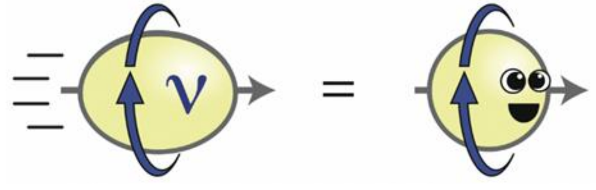
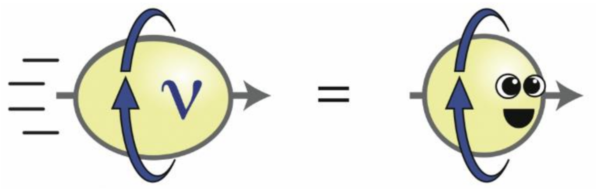
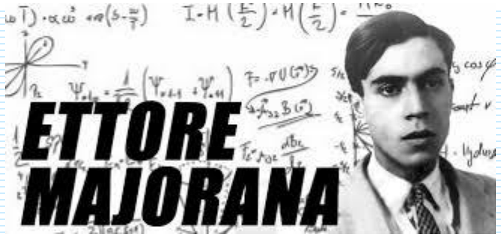
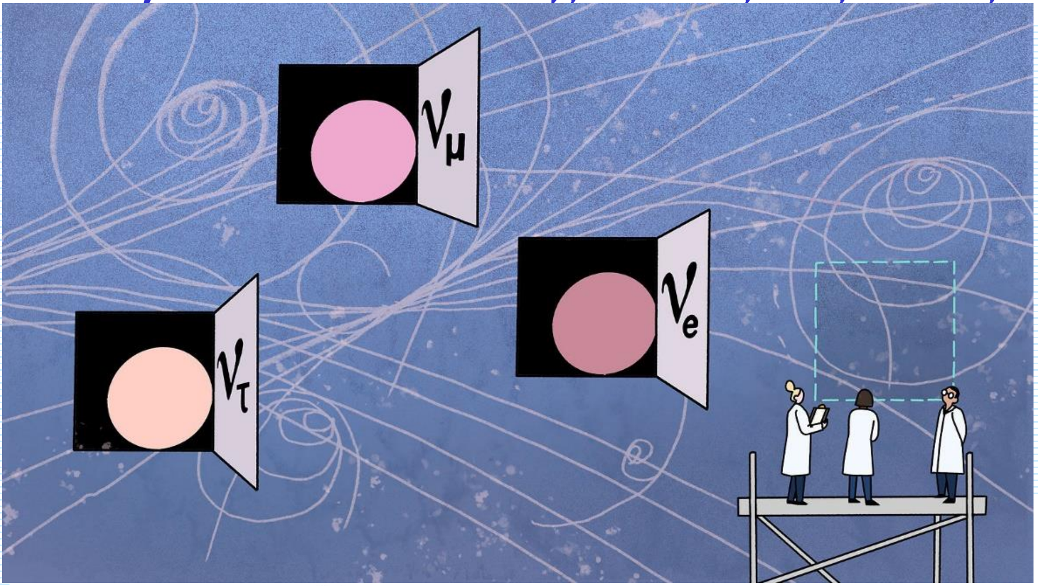


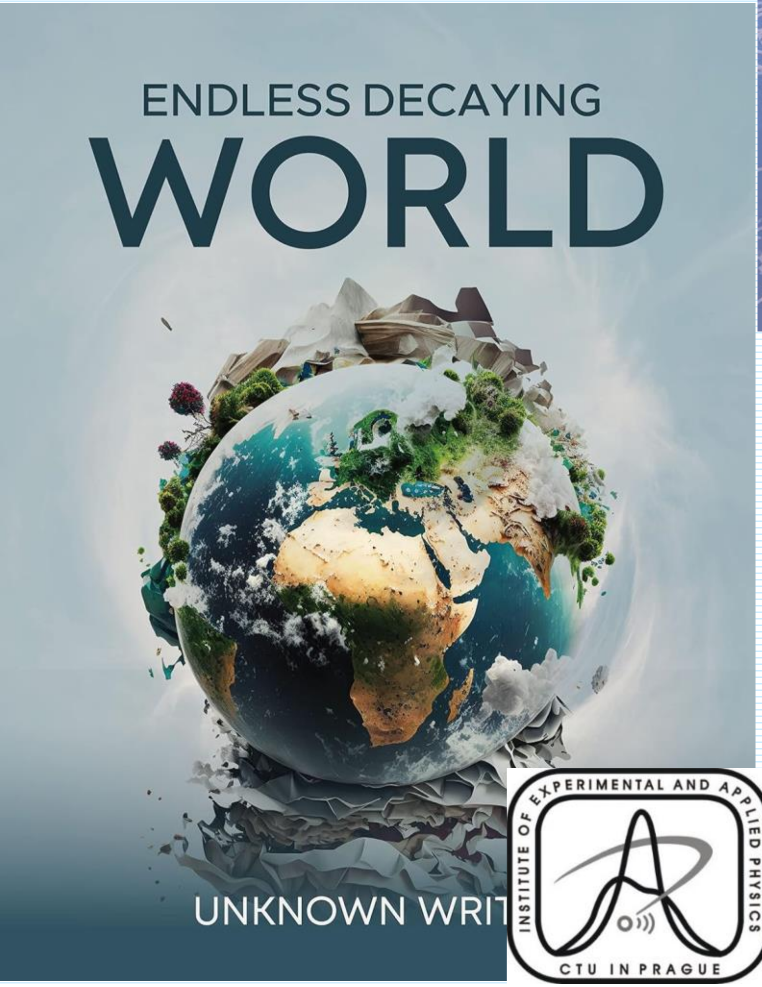
Recommended Values for Neutrinoless Double-Beta Decay Nuclear Matrix Elements (INT-26-2a)

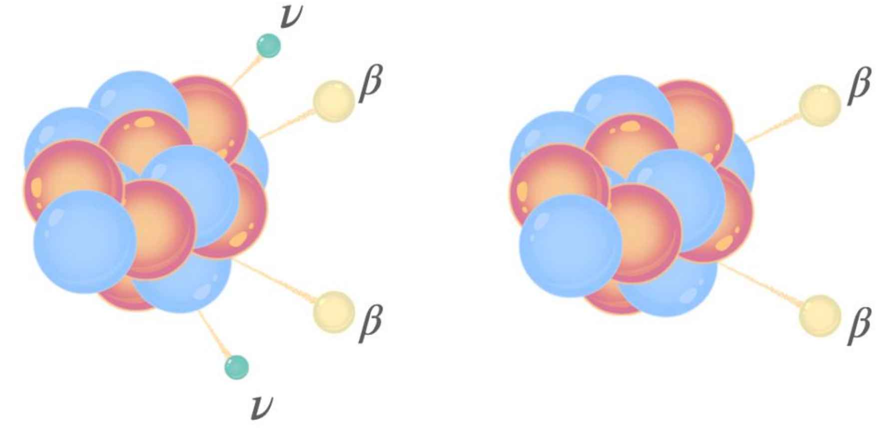
Institute for Nuclear Theory, Seattle, US, June , 2026





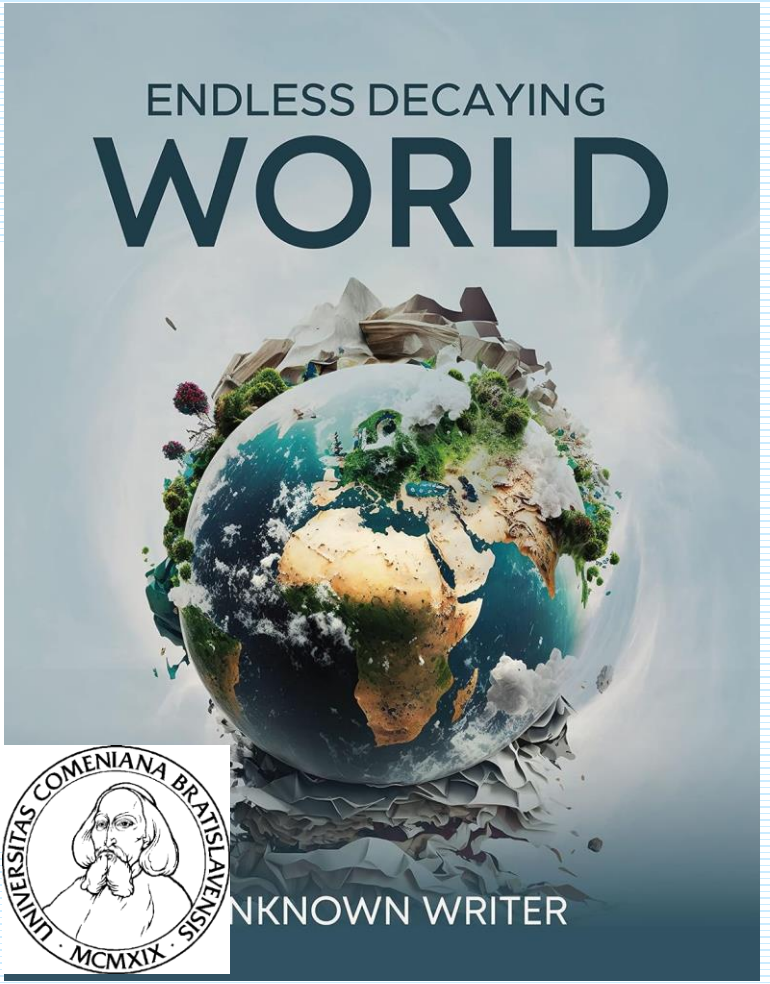






$(A,Z) \to (A,Z \pm 2) + 2\beta^{\mp} + 2\overset{(-)}{\nu}$

$(A,Z) \to (A,Z \pm 2) + 2\beta^{\mp}$

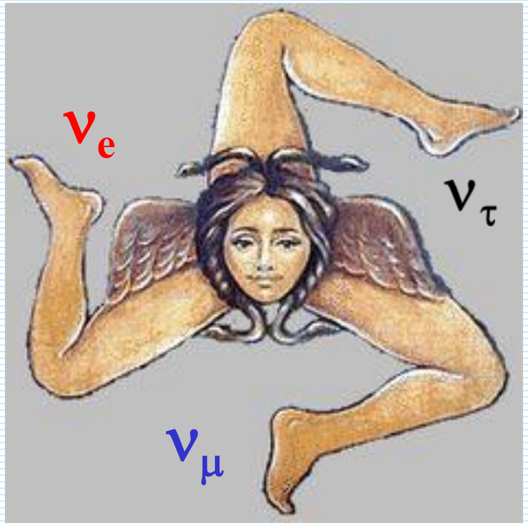


INSTITUTE OF EXPERIMENTAL AND APPLIED PHYSICS
CTU IN PRAGUE

Double-Beta Decay NMEs

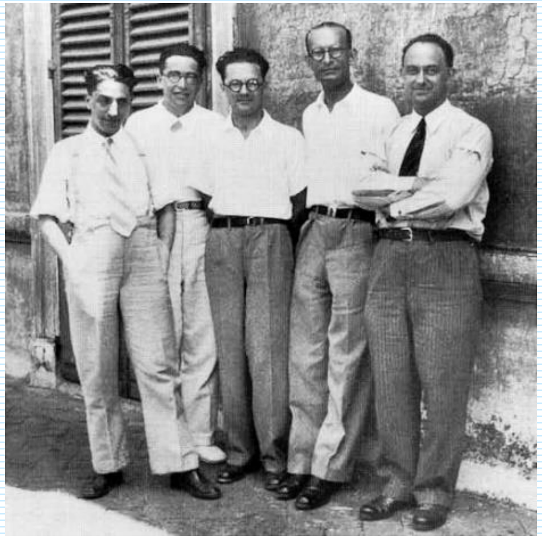
Fedor Šimkovic

UNIVERSITAS COMENIANA BRATISLAVENSIS
MCMXIX



OUTLINE

- ***I. Introduction** (DBD modes, 0nbb-NMEs)*
- ***II. $2\nu\beta\beta$–decay** (a brief derivation, the $2\nu\beta\beta$–decay NMEs – phenomenological versus theoretical approaches, role of the spin-isospin symmetry restoration)*
- ***V. Improved description of the $2\nu\beta\beta$-decay***
  (the effect of lepton energies in energy denominators, widths of intermediate nuclear states, effect of $p_{1/2}$ wave states, …)
- ***VI. Ordinary muon capture***
- ***VII. A connection of $0\nu\beta\beta$ and $2\nu\beta\beta$ NMEs***
- ***VIII. Outlook***



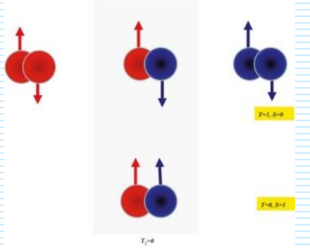
Oscar D'Agostino
Emilio Segrè
Edoardo Amaldi
Franco Rasetti
Enrico Fermi

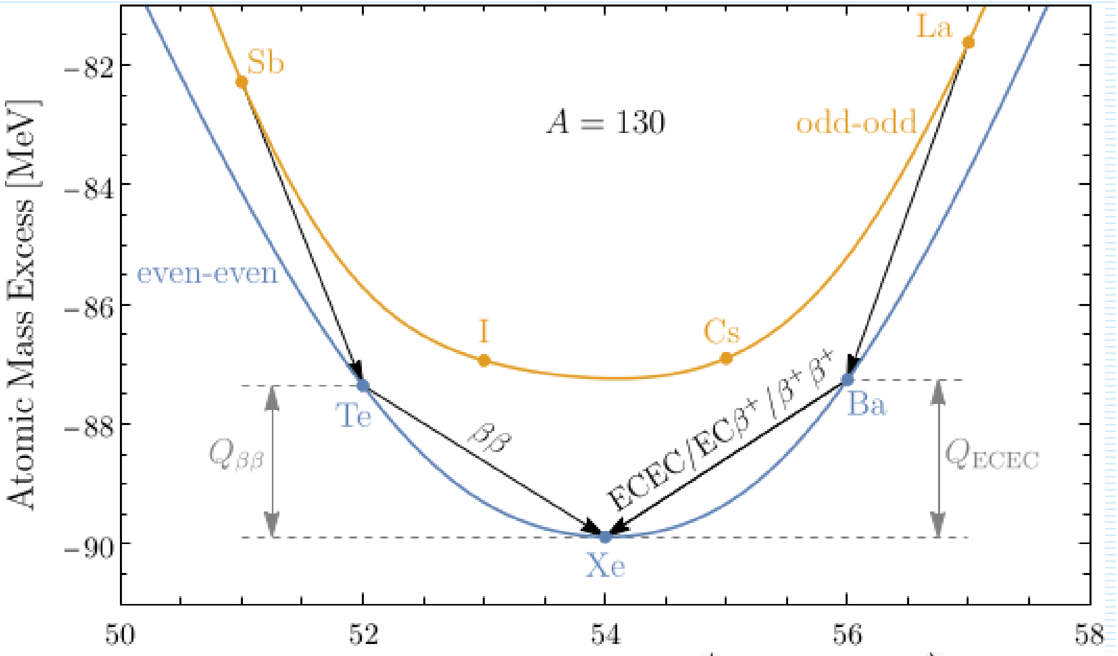
Ettore Majorana

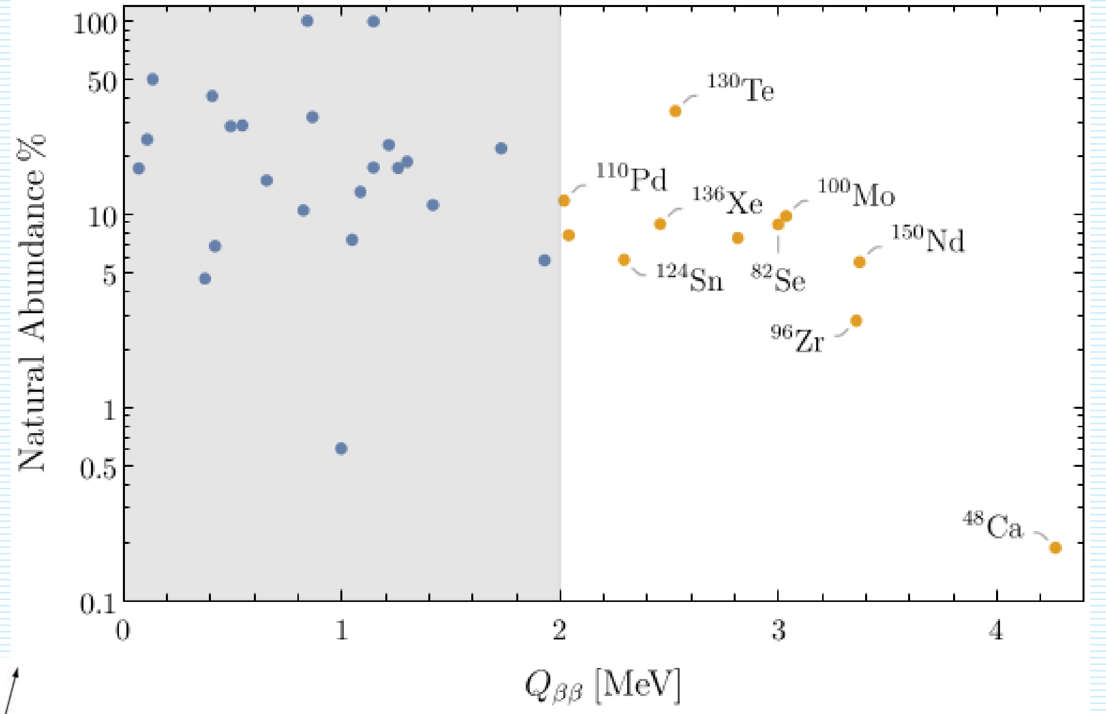
Bruno Pontecorvoa

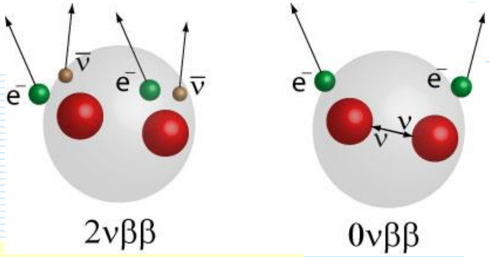
Nuclear double-β decay

(even-even nuclei, pairing int.)









Phys. Rev. 48, 512 (1935)

Two-neutrino double-β decay – LN conserved

$(A,Z) \rightarrow (A,Z+2) + e^- + e^- + \nu_e + \nu_e$

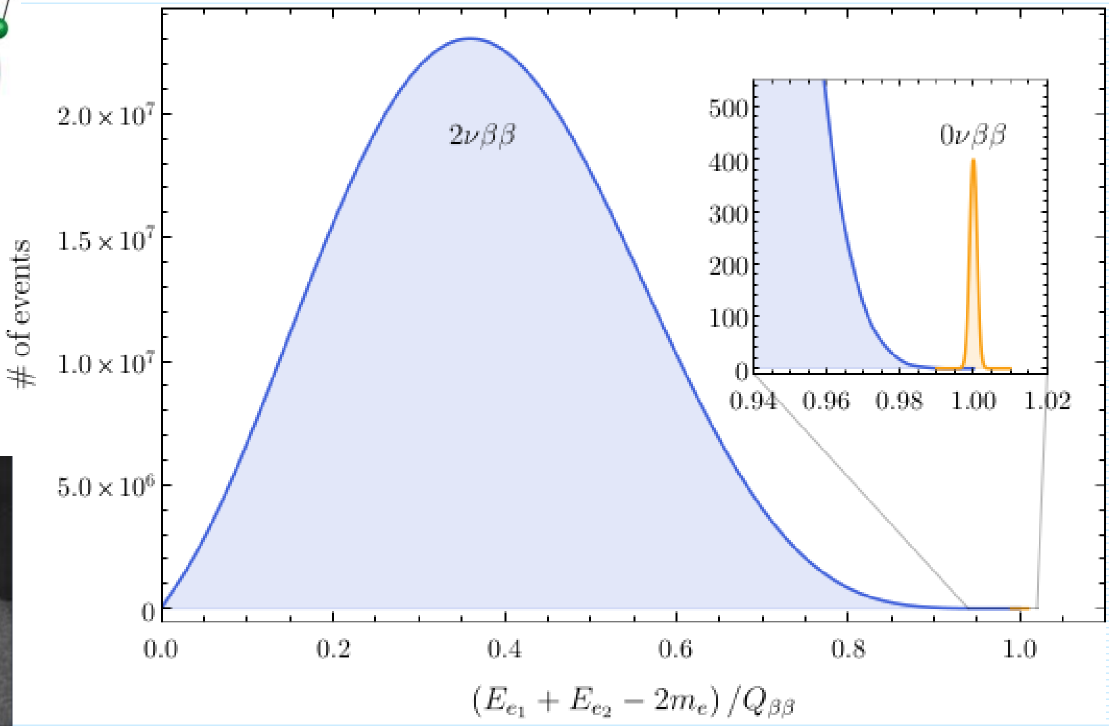
Goepert-Mayer – 1935. 1st observation in 1987

Nuovo Cim. 14, 322 (1937) **Phys. Rev. 56, 1184 (1939)**

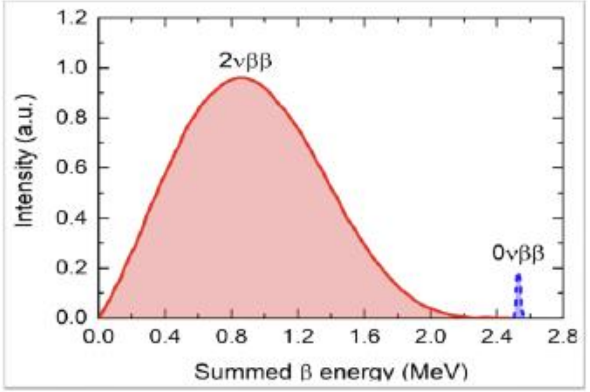
Neutrinoless double-β decay – LN violated

$(A,Z) \rightarrow (A,Z+2) + e^- + e^-$ (Furry 1937)

Not observed yet. Requires massive Majorana ν's



0νββ experiments – a worldwide competition of ideas and underground physics technologies



GERDA (^{76}Ge)

$T^{0\nu}_{1/2} > 1.8 \times 10^{26}$yrs

Phys.Rev.Lett.125(2020)252502

GUORE (^{130}Te)

$T^{0\nu}_{1/2} > 2.2 \times 10^{25}$yrs

Nature 604(2022)53

CUPID-0 (^{82}Se)

$T^{0\nu}_{1/2} > 2.4 \times 10^{24}$yrs

Phys.Rev.Lett.120(2018)232502

MAJORANA (^{76}Ge)

$T^{0\nu}_{1/2} > 8.3 \times 10^{25}$yrs

Phys.Rev.Lett.130(2023)062501

CUPID (^{100}Mo)

$T^{0\nu}_{1/2} > 1.5 \times 10^{24}$yrs

Phys.Rev.Lett.126(2021)181802

SNO+

NνDex, PandaX, CDEX, CUPID-China

NEMO-3

EXO-200 (^{136}Xe)

$T^{0\nu}_{1/2} > 3.5 \times 10^{25}$yrs

Phys.Rev.Lett.123(2019)161802

JUNO

KamLAND-Zen (^{136}Xe)

$T^{0\nu}_{1/2} > 2.3 \times 10^{26}$yrs

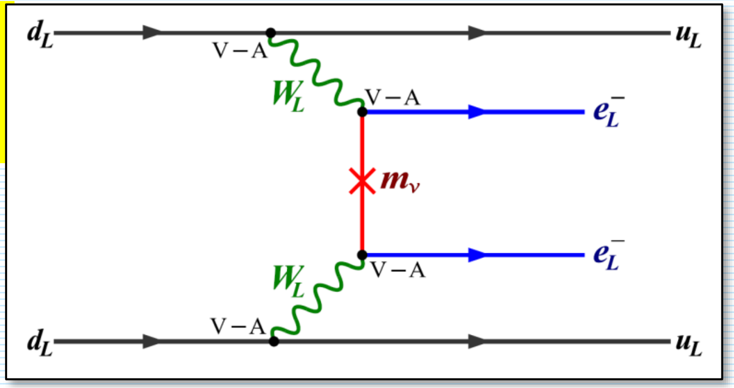
★ Phys.Rev.Lett.130(2023)051801

Isotope	$Q_{\beta\beta}$(MeV)	N.A. (%)
^{48}Ca	4.268	0.187
^{76}Ge	2.039	7.8
^{82}Se	2.998	8.8
^{96}Zr	3.356	2.8
^{100}Mo	3.034	9.7
^{110}Pd	2.017	11.7
^{116}Cd	2.813	7.5
^{124}Sn	2.293	5.8
^{130}Te	2.528	34.1
^{136}Xe	2.458	8.9
^{150}Nd	3.371	5.6

(A,Z) → (A,Z+2) + e⁻ + e⁻

$0\nu\beta\beta$-decay

(LNV at ≈ GUT scale, exchange of three light ν)



$$\left(T^{0\nu}_{1/2}\right)^{-1} = \left|\frac{m_{\beta\beta}}{m_e}\right|^2 \; g_A^4 \; \left|M^{0\nu}_\nu\right|^2 \; G^{0\nu}$$

?

Phase space factor well understood

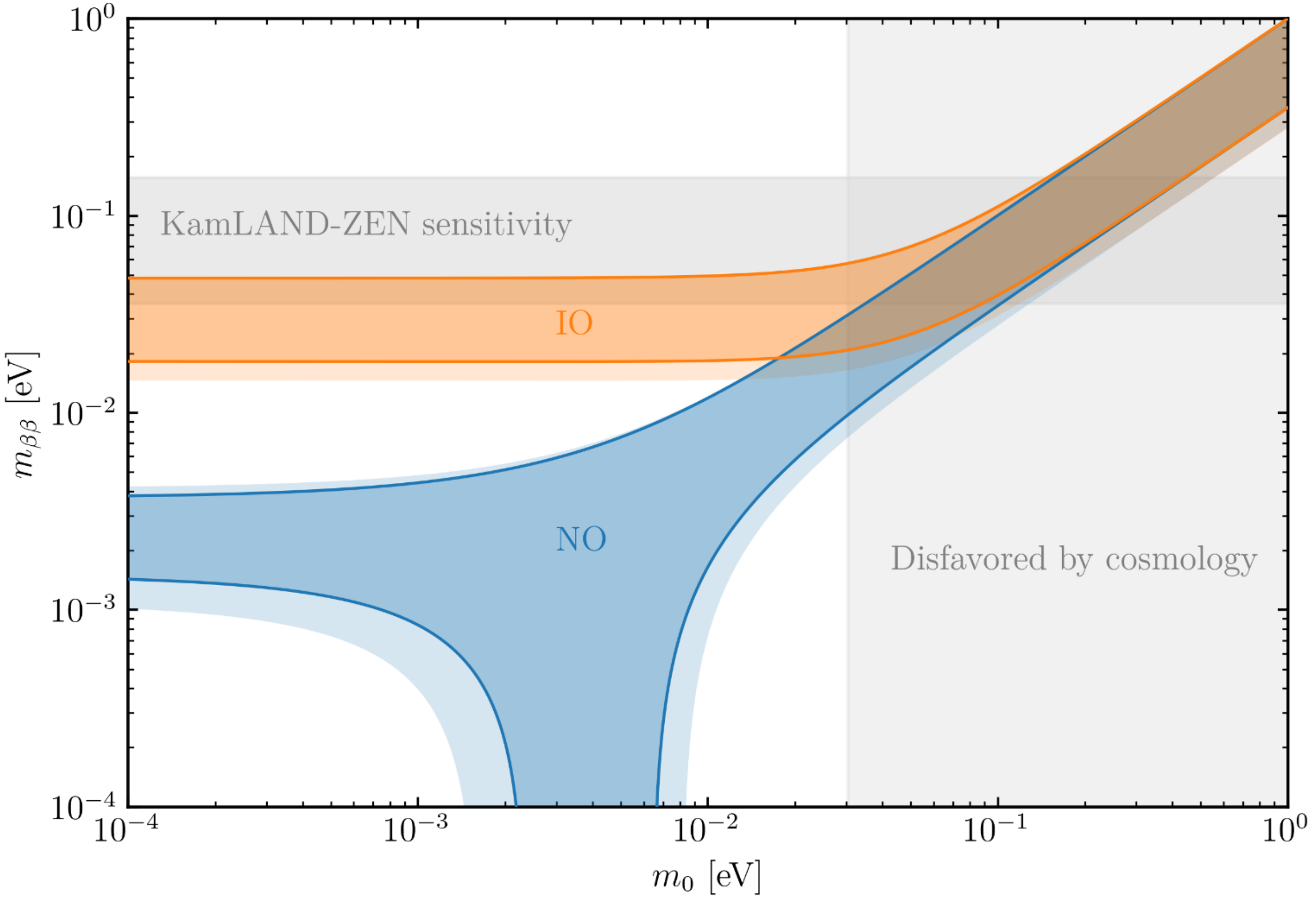
NME must be evaluated using tools of nuclear theory

$$m_{\beta\beta} = \left|c_{13}^2 c_{12}^2 e^{i\alpha_1} m_1 + c_{13}^2 s_{12}^2 e^{i\alpha_2} m_2 + s_{13}^2 m_3\right|$$

Constraint from cosmology

Σ = $m_1 + m_2 + m_3$
< 0.90 eV
< 0.26 eV (Planck coll.)
< 0.12 eV

Contrary, the constraint from $0\nu\beta\beta$-decay (KLZ)
$m_{\beta\beta}$ < 0.036-0.156 eV
implies
Σ < 0.12 eV



0νββ-decay NME status 2023(25)

