Quarkyonic duality in dense matter

Yuki Fujimoto
(University of Washington)

INSTITUTE for NUCLEAR THEORY

Based on: Y. Fujimoto, T. Kojo, L. McLerran, 2306.04304 [nucl-th]

July 27, 2023 — INT Workshop: Astrophysical neutrinos and the origin of the elements

What is Quarkyonic?

quarkyonic

English [edit]

Etymology [edit]

Blend of *quark* + *baryonic*

Adjective [edit]

quarkyonic (*not comparable*)

1. (*physics*) *Describing elements of quantum chromodynamics that have characteristics both of quarks and of baryons.*

Quarkyonic duality

Collins,Perry (1974)

Naive expectation: Free deconfined quarks at high density

However, Large-Nc QCD implies: McLerran,Pisarski (2008)

Duality between quark matter and baryonic matter

$$r_{\rm Debye}^{-1} \sim \frac{1}{N_{\rm c}} \lambda_{\rm 't\ Hooft}\, \mu^2$$ … never screened at finite μ

$$\lambda_{\rm 't\ Hooft} = g^2 N_{\rm c}$$

Deconfinement sets in when $r_{\rm Debye} < r_{\rm conf} \sim \Lambda_{\rm QCD}^{-1}$

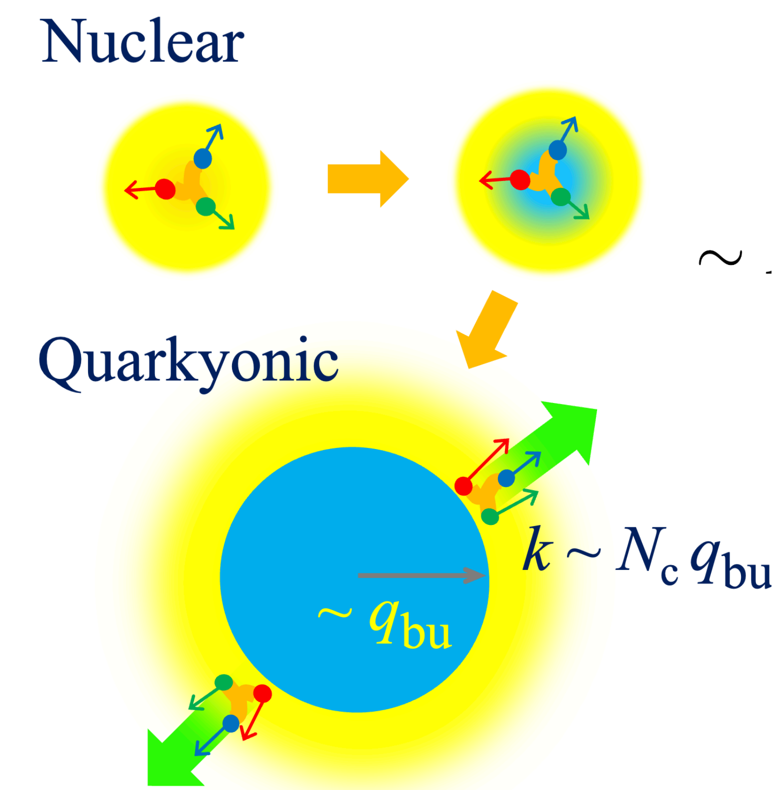
Quarkyonic regime: $\Lambda_{\rm QCD} < \mu < \sqrt{N_{\rm c}}\Lambda_{\rm QCD}$

… **High-density yet confined matter**, where we naively expect the perturbative calculation is valid [$P \sim O(N_{\rm c})$]

Fermi “shell” picture

McLerran,Pisarski (2008);
see also: Jeong,McLerran,Sen (2019)

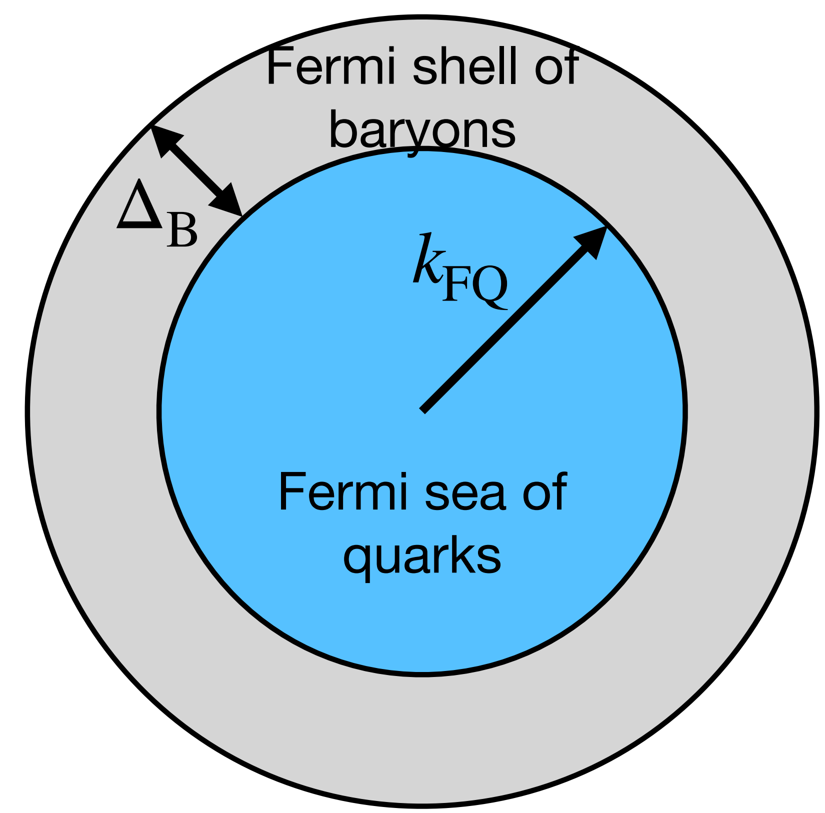
Resolution to the duality paradox is given by assuming the Fermi shell picture



Fermi sea: dominated by interaction that is less sensitive to IR → quarks

Fermi shell: interaction sensitive to IR d.o.f. → baryons, mesons, glues.

