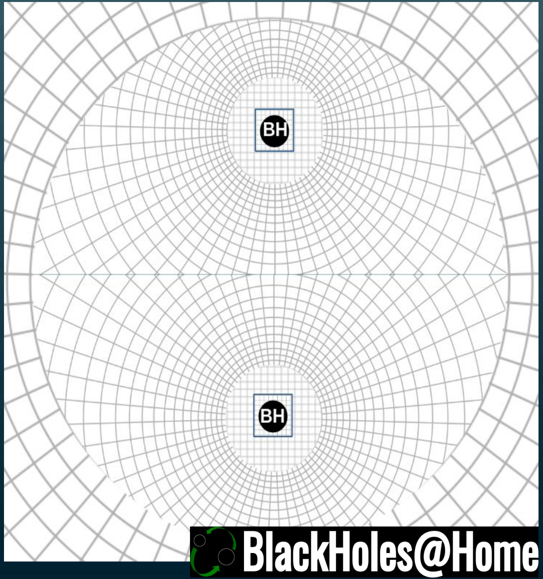
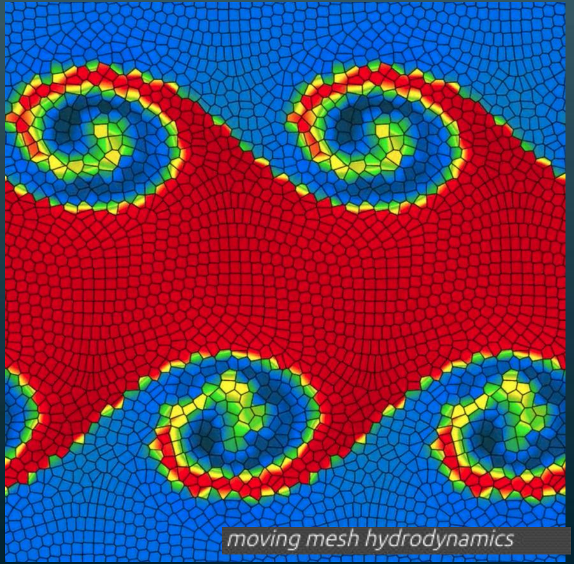
New Techniques in Numerical Relativity

Zachariah Etienne
Department of Physics
University of Idaho





Funding Acknowledgements
PHY-2110352 (Grav theory, 2021-2024)
AST-2108072 (WoU-MMA, 2021-2024)

Disclaimer: Not a review talk

New Techniques in Numerical Relativity

Why we need new techniques in numerical relativity

- Current codes don’t have all the physics we need for BNS
  - GR solver
  - Neutrino physics with advanced EOSs, nuclear physics
  - GRMHD
  - EM radiation transport

New Techniques in Numerical Relativity

Why we need new techniques in numerical relativity

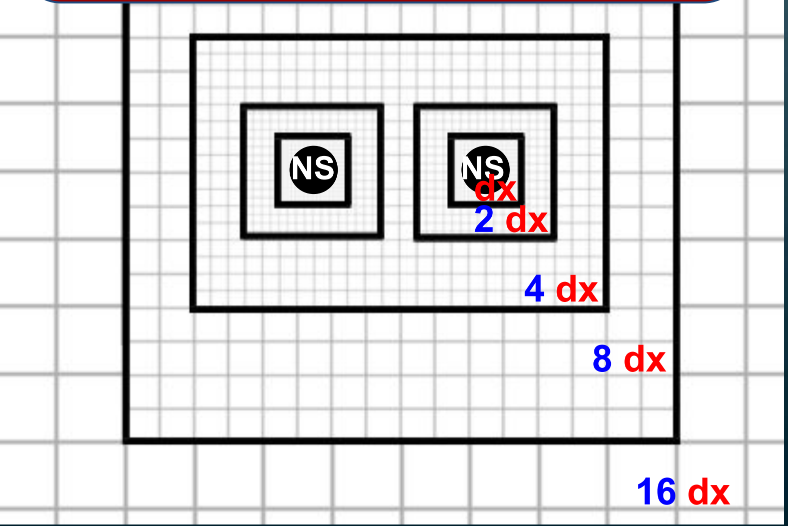
- Current codes don't have all the physics we need for BNS
  - GR solver
  - Neutrino physics with advanced EOSs, nuclear physics
  - GRMHD
  - EM radiation transport

