Trace anomaly as a measure of conformality in dense matter EoS

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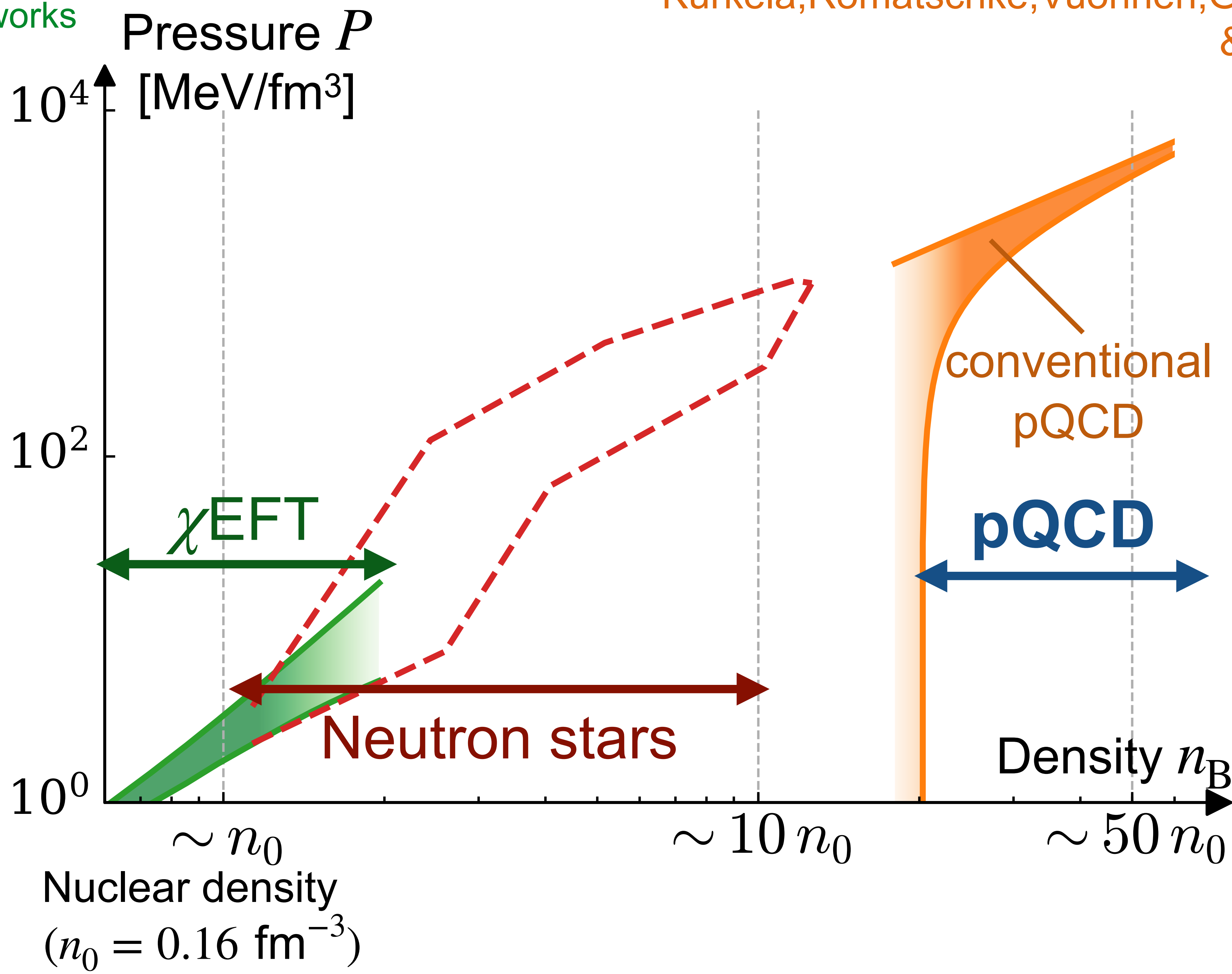
Dense matter EoS from QCD

χEFT: Tews,Krüger,Hebeler,Schwenk(2013);
Drischler,Furnstahl,Melendez,Philips(2020)
& many other works

pQCD: Freedman,McLerran(1978); Baluni(1979);
Kurkela,Romatschke,Vuorinen,Gorda,Säppi (2009-)
& many other works



Synopsis of this talk

We introduce a new quantity, **trace anomaly**:

$$\Delta = \frac{1}{3} - \frac{P}{\varepsilon}$$

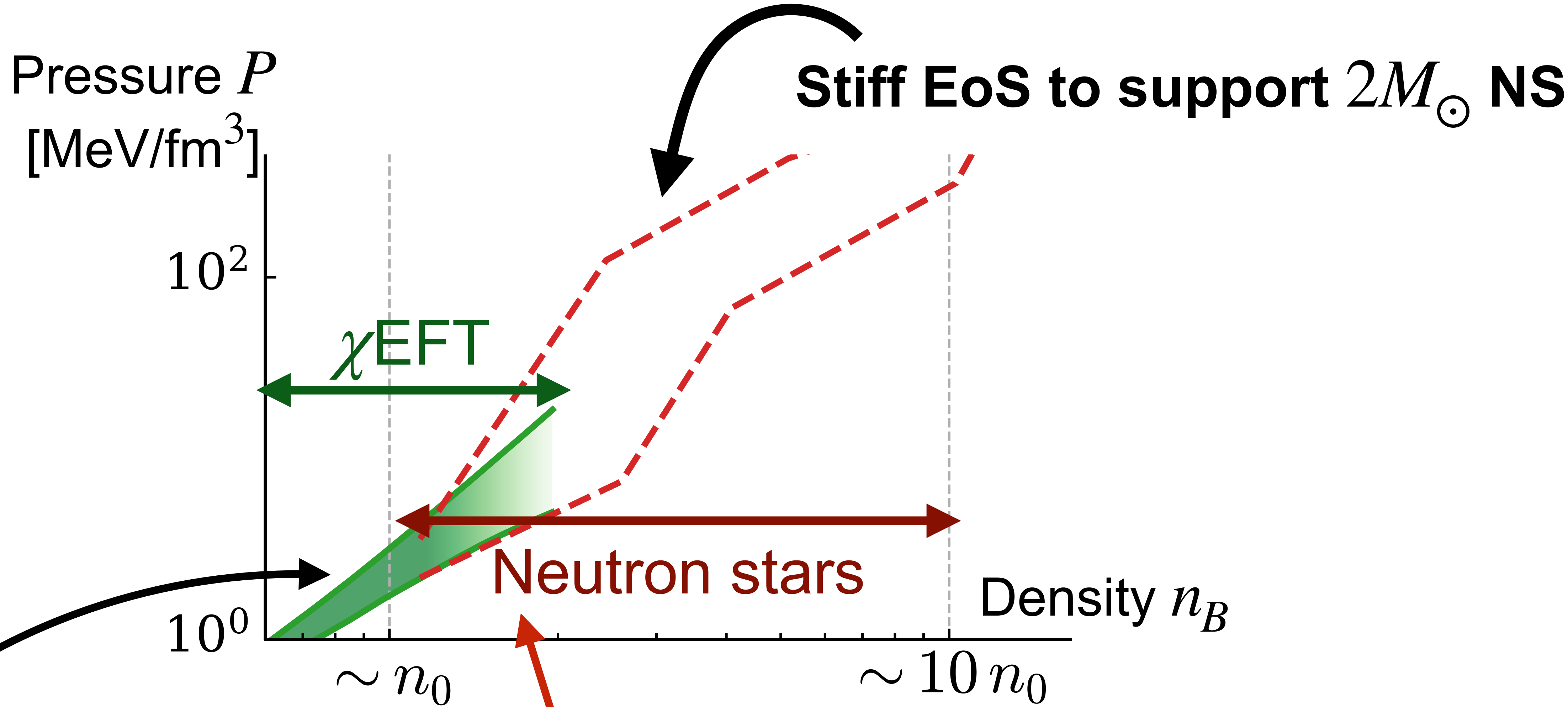
This is a measure of **conformality** (conformal EoS: $P = \frac{1}{3}\varepsilon$)

The behavior of Δ may signify the conformal matter in NSs.

From theory consideration, we conjecture $\Delta \geq 0$.

This conjecture can be tested by the future NS observations.

Rapid stiffening in EoS



Relatively soft χEFT EoS + small tidal deformability from GW170817

Rapid stiffening at $n_B \gtrsim 1.5\, n_0$

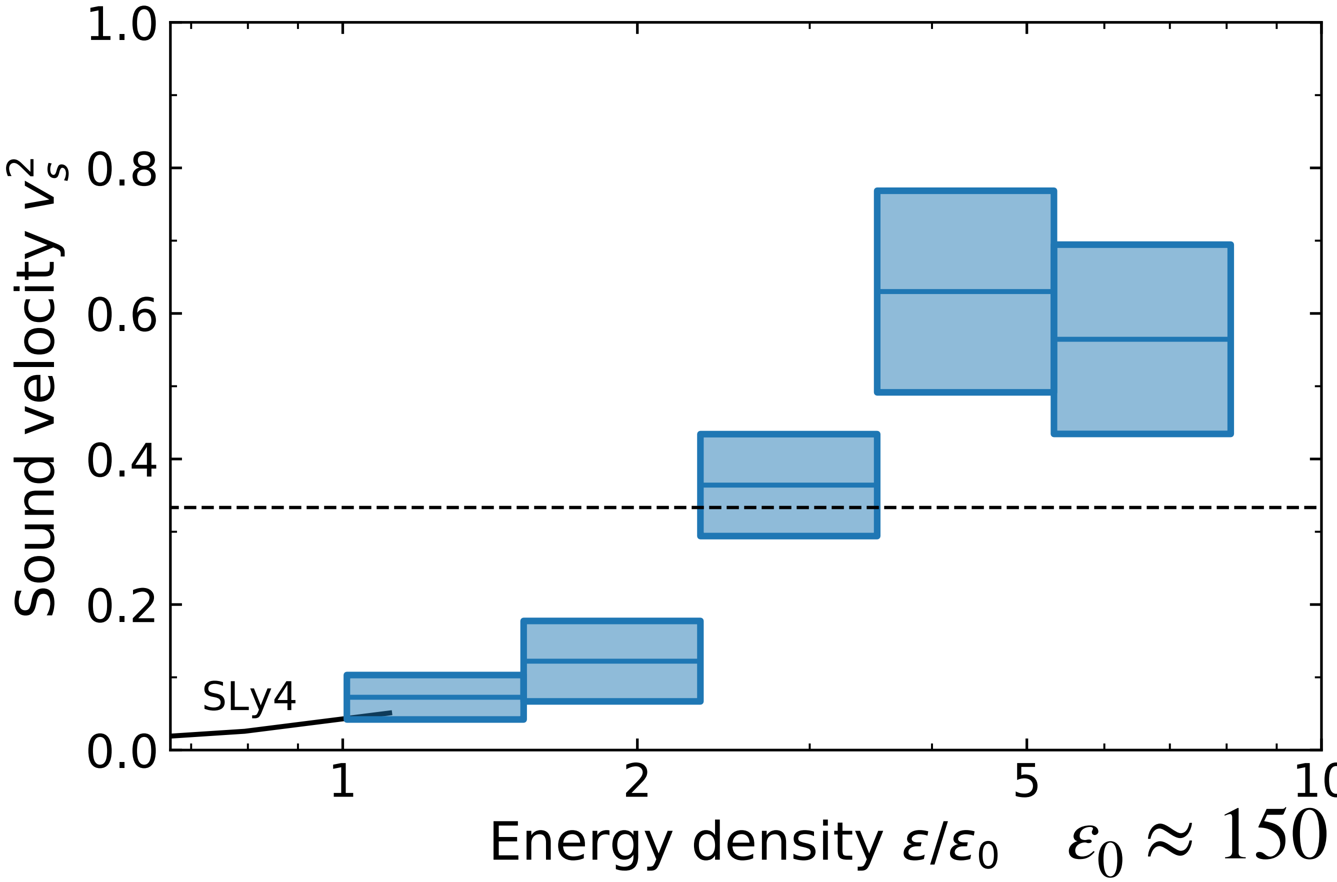
Drischler,Han,Lattimer,Prakash,Reddy,Zhao (2020)