In this talk, we give an alternative explanation to this shell structure based on an explicit duality



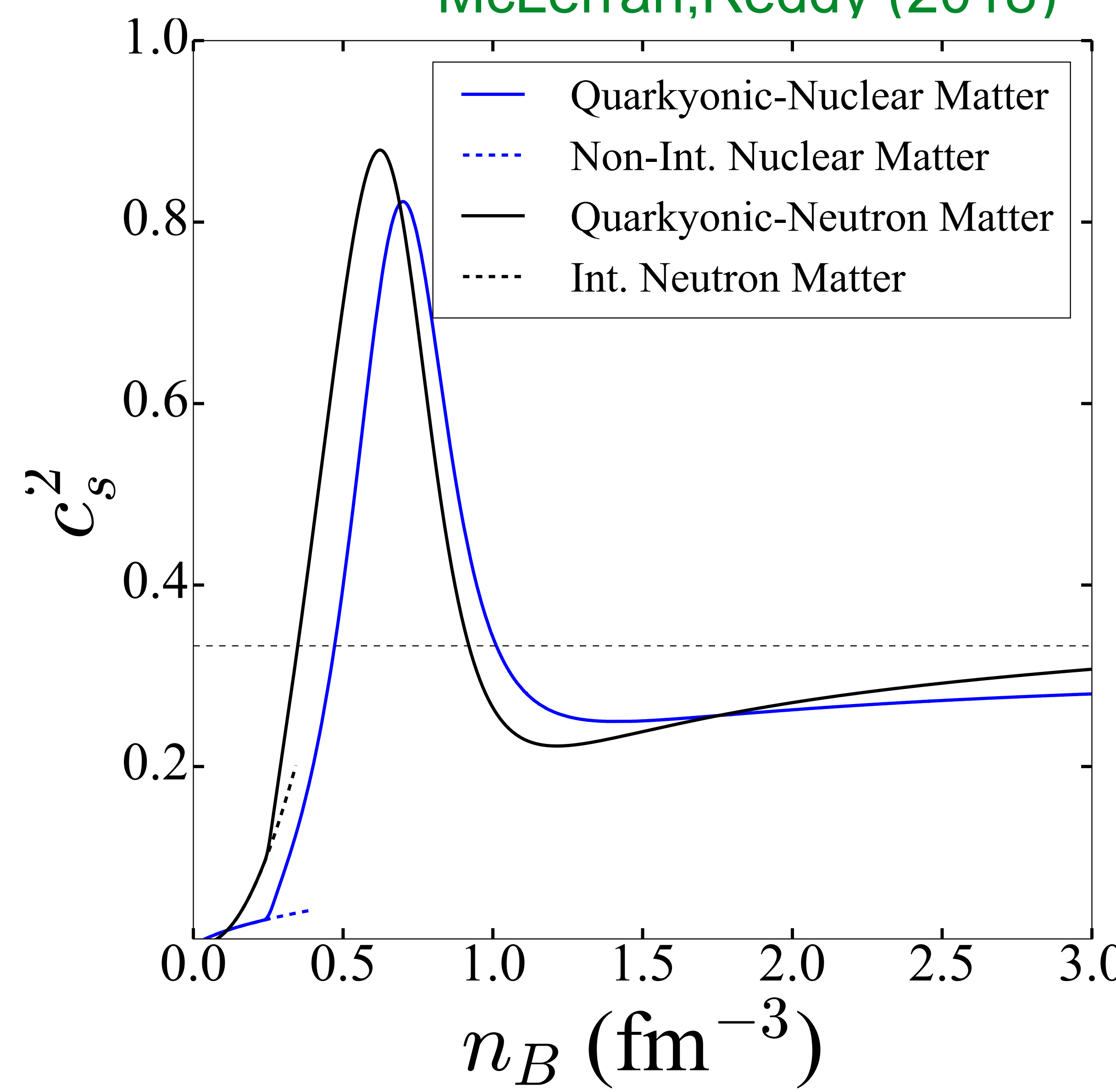
Implication to neutron-star EoS

McLerran,Pisarski (2008);
McLerran,Reddy (2018)

- Large sound speed at the onset of Quarkyonic matter
  → Transition is crossover.
  Different from the first-order phase transition.

- Rapid stiffening needed to support $2M_\odot$ neutron stars.

- Approaches to conformality at high density.



Theory with an explicit duality

Kojo (2021); Fujimoto,Kojo,McLerran (2023)

Quantum occupation of baryons and quarks in momentum space:

$$0 \le f_{\rm B}(k) \le 1, \quad 0 \le f_{\rm Q}(q) \le 1$$

- Free energy and density with an explicit duality

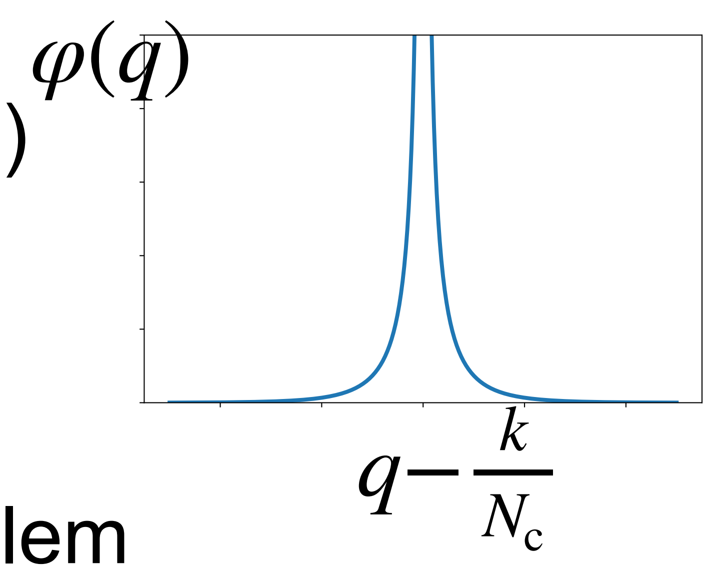
(= described in two ways, i.e., baryons and quarks)

