

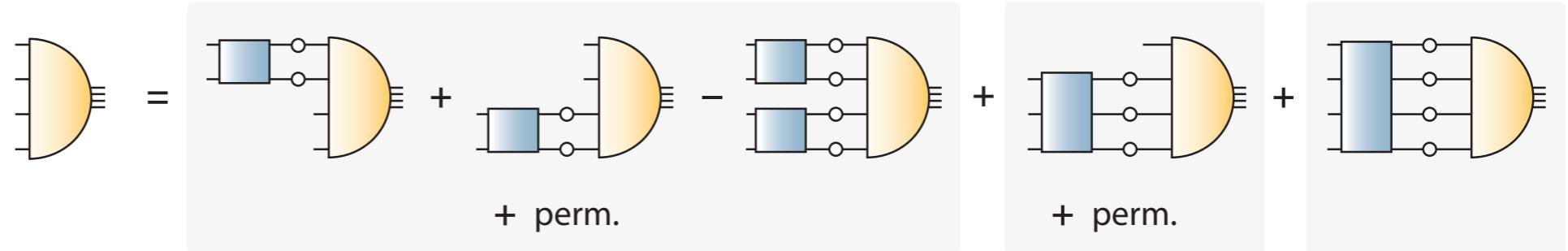


**INT Workshop
2023**

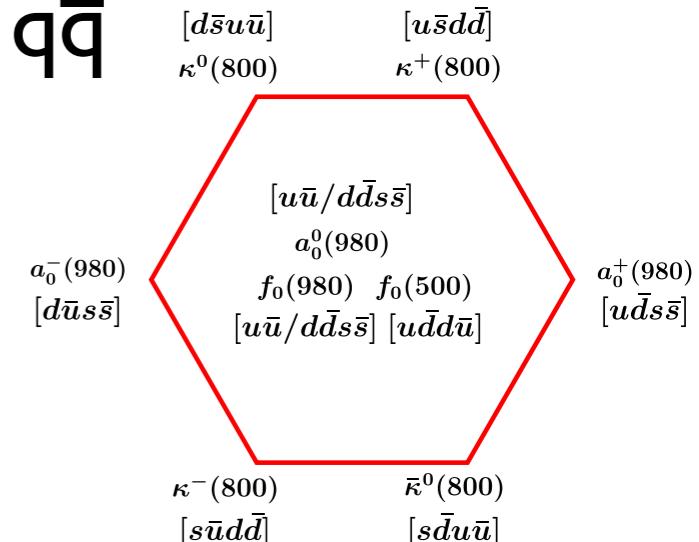
Four-quark states from functional methods

with Gernot Eichmann, Joshua Hoffer, Nico Santowsky, Paul Wallbott

I. The framework and conventional mesons

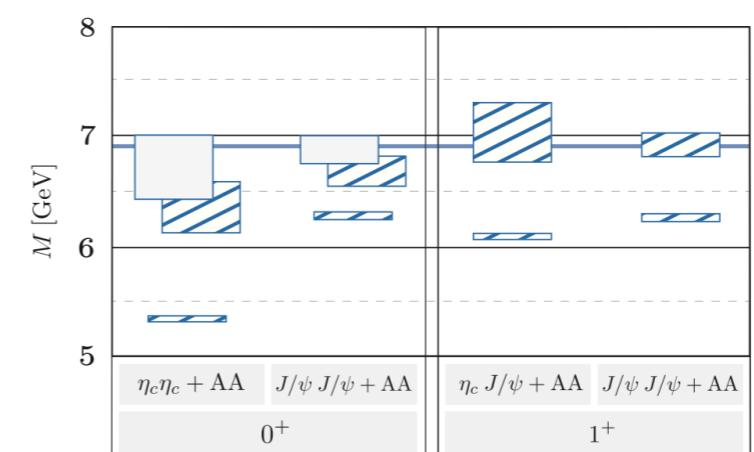


2. Detour: light four-quark states and mixing with $q\bar{q}$

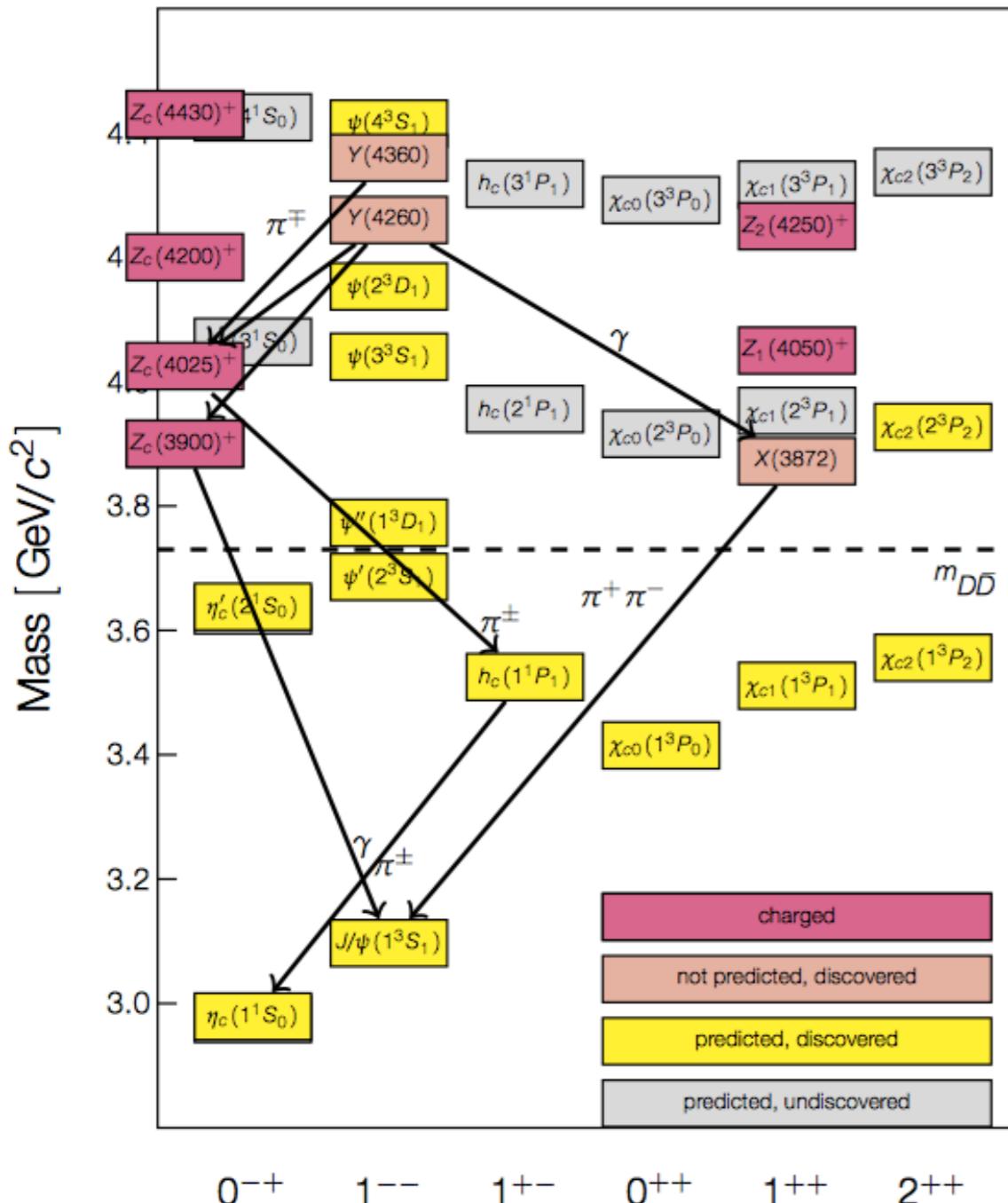


3. Heavy-light and heavy-heavy four-quark states

	$I(J^{PC})$	dominant	4-body	eff. 2-body	Exp.
hidden charm $(c\bar{c}q\bar{q})$	$0(0^{++})$	DD	3.20 (11)	3.49 (25)	
	$0(1^{++})$	DD*	3.92 (7)	3.85 (18)	$X(3872)$
	$1(1^{+-})$	DD*	3.74 (9)	3.79 (31)	$Z_c(3900)$
	$1(0^{++})$	DD		3.20 (31)	
open charm $(cc\bar{q}\bar{q})$	$0(1^+)$	DD*	3.90 (8)	3.49 (48)	$T_{cc}(3875)$
	$1(0^+)$	DD+AA	3.80 (10)	3.21 (2)	
	$1(1^+)$	DD*+AA	4.22 (44)	3.47 (24)	

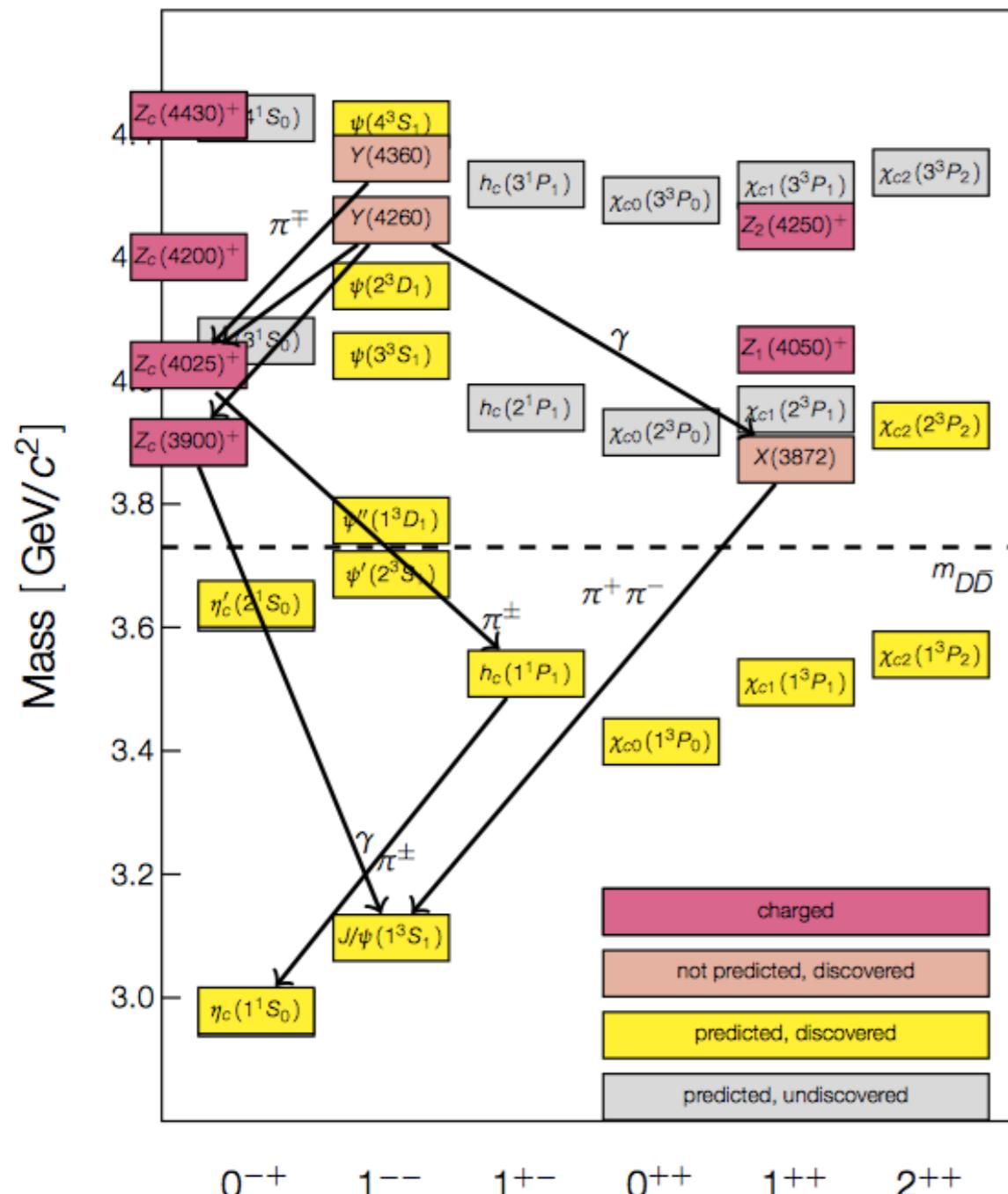


Tetraquark candidates with $c\bar{q}q\bar{c}$ -content



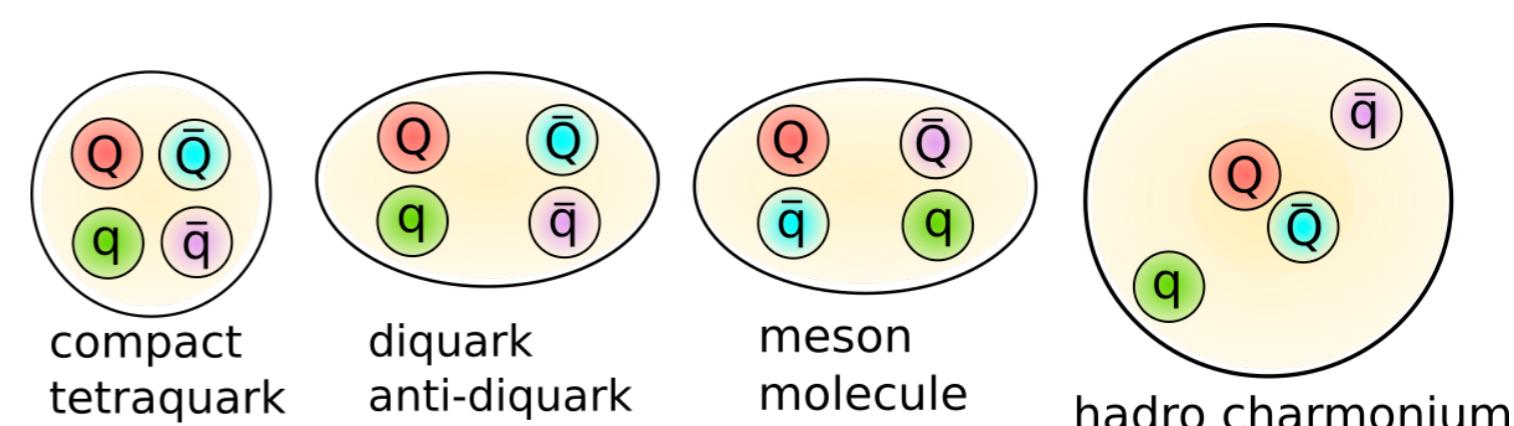
Many new unexpected states found: Belle, BABAR, BES, LHCb ...

Tetraquark candidates with $c\bar{q}\bar{q}\bar{c}$ -content



Many new unexpected states found: Belle, BABAR, BES, LHCb ...

Internal structure ??



Related to details of underlying QCD forces between quarks and gluons

Bound states and Bethe-Salpeter equations

BSEs:

$$\text{Diagram: } \text{Yellow semi-circle} = \text{Blue rectangle} + \text{Blue rectangle}$$

$$\text{Diagram: } -1 = \text{Arrow} - \text{Diagram with wavy line}$$

$$\text{Diagram: } \text{Orange semi-circle} = \text{Blue rectangle} + \text{Blue rectangle}$$

$$\text{Diagram: } \text{Yellow semi-circle with } ee \text{ labels} = \text{Blue rectangle} + \text{Blue rectangle}$$

$$\text{Diagram: } \text{Yellow semi-circle with } ee \text{ labels} = \text{Blue rectangle} + \text{Blue rectangle} - \text{Blue rectangle} + \text{Blue rectangle} + \text{Blue rectangle}$$

+ perm.

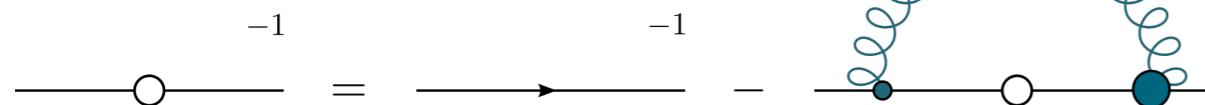
+ perm.

Eigenvalue equations: masses and wave functions

Dyson-Schwinger equations - “3PI vs RL”

$$\mathcal{Z}_{QCD} = \int \mathcal{D}[\Psi, A] \exp \left\{ - \int d^4x \left(\bar{\Psi} (i \not{D} - m) \Psi - \frac{1}{4} (F_{\mu\nu}^a)^2 \right) \right\}$$

propagators



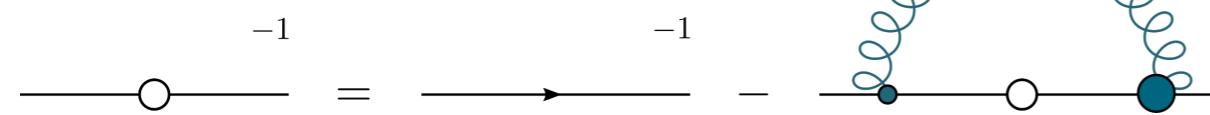
CF,Alkofer, PRD67 (2003) 094020
Williams, CF, Heupel, PRD93 (2016) 034026
Huber, PRD 101 (2020) 114009

see Talk of Markus Huber

Dyson-Schwinger equations - “3PI vs RL”

$$\mathcal{Z}_{QCD} = \int \mathcal{D}[\Psi, A] \exp \left\{ - \int d^4x \left(\bar{\Psi} (i \not{D} - m) \Psi - \frac{1}{4} (F_{\mu\nu}^a)^2 \right) \right\}$$

propagators



CF,Alkofer, PRD67 (2003) 094020

Williams, CF, Heupel, PRD93 (2016) 034026

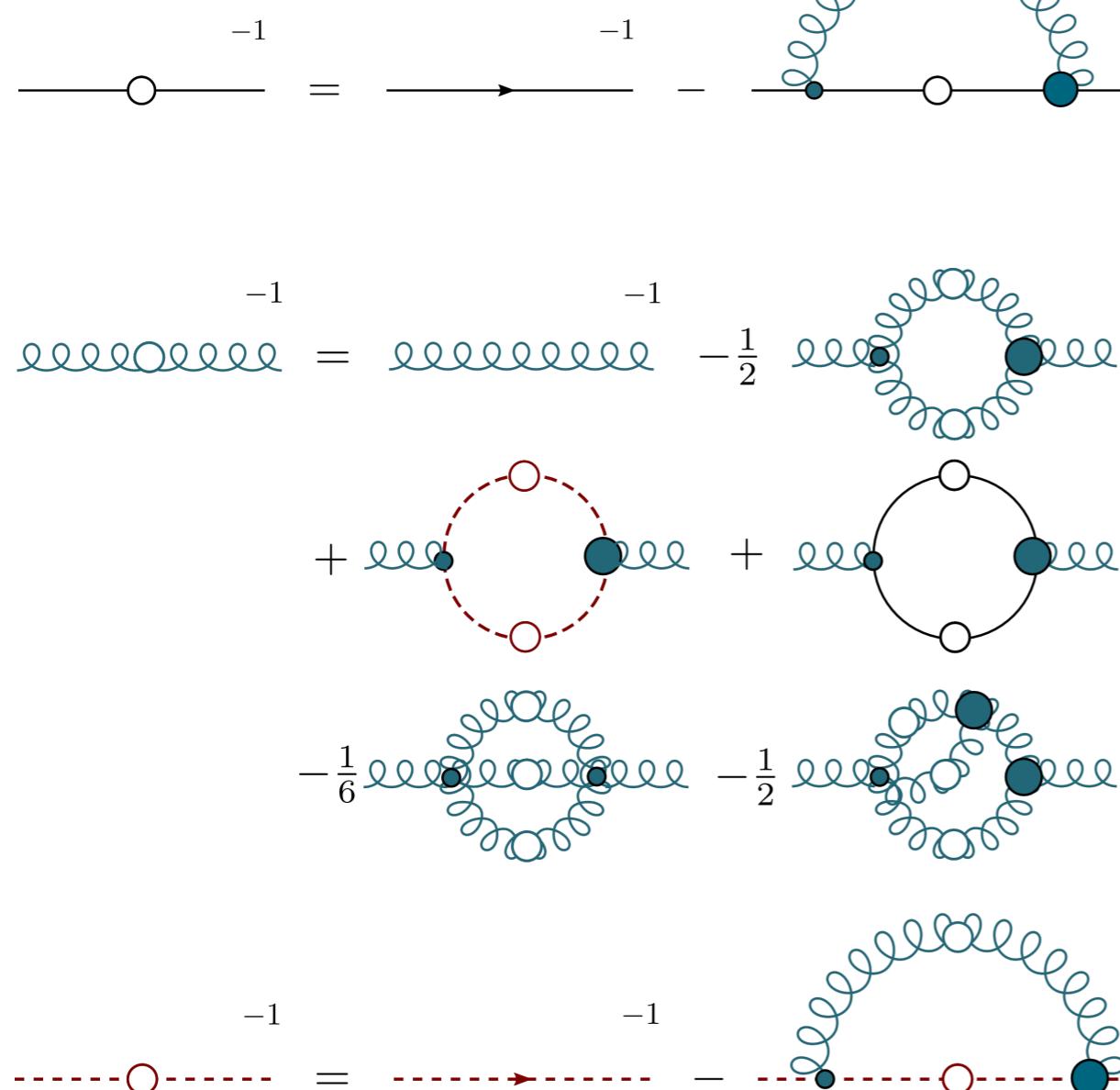
Huber, PRD 101 (2020) 114009

see Talk of Markus Huber

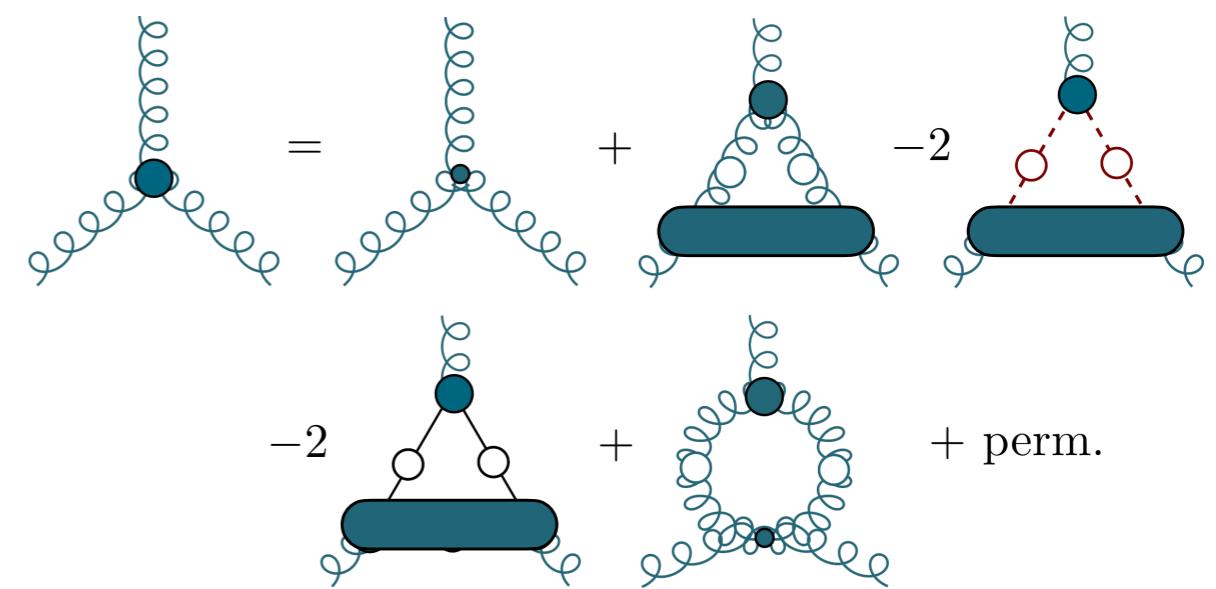
Dyson-Schwinger equations - “3PI vs RL”

$$\mathcal{Z}_{QCD} = \int \mathcal{D}[\Psi, A] \exp \left\{ - \int d^4x \left(\bar{\Psi} (i \not{D} - m) \Psi - \frac{1}{4} (F_{\mu\nu}^a)^2 \right) \right\}$$

propagators



vertices



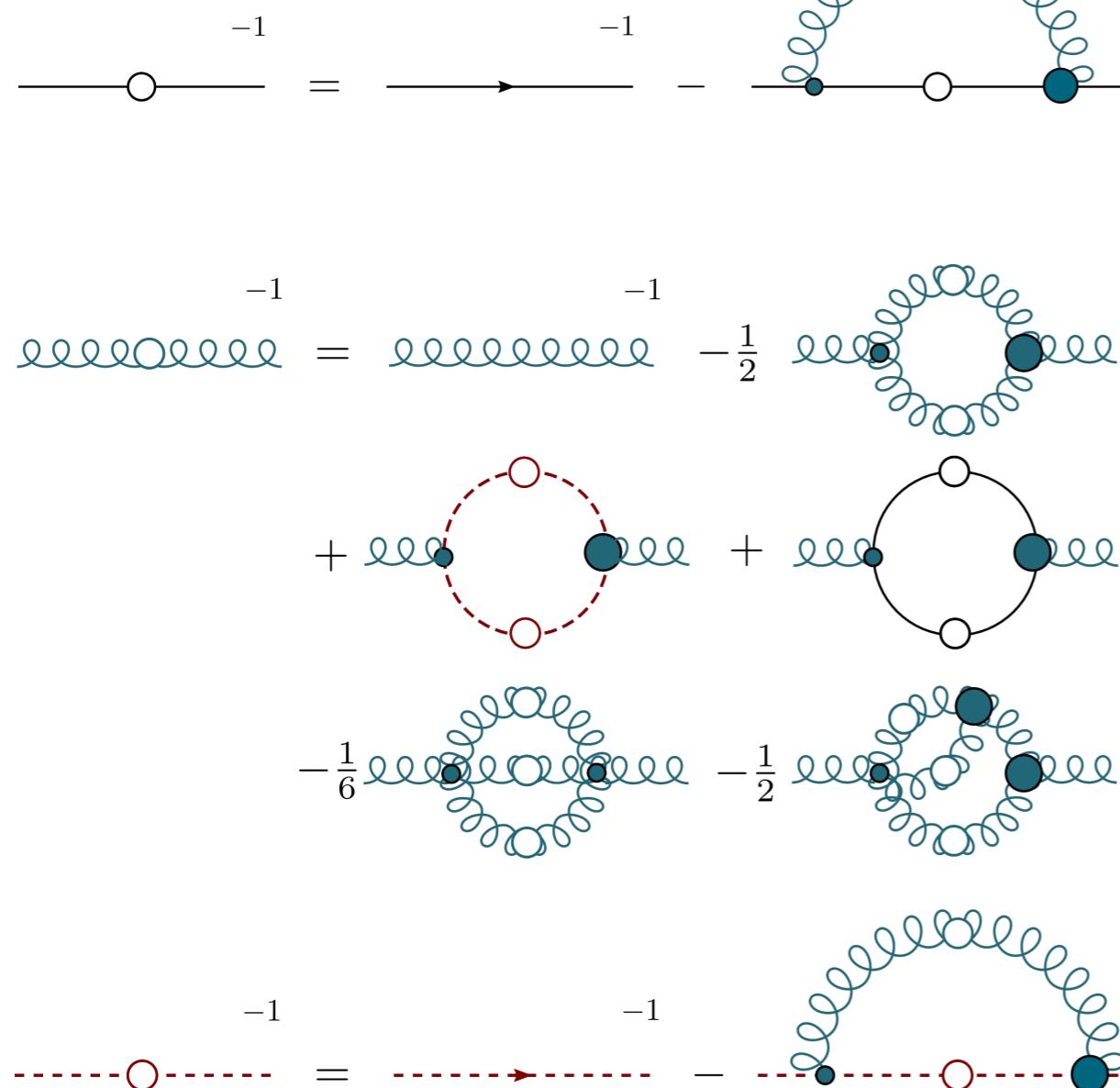
CF,Alkofer, PRD67 (2003) 094020
 Williams, CF, Heupel, PRD93 (2016) 034026
 Huber, PRD 101 (2020) 114009

