_{\min}}^{1}\frac{dx}{x}\int_{z_f}^{1}\frac{dz}{z}\,[f\otimes D\otimes\hat{\sigma}_k] \times \delta\left(\frac{q_T^2}{Q^2} - \left(\frac{1}{\hat{x}} - 1\right)\left(\frac{1}{\hat{z}} - 1\right)\right)$$

- "Collinear " region

$\Lambda_{qcd} \ll q_T \sim Q$

Two mechanisms ?

Cahn intrinsic $\boldsymbol{k}_T$



- "TMD" region

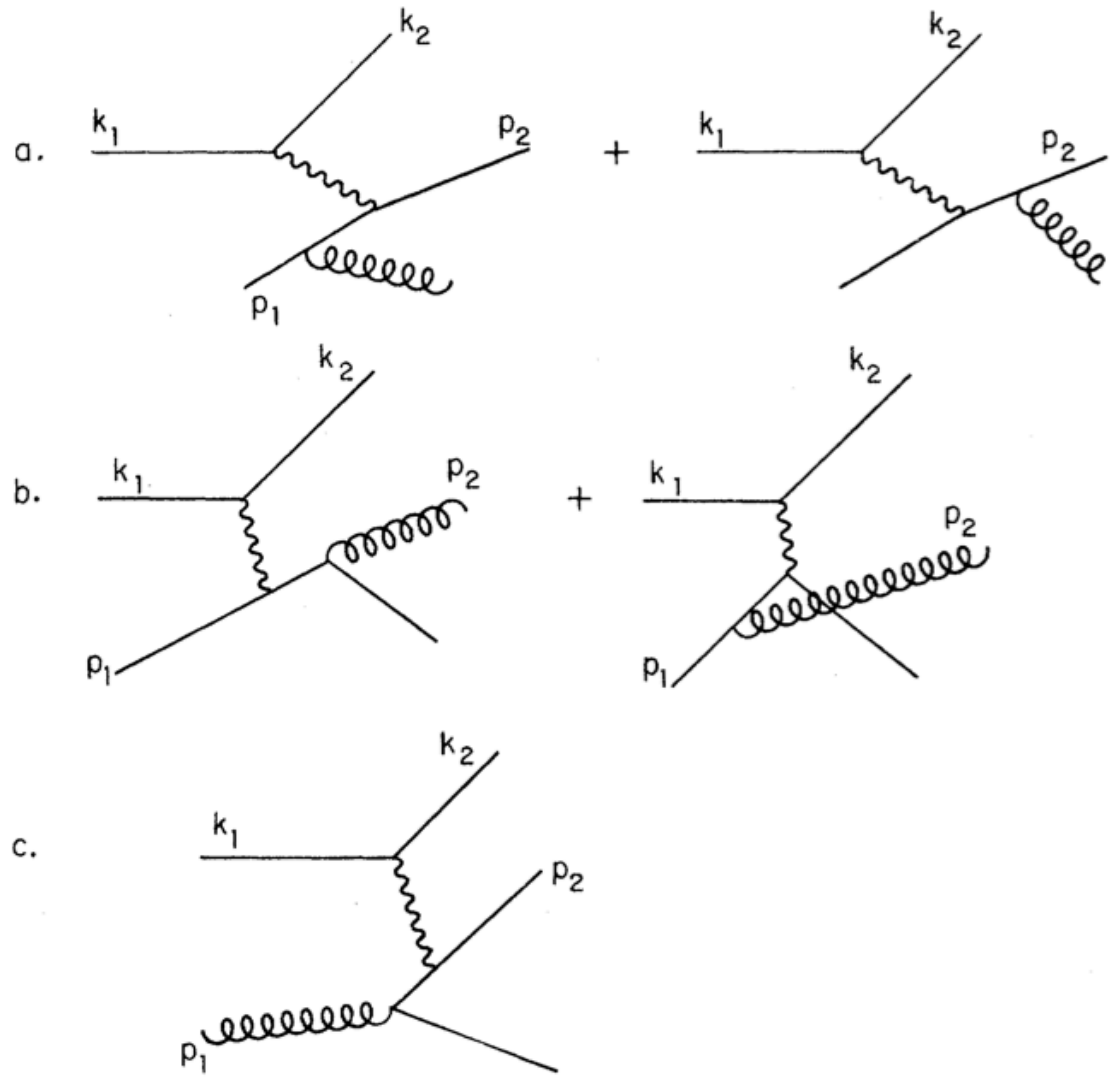
$(p_T \sim k_T) \sim q_T \ll Q$

$$\frac{d\sigma}{dx\, dy\, d\psi\, dz\, d\phi_h\, dP_{h\perp}^2} =$$

$$\frac{\alpha^2}{xyQ^2}\,\frac{y^2}{2\,(1-\varepsilon)}\left(1+\frac{\gamma^2}{2x}\right)\Big\{F_{UU,T}+\varepsilon\, F_{UU,L}+\sqrt{2\,\varepsilon(1+\varepsilon)}\,\cos\phi_h\, F_{UU}^{\cos\phi_h}$$

$$+\,\varepsilon\cos(2\phi_h)\,F_{UU}^{\cos 2\phi_h}\Big\}$$

e.g. $F_{UU}^{\cos\phi_h} \approx \frac{2M}{Q}\,\mathcal{C}\left[-\frac{\hat{\boldsymbol{h}}\cdot \boldsymbol{p}_T}{M}\, f_1 D_1\right]$



- "Collinear" region

$\Lambda_{qcd} \ll q_T \sim Q$

Data General features

$$\frac{d\sigma^{ep\rightarrow ehX}}{d\phi} = \mathcal{A} + \mathcal{B}\cos\phi + \mathcal{C}\cos 2\phi + \mathcal{D}\sin\phi + \mathcal{E}\sin 2\phi$$

EMC collaboration Phys. Lett. B 130 (1983) 118, & Z. Phys. C 34 (1987) 277

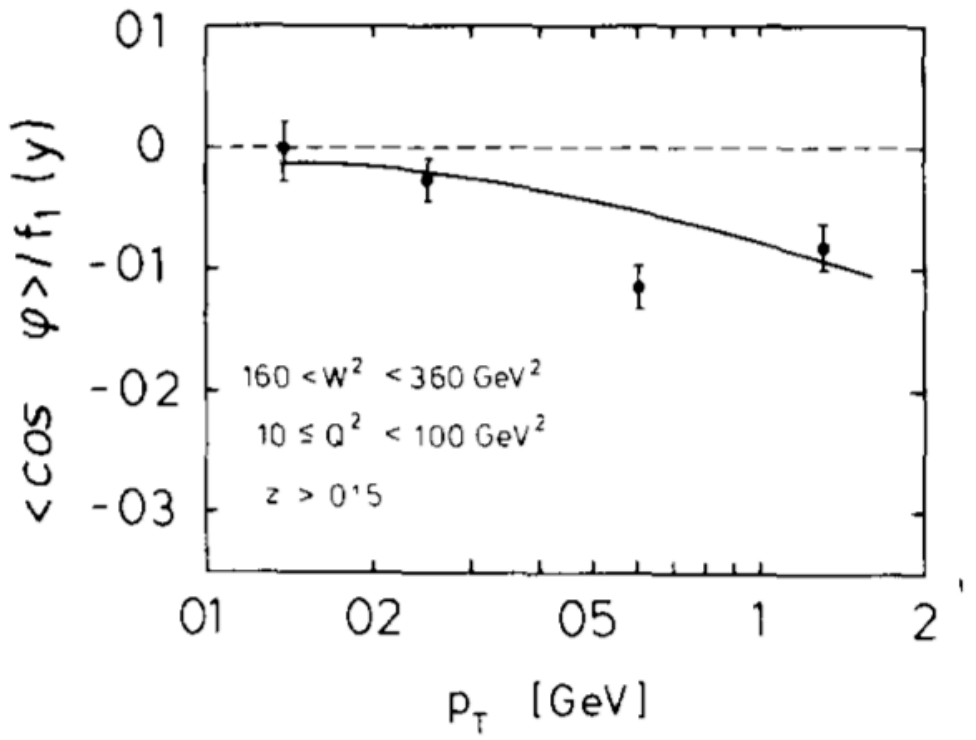
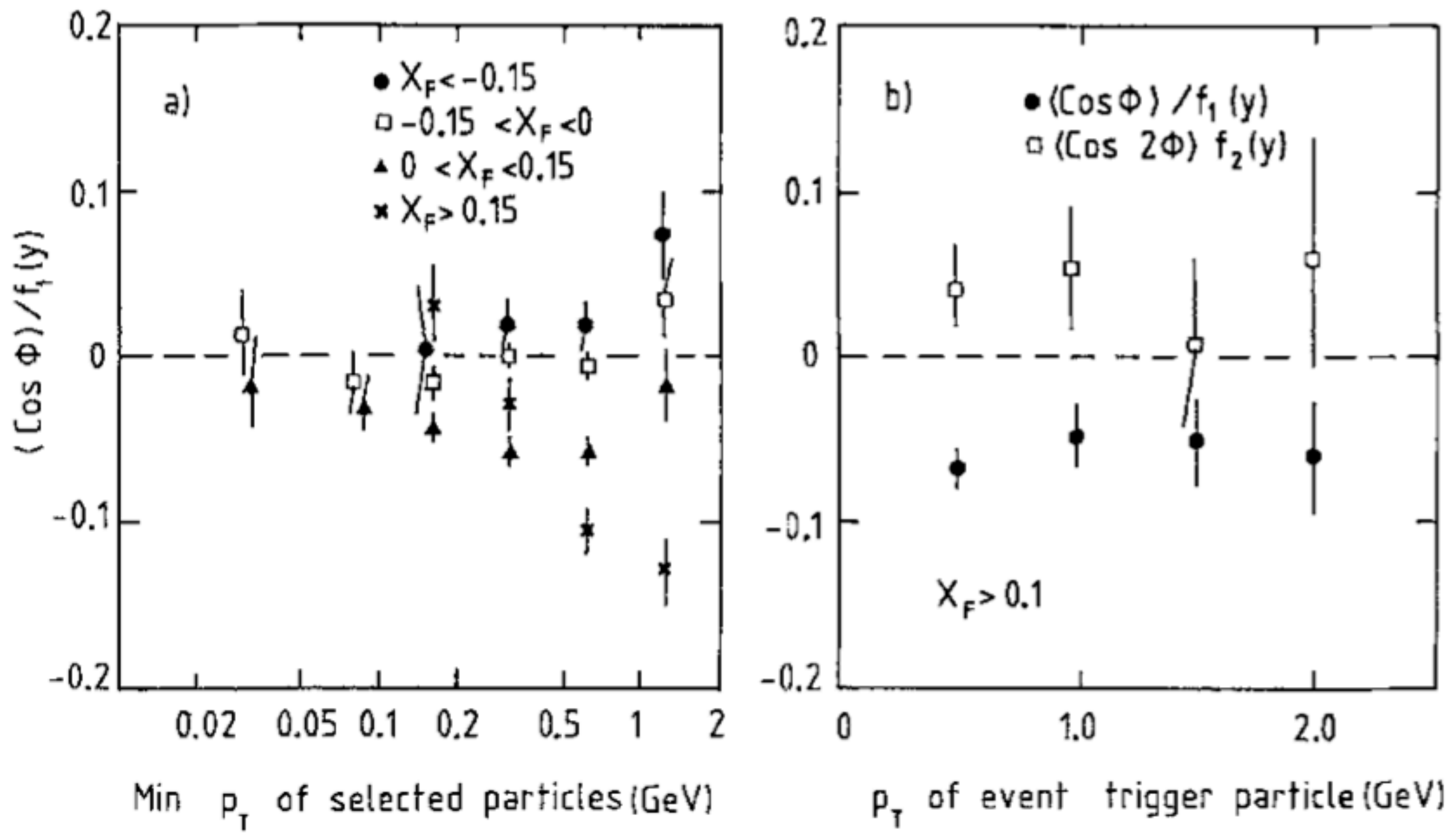
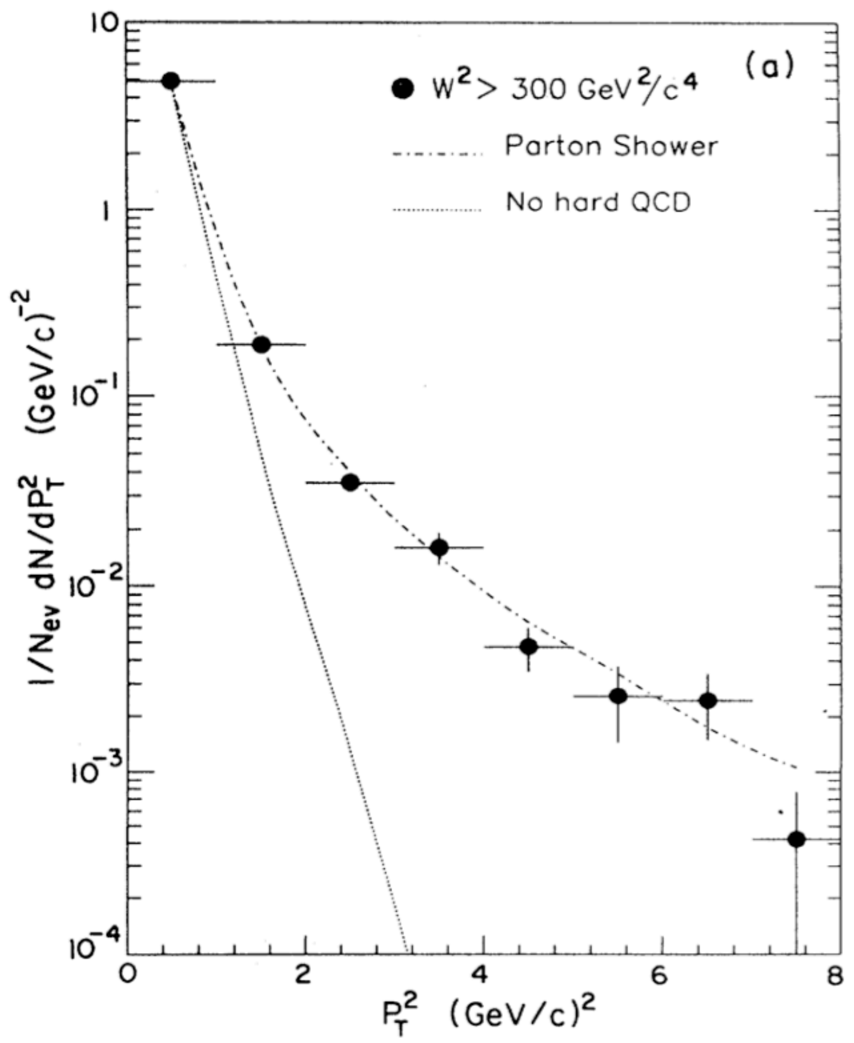


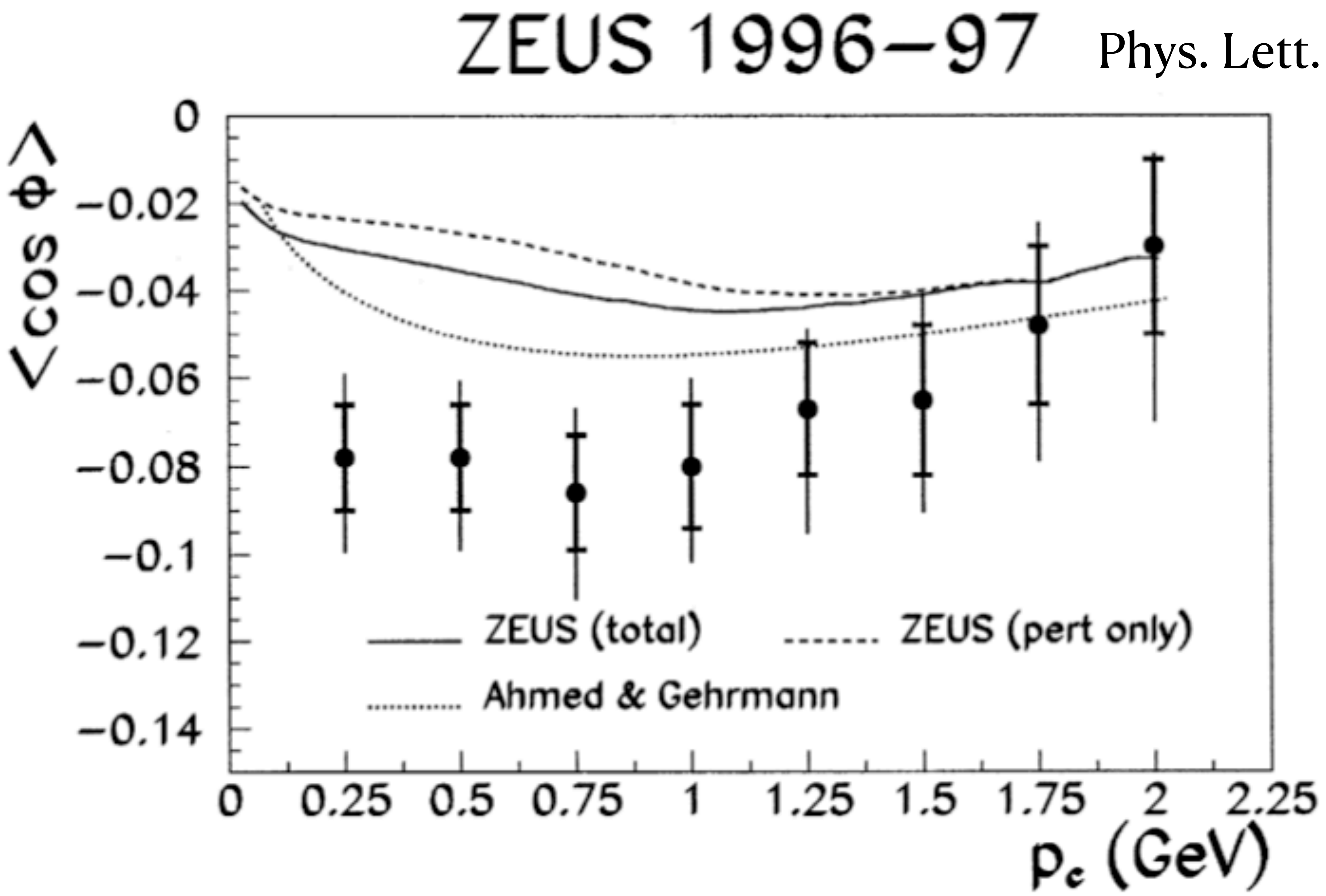
Fig 4 p_T dependence ($p_T > 50$ MeV) of cos φ moment for $160 \leqslant W^2 < 360$ GeV^2, $Q^2 > 10$ GeV^2 and $z > 0.15$ compared with model calculations described in ref [8] (statistical errors on model curve from Monte Carlo ±0 03 not shown)