$$\varepsilon = \varepsilon_{\rm B}[f_{\rm B}(k)] = \varepsilon_{\rm Q}[f_{\rm Q}(q)], \qquad n_{\rm B} = \int_{\boldsymbol{k}} f_{\rm B}(k) = \int_{\boldsymbol{q}} f_{\rm Q}(q)$$

- The duality relation between $f_{\rm B}$ and $f_{\rm Q}$

(= probability to find quarks inside a single baryon)

$$f_{\rm Q}(q) = \int_{\boldsymbol{k}} \varphi\left(\boldsymbol{q} - \frac{\boldsymbol{k}}{N_{\rm c}}\right) f_{\rm B}(k)$$



- Goal: Minimize ε w.r.t. $f_{\rm B}$ or $f_{\rm Q}$ → Variational problem

Theory with an explicit duality

Kojo (2021); Fujimoto,Kojo,McLerran (2023)

Free energy with an explicit duality

In this work, we use the **ideal** gas expression for baryons

$$\varepsilon = \varepsilon_{\mathrm{B}}[f_{\mathrm{B}}(k)] = \varepsilon_{\mathrm{Q}}[f_{\mathrm{Q}}(q)]$$

$$\varepsilon_{\mathrm{B}}[f_{\mathrm{B}}(k)] = \int_{\boldsymbol{k}} E_{\mathrm{B}}(k) f_{\mathrm{B}}(k), \qquad \left(E_{\mathrm{B}}(k) = \sqrt{k^2 + M_N^2}\right)$$

$$\varepsilon_{\mathrm{Q}}[f_{\mathrm{Q}}(q)] = \int_{\boldsymbol{q}} E_{\mathrm{Q}}(q) f_{\mathrm{Q}}(q)$$

We fix the baryon expression because we know this gives a suitable low-density description. Quark dispersion is fixed via the duality relation.

We name it as **IdylliQ** (Ideal dual Quarkyonic) **matter**

Explicitly solvable model

Fujimoto,Kojo,McLerran (2023)

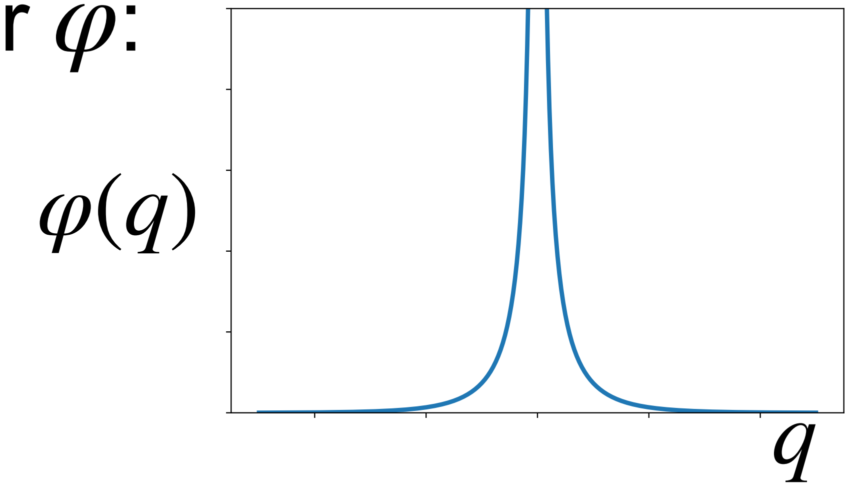
The duality relation between $f_{\rm B}$ and $f_{\rm Q}$ (quark model):

$$f_{\rm Q}(q) = \int \frac{d^d\boldsymbol{k}}{(2\pi)^d}\varphi\left(\boldsymbol{q} - \frac{\boldsymbol{k}}{N_{\rm c}}\right) f_{\rm B}(k)$$

In this work, we assume the specific form for φ:

$$\varphi(q) = \frac{2\pi^2}{\Lambda^2}\frac{e^{-q/\Lambda}}{q}$$

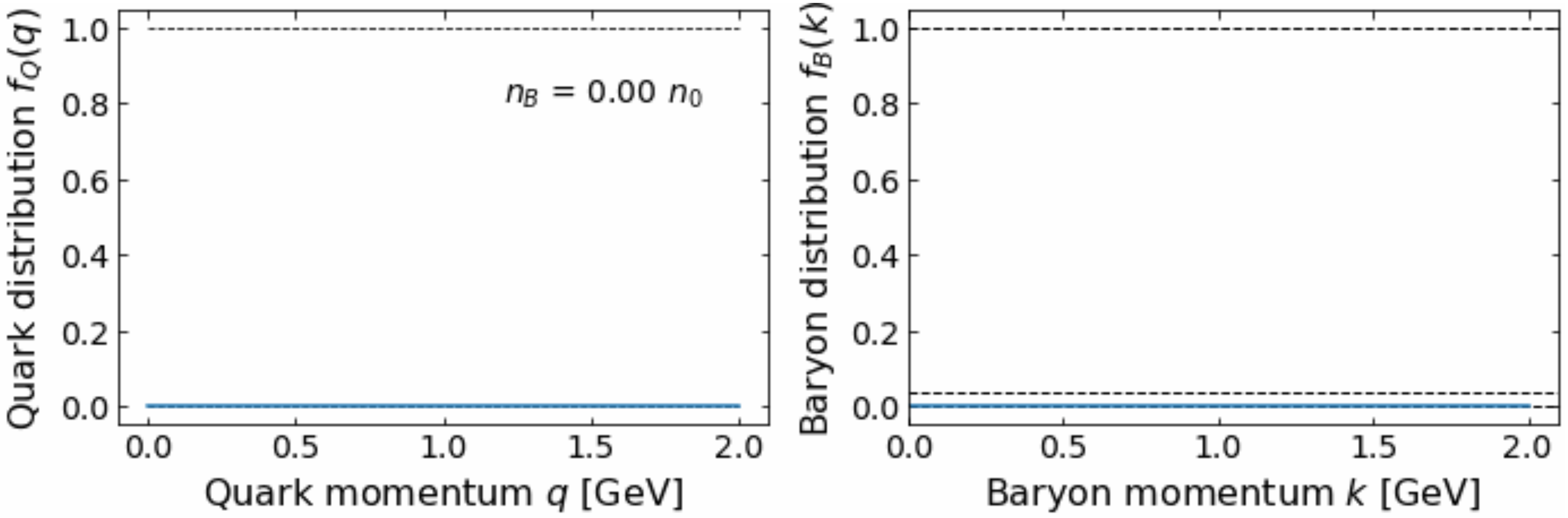
Λ: confining scale



This specific choice entails the one-to-one correspondence:

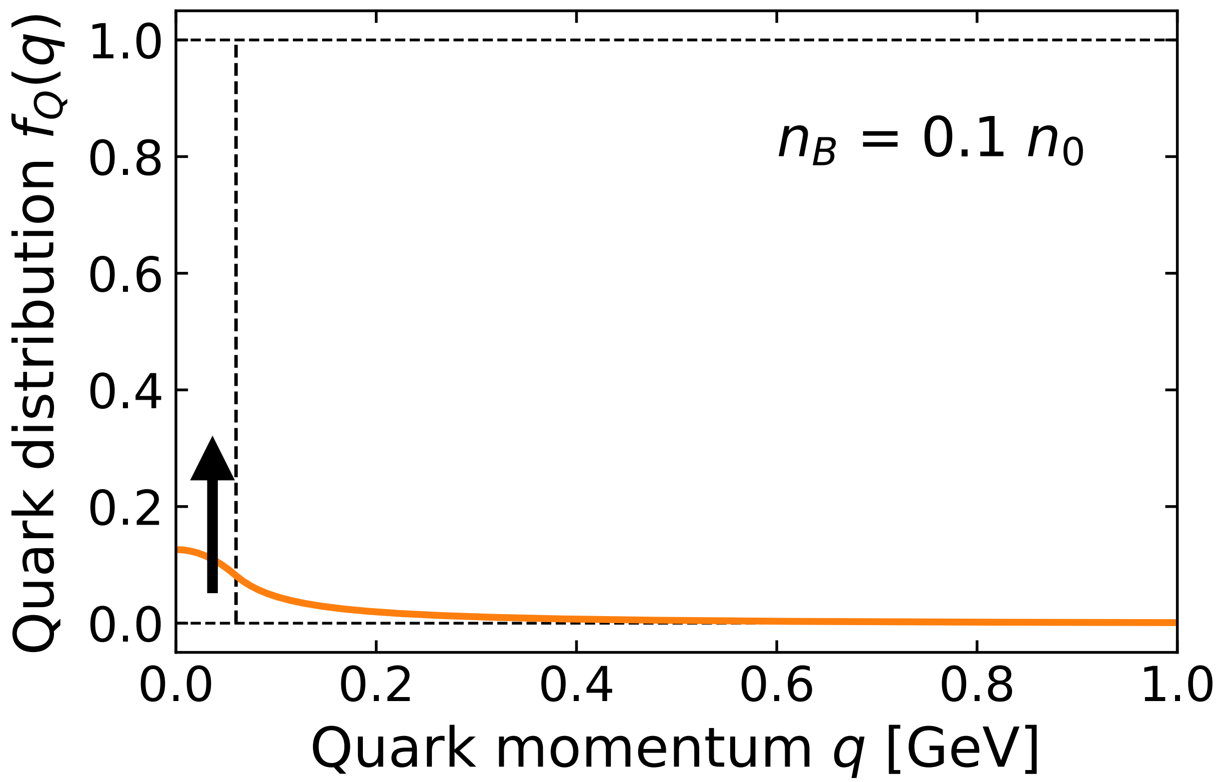
$$f_{\rm B}(N_{\rm c}q) = \frac{\Lambda^2}{N_{\rm c}^3}\left(-\nabla_q^2 + \frac{1}{\Lambda^2}\right) f_{\rm Q}(q)$$