- Current-gen codes too inefficient to survey needed BNS parameter space
  - *All else being the same*, cost ~ # of gridpoints

Most NR codes use Cartesian AMR Grids

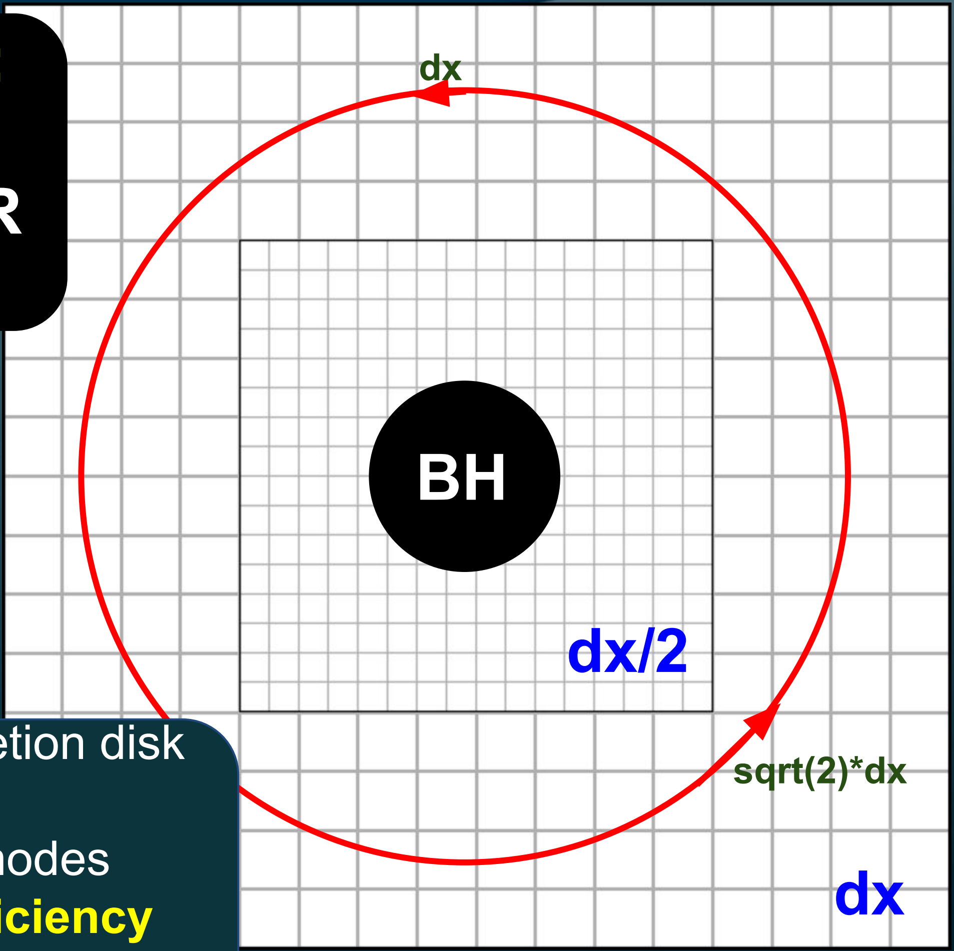
AMR Grids

Adaptive Mesh Refinement
(Most Popular Method in NR)