see Talk of Markus Huber

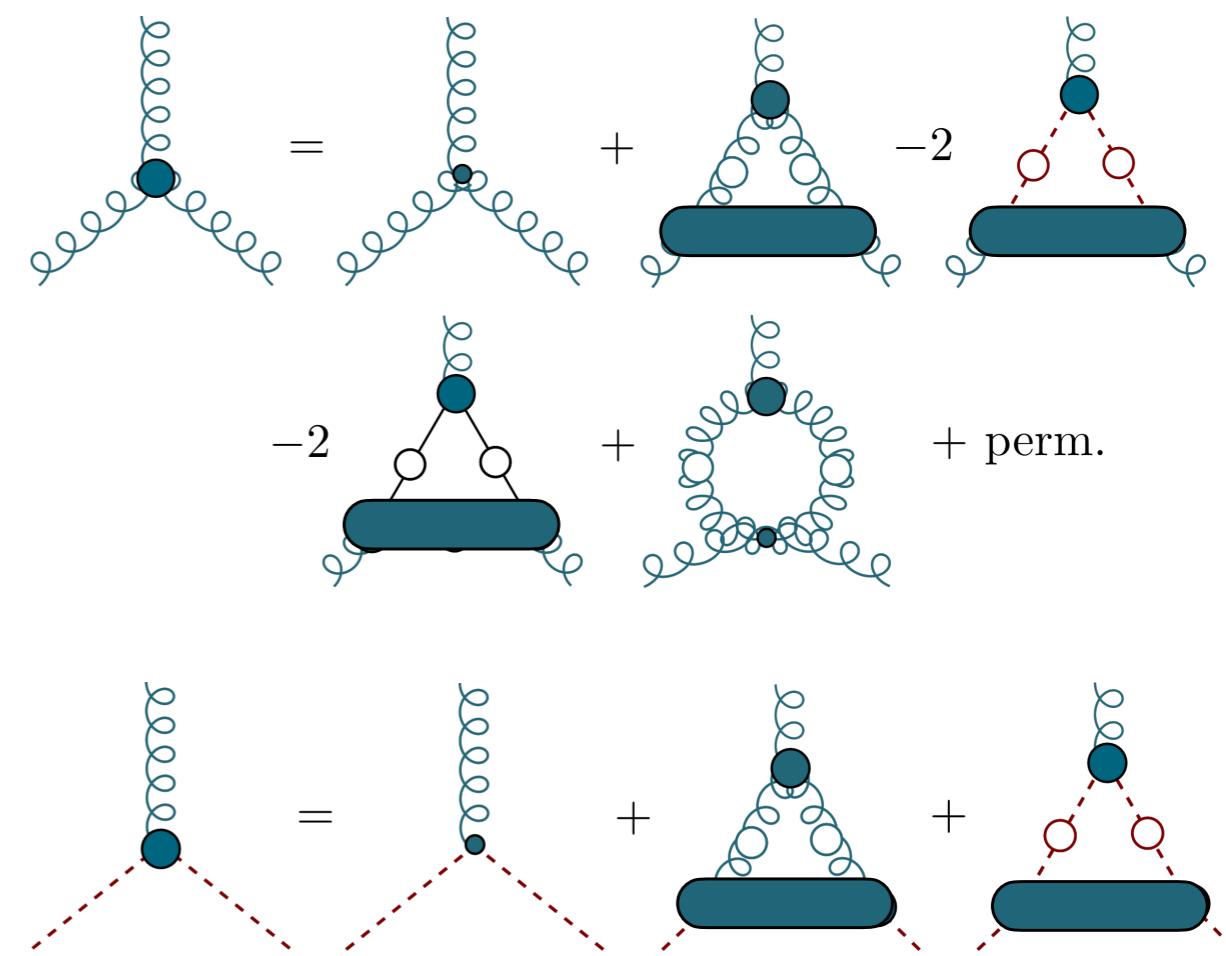
Dyson-Schwinger equations - “3PI vs RL”

$$\mathcal{Z}_{QCD} = \int \mathcal{D}[\Psi, A] \exp \left\{ - \int d^4x \left(\bar{\Psi} (i \not{D} - m) \Psi - \frac{1}{4} (F_{\mu\nu}^a)^2 \right) \right\}$$

propagators



vertices



CF,Alkofer, PRD67 (2003) 094020

Williams, CF, Heupel, PRD93 (2016) 034026

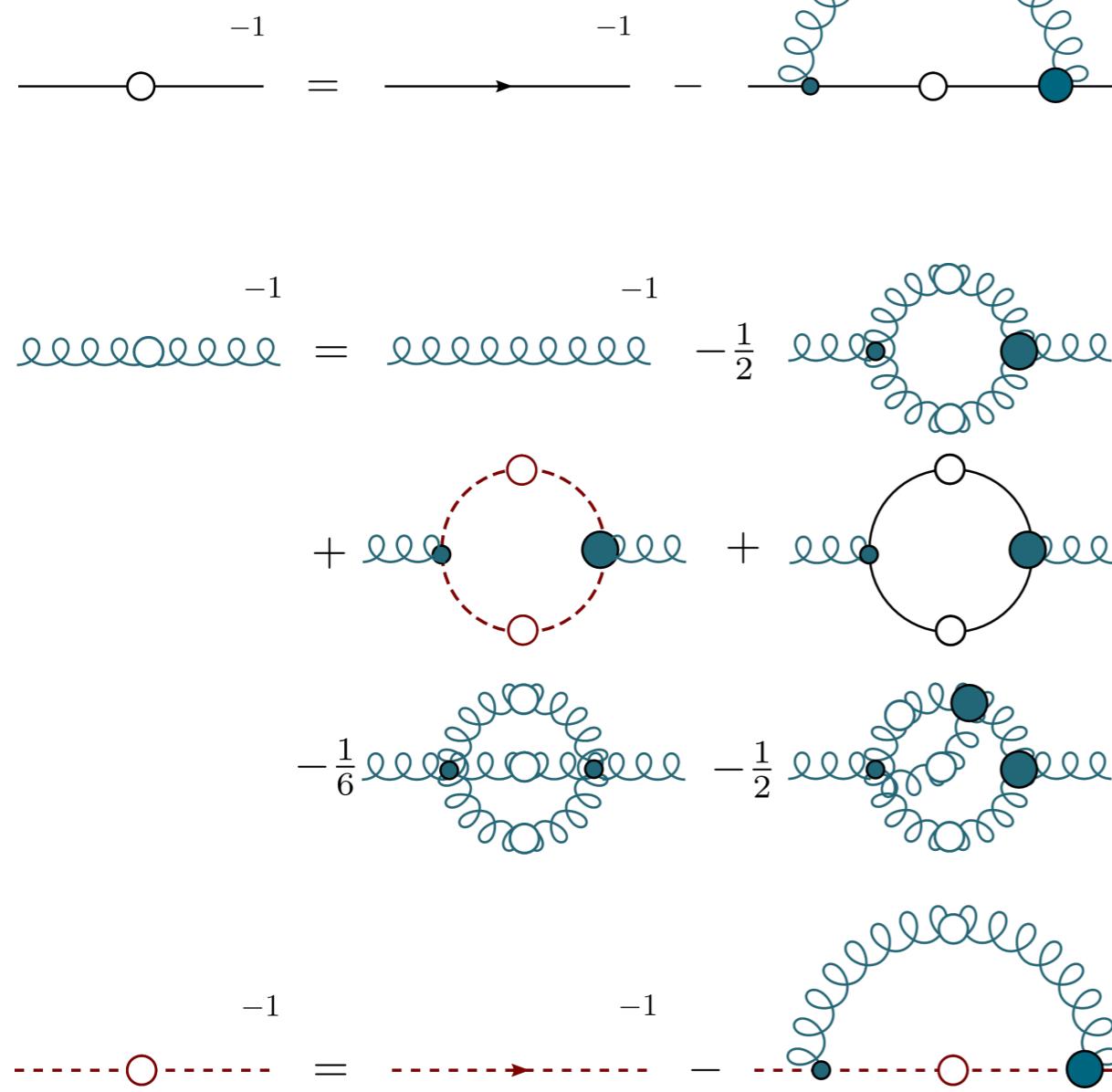
Huber, PRD 101 (2020) 114009

see Talk of Markus Huber

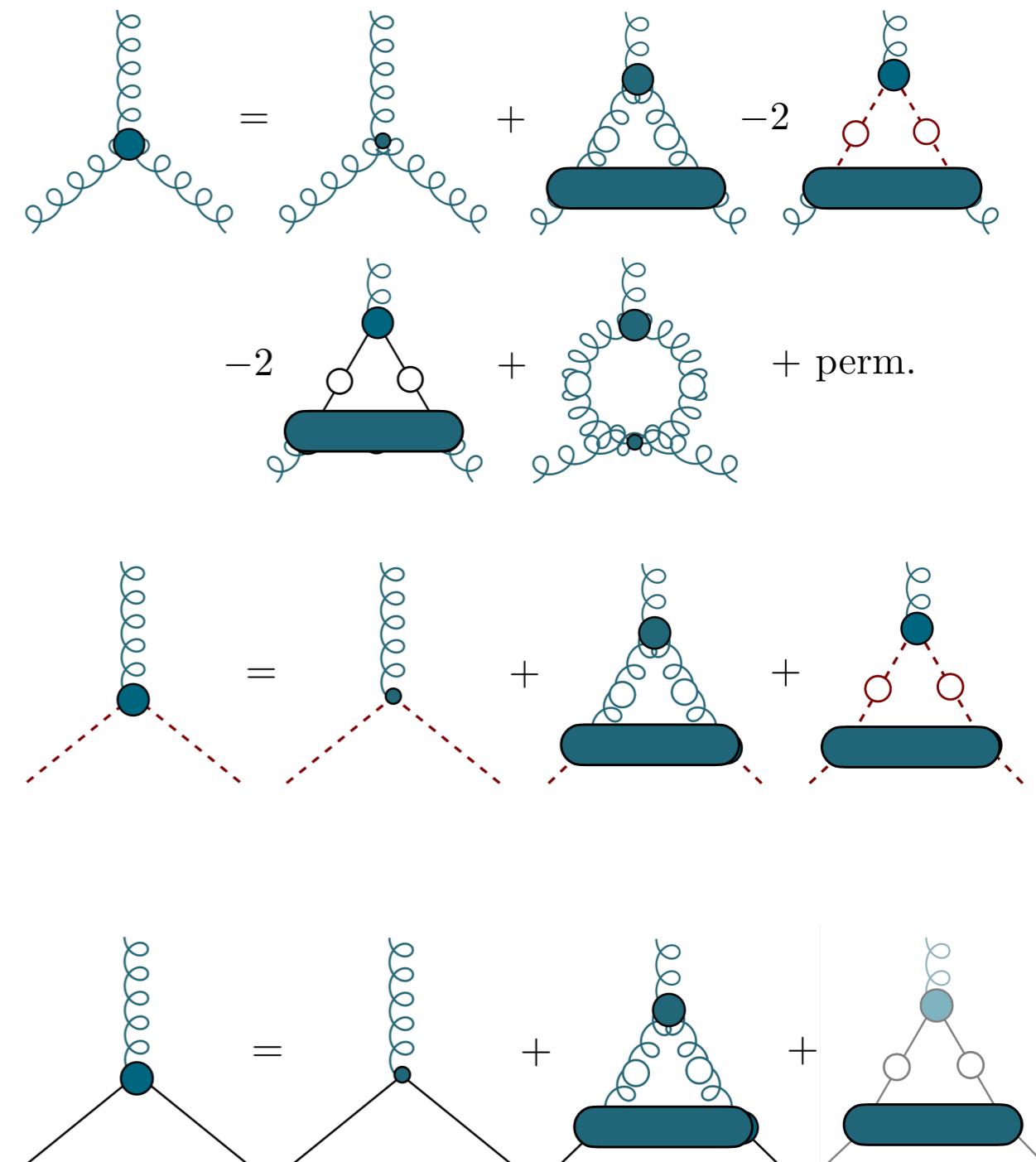
Dyson-Schwinger equations - “3PI vs RL”

$$\mathcal{Z}_{QCD} = \int \mathcal{D}[\Psi, A] \exp \left\{ - \int d^4x \left(\bar{\Psi} (i \not{D} - m) \Psi - \frac{1}{4} (F_{\mu\nu}^a)^2 \right) \right\}$$

propagators



vertices



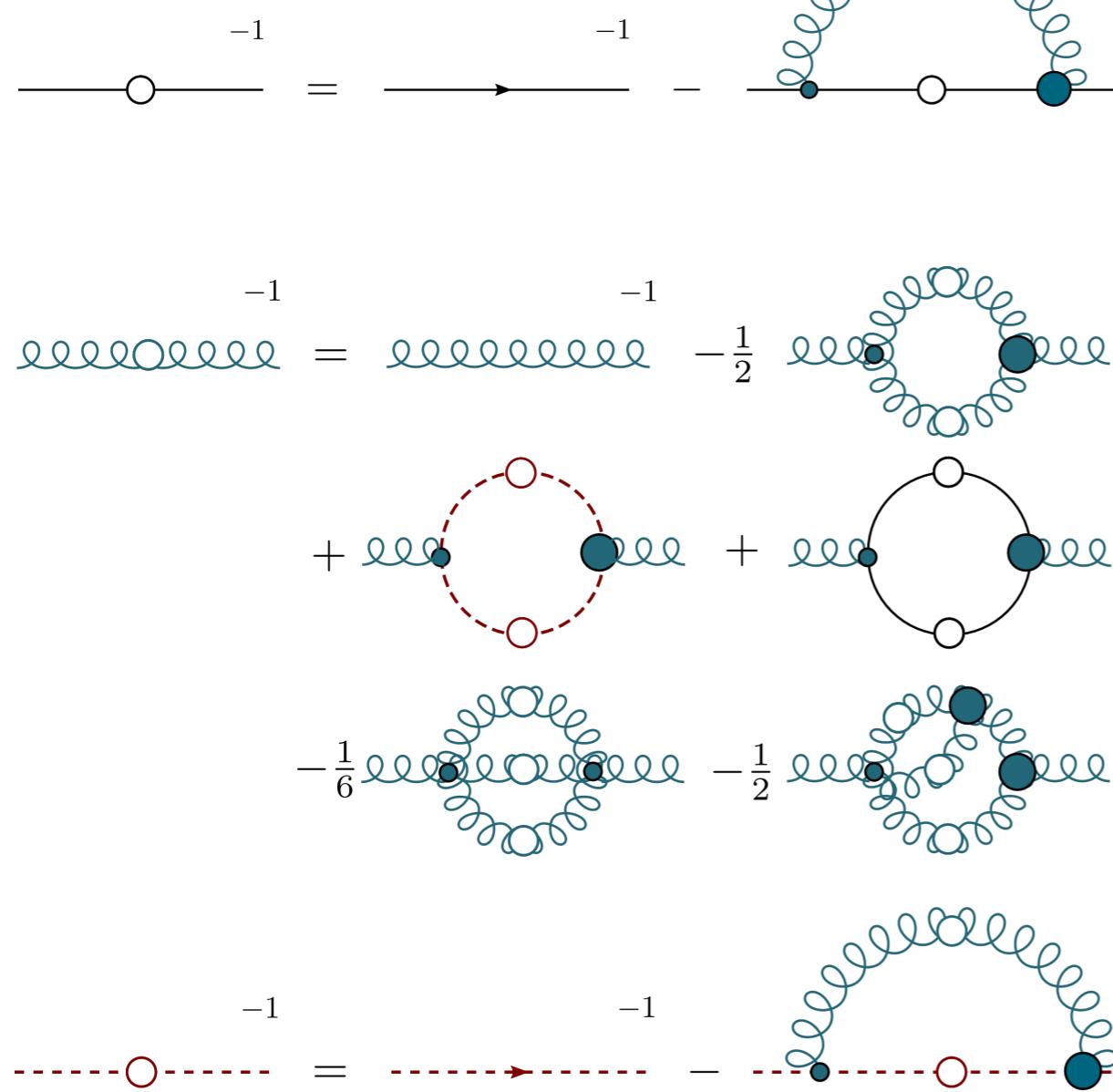
CF,Alkofer, PRD67 (2003) 094020
 Williams, CF, Heupel, PRD93 (2016) 034026
 Huber, PRD 101 (2020) 114009

see Talk of Markus Huber

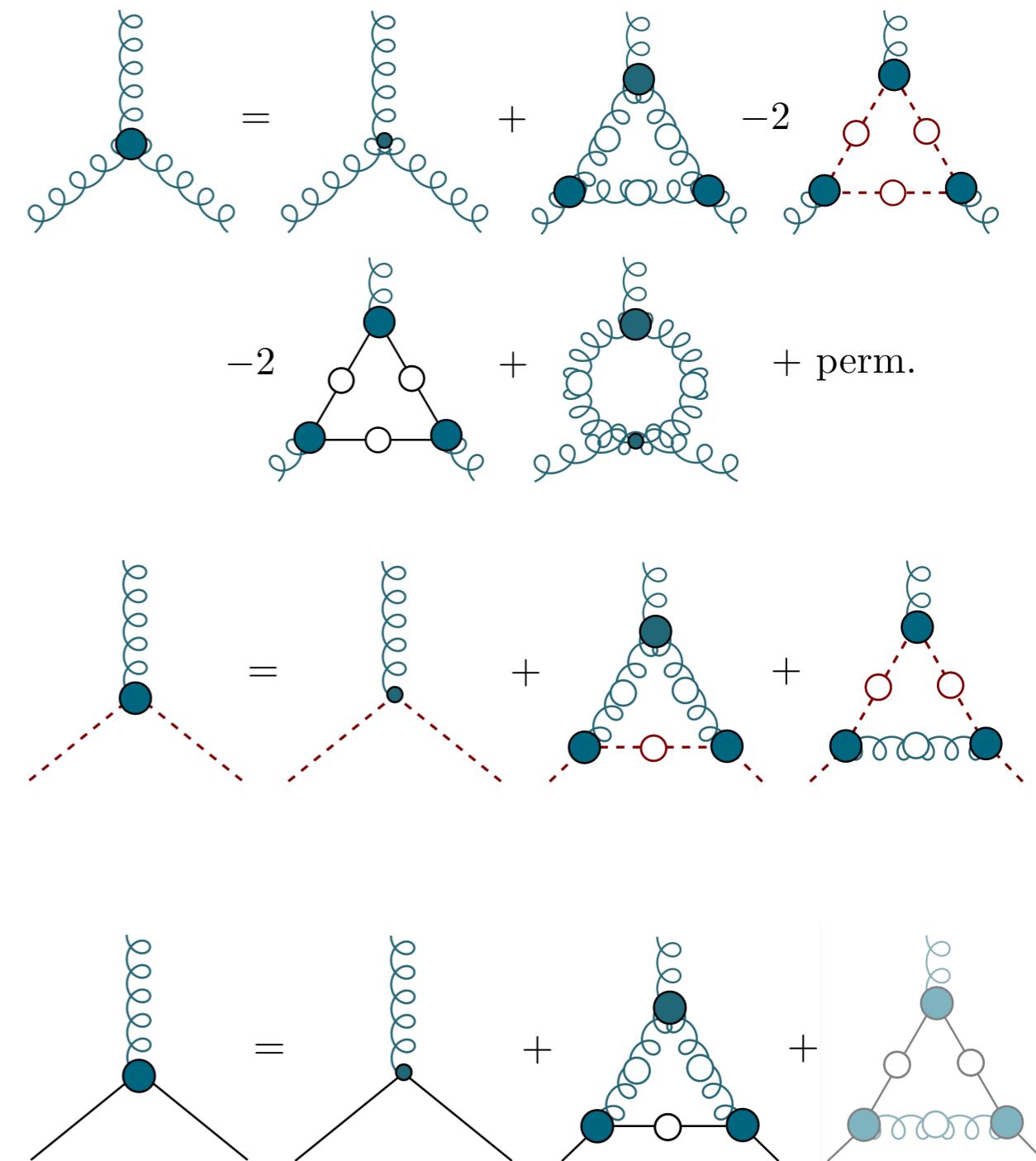
Dyson-Schwinger equations - “3PI vs RL”

$$\mathcal{Z}_{QCD} = \int \mathcal{D}[\Psi, A] \exp \left\{ - \int d^4x \left(\bar{\Psi} (i \not{D} - m) \Psi - \frac{1}{4} (F_{\mu\nu}^a)^2 \right) \right\}$$

propagators



vertices



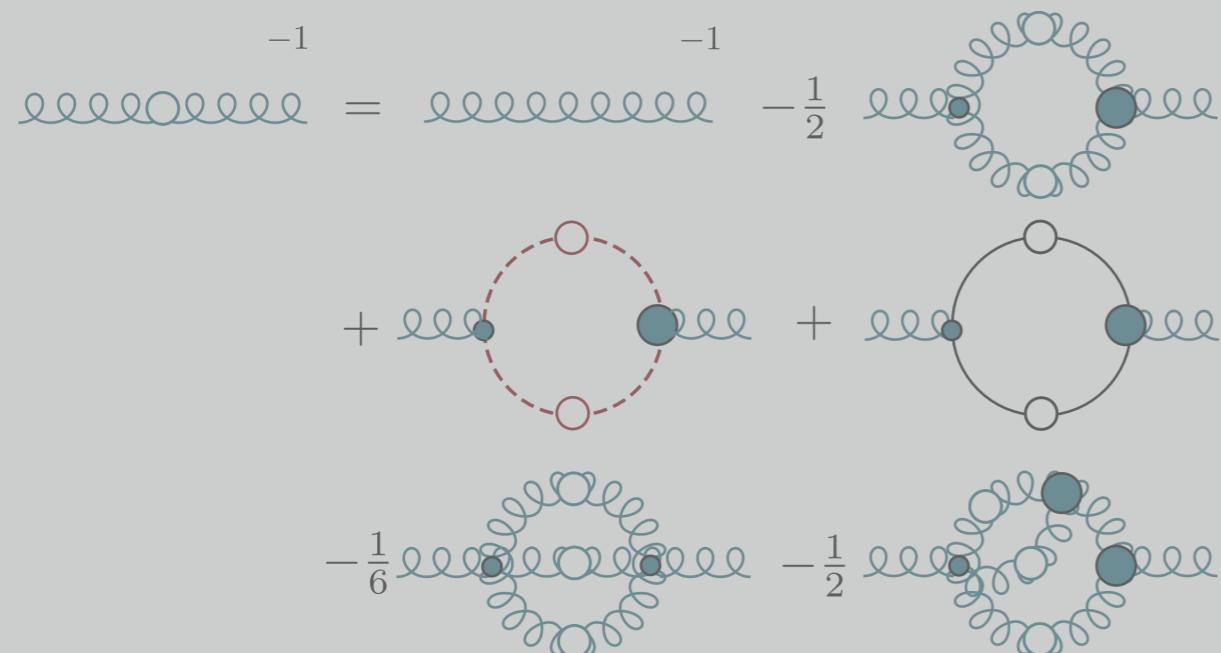
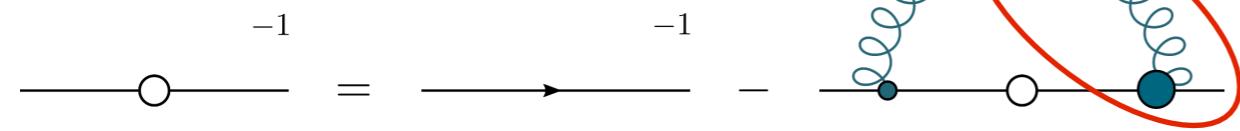
CF,Alkofer, PRD67 (2003) 094020
 Williams, CF, Heupel, PRD93 (2016) 034026
 Huber, PRD 101 (2020) 114009

see Talk of Markus Huber

Dyson-Schwinger equations - “3PI vs RL”

$$\mathcal{Z}_{QCD} = \int \mathcal{D}[\Psi, A] \exp \left\{ - \int d^4x \left(\bar{\Psi} (i \not{D} - m) \Psi - \frac{1}{4} (F_{\mu\nu}^a)^2 \right) \right\}$$

propagators



CF,Alkofer, PRD67 (2003) 094020

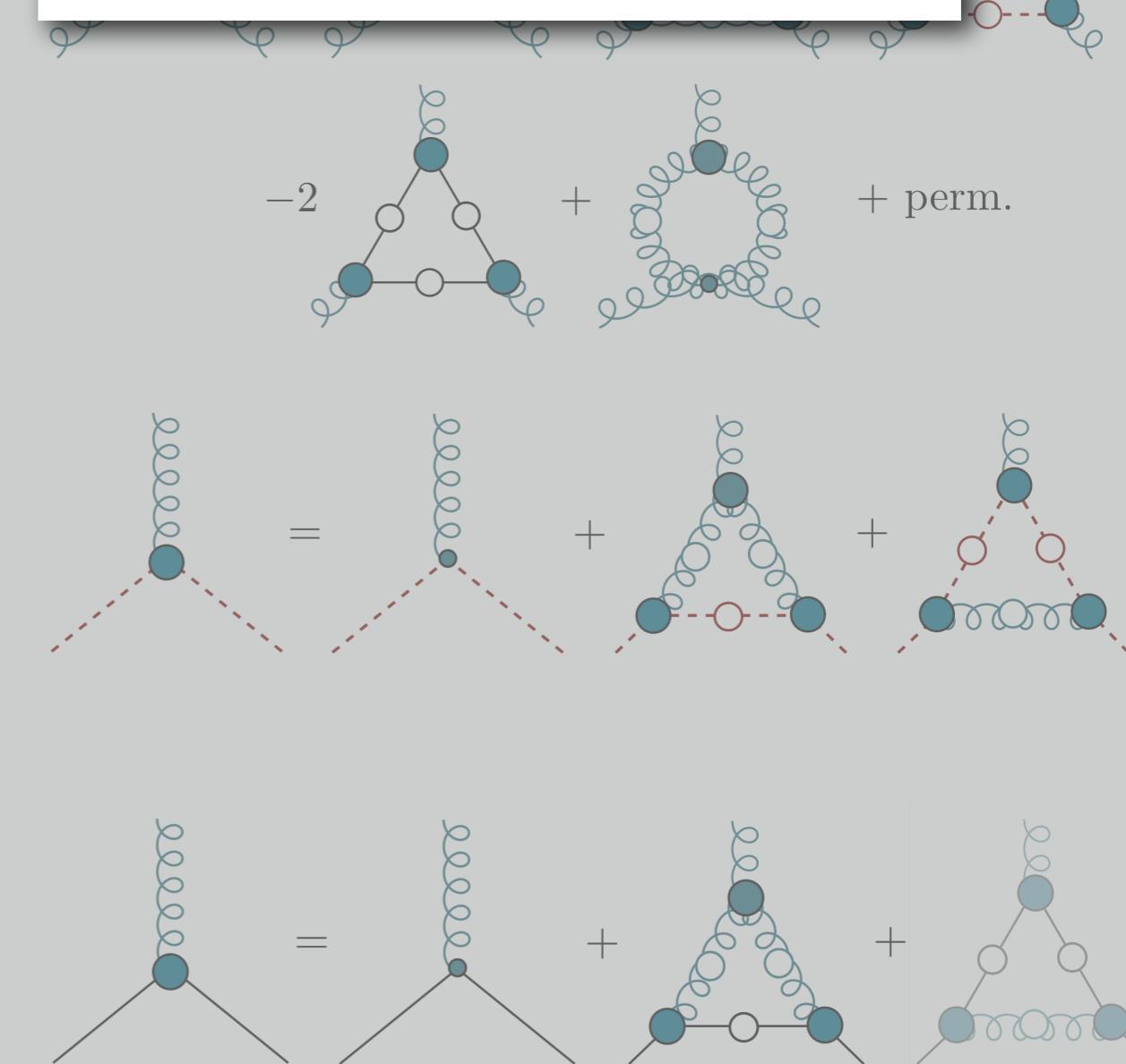
Williams, CF, Heupel, PRD93 (2016) 034026