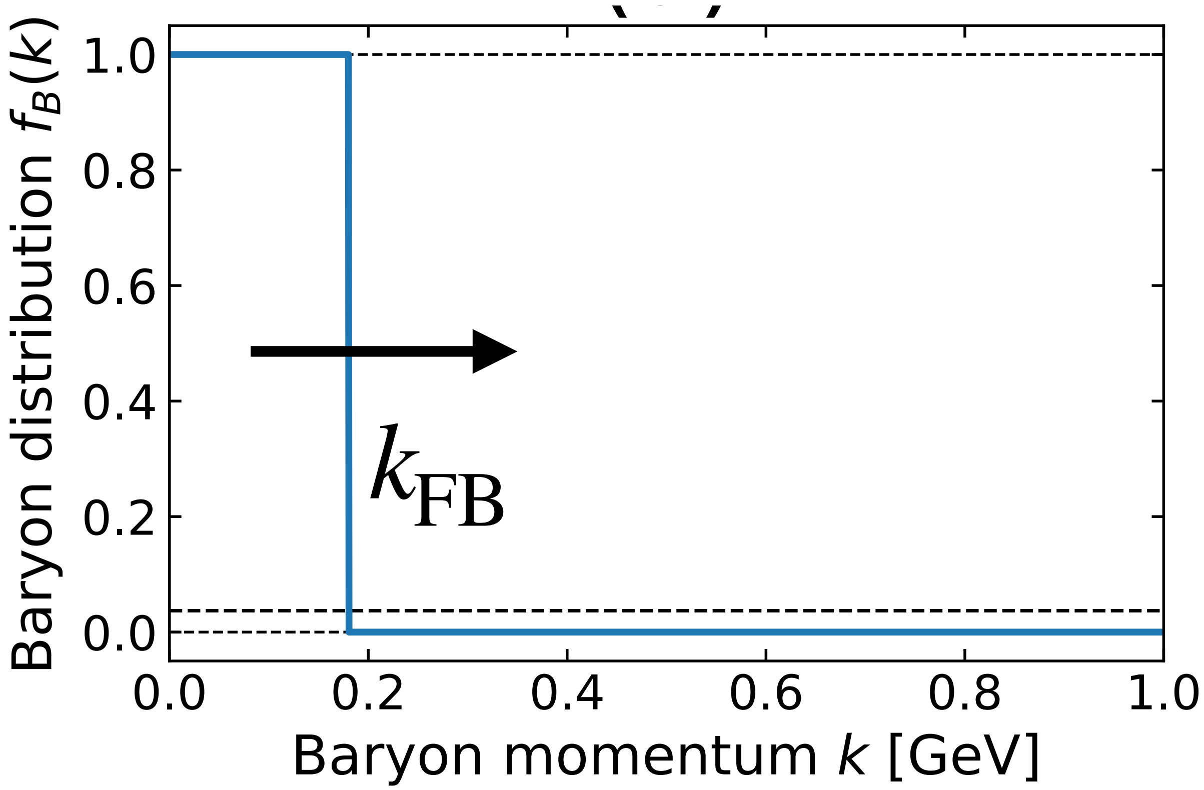
Overview on the analytic solution



Solution at low density

Kojo (2021), Fujimoto,Kojo,McLerran (2023)



