Trace anomaly as a measure of conformality in dense matter EoS

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Dense matter EoS from QCD

χEFT: Tews, Krüger, Hebeler, Schwenk (2013); Drischler, Furnstahl, Melendez, Philips (2020) & many other works

pQCD: Freedman, McLerran (1978); Baluni (1979); Kurkela, Romatschke, Vuorinen, Gorda, Säppi (2009-) & many other works

Pressure $P$ [MeV/fm$^3$]

Density $n_B$

Neutron stars

$\sim n_0$ $\sim 10 n_0$ $\sim 50 n_0$

Nuclear density ($n_0 = 0.16$ fm$^{-3}$)
We introduce a new quantity, **trace anomaly**:

\[
\Delta = \frac{1}{3} - \frac{\varepsilon}{P}
\]

This is a measure of **conformality** (conformal EoS: \( P = \frac{1}{3} \varepsilon \))

The behavior of \( \Delta \) may signify the conformal matter in NSs.

From theory consideration, we conjecture \( \Delta \geq 0 \).

This conjecture can be tested by the future NS observations.
Rapid stiffening in EoS

\[ P = \text{Pressure} \quad [\text{MeV/fm}^3] \]

\[ n_B = \text{Density} \]

Relatively soft $\chi$EFT EoS + small tidal deformability from GW170817

Rapid stiffening at $n_B \gtrsim 1.5 \, n_0$

Drischler, Han, Lattimer, Prakash, Reddy, Zhao (2020)
Rapid stiffening in EoS

NS data favors rapid increase in sound velocity, accompanied by a peak structure.

\[ v_s^2 \equiv \frac{dP}{d\varepsilon} \]

\[ v_s^2 = 1/3 \] is referred to as conformal limit.

Because \( v_s^2 \uparrow 1/3 \) when \( \varepsilon \to \infty \).

Conformal limit is violated at intermediate density.

Fujimoto, Fukushima, Murase (2019)

Bedaque, Steiner (2015); Tews, Carlson, Gandolfi, Reddy (2018);
Altiparmak, Ecker, Rezzolla (2022); Gorda, Komoltsev, Kurkela (2022) & many others.
Trace anomaly equation

Related to scale/conformal nature of matter:

\[ j_D^\nu = x_\mu T^{\mu\nu} \rightarrow \partial_\nu j_D^\nu = T_\mu^\mu \begin{cases} = 0 & \text{Classical YM} \\ \neq 0 & \text{in QFT (RG effect)} \end{cases} \]

Expectation value:

\[ \langle T_\mu^\mu \rangle = \langle T_\mu^\mu \rangle_{\mu_B} + \langle T_\mu^\mu \rangle_0 \]

Finite-\(\mu_B\) part of the trace anomaly (interaction measure):

\[ \langle T_\mu^\mu \rangle_{\mu_B} = \varepsilon - 3P \]

… related to EoS of neutron star matter
Behavior of trace anomaly

\[ \Delta \equiv \frac{\langle T^\mu_\mu \rangle_{\mu B}}{3 \varepsilon} = \frac{1}{3} - \frac{P}{\varepsilon} \]

Inference of EoSs from NS observations shows:

\[ \Delta \sim 0, \text{ i.e., } P \sim \varepsilon/3 \]

already at \( \sim 5\varepsilon_0 \)

rapid stiffening to conformality

Strongly-coupled conformal matter?
Trace anomaly and sound velocity

\( \Delta = \frac{\langle T^\mu_\mu \rangle_{\mu B}}{3 \epsilon} = \frac{1}{3} - \frac{P}{\epsilon} \)


Sound velocity:
\( v_s^2 = \frac{dP}{d\epsilon} = -\epsilon \frac{d\Delta}{d\epsilon} + \left( \frac{1}{3} - \Delta \right) \)

Conformal limits:
\( \Delta(\epsilon) \searrow 0 \)
\( v_s^2(\epsilon) \nearrow 1/3 \)

when \( \epsilon \to \infty \)

Bedaque, Steiner (2015)