Huber, PRD 101 (2020) 114009

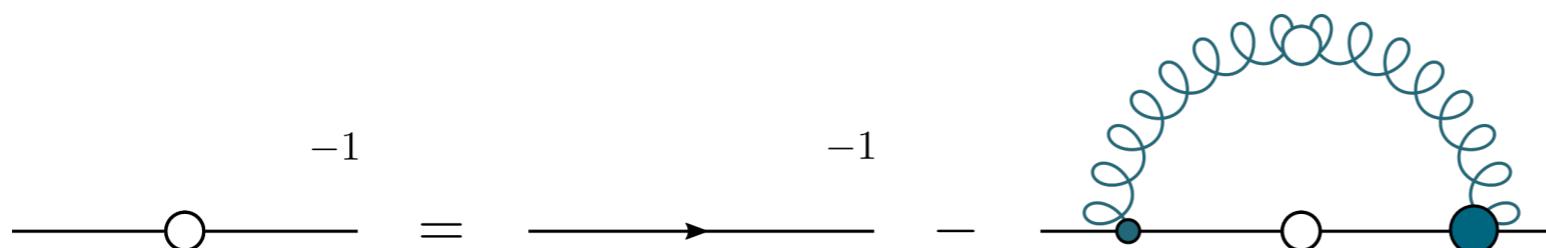
see Talk of Markus Huber

vortices

“rainbow-ladder” (RL) :
model for gluon+vertex

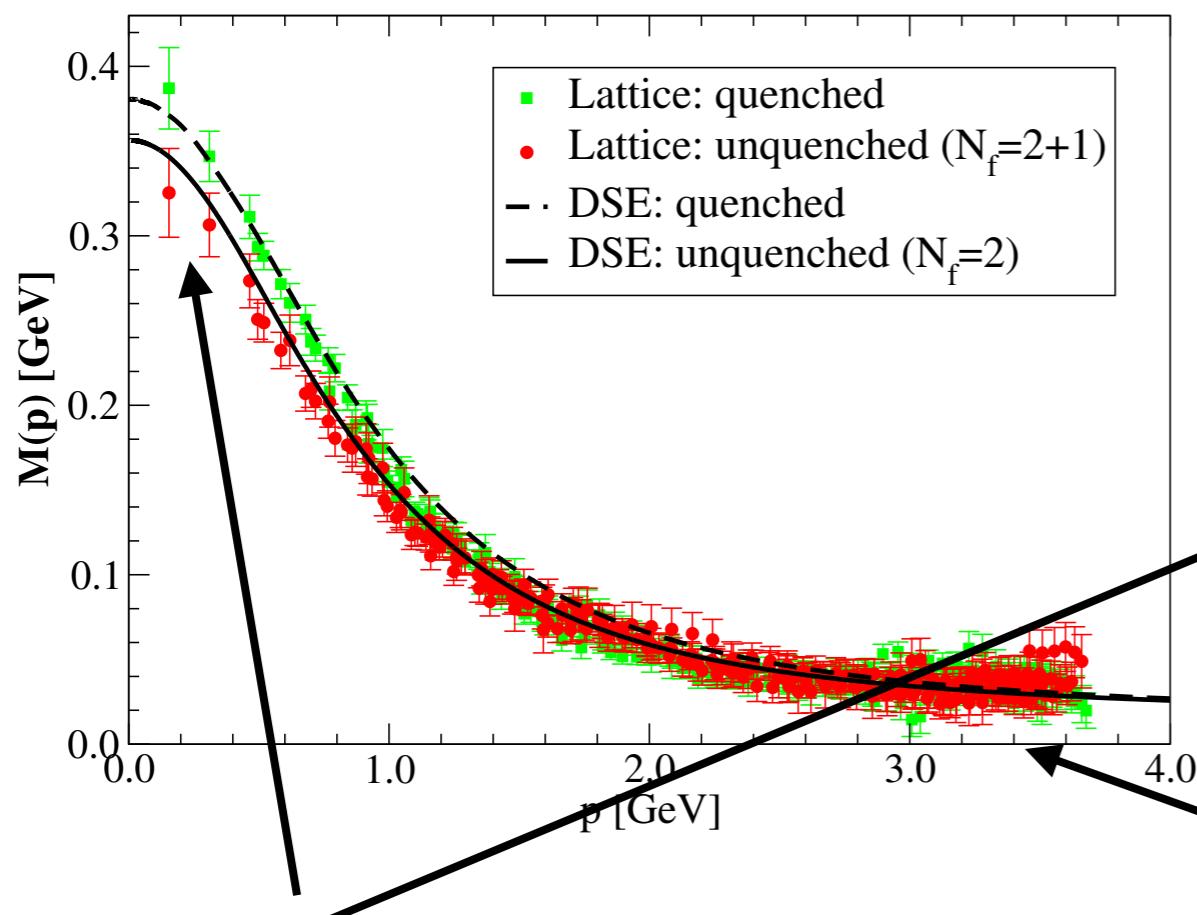


Quarks: mass from interaction

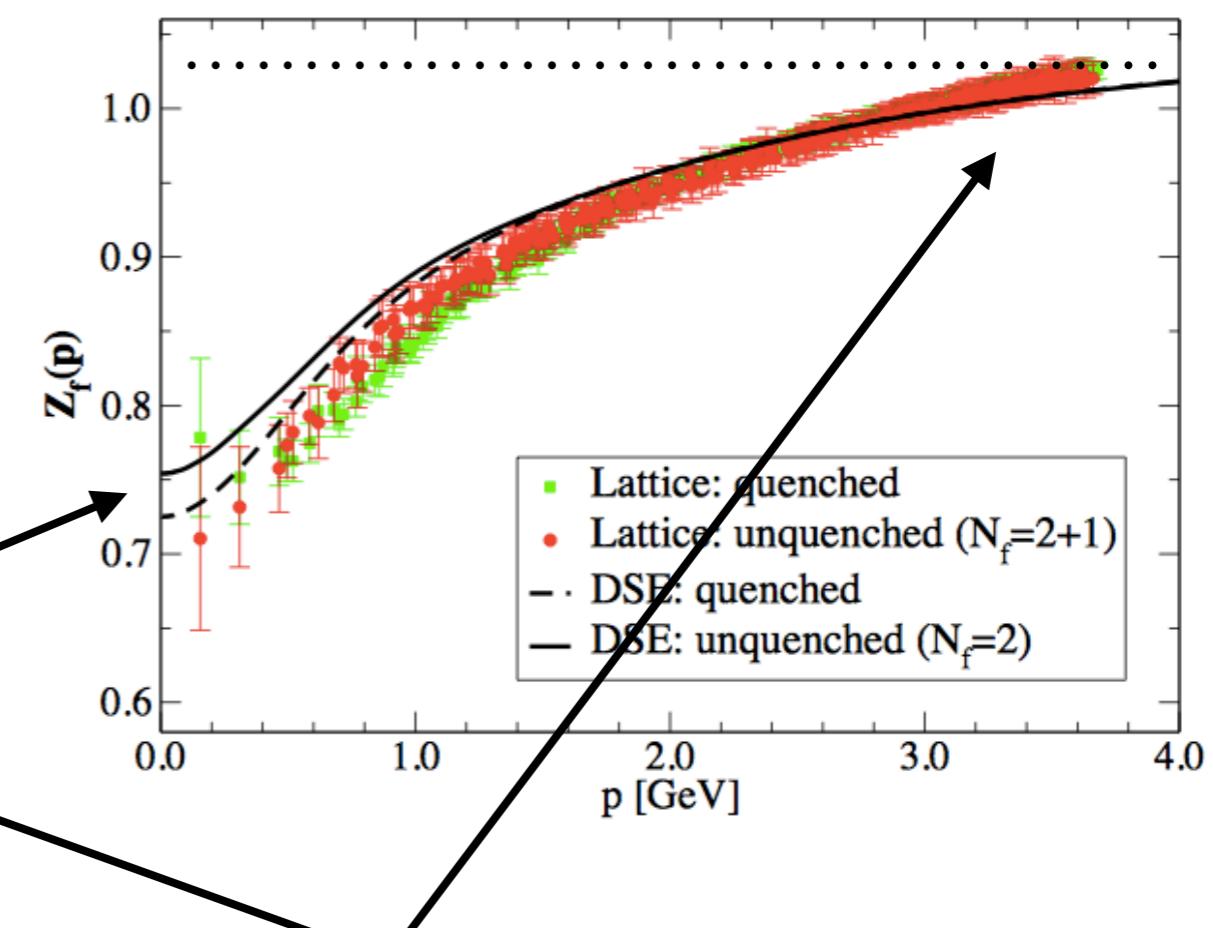


$$S(p) = Z_f(p^2) \frac{-ip + M(p^2)}{p^2 + M^2(p^2)}$$

DSE: CF, Nickel, Williams, EPJ C 60 (2009) 47
Williams, CF, Heupel, PRD 93 (2016) 034026
Lattice: P. O. Bowman, et al PRD 71 (2005) 054507

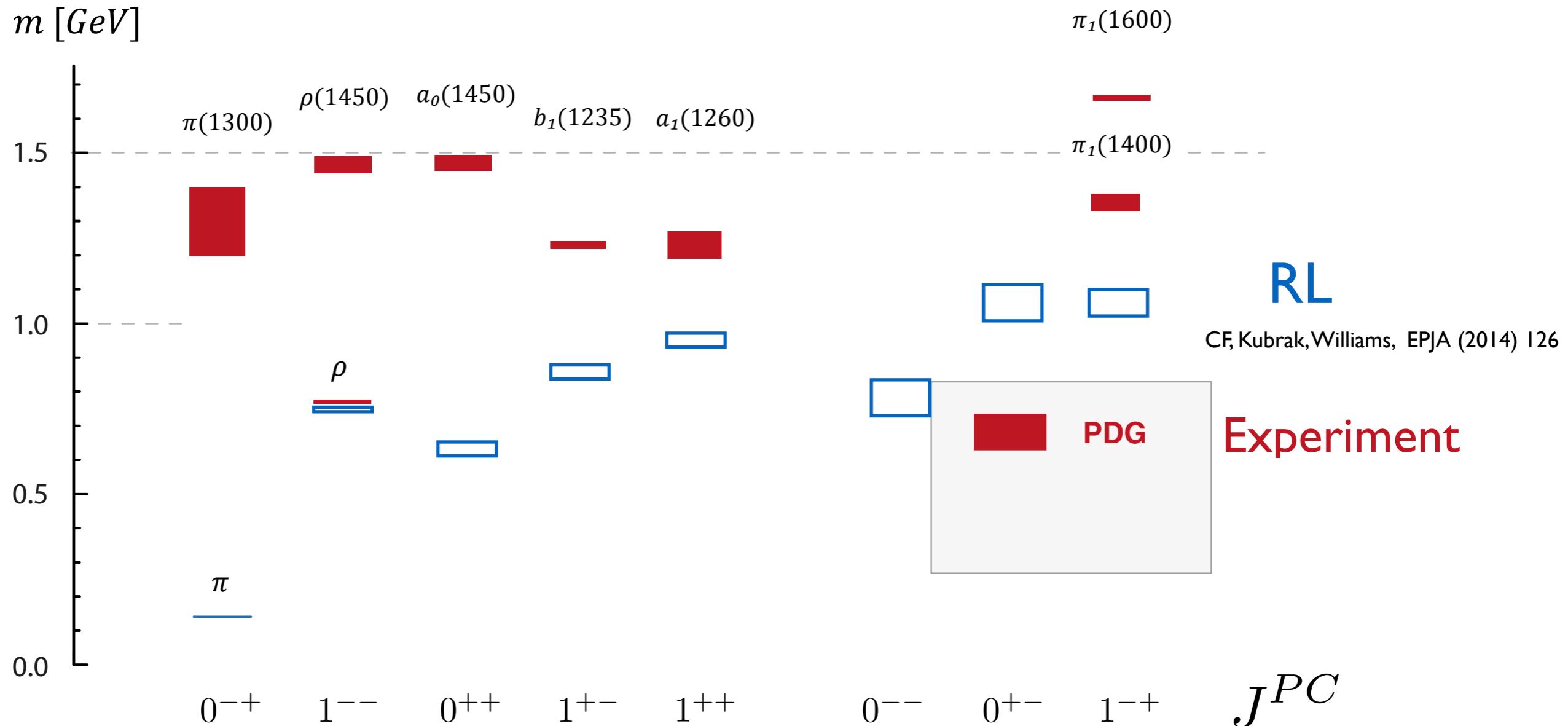


‘constituent quark’:
large mass; very composite

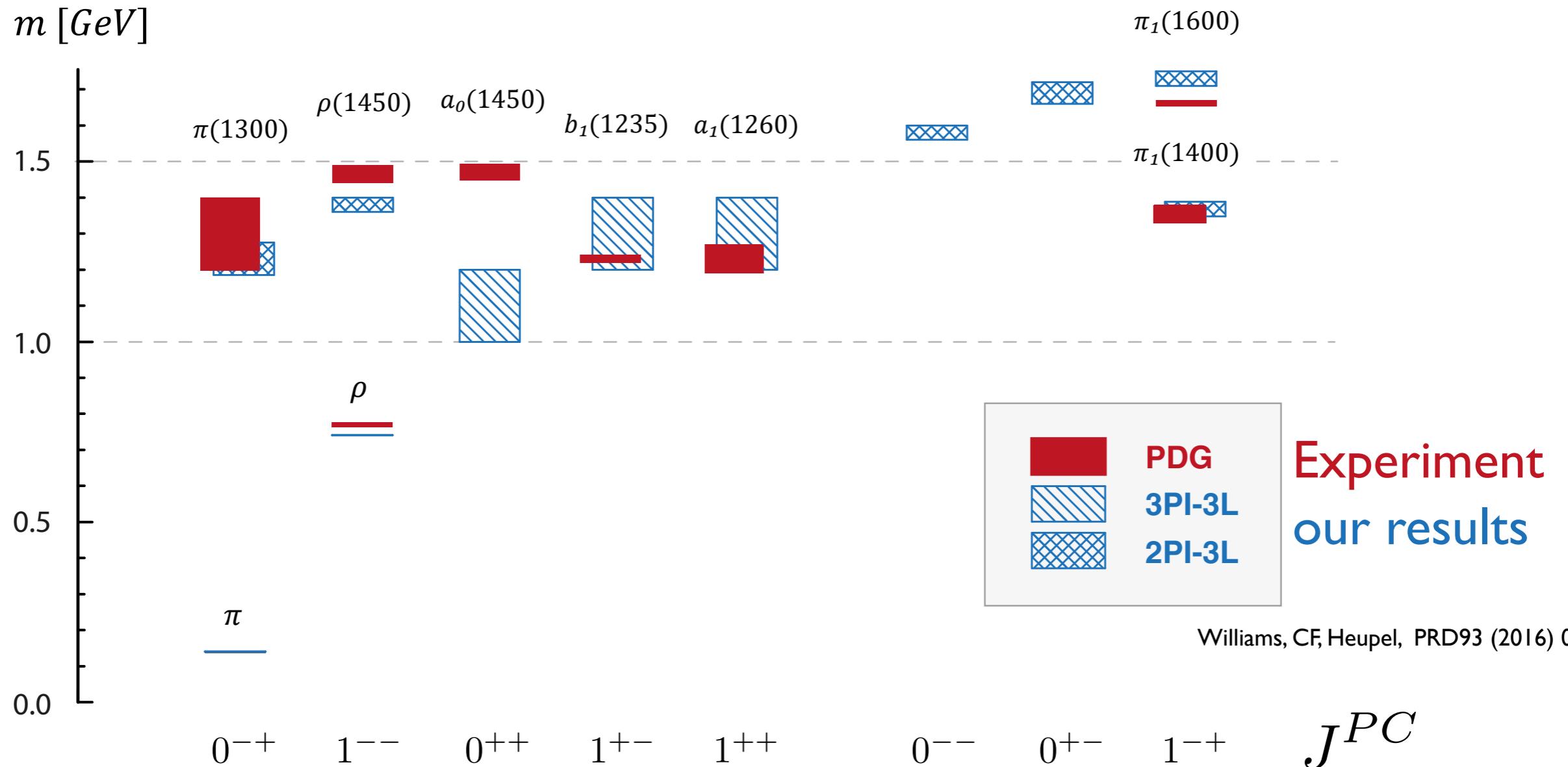


‘current quark’:
- small mass; non-composite

Light meson spectrum - full 3PI-calculation

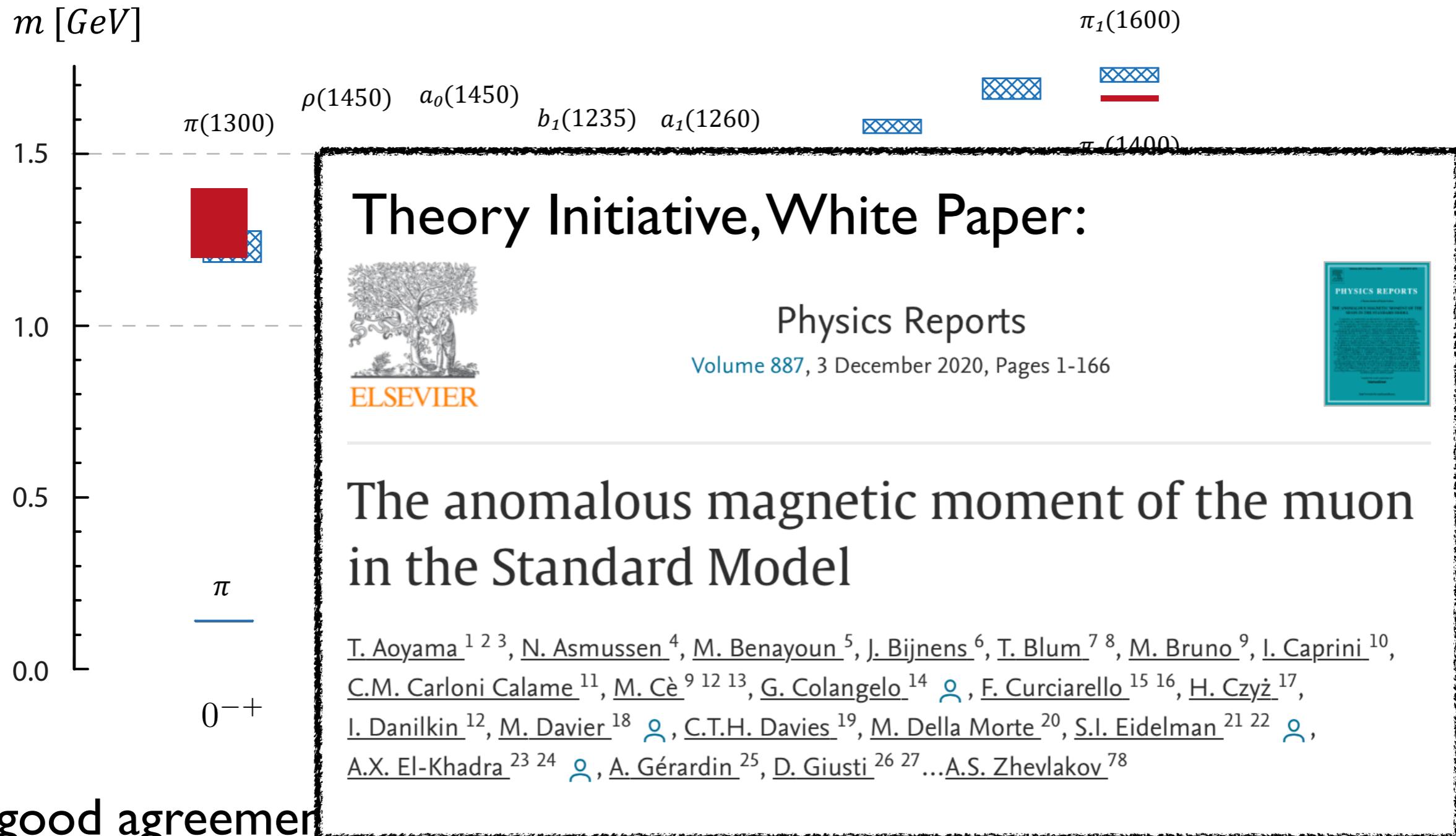


Light meson spectrum - full 3PI-calculation



- good agreement with experiment in most channels
- special channels:
 - pseudoscalar 0^{-+} : (pseudo-) Goldstone bosons
 - scalar 0^{++} : complicated channel...

Light meson spectrum - full 3PI-calculation



- good agreement
- special channels:
 - pseudoscalar 0^{-+} : (pseudo-) Goldstone bosons
 - scalar 0^{++} : complicated channel...

Tetraquarks from the four-body equation

Exact equation:

Kvinikhidze & Khvedelidze, Theor. Math. Phys. 90 (1992)
Heupel, Eichmann, CF, PLB 718 (2012) 545-549
Eichmann, CF, Heupel, PLB 753 (2016) 282-287

$$\text{Diagram} = \text{Diagram}_1 + \text{Diagram}_2 - \text{Diagram}_3 + \text{Diagram}_4 + \text{Diagram}_5 + \text{perm.}$$

+ perm.

+ perm.