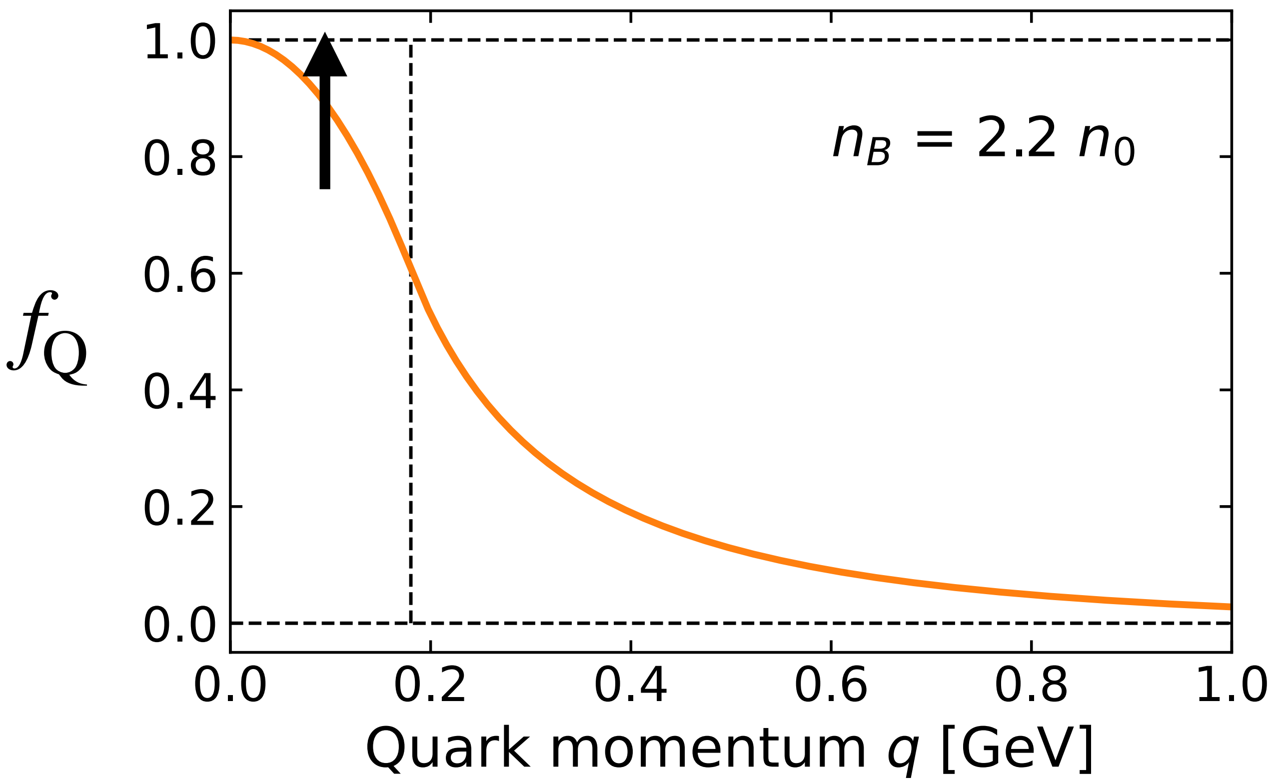


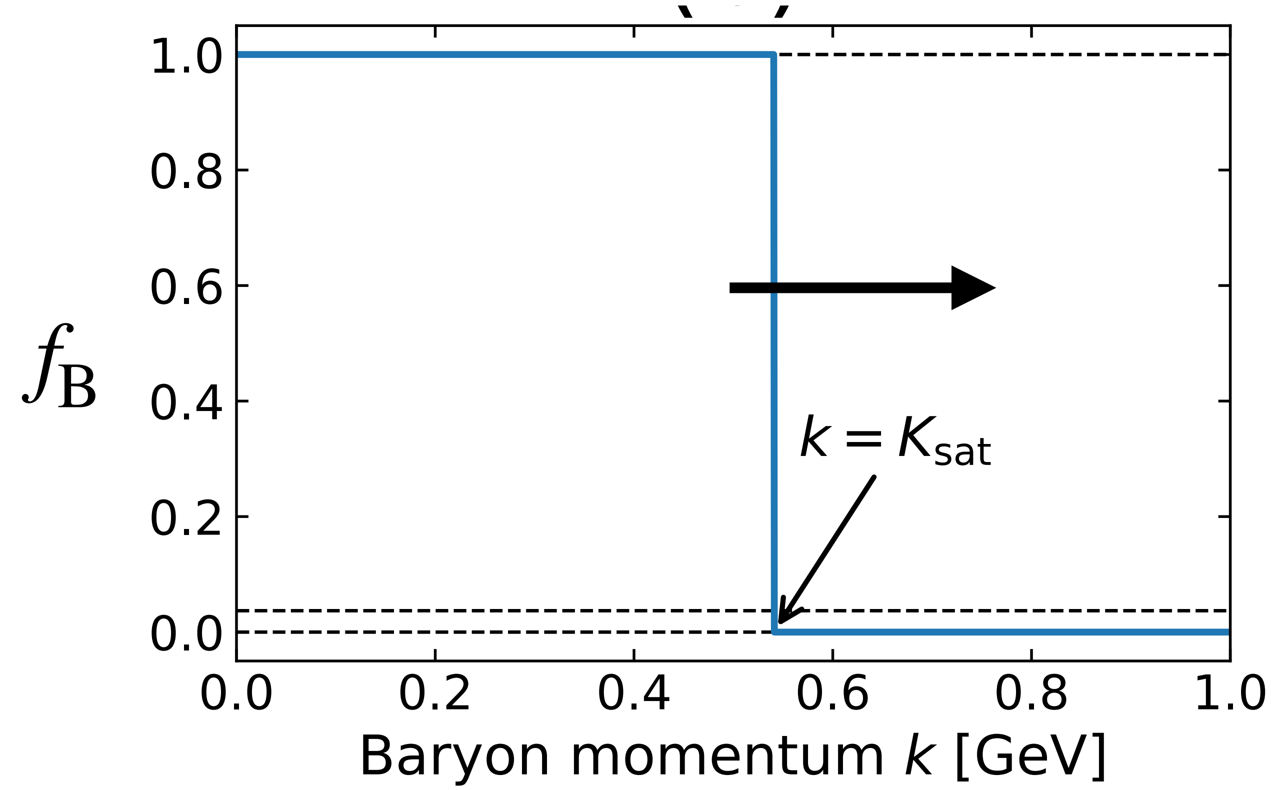
Fermi gas of baryons is formed.

Baryonic Fermi momentum $k_{\rm FB}$ grows until $f_{\rm Q}$ reaches 1

Saturation of the quark distribution

Kojo (2021), Fujimoto,Kojo,McLerran (2023)

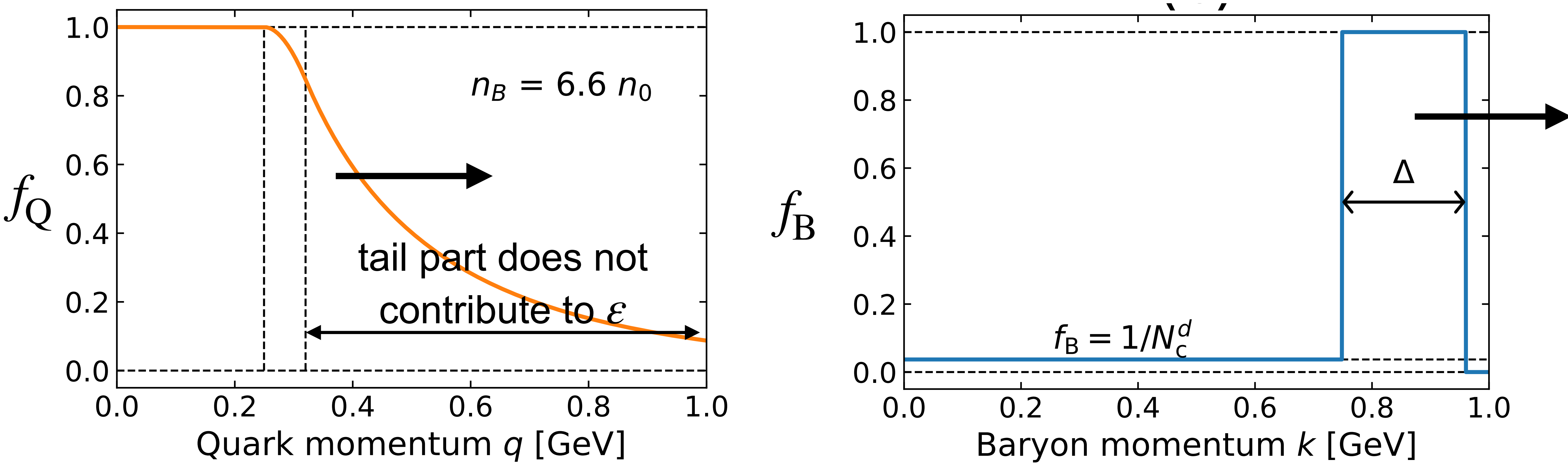




At this point, f_Q "saturates" and Pauli blocking constraint becomes essential.