Rapid stiffening in EoS

NS data favors rapid increase in **sound velocity**, accompanied by a peak structure

Fujimoto,Fukushima,Murase (2019)

$$v_s^2 \equiv \frac{dP}{d\varepsilon}$$



$\varepsilon_0 \approx 150\,\mathrm{MeV/fm^3}$

$v_s^2 = 1/3$ is referred to as **conformal limit**

Because $v_s^2 \nearrow 1/3$ when $\varepsilon \to \infty$.

Conformal limit is violated at intermediate density

Bedaque,Steiner (2015); Tews,Carlson,Gandolfi,Reddy (2018);
Altiparmak,Ecker,Rezzolla (2022); Gorda,Komoltsev,Kurkela (2022) & many others

Trace anomaly equation

Related to scale/conformal nature of matter:

$$j_D^\nu = x_\mu T^{\mu\nu} \;\to\; \partial_\nu j_D^\nu = T^\mu_\mu \begin{cases} = 0 & \text{Classical YM} \\ \neq 0 & \text{in QFT (RG effect)} \end{cases}$$

Expectation value: $\langle T^\mu_\mu \rangle = \langle T^\mu_\mu \rangle_{\mu_B} + \langle T^\mu_\mu \rangle_0$

matter (μ_B-dependent) vacuum

Finite-μ_B part of the trace anomaly (interaction measure):

$$\langle T^\mu_\mu \rangle_{\mu_B} = \varepsilon - 3P$$

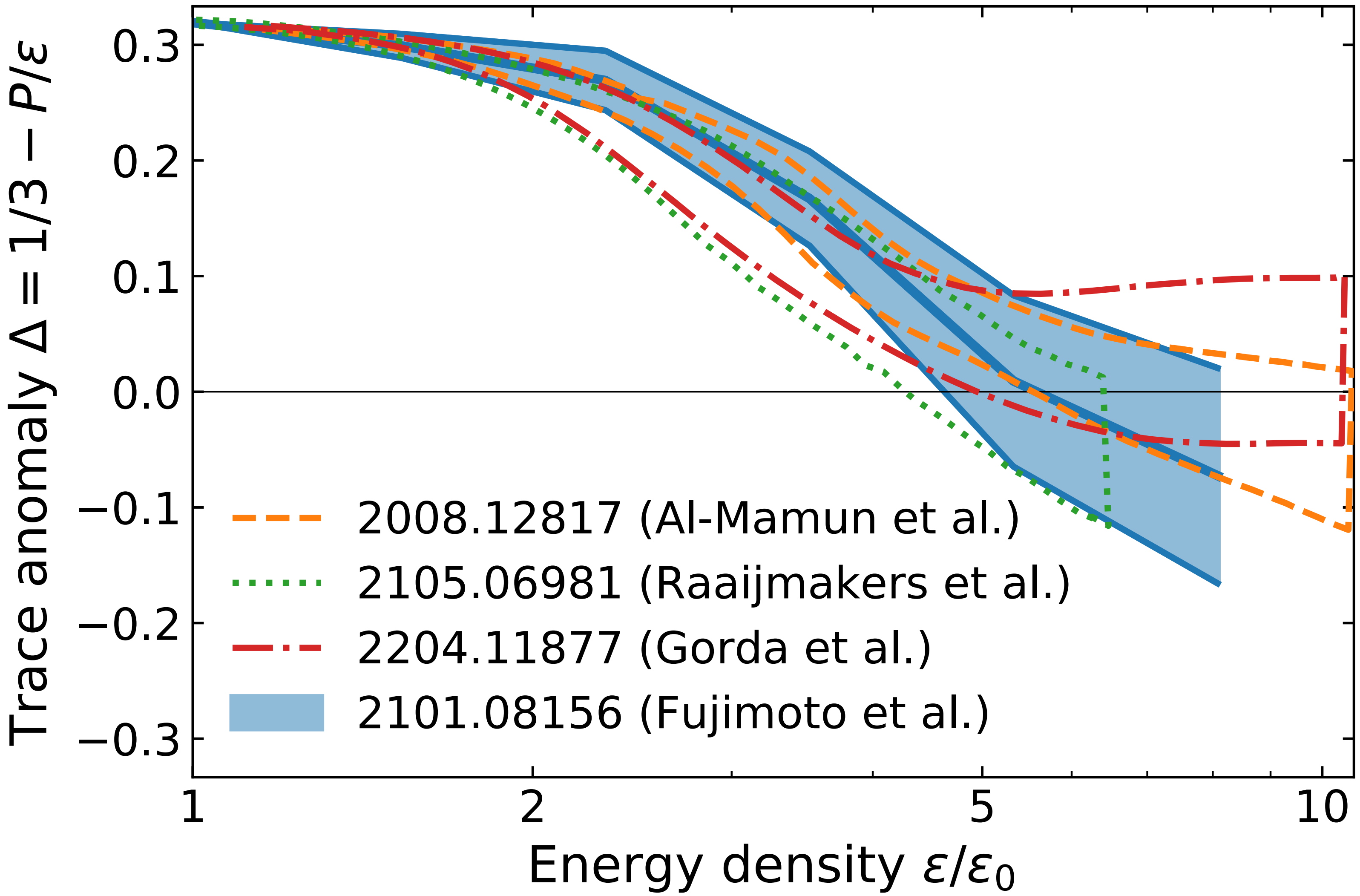
… related to EoS of neutron star matter

Behavior of trace anomaly

Fujimoto, Fukushima, McLerran, Praszalowicz, PRL129 (2022)