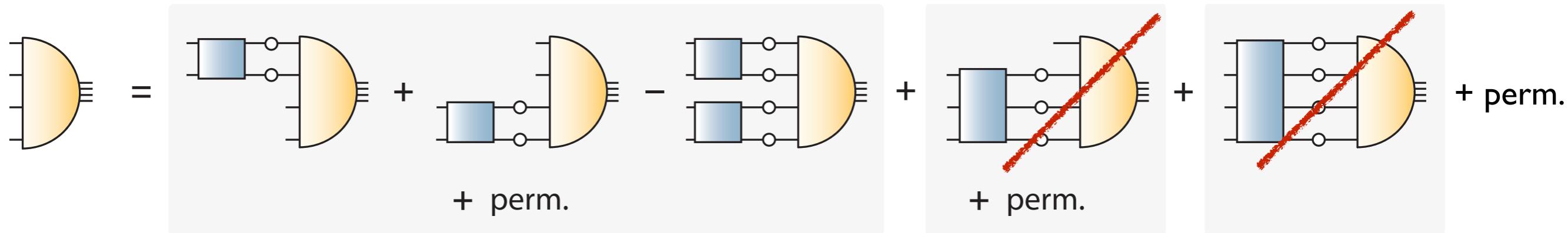
Two-body interactions

Three- and four-body interactions

Tetraquarks from the four-body equation

Exact equation:

Kvinikhidze & Khvedelidze, Theor. Math. Phys. 90 (1992)
Heupel, Eichmann, CF, PLB 718 (2012) 545-549
Eichmann, CF, Heupel, PLB 753 (2016) 282-287



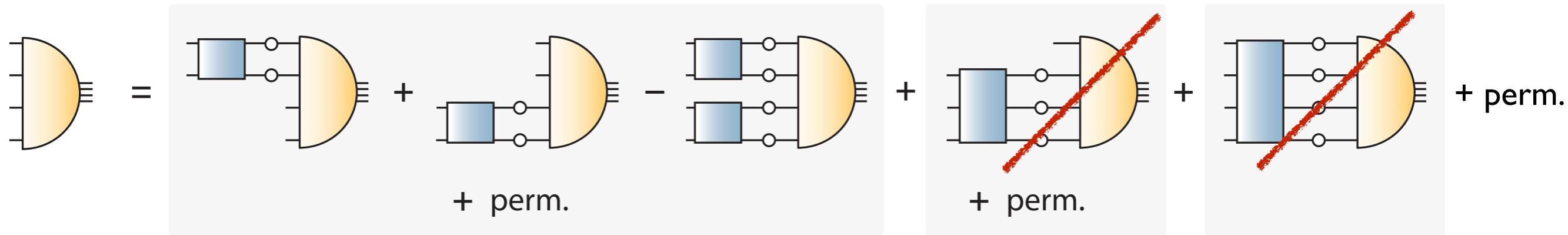
Two-body interactions

Three- and four-body interactions

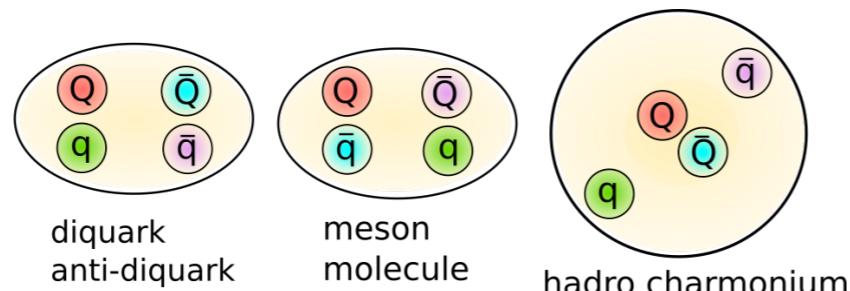
Tetraquarks from the four-body equation

Exact equation:

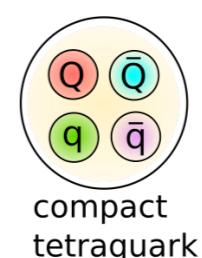
Kvinikhidze & Khvedelidze, Theor. Math. Phys. 90 (1992)
Heupel, Eichmann, CF, PLB 718 (2012) 545-549
Eichmann, CF, Heupel, PLB 753 (2016) 282-287



Two-body interactions



Three- and four-body interactions

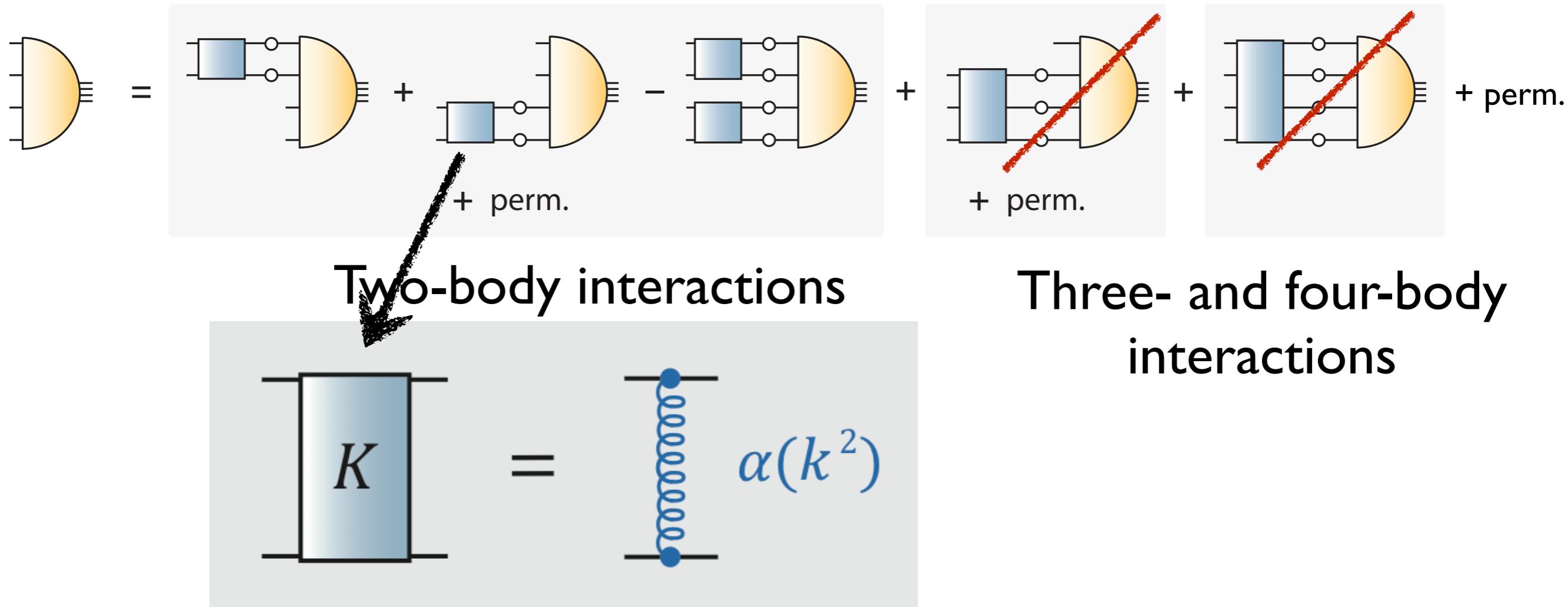


- Two-body interactions: allow for **internal clustering**
- use rainbow-ladder approximation...

Tetraquarks from the four-body equation

Exact equation:

Kvinikhidze & Khvedelidze, Theor. Math. Phys. 90 (1992)
Heupel, Eichmann, CF, PLB 718 (2012) 545-549
Eichmann, CF, Heupel, PLB 753 (2016) 282-287



- Input: Non-perturbative quark, quark-gluon interaction

$$\text{---} \circ \text{---}^{-1} = \text{---} \text{---}^{-1} + \text{---} \bullet \text{---} \circ \text{---}$$

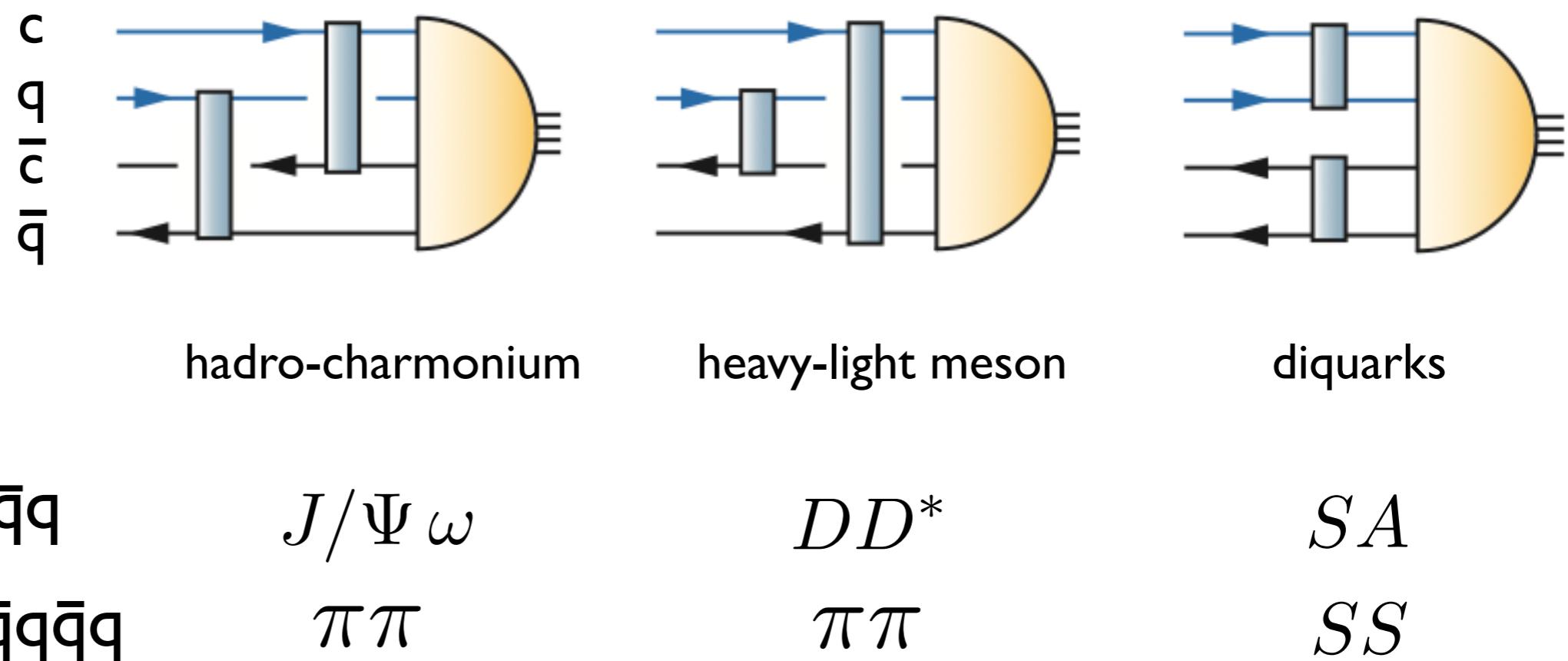
$$\alpha(k^2) = \pi \eta^7 \left(\frac{k^2}{\Lambda^2} \right) e^{-\eta^2 \left(\frac{k^2}{\Lambda^2} \right)} + \alpha_{UV}(k^2)$$

Structure of the amplitude

$$\Gamma(P, p, q, k) = \sum_i f_i(s_1, \dots, s_9) \times \tau_i(P, p, q, k) \times \text{color} \times \text{flavor}$$

↑
>256 tensor structures !!
↓

- physics-guided approximation: **s-wave tensors are most important**

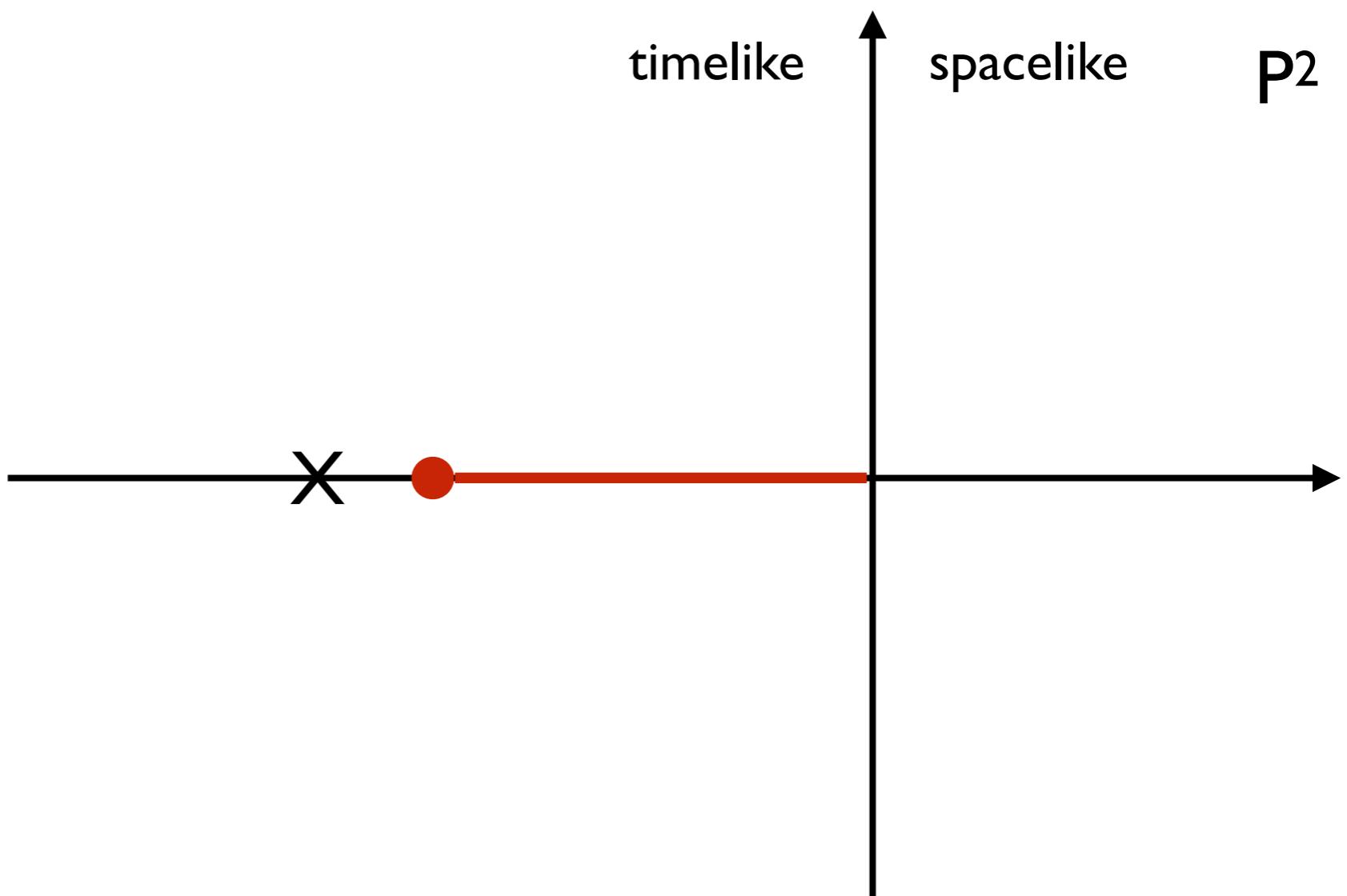


The complex P^2 -plane

$$\lambda(P^2) \circ BSA = \text{kernel} \circ BSA$$

$\lambda(P^2) \stackrel{!}{=} I$

generic situation



Williams, PLB 798 (2019) 134943, [arXiv:1804.11161]

Santowsky, Eichmann, CF, Wallbott and Williams,
PRD 102 (2020) no.5, 056014, arXiv:2007.06495.

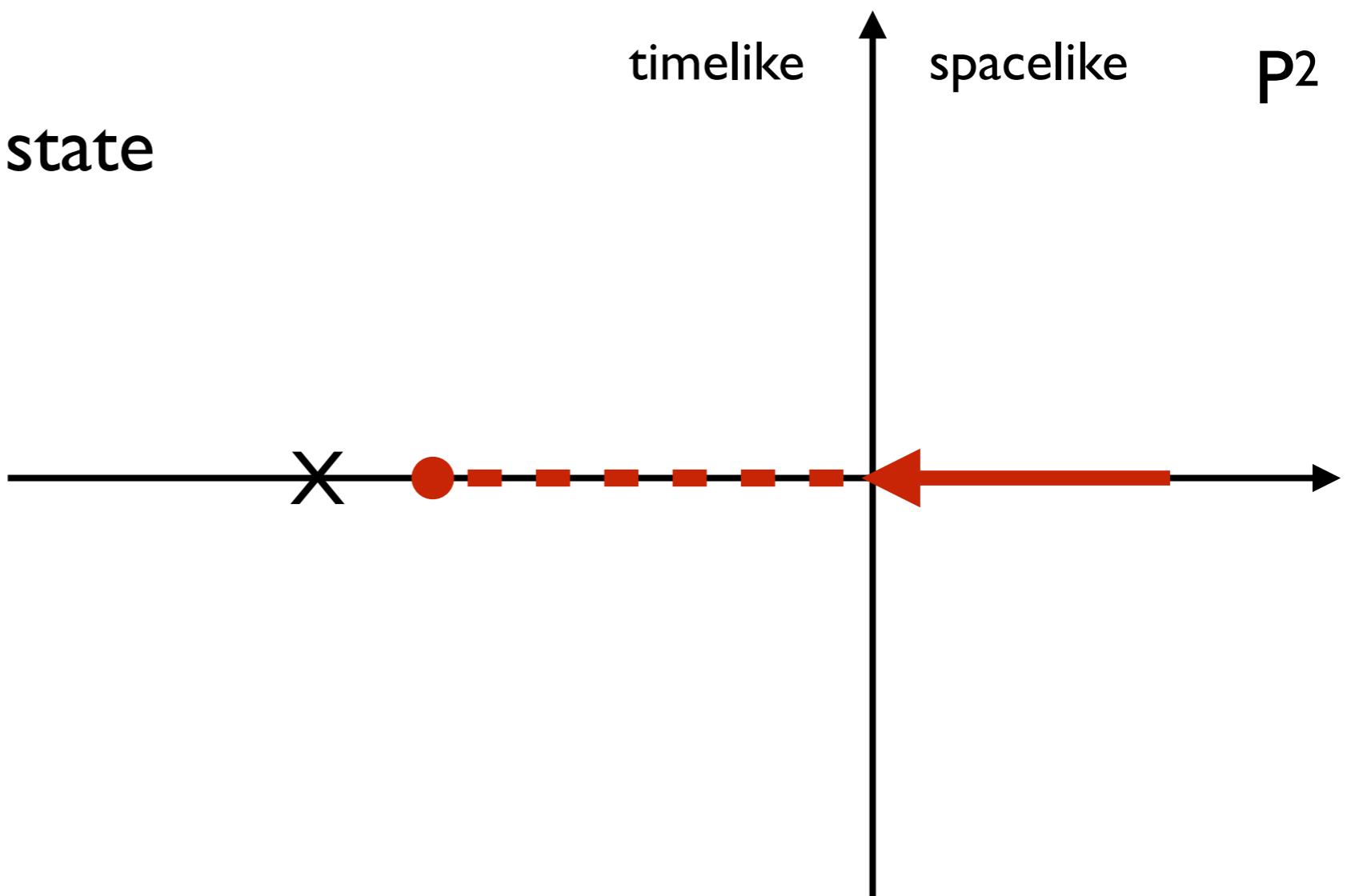
Santowsky, CF, PRD 105 (2022) 4,313; arXiv:2109.00755

The complex P²-plane

$$\lambda(P^2) \circ BSA = \text{kernel} \circ BSA$$

$\lambda(P^2) \stackrel{!}{=} I$
SPM
(see talk by Tripolt)

extrapolation to bound state



Williams, PLB 798 (2019) 134943, [arXiv:1804.11161]
Santowsky, Eichmann, CF, Wallbott and Williams,
PRD 102 (2020) no.5, 056014, arXiv:2007.06495.
Santowsky, CF, PRD 105 (2022) 4,313; arXiv:2109.00755

The complex P^2 -plane

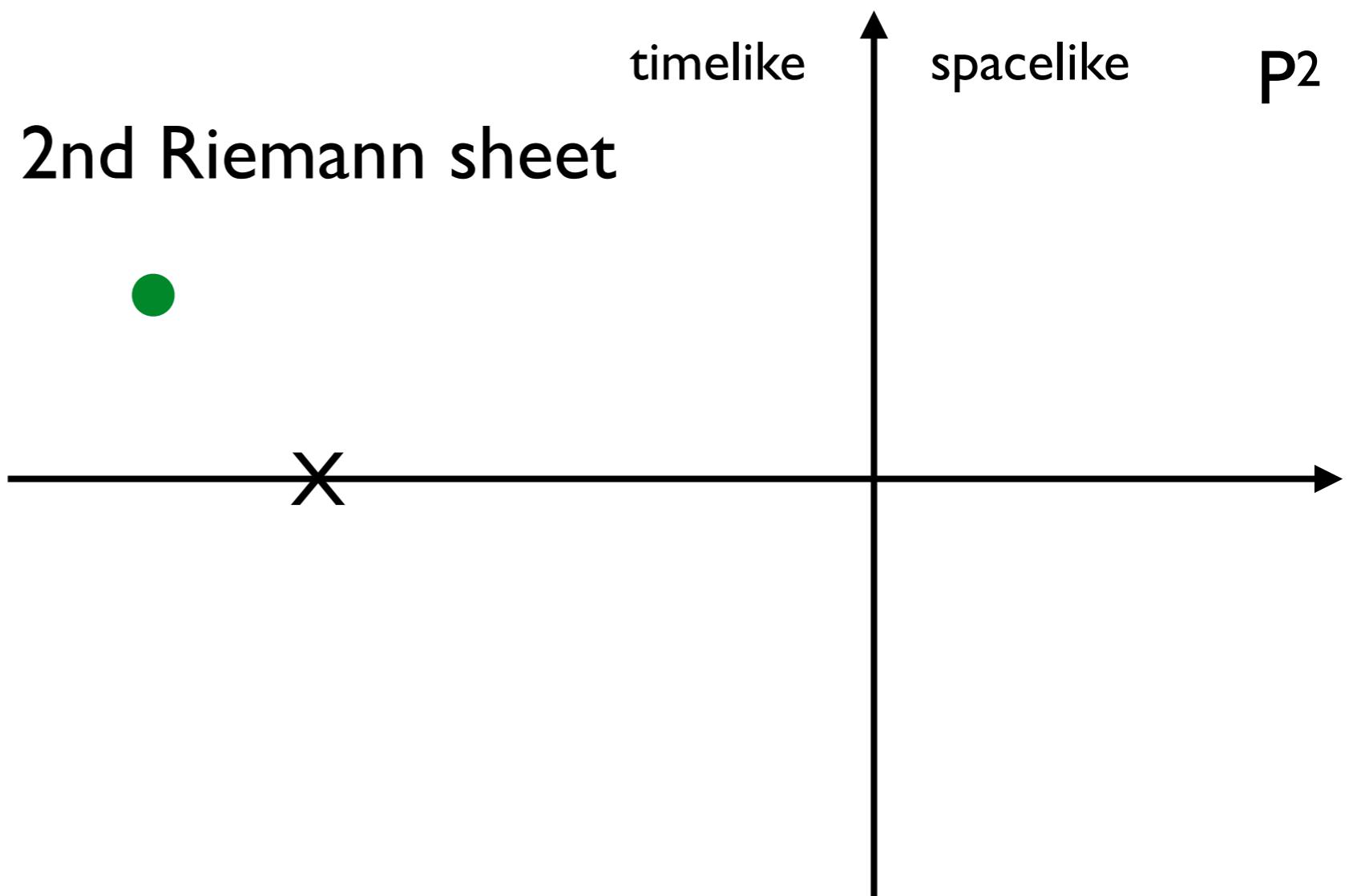
$$\lambda(P^2) \circ BSA = \text{kernel} \circ BSA$$

$\lambda(P^2) \stackrel{!}{=} I$
SPM
(see talk by Tripolt)

extrapolation to pole in 2nd Riemann sheet

$$\rho \rightarrow \pi\pi$$

$$\sigma \rightarrow \pi\pi$$



Williams, PLB 798 (2019) 134943, [arXiv:1804.11161]

Santowsky, Eichmann, CF, Wallbott and Williams,
PRD 102 (2020) no.5, 056014, arXiv:2007.06495.

Santowsky, CF, PRD 105 (2022) 4,313; arXiv:2109.00755

The complex P^2 -plane

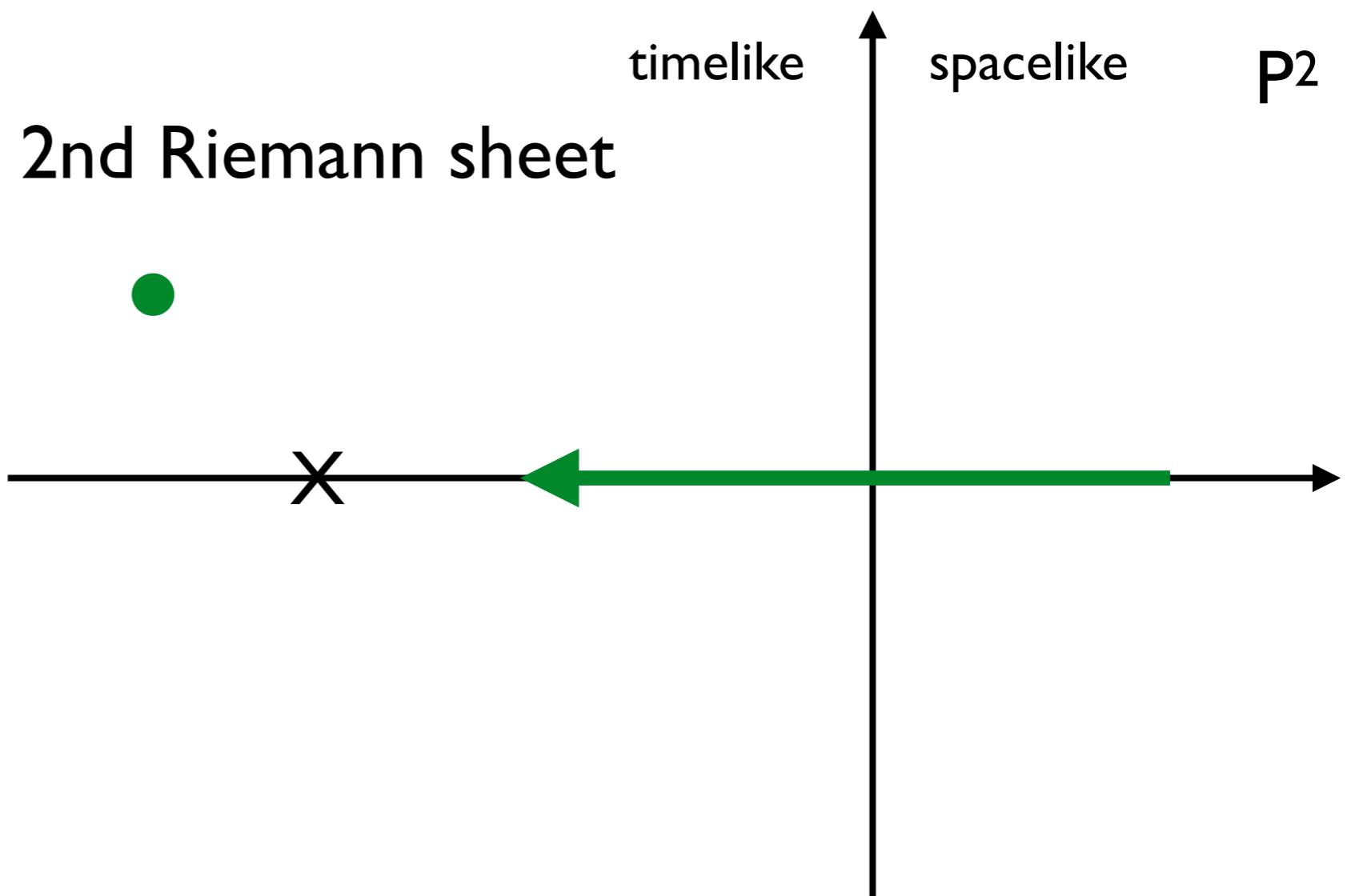
$$\lambda(P^2) \circ BSA = \text{kernel} \circ BSA$$

$\lambda(P^2) \stackrel{!}{=} I$
SPM
(see talk by Tripolt)

extrapolation to pole in 2nd Riemann sheet

$$\rho \rightarrow \pi\pi$$

$$\sigma \rightarrow \pi\pi$$



Williams, PLB 798 (2019) 134943, [arXiv:1804.11161]

Santowsky, Eichmann, CF, Wallbott and Williams,
PRD 102 (2020) no.5, 056014, arXiv:2007.06495.