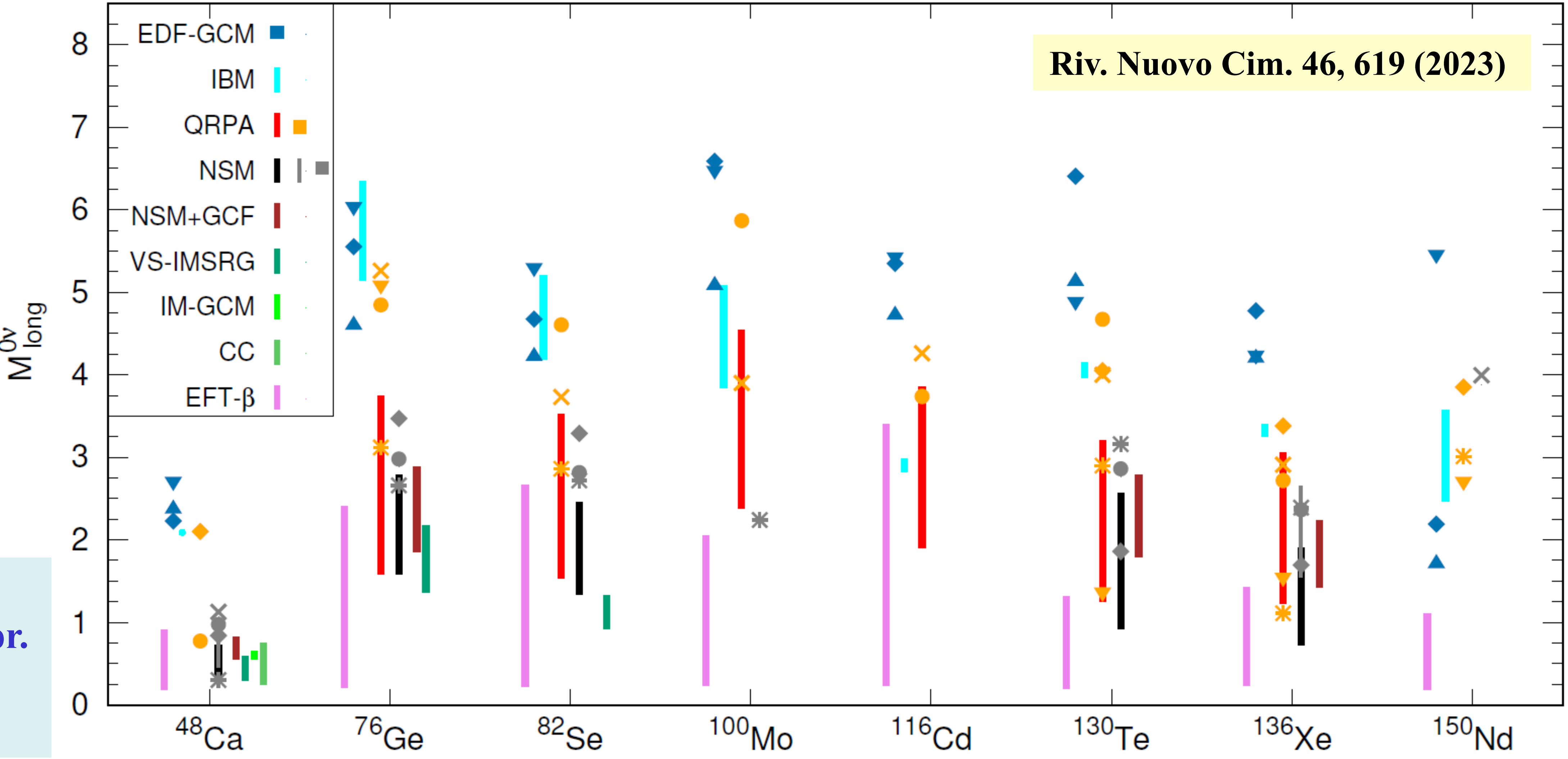
The nuclear w. f. of
(A,Z), (A,Z+1)*, (A,Z+2)
Many-body methods
of choice:

The 0νββ nuclear transition operators
(F, GT, and tensor type):

! Isospin, and spin-isospin symmetries
($M_{Fcl}\approx 0$, M_{GTcl} strongly suppressed):

All models missing essential physics

Impossible to assign rigorous uncertainties



Riv. Nuovo Cim. 46, 619 (2023)

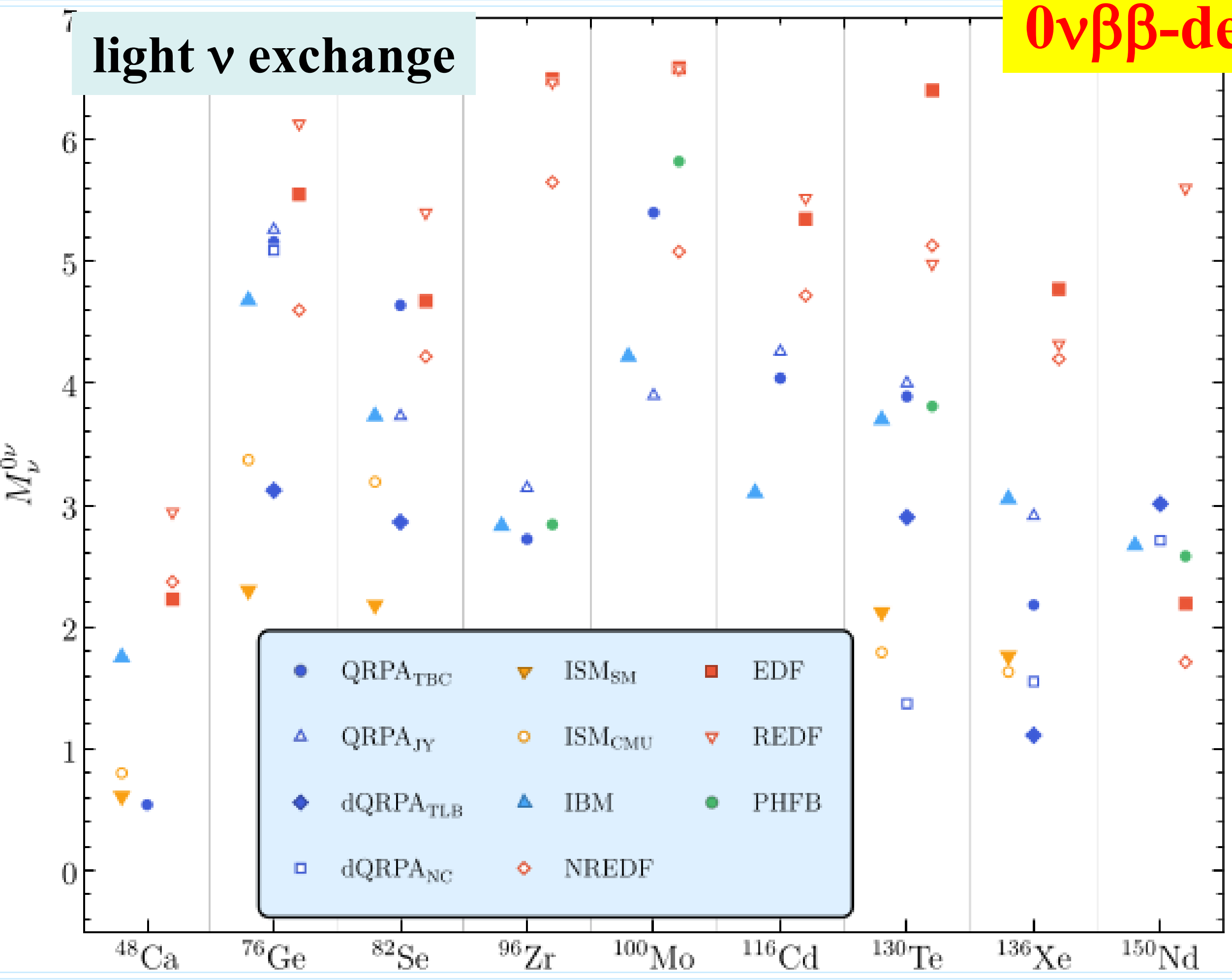
Differencies:
Many-body appr.
Size of the m.s.
residual int.

$$M'^{0\nu}(m_{\rm N}, g_{\rm A}^{\rm eff}) = \frac{1}{m_{\rm p} m_{\rm e}} \frac{R}{2\pi^2 g_A^2} \sum_n \int d^3x\, d^3y\, d^3p \times e^{i\mathbf{p}\cdot(\mathbf{x}-\mathbf{y})} \frac{\langle 0_F^+|J^{\mu\dagger}(\mathbf{x})|n\rangle\langle n|J_\mu^\dagger(\mathbf{y})|0_I^+\rangle}{\sqrt{p^2+m_N{}^2}(\sqrt{p^2+m_N{}^2}+E_n-\frac{E_I-E_F}{2})}$$

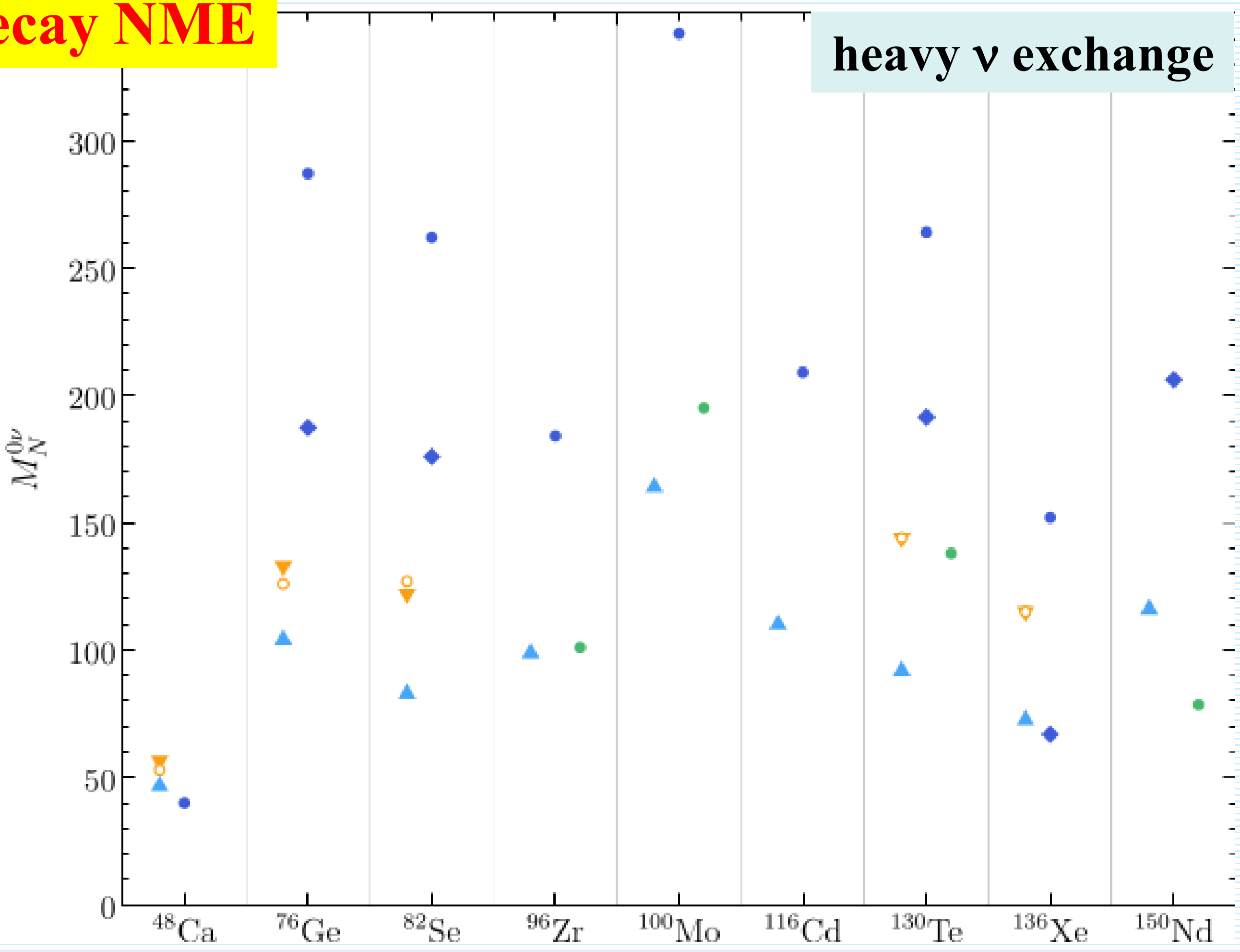
$$M'^{0\nu}(m_{\rm N}\to 0, g_{\rm A}^{\rm eff}) = \frac{1}{m_{\rm p} m_{\rm e}} M_\nu'^{0\nu}(g_{\rm A}^{\rm eff})$$

$$M'^{0\nu}(m_{\rm N}\to \infty, g_{\rm A}^{\rm eff}) = \frac{1}{m_{\rm N}^2} M_{\rm N}'^{0\nu}(g_{\rm A}^{\rm eff})$$

0νββ-decay NME



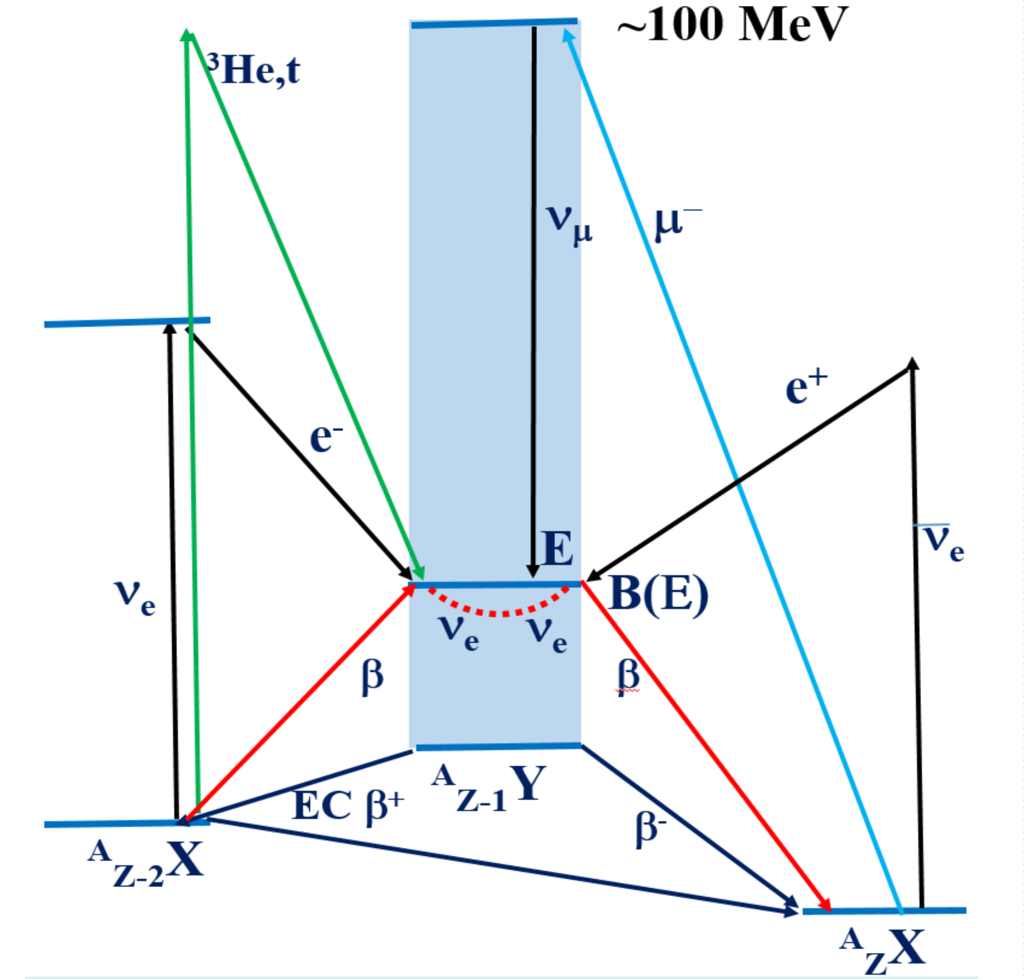
light ν exchange



heavy ν exchange

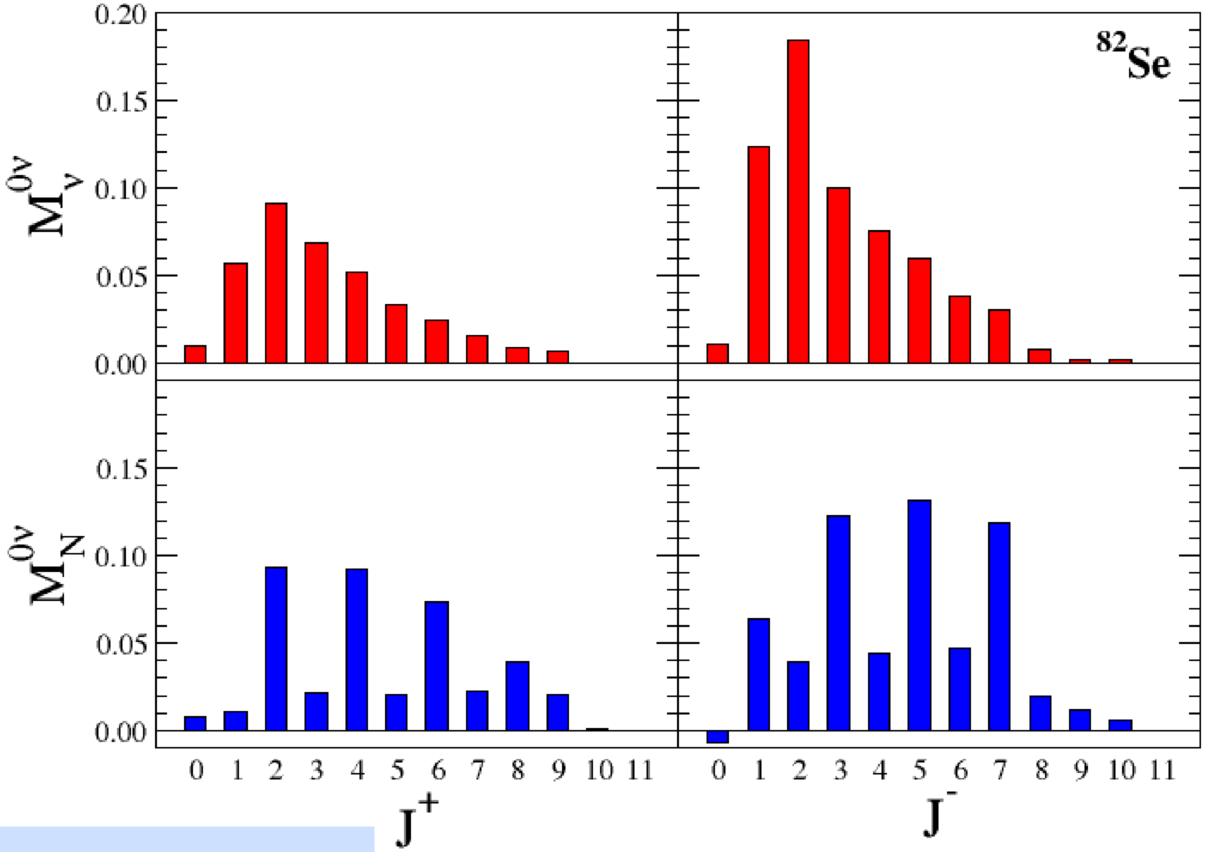
Supporting nuclear physics experiments

(Measurements still not conclusive for 0νββ NME)



- ✓ **β-decay, EC and 2νββ decay**
- ✓ **μ-capture**
- ✓ **(π$^+$, π$^-$), single charge exchange**
- ✓ **(^{3}He,t), (d,^{2}He), transfer reactions**
- ✓ **γ-ray spectroscopy, γγ-decay**
- ✓ **A promising experimental tool:**

  Heavy-Ion Double Charge-Exchange



Multipole decomposition of light and heavy 0νββ-decay NMEs normalized to unity

Higher multiploles are populated mostly due large ν-momenta transfer

Understanding of the 2νββ-decay NMEs is of crucial importance for correct evaluation of the 0νββ-decay NMEs

$$(A,Z) \rightarrow (A,Z+2) + 2e^{-} + 2\overline{\nu}_e$$

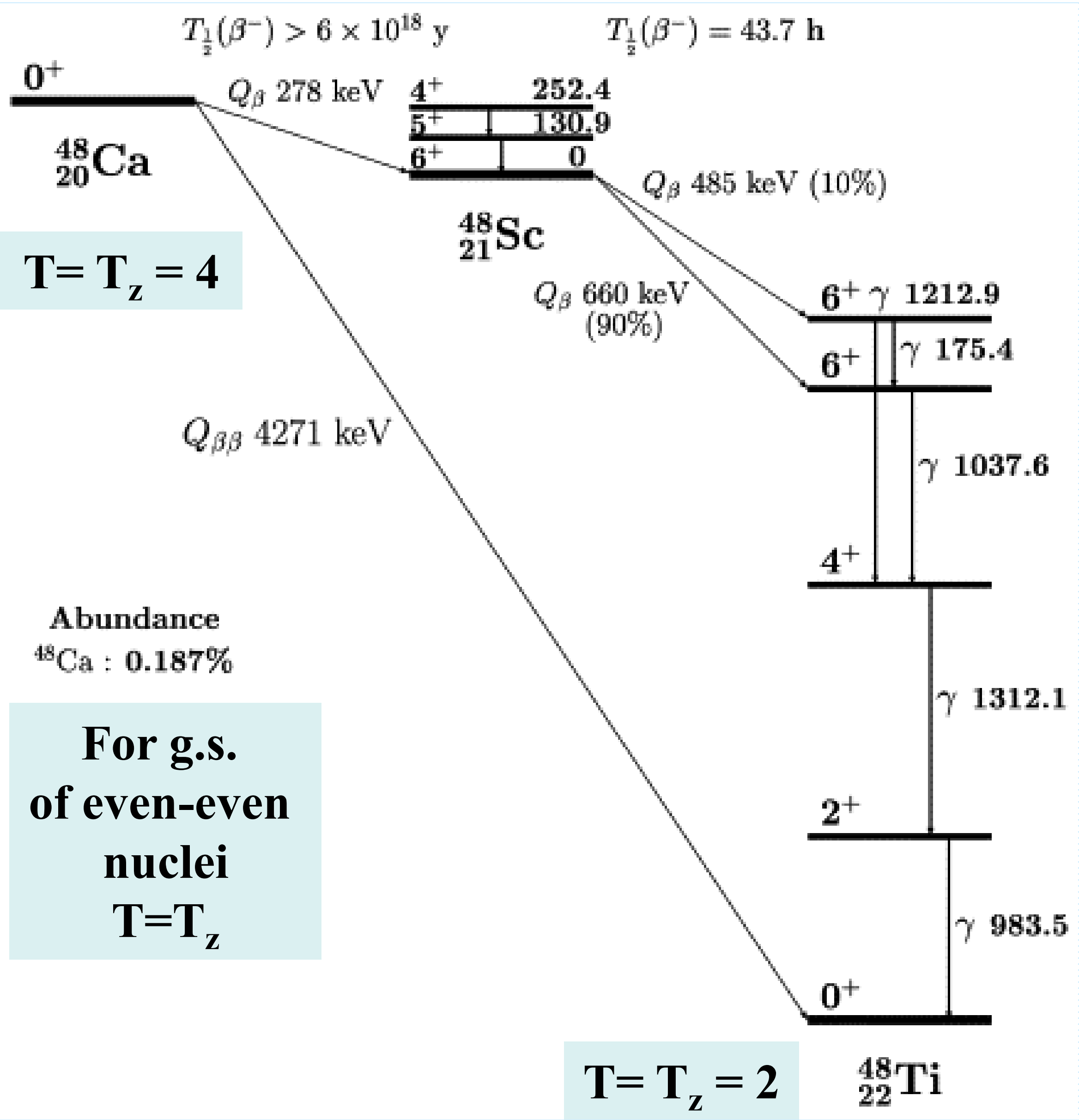
Both 2νββ and 0νββ operators connect the same states.
Both change two neutrons into two protons.

Explaining 2νββ-decay is necessary but not sufficient

Let's put credit approaches that allow to reproduce 2νββ-decay NMEs, in particular without closure approximations

QRPA calculations at the forefront:
i) importance of pairing int.; ii) importance of isospin and spin-isospin symmetry restoration (isoscalar and isovector residual int.); iii) importance of relative deformation of the initial and final nuclei

There is no reliable calculation of the 2νββ-decay NMEs yet



$T = T_z = 4$

For g.s. of even-even nuclei $T = T_z$

$T = T_z = 2$

2νββ-decay NME

Weak interaction Hamiltonian

$$\mathcal{H}^{\beta}(x) = \frac{G_F}{\sqrt{2}} 2\left[\bar{e}_L(x)\gamma_\alpha \nu_{eL}(x)\right] j_\alpha(x) + h.c.$$

Phys. Atom. Nucl. 62 (1999) 585

2νββ-decay amplitude

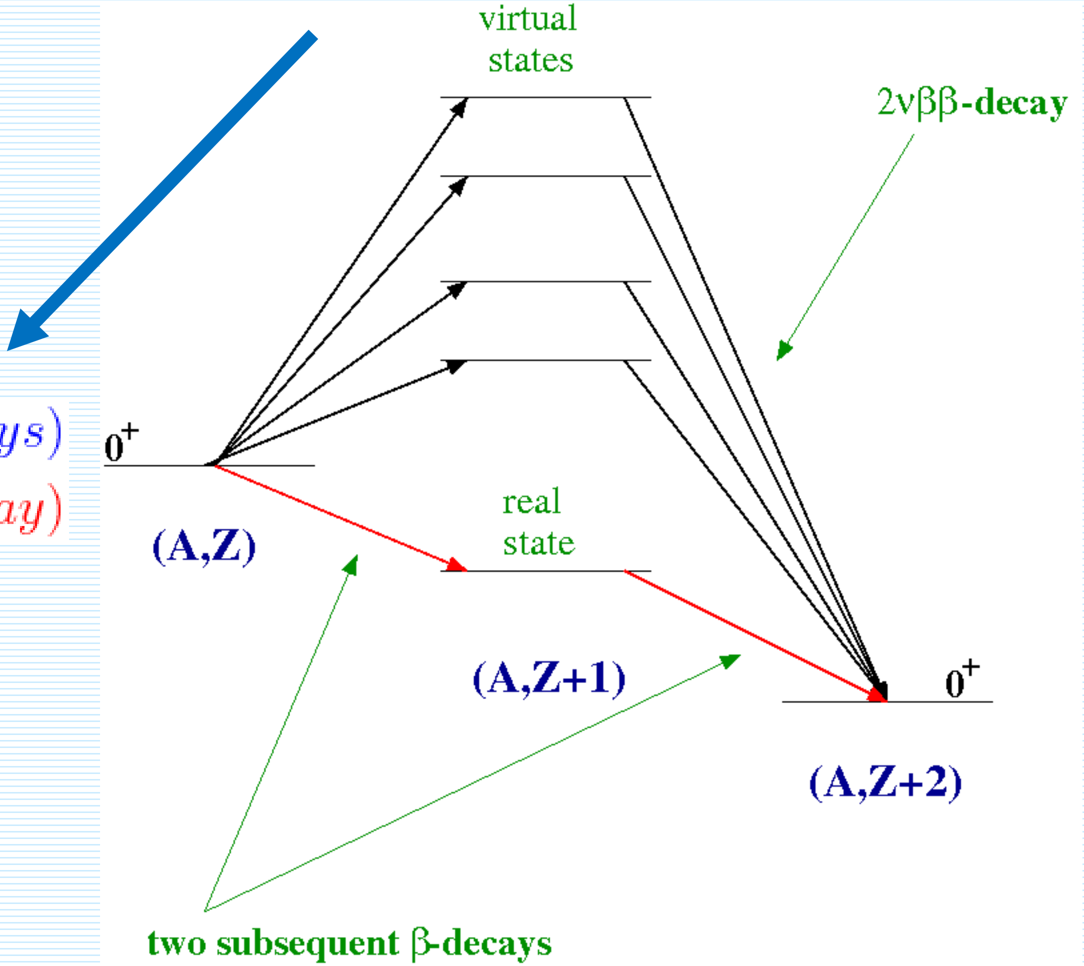
Hadron part of amplitude

$$J_{\mu\nu}(p_1,p_2,k_1,k_2) = \int e^{-i(p_1+k_1)x_1} e^{-i(p_2+k_2)x_2}$$
$${}_{out}<p_f|T(J_\mu(x_1)J_\nu(x_2))|p_i>_{in}\, dx_1 dx_2$$

$$< f|S^{(2)}|i > =$$
$$\frac{(-i)^2}{2}\left(\frac{G_F}{\sqrt{2}}\right)^2 L_{\mu\nu}(p_1,p_2,k_1,k_2) J_{\mu\nu}(p_1,p_2,k_1,k_2)$$
$$-(p_1 \leftrightarrow p_2) - (k_1 \leftrightarrow k_2) + (p_1 \leftrightarrow p_2)(k_1 \leftrightarrow k_2)$$

$$\begin{aligned} T(J_\mu(x_1)J_\nu(x_2)) &= J_\mu(x_1)J_\nu(x_2) && (two\ \beta - decays) \\ &+ \Theta(x_{20}-x_{10})[J_\nu(x_2),J_\mu(x_1)] && (2\nu\beta\beta - decay) \end{aligned}$$

F.Š., G. Pantis, Phys. Atom. Nucl. 62 (1999) 585



A sum over intermediate nuclear states represents a sum over all meson and γ-exchange correlations of two β-decaying nucleons inside nucleus

Due to $0^+ \rightarrow 0^+$ nuclear transition

$$J^{2\beta 2\nu}_{\mu\nu}(p_1, p_2, k_1, k_2) = -i2M_{GT}\delta_{\mu k}\delta_{\nu k}$$
$$\times 2\pi\delta(E_f - E_i + p_{10} + k_{10} + p_{20} + k_{20}), \ \ k = 1, 2, 3,$$

Nuclear current (impulse approx.)

$$J_\alpha(0, \vec{x}) = \sum_n \tau^+_n (\delta_{\alpha 4} + i g_A (\vec{\sigma})_k \delta_{\alpha k}) \delta(\vec{x} - \vec{x}_n)$$

2νββ-decay NME in time integral representation

$$M_{GT} = \frac{i}{2}\int_0^\infty (e^{i(p_{10}+k_{10}-\Delta)t} + e^{i(p_{20}+k_{20}-\Delta)t}) M_{AA}(t) dt$$

with

$$M_{AA}(t) =< 0^+_f | \frac{1}{2}[A_k(t/2), A_k(-t/2)] | 0^+_i >$$

time dependent axial current

$$A_k(t) = e^{iHt} A_k(0) e^{-iHt}, \qquad A_k = \sum_i \tau^+_i (\vec{\sigma}_i)_k, \ \ k = 1, 2, 3.$$

$$A_k(t) = e^{itH} A_k(0) e^{-itH} = \sum_{n=0}^\infty \frac{(it)^n}{n!} \overbrace{[H[H \ldots [H, A_k(0)] \ldots]]}^{n\ times}$$

Completeness: Σ_n |n><n|=1

$$< A' | J_\alpha(x_1) J_\beta(x_2) | A > \ = \ \sum_n < A' | J_\alpha(0, \vec{x}_1) | n >< n | J_\beta(0, \vec{x}_2) | A > \times$$
$$e^{-i(E' - E_n)x_{10}} e^{-i(E_n - E)x_{20}}$$

integration over time variable

$$\int_0^\infty e^{-iat} dt \Rightarrow \lim_{\epsilon \to 0} \int_0^\infty e^{-i(a - i\epsilon)t} dt = \lim_{\epsilon \to 0} \frac{-i}{a - i\epsilon}$$

dor S

Standard form of 2νββ-decay NME

$$M_{GT} = \sum_n \frac{< 0^+_f | A(0)_k | 1^+_n >< 1^+_n | A(0)_k | 0^+_i >}{E_n - E_i + \boxed{\Delta}}$$

$\Delta = (E_i - E_f)/2$, stands for sum of lepton energies

Double beta decay is a two-body process

$$\mathbf{H = one - body + two - body}, \quad \mathrm{A}_k(0) = \text{one–body}$$

$$A_k(t) = \sum_{n=0}^{\infty} \frac{(it)^n}{n!} \overbrace{[H[H...[H}^{n\ times}, A_k(0)]...]]$$

$$\begin{tikzcd}
\mathrm{n}=0 & {[H^{(0)}A]} & {[1]} \arrow[d] \arrow[dr] & & & \\
\mathrm{n}=1 & {[H^{(1)}, A]} & {[1]} \arrow[d] \arrow[dr] & {[2]} \arrow[d] \arrow[dr] & & \\
\mathrm{n}=2 & {[H^{(2)}, A]} & {[1]} \arrow[d] \arrow[dr] & {[2]} \arrow[d] \arrow[dr] & {[3]} \arrow[d] \arrow[dr] & \\
\mathrm{n}=3 & {[H^{(3)}, A]} & {[1]} & {[2]} & {[3]} & {[4]}
\end{tikzcd}$$

$$If \quad \mathrm{H} \approx \text{one–body op.} \Longrightarrow \mathbf{A_k(t)} \text{ is one–body op.}$$

The two-nucleon β-decays in 2νββ-decay are connected through strong and EM interactions.