M. Arneodo et al.: Measurement of Hadron Azimuthal Distributions



E665 Phys. Rev. D 48 (1993) 5057



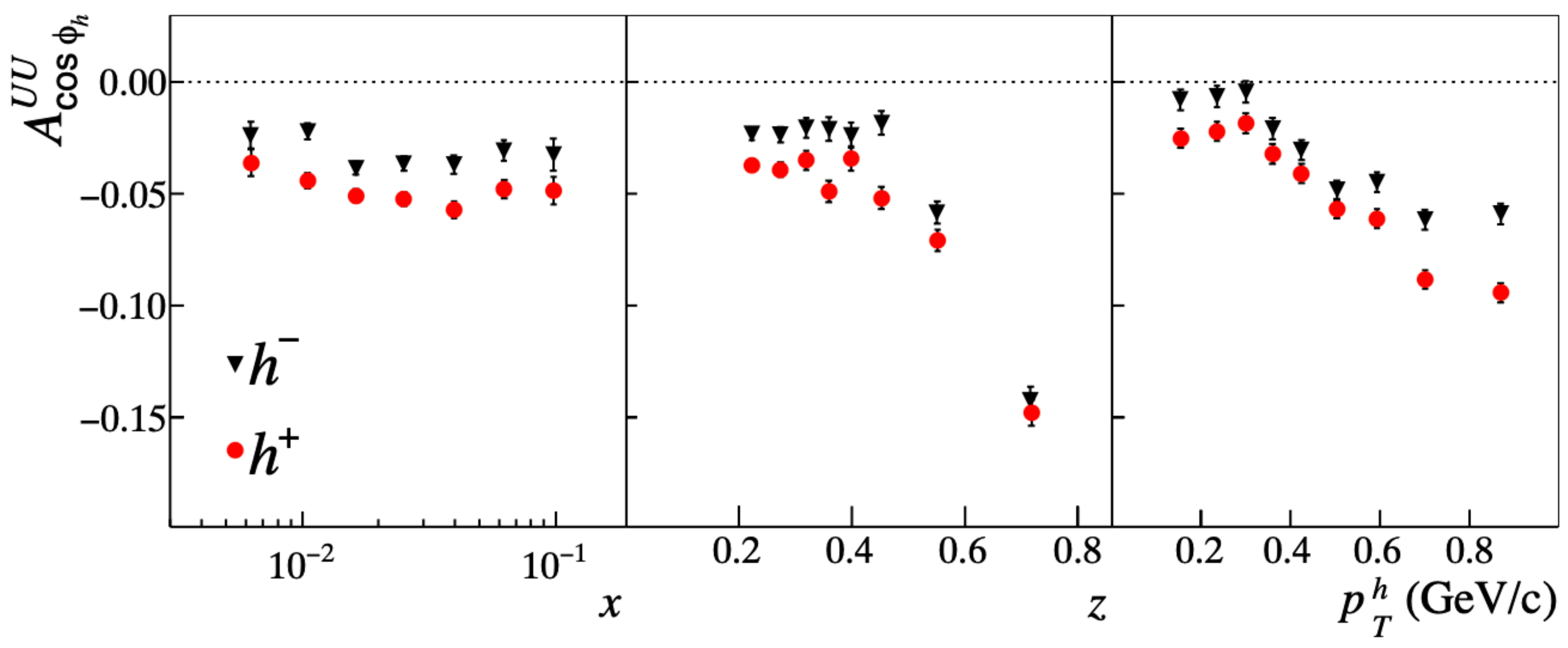


Phys. Lett. B 481 (2000)

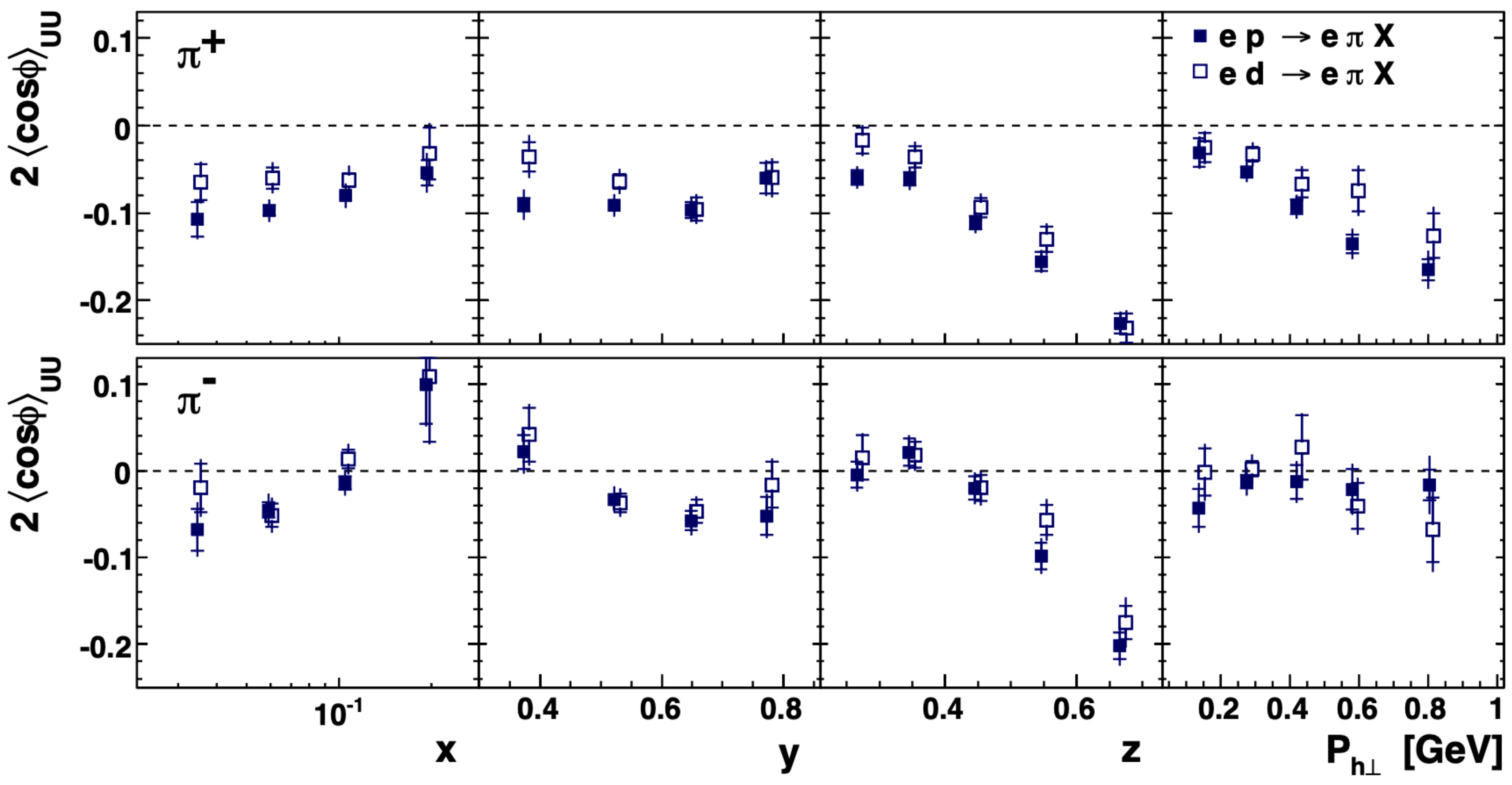
Fig. 4. The values of $\langle\cos\phi\rangle$ and $\langle\cos 2\phi\rangle$ are shown as a function of p_c in the kinematic region $0.01 < x < 0.1$ and $0.2 < y < 0.8$ for charged hadrons with $0.2 < z_h < 1.0$. The inner error bars represent the statistical errors, the outer are statistical and systematic errors added in quadrature. The lines are the LO predictions from ZEUS with perturbative and non-perturbative contributions (full line), ZEUS with the perturbative contribution only (dashed line) and Ahmed & Gehrmann (dotted line – see text

$$\frac{d\sigma^{ep\to ehX}}{d\phi} = \mathcal{A} + \mathcal{B}\cos\phi + \mathcal{C}\cos 2\phi + \mathcal{D}\sin\phi + \mathcal{E}\sin 2\phi$$

COMPASS, Nucl. Phys. B 886 (2014) 1046



HERMES, Phys. Rev. D 87 (2013) 012010



Parton model pheno ?

$$\langle \cos\phi \rangle = \frac{\int d\sigma^{(0)} \cos\phi + \int d\sigma^{(1)} \cos\phi}{\int d\sigma^{(0)} + \int d\sigma^{(1)}}$$

larger than 2 GeV, $\langle \cos\phi \rangle$ is,

$$\langle \cos\phi \rangle \approx \frac{\int d\sigma^{(1)} \cos\phi}{\int d\sigma^{(1)}}.$$

Chay, S.D. Ellis, Stirling, Phys. Lett. B (1991)

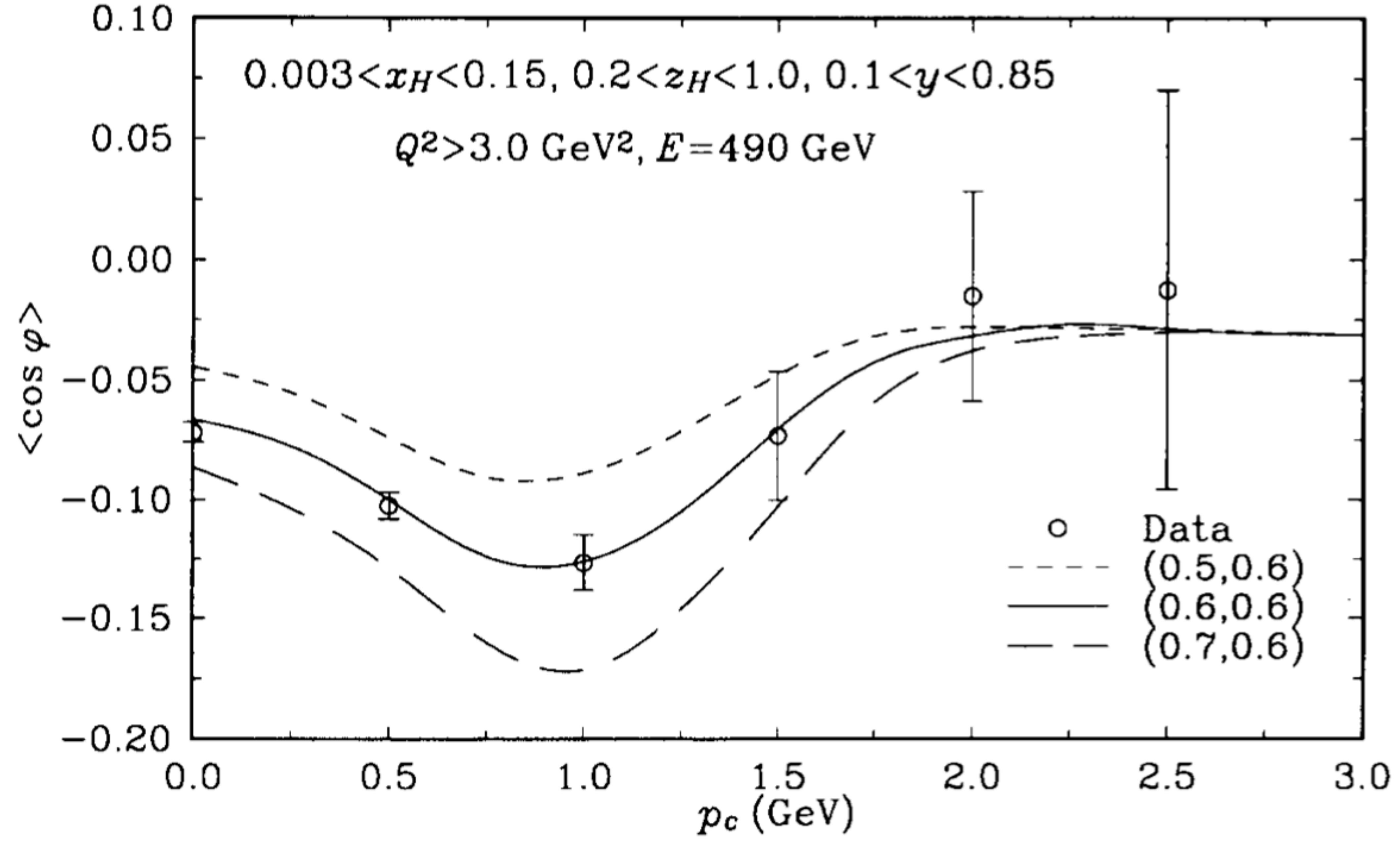
Oganessyan, Avakian, Bianchi, EPJC (1998)

$$\int d\sigma^{(0)} = 2\pi \frac{\alpha^2}{Q^2} \sum_j Q_j^2 F_j(x_H) D_j(z_H) \exp\left(-\frac{p_c^2}{b^2 + z_H^2 a^2}\right) \times \left\{ \frac{1+(1-y)^2}{y} + 4\frac{1-y}{yQ^2}\left[\frac{a^2 b^2}{b^2 + z_H^2 a^2} + \left(\frac{z_H a^2}{b^2 + z_H^2 a^2}\right)^2 (p_c^2 + b^2 + z_H^2 a^2)\right]\right\}$$

$$\int d\sigma^{(1)} \cos\phi = \int d^2 P_T \cos\phi \frac{d\sigma}{dx_H\, dy\, dz_H\, d^2 P_T} = \frac{8}{3}\frac{\alpha_s \alpha^2}{Q^2}\frac{(2-y)\sqrt{1-y}}{y}\int_{x_H}^1 \frac{dx}{x}\int_{z_H}^1 \frac{dz}{z}\sum_j Q_j^2 (A_j + B_j + C_j)$$

$$A_j = -\sqrt{\frac{xz}{(1-x)(1-z)}}\,[xz + (1-x)(1-z)]\, F_j\left(\frac{x_H}{x}, Q^2\right) D_j\left(\frac{z_H}{z}, Q^2\right)$$

Etc. ...