Santowsky, CF, PRD 105 (2022) 4,313; arXiv:2109.00755

The complex P^2 -plane

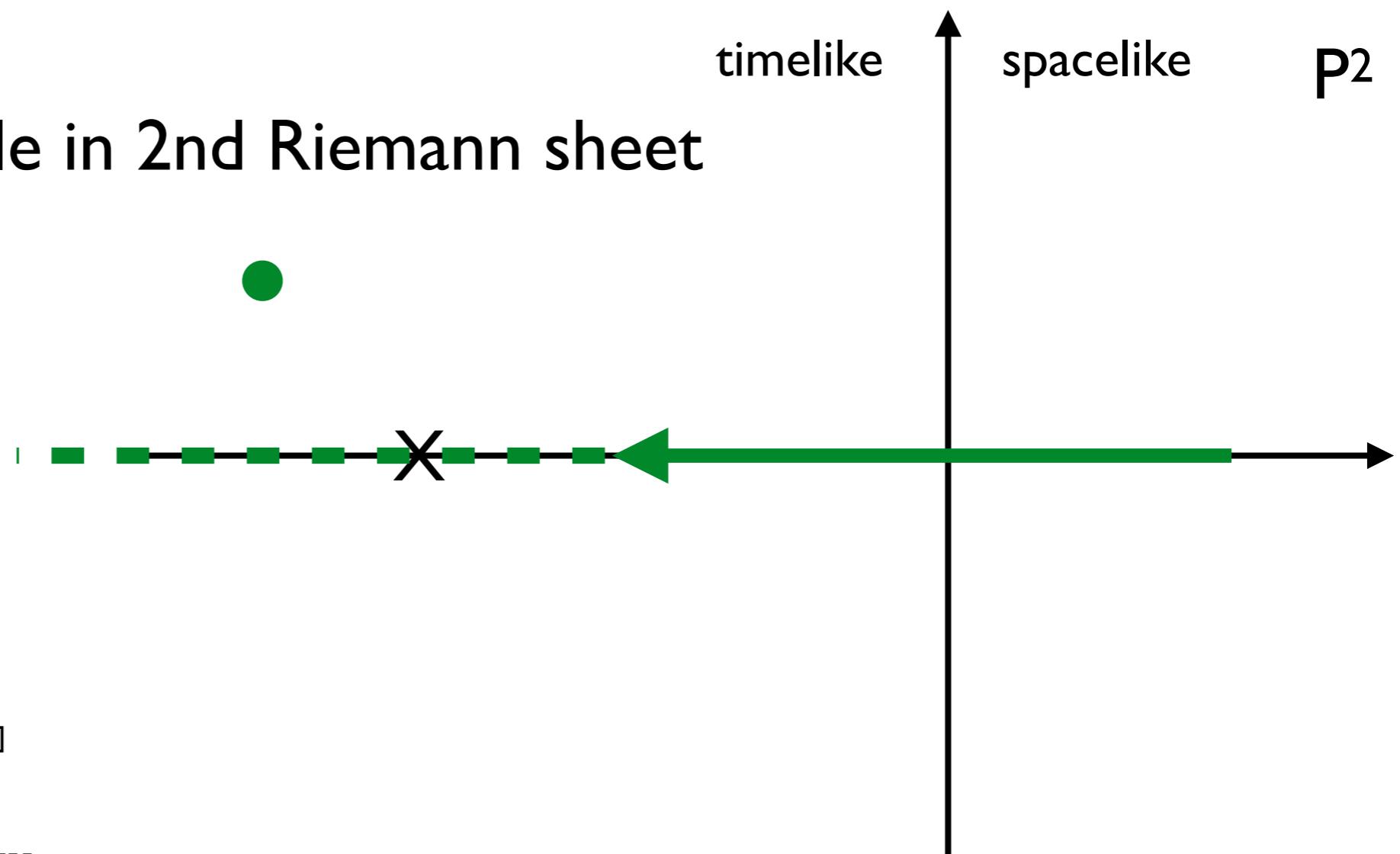
$$\lambda(P^2) \circ BSA = \text{kernel} \circ BSA$$

$\lambda(P^2) \stackrel{!}{=} I$
SPM
(see talk by Tripolt)

extrapolation to pole in 2nd Riemann sheet

$$\rho \rightarrow \pi\pi$$

$$\sigma \rightarrow \pi\pi$$



Williams, PLB 798 (2019) 134943, [arXiv:1804.11161]

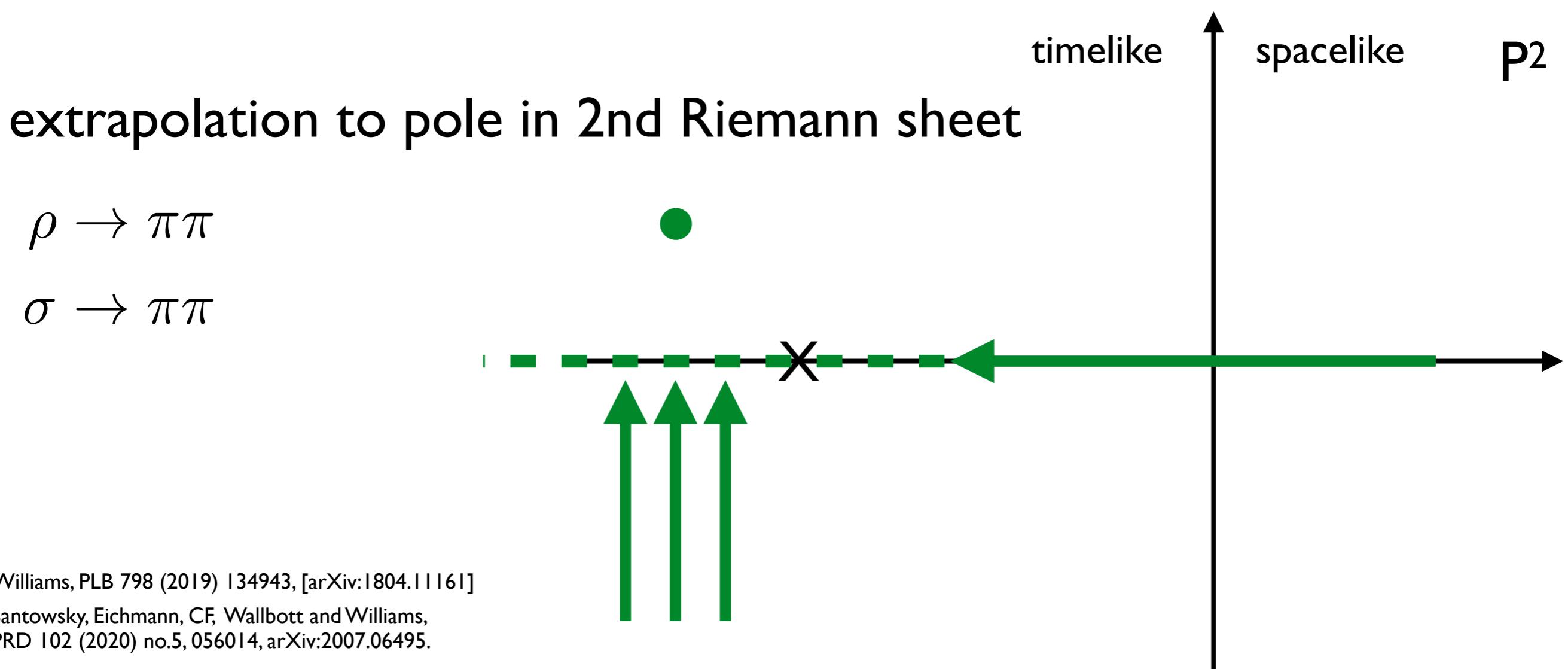
Santowsky, Eichmann, CF, Wallbott and Williams,
PRD 102 (2020) no.5, 056014, arXiv:2007.06495.

Santowsky, CF, PRD 105 (2022) 4,313; arXiv:2109.00755

The complex P^2 -plane

$$\lambda(P^2) \circ BSA = \text{kernel} \circ BSA$$

$\lambda(P^2) \stackrel{!}{=} I$
SPM
(see talk by Tripolt)



The complex P^2 -plane

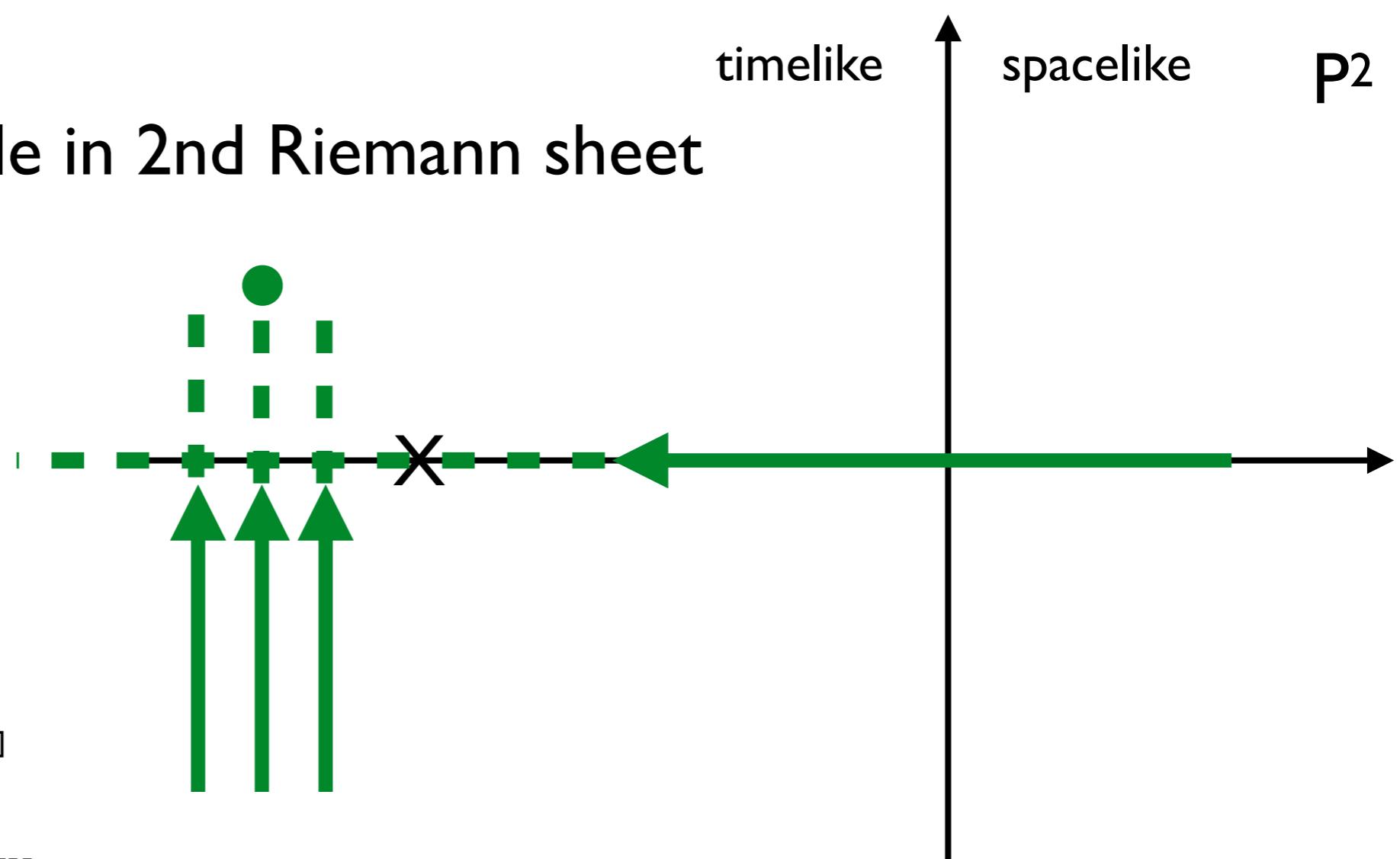
$$\lambda(P^2) \circ BSA = \text{kernel} \circ BSA$$

$\lambda(P^2) \stackrel{!}{=} I$
SPM
(see talk by Tripolt)

extrapolation to pole in 2nd Riemann sheet

$$\rho \rightarrow \pi\pi$$

$$\sigma \rightarrow \pi\pi$$



Williams, PLB 798 (2019) 134943, [arXiv:1804.11161]

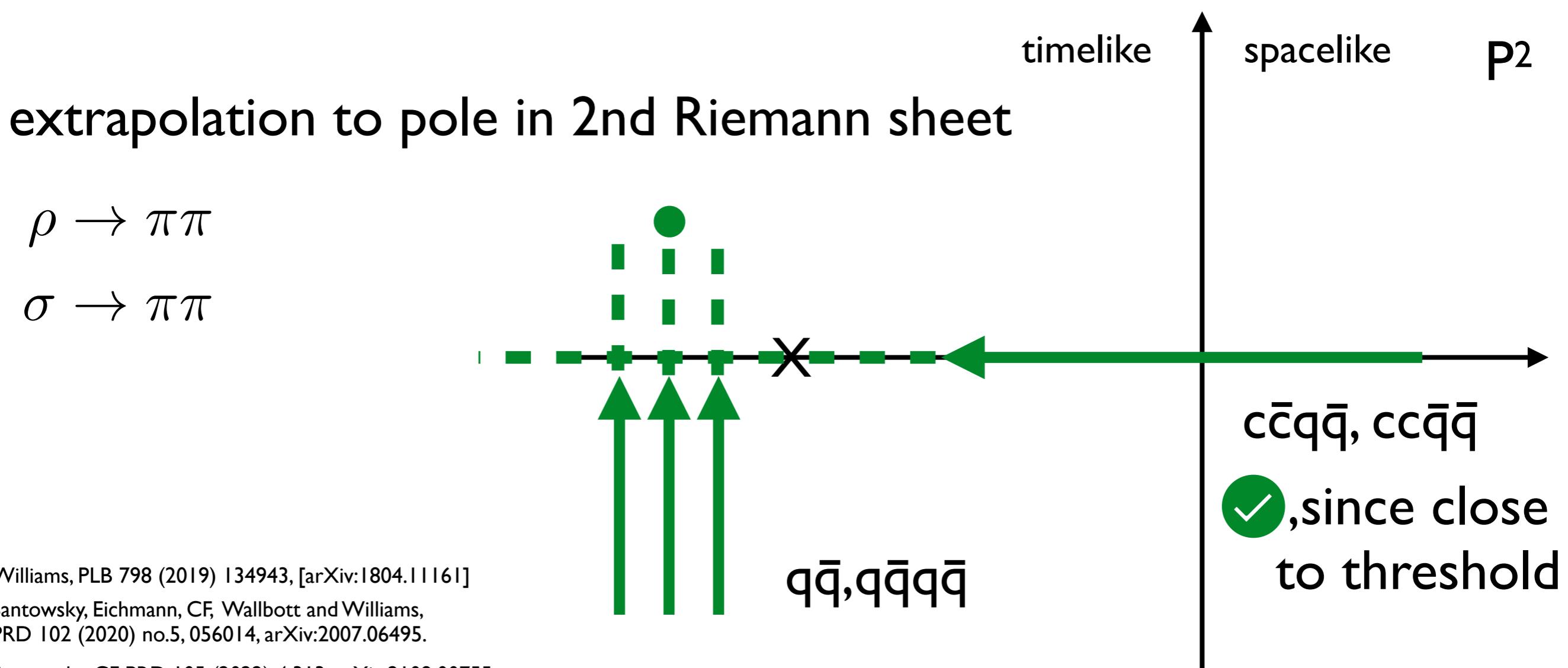
Santowsky, Eichmann, CF, Wallbott and Williams,
PRD 102 (2020) no.5, 056014, arXiv:2007.06495.

Santowsky, CF, PRD 105 (2022) 4,313; arXiv:2109.00755

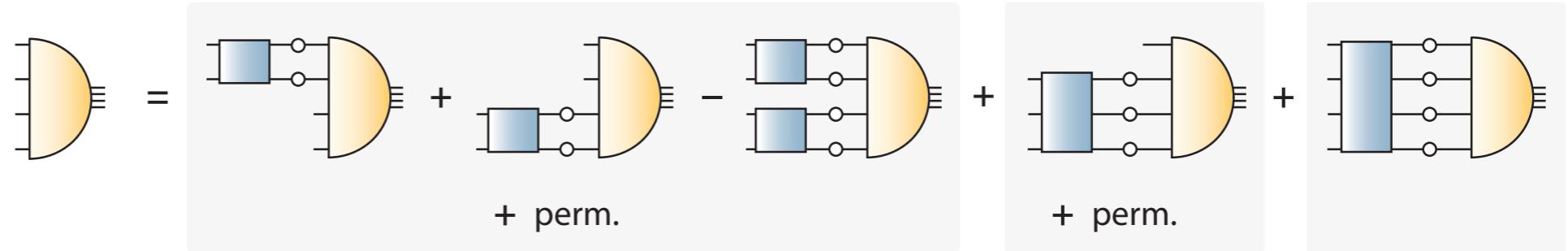
The complex P²-plane

$$\lambda(P^2) \circ BSA = \text{kernel} \circ BSA$$

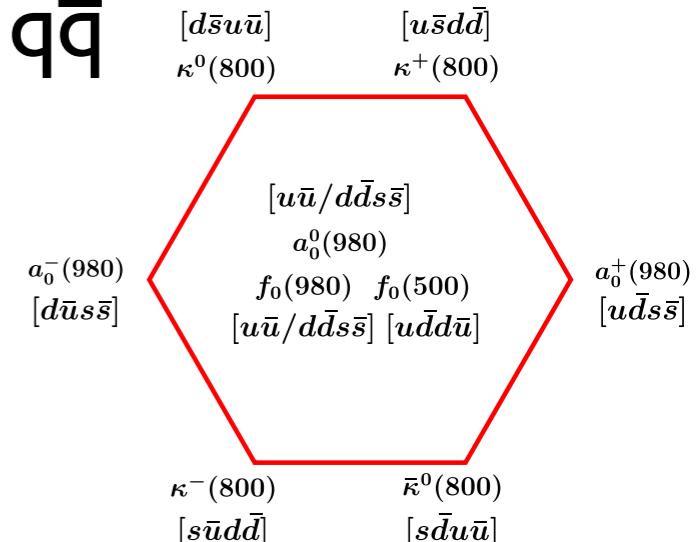
$\lambda(P^2) \stackrel{!}{=} I$
SPM
(see talk by Tripolt)



I. The framework and conventional mesons

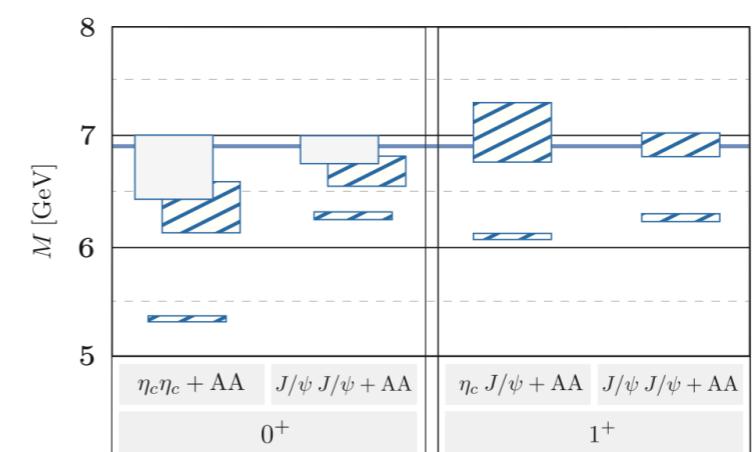


2. Detour: light four-quark states and mixing with $q\bar{q}$



3. Heavy-light and heavy-heavy four-quark states

	$I(J^{PC})$	dominant	4-body	eff. 2-body	Exp.
hidden charm $(c\bar{c}q\bar{q})$	0(0^{++})	DD	3.20 (11)	3.49 (25)	
	0(1^{++})	DD*	3.92 (7)	3.85 (18)	$X(3872)$
	1(1^{+-})	DD*	3.74 (9)	3.79 (31)	$Z_c(3900)$
	1(0^{++})	DD		3.20 (31)	
open charm $(cc\bar{q}\bar{q})$	0(1^+)	DD*	3.90 (8)	3.49 (48)	$T_{cc}(3875)$
	1(0^+)	DD+AA	3.80 (10)	3.21 (2)	
	1(1^+)	DD*+AA	4.22 (44)	3.47 (24)	



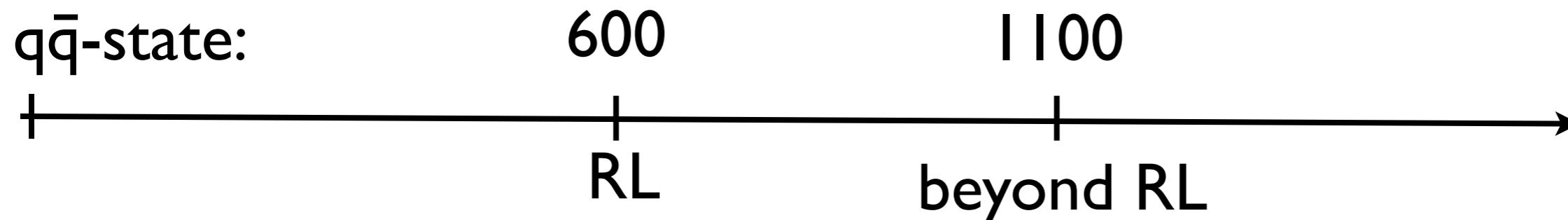
Bound state vs resonance: light scalars

$q\bar{q}$ -state:



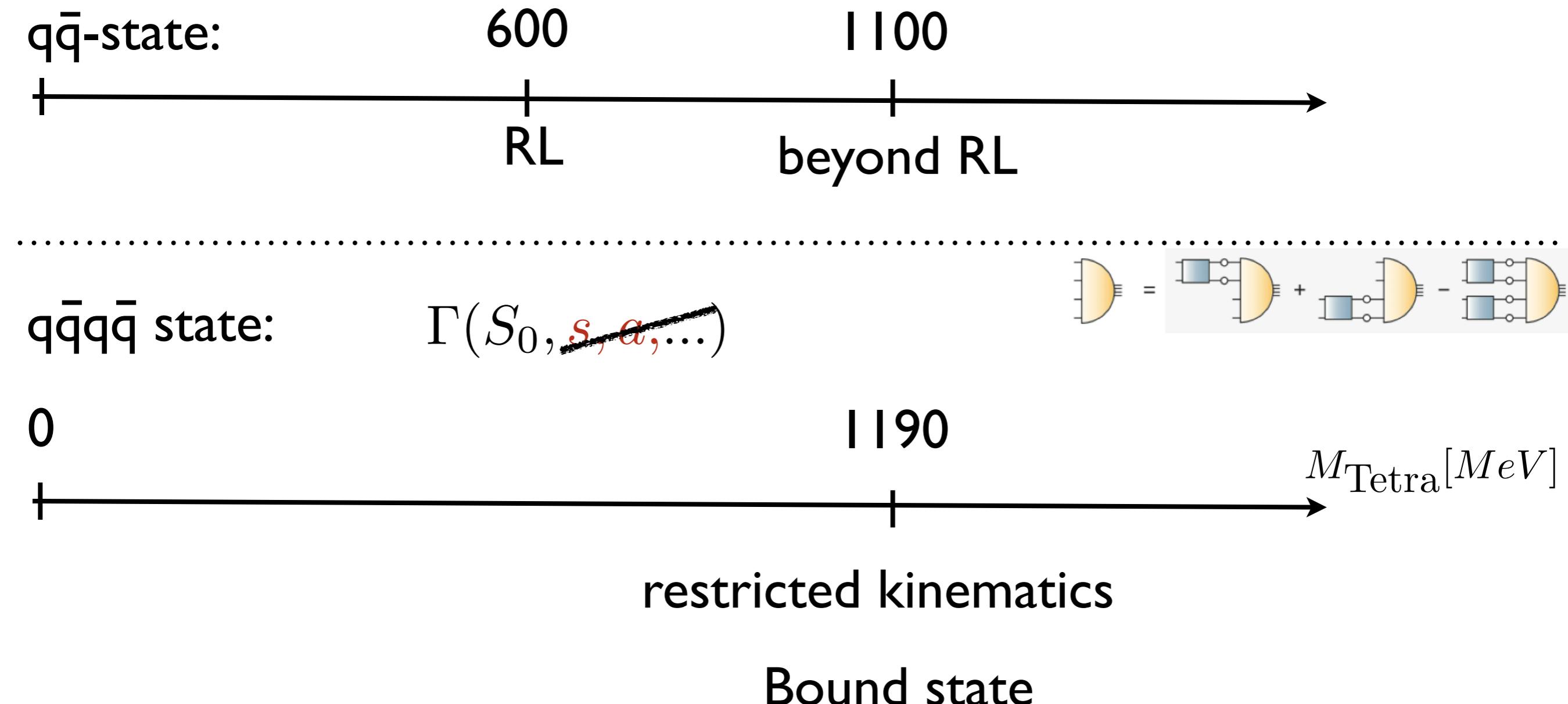
Eichmann, CF, Heupel, PLB 753 (2016) 282-287
Santowsky, CF, PRD 105 (2022) 4,313