$$\Delta \equiv \frac{\langle T^{\mu}{}_{\mu} \rangle_{\mu_B}}{3\varepsilon} = \frac{1}{3} - \frac{P}{\varepsilon}$$

Inference of EoSs from NS observations shows:



$\Delta \sim 0$, i.e., $P \sim \varepsilon/3$

already at $\sim 5\varepsilon_0$

rapid stiffening to conformality

Strongly-coupled conformal matter?

Trace anomaly and sound velocity

Fujimoto,Fukushima,McLerran,Praszalowicz, PRL129 (2022)

(Normalized) trace anomaly: $\Delta = \frac{\langle T^\mu_\mu \rangle_{\mu_B}}{3\varepsilon} = \frac{1}{3} - \frac{P}{\varepsilon}$

cf. Gavai,Gupta,Mukherjee (2004)

$-\frac{2}{3} \lesssim \Delta \leq \frac{1}{3}$

Sound velocity: $v_s^2 = \frac{dP}{d\varepsilon} = -\varepsilon \frac{d\Delta}{d\varepsilon} + \left(\frac{1}{3} - \Delta\right)$

Conformal limits:

$\Delta(\varepsilon) \searrow 0$

$v_s^2(\varepsilon) \nearrow 1/3$

when $\varepsilon \to \infty$

Bedaque,Steiner (2015)

It is very likely that $v_s^2 > 1/3$. Simultaneously, $\Delta \gtrsim 0$.

Meaning of the sound velocity peak

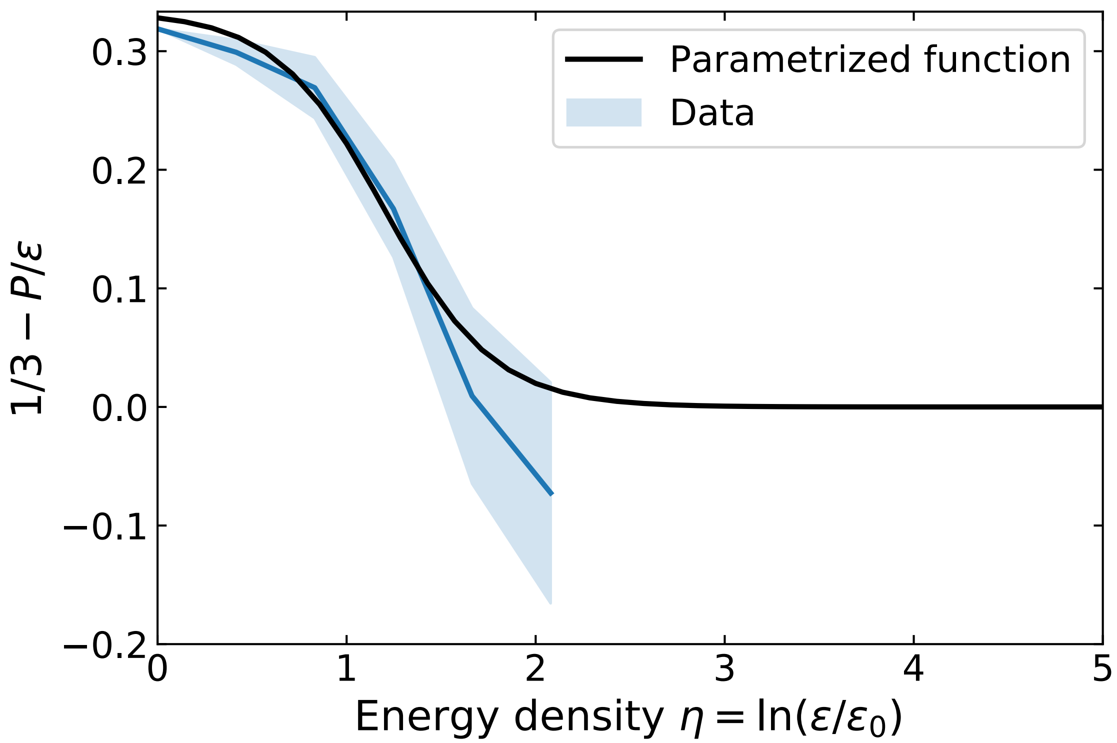
Fujimoto,Fukushima,McLerran,Praszalowicz, PRL129 (2022)