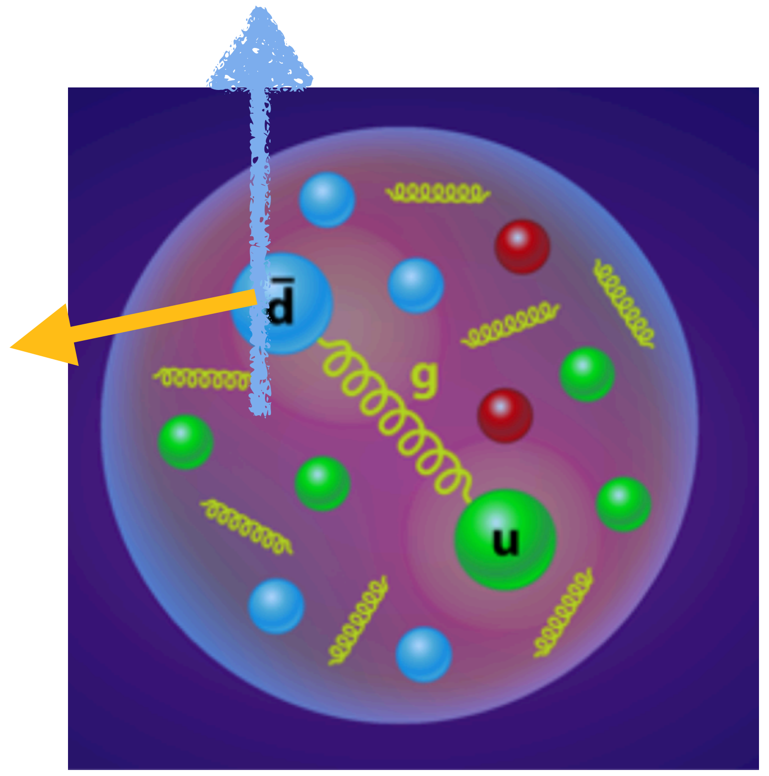


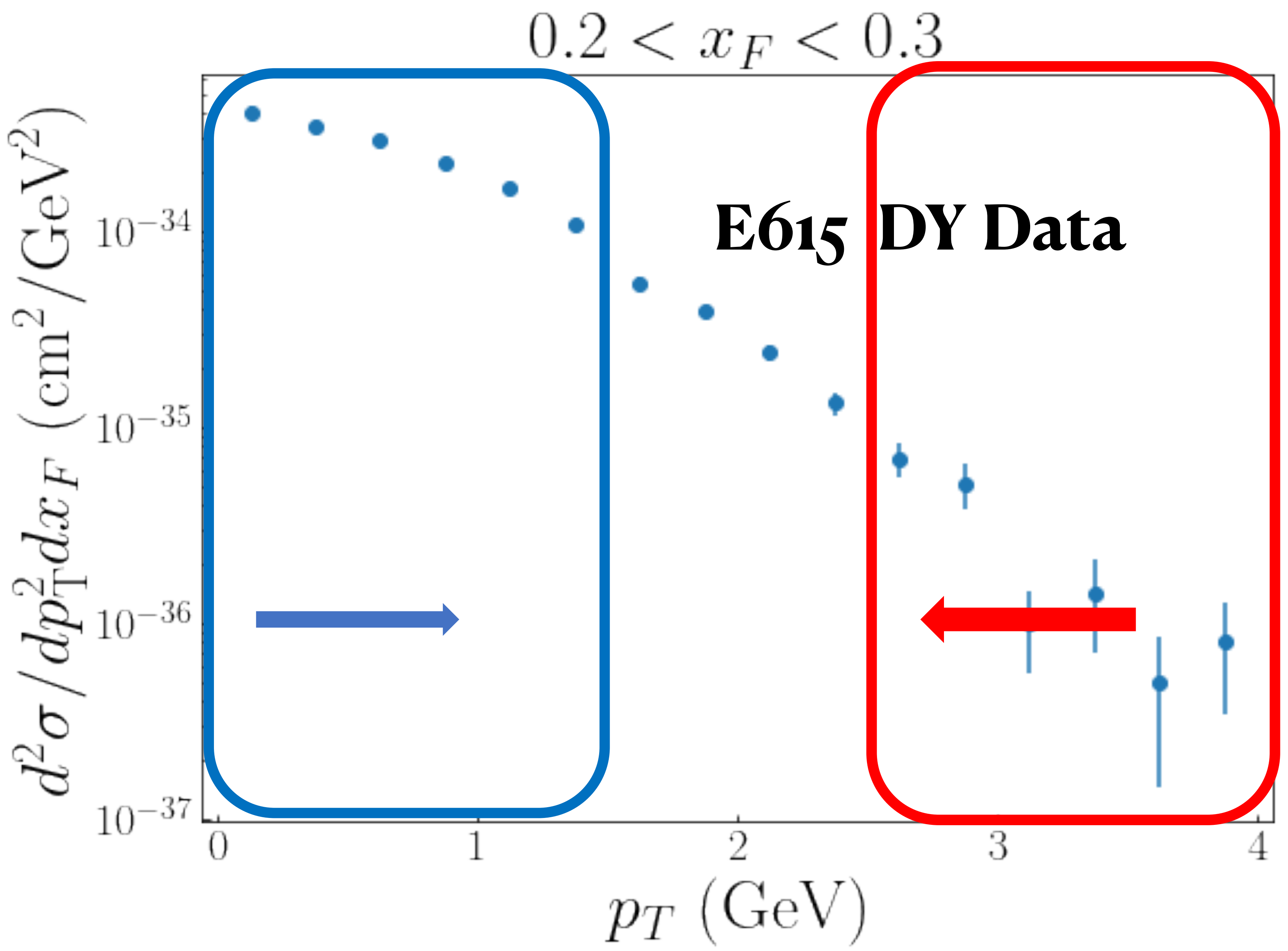
$\langle \cos\phi \rangle$ as a function of transverse momentum cutoff non-perturbative Cahn-like effect negligible at large values of p_c because assumed intrinsic transverse momentum in distribution and fragmentation functions are too small to produce $P_T > p_c$ (data E665 Fermi-lab).

Matching—TMD & collinear factorization

To describe the asymptotic "region" $\Lambda_{QCD} \ll q_T \ll Q$ is the subject of "matching" SIDS/ Drell-Yan cross section/ e^+e^- CSS NPB 1985, Catani et. al., $W+Y$ formalism-*unpolarized* Bacchetta Boer Diehl Mulders (BBDM) matches & mismatches JHEP 2008 azimuthal & leading subheading power

PRD (2016) Collins, Gamberg, Prokudin, Rogers, Sato, Wang





TMD Factorization intrinsic k_T

Fixed Order Collinear Factorization hard q_T radiation

"More granular" matching TMD w/ the collinear factorization

Overview comments Matching

✦ PRD (2016) **Collins, Gamberg, Prokudin, Rogers, Sato, Wang** modify the "*standard matching prescription*" traditionally used in CSS formalism relating low & high q_T behavior cross section @ moderate Q in particular where studies of TMDs are relevant

Matching studies in CSS related approaches

…

NPB Collins & Soper(1982), & Sterman 1985

NPB (1991) Arnold, Kauffman

PRD (1998) Nadolsky Stump Yuan

PRL (2001) Qiu, Zhang

PRD (2003) Berger, Qiu

NPB (2006) Bozzi, Catani, DeFlorian, Grazzini …

NPB (2006) Y. Koike, J. Nagashima, W. Vogelsang

JHEP (2008) Bacchetta et al.

arXiv (2014) Sun, Isacson, Yuan-CP,Yuan-F

JHEP (2015) Boglione, Hernandez, Melis Prokudin

PRD (2016) Collins, Gamberg, Prokudin, Rogers, Sato, Wang

PLB (2018) Gamberg , Metz, Pitonyak, Prokudin

PLB (2018) Echevarria, Kasemets, Lansberg, Pisano, Signori

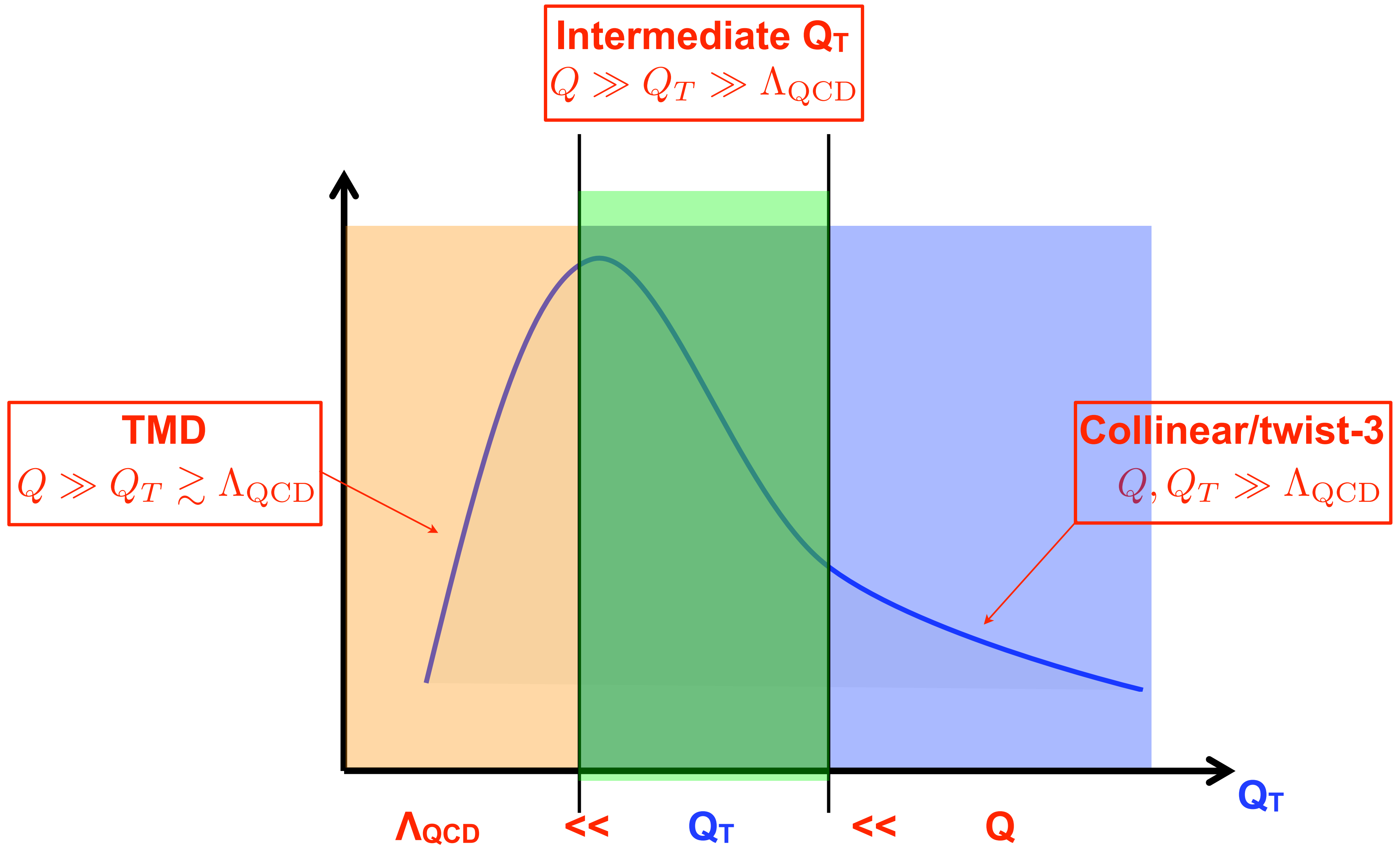
EJPA (2018) Scimemi, Vladimirov

JHEP 05 (2019) Scimemi , Tarasov, Vladimirov….

Series of papers on matching TMD and collinear ETQS transv. Spin

Ji, Qiu, Vogelsang, Yuan PRL PRD 2006, …

Kang, Xiao, Yuan PRL 2011



A comprehensive study of matching the hi & low Q_T in the overlap region in SIDIS was carried out by JHEP (2008) Bacchetta et al.
where attention was given to azimuthal and polarization dependence

One finds the definition of the Y term via "approximators" CSS

$$Y(q_T, Q) \equiv T_{coll}\, d\sigma(q_T, Q) - T_{coll} T_{TMD}\, d\sigma(q_T, Q)$$

$$Y(q_T, Q) = FO(q_T, Q) - ASY(q_T, Q)$$

Engineer matching with the AY term which cancels double counting in CSS

- It is the difference of the cross section calculated with collinear pdfs and ffs at fixed order FO and the asymptotic contribution of the cross section

- ***nb at small q_T the FO and ASY are dominated by the same diverging terms***

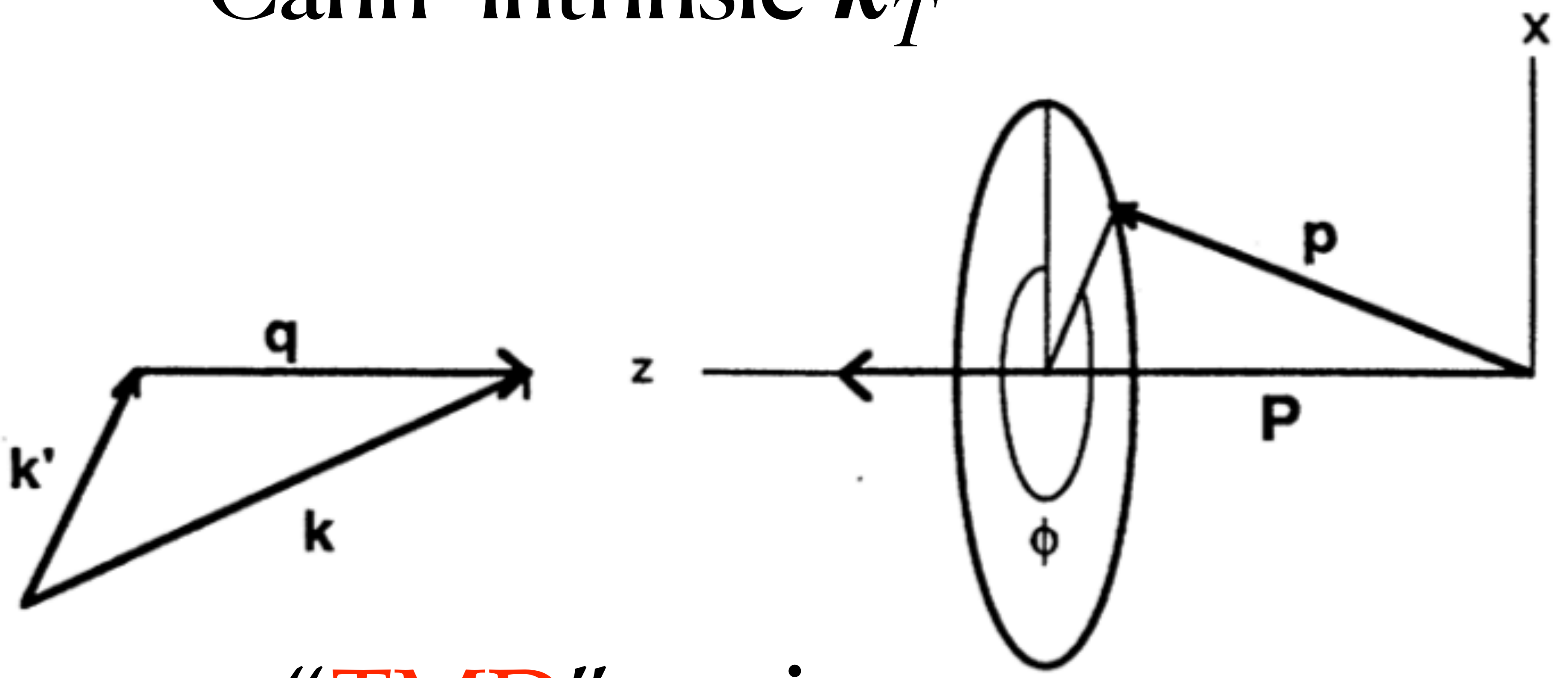
$$\frac{1}{q_T^2} \quad \text{and} \quad \frac{1}{q_T^2} \log \frac{Q^2}{q_T^2}$$

- ***Thus its expected that the Y term is small or zero leaving***

$$d\sigma(q_T \ll Q, Q) \approx W(q_T, Q)$$

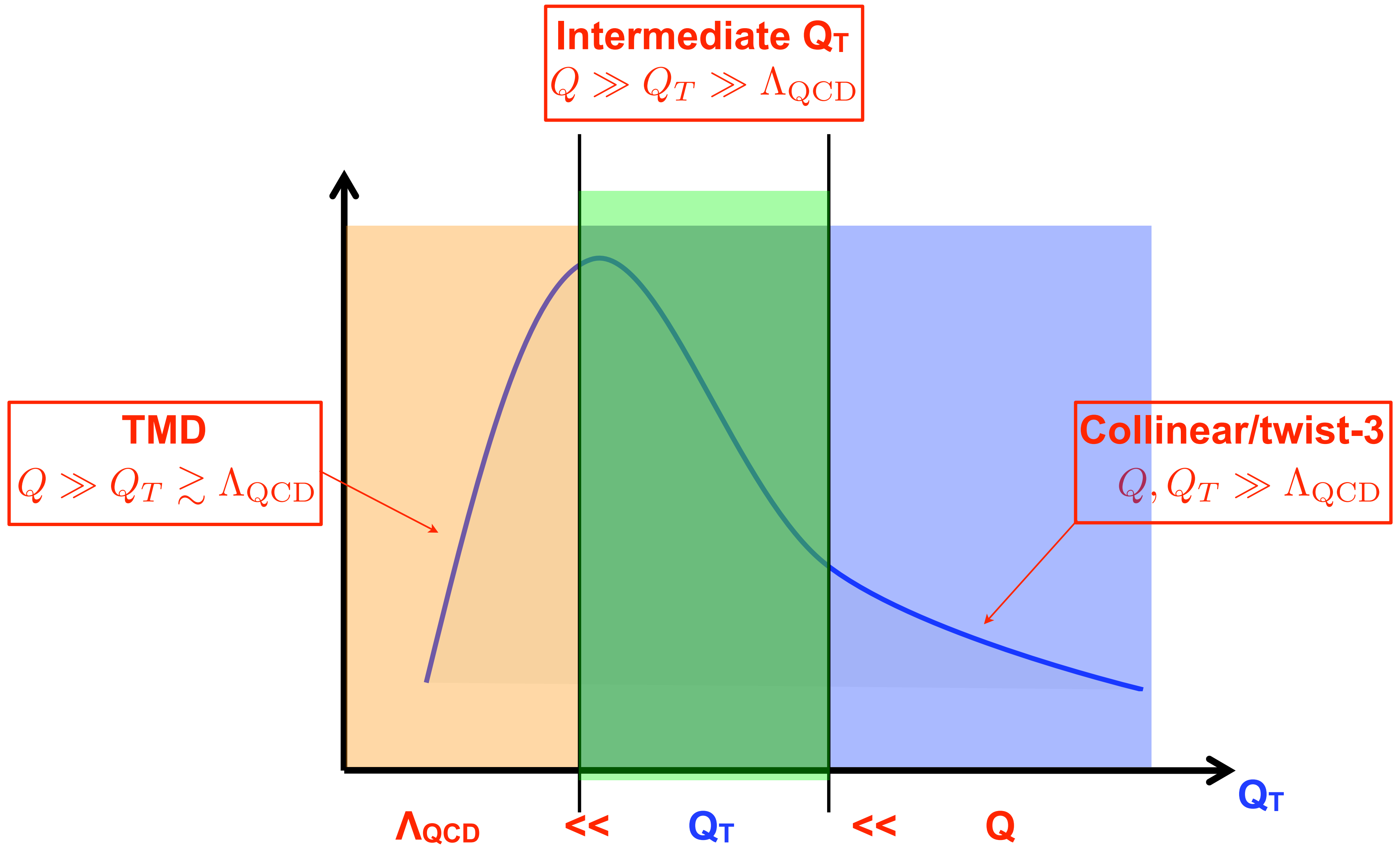
However! Factorization @ sub-leading power