Bound state vs resonance: light scalars



Eichmann, CF, Heupel, PLB 753 (2016) 282-287
Santowsky, CF, PRD 105 (2022) 4,313

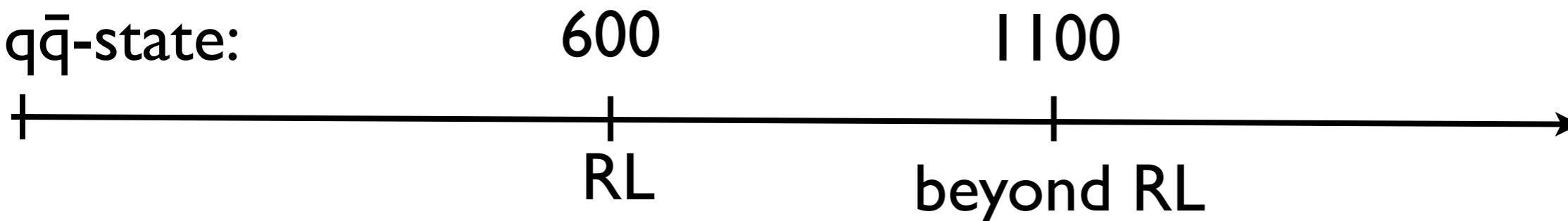
Bound state vs resonance: light scalars



Eichmann, CF, Heupel, PLB 753 (2016) 282-287
Santowsky, CF, PRD 105 (2022) 4,313

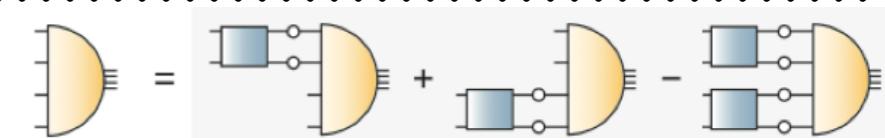
Bound state vs resonance: light scalars

q \bar{q} -state:



$q\bar{q}q\bar{q}$ state:

$$\Gamma(S_0, s, a, \dots)$$



0 400



full kinematics

restricted kinematics

Two-pion resonance

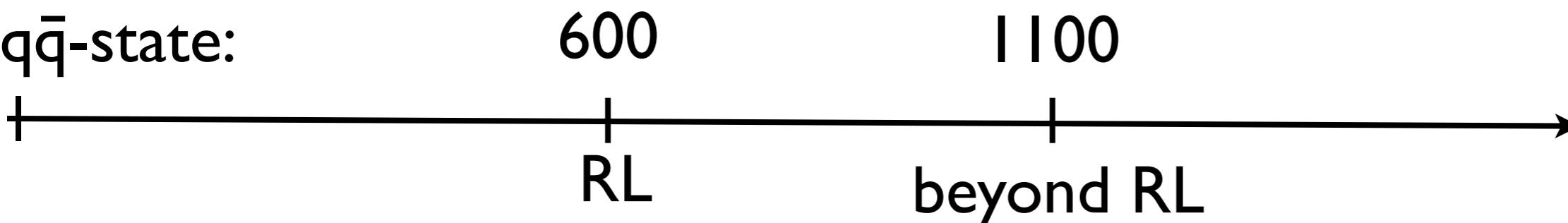
Bound state

→ identify with $f_0(500)$ (' σ -meson')

Eichmann, CF, Heupel, PLB 753 (2016) 282-287
Santowsky, CF, PRD 105 (2022) 4,313

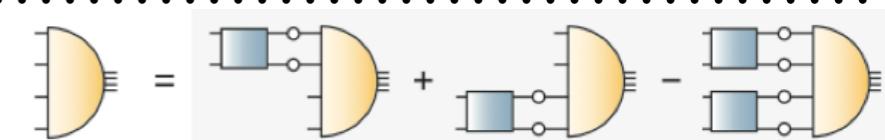
Bound state vs resonance: light scalars

$q\bar{q}$ -state:



$q\bar{q}q\bar{q}$ state:

$$\Gamma(S_0, s, a, \dots)$$



0

400

1190

$M_{\text{Tetra}} [MeV]$

+

|

full kinematics

restricted kinematics

Two-pion resonance

Bound state

→ identify with $f_0(500)$ (' σ -meson')

with strange quarks: $m(a_0, f_0) \approx 1 GeV$

Eichmann, CF, Heupel, PLB 753 (2016) 282-287
Santowsky, CF, PRD 105 (2022) 4,313

Mass evolution of four-quark state: 0++

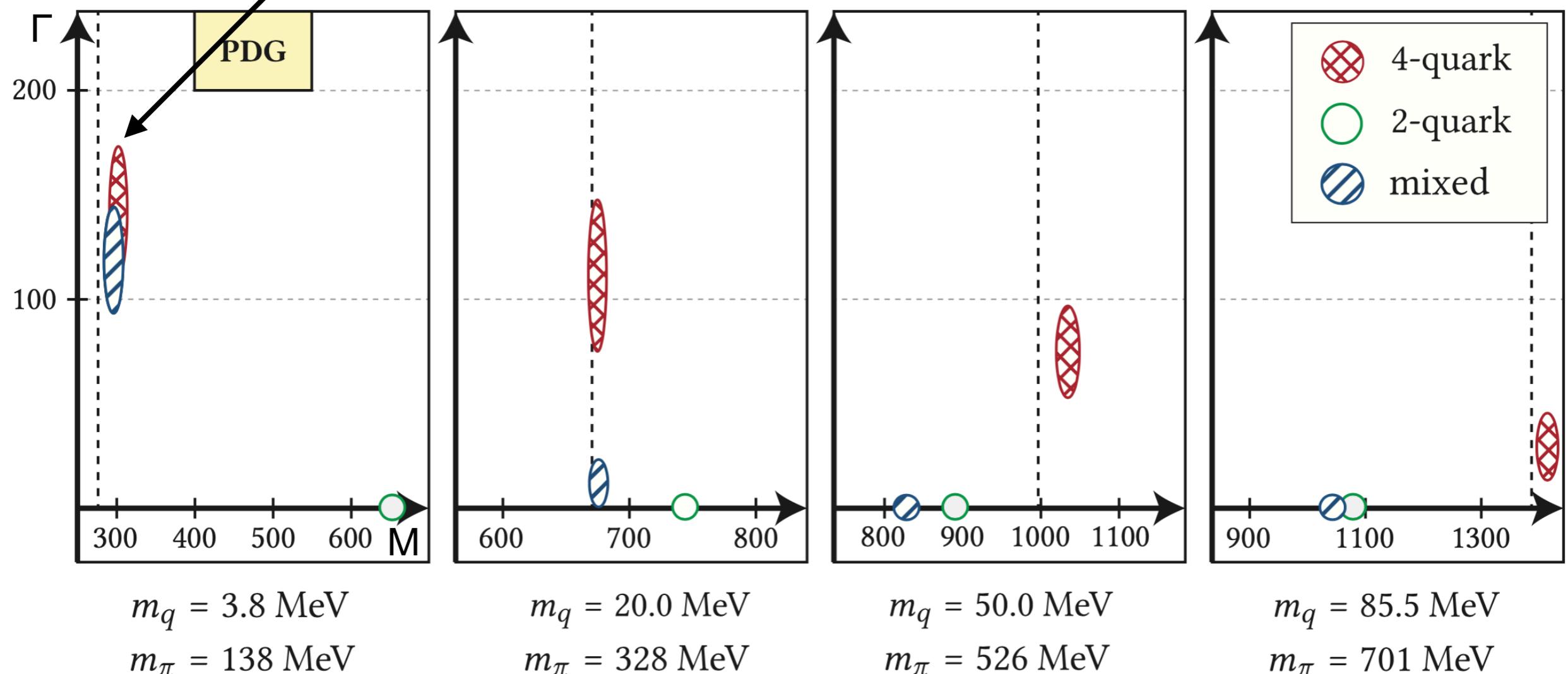
$f_0(500) : \pi\pi$ – component dominates!



Santowsky, CF, PRD 105 (2022) 4,313; arXiv:2109.00755

Mass evolution of four-quark state: 0++

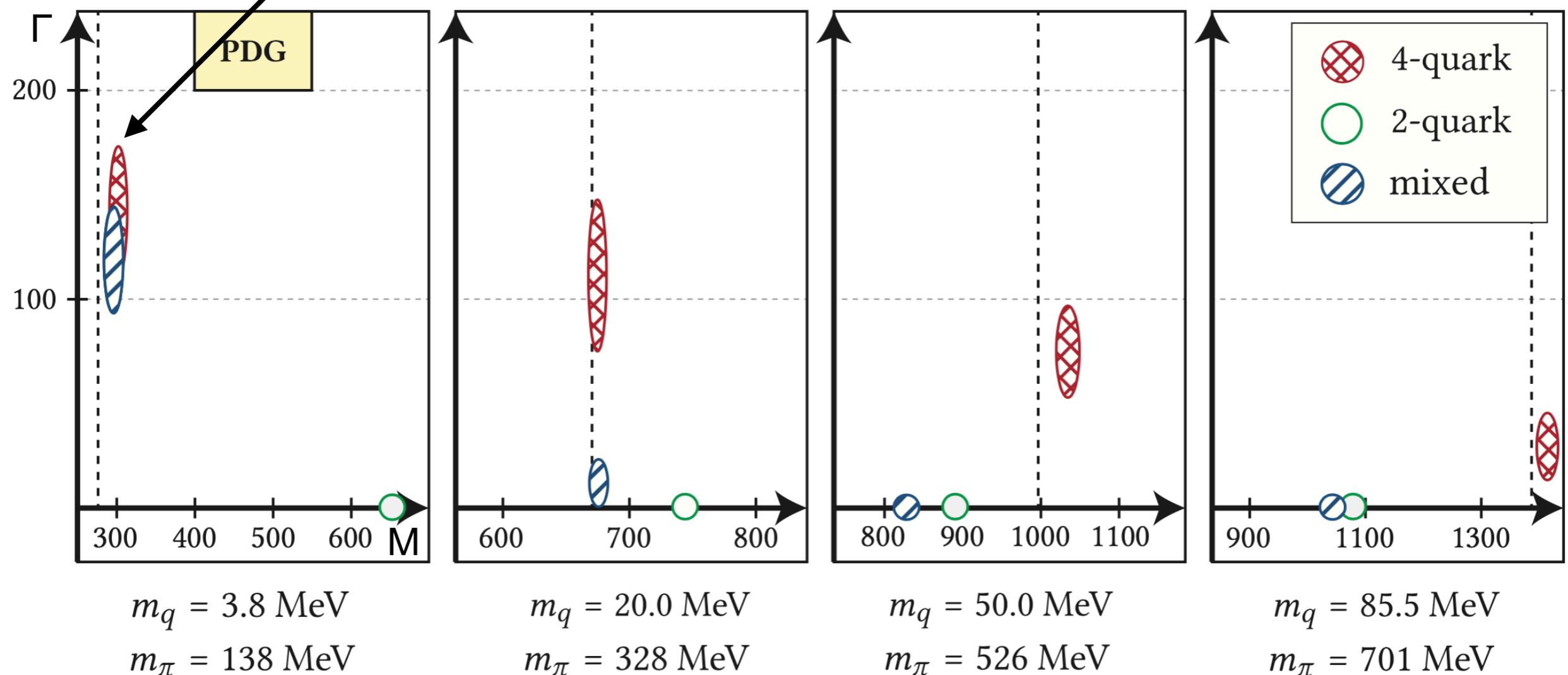
$f_0(500) : \pi\pi$ – component dominates!



Santowsky, CF, PRD 105 (2022) 4,313; arXiv:2109.00755

Mass evolution of four-quark state: 0^{++}

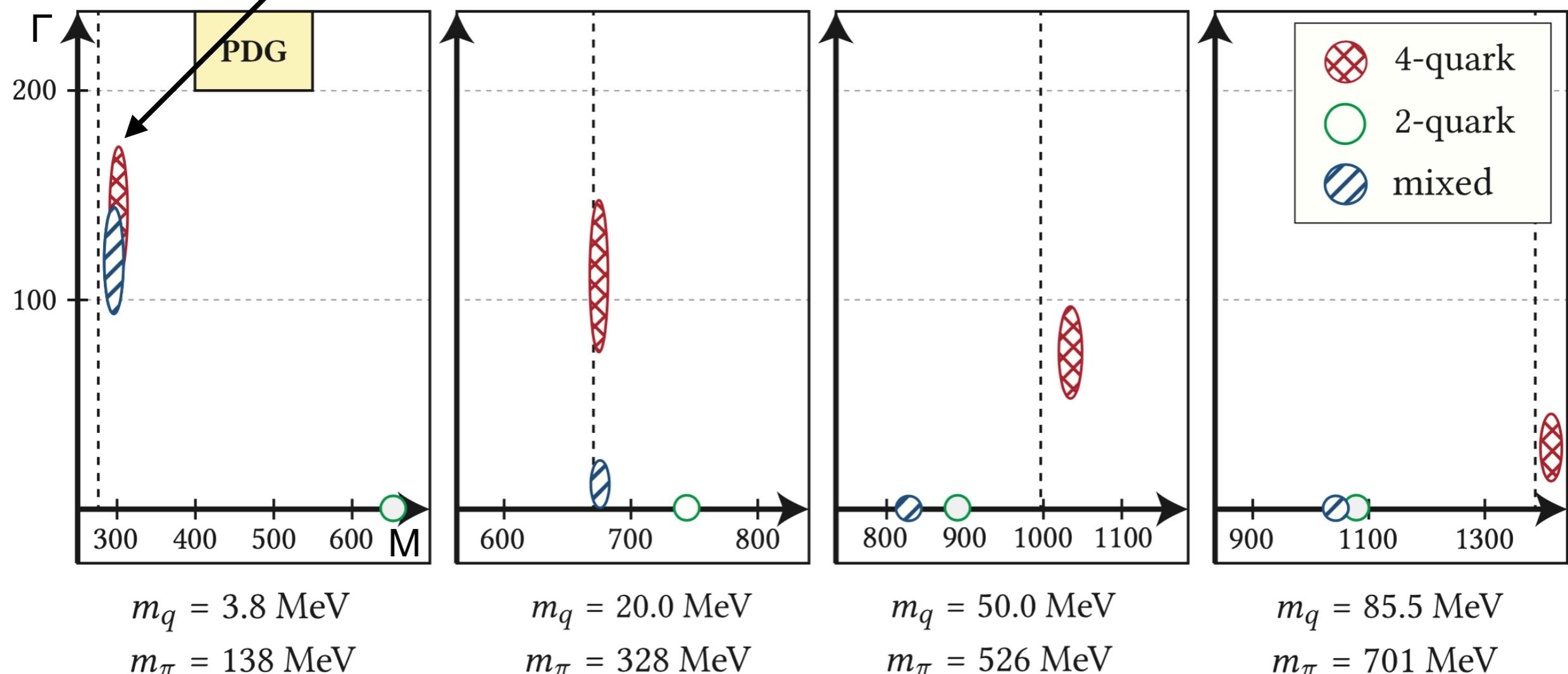
$f_0(500) : \pi\pi$ – component dominates!



- mixed state becomes qq-dominated for large m_q
- dynamical decision !

Mass evolution of four-quark state: 0++

$f_0(500) : \pi\pi$ – component dominates!

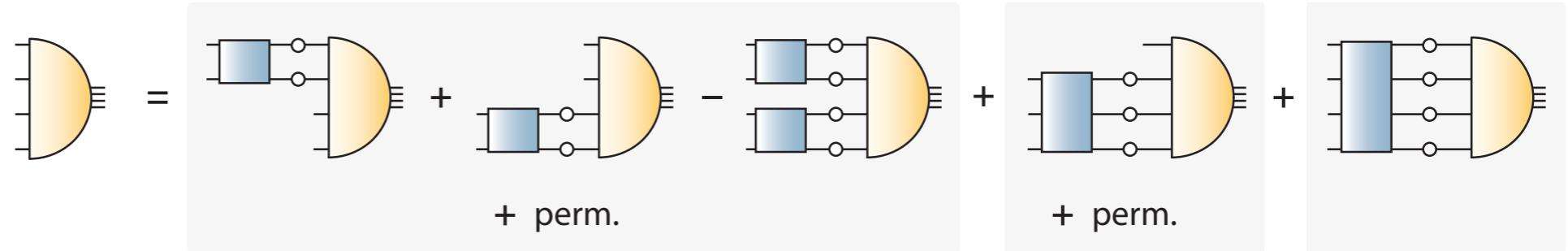


- mixed state becomes qq-dominated for large m_q
- dynamical decision !

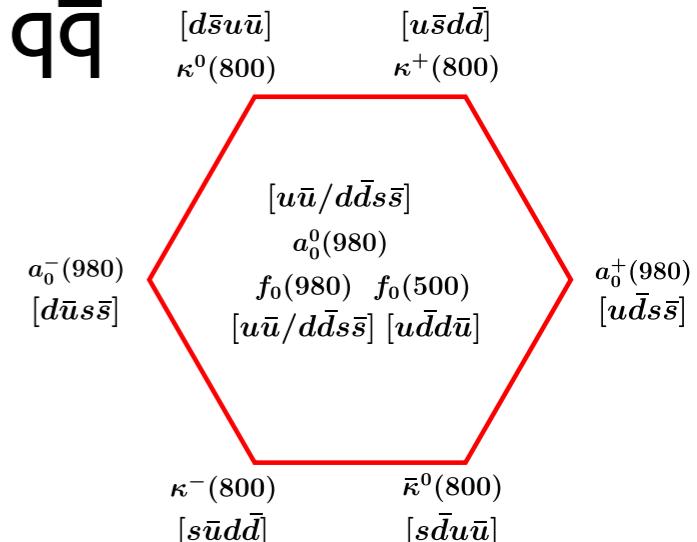
→ consequences for ccqq, ccss, bbqq, bbss, bbcc ?
work to be done!

Santowsky, CF, PRD 105 (2022) 4,313; arXiv:2109.00755

I. The framework and conventional mesons

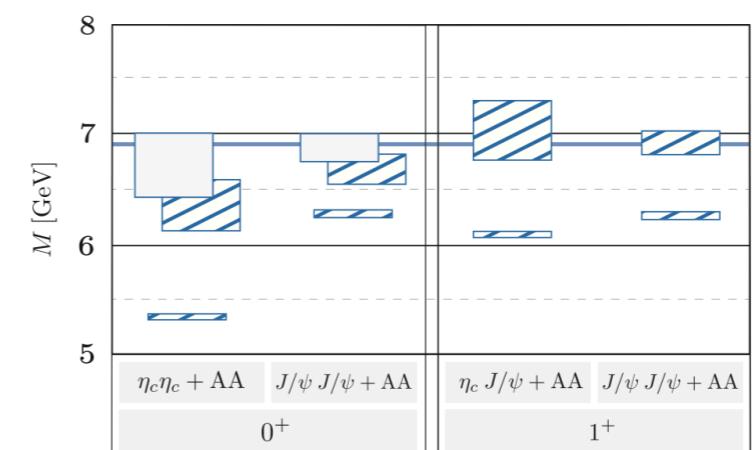


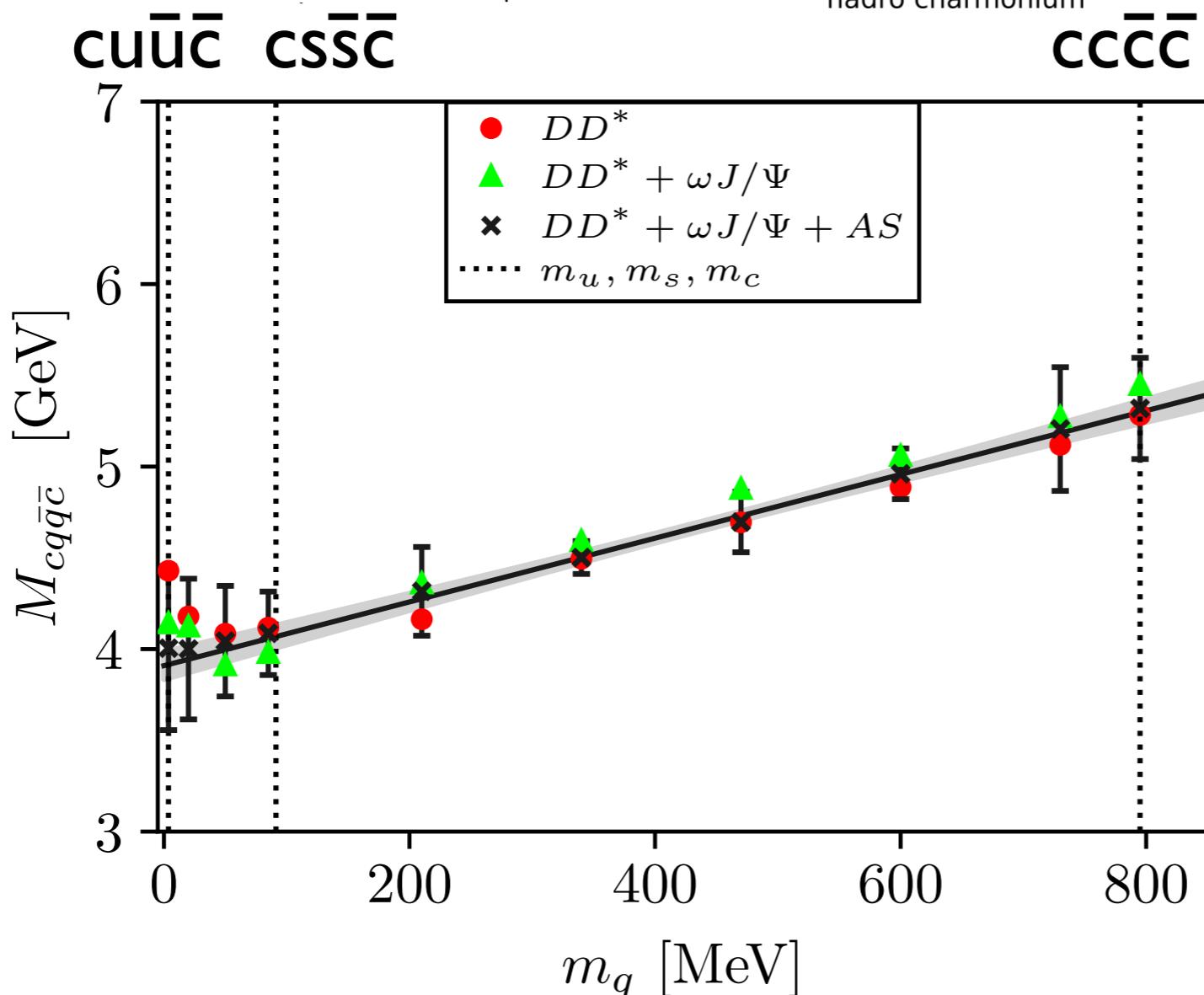
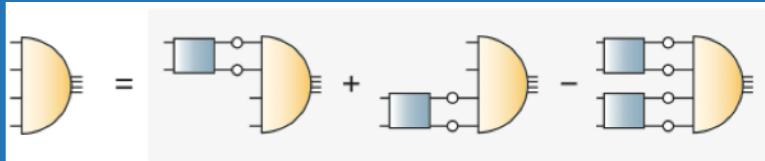
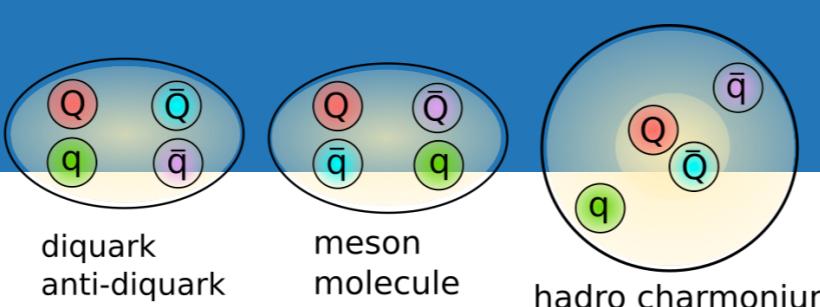
2. Detour: light four-quark states and mixing with $q\bar{q}$



3. Heavy-light and heavy-heavy four-quark states

	$I(J^{PC})$	dominant	4-body	eff. 2-body	Exp.
hidden charm $(c\bar{c}q\bar{q})$	$0(0^{++})$	DD	3.20 (11)	3.49 (25)	$X(3872)$ $Z_c(3900)$
	$0(1^{++})$	DD*	3.92 (7)	3.85 (18)	
	$1(1^{+-})$	DD*	3.74 (9)	3.79 (31)	
	$1(0^{++})$	DD		3.20 (31)	
open charm $(cc\bar{q}\bar{q})$	$0(1^+)$	DD*	3.90 (8)	3.49 (48)	$T_{cc}(3875)$
	$1(0^+)$	DD+AA	3.80 (10)	3.21 (2)	
	$1(1^+)$	DD*+AA	4.22 (44)	3.47 (24)	





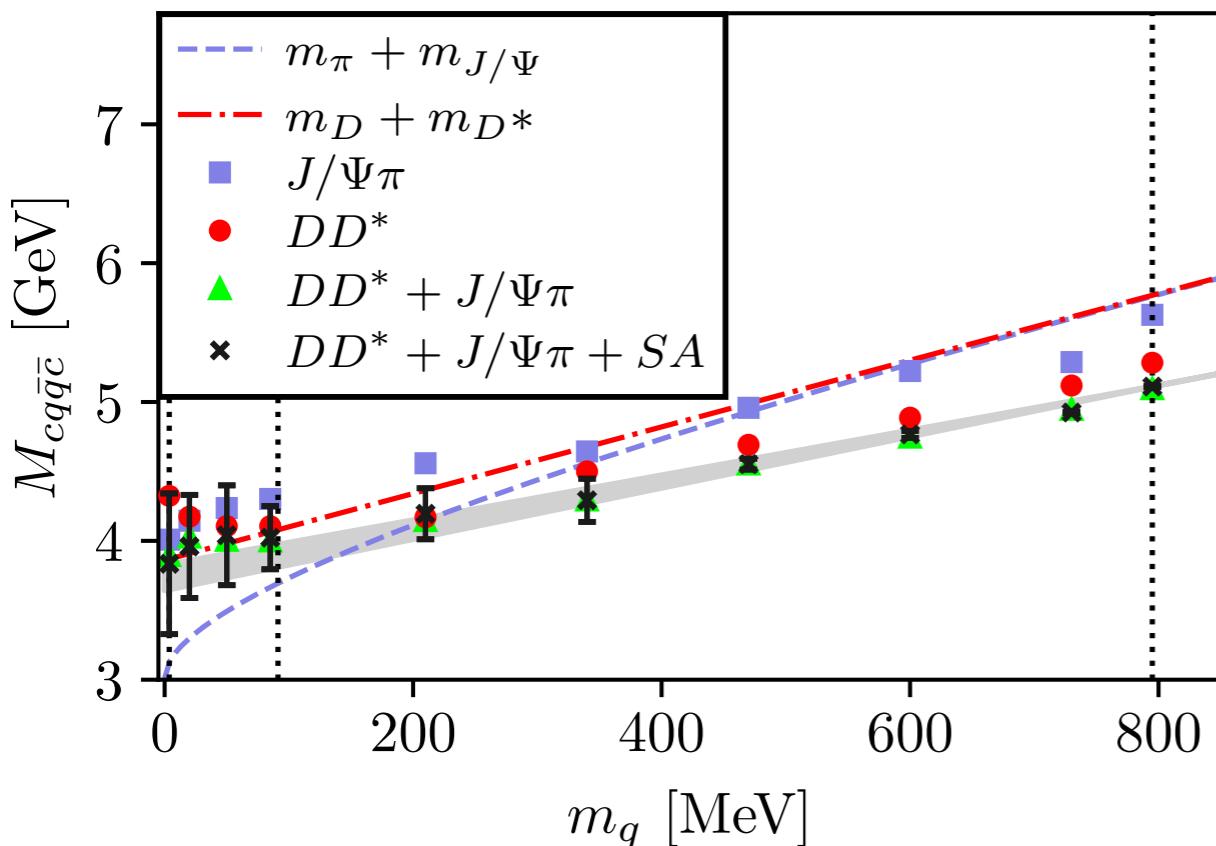
m_c fixed
 m_q varied

- DD* components dominate !