Solution at high density

Fujimoto,Kojo,McLerran (2023)



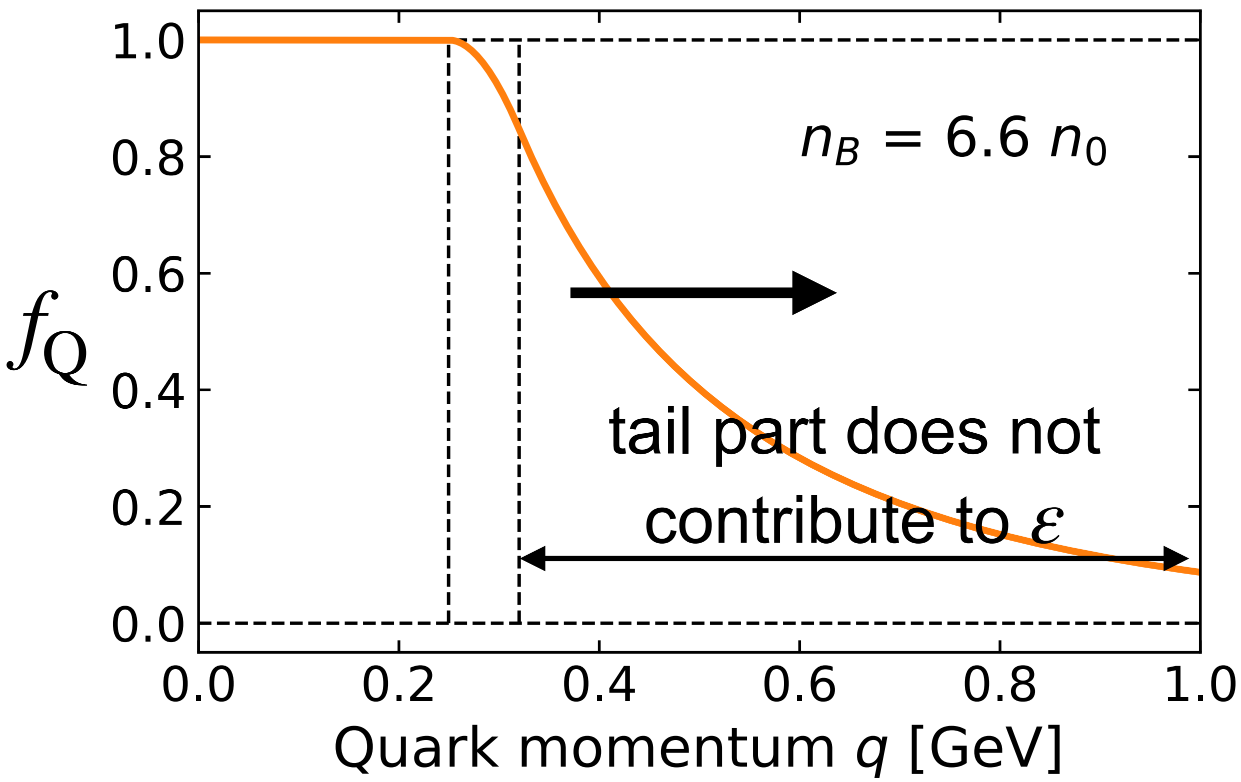
Saturation in low-momentum f_Q → Depletion in $f_B \sim 1/N_c^3$ (in 3d)