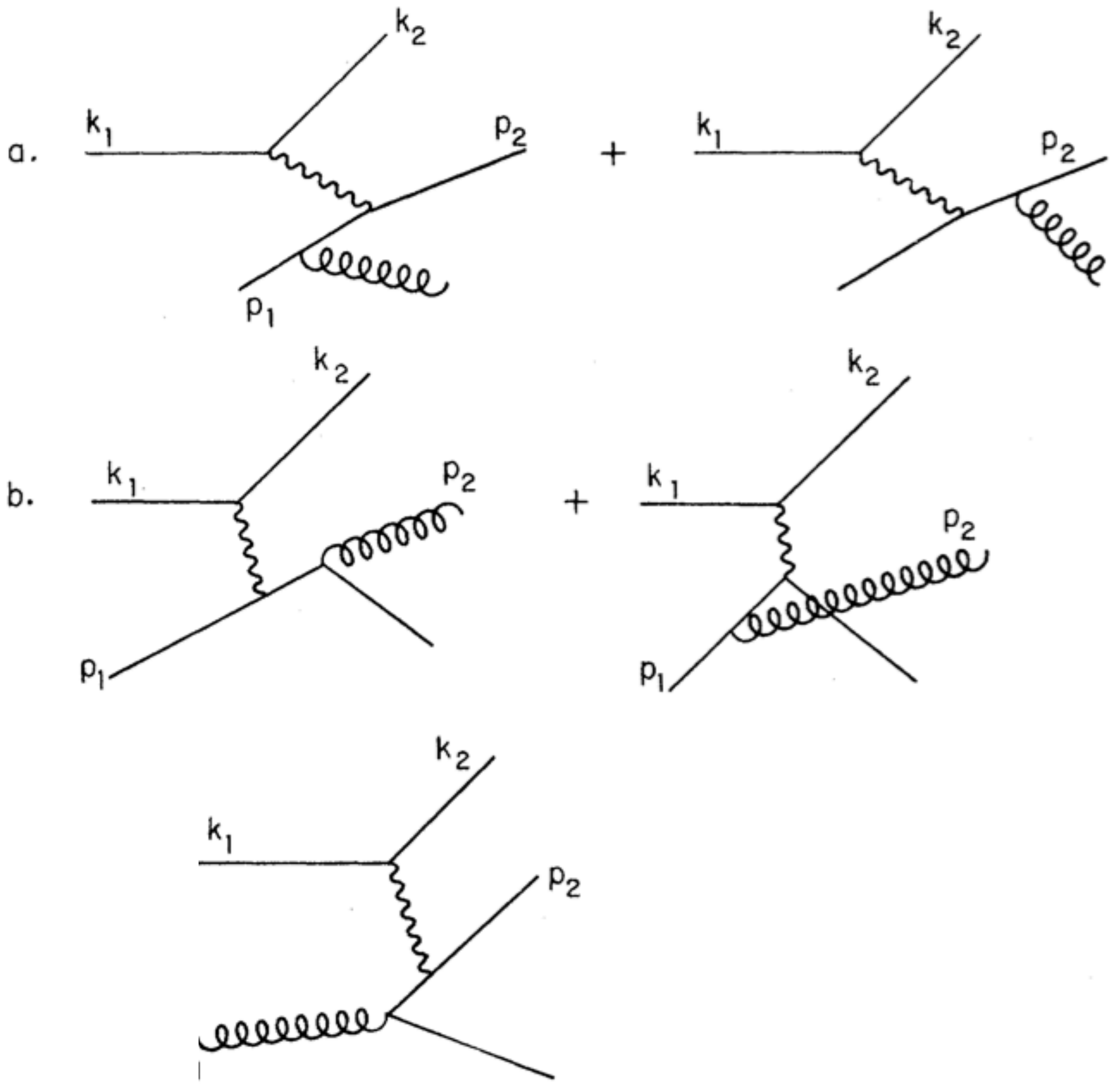
Cahn intrinsic $\boldsymbol{k}_T$



- “TMD” region

$(p_T \sim k_T) \sim q_T \ll Q$





“Collinear ” region

$\Lambda_{qcd} \ll q_T \sim Q$

Factorization at sub-leading power … revisit Tree level

- "TMD" region $(p_T \sim k_T) \sim q_T \ll Q$

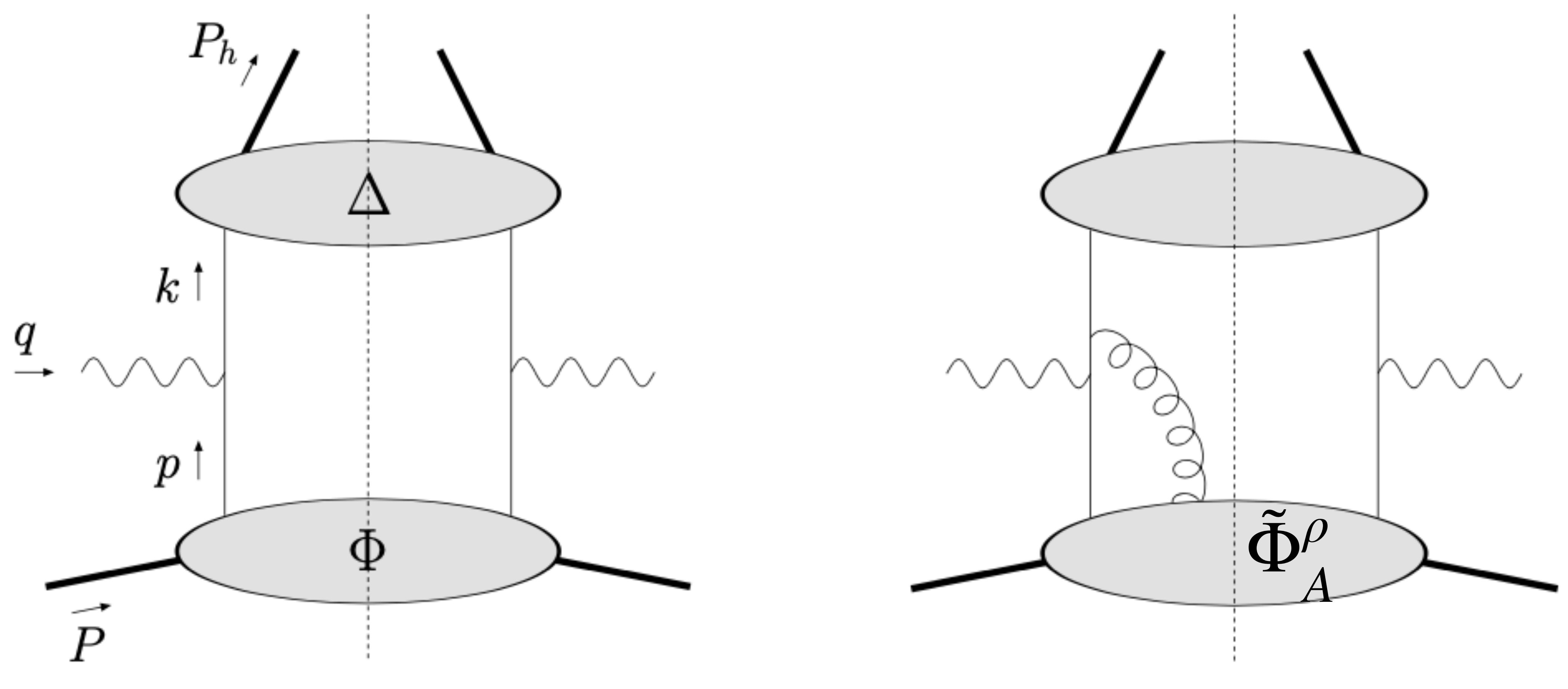
Factorization at sub-leading power … revisit Tree level

- "TMD" region $(p_T \sim k_T) \sim q_T \ll Q$

- Factorization beyond leading order and leading power via Collins, Aybat & Rogers 2011
- To do this at sub-leading power; revisit tree level build RG consistency
- Develop RG and rapidity renormalization group Eqs. CS equation

Factorization at sub-leading power … revisit Tree level

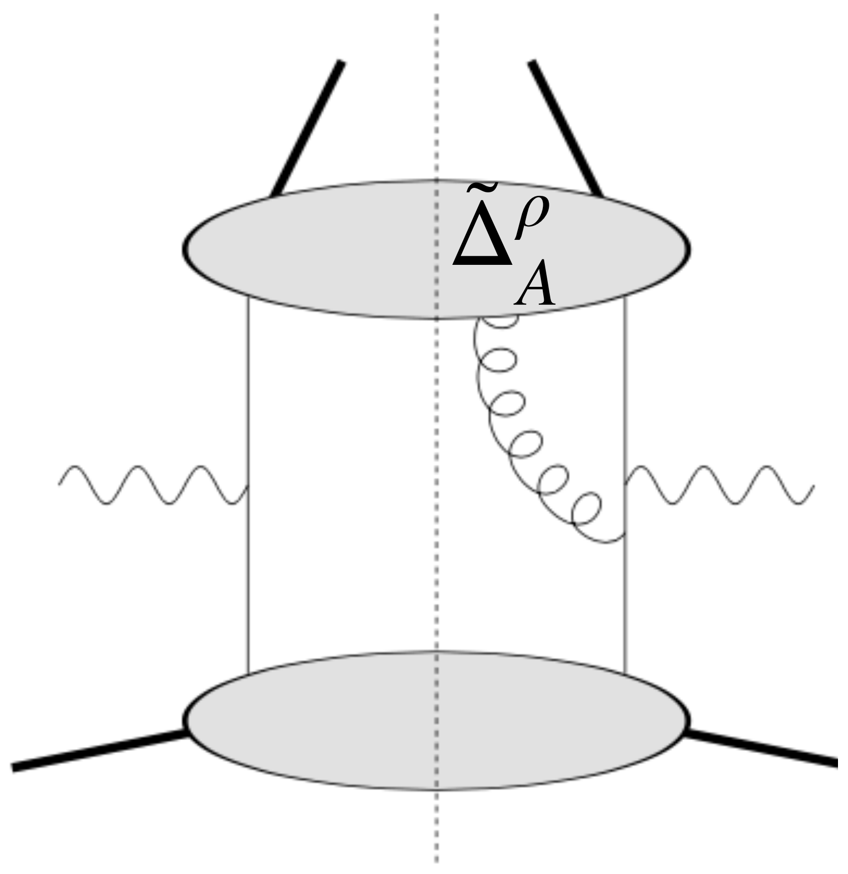
- "TMD" region $(p_T \sim k_T) \sim q_T \ll Q$



(a) (b)

SIDIS tree-level diagrams relevant for sub-leading-power observables.
Upper left contain *"intrinsic and kinematical"* contributions, the other two diagrams "*dynamical*" contributions with

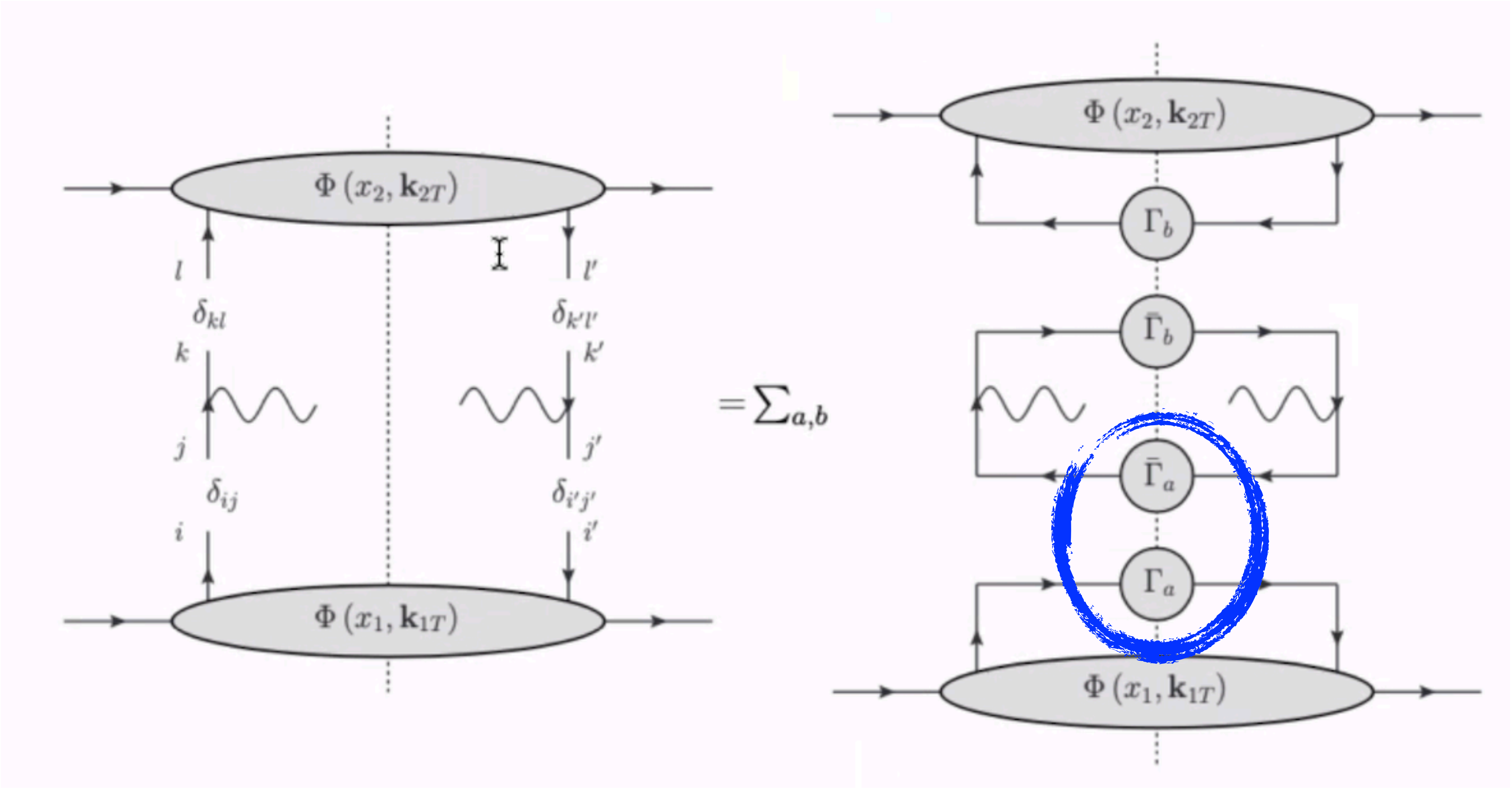
$$A_T = n_T \cdot A$$



(c)

$$\begin{aligned}
2M\mathcal{W}^{\mu\nu} =& e^2 \int d^2\boldsymbol{p}_T d^2\boldsymbol{k}_T \delta^2(\boldsymbol{p}_T + \boldsymbol{q}_T - \boldsymbol{k}_T) \Big\{ \mathrm{Tr}\left[\Phi(p)\gamma_\mu \Delta(k)\gamma_\nu\right] \\
&- \frac{1}{Q2\sqrt{2}} \mathrm{Tr}\left[\gamma_\alpha \not{\bar{n}} \gamma_\nu \Phi^\alpha_A(p)\gamma_\mu \Delta(k)\right] - \frac{1}{Q2\sqrt{2}} \mathrm{Tr}\left[\gamma_\mu \not{\bar{n}} \gamma_\alpha \Delta(k)\gamma_\nu \Phi^{\alpha\dagger}_A(p)\right] \\
&- \frac{1}{Q2\sqrt{2}} \mathrm{Tr}\left[\gamma_\nu \not{n} \gamma_\alpha \Phi(p)\gamma_\mu \Delta^{\alpha\dagger}_A(k)\right] - \frac{1}{Q2\sqrt{2}} \mathrm{Tr}\left[\gamma_\alpha \not{n} \gamma_\mu \Delta^\alpha_A(k)\gamma_\nu \Phi(p)\right] \Big\}
\end{aligned}$$