$$S^{(2)} = -\frac{(-i)^2}{2} 4 \left(\frac{G_F}{\sqrt{2}}\right)^2 \times$$

$$\int N[\overline{e}_L(x_1)\gamma\nu_{eL}(x_1)\ \overline{e}_L(x_2)\gamma\nu_{eL}(x_2)] \times$$

$$T\left[j_\alpha(x_1)\ j_\beta(x_2)\ exp\left\{-i\int \mathcal{H}^{\text{str+em}}(x)dx\right\}\right]\ dx_1\ dx_2$$

$$[j_\alpha(x_1), j_\beta(x_2)] \ =$$

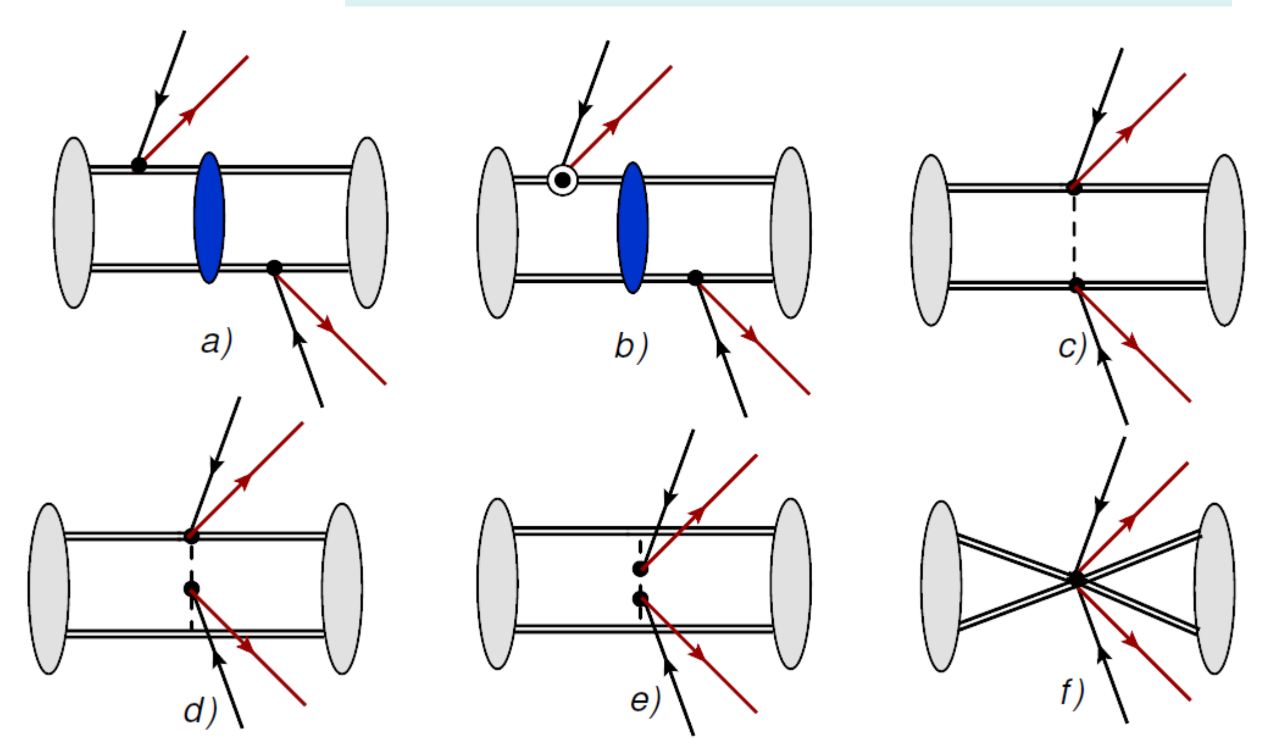
$$[\overline{p}(x_1)\gamma_\alpha(1+\gamma_5)n(x_1), \overline{p}(x_2)\gamma_\alpha(1+\gamma_5)n(x_2)$$

$$= 0$$

$$J_\alpha(x) = U^\dagger(x_0,0)\ j_\alpha(x)\ U(x_0,0)$$

$$U(t,t_0) = T\left(exp\left[-i\int_{t_0}^{t}\mathcal{H}^{str+em}(t)dt\right]\right)$$

c, d, e pion-exchange contributions are already included ín a and b diagrams



2νββ-decay rate

$$\left[T_{1/2}^{2\nu\beta\beta}(0^+)\right]^{-1} = \frac{m_e}{8\pi^7 \ln 2}(G_\beta m_e^2)^4 I^{2\nu}(0^+),$$

$$I^{2\nu}(0^+) = \frac{1}{m_e^9}\int_{m_e}^{E_i-E_f-m_e} F_0(Z_f,E_{e_1})p_{e_1}E_{e_1}dE_{e_1}$$
$$\times\int_{m_e}^{E_i-E_f-E_{e_1}} F_0(Z_f,E_{e_2})p_{e_2}E_{e_2}dE_{e_2}$$
$$\times\int_{0}^{E_i-E_f-E_{e_1}-E_{e_2}} E_{\nu_1}^2 E_{\nu_2}^2 \mathcal{A}^{2\nu} dE_{\nu_1}.$$

$$\mathcal{A}^{2\nu} = g_V^4\left[\frac{1}{4}|M_F^K + M_F^L|^2 + \frac{3}{4}|M_F^K - M_F^L|^2\right]$$
$$-g_V^2 g_A^2 \mathrm{Re}\left\{M_F^{K*}M_{GT}^L + M_{GT}^{K*}M_F^L\right\}$$
$$+\frac{g_A^4}{3}\left[\frac{3}{4}|M_{GT}^K + M_{GT}^L|^2 + \frac{1}{4}|M_{GT}^K - M_{GT}^L|^2\right]$$

$$M_F^K = \sum_n \frac{K(0_n^+)}{2}F_n, \quad M_F^L = \sum_n \frac{L(0_n^+)}{2}F_n,$$
$$M_{GT}^K = \sum_n \frac{K(1_n^+)}{2}G_n, \quad M_{GT}^L = \sum_n \frac{L(1_n^+)}{2}G_n.$$

$$K_n(J^+) = \frac{2}{(2E_n(J^+) - E_i - E_f) + \epsilon_K} + \frac{2}{(2E_n(J^+) - E_i - E_f) - \epsilon_K}$$

$$L_n(J^+) = \frac{2}{(2E_n(J^+) - E_i - E_f) + \epsilon_L} + \frac{2}{(2E_n(J^+) - E_i - E_f) - \epsilon_L}$$

$$F_n = \langle 0_f^+ \parallel \sum_m \tau_m^- \parallel 0_n^+\rangle\langle 0_n^+ \parallel \sum_m \tau_m^- \parallel 0_i^+\rangle,$$
$$G_n = \langle 0_f^+ \parallel \sum_m \tau_m^-\sigma_m \parallel 1_n^+\rangle\langle 1_n^+ \parallel \sum_m \tau_m^-\sigma_m \parallel 0_i^+\rangle$$

$$\epsilon_K = E_{e_2} + E_{\nu_2} - E_{e_1} - E_{\nu_1}$$

$$\epsilon_L = E_{e_1} + E_{\nu_2} - E_{e_2} - E_{\nu_1}$$

In the closure approximation, by assuming

$$2E_n - E_i - E_f = 0$$

$$\Rightarrow \mathcal{A}^{2\nu} = 0$$

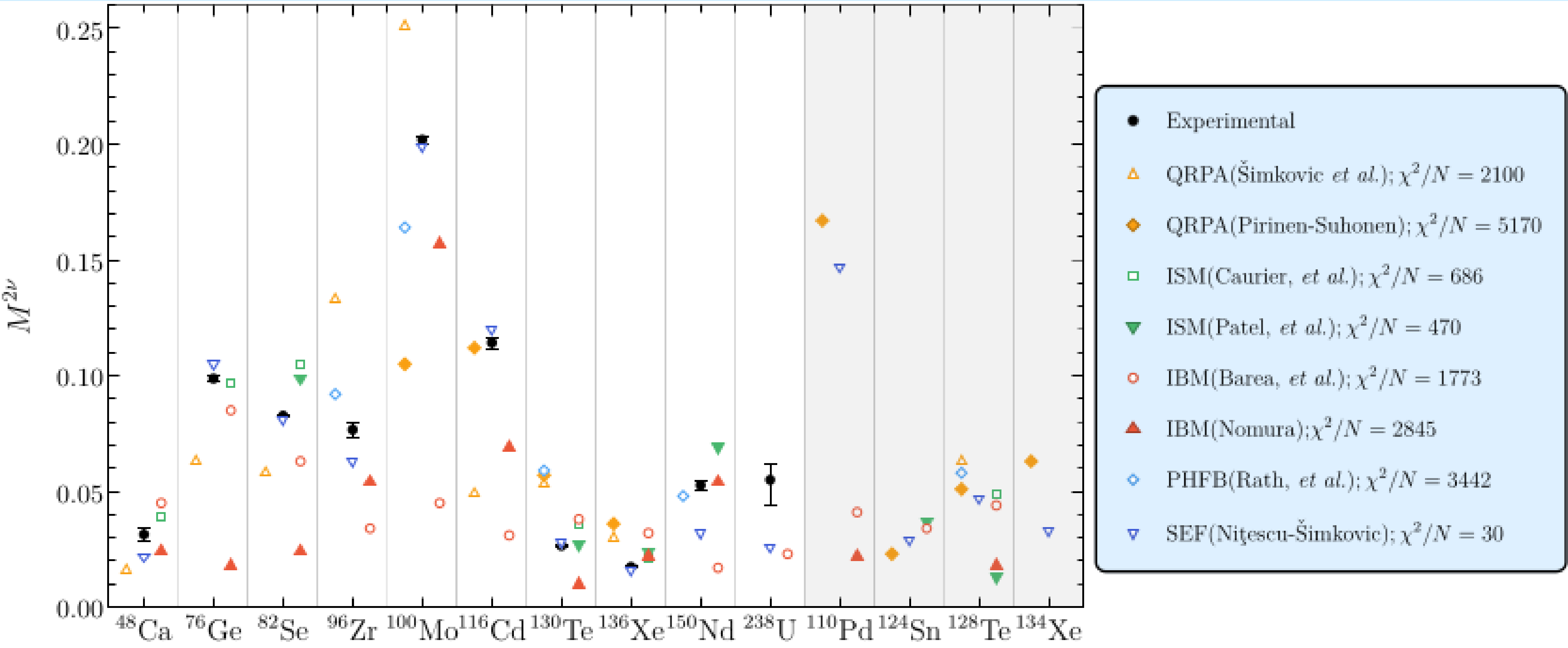
Is the real closure value close?

Cancellation of contributions from higher and lower lying states of intermediate nucleus

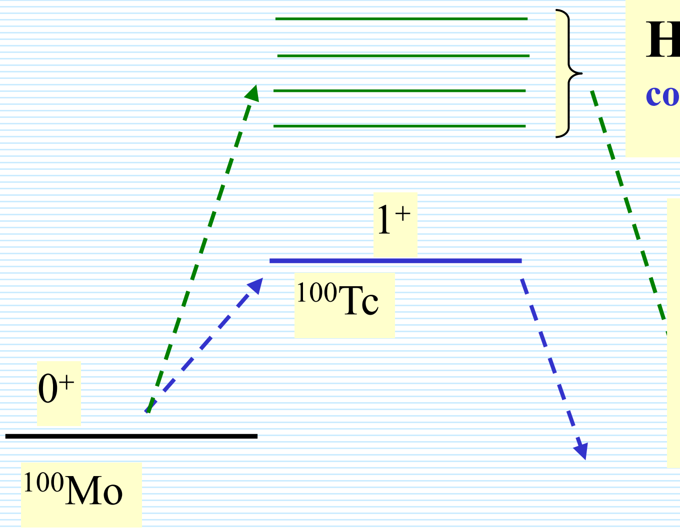
2νββ-decay
(theory versus experiment)

$$(T^{2\nu}_{1/2})^{-1} = G^{2\nu}|M^{2\nu}_{GT}|^2$$

$$M^{2\nu}_{GT} = \sum_m \frac{< 0_f^+||\tau^+\sigma||1_m^+ >< 1_m^+||\tau^+\sigma||0_i^+ >}{E_m - E_i + \Delta}$$



Single State Dominance (^{100}Mo, ^{106}Cd, ^{116}Cd, ^{128}Te …)



HSD, higher levels contribute to the decay

SSD, 1$^+$ level dominates in the decay

(Abad et al., 1984, Ann. Fis. A 80, 9)

Isotope	f.s.	$T_{1/2}$(SSD)[y]	$T_{1/2}$(exp.)[y]
		$2\nu\beta^-\beta^-$	
^{100}Mo	$0_{g.s.}$	7.3 10^{18}	7.07 10^{18}
	0_1	4.2 10^{20}	6.1 10^{20}
^{116}Cd	$0_{g.s.}$	1.1 10^{19}	2.63 10^{19}
^{128}Te	$0_{g.s.}$	1.1 10^{25}	2.49 10^{24}
		EC/EC	
^{106}Cd	$0_{g.s.}$	>4.4 10^{21}	>4.7 10^{20}
^{130}Ba	$0_{g.s.}$	5.0 10^{22}	2.2 10^{21}

$$M^K_{GT} = \sum_m \left(\frac{M^i_m(1^+)M^f_m(1^+)}{E_m - E_i + e_{10} + \nu_{10}} + \frac{M^i_m(1^+)M^f_m(1^+)}{E_m - E_i + e_{20} + \nu_{20}} \right)$$

$$\Rightarrow \frac{M^i_1(1^+)M^f_1(1^+)}{E_1 - E_i + e_{10} + \nu_{10}} + \frac{M^i_1(1^+)M^f_1(1^+)}{E_1 - E_i + e_{20} + \nu_{20}} \Rightarrow 2\frac{M^i_1(1^+)M^f_1(1^+)}{E_1 - E_i + \Delta}$$

SSD

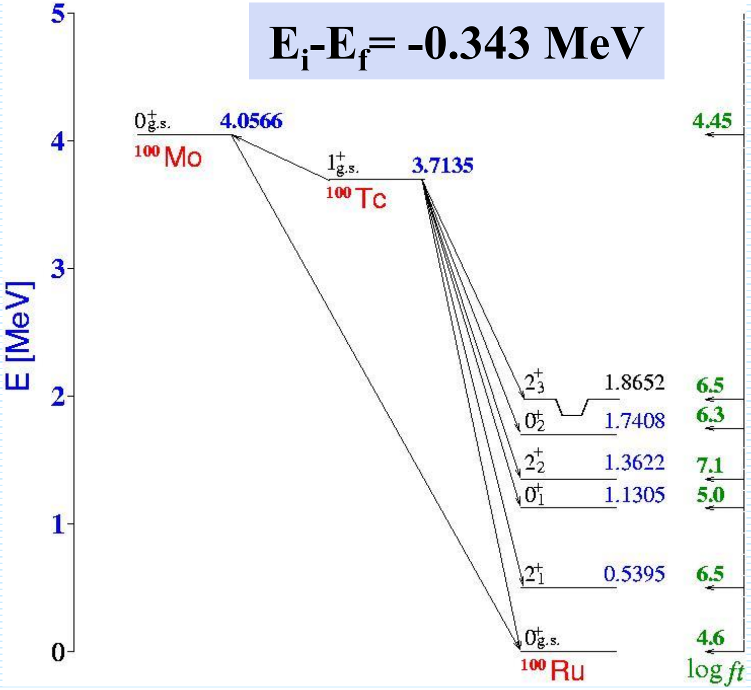
↑ **common approx.**

$$M^K_{GT} = M^L_{GT}(\nu_{10} \leftrightarrow \nu_{20})$$

$$e_{10} + \nu_{10} \approx e_{20} + \nu_{20} \approx (E_i - E_f)/2 \equiv \Delta$$

Domin, Kovalenko, F.Š., Semenov, NPA 753, 337 (2005)

E_i-E_f= -0.343 MeV

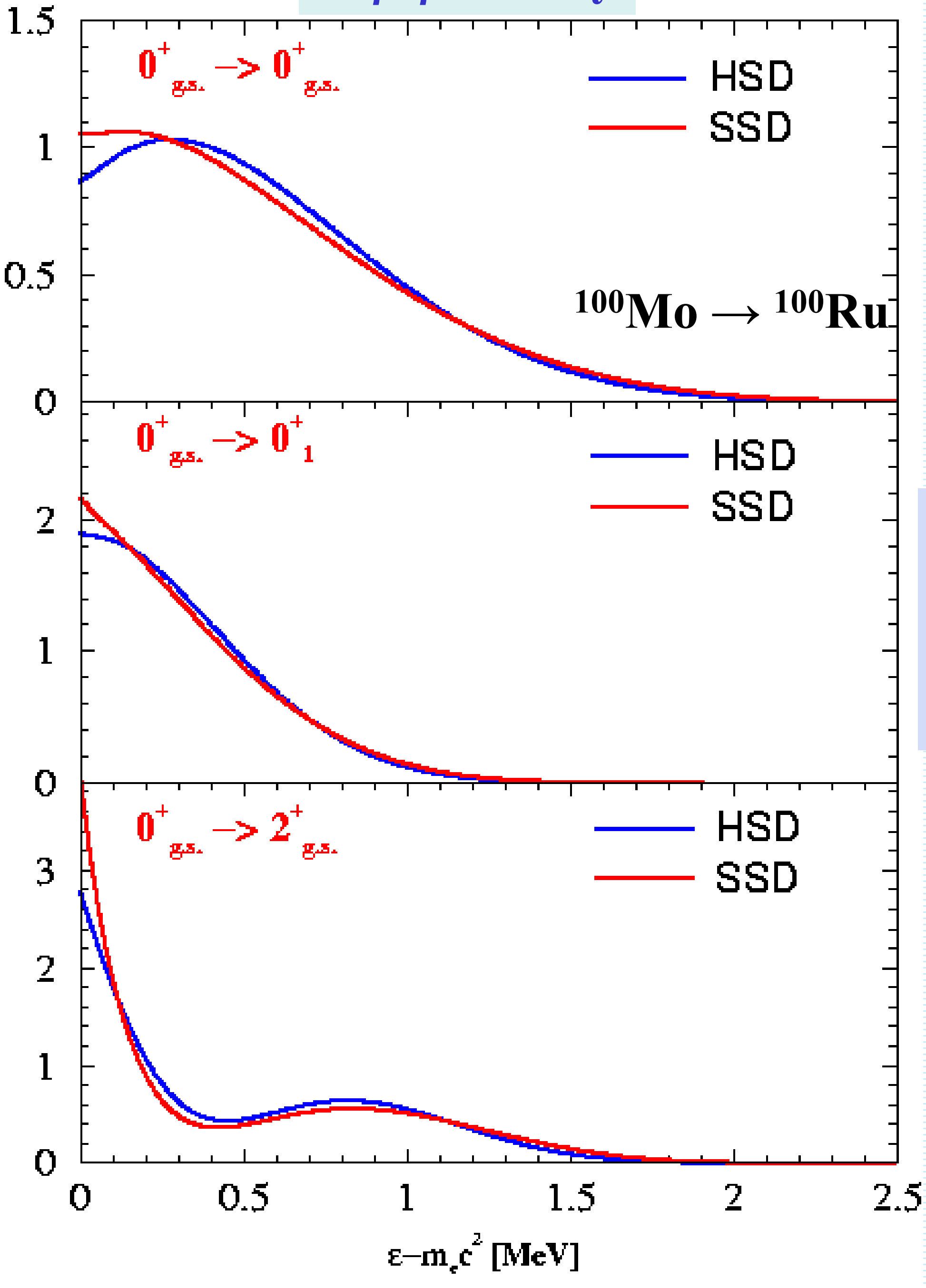


Study of differential characteristics

E_1-E_i ≈ 0 or neg. ⇒ sensitivity to lepton energies in energy Denominators

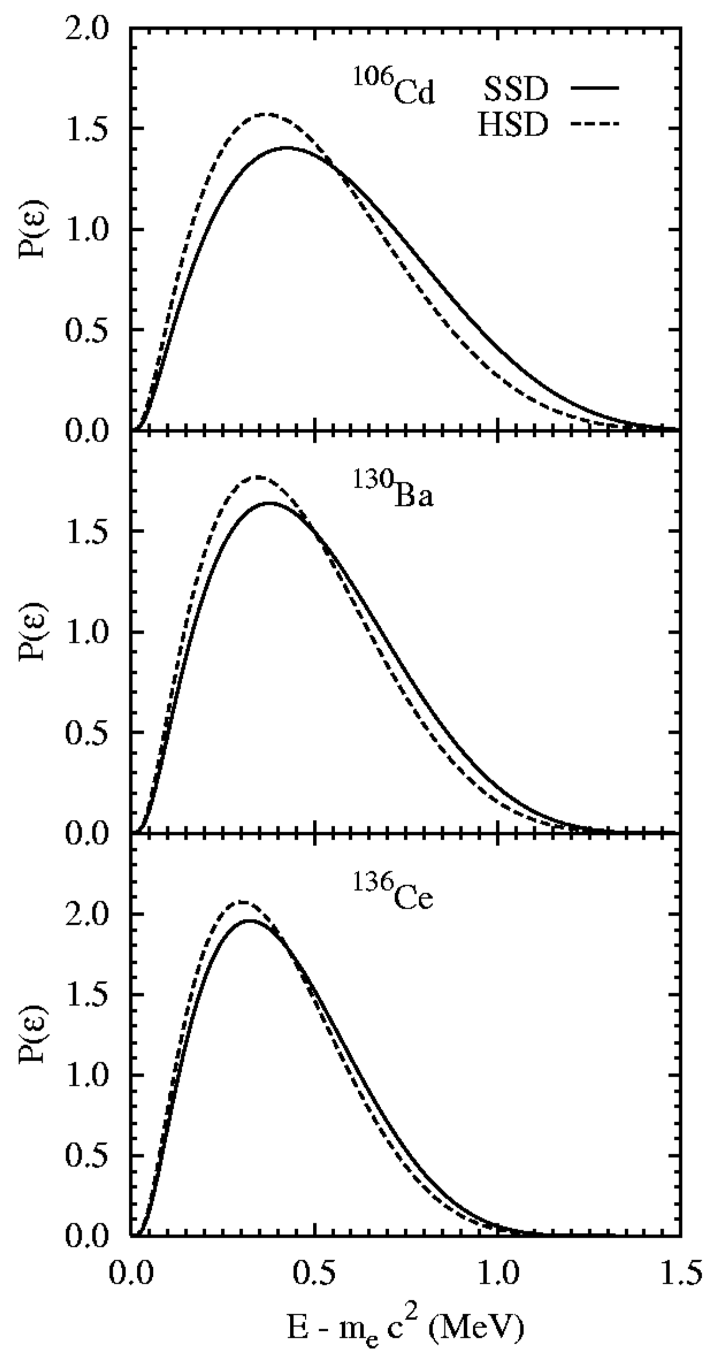
⇒ **SSD** and **HSD** offer different differential characteristics

$2\nu\beta^-\beta^-$–decay



Do not depend on M^iM^f

$2\nu EC/\beta^+$–decay



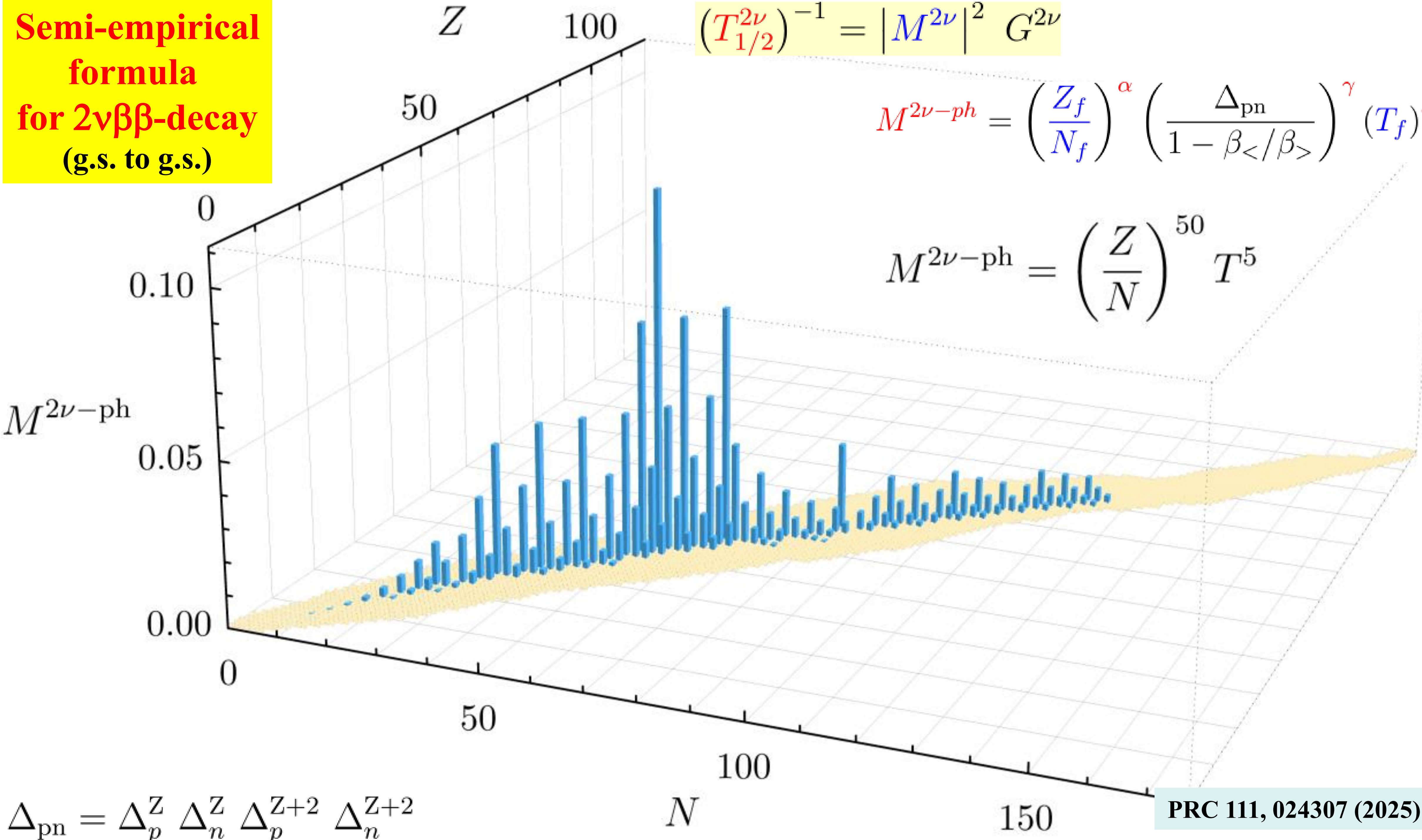
F.Š., Šmotlák, Semenov
J. Phys. G, 27, 2233, 2001

Semi-empirical formula for 2νββ-decay

(g.s. to g.s.)

$$\left(T^{2\nu}_{1/2}\right)^{-1} = \left|M^{2\nu}\right|^{2} \ G^{2\nu}$$

$$M^{2\nu-ph} = \left(\frac{Z_f}{N_f}\right)^{\alpha} \left(\frac{\Delta_{\mathrm{pn}}}{1-\beta_{<}/\beta_{>}}\right)^{\gamma} (T_f)^{\sigma}$$



$$M^{2\nu-\mathrm{ph}} = \left(\frac{Z}{N}\right)^{50} T^{5}$$

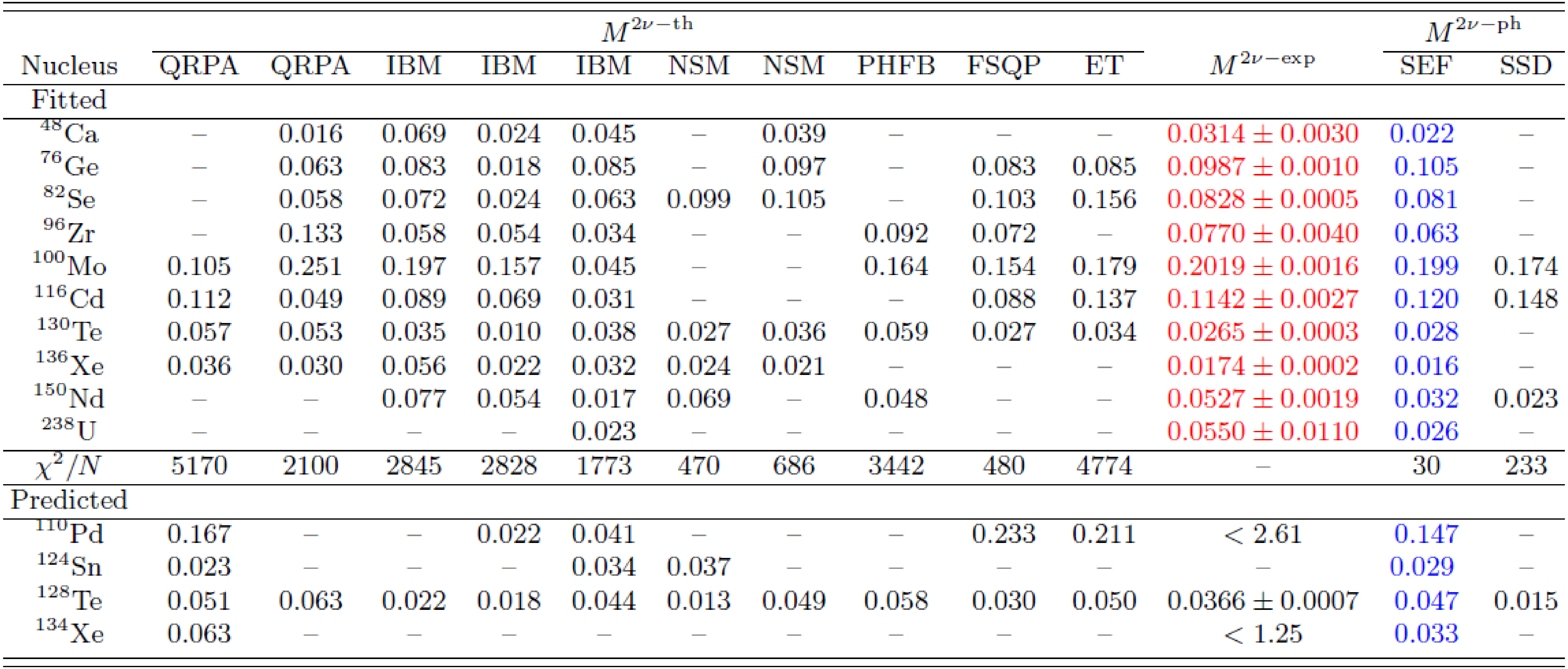
$$\Delta_{\mathrm{pn}} = \Delta_p^{\mathrm{Z}} \ \Delta_n^{\mathrm{Z}} \ \Delta_p^{\mathrm{Z+2}} \ \Delta_n^{\mathrm{Z+2}}$$

PRC 111, 024307 (2025)

$$\chi^2 = \sum_{i=1}^{N} \frac{(O_i - P_i)^2}{\sigma_i^2}$$

There is no reliable calculation of the *2νββ-decay NMEs yet*

PRC 111, 024307 (2025)