Most important MMA system: BH accretion disk in full 3D

Comparison of Cartesian AMR vs spherical grids



Red circle: path of fluid element in BH accretion disk

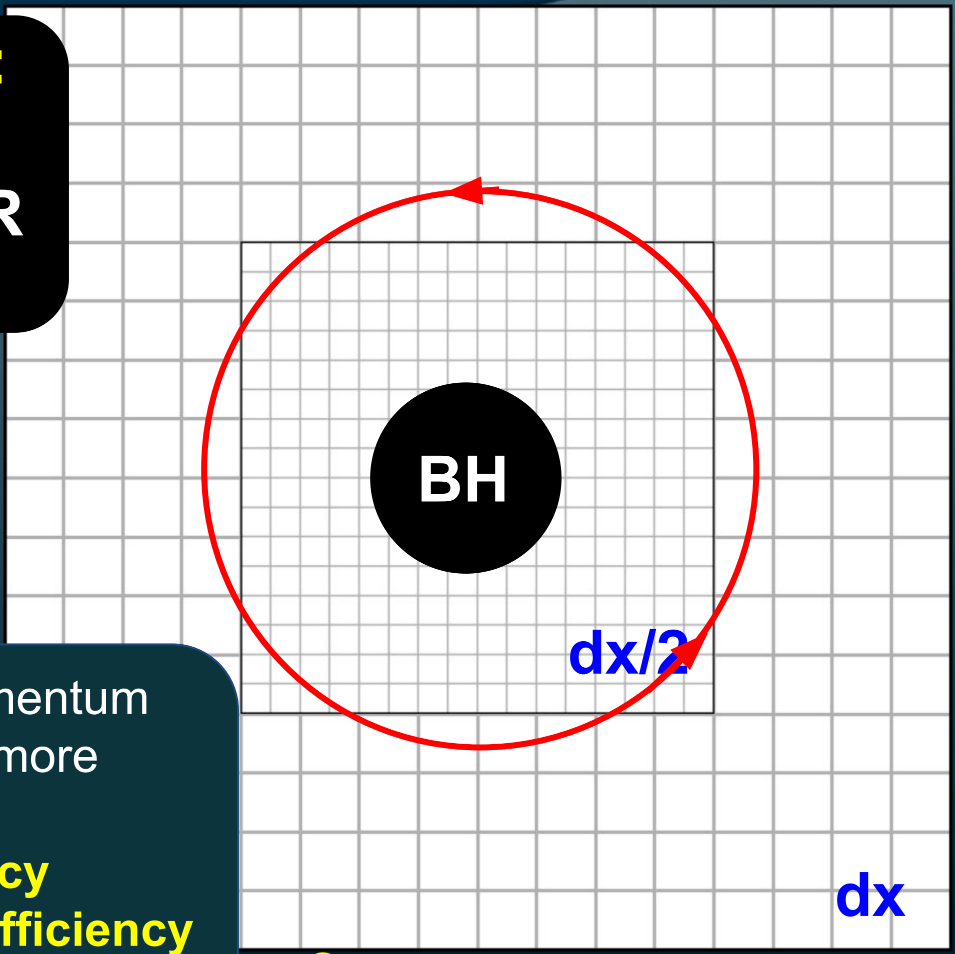
- Resolution changes by 1.4x over path
- Induces artificial high-order multipole modes

Azimuthal resolution ~1.4x lower: **~2x inefficiency (over spherical grids)**

~2x jumps in dr vs smooth dr: **~2x inefficiency**

Most important MMA system: BH accretion disk in full 3D

Comparison of Cartesian AMR vs spherical grids



Situation becomes far worse if angular momentum transport causes fluid element to orbit more closely!

Sharp AMR corners *wasted*: **~2x inefficiency**

Coarse grid underneath fine grid: **~1.2x inefficiency**

Fine grids' wide AMR boundary: **~1.5x inefficiency**

Summary:
Cart AMR ~15x inefficient

New Techniques in Numerical Relativity

Why **we need new techniques in numerical relativity**

- Current codes don’t have all the physics we need for BNS
  - GR solver
  - Neutrino physics with advanced EOSs, nuclear physics
  - GRMHD
  - EM radiation transport

- Current-gen codes too inefficient to survey needed BNS parameter space
  - *All else being the same*, cost ~ # of gridpoints
  - Cartesian AMR: ~15x more gridpoints than needed
    - Next-gen AMR: maybe ~1.5x improvement at best
    - *Thinking outside the box* is probably optimal

New Techniques in Numerical Relativity: Next-Generation NR Codes

<u>Core need for next-gen NR:</u>
Move beyond proof-of-principle simulations & into realm of connecting with observations