Next step factorization for subleading power via Fierz



Twist 2	Twist 3	Twist 4
$\frac{1}{2}\not{n}$, $\frac{1}{4}\not{\bar{n}}$	$\frac{1}{2}, \frac{1}{2}$	$\frac{1}{2}\not{\bar{n}}$, $\frac{1}{4}\not{n}$
$\frac{1}{2}\not{n}\gamma^5$, $\frac{1}{4}\gamma^5\not{\bar{n}}$	$\frac{1}{2}\gamma^5$, $\frac{1}{2}\gamma^5$	$\frac{1}{2}\not{\bar{n}}\gamma^5$, $\frac{1}{4}\gamma^5\not{n}$
$\frac{i}{2}\sigma^{k+}\gamma^5$, $\frac{i}{4}\gamma^5\sigma_{-k}$	$\frac{1}{2}\gamma^k$, $\frac{1}{2}\gamma_k$	$\frac{i}{2}\sigma^{k-}\gamma^5$, $\frac{i}{4}\gamma^5\sigma_{+k}$
	$\frac{1}{2}\gamma^k\gamma^5$, $\frac{1}{2}\gamma^5\gamma_k$	
	$\frac{i}{2}\sigma^{kl}\gamma^5$, $\frac{i}{4}\gamma^5\sigma_{lk}$	
	$\frac{i}{4}\sigma^{+-}\gamma^5$, $\frac{i}{4}\gamma^5\sigma_{+-}$	

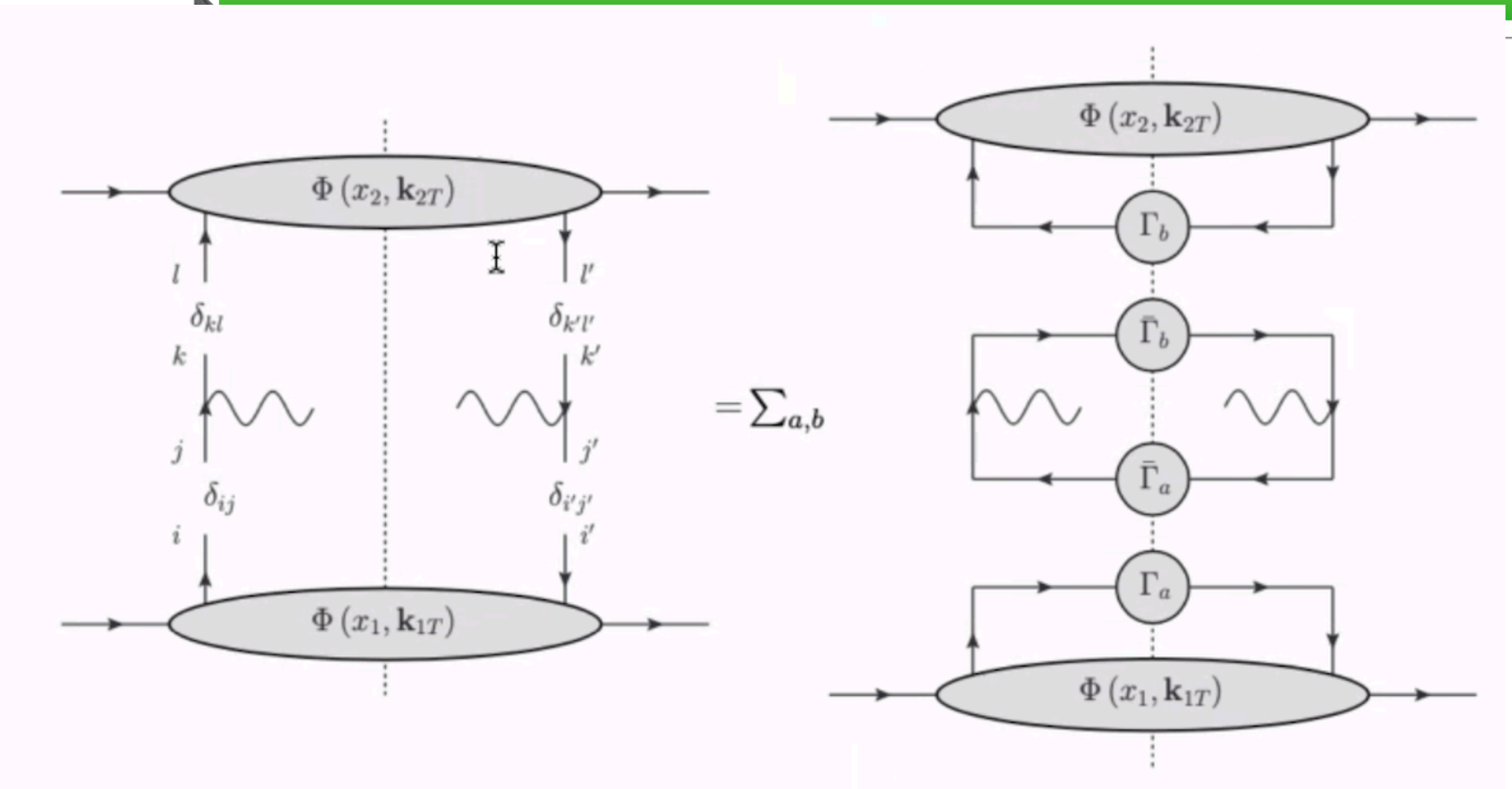
$$\delta_{ij}\delta_{kl} = \sum_a \Gamma^a_{il}\,\bar{\Gamma}^a_{kj}$$

$$\Phi^{\Gamma^a}(x_1, \boldsymbol{k}_T, \boldsymbol{S}) \equiv \mathrm{Tr}\left[\Phi(x_1, \boldsymbol{k}_T, \boldsymbol{S})\,\Gamma^a\right]$$

$$\Gamma^a \in \{1, \gamma_5, \gamma^\mu, \gamma^\mu\gamma^5, \sigma^{\mu\nu}\gamma^5\}$$

At subleading twist, for intrinsic functions get mixing

Organize via Fierz decomp motivate TMD factorization framework



Twist 2	Twist 3	Twist 4
$\frac{1}{2}\not{n}$, $\frac{1}{4}\not{\bar{n}}$	$\frac{1}{2}, \frac{1}{2}$	$\frac{1}{2}\not{\bar{n}}$, $\frac{1}{4}\not{n}$
$\frac{1}{2}\not{n}\gamma^5$, $\frac{1}{4}\gamma^5\not{\bar{n}}$	$\frac{1}{2}\gamma^5$, $\frac{1}{2}\gamma^5$	$\frac{1}{2}\not{\bar{n}}\gamma^5$, $\frac{1}{4}\gamma^5\not{n}$
$\frac{i}{2}\sigma^{k+}\gamma^5$, $\frac{i}{4}\gamma^5\sigma_{-k}$	$\frac{1}{2}\gamma^k$, $\frac{1}{2}\gamma_k$	$\frac{i}{2}\sigma^{k-}\gamma^5$, $\frac{i}{4}\gamma^5\sigma_{+k}$
	$\frac{1}{2}\gamma^k\gamma^5$, $\frac{1}{2}\gamma^5\gamma_k$	
	$\frac{i}{2}\sigma^{kl}\gamma^5$, $\frac{i}{4}\gamma^5\sigma_{lk}$	
	$\frac{i}{4}\sigma^{+-}\gamma^5$, $\frac{i}{4}\gamma^5\sigma_{+-}$	

$$W^{\mu\nu}_{\{2,3\,\mathrm{intrinsic}\}} = \frac{1}{N_c}\sum_{a1,a2}\sum_{q} e_q^2 \int d^2k_{1T}\, d^2k_{2T}\, \delta^{(2)}\left(\boldsymbol{q}_T - \boldsymbol{k}_{1T} - \boldsymbol{k}_{2T}\right)$$
$$\times\, \mathrm{Tr}\left[\gamma^\mu\, \bar{\Gamma}_1^{a1}\, \gamma^\nu\, \bar{\Gamma}_1^{a2}\right]\, \Phi^{[\Gamma^{a1}]}\left(x_1, \boldsymbol{k}_{1T}, \boldsymbol{S}_1\right)\, \bar{\Phi}^{[\Gamma^{a2}]}\left(x_2, \boldsymbol{k}_{2T}, \boldsymbol{S}_2\right) .$$

$$\Phi^{\Gamma^a}\left(x_1, \boldsymbol{k}_T, \boldsymbol{S}\right) \equiv \mathrm{Tr}\left[\Phi\left(x_1, \boldsymbol{k}_T, \boldsymbol{S}\right)\Gamma^a\right]$$

Representation of the Fierz decompostion of the hadronic tensor.
Left: broken lines used to separate the hard interaction from the definition of the *qq* correlation function.
Right: The Fierz decomposition where Γ_a represent the operators which give rise to the parton densities while $\bar{\Gamma}_a$ represent the operators which enter into the hard function.

Tree level factorization sub-leading power

$$\Phi(x, \boldsymbol{k}_T)$$

SIDIS tree-level diagrams relevant for sub-leading-power observables.
"intrinsic"

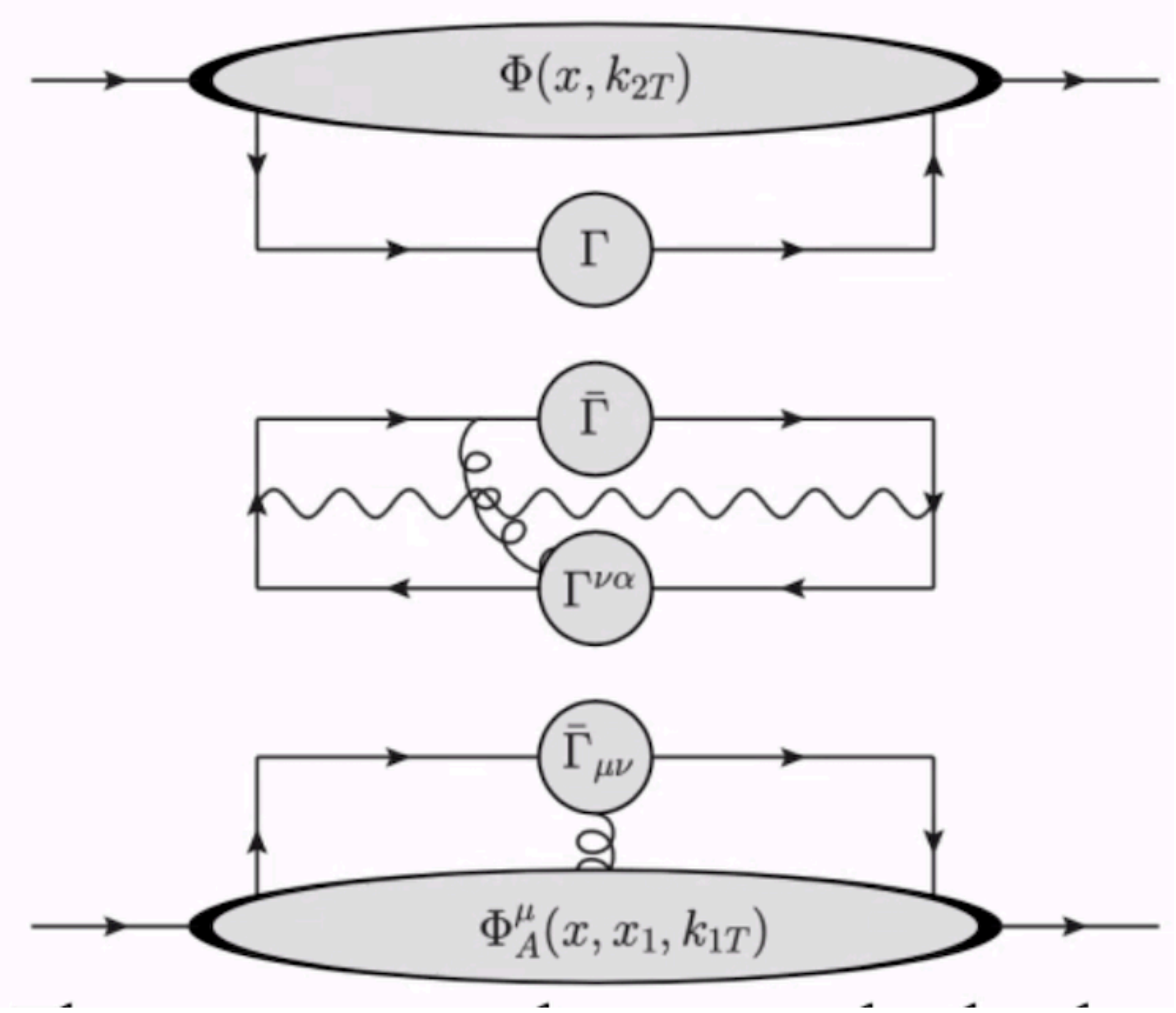
Subleading Quark TMDPDFs

		Quark Chirality	
		Chiral Even	Chiral Odd
Nucleon Polarization	U	$f^{\perp}$, $g^{\perp}$	e , h
	L	$f_L^{\perp}$, $g_L^{\perp}$	e_L, h_L
	T	f_T, $f_T^{\perp}$, g_T, $g_T^{\perp}$	e_T, $e_T^{\perp}$, h_T, $h_T^{\perp}$

- ✦Mulders Tangerman NPB1995
- ✦Goeke Metz Schlegel PLB 2005
- ✦Bacchetta et al 2007 JHEP

$$\begin{aligned}\Phi^{(3)}(x, \boldsymbol{k}_T, \boldsymbol{S}) = \frac{M}{P^+}\Bigg[& \left(e - \frac{\epsilon_T^{\rho\sigma} k_{T\rho} S_{T\sigma}}{M} e_T^{\perp}\right)\frac{1}{2} - i\left(\lambda_g e_L - \frac{\boldsymbol{k}_T \cdot \boldsymbol{S}_T}{M} e_T\right)\frac{\gamma^5}{2} \\ & + \left(\frac{k_T^k}{M} f^{\perp} - \epsilon_T^{kl} S_{Tl} f_T' - \frac{\epsilon_T^{kl} k_{Tl}}{M}\left(\lambda_g f_L^{\perp} - \frac{\boldsymbol{k}_T \cdot \boldsymbol{S}_T}{M} f_T^{\perp}\right)\right)\frac{\gamma_k}{2} \\ & + \left(g_T' S_T^k - \frac{\epsilon_T^{kl} k_{Tl}}{M} g^{\perp} + \frac{k_T^k}{M}\left(\lambda_g g_L^{\perp} - \frac{\boldsymbol{k}_T \cdot \boldsymbol{S}_T}{M} g_T^{\perp}\right)\right)\frac{\gamma^5 \gamma_k}{2} \\ & + \left(\frac{S_T^k k_T^l}{M} h_T^{\perp}\right)\frac{i\gamma^5 \sigma_{lk}}{4} + \left(h + \lambda_g h - \frac{\boldsymbol{k}_T \cdot \boldsymbol{S}_T}{M} h^{\perp}\right)\frac{i\gamma^5 \sigma_{+-}}{4}\Bigg]\end{aligned}$$

Organize via Fierz decomp motivate TMD factorization framework



SIDIS tree-level diagrams relevant for sub-leading-power observables. diagrams "*dynamical*" contributions with

$$A_T = n_T \cdot A$$

$$\tilde{\Phi}^{\rho}_{A}(x, \boldsymbol{k}_T) \longrightarrow$$

Tree level factorization sub-leading power

$$\tilde{\Phi}_A^{\rho}(x, \boldsymbol{k}_T)$$

SIDIS tree-level diagrams relevant for sub-leading-power observables.
diagrams “*dynamical*” contributions with

$$A_T = n_T \cdot A$$

✦Mulders Tangerman NPB1995
✦ Boer Pijlman Mulders NPB 2003
✦Bacchetta et al 2007 JHEP

Subleading Quark-Gluon-Quark TMDPDFs

Nucleon Polarization		Quark Chirality: Chiral Even	Quark Chirality: Chiral Odd
	U	$\tilde{f}^{\perp}, \tilde{g}^{\perp}$	$\tilde{e}, \tilde{h}$
	L	$\tilde{f}_L^{\perp}, \tilde{g}_L^{\perp}$	$\tilde{e}_L, \tilde{h}_L$
	T	$\tilde{f}_T, \tilde{f}_T^{\perp}, \tilde{g}_T, \tilde{g}_T^{\perp}$	$\tilde{e}_T, \tilde{e}_T^{\perp}, \tilde{h}_T, \tilde{h}_T^{\perp}$

$$\tilde{\Phi}_A^{\alpha}(x, p_T) =$$
$$\frac{xM}{2}\left\{\left[(\tilde{f}^{\perp} - i\,\tilde{g}^{\perp})\frac{p_{T\rho}}{M} - (\tilde{f}_T' + i\,\tilde{g}_T')\,\epsilon_{T\rho\sigma}S_T^{\sigma} - (\tilde{f}_s^{\perp} + i\,\tilde{g}_s^{\perp})\frac{\epsilon_{T\rho\sigma}p_T^{\sigma}}{M}\right](g_T^{\alpha\rho} - i\epsilon_T^{\alpha\rho}\gamma_5)\right.$$
$$\left. - (\tilde{h}_s + i\,\tilde{e}_s)\gamma_T^{\alpha}\gamma_5 + \left[(\tilde{h} + i\,\tilde{e}) + (\tilde{h}_T^{\perp} - i\,\tilde{e}_T^{\perp})\frac{\epsilon_T^{\rho\sigma}p_{T\rho}S_{T\sigma}}{M}\right]i\gamma_T^{\alpha} + \ldots (g_T^{\alpha\rho} + i\epsilon_T^{\alpha\rho}\gamma_5)\right\}\frac{\not{n}_+}{2}$$