Nucleus	$M^{2\nu-\mathrm{th}}$										$M^{2\nu-\mathrm{exp}}$	$M^{2\nu-\mathrm{ph}}$	
	QRPA	QRPA	IBM	IBM	IBM	NSM	NSM	PHFB	FSQP	ET		SEF	SSD
Fitted													
^{48}Ca	–	0.016	0.069	0.024	0.045	–	0.039	–	–	–	0.0314 ± 0.0030	0.022	–
^{76}Ge	–	0.063	0.083	0.018	0.085	–	0.097	–	0.083	0.085	0.0987 ± 0.0010	0.105	–
^{82}Se	–	0.058	0.072	0.024	0.063	0.099	0.105	–	0.103	0.156	0.0828 ± 0.0005	0.081	–
^{96}Zr	–	0.133	0.058	0.054	0.034	–	–	0.092	0.072	–	0.0770 ± 0.0040	0.063	–
^{100}Mo	0.105	0.251	0.197	0.157	0.045	–	–	0.164	0.154	0.179	0.2019 ± 0.0016	0.199	0.174
^{116}Cd	0.112	0.049	0.089	0.069	0.031	–	–	–	0.088	0.137	0.1142 ± 0.0027	0.120	0.148
^{130}Te	0.057	0.053	0.035	0.010	0.038	0.027	0.036	0.059	0.027	0.034	0.0265 ± 0.0003	0.028	–
^{136}Xe	0.036	0.030	0.056	0.022	0.032	0.024	0.021	–	–	–	0.0174 ± 0.0002	0.016	–
^{150}Nd	–	–	0.077	0.054	0.017	0.069	–	0.048	–	–	0.0527 ± 0.0019	0.032	0.023
^{238}U	–	–	–	–	0.023	–	–	–	–	–	0.0550 ± 0.0110	0.026	–
χ^2/N	5170	2100	2845	2828	1773	470	686	3442	480	4774	–	30	233
Predicted													
^{110}Pd	0.167	–	–	0.022	0.041	–	–	–	0.233	0.211	< 2.61	0.147	–
^{124}Sn	0.023	–	–	–	0.034	0.037	–	–	–	–	–	0.029	–
^{128}Te	0.051	0.063	0.022	0.018	0.044	0.013	0.049	0.058	0.030	0.050	0.0366 ± 0.0007	0.047	0.015
^{134}Xe	0.063	–	–	–	–	–	–	–	–	–	< 1.25	0.033	–

$M^{2\nu}_{F\text{-}cl} = 0$

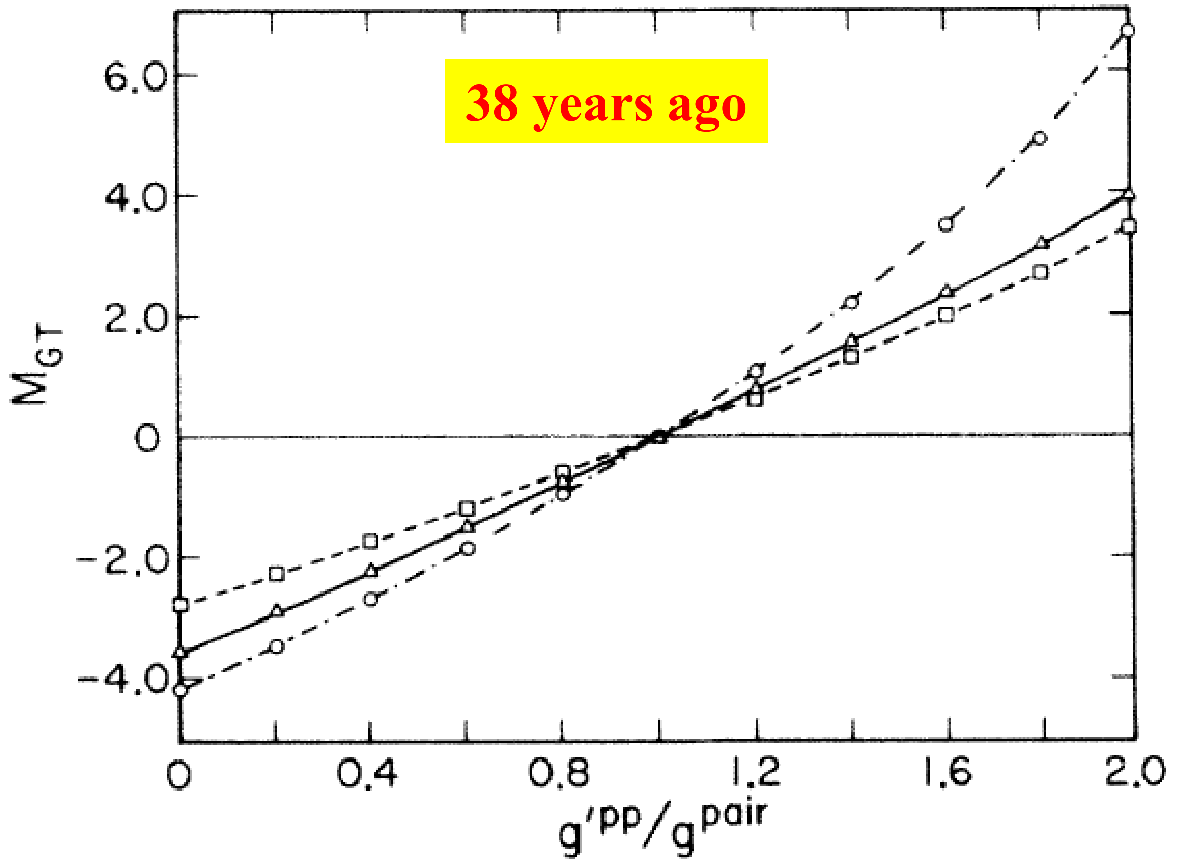
The DBD Nuclear Matrix Elements and the SU(4) symmetry restoration)

$M^{2\nu}_{GT\text{-}cl} = 0$

Suppression of the Two Neutrino Double Beta Decay by Nuclear Structure Effects

P. Vogel, M.R. Zirnbauer, PRL (1986) 3148

O. Civitarese, A. Faessler, T. Tomoda, PLB 194 (1987) 11
E. Bender, K. Muto, H.V. Klapdor, PLB 208 (1988) 53
…



38 years ago

The isospin is known to be a good approximation in nuclei

In heavy nuclei the SU(4) symmetry is strongly broken by the spin-orbit splitting.

What is beyond this behavior?
Is it an artifact of the QRPA?

Study of $2\nu\beta\beta$ NME within the schematic model

s.p. mean-field

Conserves SU(4) symmetry

$$H = \underbrace{e_n N_n + e_p N_p - g_{pair}\left(\sum_{M_T=-1,0,1} A^\dagger_{0,1}(0,M_T)A_{0,1}(0,M_T) + \sum_{M_S=-1,0,1} A^\dagger_{1,0}(M_S,0)A_{1,0}(M_S,0)\right) + g_{ph}\sum_{a,b} E^\dagger_{a,b}E_{a,b}}_{H_0}$$

$$\underbrace{+ (g_{pair} - g_{pp}^{T=0}) \sum_{M_S=-1,0,1} A^\dagger_{1,0}(M_S,0)A_{1,0}(M_S,0) + (g_{pair} - g_{pp}^{T=1}) A^\dagger_{0,1}(0,0)A_{0,1}(0,0)\,.}$$

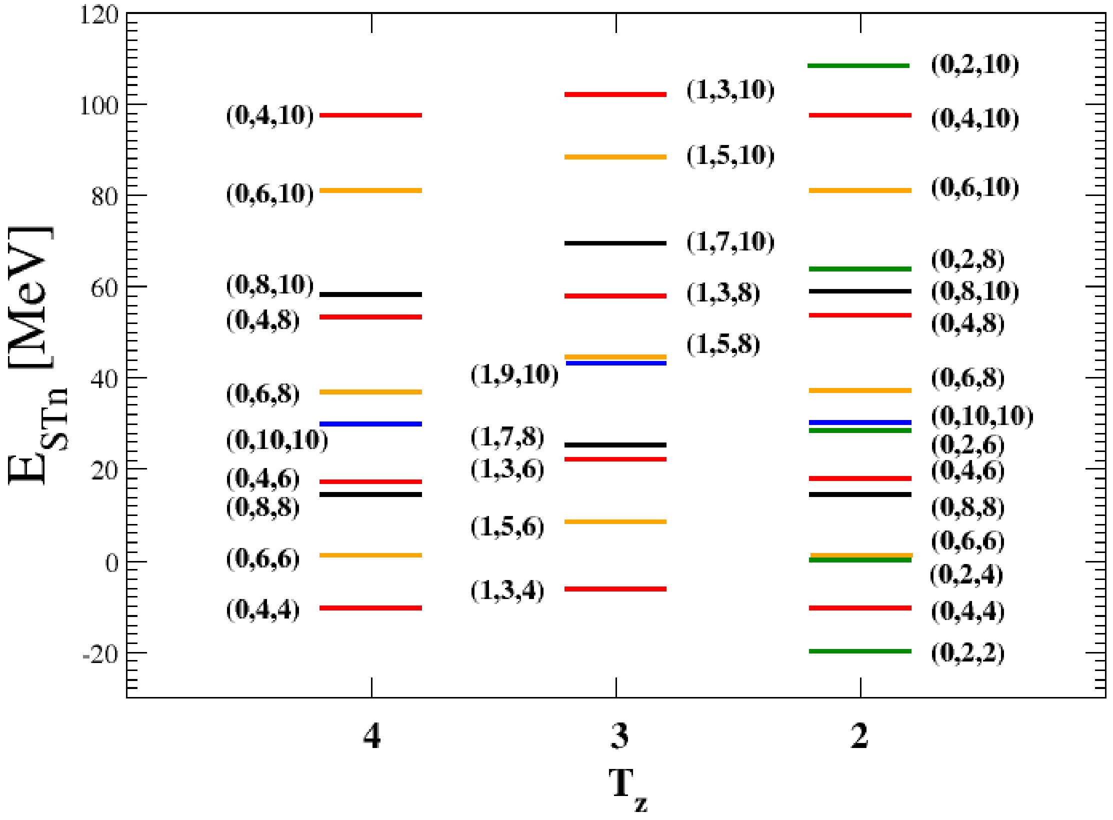
H_I violates SU(4) symmetry

g_{pair}- strength of isovector like nucleon pairing (L=0, S=0, T=1, M_T=±1)

$g_{pp}^{T=1}$- strength of isovector spin-0 pairing (L=0, S=0, T=1, M_T=0)

$g_{pp}^{T=0}$- strength of isoscalar spin-1 pairing (L=0, S=1, T=0)

g_{ph}- strength of particle-hole force



Energies of excited states for the case of conserved SU(4) symmetry

M_F=0, M_{GT}=0 (see SU(4) multiplets)

D. Štefánik, F.Š., A. Faessler, PRC 91, 064311 (2015)

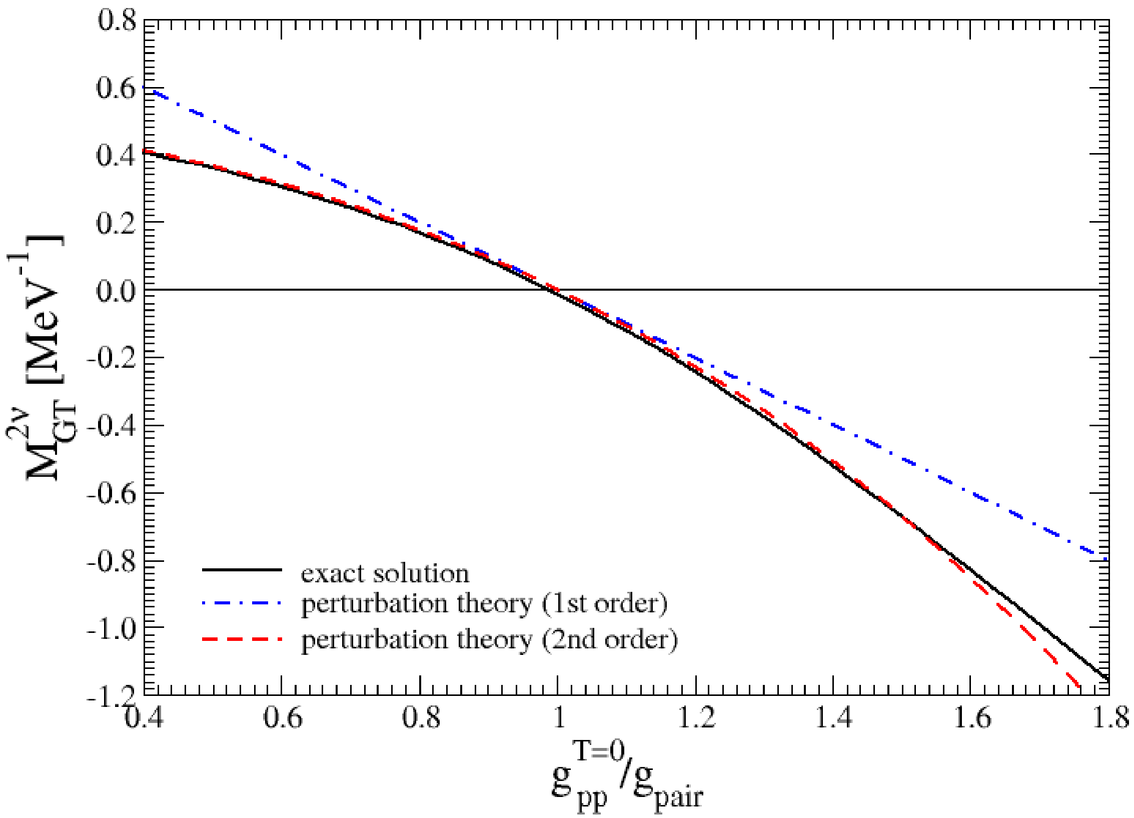
M_F and M_{GT} do not depend on the mean-field part of H

M_F and M_{GT} are governed by a weak violation of the SU(4) symmetry by the article-particle interaction of H

$$M_F^{2\nu} = -\frac{48\sqrt{\frac{33}{5}}\boxed{(g_{pair} - g_{pp}^{T=1})}}{(5g_{pair} + 3g_{ph})(10g_{pair} + 6g_{ph})}$$

$$M_{GT}^{2\nu} = \frac{144\sqrt{\frac{33}{5}}}{5g_{pair} + 9g_{ph}} \left\{ \frac{\boxed{(g_{pair} - g_{pp}^{T=0})}}{(10g_{pair} + 20g_{ph})} + \frac{2g_{ph}\boxed{(g_{pair} - g_{pp}^{T=1})}}{(10g_{pair} + 20g_{ph})(10g_{pair} + 6g_{ph})} \right\}$$

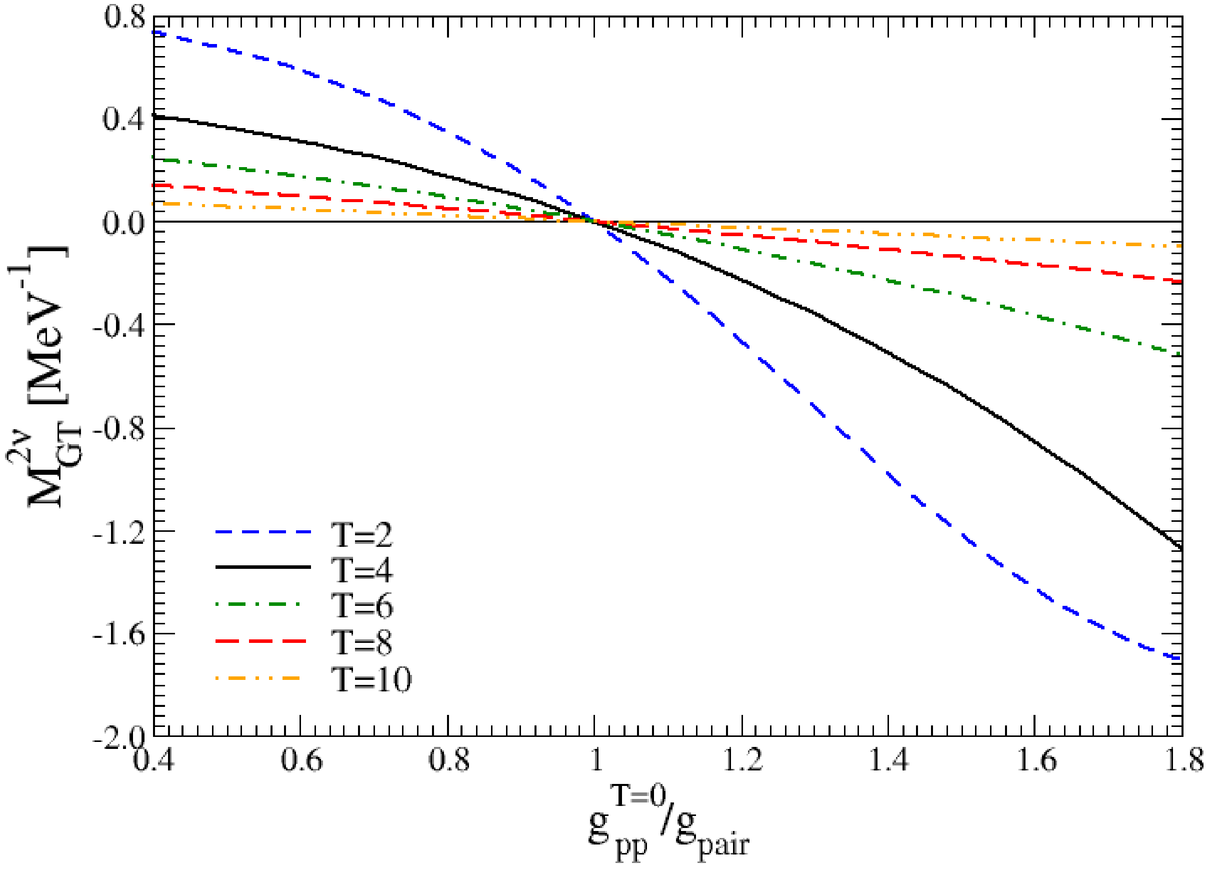
M_{GT} up to the second order of perturbation theory due to the violation of the SU(4) symmetry by the particle-particle interaction of H

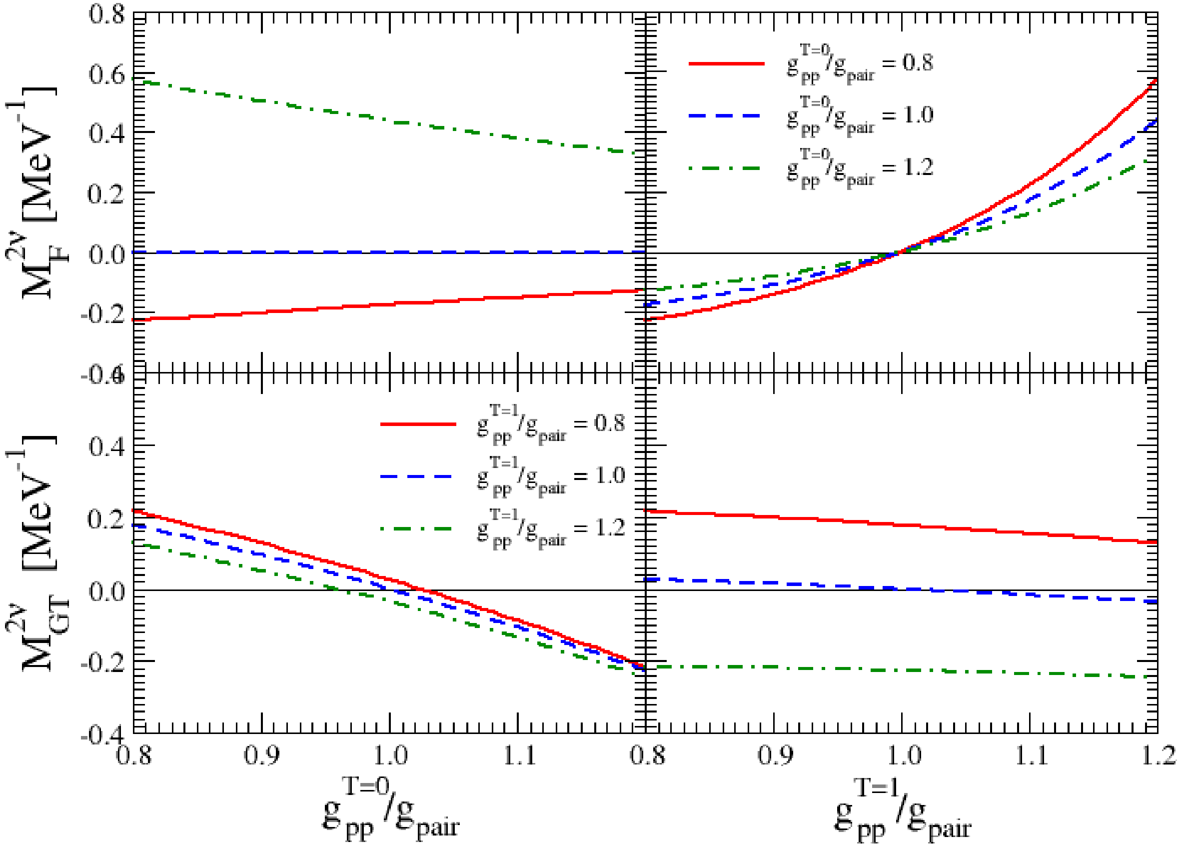


Results confirm the dependence of M_F and M_{GT} on $g_{pp}^{T=0}$ and $g_{pp}^{T=1}$ by the QRPA

By assuming a fixed violation of the SU(4) symmetry by particle-particle int.
M_{GT} decreases by increase of isospin of the ground state

Results confirm dependence of M_F and M_{GT} on $g_{pp}^{T=0}$ and $g_{pp}^{T=1}$ by the QRPA





Improved description of the 0νββ–decay rate (a way to fix g_A^{eff})

Both 2νββ and 0νββ operators connect the same states.
Both change two neutrons into two protons.
Explaining 2νββ-decay is necessary but not sufficient

PRC 97, 034315 (2018)

$$M_{GT}^{K,L} = m_e \sum_n M_n \frac{E_n - (E_i + E_f)/2}{[E_n - (E_i + E_f)/2]^2 - \varepsilon^2_{K,L}}$$

$$\epsilon_K = (E_{e_2} + E_{\nu_2} - E_{e_1} - E_{\nu_1})/2$$
$$\epsilon_L = (E_{e_1} + E_{\nu_2} - E_{e_2} - E_{\nu_1})/2$$

Taylor expansion $\frac{\varepsilon_{K,L}}{E_n - (E_i + E_f)/2}$

$$\epsilon_{K,L} \in (-\frac{Q}{2}, \frac{Q}{2})$$

We get

$$\left[T^{2\nu}_{1/2}\right]^{-1} \simeq (g_A^{\text{eff}})^4 \left|M_{GT-1}\right|^2 \left[G_0^{2\nu} + \xi_{31} G_2^{2\nu} + (\xi_{31})^2 G_{22}^{2\nu} + \left(\frac{1}{3}(\xi_{31})^2 + \xi_{51}\right) G_4^{2\nu} + \frac{1}{3}\xi_{31}\xi_{51} G_{42}^{2\nu} + \frac{2}{3}\xi_{31}\xi_{51} G_6^{2\nu}\right]$$

$$\xi_{31} = \frac{M^{2\nu}_{GT-3}}{M^{2\nu}_{GT-1}}$$
$$\xi_{51} = \frac{M^{2\nu}_{GT-5}}{M^{2\nu}_{GT-1}}$$

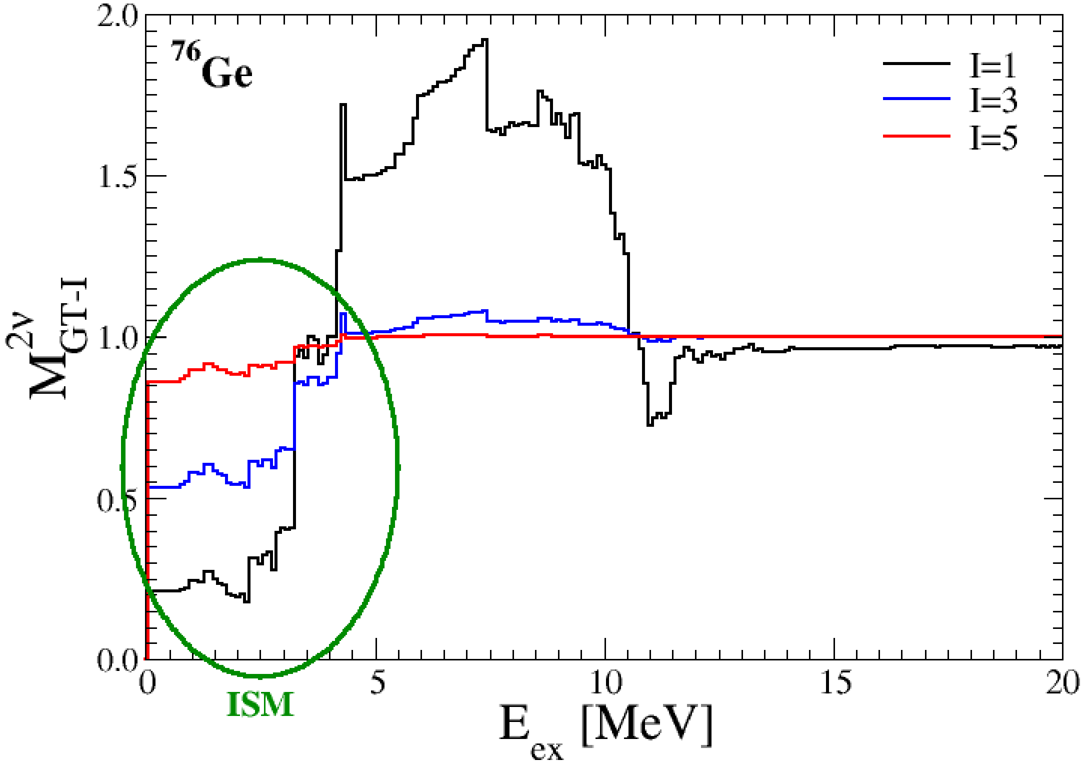
$$M^{2\nu}_{GT-1} = \sum_n M_n(0^+) \frac{m_e}{E_n(1^+) - (E_i + E_f)/2}$$
$$M^{2\nu}_{GT-3} = \sum_n M_n(0^+) \frac{4\ m_e^3}{(E_n(1^+) - (E_i + E_f)/2)^3}$$
$$M^{2\nu}_{GT-5} = \sum_n M_n(0^+) \frac{16\ m_e^5}{(E_n(1^+) - (E_i + E_f)/2)^5}$$

The g_A^{eff} can be deterimed with measured half-life and ratio of NMEs and calculated NME dominated by transitions through low lying states of the intermediate nucleus (ISM)

The running sum of the 2νββ–decay NMEs (QRPA)

$$M^{2\nu}_{GT-1} = \sum_n M_n \frac{1}{(E_n - (E_i + E_f)/2} \quad \xi^{2\nu}_{13} = \frac{M^{2\nu}_{GT-3}}{M^{2\nu}_{GT-1}}$$

$$M^{2\nu}_{GT-3} = \sum_n M_n \frac{4\ m_e^3}{(E_n - (E_i + E_f)/2)^3} \quad \xi^{2\nu}_{15} = \frac{M^{2\nu}_{GT-5}}{M^{2\nu}_{GT-1}}$$

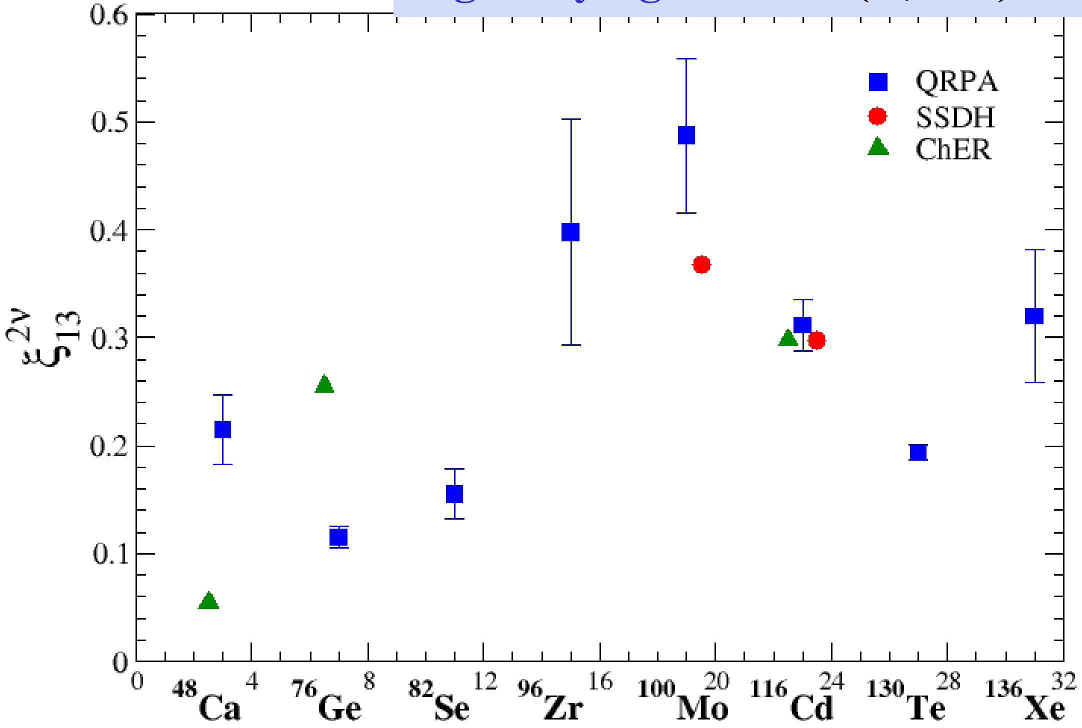


ξ_{13} tell us about importance of higher lying states of int. nucl.