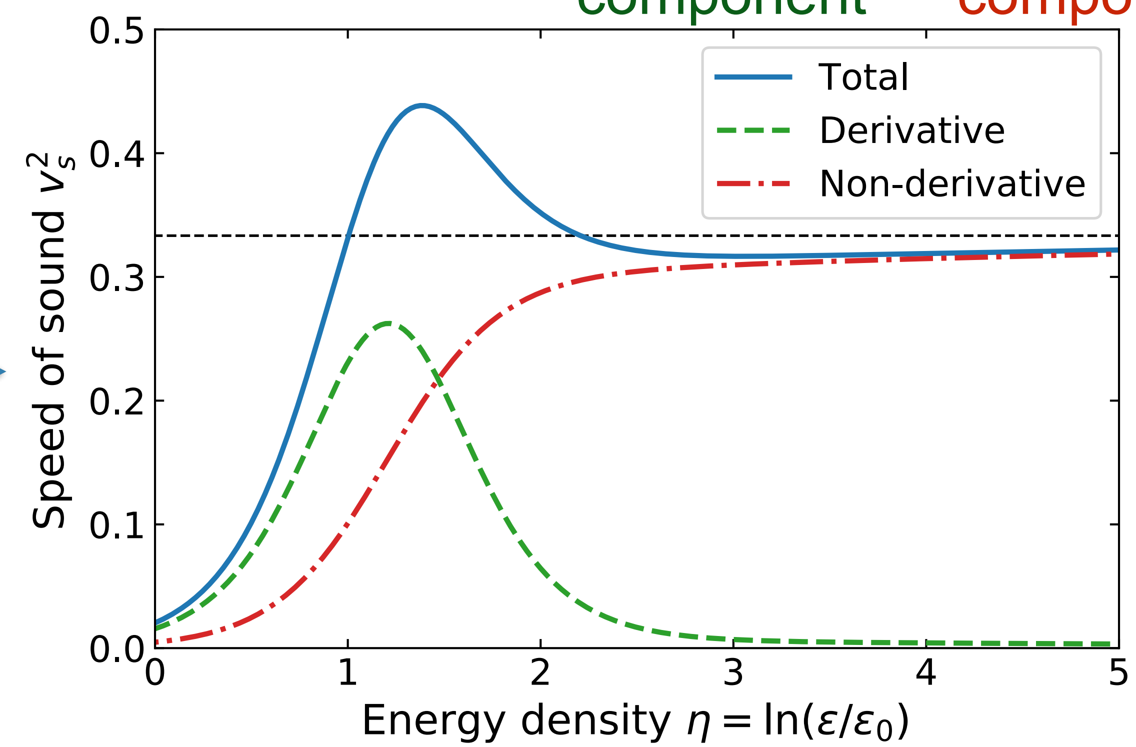
Rapid approach to $\Delta \to 0$ naturally spikes v_s^2

Sound velocity $v_s^2 = -\varepsilon \frac{d\Delta}{d\varepsilon} + (\frac{1}{3} - \Delta)$

Derivative component — Non-derivative component

Trace anomaly $\Delta = \frac{1}{3} - \frac{P}{\varepsilon}$





$\Delta \gtrsim 0$ $\qquad$ $v_s^2 > 1/3$

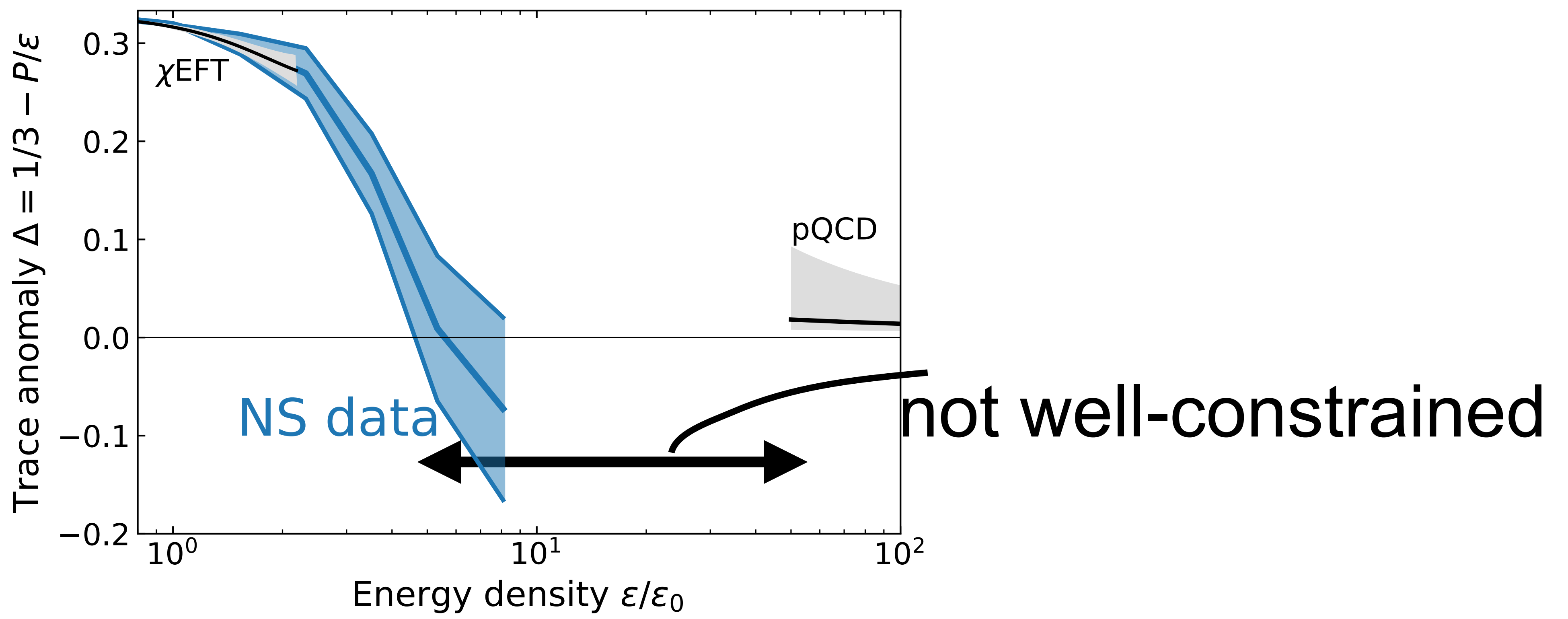
Monotonic Δ gives rise to non-monotonic v_s^2 and violation of $v_s^2 \leq 1/3$