Summary tree level factorization sub-leading power

$$\Phi(x, p_T) = \frac{1}{4} f_1 \not{\bar{n}} + \frac{1}{2P^+} f^\perp \not{p}_T + \cdots ,$$

$$\Delta(z, k_T) = \frac{1}{4} D_1 \not{n} + \frac{1}{2P_h^-} D^\perp \not{k}_T + \cdots ,$$

$$\tilde{\Phi}_A^\alpha(x, p_T) = \frac{x p_{T\rho}}{4} \left(\tilde{f}^\perp - i\tilde{g}^\perp\right) \left(g_T^{\alpha\rho} - i\epsilon_T^{\alpha\rho}\right) \not{\bar{n}} + \cdots ,$$

$$\tilde{\Delta}_A^\alpha(z, k_T) = \frac{k_{T\rho}}{4z} \left(\tilde{D}^\perp - i\tilde{G}^\perp\right) \left(g_T^{\alpha\rho} + i\epsilon_T^{\alpha\rho}\right) \not{n} + \cdots .$$

Summary tree level factorization sub-leading power

$$\Phi(x, p_T) = \frac{1}{4} f_1 \not{\bar{n}} + \frac{1}{2P^+} f^\perp \not{p}_T + \cdots \,,$$

$$\Delta(z, k_T) = \frac{1}{4} D_1 \not{n} + \frac{1}{2P_h^-} D^\perp \not{k}_T + \cdots \,,$$

$$\tilde{\Phi}_A^\alpha(x, p_T) = \frac{x p_{T\rho}}{4} \left(\tilde{f}^\perp - i\tilde{g}^\perp\right) \left(g_T^{\alpha\rho} - i\epsilon_T^{\alpha\rho}\right) \not{\bar{n}} + \cdots \,,$$

$$\tilde{\Delta}_A^\alpha(z, k_T) = \frac{k_{T\rho}}{4z} \left(\tilde{D}^\perp - i\tilde{G}^\perp\right) \left(g_T^{\alpha\rho} + i\epsilon_T^{\alpha\rho}\right) \not{n} + \cdots \,.$$

Using the EoM relations $x f^\perp = x\tilde{f}^\perp + f_1$ and $D^\perp/z = \tilde{D}^\perp/z + D_1$, one has

$$\mathrm{Tr}\left[\Phi(p)\gamma^\mu \Delta(k)\gamma^\nu\right] = -\, g_\perp^{\mu\nu} f_1 D_1 + \frac{\sqrt{2}}{2Q} \frac{f_1 \tilde{D}^\perp}{z} \bar{n}^{\{\mu} k_T^{\nu\}}$$

$$+ \frac{\sqrt{2}}{2Q} (\bar{n} + n)^{\{\mu} p_T^{\nu\}} x f^\perp D_1 - \frac{\sqrt{2}}{2Q} \bar{n}^{\{\mu} p_T^{\nu\}} x \tilde{f}^\perp D_1$$

Summary tree level factorization sub-leading power

$$\Phi(x,p_T) = \frac{1}{4} f_1 \bar{\not{n}} + \frac{1}{2P^+} f^\perp \not{p}_T + \cdots ,$$

$$\Delta(z,k_T) = \frac{1}{4} D_1 \not{n} + \frac{1}{2P_h^-} D^\perp \not{k}_T + \cdots ,$$

$$\tilde{\Phi}_A^\alpha(x,p_T) = \frac{x p_{T\rho}}{4} \left(\tilde{f}^\perp - i\tilde{g}^\perp\right)\left(g_T^{\alpha\rho} - i\epsilon_T^{\alpha\rho}\right) \bar{\not{n}} + \cdots ,$$

$$\tilde{\Delta}_A^\alpha(z,k_T) = \frac{k_{T\rho}}{4z} \left(\tilde{D}^\perp - i\tilde{G}^\perp\right)\left(g_T^{\alpha\rho} + i\epsilon_T^{\alpha\rho}\right) \not{n} + \cdots .$$

From three parton correlators get dynamic contributions

$$- \frac{1}{Q2\sqrt{2}} \mathrm{Tr}\left[\gamma_\alpha \bar{\not{n}} \gamma_\nu \Phi_A^\alpha(p) \gamma_\mu \Delta(k)\right] - \frac{1}{Q2\sqrt{2}} \mathrm{Tr}\left[\gamma_\mu \bar{\not{n}} \gamma_\alpha \Delta(k) \gamma_\nu \Phi_A^{\alpha\dagger}(p)\right]$$

$$- \frac{1}{Q2\sqrt{2}} \mathrm{Tr}\left[\gamma_\nu \not{n} \gamma_\alpha \Phi(p) \gamma_\mu \Delta_A^{\alpha\dagger}(k)\right] - \frac{1}{Q2\sqrt{2}} \mathrm{Tr}\left[\gamma_\alpha \not{n} \gamma_\mu \Delta_A^\alpha(k) \gamma_\nu \Phi(p)\right]$$

$$= \frac{\sqrt{2}}{2Q} \bar{n}^{\{\mu} p_T^{\nu\}} x \tilde{f}^\perp D_1 + \frac{\sqrt{2}}{2Q} n^{\{\mu} k_T^{\nu\}} \frac{f_1 \tilde{D}^\perp}{z}$$

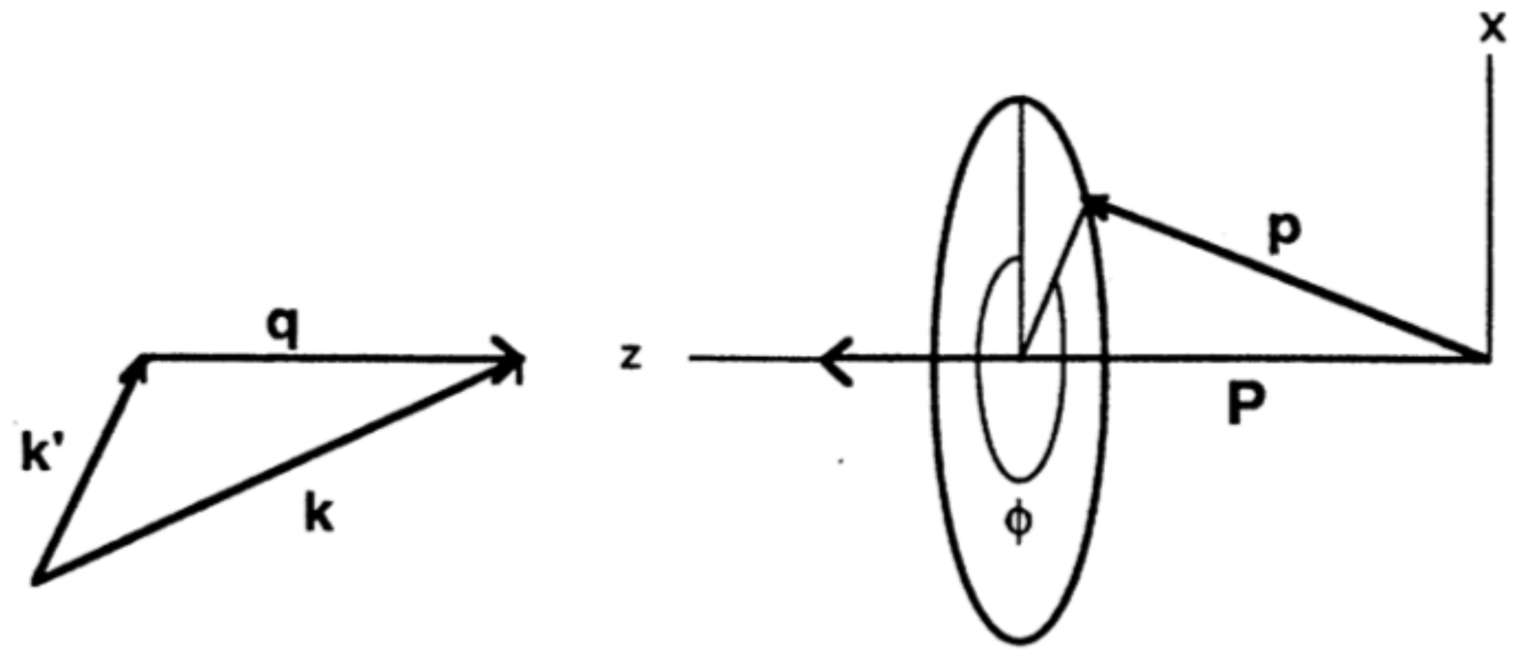
Tree level factorization sub-leading power

Combining these contributions and multiplying by leptonic tensor
get factorized Cahn and more ....

$$\frac{1}{Q}\hat{t}^{\{\mu}k_T^{\nu\}}\frac{f_1\tilde{D}^{\perp}}{z}L_{\mu\nu} = -\frac{4Q^2}{y^2}(2-y)\sqrt{1-y}\left[\frac{1}{Q}\,\hat{\boldsymbol{h}}\cdot\boldsymbol{k}_T\frac{f_1\tilde{D}^{\perp}}{z}\right]\cos\phi_h$$

$$\frac{2}{Q}\hat{t}^{\{\mu}p_T^{\nu\}}xf^{\perp}D_1L_{\mu\nu} = -\frac{4Q^2}{y^2}(2-y)\sqrt{1-y}\left[\frac{2}{Q}\,\hat{\boldsymbol{h}}\cdot\boldsymbol{p}_T xf^{\perp}D_1\right]\cos\phi_h$$

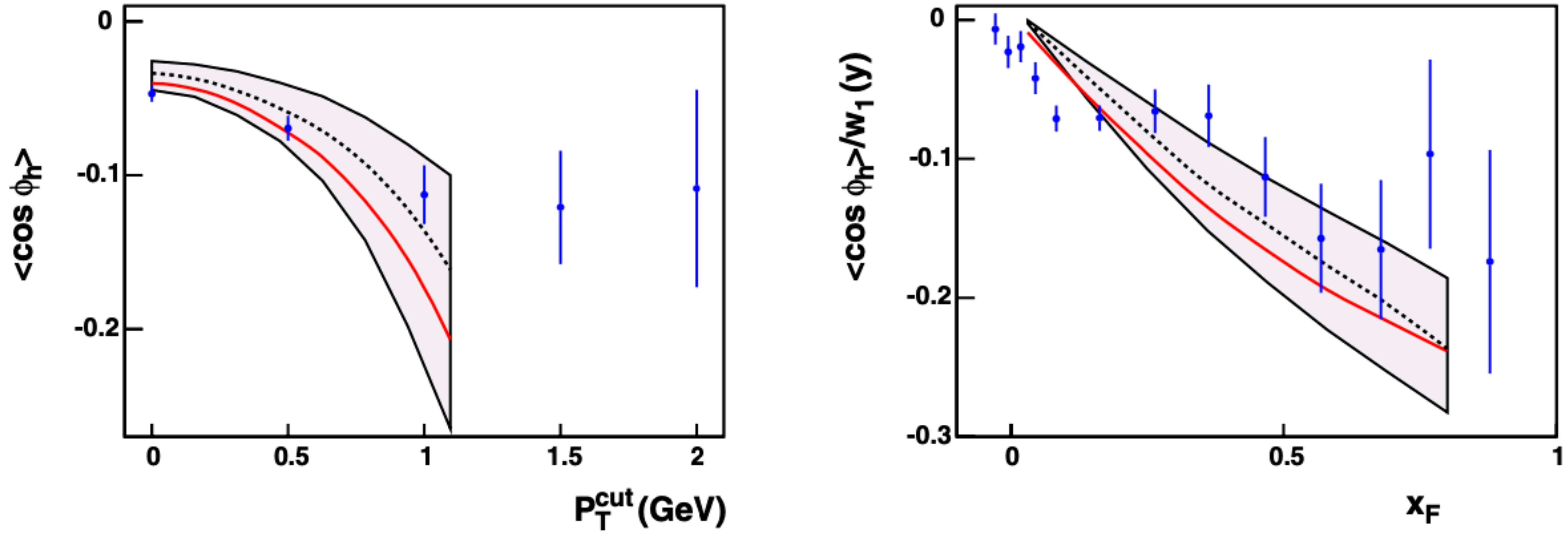
Cahn intrinsic $\boldsymbol{k}_T$



Role of Cahn effect in SIDIS from TMD framework
Modeling tree level result comparing w/ E665 data

Anselmino, Boglione,D'Alesio, Kotzinian, Murgia, Prokudin

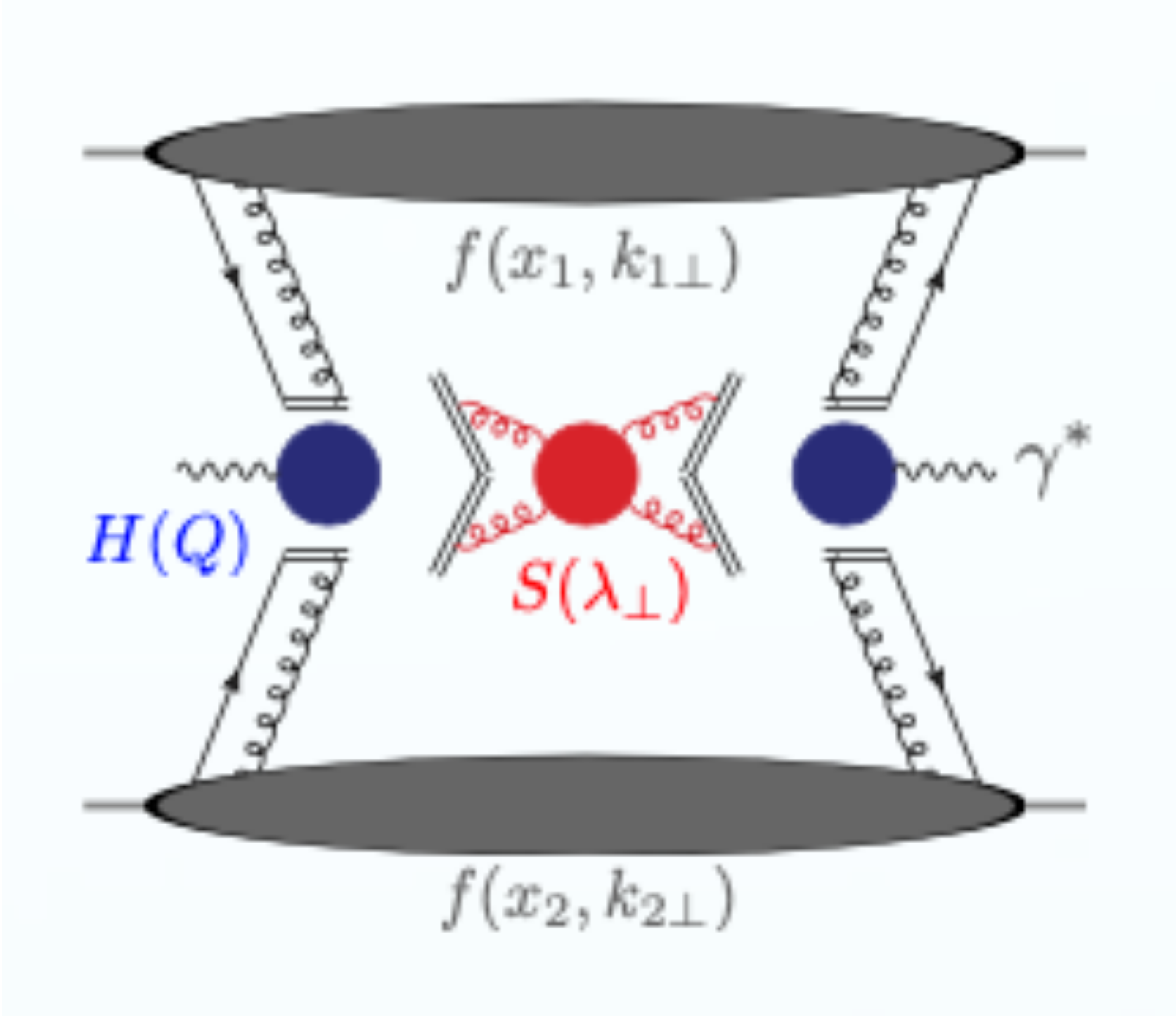
PHYSICAL REVIEW D **71,** 074006 (2005)



$$F_{UU}^{\cos\phi_h} \approx \frac{2M}{Q}\,\mathcal{C}\left[-\frac{\hat{\boldsymbol{h}}\cdot\boldsymbol{p}_T}{M}\,f_1 D_1\right].$$

$$\frac{d^5\sigma^{\ell p\to\ell hX}}{dx_B dQ^2 dz_h d^2\boldsymbol{P}_T} \simeq \sum_q \frac{2\pi\alpha^2 e_q^2}{Q^4} f_q(x_B) D_q^h(z_h)\left[1+(1-y)^2 - 4\frac{(2-y)\sqrt{1-y}\langle k_\perp^2\rangle z_h P_T}{\langle P_T^2\rangle Q}\cos\phi_h\right]\frac{1}{\pi\langle P_T^2\rangle}e^{-P_T^2/\langle P_T^2\rangle},$$