HSD: $\xi_{13} \approx 0$

$\xi_{13} \approx 1$ (large)

Possible because of a large cancellation of contributions through lower and higher-lying states of (A,Z+1) nucleus



ξ_{13} can be determined phenomenologically from the shape of energy distributions of emitted electrons

F.Š., Šmotlák, Semenov, J. Phys. G, 27, 2233, 2001

KamLAND-Zen Exp. : $\xi_{13}<0.26$ (^{136} Xe)

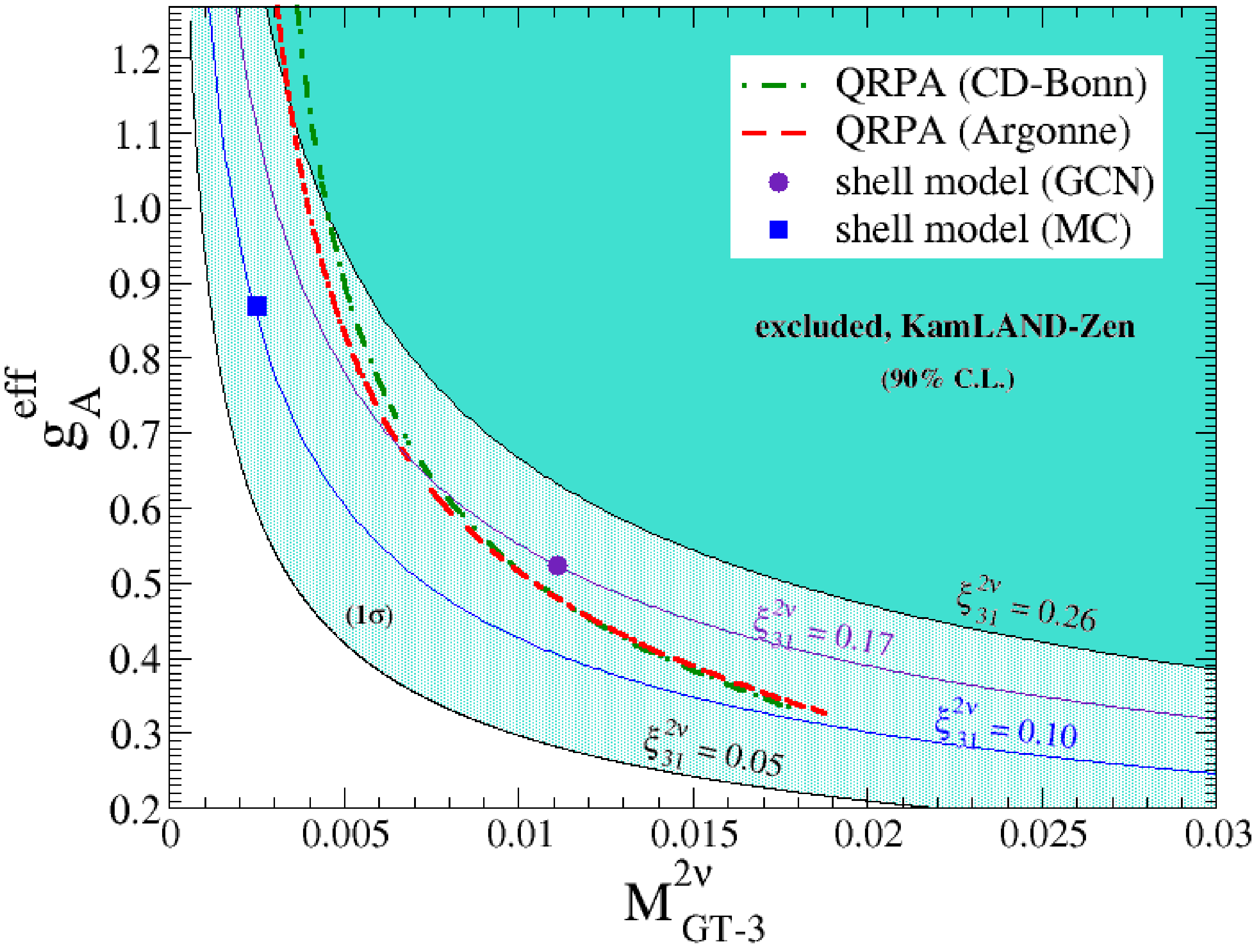
KamLAND-Zen Coll. (+J. Menendez, F.Š.),
Phys.Rev.Lett. 122, 192501 (2019)

ξ_{13} can be determined phenomenologically from the shape of energy distributions of emitted electrons

The g_A^{eff} can be deterimed with measured **half-life, ratio of NMEs ξ_{31}** and **calculated NME,** dominated by transitions through low lying states of the intermediate nucleus.

$$\left(g_A^{\rm eff}\right)^2 = \frac{1}{\left|M^{2\nu}_{GT-3}\right|}\frac{\left|\xi^{2\nu}_{13}\right|}{\sqrt{T^{2\nu-exp}_{1/2}\left(G^{2\nu}_0+\xi^{2\nu}_{13}G^{2\nu}_2\right)}}$$

M_{GT-3} have to be calculated by nuclear theory - ISM

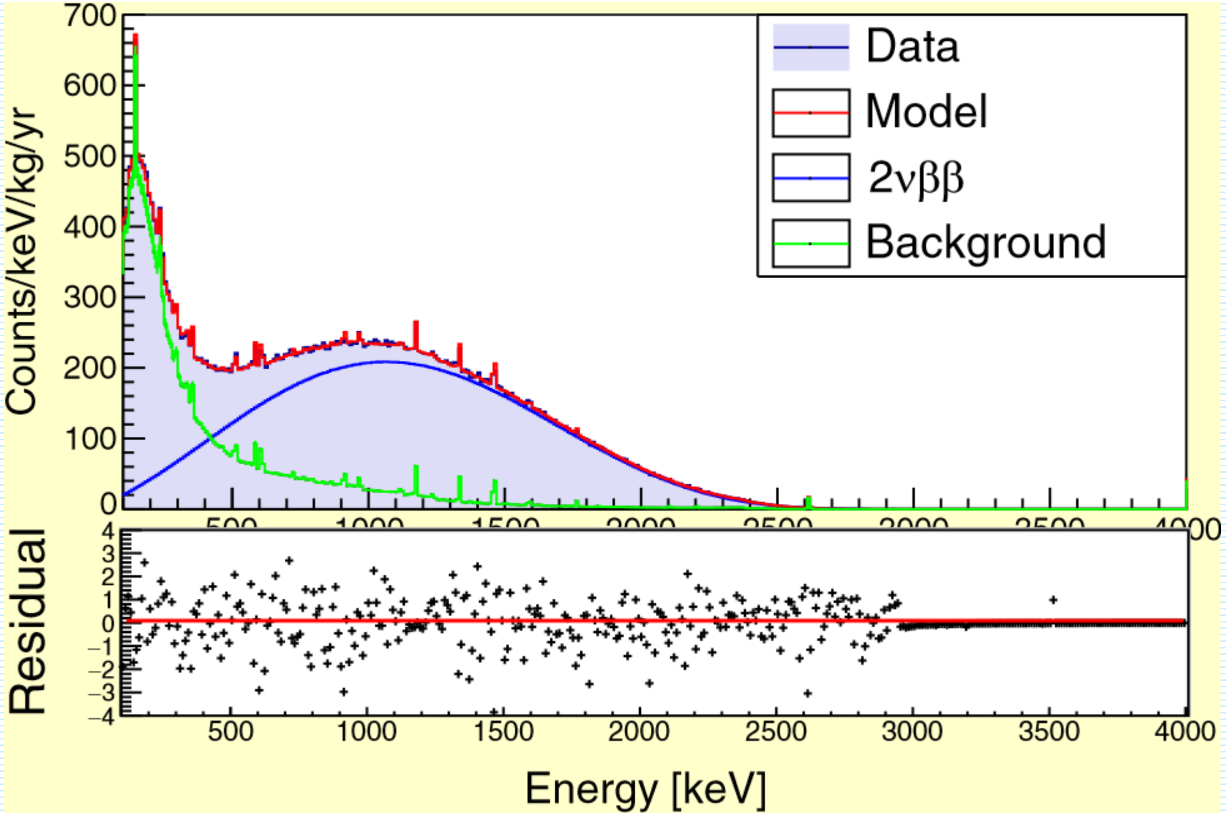


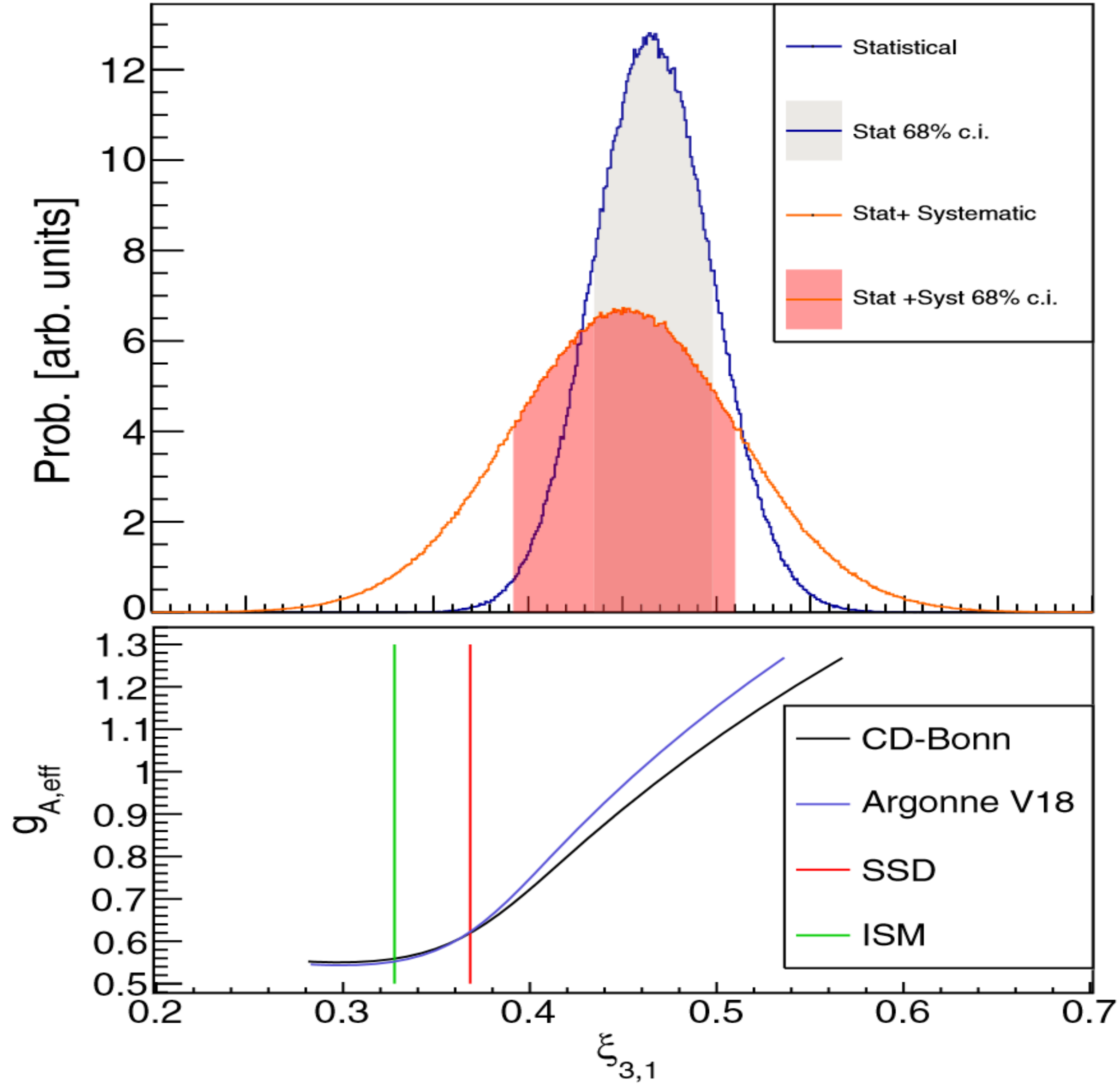
PandaX-4T data (37.8 kg y): $\xi_{13}=0.59^{+0.41}_{-0.38}$ (^{136} Xe)

PandaX Coll.,
arXiv:2512.04849 [nucl-ex], (2025)

CUPID-Mo Exp. : ξ_{13}=0.45±0.03 (stat) ±0.05 (syst) (^{100} Mo)

Phys.Rev.Lett. 131, 162501 (2023)





$T_{1/2}$ = [7.07±0.02(stat)±0.11(syst)] x 10^{18} yr

ξ_{51}/ξ_{31} = 0.364-0.368 (QRPA), 0.367 (SSD), 0.349 (ISM)

g_A^{eff} (pnQRPA) = 1.0 ±0.1(stat)±0.2(syst)

g_A^{eff} (ISM) = 1.11 ±0.03(stat)±0.05(syst)

Determining g_A from the 2νββ differential characteristics
CUPID-Mo, NEMO3

Measurement of the 2νββ decay rate and shape of ^{100}Mo from the CUPID-Mo experiment

CUPID-Mo Coll., C. Augier, et al., 2307.14086 [nucl-ex]

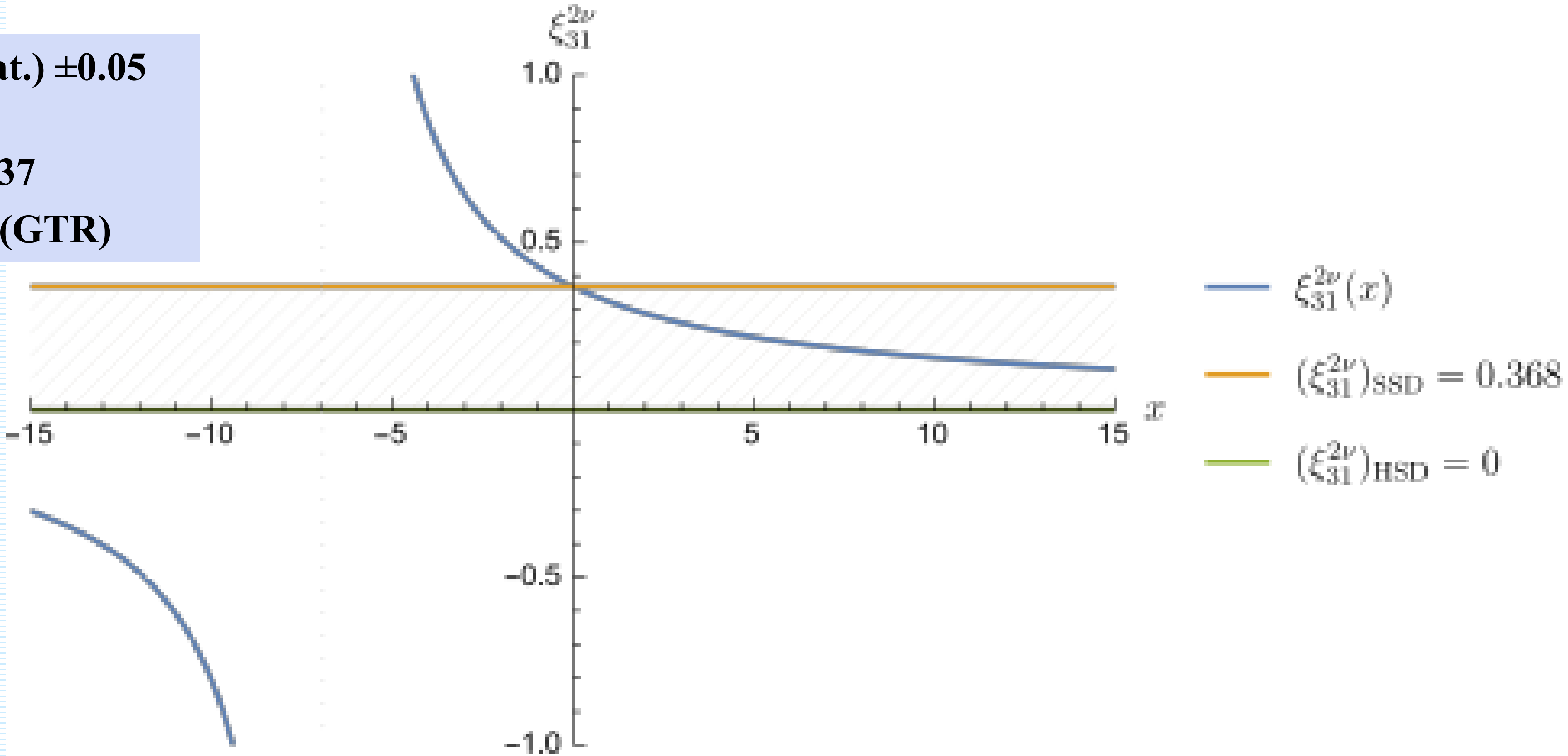
Toy model

$$M_{GT} = M_{GT}(SSD) + x\, M_{GT}(HSD)$$

ξ_{31} = 0.45 ±0.03 (stat.) ±0.05 (syst.)

ξ_{31}(SSD) = 0.37

ξ_{31}(HSD) = 0.008 (GTR)



CUORE Exp. (^{130}Te)

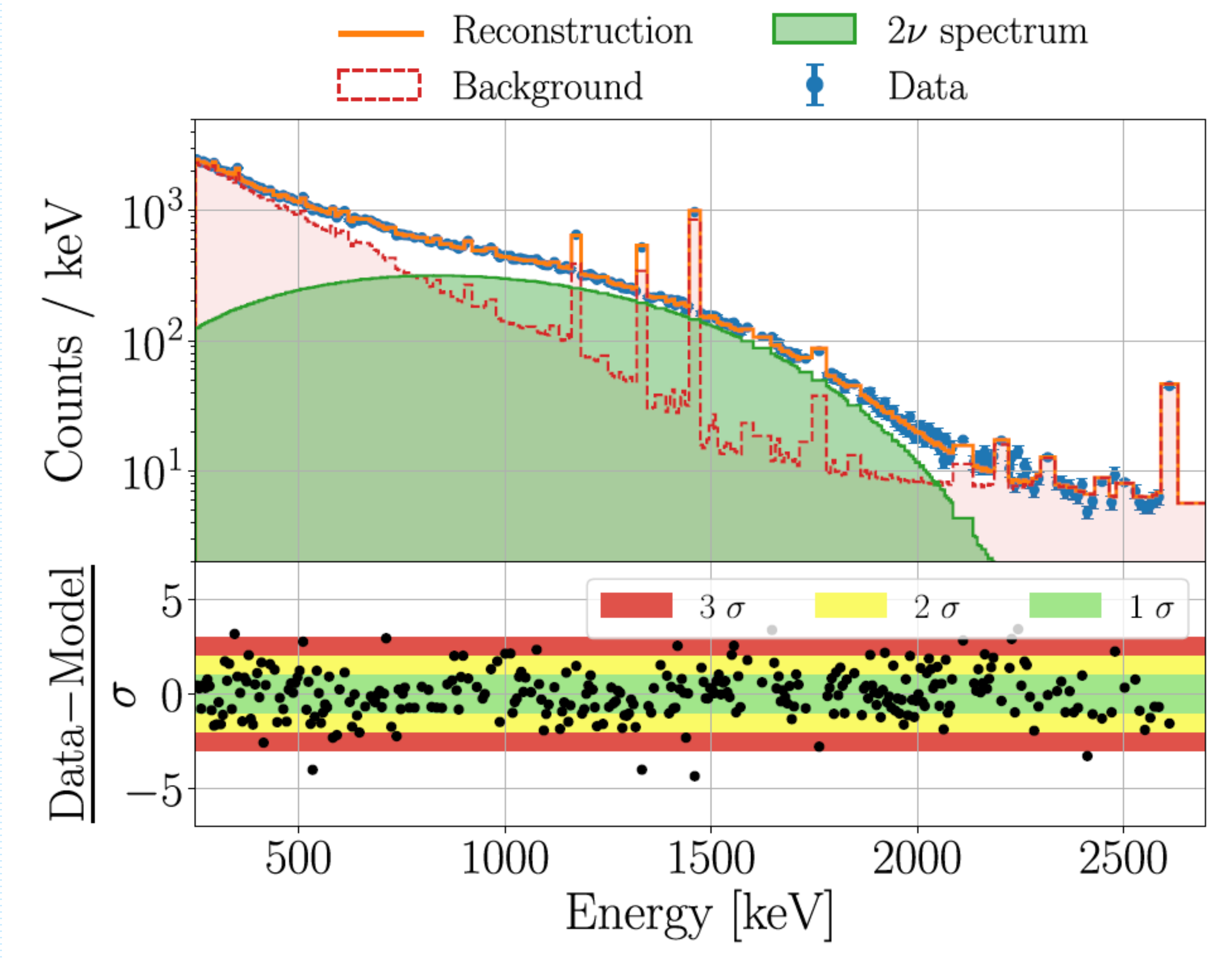
arXive: 2503.24137 [nucl-ex]

$$\xi_{31} = 0.01^{+0.31}_{-0.01}$$

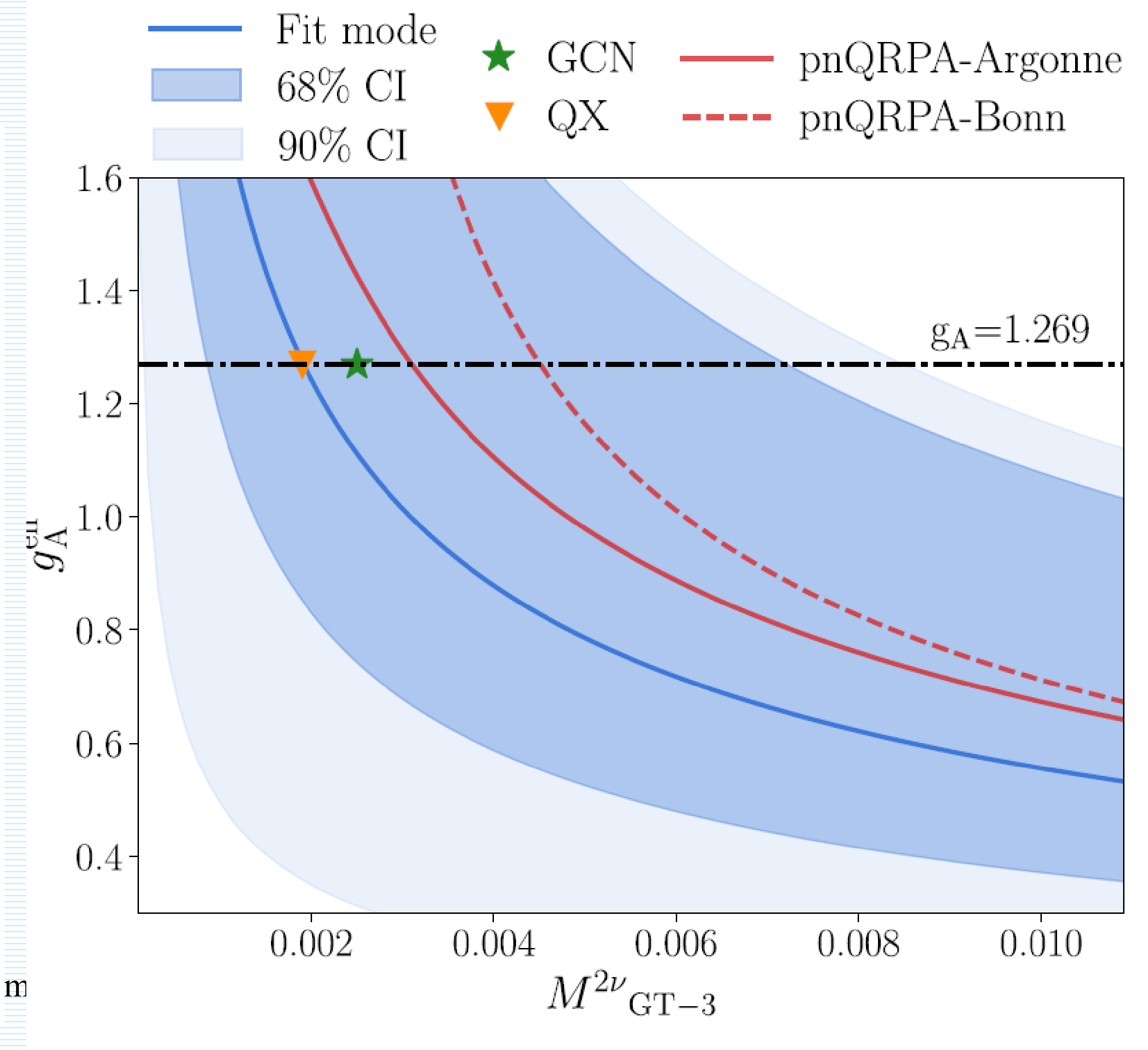
$$\xi_{51} = 1.46^{+0.33}_{-0.62}$$

It does not make sense(!?)
We are confused …

$$T^{2\nu}_{1/2} = \left(9.32^{+0.05}_{-0.04}\text{stat.}{}^{+0.07}_{-0.07}\text{syst.}\right) \times 10^{20}\ \text{yr}$$



m



The role of the width of the states of the intermediate nucleus
$(E_n \rightarrow E_n + \Gamma_n/2)$

$$\left[T_{1/2}^{2\nu\beta\beta}\right]^{-1} = \left(g_A^{\rm eff}\right)^4 \left|M_{GT}^{R1}\right|^2 \Big[\left(1+\xi_{IR11}{}^2\right) G_0^{2\nu} + \left(\xi_{RR31} + \xi_{IR11}\xi_{IR31}\right) G_2^{2\nu} + \frac{1}{3}\left(\xi_{IR31}{}^2 + \xi_{RR31}{}^2\right)\left(G_{22}^{2\nu} + G_4^{2\nu}\right)\Big]$$

$$\xi_{RR31} = \frac{M_{GT}^{R3}}{M_{GT}^{R1}} \qquad \xi_{IR11} = \frac{M_{GT}^{I1}}{M_{GT}^{R1}} \qquad \xi_{IR31} = \frac{M_{GT}^{I3}}{M_{GT}^{R1}}$$

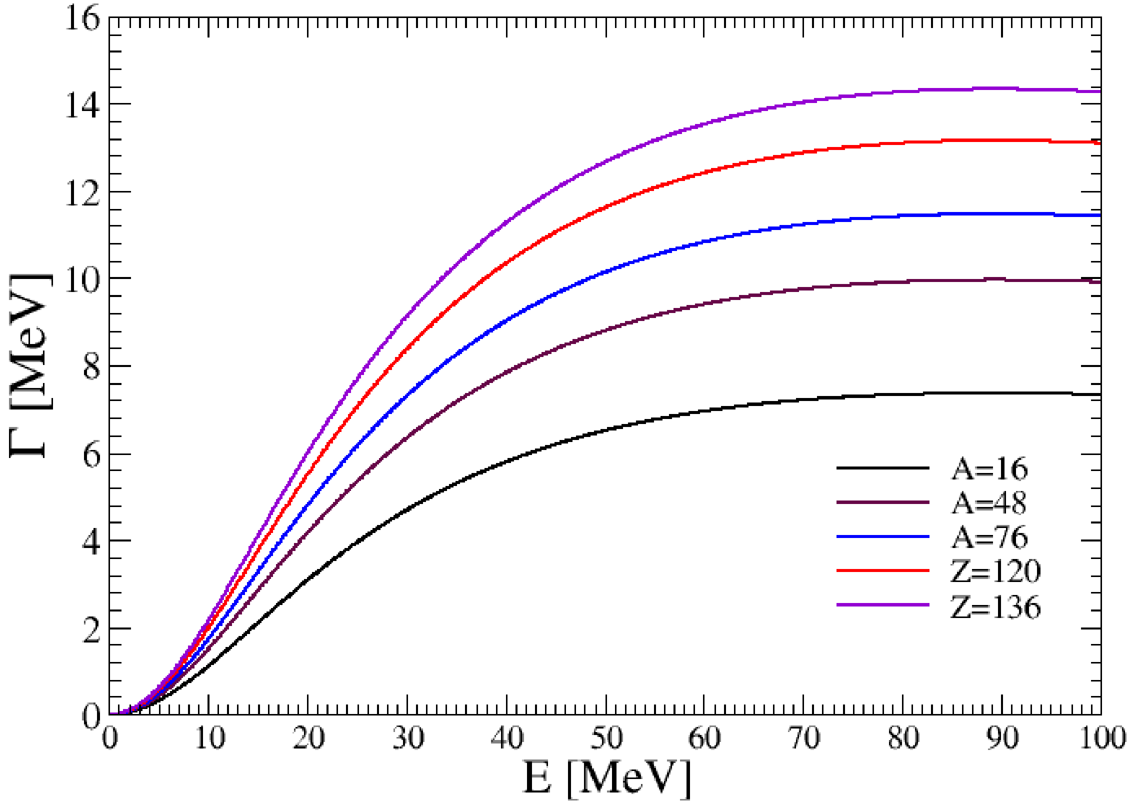
$$M_{GT}^{R1} = m_e \sum_n \frac{M_n D_n}{\left(D_n^2 + \Gamma_n^2/4\right)}$$

$$M_{GT}^{R3} = 4 m_e^3 \sum_n \frac{M_n D_n \left(D_n^2 - 3\Gamma_n^2/4\right)}{\left(D_n^2 + \Gamma_n^2/4\right)^3}$$

$$M_{GT}^{I1} = m_e \sum_n \frac{M_n \Gamma_n/2}{\left(D_n^2 + \Gamma_n^2/4\right)}$$

$$M_{GT}^{I3} = 4 m_e^3 \sum_n \frac{M_n \Gamma_n/2 \left(3D_n^2 - \Gamma_n^2/4\right)}{\left(D_n^2 + \Gamma_n^2/4\right)^3}$$

$$D_n = E_n - (E_i + E_f)/2$$

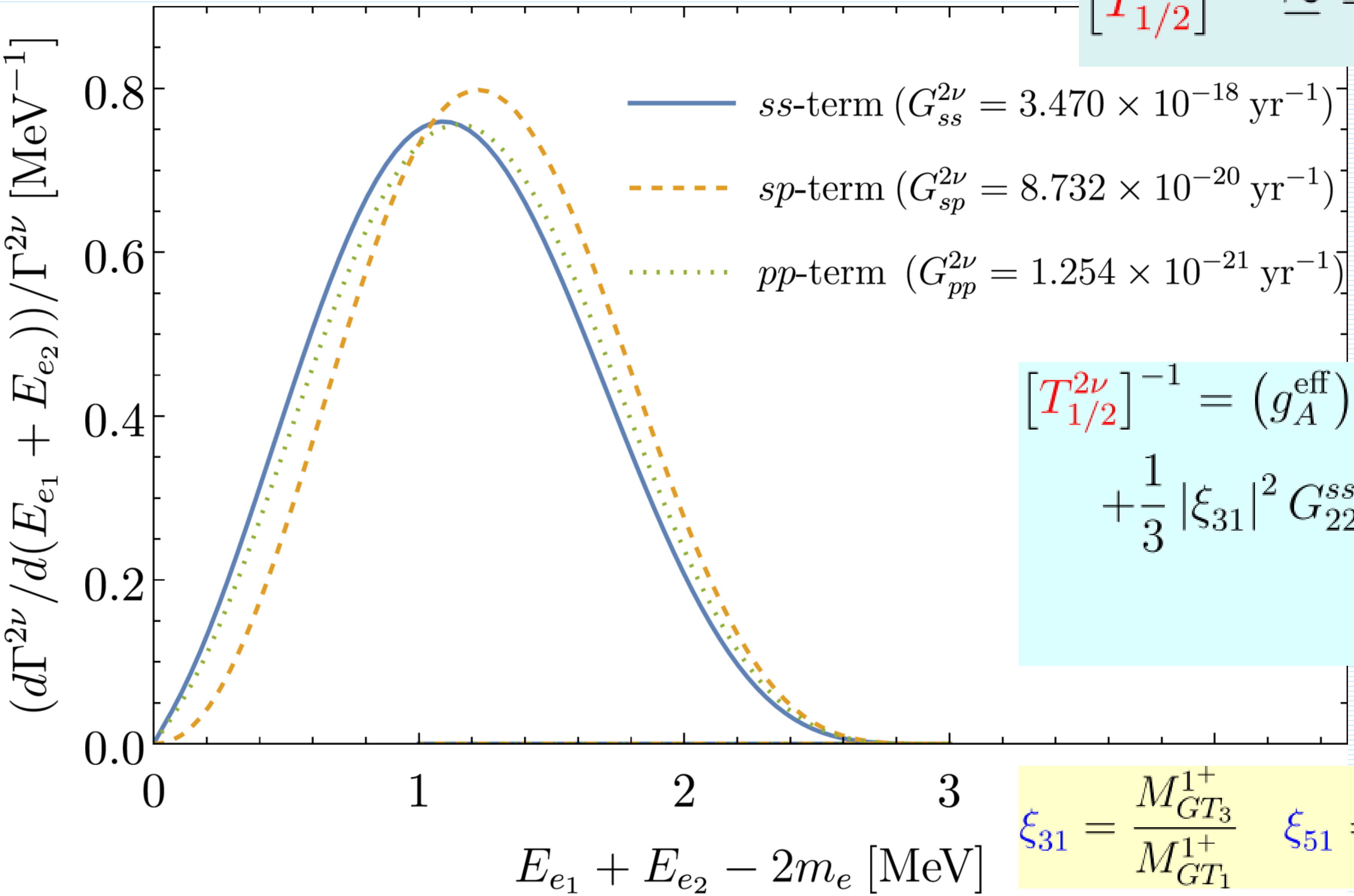


We found that ξ_{IR11} and ξ_{IR31} are negligibly small

Effect of the electron $p_{1/2}$-wave state in $2\nu\beta\beta$-decay (preliminary)

$$A^{2\nu} = A^{ss} + A^{p_{1/2}p_{1/2}} + \cdots$$

$$\left[T^{2\nu}_{1/2}\right]^{-1} \simeq \frac{\Gamma_0^{ss} + \Gamma_2^{ss} + \Gamma_4^{ss} + \Gamma_0^{sp_{1/2}}}{\ln(2)}$$



$$\left[T^{2\nu}_{1/2}\right]^{-1} = \left(g_A^{\rm eff}\right)^4 \left|M^{ss}_{GT_1}\right|^2 \left(G_0^{ss} + \Re\{\xi_{31}\}G_2^{ss} + \frac{1}{3}\left|\xi_{31}\right|^2 G_{22}^{ss} + \left(\frac{1}{3}\left|\xi_{31}\right|^2 + \Re\{\xi_{51}\}\right) G_4^{ss} + G_0^{sp_{1/2}}\ \xi_{p_{1/2}s}\right)$$

$$\xi_{31} = \frac{M^{1^+}_{GT_3}}{M^{1^+}_{GT_1}} \qquad \xi_{51} = \frac{M^{1^+}_{GT_5}}{M^{1^+}_{GT_1}} \qquad \xi_{p_{1/2}s} = \frac{M^{0^-} + M^{1^-}}{M^{1^+}_{GT_1}}$$

2νββ probes New Beyond SM Physics

All 100 kg- and ton-class 0νββ experiments can also study a diverse range of exotic phenomena, e.g. through spectral distortion in $2\nu\beta\beta$.
Future searches will probe the $2\nu\beta\beta$ with high statistics about 10^5-10^6 events.