Rapid stiffening to the conformal EoS creates a peak

Positive trace anomaly conjecture

Fujimoto,Fukushima,McLerran,Praszalowicz, PRL129 (2022)

Combining QCD ab-initio calculations:



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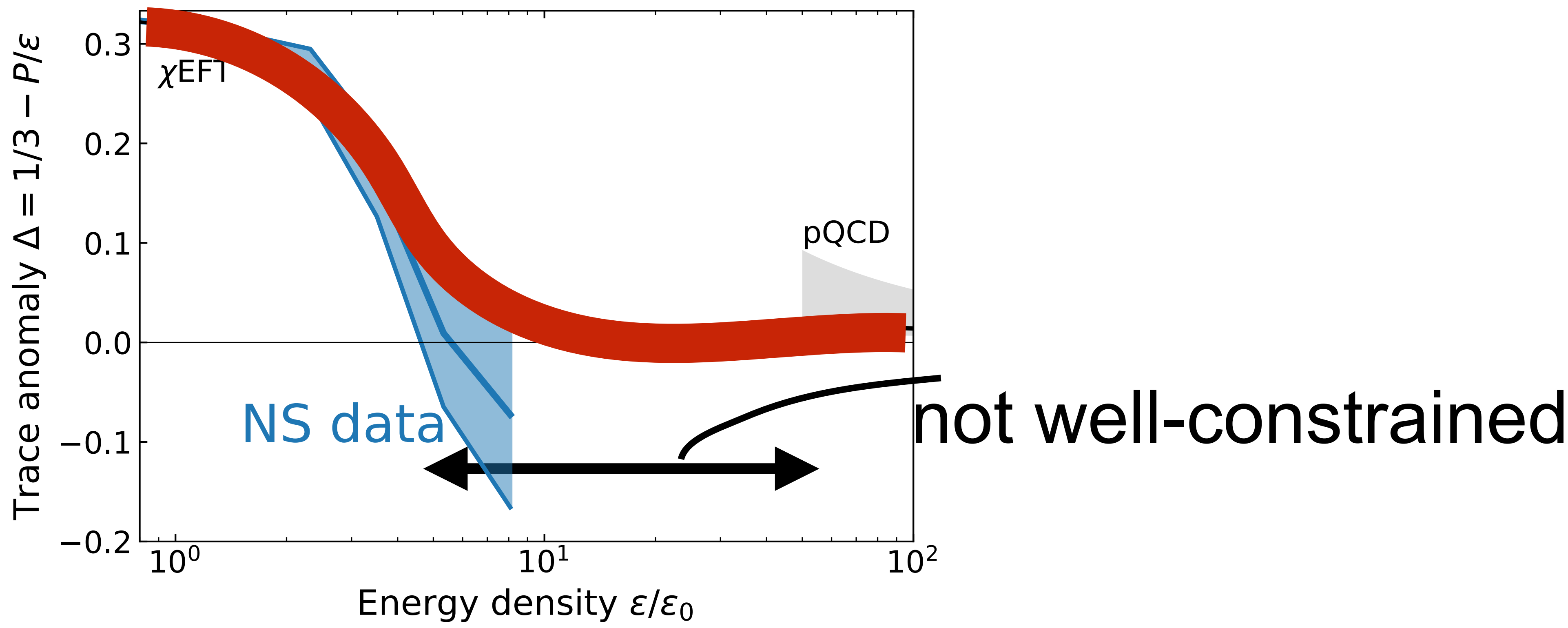

Our conjecture: $\Delta \gtrsim 0$ at any ε for NS matter

Positive trace anomaly conjecture

Fujimoto,Fukushima,McLerran,Praszalowicz, PRL129 (2022)

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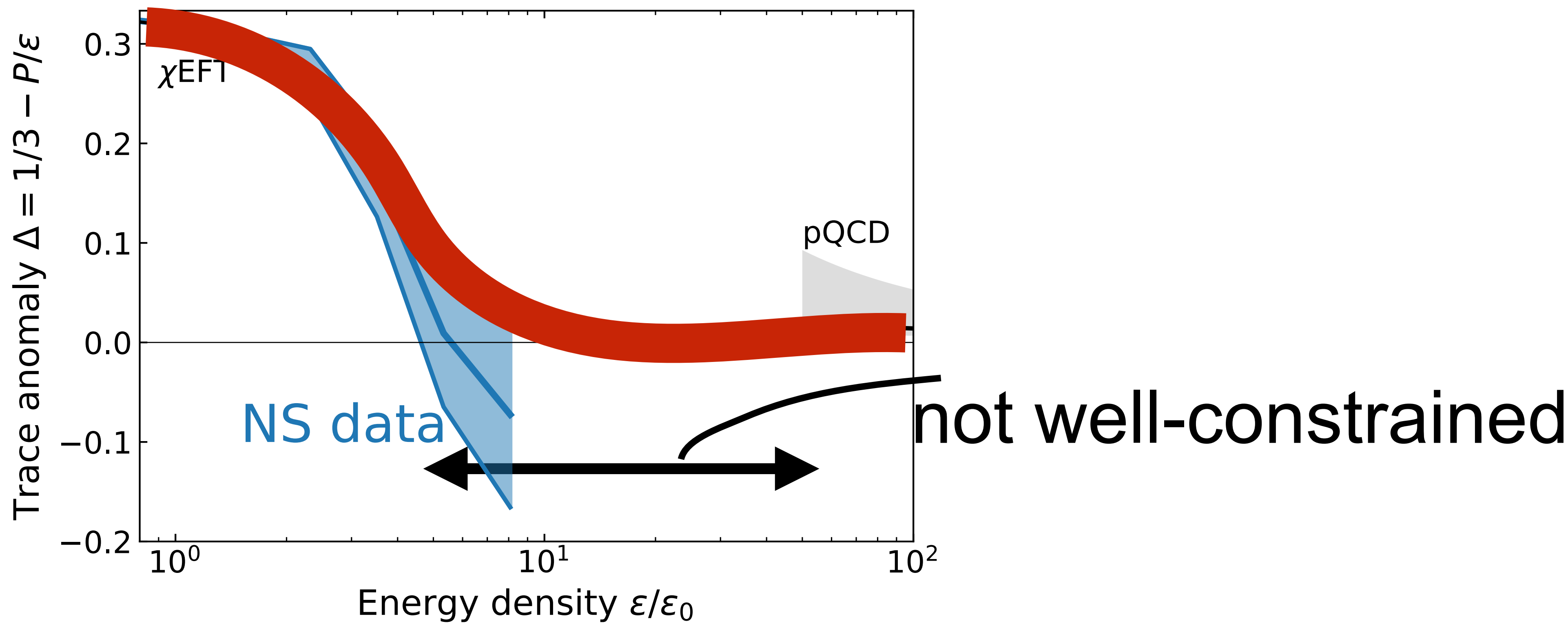