Extend TMD factorization, renormalization & evolution to sub leading power



$q_T \sim k_T \ll Q$

TMD Factorization

- ✦*Collins Soper Sterman NPB 1985*
- ✦*Ji Ma Yuan PRD PLB …2004, 2005*
- ✦*Aybat Rogers PRD 2011*
- ✦*Collins 2011 Cambridge Press*
- ✦*Echevarria, Idilbi, Scimemi JHEP 2012, …*
- ✦*SCET Becher & Neubert, 2011 EJPC*

$$\frac{\mathrm{d}\sigma^W}{\mathrm{d}Q^2\,\mathrm{d}x_F\,\mathrm{d}p_\mathrm{T}^2} = \int \frac{\mathrm{d}^2\boldsymbol{b}_\mathrm{T}}{(2\pi)^2}\, e^{i\boldsymbol{p}_\mathrm{T}\cdot\boldsymbol{b}_\mathrm{T}}\tilde{W}(x_F, b_T, Q)$$

$$\tilde{W}(x_F, b_T, Q) = \sum_j H^{\mathrm{DY}}_{j\bar{j}}(Q, \mu, a_s(\mu))\tilde{f}_{j/A}(x_A, b_\mathrm{T}; \zeta_A, \mu)\tilde{f}_{\bar{j}/B}(x_B, b_\mathrm{T}; \zeta_B, \mu)$$

TMD factorization

$W - term$ – *leading power*

- In small-p_T region, Use the CSS formalism for TMD evolution

$$\frac{d\sigma}{dQ^2\,dy\,dq_T^2} = \frac{4\pi^2\alpha^2}{9Q^2 s}\sum_{j,j_A,j_B} H^{DY}_{j\bar{j}}(Q,\mu_Q,a_s(\mu_Q))\int\frac{d^2\boldsymbol{b}_T}{(2\pi)^2}e^{i\boldsymbol{q}_T\cdot\boldsymbol{b}_T}$$
$$\times\, e^{-\boxed{g_{j/A}(x_A,b_T;b_{max})}}\int_{x_A}^{1}\frac{d\xi_A}{\xi_A} f_{j_A/A}(\xi_A;\mu_{b_*})\,\tilde{C}^{PDF}_{j/j_A}\left(\frac{x_A}{\xi_A},b_*;\mu_{b_*}^2,\mu_{b_*},a_s(\mu_{b_*})\right)$$
$$\times\, e^{-\boxed{g_{\bar{j}/B}(x_B,b_T;b_{max})}}\int_{x_B}^{1}\frac{d\xi_B}{\xi_B} f_{j_B/B}(\xi_B;\mu_{b_*})\,\tilde{C}^{PDF}_{\bar{j}/j_B}\left(\frac{x_B}{\xi_B},b_*;\mu_{b_*}^2,\mu_{b_*},a_s(\mu_{b_*})\right)$$
$$\times\,\exp\left\{-\boxed{g_K(b_T;b_{max})}\ln\frac{Q^2}{Q_0^2}+\tilde{K}(b_*;\mu_{b_*})\ln\frac{Q^2}{\mu_{b_*}^2}+\int_{\mu_{b_*}}^{\mu_Q}\frac{d\mu'}{\mu'}\left[2\gamma_j(a_s(\mu'))-\ln\frac{Q^2}{(\mu')^2}\gamma_K(a_s(\mu'))\right]\right\}$$

Non-perturbative TMDs to extract

TMD Factorization

- ✦ *Collins Soper Sterman NPB 1985*
- ✦ *Ji Ma Yuan PRD PLB …2004, 2005*
- ✦ *Aybat Rogers PRD 2011*
- ✦ *Collins 2011 Cambridge Press*
- ✦ *Echevarria, Idilbi, Scimemi JHEP 2012, …*
- ✦ *SCET Becher & Neubert, 2011 EJPC*

- Perturbative content calculated from first principles of QFT
- Non-perturbative Collinear pdfs &
- TMD to be fit to data

Renormalization and TMD Evolution- $\{\zeta, \mu\}$

- Collins Soper Eq. $$\frac{\partial \ln \tilde{f}_{j/H}(x, b_T; \mu, \zeta)}{\partial \ln \sqrt{\zeta}} = \tilde{K}(b_T, \mu)$$

$$\tilde{K}(b_T, \mu) \equiv \frac{1}{2} \frac{\partial}{\partial y_n} \ln \frac{S(b_T, y_n, -\infty)}{S(b_T, y_n, -\infty)}$$

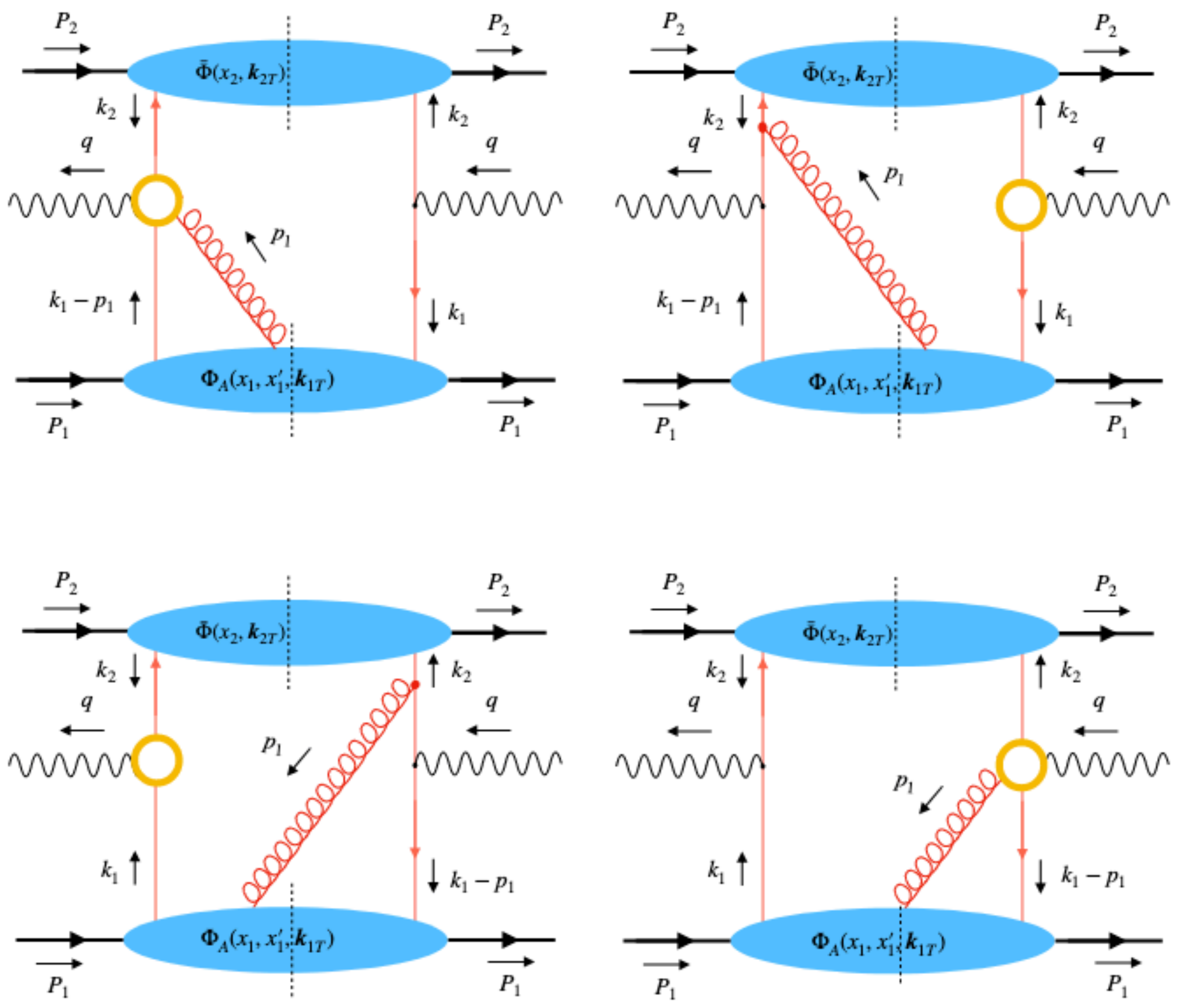
- RGE for C.S. kernel

$$\frac{d\tilde{K}(b_T; \mu)}{d \ln \mu} = -\gamma_k(\alpha_s(\mu))$$

- RGE for TMD $$\frac{d \ln \tilde{f}_{j/H}(x, b_T; \mu, \zeta)}{d \ln \mu} = -\gamma_F(\alpha_s(\mu), \zeta/\mu)$$

Solve simultaneously and get evolved renormalized TMD $\rightarrow \zeta = Q^2, \quad \mu = \mu_Q \sim Q$

Calculate "hard" "soft" and TMDs at NLO and establish RG consistency



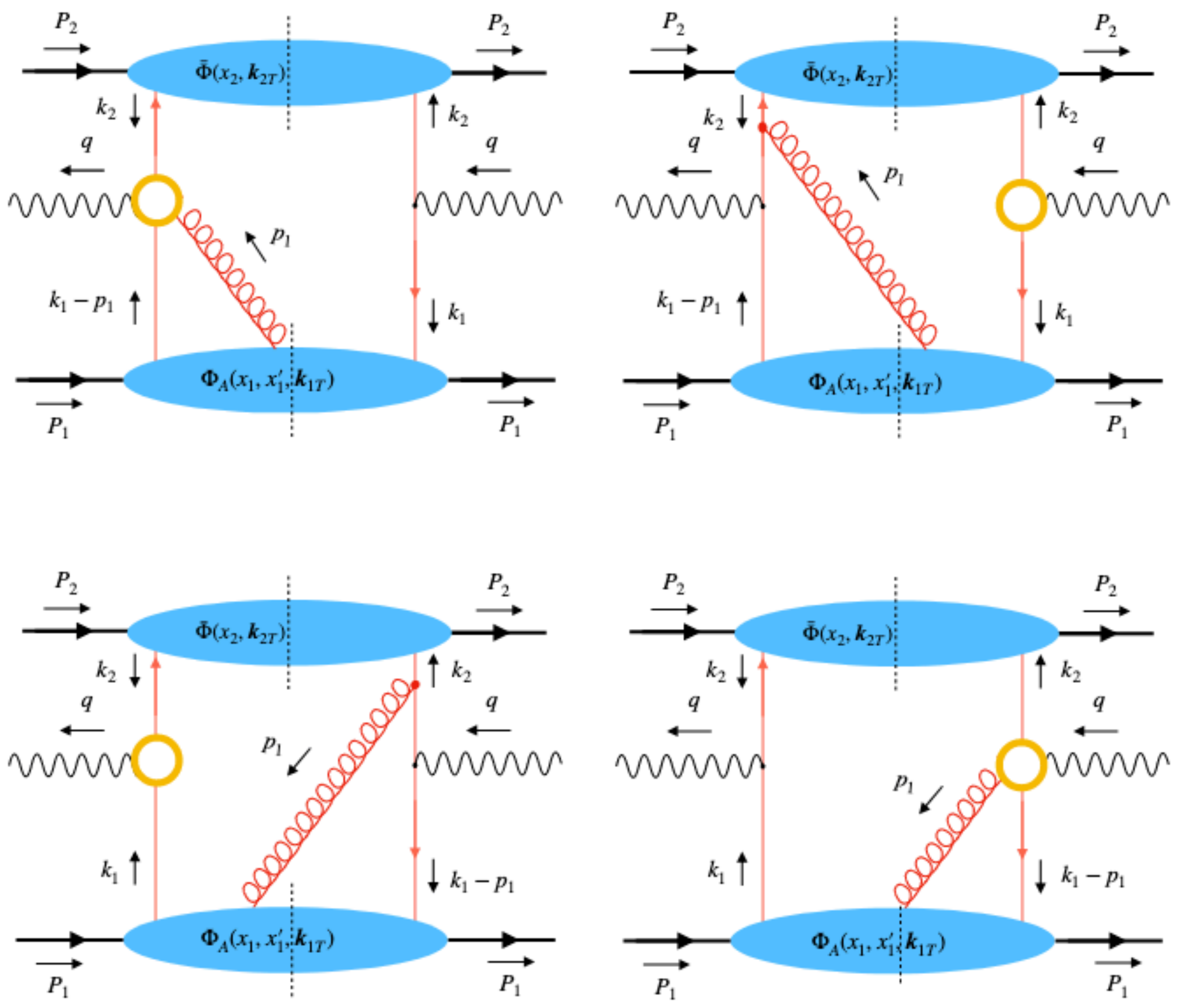
Twist 2	Twist 3	Twist 4
$\frac{1}{2}\not{n}\,,\ \frac{1}{4}\not{\bar{n}}$	$\frac{1}{2},\frac{1}{2}$	$\frac{1}{2}\not{\bar{n}}\,,\ \frac{1}{4}\not{n}$
$\frac{1}{2}\not{n}\gamma^5,\ \frac{1}{4}\gamma^5\not{\bar{n}}$	$\frac{1}{2}\gamma^5,\ \frac{1}{2}\gamma^5$	$\frac{1}{2}\not{\bar{n}}\gamma^5,\ \frac{1}{4}\gamma^5\not{n}$
$\frac{i}{2}\sigma^{k+}\gamma^5,\ \frac{i}{4}\gamma^5\sigma_{-k}$	$\frac{1}{2}\gamma^k,\ \frac{1}{2}\gamma_k$	$\frac{i}{2}\sigma^{k-}\gamma^5\,,\ \frac{i}{4}\gamma^5\sigma_{+k}$
	$\frac{1}{2}\gamma^k\gamma^5,\ \frac{1}{2}\gamma^5\gamma_k$	
	$\frac{i}{2}\sigma^{kl}\gamma^5,\ \frac{i}{4}\gamma^5\sigma_{lk}$	
	$\frac{i}{4}\sigma^{+-}\gamma^5,\ \frac{i}{4}\gamma^5\sigma_{+-}$	

$$\mathrm{Tr}\left[\gamma^\mu\,\bar{\Gamma}_1^{a1}\,\gamma^\nu\,\bar{\Gamma}_1^{a2}\right] \to \mathrm{Tr}\left[\gamma^\mu\,\left(1+F(Q;\mu)\right)\bar{\Gamma}_1^{a1}\,\gamma^\nu\,\left(1+F(Q;\mu)\right)\bar{\Gamma}_1^{a2}\right]$$

Hard corrections at NLO for quark quark correlator

$$W^{\mu\nu}_{\{2,3\,\mathrm{intrinsic}\}} = \frac{1}{N_c}\sum_{a1,a2}\sum_q e_q^2 \int d^2\boldsymbol{k}_{1T}\,d^2\boldsymbol{k}_{2T}\,\delta^{(2)}\left(\boldsymbol{q}_T-\boldsymbol{k}_{1T}-\boldsymbol{k}_{2T}-\boldsymbol{\lambda}_T\right)S(\boldsymbol{\lambda}_T;\mu,\nu)$$
$$\times\,\mathrm{Tr}\left[\gamma^\mu\,\bar{\Gamma}_1^{a1}\,\gamma^\nu\,\bar{\Gamma}_1^{a2}\right]\,\Phi^{[\Gamma^{a1}]}\left(x_1,\boldsymbol{k}_{1T},\boldsymbol{S}_1;\mu,\zeta_1/\nu^2\right)\,\bar{\Phi}^{[\Gamma^{a2}]}\left(x_2,\boldsymbol{k}_{2T},\boldsymbol{S}_2;\mu,\zeta_2/\nu^2\right)\,.$$