Common subjects:

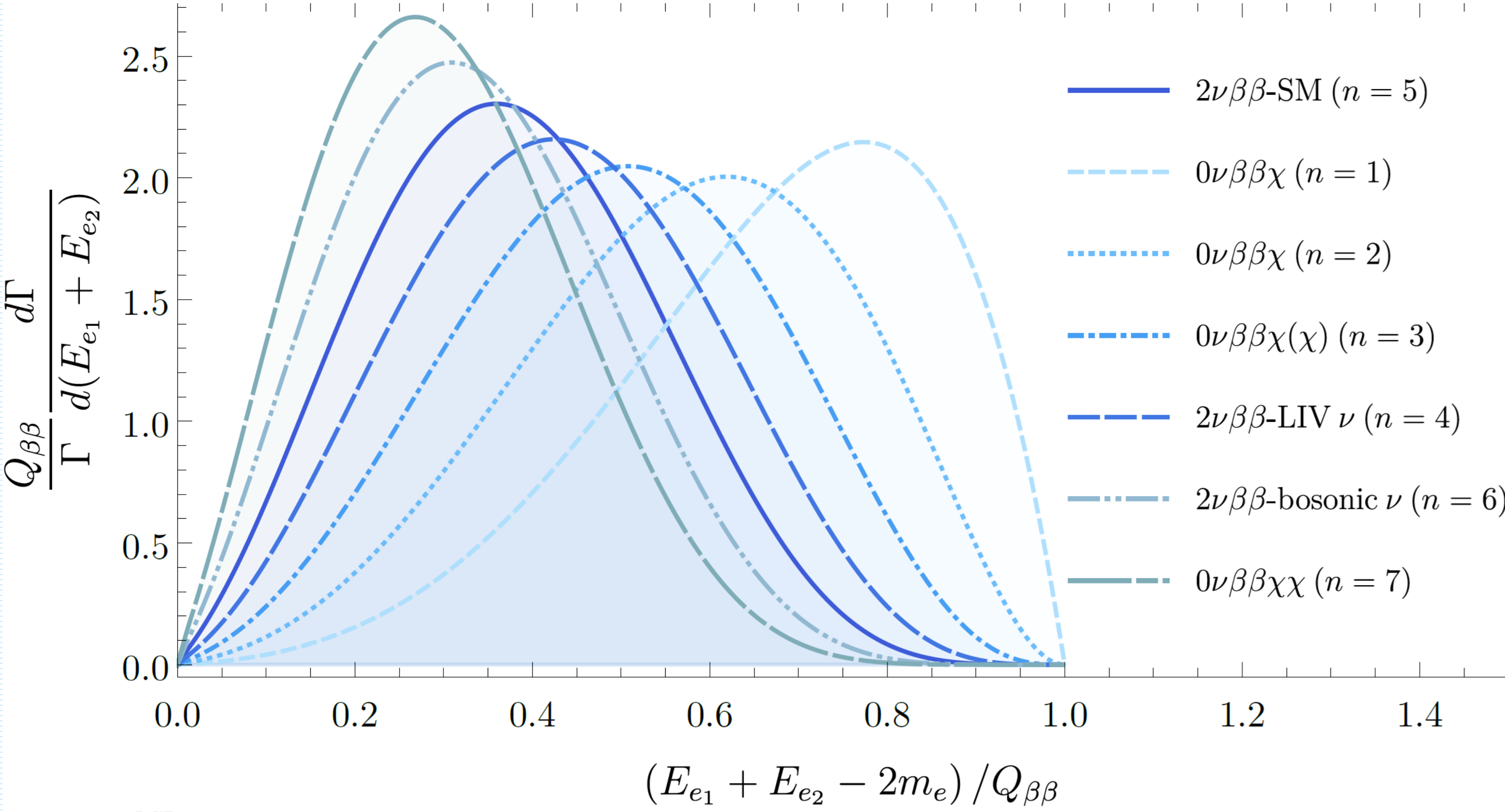
Majoron(s) emission
(partly)bosonic neutrinos,
Lorentz invariance violation

Recent subjects:

Lepton-number conserving right-handed currents
(PRL 125 (2020) 17, 171801)

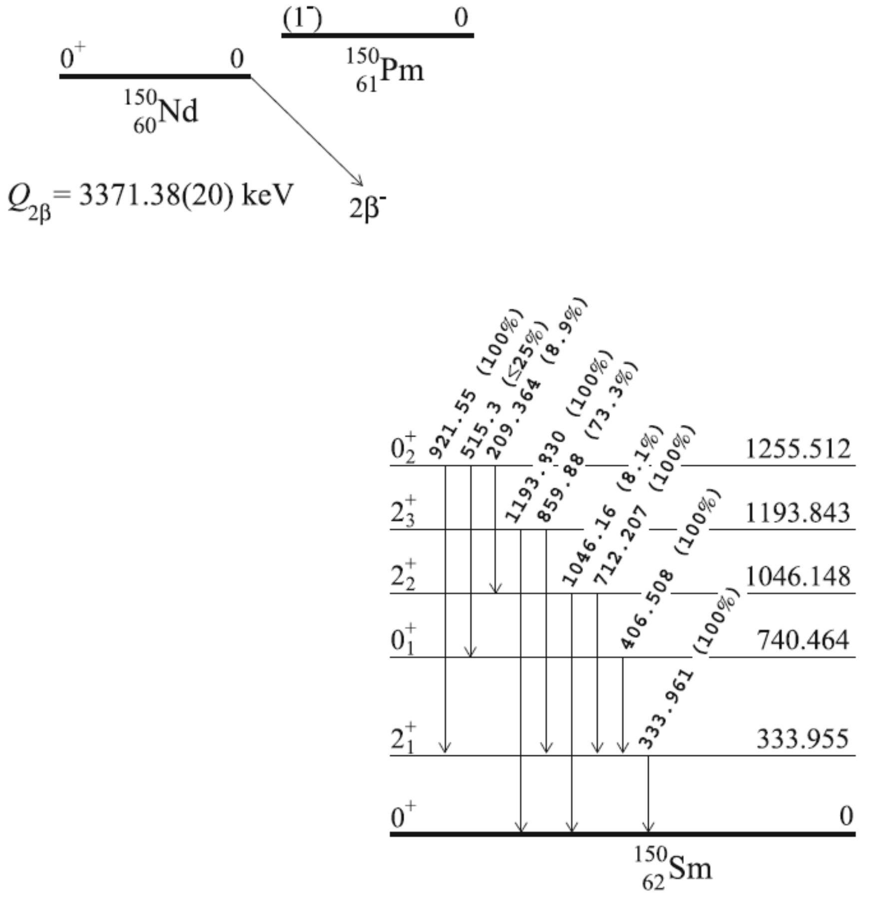
Neutrino self-interactions
(PRD 102 (2020) 5, 051701)

Sterile neutrino and light fermion searches through energy end point
(PRD 103 (2021) 5, 055019;
PLB 815 (2021) 136127)



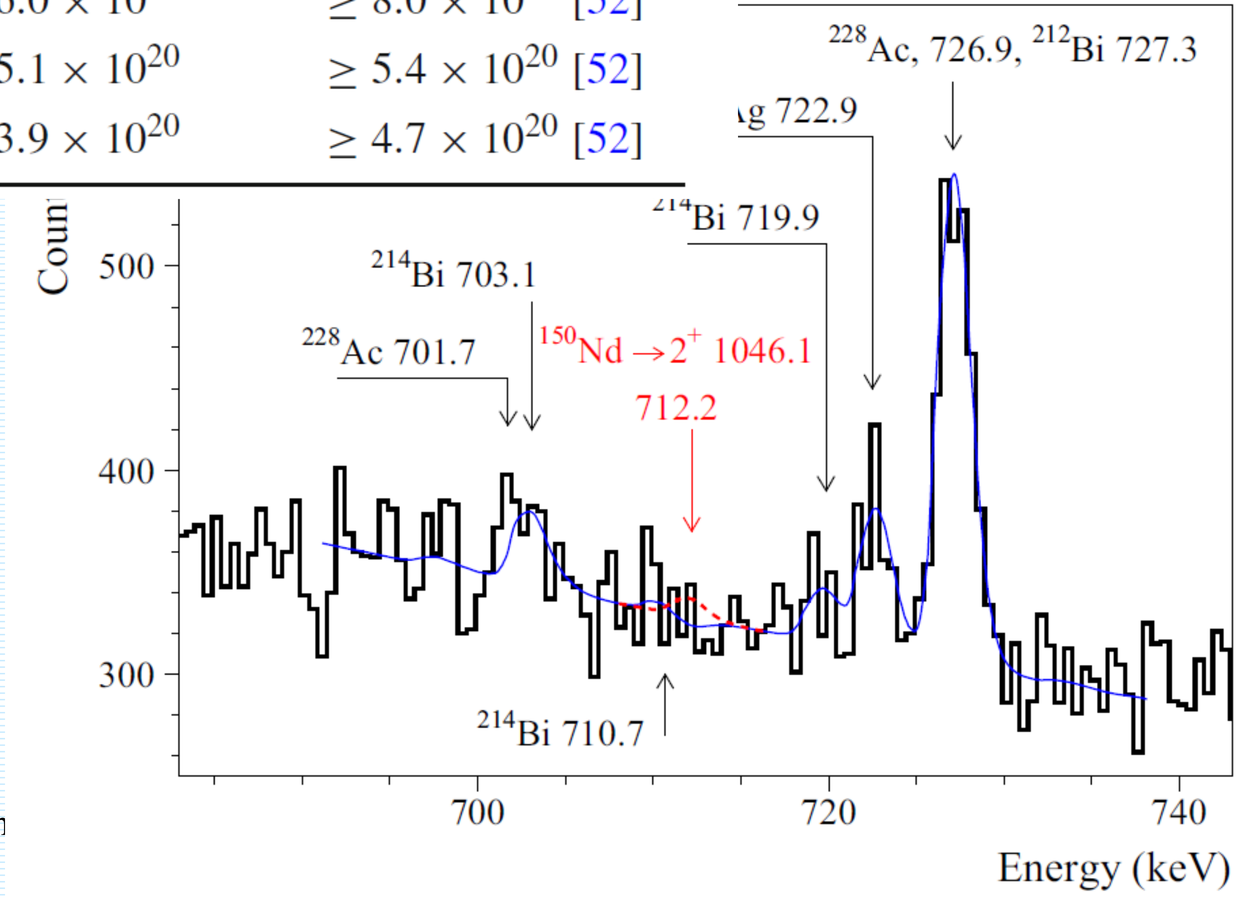
$$\frac{d\Gamma}{d\varepsilon_1 d\varepsilon_2} = C(Q-\varepsilon_1-\varepsilon_2)^n \left[p_1\varepsilon_1 F(\varepsilon_1)\right]\left[p_2\varepsilon_2 F(\varepsilon_2)\right]$$

Spectral index n



Level of ^{150}Sm (keV)	Experimental limit, lim $T_{1/2}$ (year)	
	Present work	Previous results
334.0 2_1^+	$1.5^{+2.3}_{-0.7} \times 10^{20}$	$\geq 2.42 \times 10^{20}$ [28]
334.0 2_1^+	$\geq 7.3 \times 10^{19}$	$\geq 1.26 \times 10^{23}$ [28]^a
740.5 0_1^+	$1.03^{+0.38}_{-0.29} \times 10^{20}$	See Table 1
740.5 0_1^+	$\geq 7.1 \times 10^{19}$	$\geq 1.36 \times 10^{22}$ [28]^a
1046.1 2_2^+	$\geq 6.0 \times 10^{20}$	$\geq 8.0 \times 10^{20}$ [52]
1193.8 2_3^+	$\geq 5.1 \times 10^{20}$	$\geq 5.4 \times 10^{20}$ [52]
1255.5 0_2^+	$\geq 3.9 \times 10^{20}$	$\geq 4.7 \times 10^{20}$ [52]

$2\nu\beta\beta$-decay
to 2^+ state
(First observation)



$$T_{1/2}^{2\nu2\beta}(^{150}\mathrm{Nd} \rightarrow \ ^{150}\mathrm{Sm}(0_1^+))$$
$$= [0.83^{+0.18}_{-0.13}(\mathrm{stat})^{+0.16}_{-0.19}(\mathrm{syst})] \times 10^{20}\mathrm{year}.$$

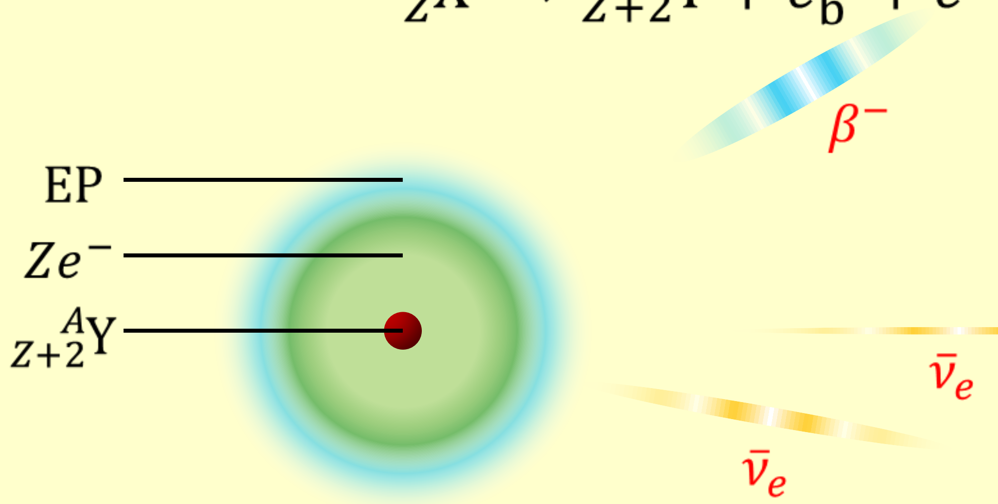
Eur. Phys. J. C 85, 174 (2025)

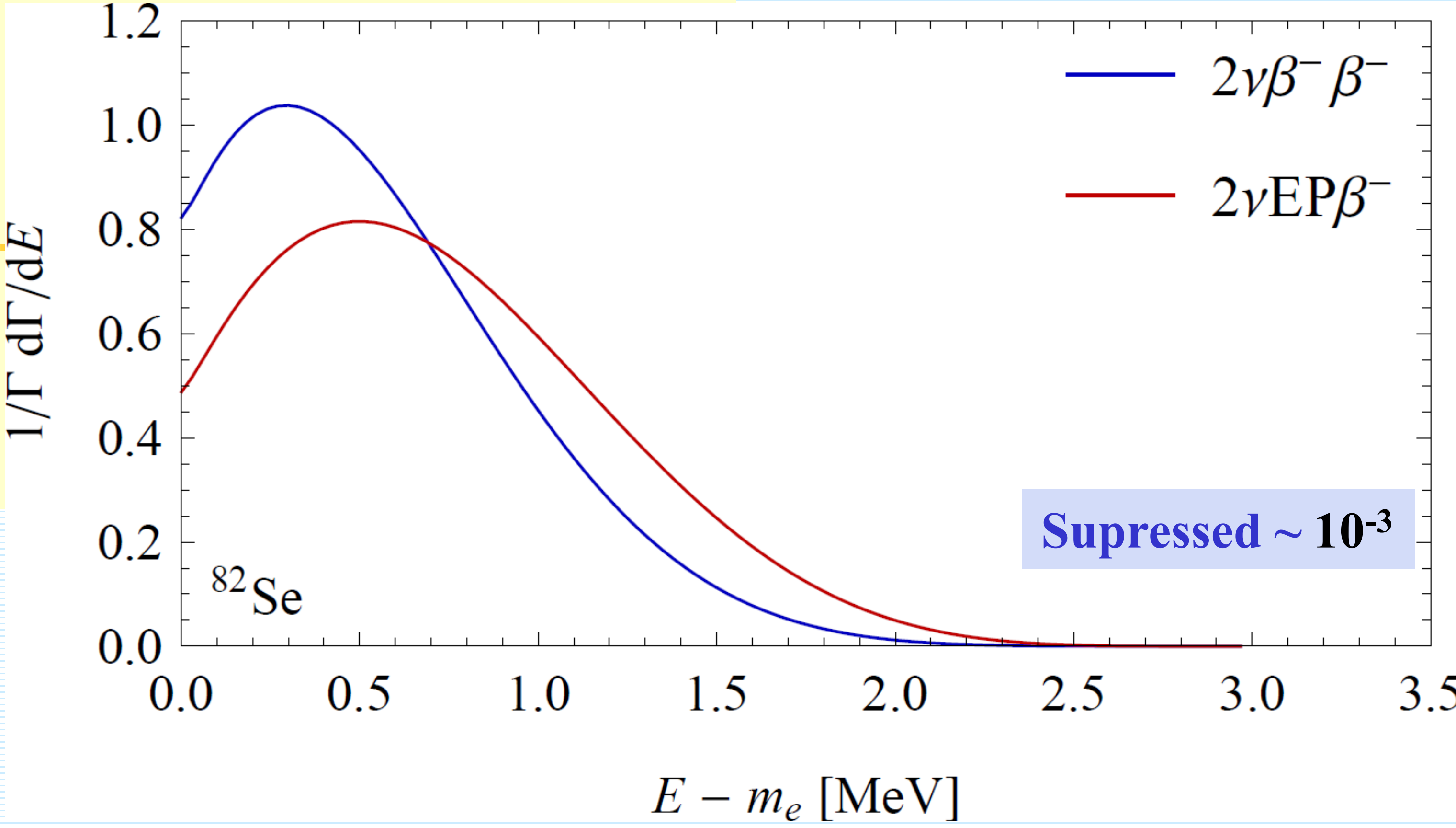
[Jung *et al.* (GSI), 1992] observed beta decay of $^{163}_{66}\mathrm{Dy}^{66+}$ ions with Electron Production (EP) in K or L shells: $T_{1/2}^{\mathrm{EP}} = 47$ d

Double Beta Decay with emission of a single electron

Bound-state double-beta decay $0\nu\mathrm{EP}\beta^-$ ($2\nu\mathrm{EP}\beta^-$) with EP in available $\mathrm{s}_{1/2}$ or $\mathrm{p}_{1/2}$ subshell of daughter 2+ ion:

$$^{A}_{Z}\mathrm{X} \longrightarrow {}_{Z+2}^{\;\;A}\mathrm{Y} + e_{\mathrm{b}}^{-} + e^{-} + (\bar{\nu}_e + \bar{\nu}_e)$$



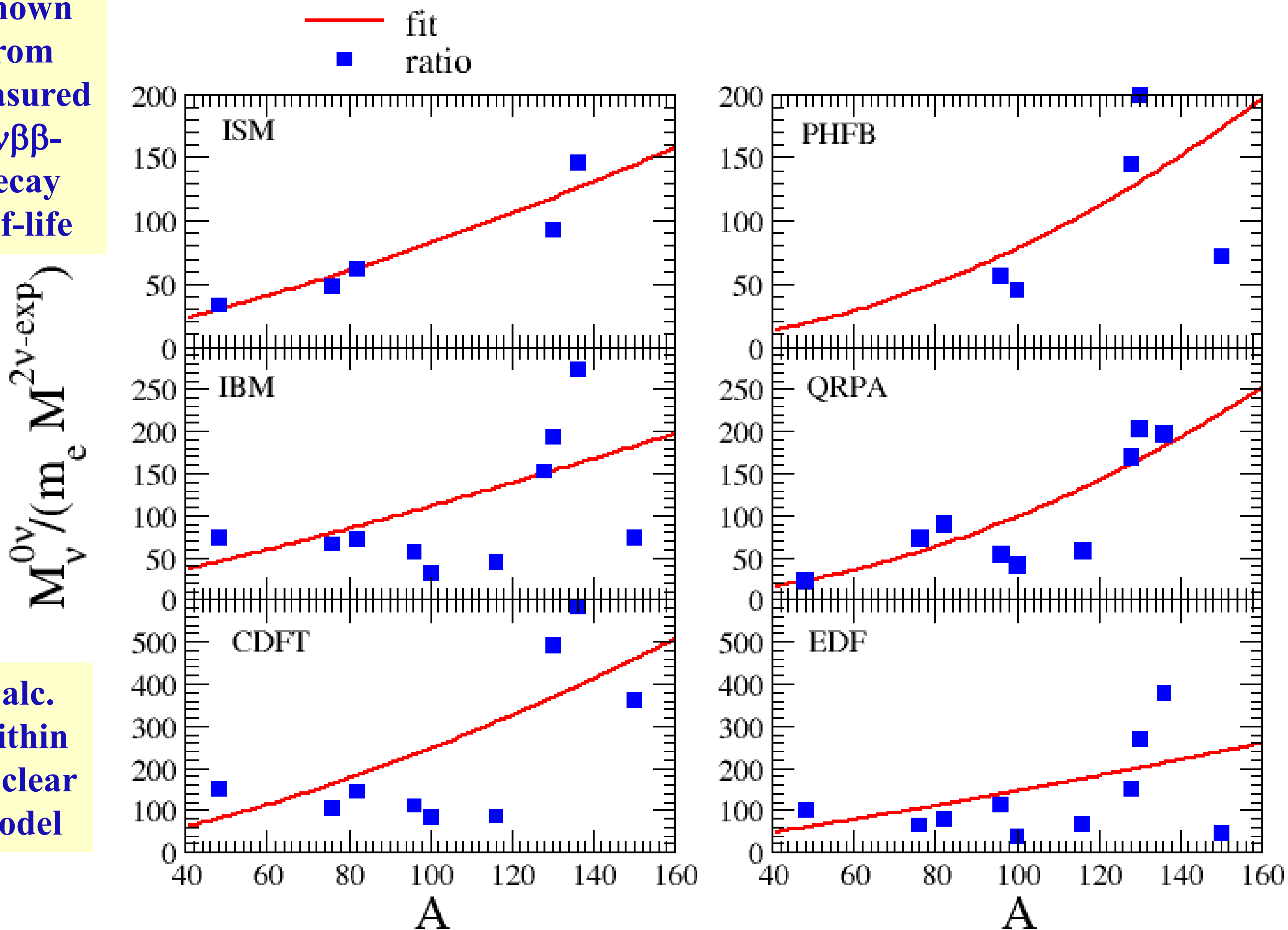


Supressed ~ 10^{-3}

2νEPβ can be detected
Half-life predictions are independent on value of gA and Value of 2νββ NME

Is there a proportionality between 0νββ- and 2νββ-decay NMEs?

Known krom measured 2νββ-decay half-life

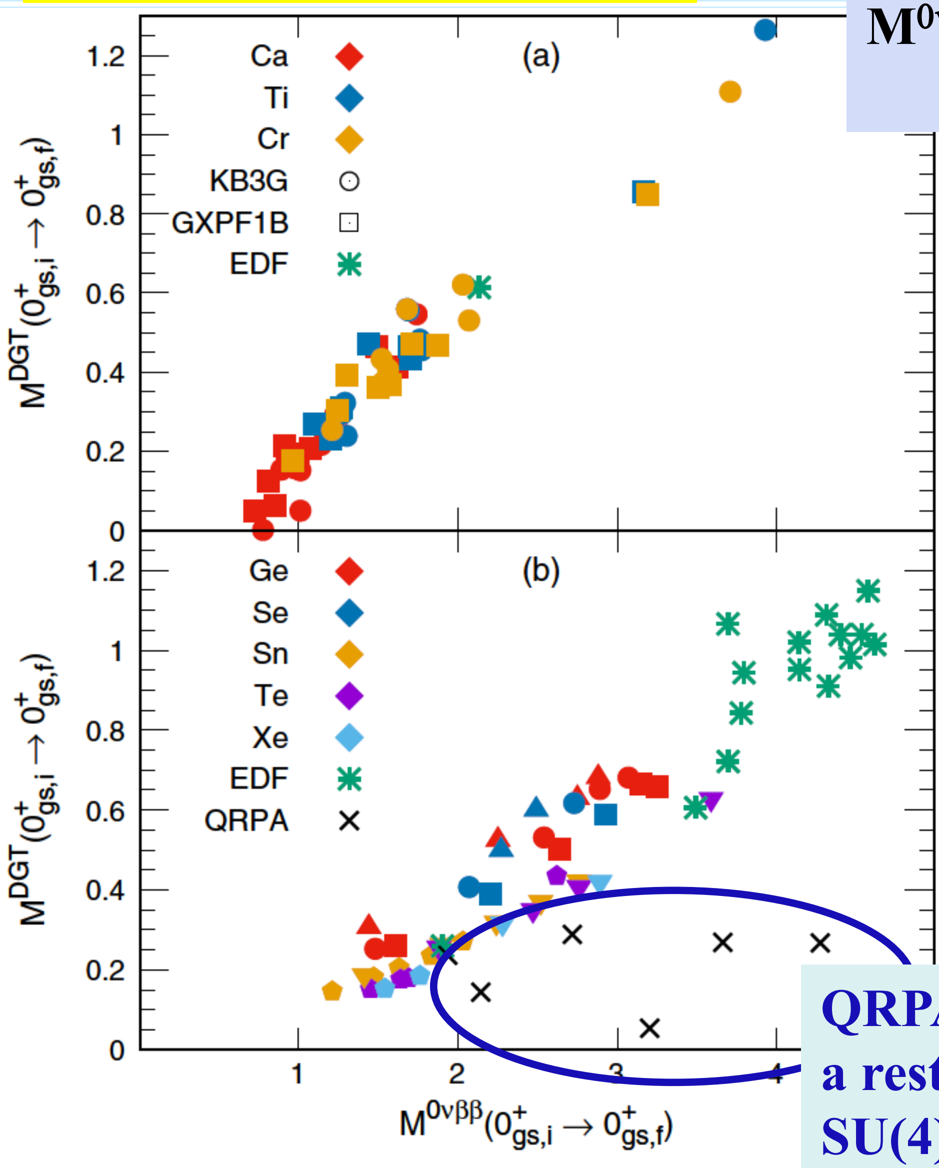


Calc. within nuclear model

ISM, EDF : $M^{0\nu} \propto M^{2\nu}_{GT\text{-}cl}$

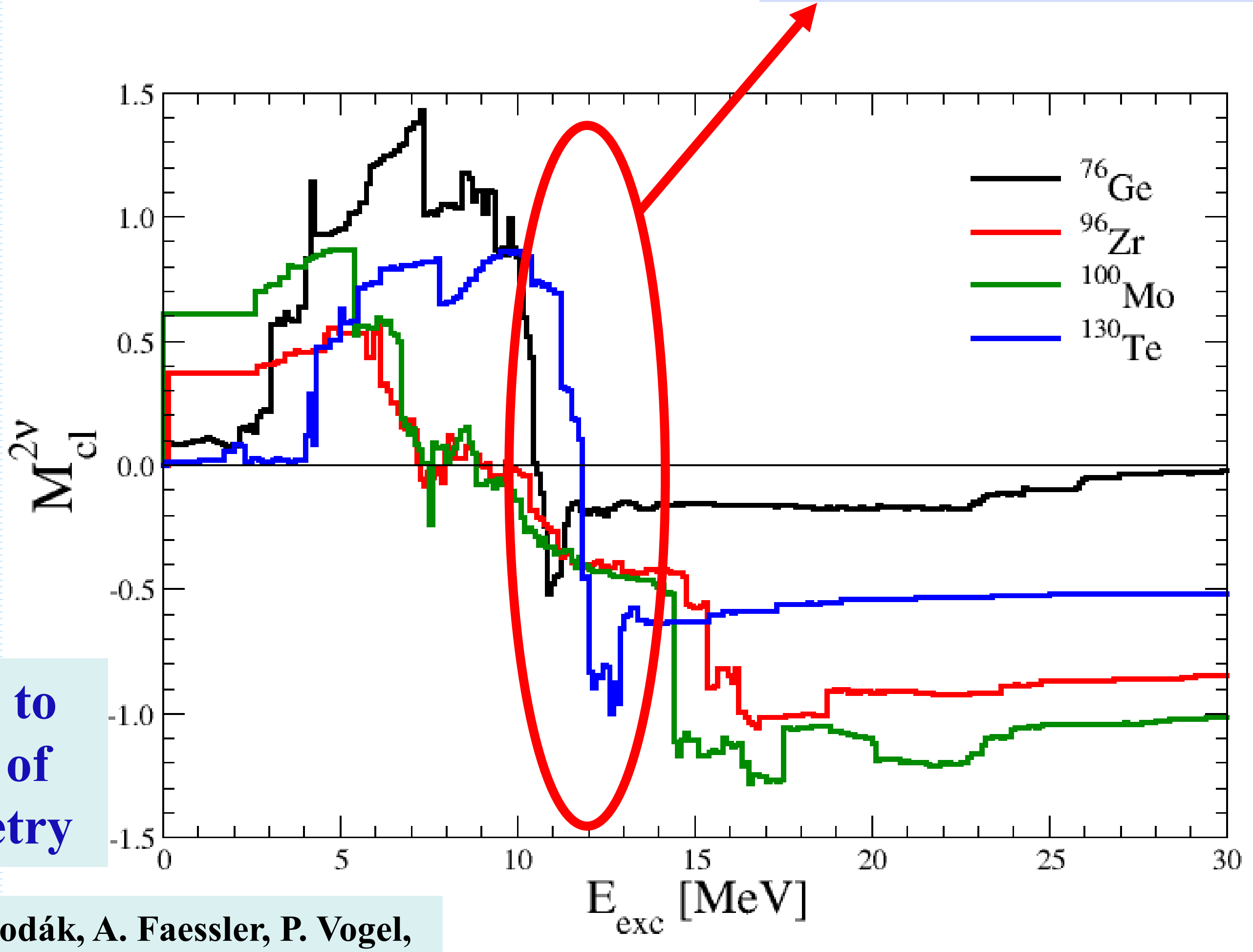
$M^{DGT} \equiv M_{GT\text{-}cl}$ – only 1^+
$M^{0\nu}$ - contribution
from many J^π (!)