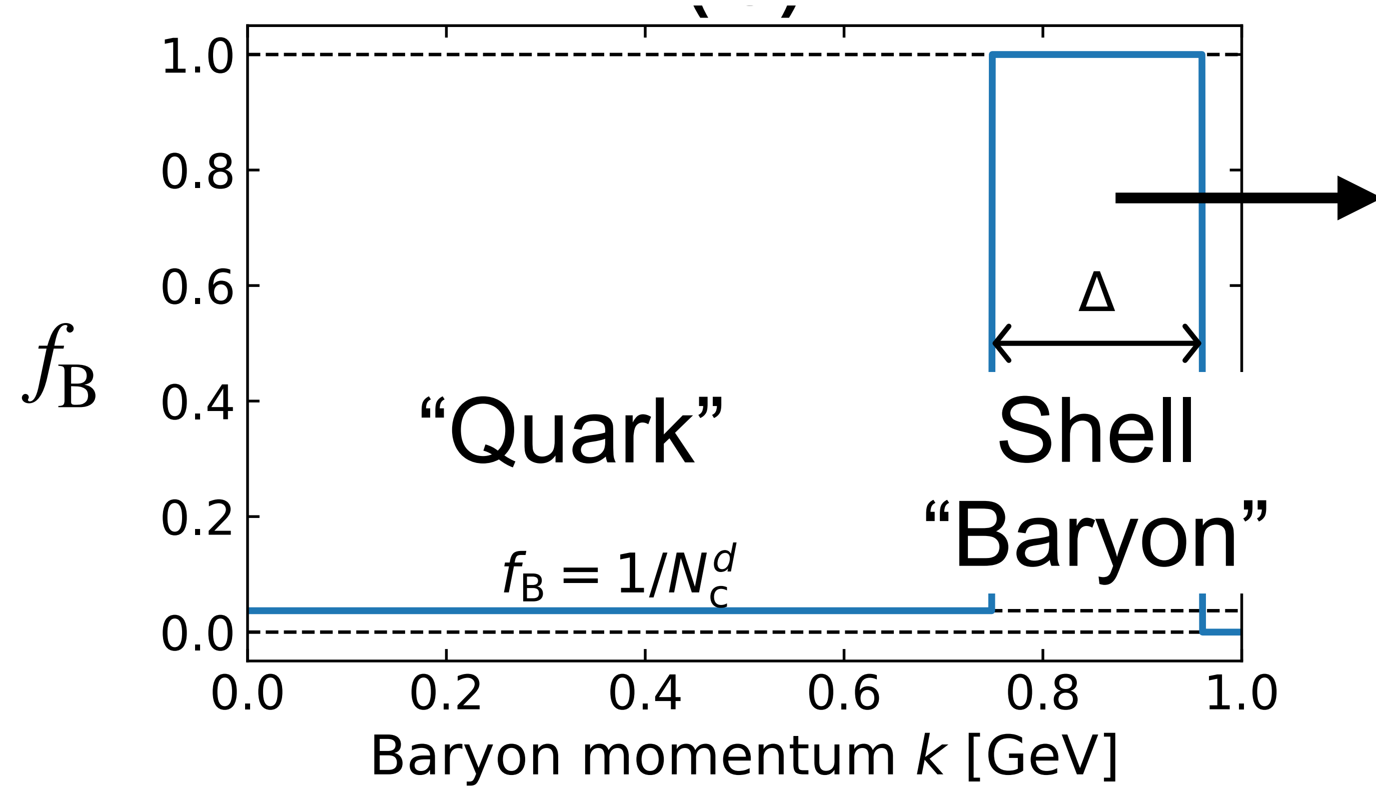
Tail in f_Q w/ a width $\sim \Lambda$

→ Shell formation in high-momentum f_B w/ $\Delta_B \sim \dfrac{\Lambda}{N_c^2}$

Solution at high density

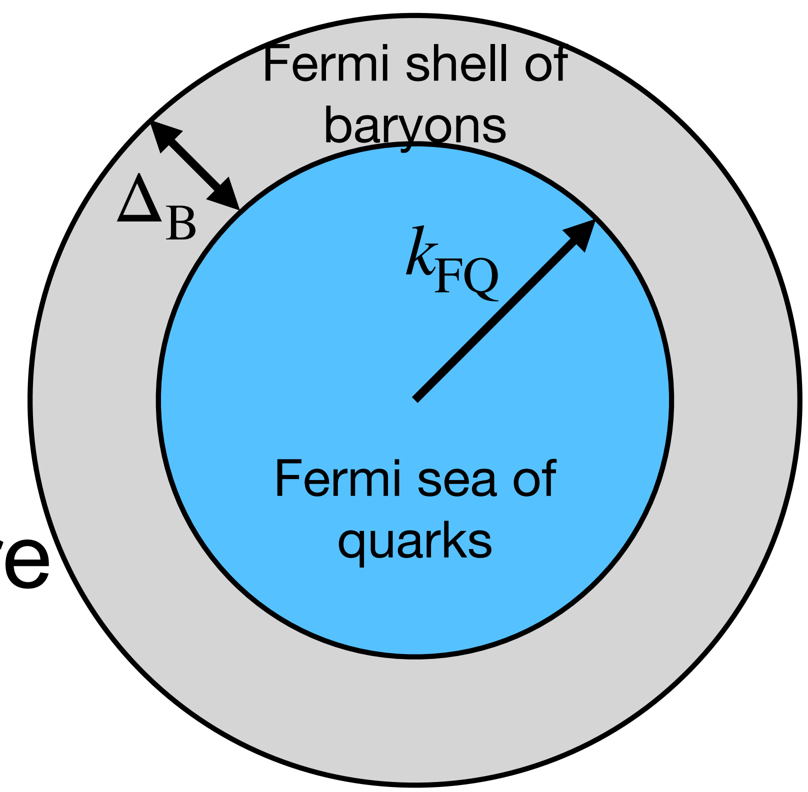
Fujimoto,Kojo,McLerran (2023)





Fermi shell structure arises in f_B

This picture is equivalent to McLerran-Reddy model of the EoS based on the McLerran-Pisarski shell picture apart from the behavior of Δ_B



Underoccupied f_{B} and occupied f_{Q}

Baryon number in the bulk “quark” region in the quark language:

$$n_{\mathrm{B}} = \int_0^{k_{\mathrm{FQ}}} \frac{d^3 q}{(2\pi)^3} f_{\mathrm{Q}}(q) \sim k_{\mathrm{FQ}}^3 f_{\mathrm{Q}}$$

In the baryon language:

$$n_{\mathrm{B}} = \int_0^{k_{\mathrm{FB}}} \frac{d^3 k}{(2\pi)^3} f_{\mathrm{B}}(k) \sim k_{\mathrm{FB}}^3 f_{\mathrm{B}} \sim N_{\mathrm{c}}^3 k_{\mathrm{FQ}}^3 f_{\mathrm{B}}$$

where the Fermi momenta are related as $k_{\mathrm{FB}} \sim N_{\mathrm{c}} k_{\mathrm{FQ}}$.

Because $f_{\mathrm{Q}} \leq 1, \quad f_{\mathrm{B}} \sim 1/N_{\mathrm{c}}^3$ … composite baryon states are underoccupied

Rapid stiffening in the EoS

A partial occupation of available baryon phase space leads to the **large speed of sound**.

$$v_s^2 = \frac{n_{\rm B}}{\mu_{\rm B} dn_{\rm B}/d\mu_{\rm B}} \to \frac{\delta\mu_{\rm B}}{\mu_{\rm B}} \sim v_s^2 \frac{\delta n_{\rm B}}{n_{\rm B}}$$

If baryon is underoccupied, the density is not changing much, but the Fermi energy changes a lot:

Summary

- We formulate the quantum-mechanical theory of Idylliq matter — Quarkyonic matter with an explicit duality and ideal baryonic dispersion relation

- Previously proposed Fermi “shell” structure of Quarkyonic matter naturally arises in the baryon distribution

- Rapid rise in the sound speed is still there — the observational signature of the Quarkyonic matter