New Techniques in Numerical Relativity: Next-Generation NR Codes

Core need for next-gen NR:

Move beyond proof-of-principle simulations & into realm of connecting with observations

Keys to unlocking next-gen NR

1. Efficiency
2. Efficiency
3. Efficiency

New Techniques in Numerical Relativity: Next-Generation NR Codes

Core need for next-gen NR:
Move beyond proof-of-principle simulations & into realm of connecting with observations

Keys to unlocking next-gen NR

1. Efficiency, so we can add more physics
2. Efficiency, so we can span param space
3. Efficiency, so that others can afford using & connecting to their own codes
    a. *OSS & good documentation!*

New Techniques in Numerical Relativity: Next-Generation NR Codes

Core need for next-gen NR:
Move beyond proof-of-principle simulations & into realm of connecting with observations

Keys to unlocking next-gen NR

1. Efficiency, so we can add more physics
2. Efficiency, so we can span param space
3. Efficiency, so that others can afford using & connecting to their own codes
    a. *OSS & good documentation!*

Higher order methods
(Higher effective resolution)
DG, WENO, high-order FD

Characteristics of next-gen NR

New Techniques in Numerical Relativity: Next-Generation NR Codes

Core need for next-gen NR:
Move beyond proof-of-principle simulations & into realm of connecting with observations

Keys to unlocking next-gen NR

1. Efficiency, so we can add more physics
2. Efficiency, so we can span param space
3. Efficiency, so that others can afford using & connecting to their own codes
    a. *OSS & good documentation!*

Higher order methods
(Higher effective resolution)
DG, WENO, high-order FD

Characteristics of next-gen NR

Excellent single-CPU performance,
GPU capabilities,
Task-based parallelism

New Techniques in Numerical Relativity: Next-Generation NR Codes

Core need for next-gen NR:
Move beyond proof-of-principle simulations & into realm of connecting with observations

Keys to unlocking next-gen NR

1. **Efficiency, so we can add more physics**
2. **Efficiency, so we can span param space**
3. **Efficiency, so that others can afford using & connecting to their own codes**
    a. ***OSS & good documentation!***

Higher order methods
(Higher effective resolution)
DG, WENO, high-order FD

Characteristics of next-gen NR

Improved grids:
Moving mesh, SPH,
Non-Cartesian coords

Excellent single-CPU performance,
GPU capabilities,
Task-based parallelism

New Techniques in Numerical Relativity: Next-Generation NR Codes

Core need for next-gen NR:
Move beyond proof-of-principle simulations & into realm of connecting with observations

Keys to unlocking next-gen NR

1. Efficiency, so we can add more physics
2. Efficiency, so we can span param space
3. Efficiency, so that others can afford using & connecting to their own codes
   a. *OSS & good documentation!*

Interoperability w/other codes

Characteristics of next-gen NR

Higher order methods
(Higher effective resolution)
DG, WENO, high-order FD

Improved grids:
Moving mesh, SPH,
Non-Cartesian coords

Excellent single-CPU performance,
GPU capabilities,
Task-based parallelism

New Techniques in Numerical Relativity: Next-Generation NR Codes

Core need for next-gen NR:
Move beyond proof-of-principle simulations & into realm of connecting with observations

Keys to unlocking next-gen NR

1. Efficiency, so we can add more physics
2. Efficiency, so we can span param space
3. Efficiency, so that others can afford using & connecting to their own codes
   a. *OSS & good documentation!*

Interoperability w/other codes

Characteristics of next-gen NR

Higher order methods
(Higher effective resolution)
DG, WENO, high-order FD

Improved grids:
Moving mesh, SPH,
Non-Cartesian coords

Excellent single-CPU performance,
GPU capabilities,
Task-based parallelism

New Techniques in Numerical Relativity: Next-Generation NR Codes

Core need for next-gen NR:
Move beyond proof-of-principle simulations & into realm of connecting with observations

Keys to unlocking next-gen NR

1. Efficiency, so we can add more physics
2. Efficiency, so we can span param space
3. Efficiency, so that others can afford using & connecting to their own codes
    a. *OSS & good documentation!*