QRPA: There is no proportionality between
0νββ- and 2νββ-decay NMEs
(change of sign at GTR level)



QRPA - close to a restoration of SU(4) symmetry

Region of GT resonance



ISM: N. Shimizu, J. Menendez, K. Yako, PRL 120, 142502 (2018)

QRPA: F.Š., R. Hodák, A. Faessler, P. Vogel, PRC 83, 015502 (2011)

A connection between closure $2\nu\beta\beta$ and $0\nu\beta\beta$ GT NMEs

F.Š., R. Hodák, A. Faessler, P. Vogel, PRC 83, 015502 (2011)
F. Š., A. Smetana, P. Vogel, PRC 98, 064325 (2018)

Going to relative coordinates:

$$M^{0\nu}_{\nu,N-I} = \int_0^\infty P^{\nu,N}_{I-src}(r)\; C^{2\nu}_{I-cl}(r)\; dr = \int_0^\infty f^2_{src}(r)\; P^{\nu,N}_I(r)\; \boxed{C^{2\nu}_{I-cl}(r)}\; dr$$

$$I = F,\; GT \text{ and } T$$

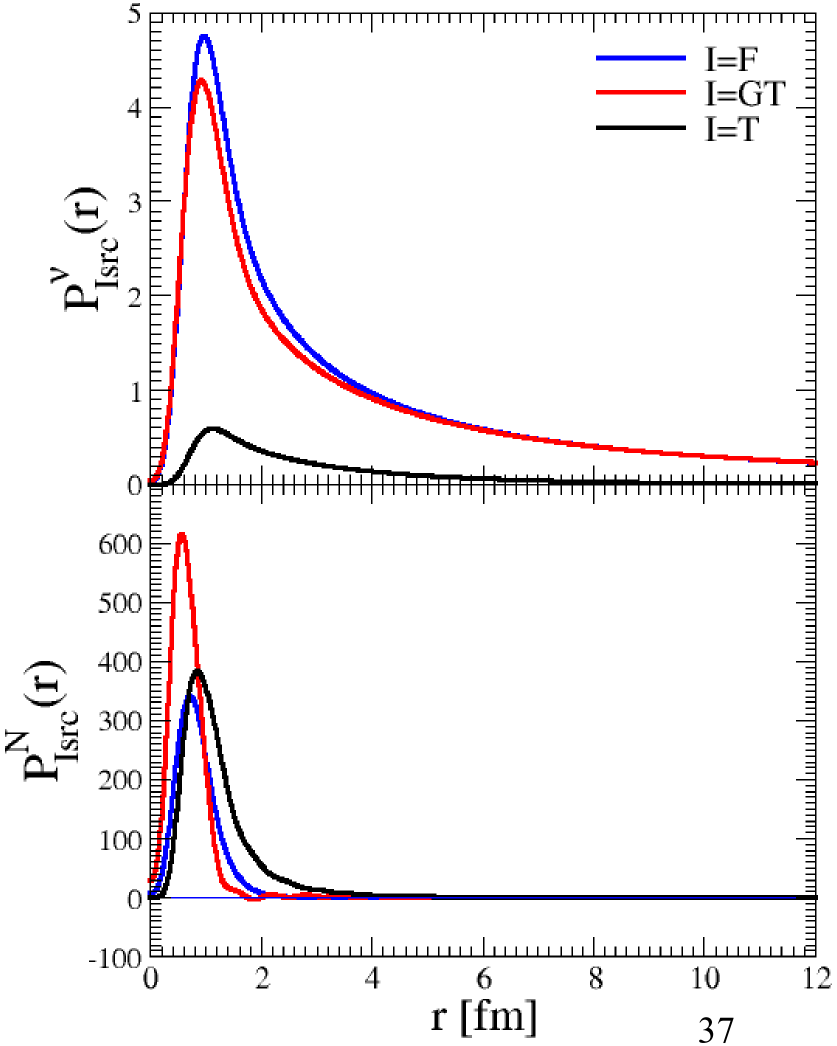
***r**- relative distance of two decaying nucleons*

$$M^{2\nu}_{GT-cl} = \int_0^\infty \boxed{C^{2\nu}_{GT-cl}(r)}\, dr$$

$$M^{2\nu}_{GT-cl} = \sum_{J^\pi,m} < 0^+_f|\tau^+\vec{\sigma}|J^\pi,m > \cdot < J^\pi,m|\tau^+\vec{\sigma}|0^+_i >$$

$$\sum_m < 0^+_f|\tau^+\vec{\sigma}|1^+,m > \cdot < 1^+,m|\tau^+\vec{\sigma}|0^+_i >$$ mkovic

ν– and N- exchange potential (x $f^2_{src}(r)$)



Neutrino potential prefers short distances

$$M^{0\nu}_{GT} = \int_0^\infty H^{0\nu}_{GT}(r)\, C^{2\nu}_{GT-cl}(r)\, dr$$

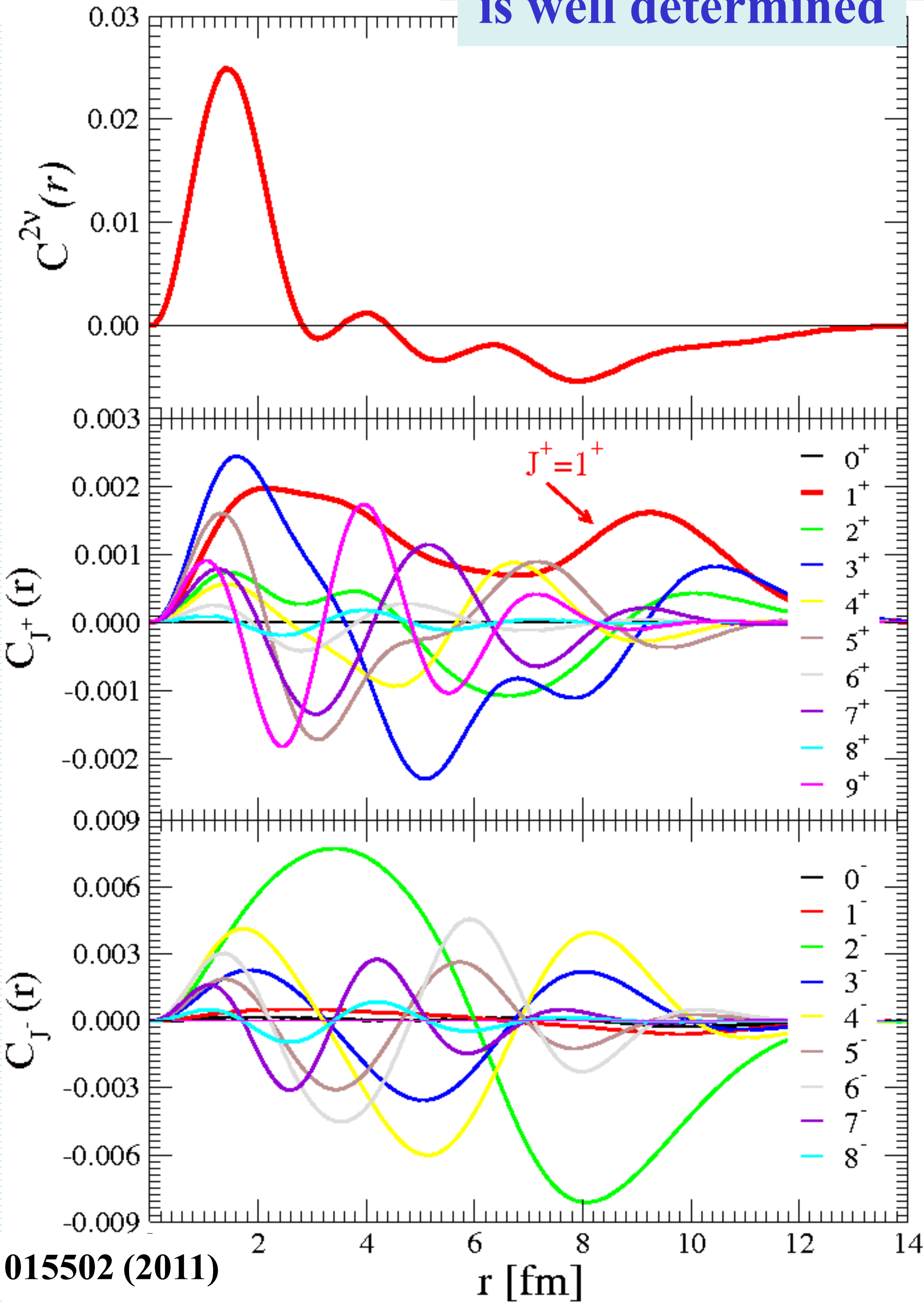
$$H(r) = R\frac{2}{\pi}\int_0^\infty j_0(qr)\frac{q}{q+\overline{E}} f^2_{FNS}(q^2)\, g_{HOT}(q^2)\, dr$$

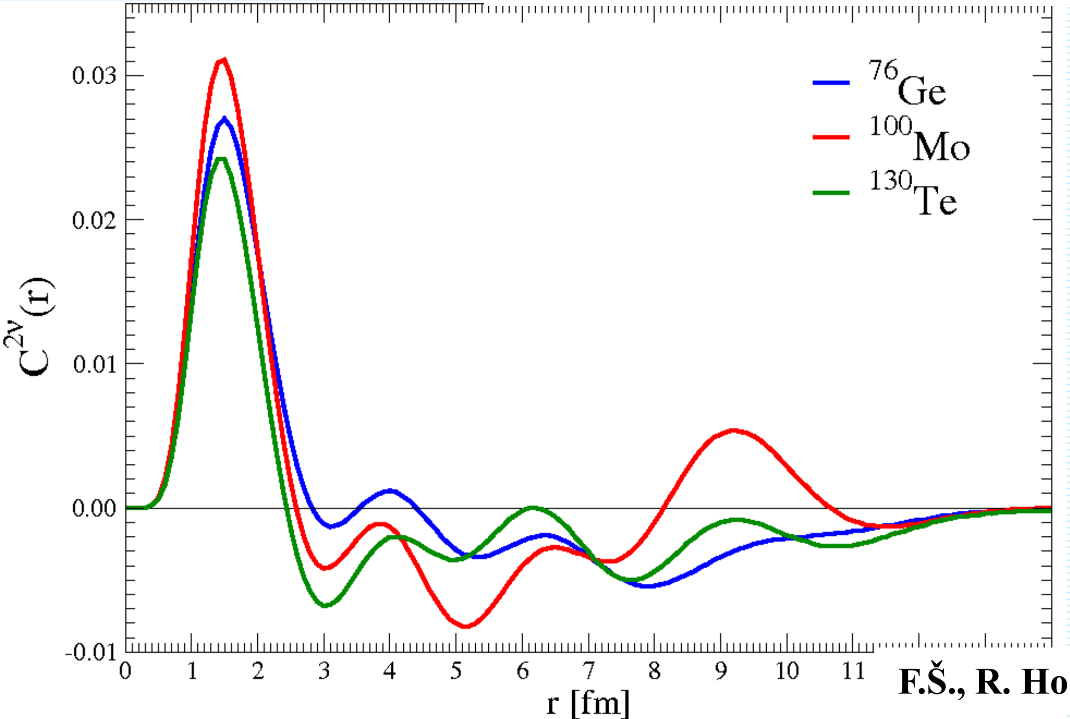
Potential is well determined

$$M^{2\nu}_{GT} = \int C^{2\nu}(r)\, dr$$

A connection between closure 2νββ and 0νββ GT NMEs

$$M^{2\nu}_{GT} = \sum_{J^\pi,k_i,k_f,\mathcal{J}} \sum_{pnp'n'} (-1)^{j_n+j_{p'}+J+\mathcal{J}} \times$$
$$\sqrt{2\mathcal{J}+1}\begin{Bmatrix} j_p & j_n & J \\ j_{n'} & j_{p'} & \mathcal{J}\end{Bmatrix} \times$$
$$\langle p(1), p'(2);\mathcal{J} \parallel \sigma(1)\cdot\sigma(2) \parallel n(1), n'(2);\mathcal{J}\rangle \times$$
$$\langle 0_f^+||\widetilde{[c^+_{p'}\tilde{c}_{n'}]_J}||J^\pi k_f\rangle\langle J^\pi k_f|J^\pi k_i\rangle\langle J^\pi k_f i||[c^+_p\tilde{c}_n]_J||0^+_i\rangle$$





for $J^\pi = 1^+$

$\int C_J(r)\, dr = M_{GT}^{2\nu}$

for $J^\pi \neq 1^+$

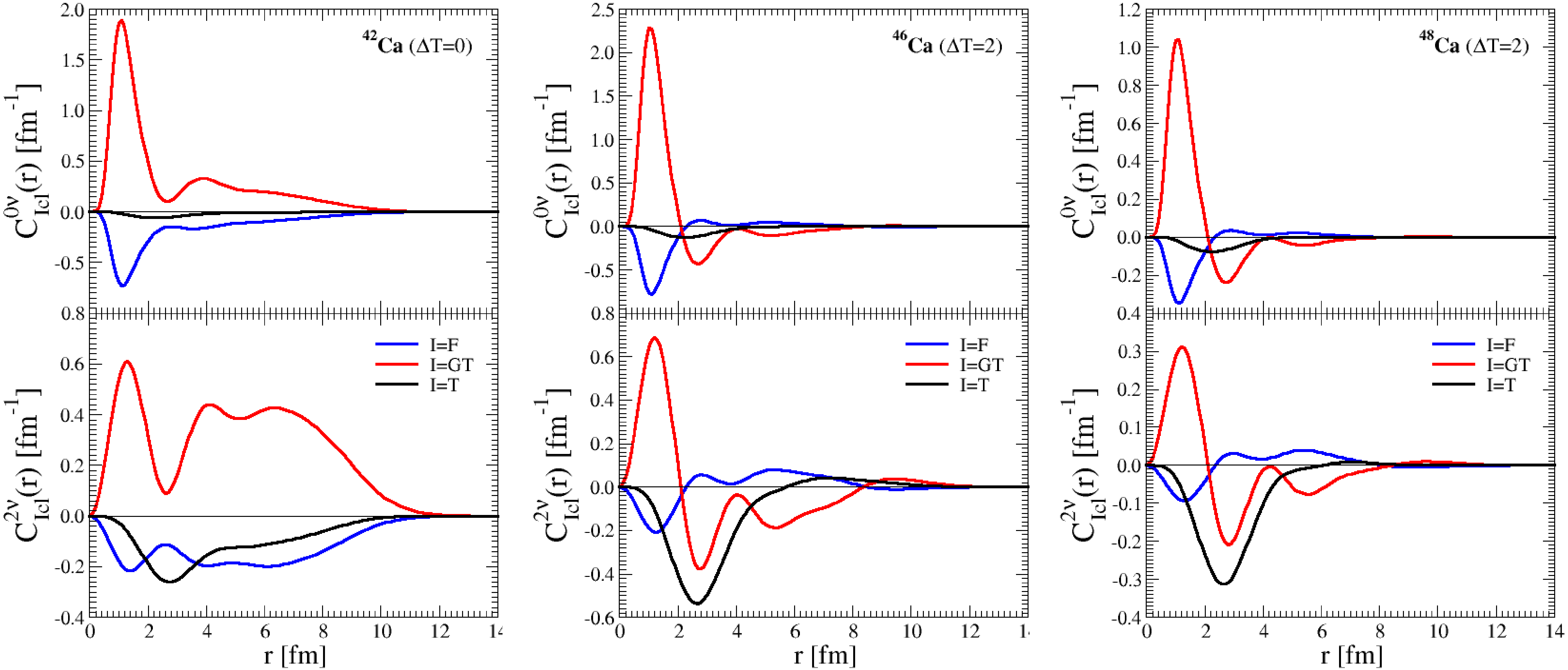
$\int C_J(r)\, dr = 0$

Fedor Simkovic

F.Š., R. Hodák, A. Faessler, P. Vogel, PRC 83, 015502 (2011)

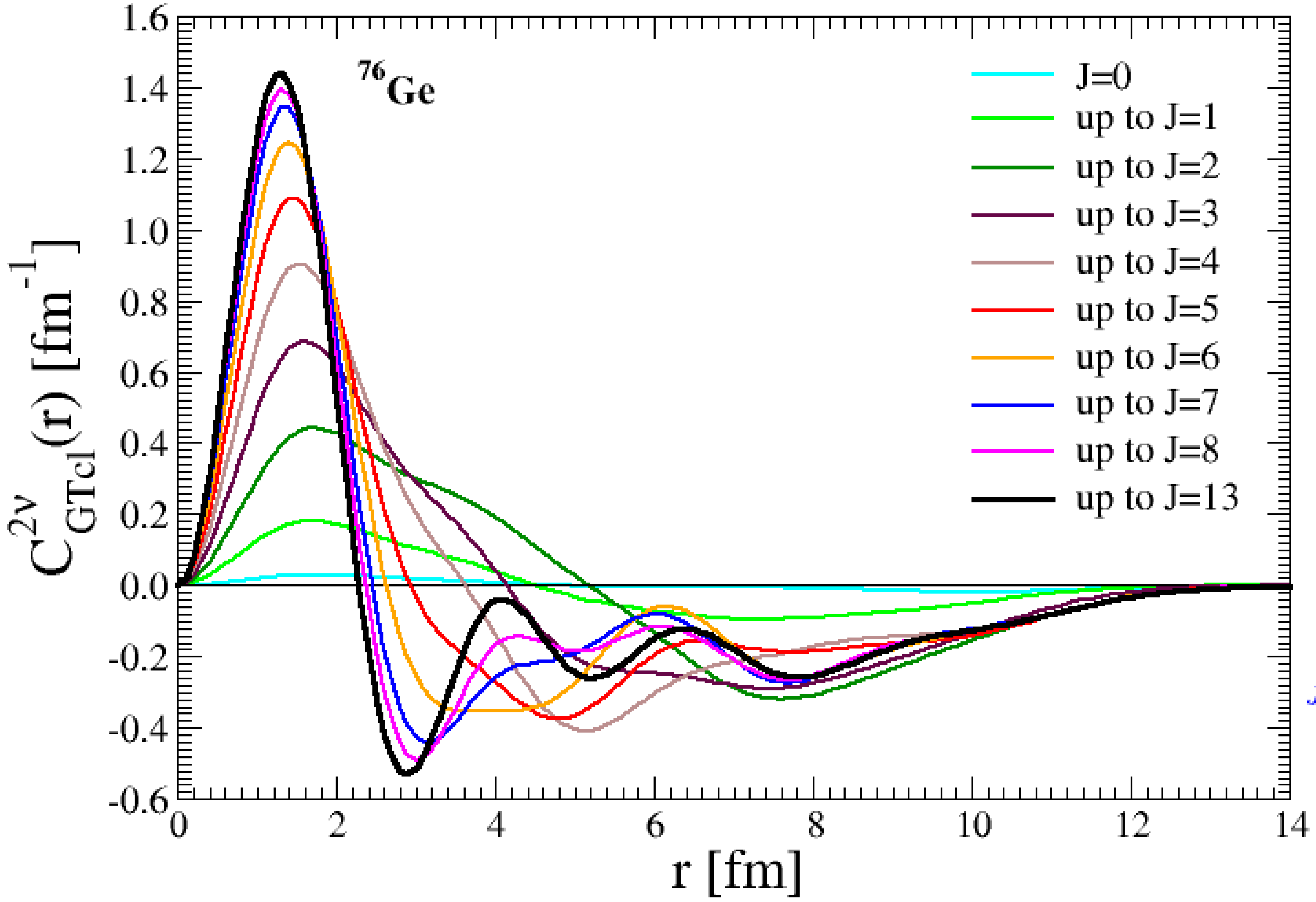
The role of the difference in isospin between the initial and final nuclei

(Δ T = 2 versus Δ T=0)



Closure $2\nu\beta\beta$ GT NME
(role of the higher multipolarities)

The only non-zero contribution from $J^\pi=1^+$



$$M^{2\nu}_{GT-cl} = \sum_{J^\pi} \int_0^\infty C^{2\nu}_{GT-J^\pi}(r)dr$$

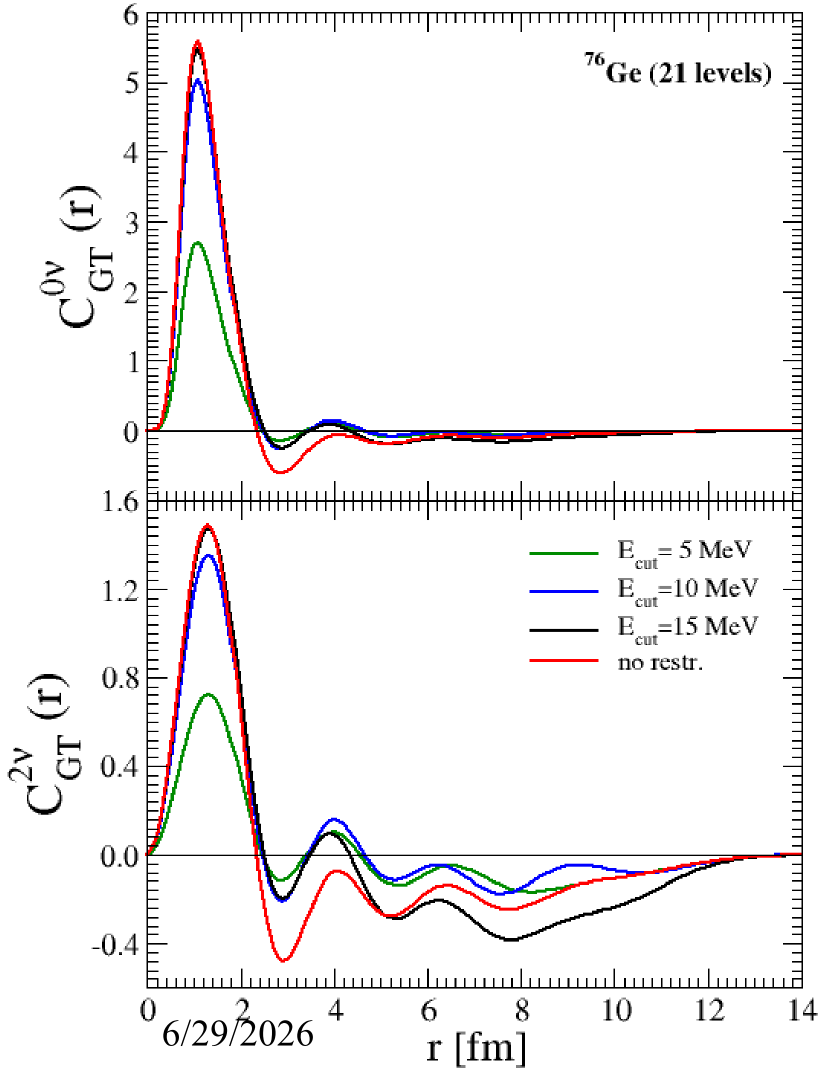
Many multipole contributions not included within the ISM due to truncation of the model space

$$M^{2\nu}_{GT-cl} =$$

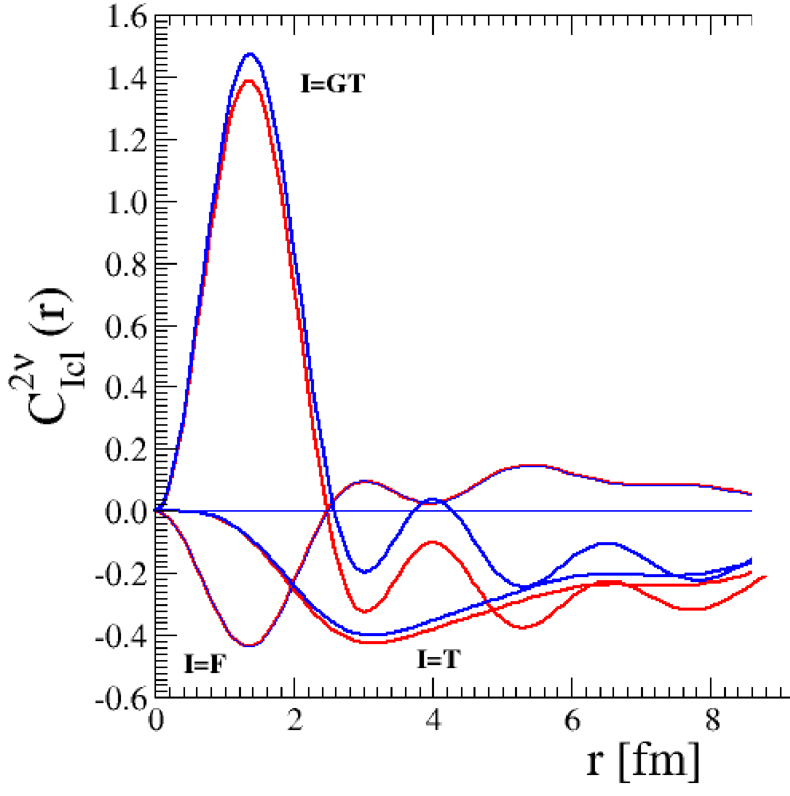
$$\sum_{J^\pi,m} < 0^+_f|\tau^+\vec{\sigma}|J^\pi, m > \cdot < J^\pi, m|\tau^+\vec{\sigma}|0^+_i >$$

$$\sum_{m} < 0^+_f|\tau^+\vec{\sigma}|1^+, m > \cdot < 1^+, m|\tau^+\vec{\sigma}|0^+_i >$$

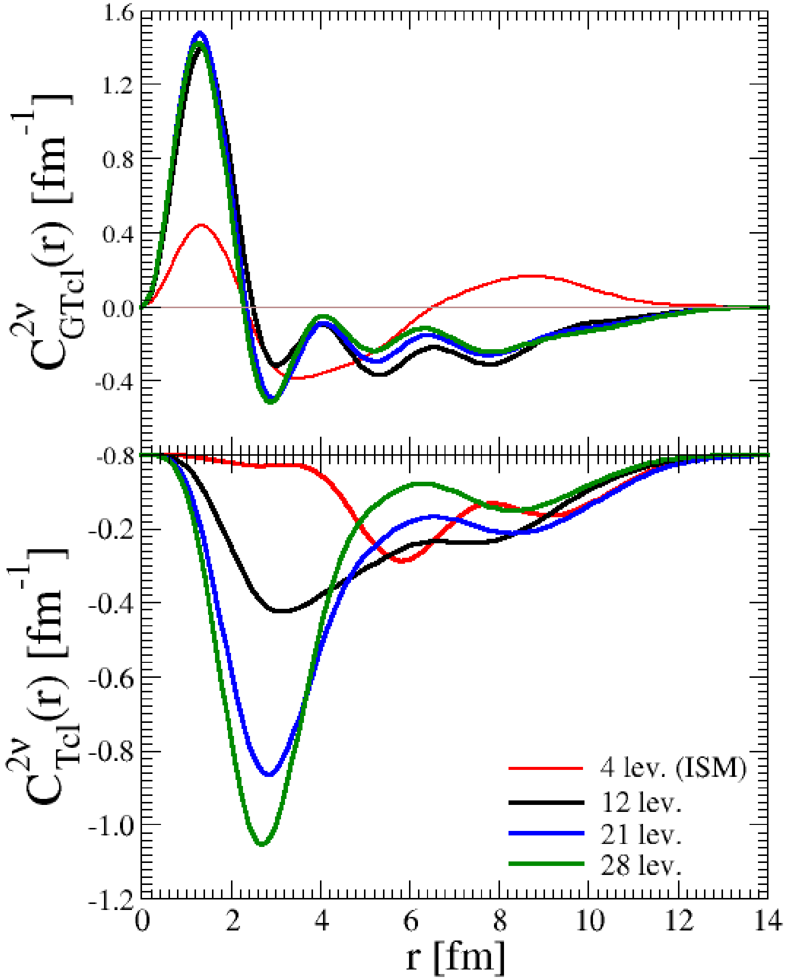
Restriction on the energy range of intermediate states



Dependence on T=0 particle-particle strength (for g_A =0.80 and 1.27)



The size of model space



Measurement of GT strength via μ-capture

$$\mu^-_b + (A,Z) \rightarrow (A,Z\text{-}1) + \nu_\mu$$

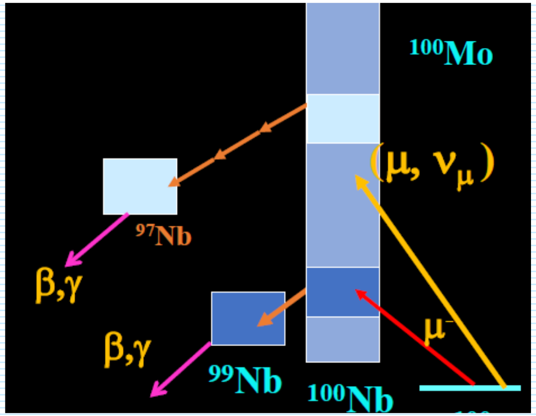
Contradicting QRPA results:

- Strong quenching ($g_A \approx 0.6$)

PRC 100, 014619 (2019)

- Weak quenching ($g_A \approx 1.1$)

PRC 74, 024326 (2006)
PRC 79, 054323 (2009)
PRC 102, 034301 (2020).



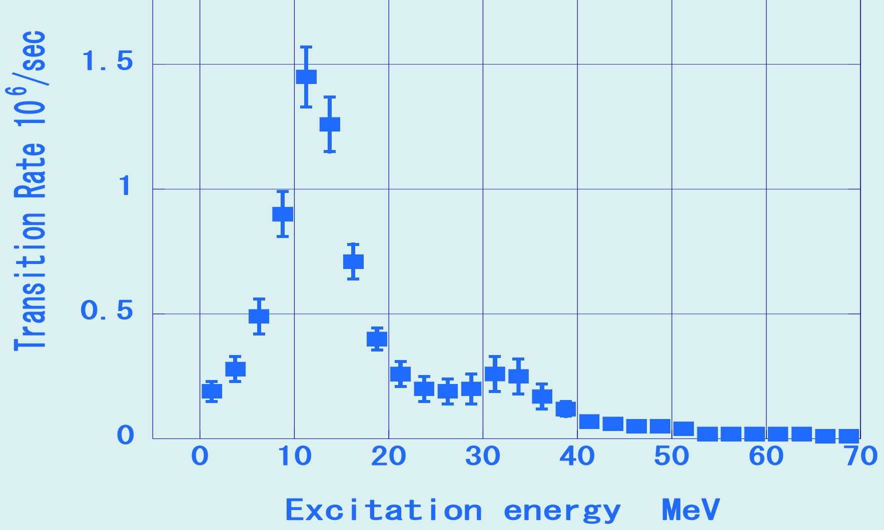
J-PARC 3-50 GeV p, ν, μ

PSI

Monument Exp.