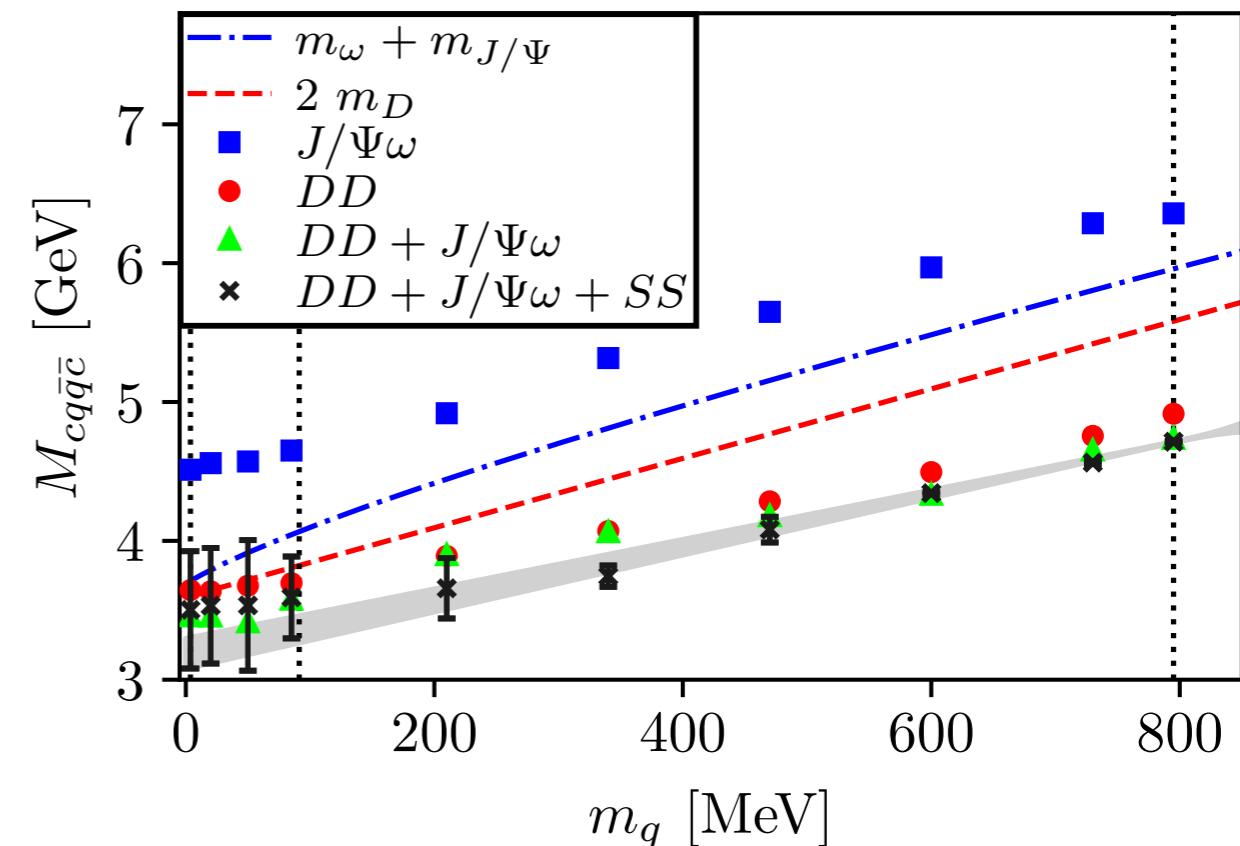
$$M_{1^{++}}^{cq\bar{q}\bar{c}} = 3916(74) \text{ MeV} \longrightarrow X(3872)$$

$J^{PC} = 1^{+-}$ and 0^{++}

$1(1^{+-})\ cq\bar{q}\bar{c}$



$0(0^{++})\ cq\bar{q}\bar{c}$

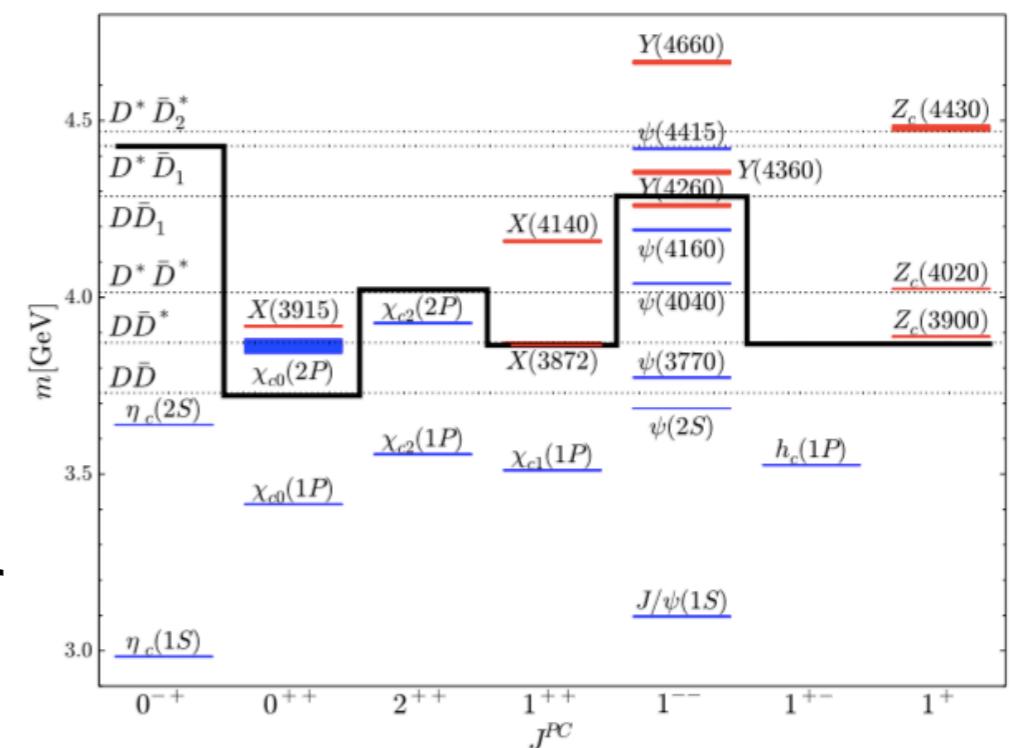


Wallbott, Eichmann and CF, PRD 102 (2020) no.5, 051501, arXiv:2003.12407

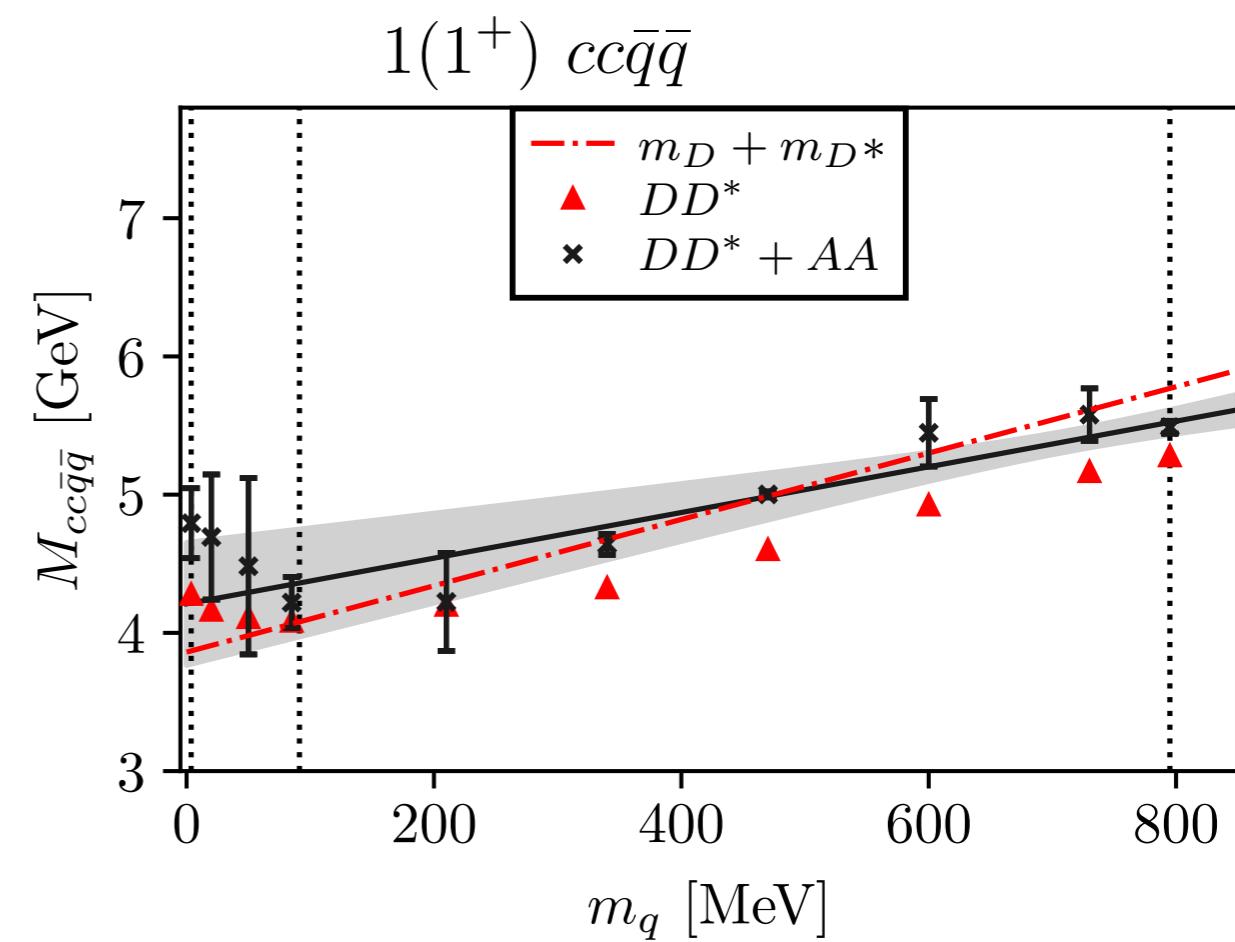
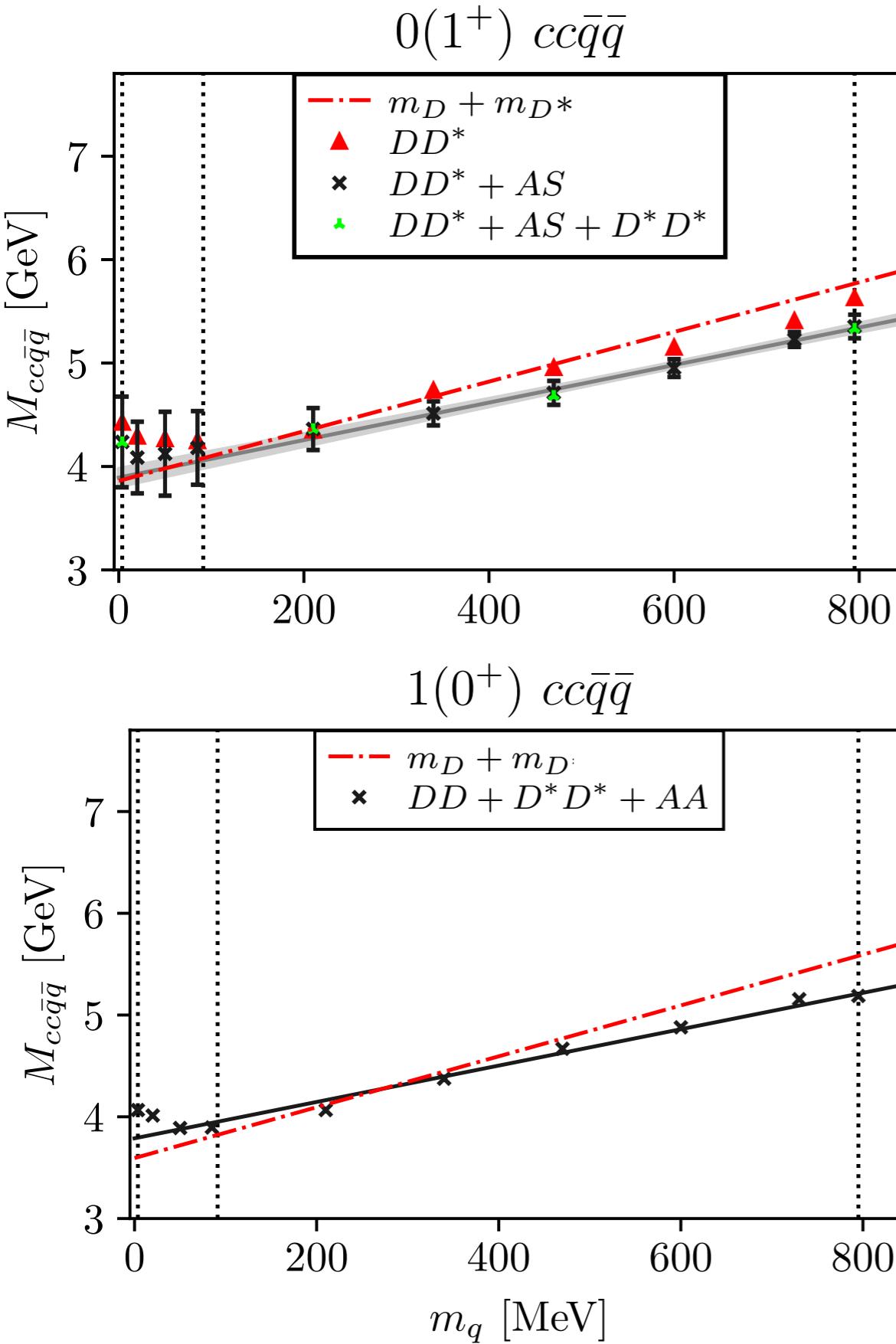
$$M_{1^{+-}}^{cq\bar{q}\bar{c}} = 3741(91) \rightarrow Z(3900)$$

$$M_{0^{++}}^{cq\bar{q}\bar{c}} = 3195(107) \rightarrow ?$$

mass pattern matches molecule picture of
Cleven et al. PRD 92 (2015) 014005:



Open charm four-quark states



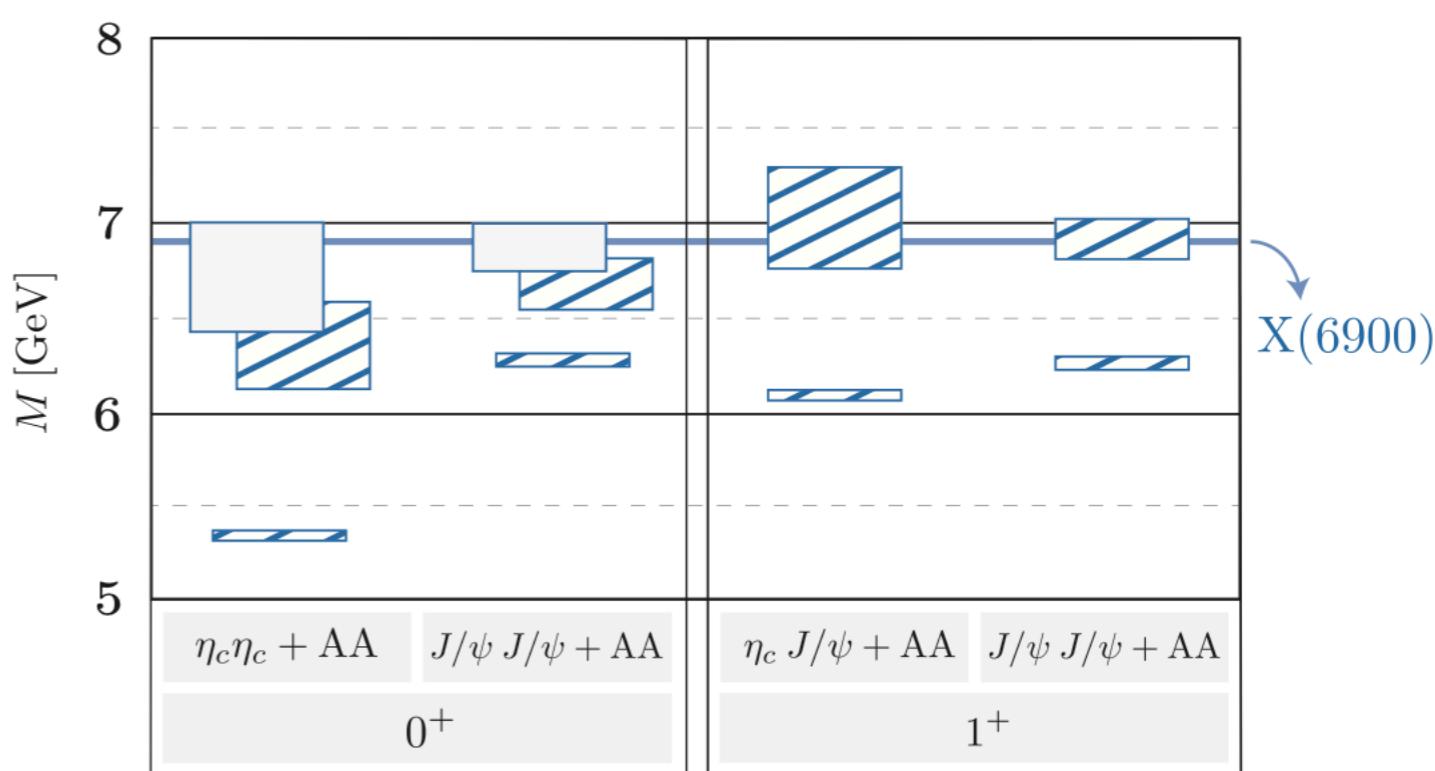
● DD(*) and diquarks important!

Wallbott, Eichmann and CF, PRD 102 (2020) no.5, 051501, arXiv:2003.12407

Heavy four-quark states from DSE/BSEs

	$I(J^{PC})$	dominant	4-body	eff. 2-body	Exp.
hidden charm $(c\bar{c}q\bar{q})$	0(0^{++})	DD	3.20 (11)	3.49 (25)	
	0(1^{++})	DD*	3.92 (7)	3.85 (18)	$X(3872)$
	1(1^{+-})	DD*	3.74 (9)	3.79 (31)	$Z_c(3900)$
	1(0^{++})	DD		3.20 (31)	
open charm $(cc\bar{q}\bar{q})$	0(1^+)	DD*	3.90 (8)	3.49 (48)	$T_{cc}(3875)$
	1(0^+)	DD+AA	3.80 (10)	3.21 (2)	
	1(1^+)	DD*+AA	4.22 (44)	3.47 (24)	

all charm
 $(\bar{c}\bar{c}cc)$



Wallbott, Eichmann and CF, PRD 100 (2019) 014033, [1905.02615]

Wallbott, Eichmann and CF, PRD 102 (2020), 051501, [2003.12407]

Santowsky, CF, EPJC 82 (2022) 4, 313

Summary

Dynamical decision between meson-meson or diquark dominance

- Dynamical description of σ : $\pi\text{-}\pi$ resonance
—
Eichmann, CF, Heupel, PLB 753 (2016) 282-287
- Mixing with qq studied for light mesons
Santowsky, Eichmann, CF, Wallbott and Williams, PRD 102 (2020) no.5, 056014, [2007.06495].
- Dynamical description of $X(3872)$ and $Z(3900)$: DD^* dominated
Wallbott, Eichmann and CF, PRD 100 (2019) 014033,
Wallbott, Eichmann and CF, PRD 102 (2020) 051501,
- First results in open charm / all charm channels
Santowsky, CF, EPJC 82 (2022) 4, 313

Work in progress:

Mini-Review: Eichmann, CF, Heupel, Santowsky, Wallbott, FBS 61 (2020) 4 38.

- Extend range of calc. to bottom quarks
- Include repulsive color channels
- Systematically compare with BO-Lattice-results
- Hybrids



INTERNATIONAL SCHOOL OF NUCLEAR PHYSICS 44th Course

From quarks and gluons to hadrons and nuclei

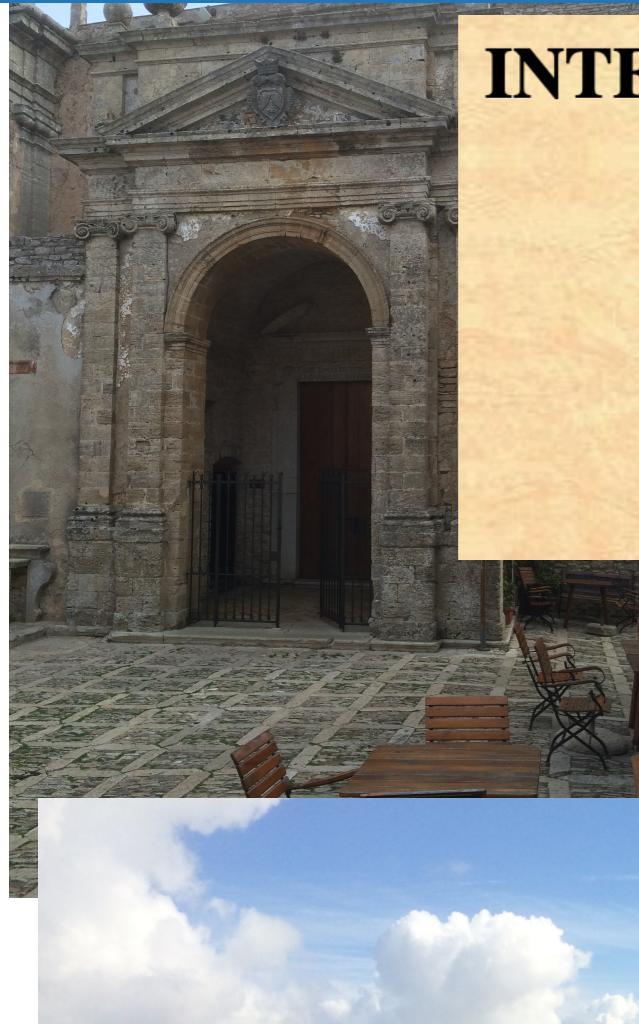
Erice, Sicily, September 18-24, 2023

Directors of the school

[Michael Buballa and Christian Fischer](#)

In detail, the following topics will be discussed:

- properties of quarks and gluons
- mass generation and confinement
- spectra and decays of conventional baryons
- spectra and decays of exotic baryons (hybrids, pentaquarks)
- spectra and decays of exotic mesons (glueballs, four-quark states, hybrids)
- form factors and structure functions of hadrons
- emergence of nuclear forces from QCD



Backup Slides

Extended linear sigma model



Physics Letters B

Volume 834, 10 November 2022, 137478



The phenomenology of the exotic hybrid nonet with $\pi_1(1600)$ and $\eta_1(1855)$

Vanamali Shastry^a   , Christian S. Fischer^{b c}  , Francesco Giacosa^{a d} 

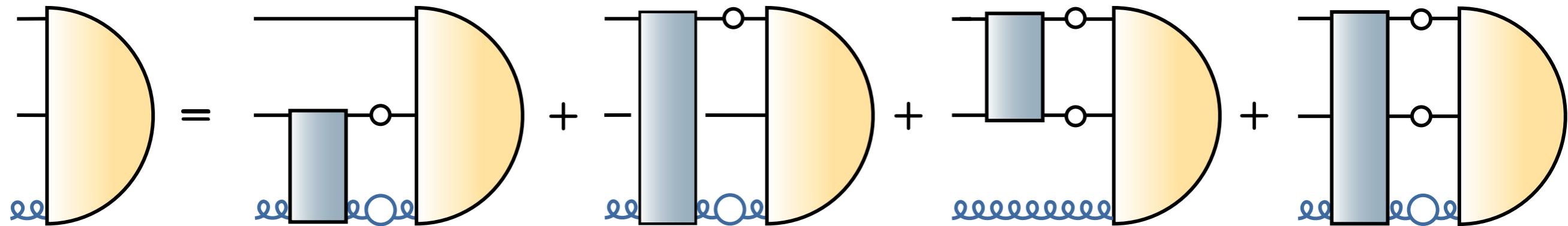
Show more ▾

 Share  Cite

- simple model
- input from lattice QCD
- predictions for various decays rates

A.~J.~Woss et al. [Hadron Spectrum],
"Decays of an exotic $1\{-+\}$ hybrid meson resonance in QCD,"
Phys. Rev. D 103 (2021) no.5, 054502