Our conjecture: $\Delta \gtrsim 0$ at any ε for NS matter

Caveat: QCD at finite-μ_I or SU(2) QCD gives $\Delta < 0$

e.g., Cotter,Giudice,Hands,Skullerud (2012); Iida,Itou (2022)
Son,Stephanov (2001); Brandt,Endrodi+ (2018-)…

Justification of the $\Delta \geq 0$ bound

Trace anomaly is related to the counting of the effective degrees of freedom in pressure, $\nu \equiv P/\mu_B^4$:

$$\Delta \propto \frac{\langle T^\mu_\mu \rangle_{\mu_B}}{\mu_B^4} = \mu_B \frac{d\nu}{d\mu_B} \geq 0$$

If ν keeps increasing, we get $\Delta \geq 0$

Open question: the effect of color superconductivity?

$\rightarrow$ Cooper pairing reduces the d.o.f.
but this is only a phenomenon around
the Fermi surface

Justification of the $\Delta \geq 0$ bound

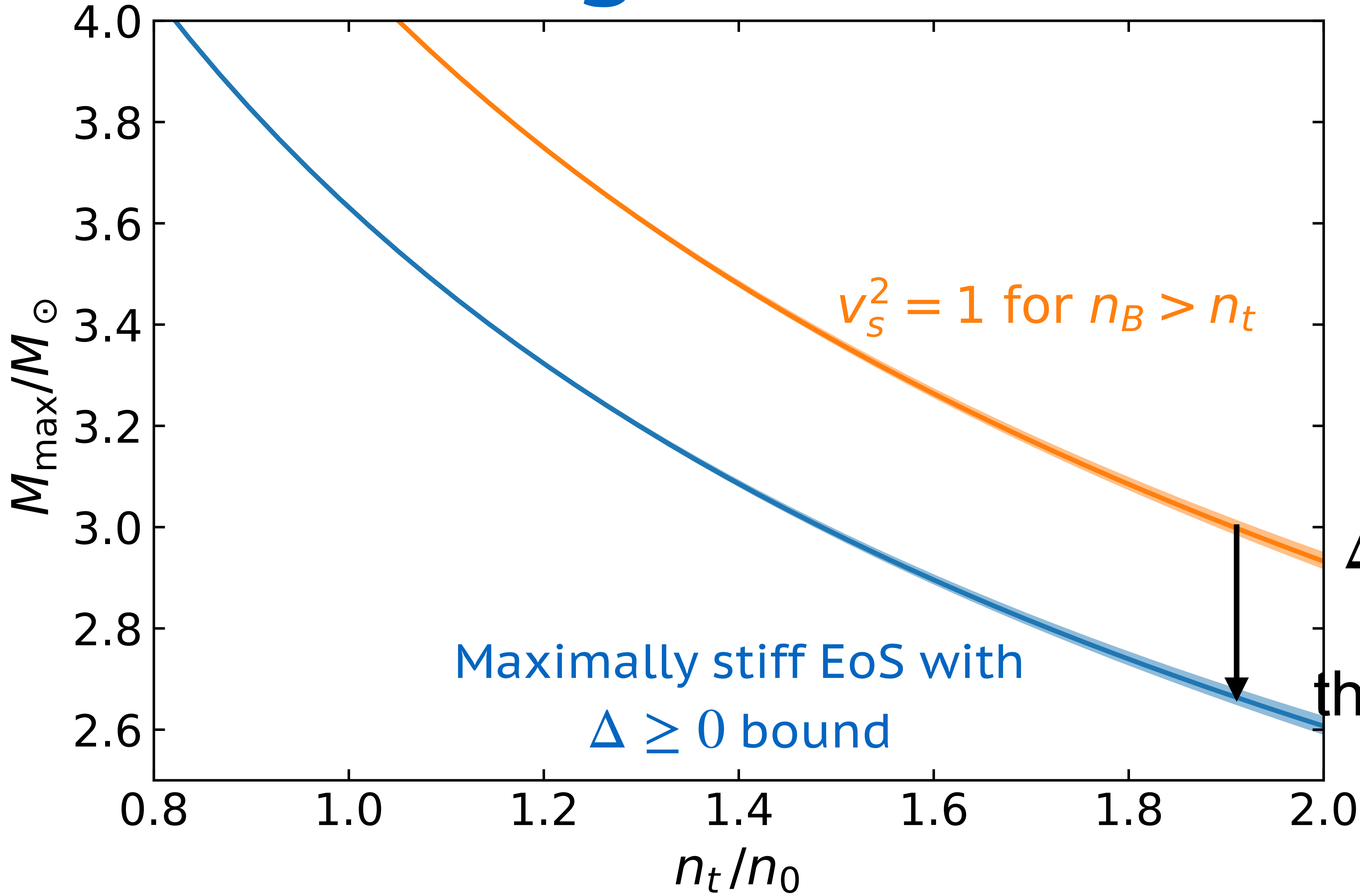
From the QCD energy-momentum tensor in the chiral limit:

$$\Delta \propto \langle T^{\mu}_{\ \mu} \rangle = \frac{\beta}{2g} \langle \boldsymbol{B}^2 - \boldsymbol{E}^2 \rangle_{\mu_B}$$

Interaction between quarks are mediated dominantly by chromo-electric fields → gluon condensate < 0

Given $\beta < 0$ (asymptotic freedom), the total trace anomaly is positive.