⇒ Small basis nuclear structure calculations (NSM, IBM) are disfavored. ⇒

Momentum transfer
q~80 MeV



I. Hashim H. Ejiri , MXG16, PR C 97 2018

$$\Gamma = m_\mu \frac{(G_\beta m_\mu^2)^2}{2\pi} \times (g_A^{\rm eff})^2 \left(C_F \frac{B_{\Phi F}}{(g_A^{\rm eff})^2} + C_{GT} B_{\Phi GT} + C_T B_{\Phi T} \right)$$

New formalism (derivation as for 0νββ-decay

$$B^k_{\Phi K}(p_{\nu_k}) = \frac{1}{\hat{J}_i} \sum_{M_i M_k} \int \frac{d\Omega_\nu}{4\pi} \times |\langle J_k M_k | \sum_{j=1}^{A} \tau_j^- e^{i\mathbf{p}_{\nu_k}\cdot\mathbf{r}_i} O_K \frac{\Phi_g(r_i)}{m_\mu^{3/2}} |J_i M_i\rangle|^2$$

Muon capture rates evaluated within QRPA

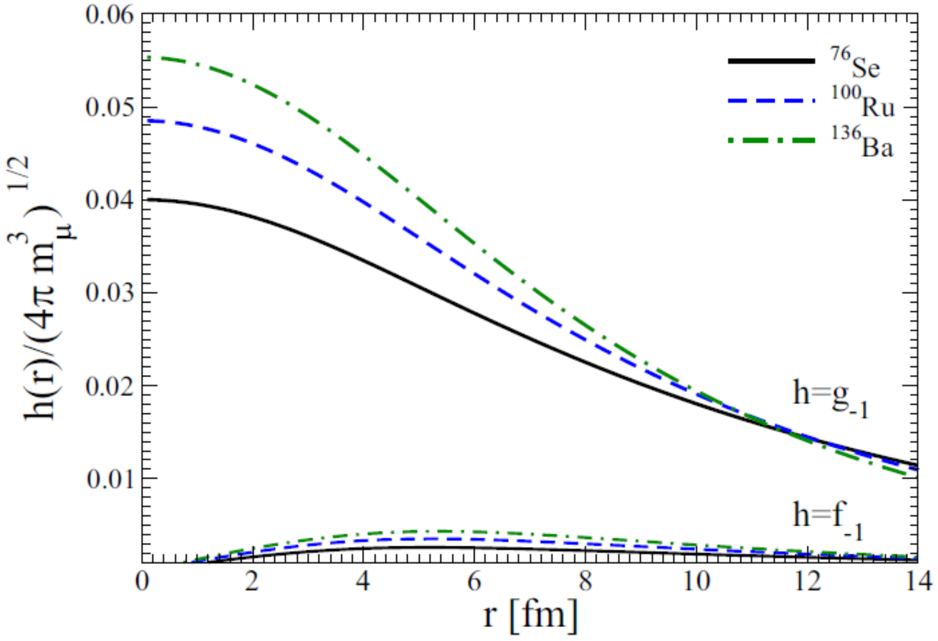
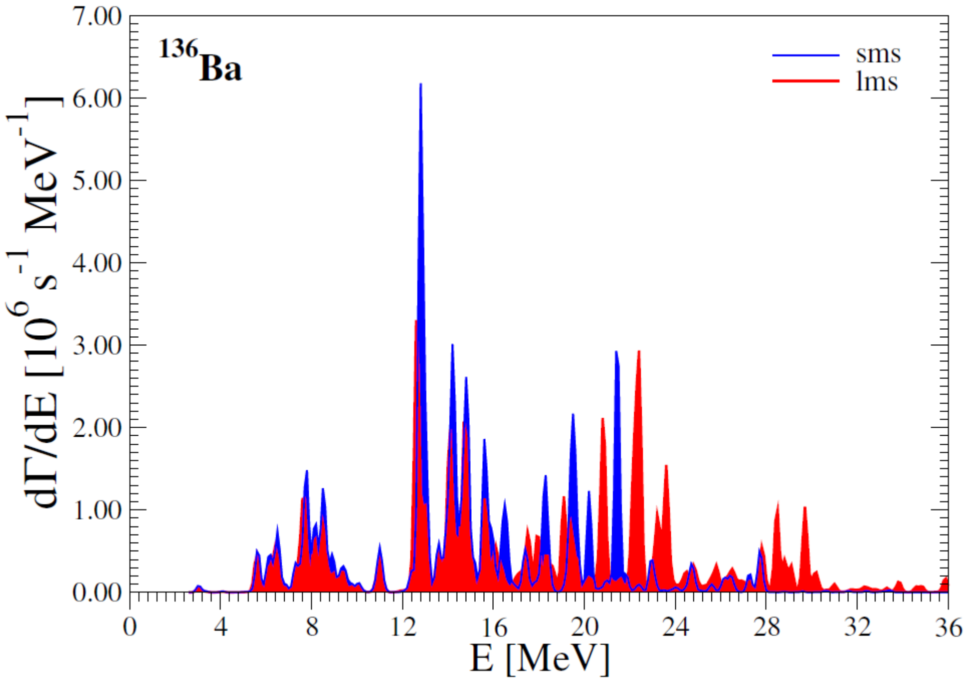
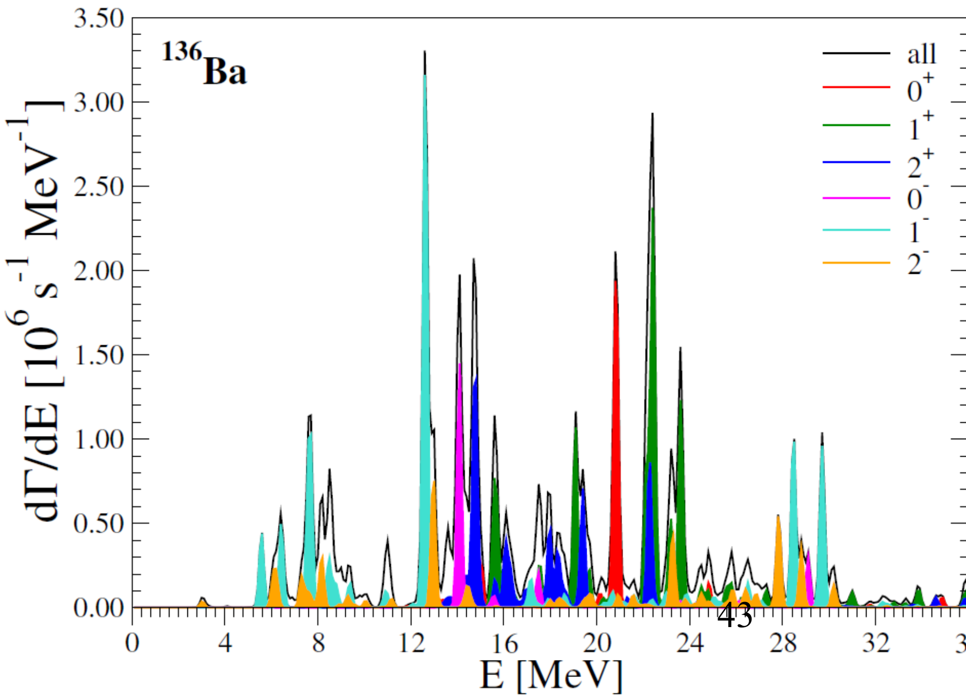


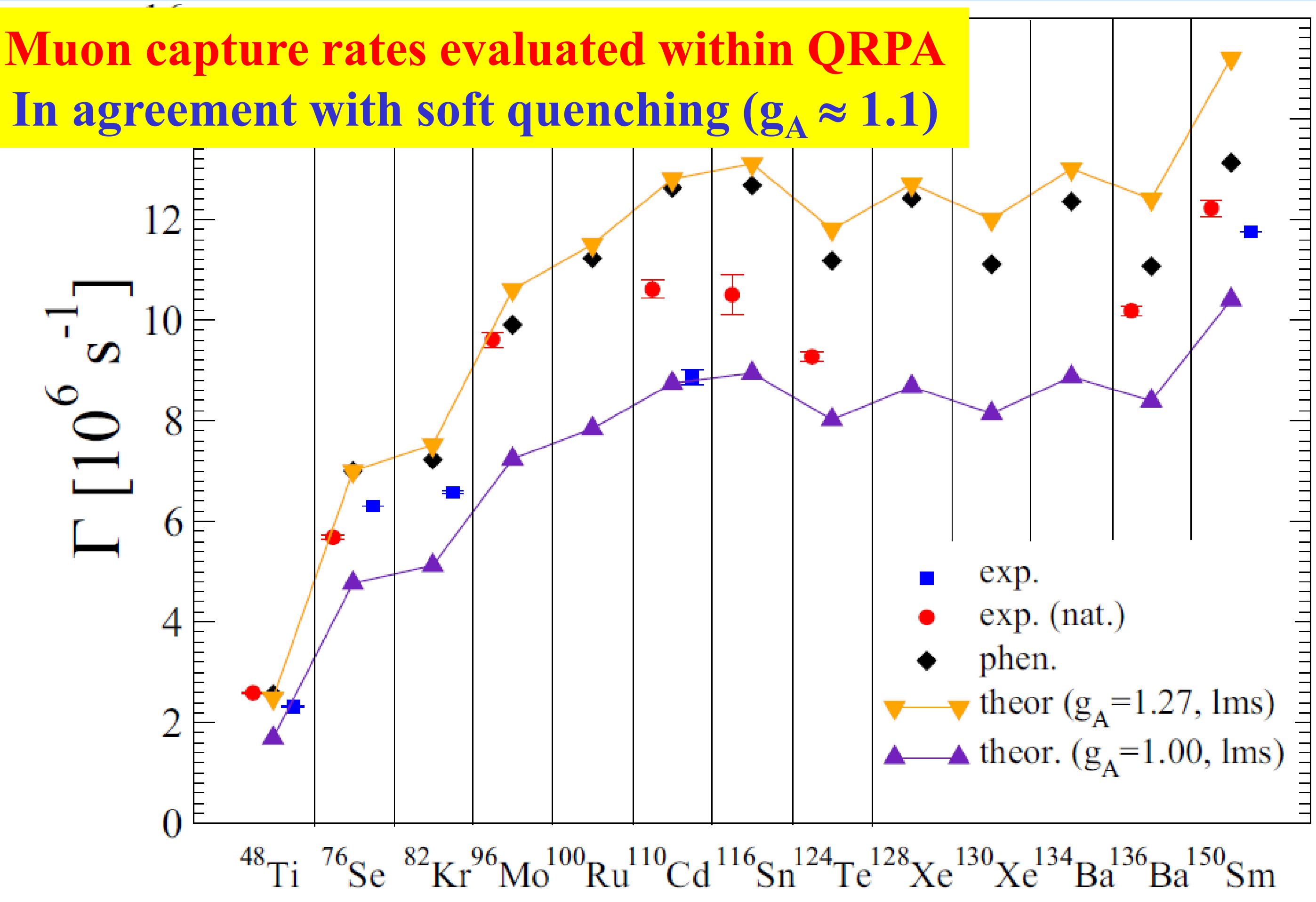
FIG. 1. The radial dependence of the bound muon wave functions $g_{-1}(r)$ and $f_{-1}(r)$ for ^{76}Se, ^{100}Ru, and ^{136}Ba.

F. Š, R. Dvornický,
P. Vogel,
PRC 102, 034301 (2020).





Muon capture rates evaluated within QRPA
In agreement with soft quenching ($g_A \approx 1.1$)



Monument - ^{76}Se (*Phys.Rev.C* 113 (2026) 6):

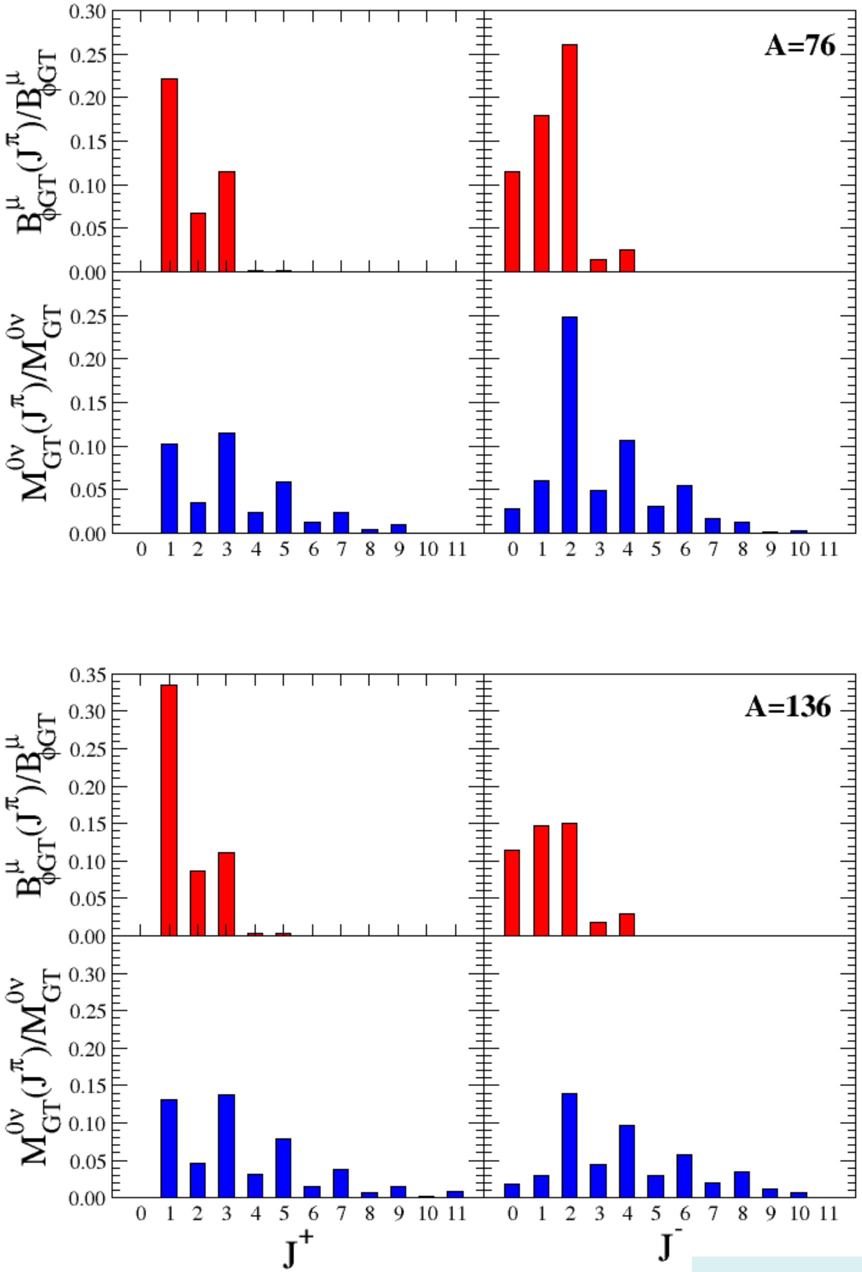
τ = 135.1 ns

Our result - ^{76}Se (PRC 102, 034301 (2020)):

τ = 134.5 ns (g_A = 1,27)

τ = 192.1 ns (g_A = 1,00)

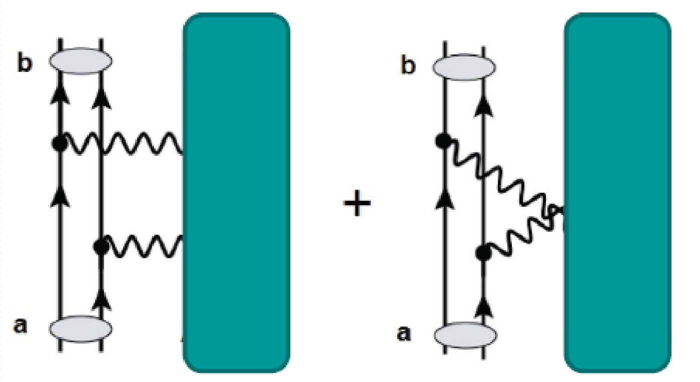
τ = 254.1 ns (g_A = 0.80)

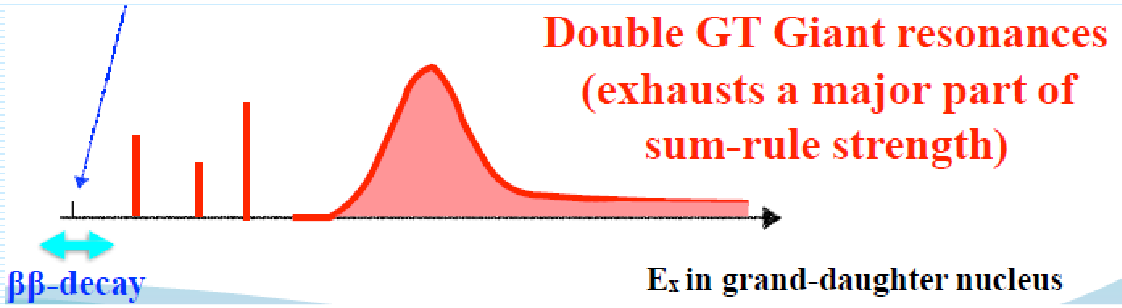


Multipole decomposition of B^μ_{GT} and $M^{0\nu}_{GT}$

PRC 102, 034301 (2020).

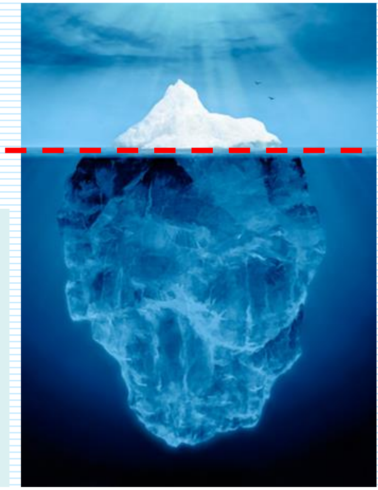
Heavy-ion DCE as surrogate processes of ββ-decay





- ✓ Induced by strong interaction
- ✓ Sequential nucleon transfer mechanism 4th order: Kinematical matching
- ✓ Meson exchange mechanism 1st or 2nd order
- ✓ Possibility to go in both directions
- ✓ Low cross section

Tiny amount of DGT strenght for low lying states

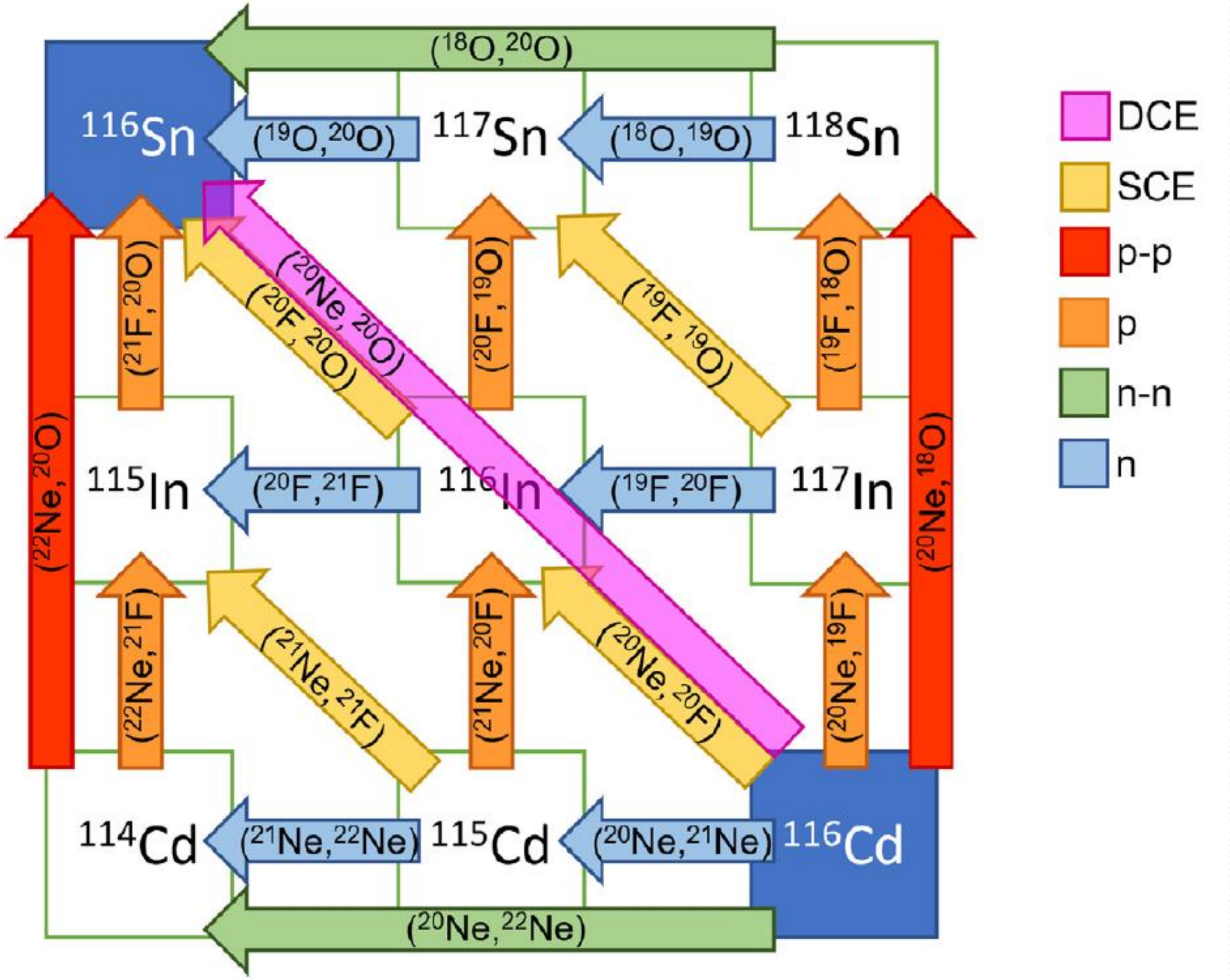


Sum rule almost exhausted by DGT Giant Mode, still not observed

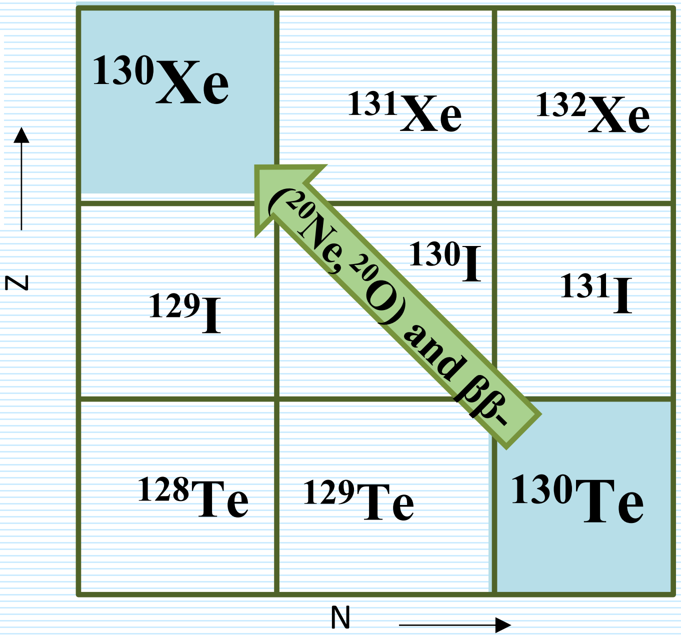
RIKEN

RCNP

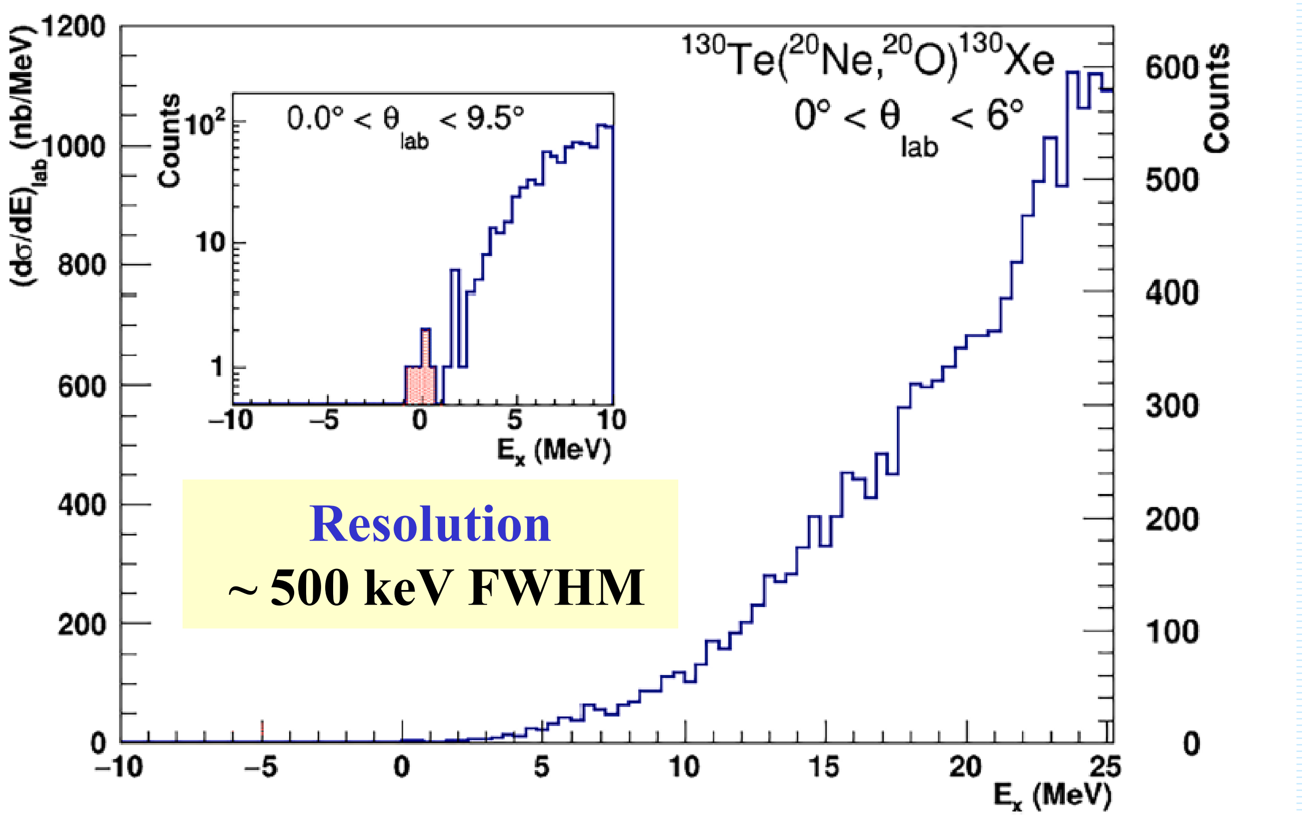
Future: INFN-LN



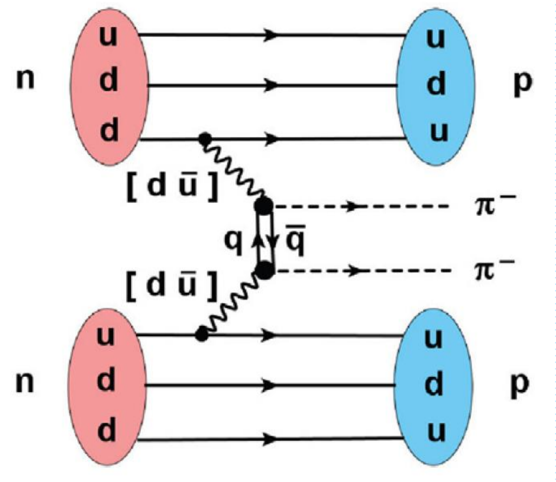
The ^{130}Te(^{20}Ne,^{20}O)^{130}Xe DCE reaction

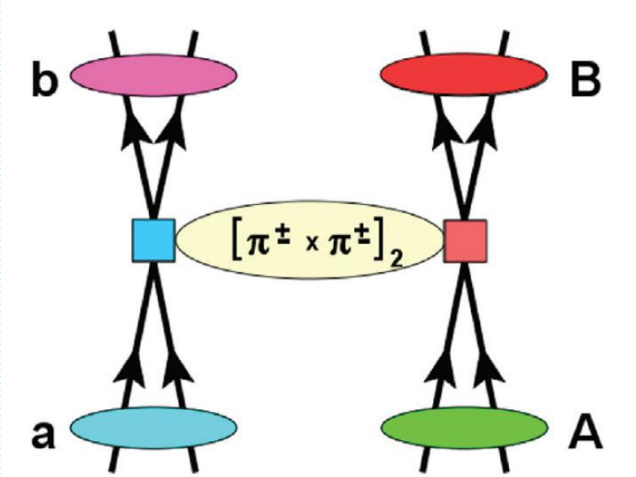


- **g.s. → g.s. transition can be isolated**
- **Absolute cross section measured**



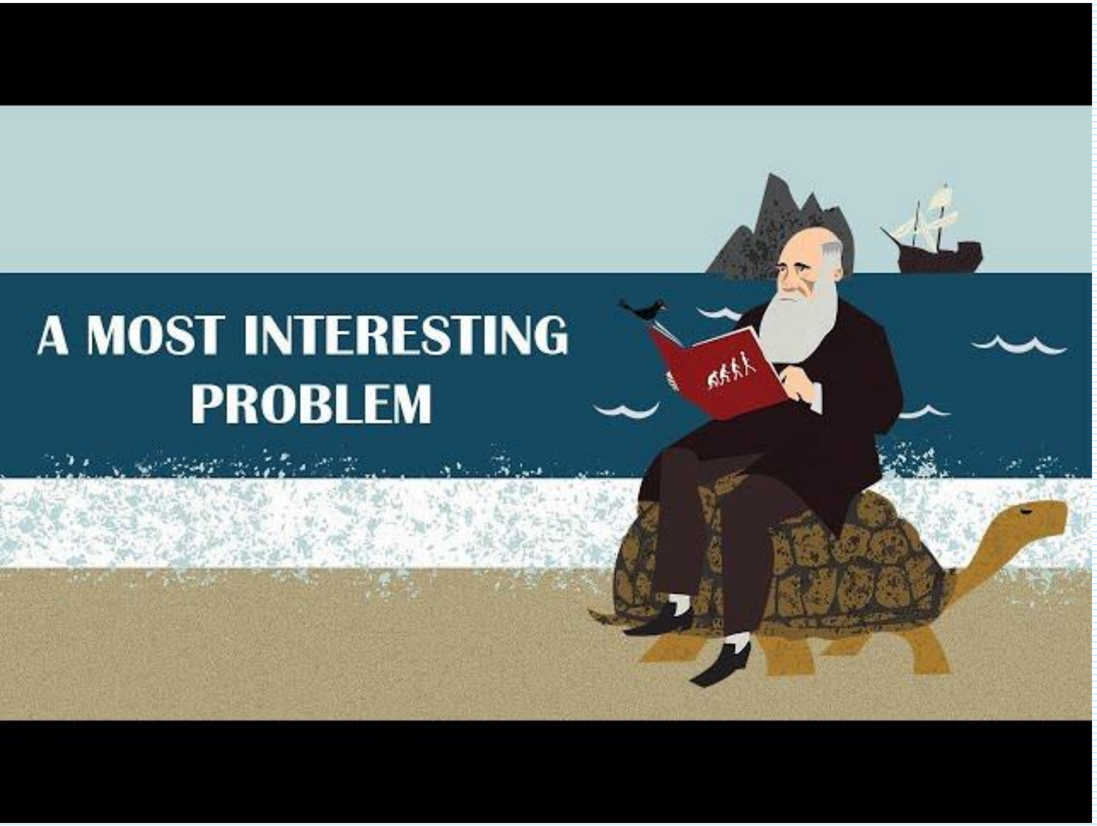
Resolution
~ 500 keV FWHM





State (MeV)	Counts	Absolute coss section (nb)	Cross section 95% limit (nb)
g.s. (0^+) + 2^+ (536 keV)	5	13	[3--18]

Analysis of cross-section sensitivity < 0.1 nb in the Region Of Interest




Time flies when you are having fun.

Albert Einstein

quotefancy