Calculate "hard" "soft" and TMDs at NLO and establish RG consistency



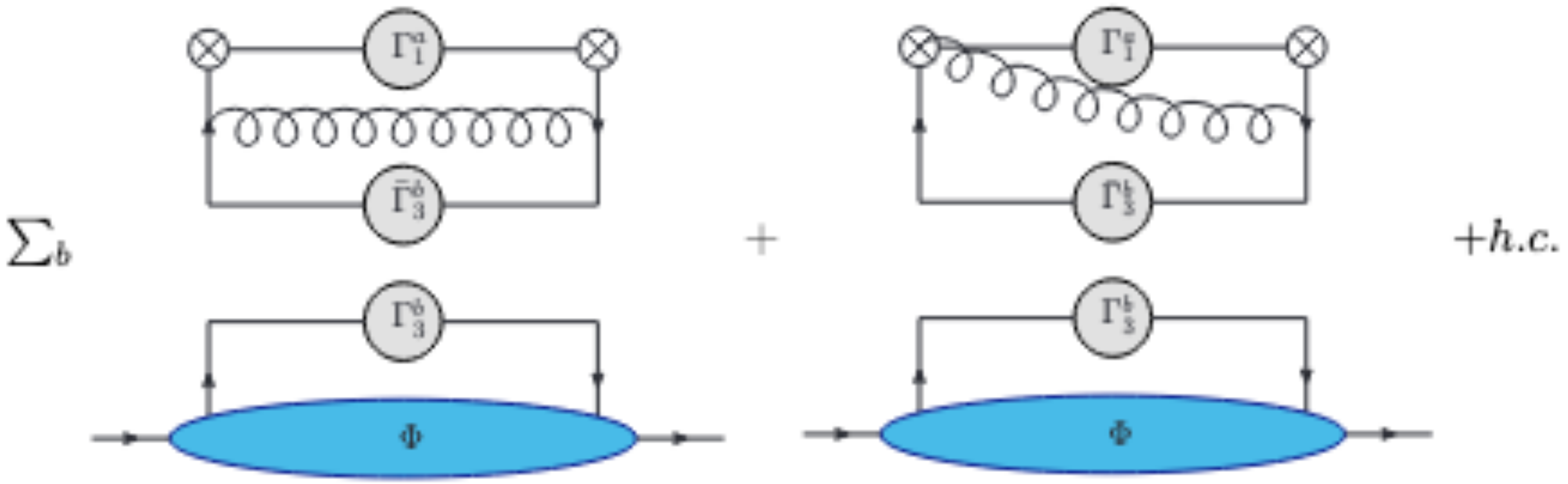
Twist 2	Twist 3	Twist 4
$\frac{1}{2}\not{n}$, $\frac{1}{4}\not{\bar{n}}$	$\frac{1}{2}, \frac{1}{2}$	$\frac{1}{2}\not{\bar{n}}$, $\frac{1}{4}\not{n}$
$\frac{1}{2}\not{n}\gamma^5$, $\frac{1}{4}\gamma^5\not{\bar{n}}$	$\frac{1}{2}\gamma^5$, $\frac{1}{2}\gamma^5$	$\frac{1}{2}\not{\bar{n}}\gamma^5$, $\frac{1}{4}\gamma^5\not{n}$
$\frac{i}{2}\sigma^{k+}\gamma^5$, $\frac{i}{4}\gamma^5\sigma_{-k}$	$\frac{1}{2}\gamma^k$, $\frac{1}{2}\gamma_k$	$\frac{i}{2}\sigma^{k-}\gamma^5$, $\frac{i}{4}\gamma^5\sigma_{+k}$
	$\frac{1}{2}\gamma^k\gamma^5$, $\frac{1}{2}\gamma^5\gamma_k$	
	$\frac{i}{2}\sigma^{kl}\gamma^5$, $\frac{i}{4}\gamma^5\sigma_{lk}$	
	$\frac{i}{4}\sigma^{+-}\gamma^5$, $\frac{i}{4}\gamma^5\sigma_{+-}$	

Hard corrections at NLO for qgq correlator

$$W_3^{\mu\nu} = \frac{1}{N_c}\sum_q e_q^2 \int d^2\boldsymbol{k}_{1T}\, d^2\boldsymbol{k}_{2T}\, d^2\boldsymbol{\lambda}_T\, \delta^{(2)}(\boldsymbol{k}_{1T}+\boldsymbol{k}_{2T}+\boldsymbol{\lambda}_T-\boldsymbol{q}_T) S'(\boldsymbol{\lambda}_T)$$

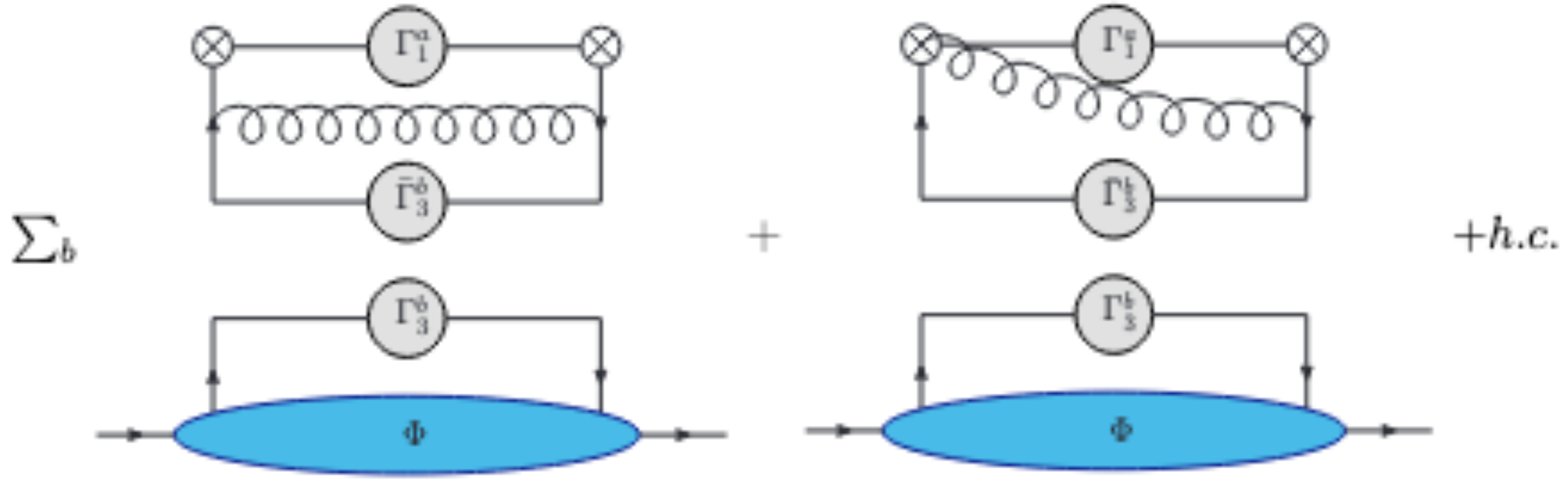
$$\times \left\{ \int dx_1' \,\mathrm{Tr}\left[\gamma_\rho \frac{\gamma}{\sqrt{Q}} \Gamma_3^\nu(Q;\mu) \Phi_F^\rho(x_1, x_1', \boldsymbol{k}_{1T}) \Gamma_3^\mu(Q;\mu) \bar{\Phi}(x_2, \boldsymbol{k}_{2T})\right] + \begin{matrix}(k_1 \leftrightarrow k_2)\\ (\mu \leftrightarrow \nu)^*\end{matrix} \right\},$$

Look under the hood rapidity and UV subtracted TMDs
Collins-Soper Equations determine rapidity and UV anomalous dimensions



To carry out analysis for intrinsic sub-leading distributions, we re-express quark fields in correlator in terms of good and bad field components $\psi(x) = \chi(x) + \phi(x)$
Impacts calculation of the gauge links and soft factors at NLP

As a consequence rapidity and UV subtracted TMDs obey Collins-Soper Equations & we can determine rapidity and UV anomalous dimensions

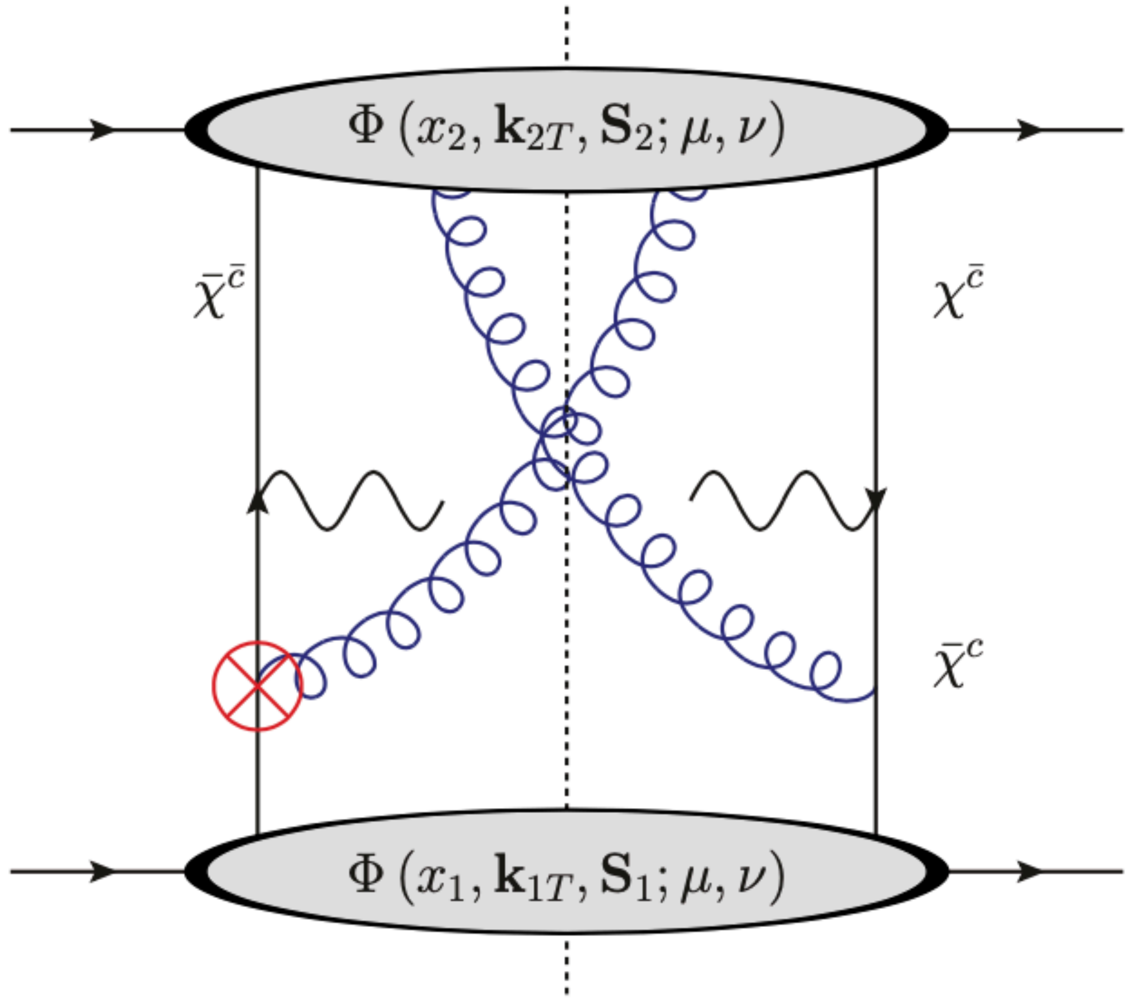


$$\frac{\partial}{\partial \ln\nu}\begin{bmatrix}\Phi^{[\not n]}\\ \Phi^{[\not n\gamma^5]}\\ \Phi^{\left[i\sigma^{k'+}\gamma^5\right]}\\ \Phi^{[1]}\\ \Phi^{[\gamma^5]}\\ \Phi^{\left[\gamma^{k'}\right]}\\ \Phi^{\left[\gamma^{k'}\gamma^5\right]}\\ \Phi^{\left[i\sigma^{k'l'}\gamma^5\right]}\\ \Phi^{\left[i\sigma^{+-}\gamma^5\right]}\end{bmatrix} = \frac{\alpha_s C_F}{2\pi}\mathbf{\Gamma}^{\nu}\begin{bmatrix}\Phi^{[\not n]}\\ \Phi^{[\not n\gamma^5]}\\ \Phi^{\left[i\sigma^{k+}\gamma^5\right]}\\ \Phi^{[1]}\\ \Phi^{[\gamma^5]}\\ \Phi^{\left[\gamma^{k}\right]}\\ \Phi^{\left[\gamma^{k}\gamma^5\right]}\\ \Phi^{\left[i\sigma^{kl}\gamma^5\right]}\\ \Phi^{\left[i\sigma^{+-}\gamma^5\right]}\end{bmatrix}$$

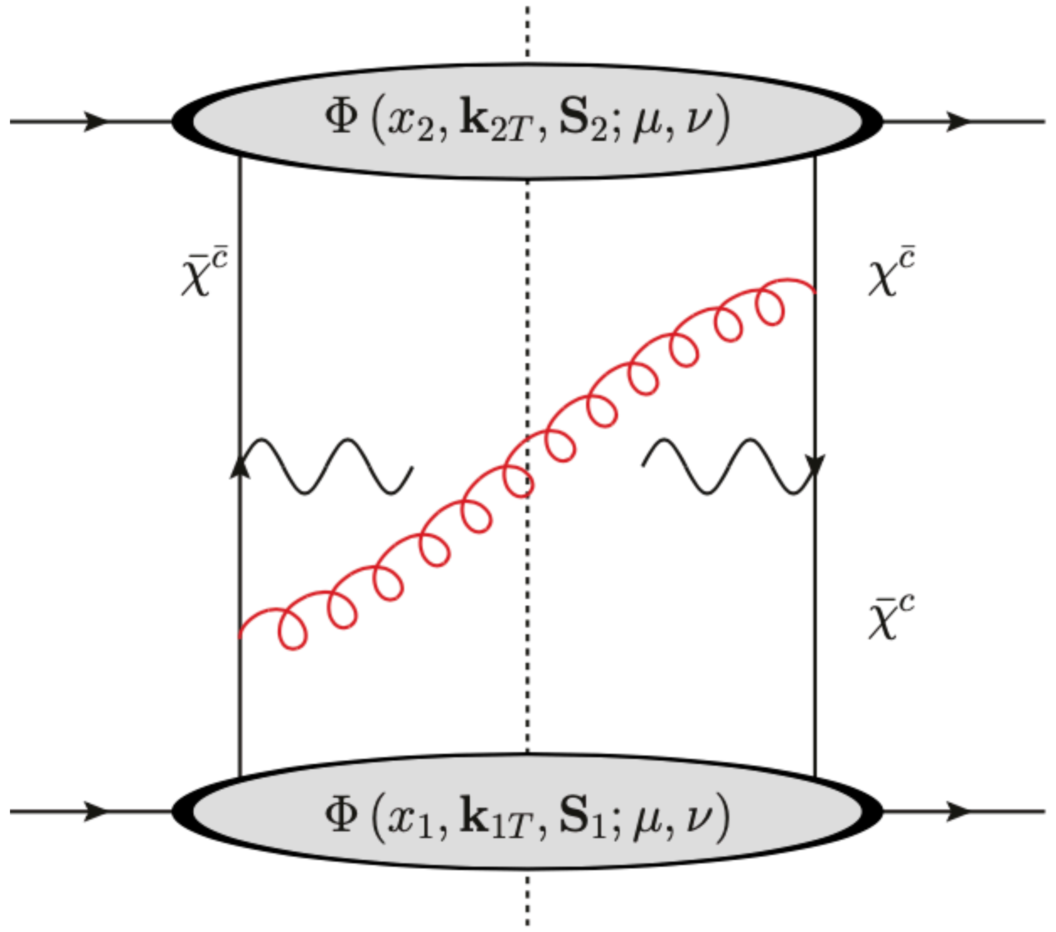
$$\frac{\partial}{\partial \ln\mu}\begin{bmatrix}\Phi^{[\not n]}\\ \Phi^{[\not n\gamma^5]}\\ \Phi^{\left[i\sigma^{k'+}\gamma^5\right]}\\ \Phi^{[1]}\\ \Phi^{[\gamma^5]}\\ \Phi^{\left[\gamma^{k'}\right]}\\ \Phi^{\left[\gamma^{k'}\gamma^5\right]}\\ \Phi^{\left[i\sigma^{k'l'}\gamma^5\right]}\\ \Phi^{\left[i\sigma^{+-}\gamma^5\right]}\end{bmatrix} = \frac{\alpha_s C_F}{2\pi}\mathbf{\Gamma}^{\mu}\begin{bmatrix}\Phi^{[\not n]}\\ \Phi^{[\not n\gamma^5]}\\ \Phi^{\left[i\sigma^{k+}\gamma^5\right]}\\ \Phi^{[1]}\\ \Phi^{[\gamma^5]}\\ \Phi^{\left[\gamma^{k}\right]}\\ \Phi^{\left[\gamma^{k}\gamma^5\right]}\\ \Phi^{\left[i\sigma^{kl}\gamma^5\right]}\\ \Phi^{\left[i\sigma^{+-}\gamma^5\right]}\end{bmatrix}$$

RG consistency established

$$\frac{d\sigma}{d\ln\nu} = 0 \quad \& \quad \frac{d\sigma}{d\ln\mu} = 0$$



$$\gamma^{\mu}_{H_{NL}} + \gamma^{\mu}_{S_{NLP}} + \gamma^{\mu}_{f_1^{kin}} + \gamma^{\mu}_{f^{\perp}} = 0$$



$$\gamma^{\nu}_{S_{NLP}} + \gamma^{\nu}_{f_1^{kin}} + \gamma^{\nu}_{f^{\perp}} = 0$$

The diagrams which give rise to the soft function at NLO+LP in Drell-Yan.

We have also evaluated the EOM beyond leading order and find RG consistency

$$xf^{\perp}(x,k_T) = x\tilde{f}^{\perp}(x,k_T) + f_1(x,k_T)$$

Summary

- We have revisited TMD factorization beyond leading power and beyond leading order in terms of intrinsic TMDs
- We are able to establish RG consistency and consistency of the EOM beyond leading order
- In doing so, we provide the basis for improved phenomenology of one the earliest
  observables used to study the intrinsic 3-D momentum structure of the nucleon—important observables for EIC study of nucleon
- Comparison of the work of Bacchetta et al. 2019 in process