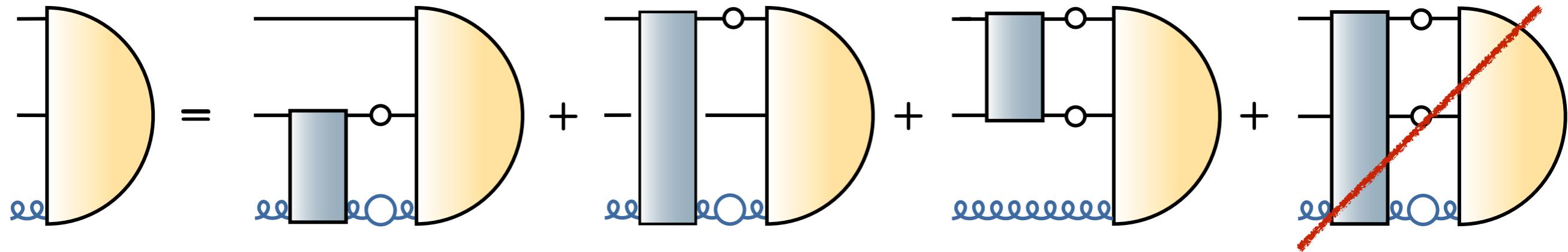
Hybrids from DSE/BSEs: work in progress



- Tensor structures for $1^{-+}, 1^{--}, 0^{-+}, 0^{++}$ constructed
- all diagrams implemented
- two codes: comparison/debugging in process

with Franziska Münster

Hybrids from DSE/BSEs: work in progress

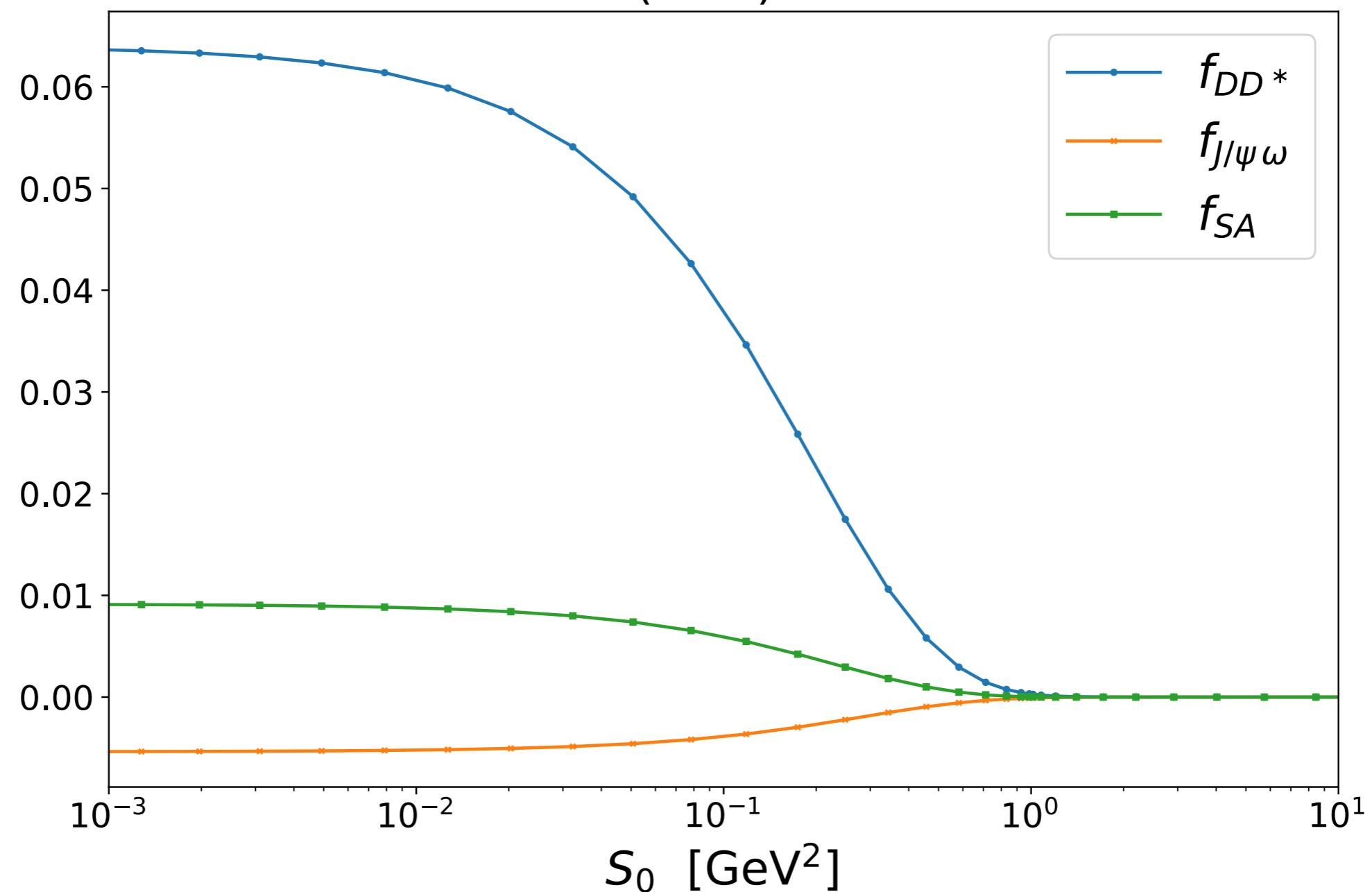


- Tensor structures for $1^{-+}, 1^{--}, 0^{-+}, 0^{++}$ constructed
- all diagrams implemented
- two codes: comparison/debugging in process

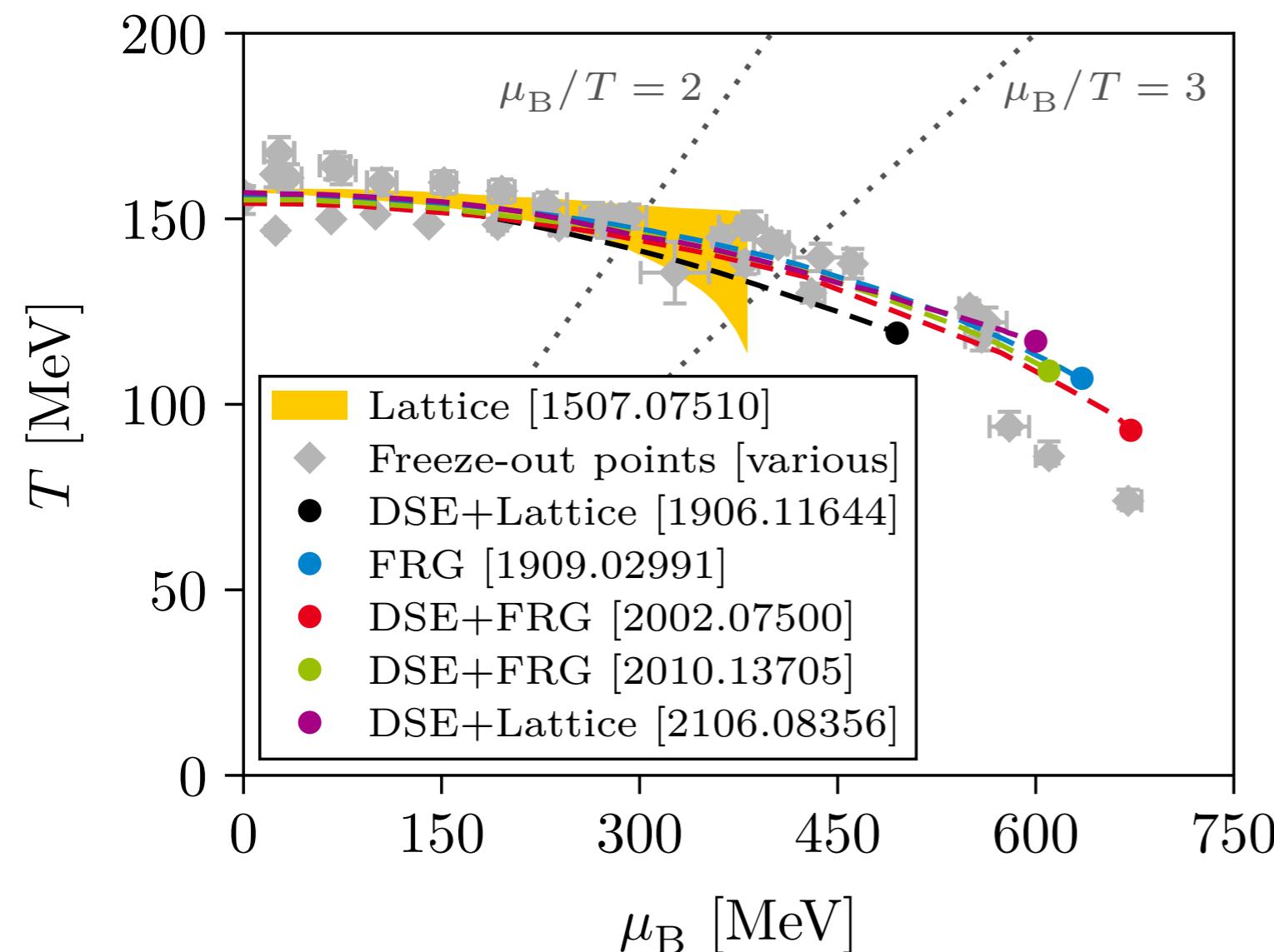
with Franziska Münster

Amplitudes of four-quark states: $\mathbb{X}(3872)$

$$M^2 = (3.54)^2 \text{ MeV}^2$$



QCD phase diagram and heavy ion collisions



Bernhardt, CF, Isserstedt, 2208.01981

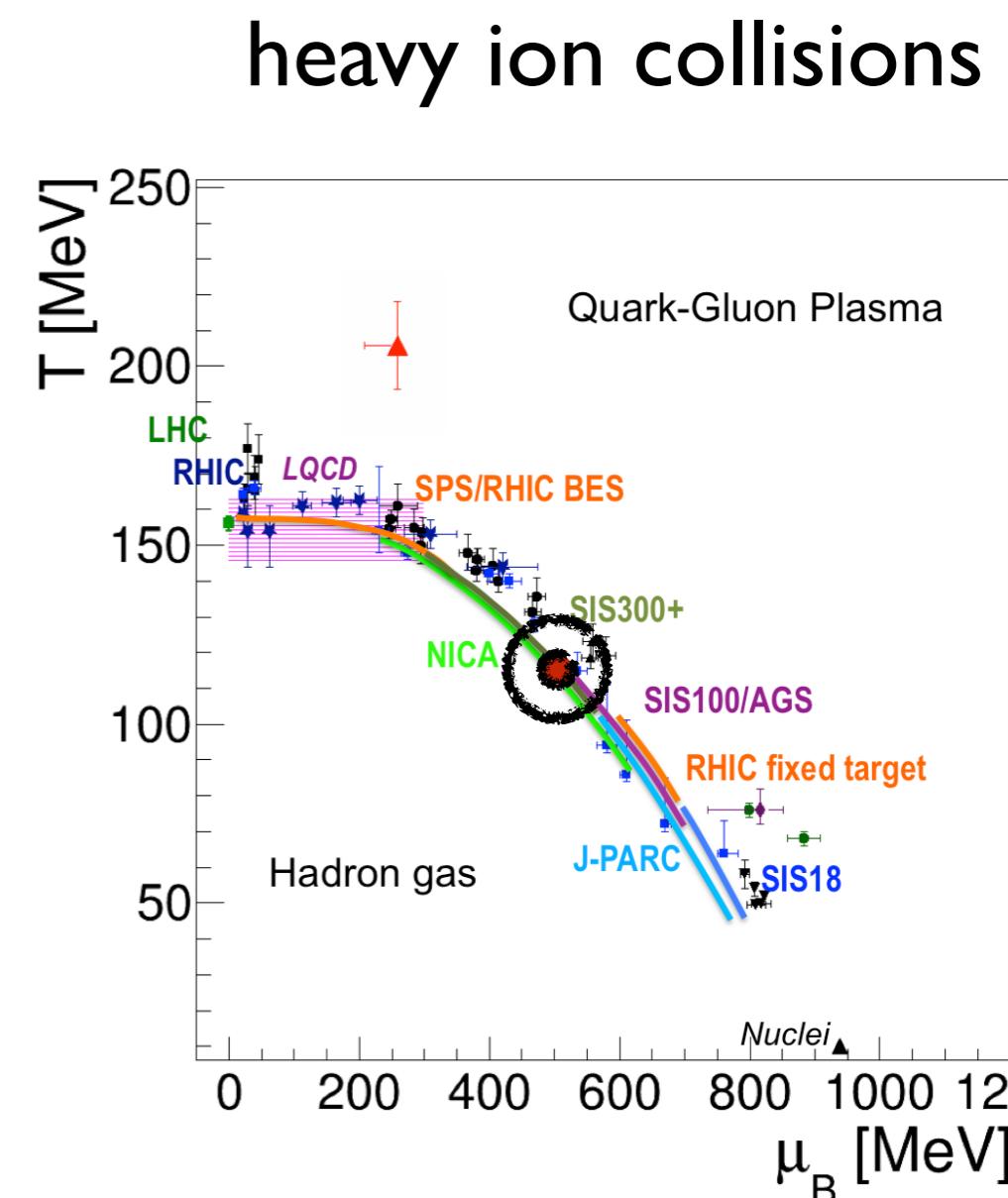
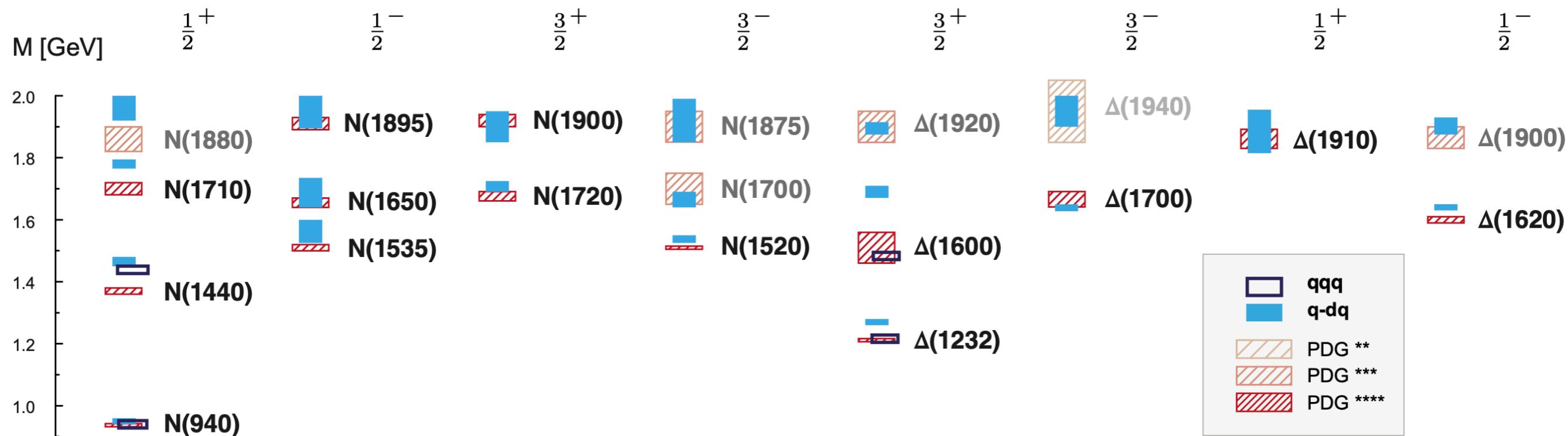


Figure adapted from talk of T. Galatyuk, Erice 2016

Light baryon spectrum: DSE-RL

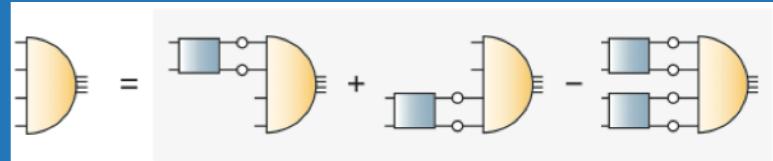
■ 3 parameters + $m_{u,d,s}$



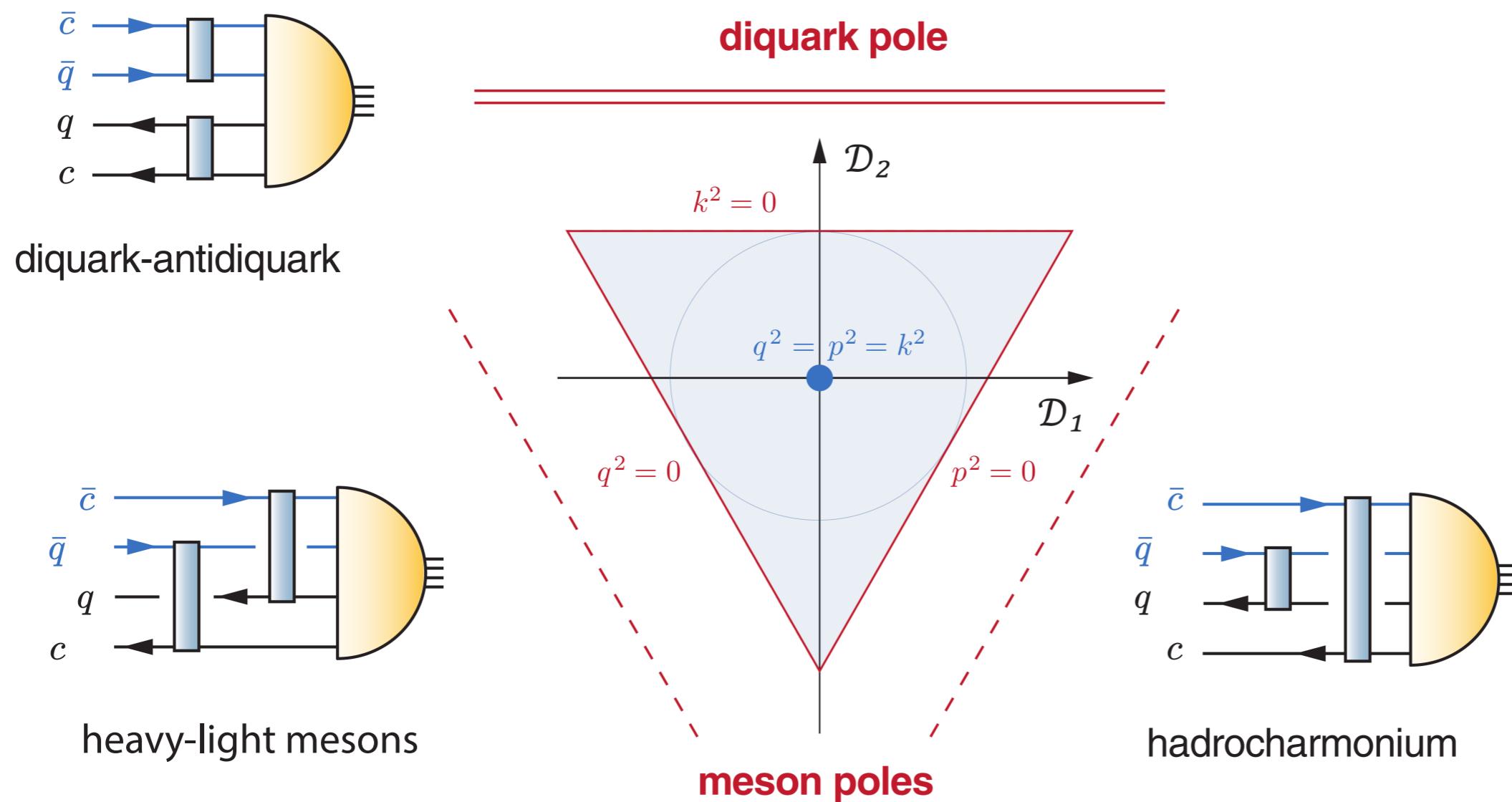
Eichmann, CF, Sanchis-Alepuz, PRD 94 (2016) [[1607.05748](#)]
 Eichmann, CF, Few Body Syst. 60 (2019) no.1, 2
 Eichmann, Few Body Syst. 63 (2022) no.3,

- spectrum in one to one agreement with experiment
- correct level ordering (without coupled channel effects...)
- three-body agrees with diquark-quark where applicable

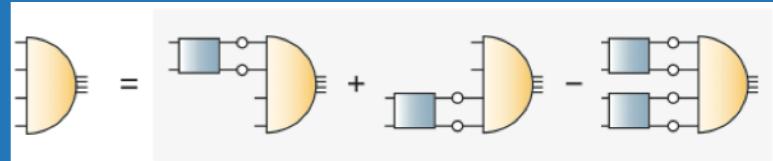
Four-body equation: permutations



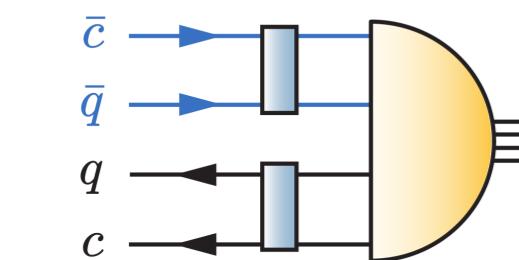
- **Singlet:** $S_0 = (p^2 + q^2 + k^2)/4$ p, q, k : relative momenta
- **Doublet:** $\mathcal{D}_1 \sim p^2 + q^2 - 2k^2$
 $\mathcal{D}_2 \sim q^2 - p^2$



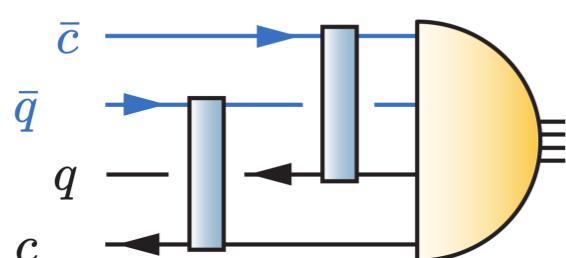
Four-body equation: permutations



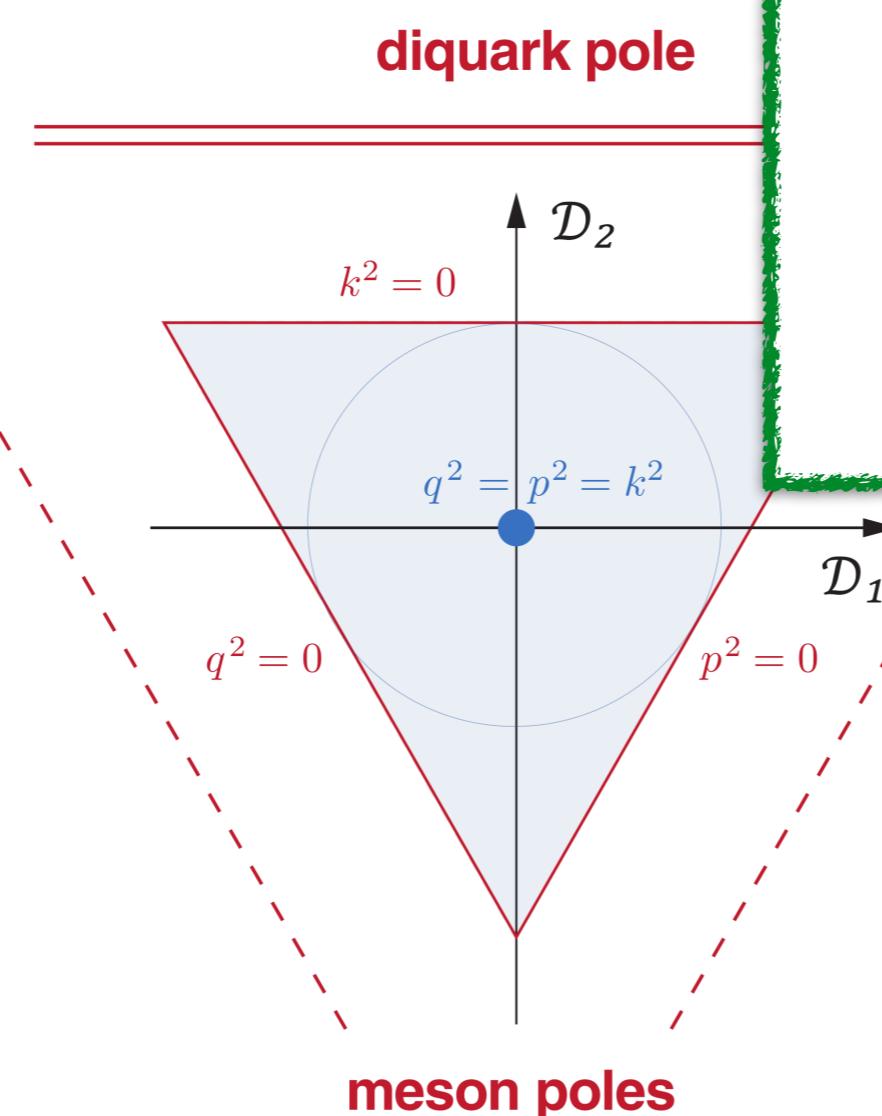
- **Singlet:** $S_0 = (p^2 + q^2 + k^2)/4$ p, q, k : relative momenta
- **Doublet:** $\mathcal{D}_1 \sim p^2 + q^2 - 2k^2$
 $\mathcal{D}_2 \sim q^2 - p^2$



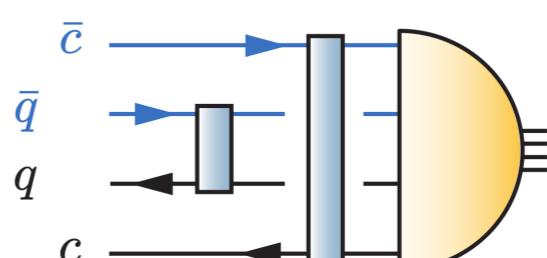
diquark-antidiquark



heavy-light mesons



- model independent:
heavy-light meson poles
more important than
diquark poles
(color factor !)



hadrocharmonium