It is very likely that \( v_s^2 > 1/3 \). Simultaneously, \( \Delta \gtrsim 0 \).
Meaning of the sound velocity peak

\[ \Delta \rightarrow 0 \text{ naturally spikes } v_s^2 \]

\[ \Delta = \frac{1}{3} - \frac{P}{\epsilon} \]

\[ v_s^2 = -\varepsilon \frac{d\Delta}{d\varepsilon} + \left( \frac{1}{3} - \Delta \right) \]

\[ \Delta \geq 0 \]

Monotonic \( \Delta \) gives rise to non-monotonic \( v_s^2 \) and violation of \( v_s^2 \leq 1/3 \)

Rapid stiffening to the conformal EoS creates a peak
Positive trace anomaly conjecture

Fujimoto, Fukushima, McLerran, Praszalowicz, PRL129 (2022)

Combining QCD ab-initio calculations:

![Graph showing trace anomaly and energy density relationship]

Trace anomaly $\Delta = 1/3 - P/\epsilon$ vs. energy density $\epsilon/\epsilon_0$

NS data not well-constrained
Positive trace anomaly conjecture

Fujimoto, Fukushima, McLerran, Praszalowicz, PRL129 (2022)

Combining QCD ab-initio calculations:

Our conjecture: $\Delta \gtrsim 0$ at any $\varepsilon$ for NS matter
Positive trace anomaly conjecture

Combining QCD ab-initio calculations:

Our conjecture: $\Delta \geq 0$ at any $\varepsilon$ for NS matter

Caveat: QCD at finite-$\mu_f$ or SU(2) QCD gives $\Delta < 0$

e.g., Cotter,Giudice,Hands,Skullerud (2012); Iida,Itou (2022)
Son,Stephanov (2001); Brandt,Endrodi+ (2018-)...
Justification of the $\Delta \geq 0$ bound

Trace anomaly is related to the counting of the effective degrees of freedom in pressure, $\nu \equiv P/\mu_B^4$:

$$\Delta \propto \frac{\langle T^\mu_\mu \rangle_{\mu_B}}{\mu_B^4} = \mu_B \frac{d\nu}{d\mu_B} \geq 0$$

If $\nu$ keeps increasing, we get $\Delta \geq 0$

Open question: the effect of color superconductivity?

$\rightarrow$ Cooper pairing reduces the d.o.f. but this is only a phenomenon around the Fermi surface
Justification of the $\Delta \geq 0$ bound

From the QCD energy-momentum tensor in the chiral limit:

$$\Delta \propto \langle T^\mu_\mu \rangle = \frac{\beta}{2g} \langle B^2 - E^2 \rangle_{\mu B}$$

Interaction between quarks are mediated dominantly by chromo-electric fields $\rightarrow$ gluon condensate $< 0$

Given $\beta < 0$ (asymptotic freedom), the total trace anomaly is positive.
Testing the $\Delta \geq 0$ bound

Maximally stiff EoS with $\Delta \geq 0$ bound

Density up to which I use ChEFT EoS

$\nu_s^2 = 1$ for $n_B > n_t$

$\Delta \geq 0$ condition pushes down the maximum mass bound of NSs

ChEFT data: Drischler, Han, Lattimer, Prakash, Reddy, Zhao (2020)
Testing the $\Delta \geq 0$ bound

Combining the radius observation:

- $M_{\text{max}}(R)$ with the $\Delta \geq 0$ bound
- Causality bound

See also: Drischler, Han, Lattimer, Prakash, Reddy, Zhao (2020)
Rhoades Jr., Ruffini (1974)
Koranda, Stergioulas, Friedman (1995)
Riley et al. (2021); Miller et al. (2021)
Summary

- Trace anomaly $\Delta$ is a measure of conformality. It is a complement to the speed of sound $v_s^2$.

- NS data suggest $\Delta$ rapidly approach to the conforal limit and $\Delta \to 0$ naturally gives rise to the sound velocity peak.

- Strongly-interacting conformal matter may be inside NSs.

- The trace anomaly may be positive (not proven). It can be tested by, e.g., the bound on the maximum mass of NSs.

- Quarkyonic matter can be responsible for rapid stiffening to conformality.

(see, e.g., Fujimoto, Kojo, McLerran [2306.04304])