Testing the $\Delta \geq 0$ bound



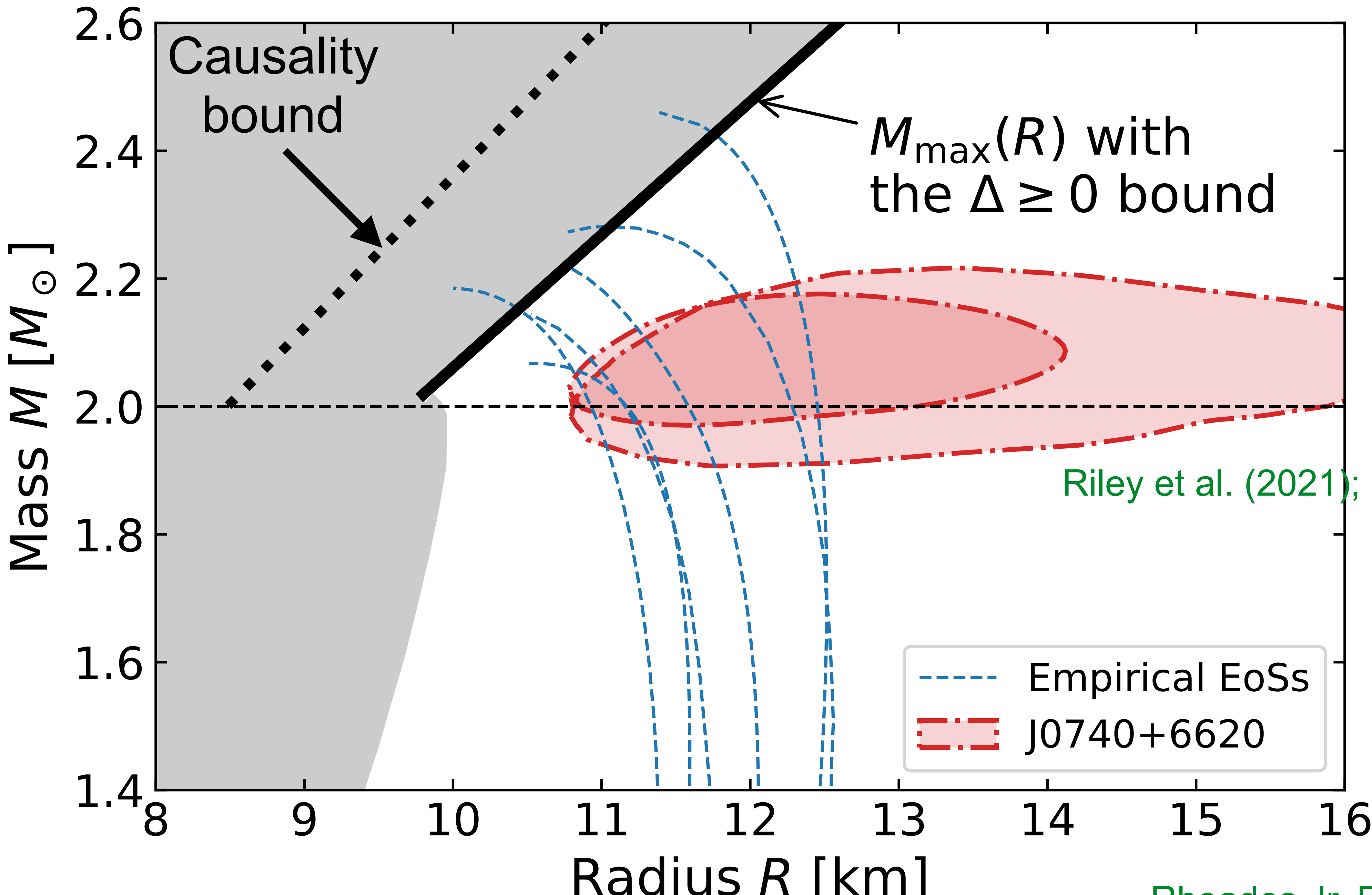
$\Delta \geq 0$ **condition**
pushes down
the maximum mass
bound of NSs

Density up to which I use ChEFT EoS

ChEFT data: Drischler,Han,Lattimer,Prakash,Reddy,Zhao (2020)

Testing the $\Delta \geq 0$ bound

Combining the radius observation:



Riley et al. (2021); Miller et al. (2021)

Rhoades Jr.,Ruffini (1974)

Koranda,Stergioulas,Friedman (1995)

See also: Drischler,Han,Lattimer,Prakash,Reddy,Zhao (2020)

Summary

- Trace anomaly Δ is a measure of conformality. It is a complement to the speed of sound v_s^2

- NS data suggest Δ rapidly approach to the conforal limit and $\Delta \to 0$ naturally gives rise to the sound velocity peak

- Strongly-interacting conformal matter may be inside NSs

- The trace anomaly may be positive (not proven). It can be tested by, e.g., the bound on the maximum mass of NSs

- Quarkyonic matter can be responsible for rapid stiffening to conformality

see, e.g., Fujimoto,Kojo,McLerran [2306.04304]