Interoperability w/other codes

Higher order methods
(Higher effective resolution)
DG, WENO, high-order FD

AI/ML? Probably next-next-gen NR

Characteristics of next-gen NR

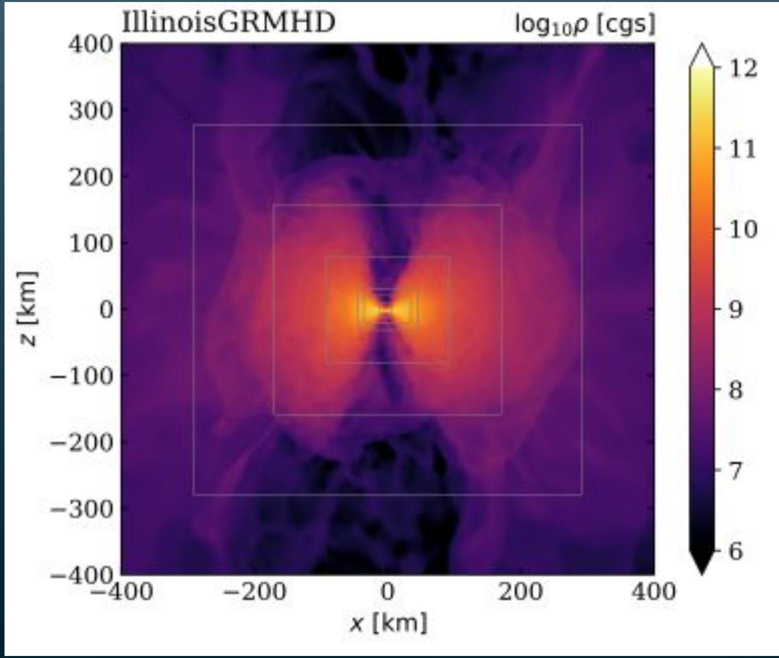
Improved grids:
Moving mesh, SPH,
Non-Cartesian coords

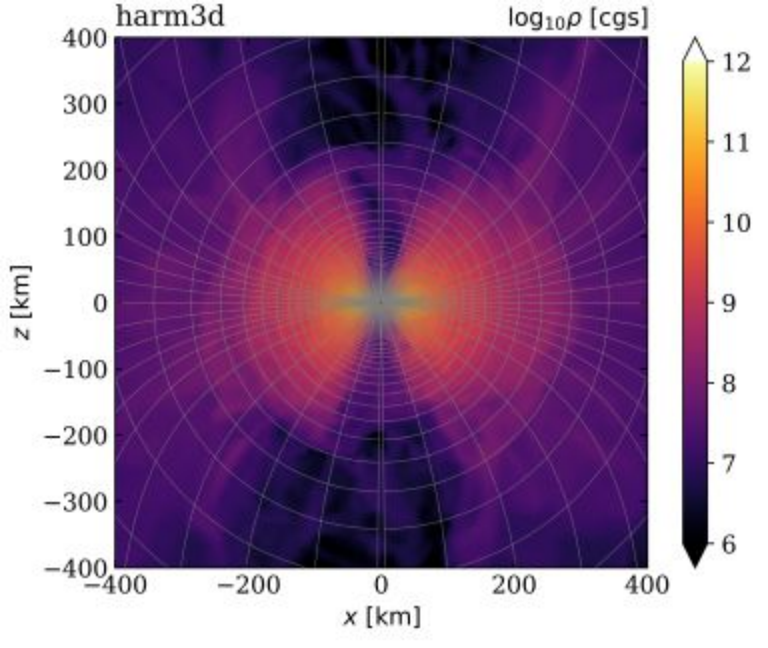
Excellent single-CPU performance,
GPU capabilities,
Task-based parallelism

Improved post-merger BNS simulations:
Inspiral+merger using Cartesian AMR GRMHD
Very long post-merger using spherical GRMHD

Lopez Armengol, ZBE, *et al.* https://arxiv.org/abs/2112.09817

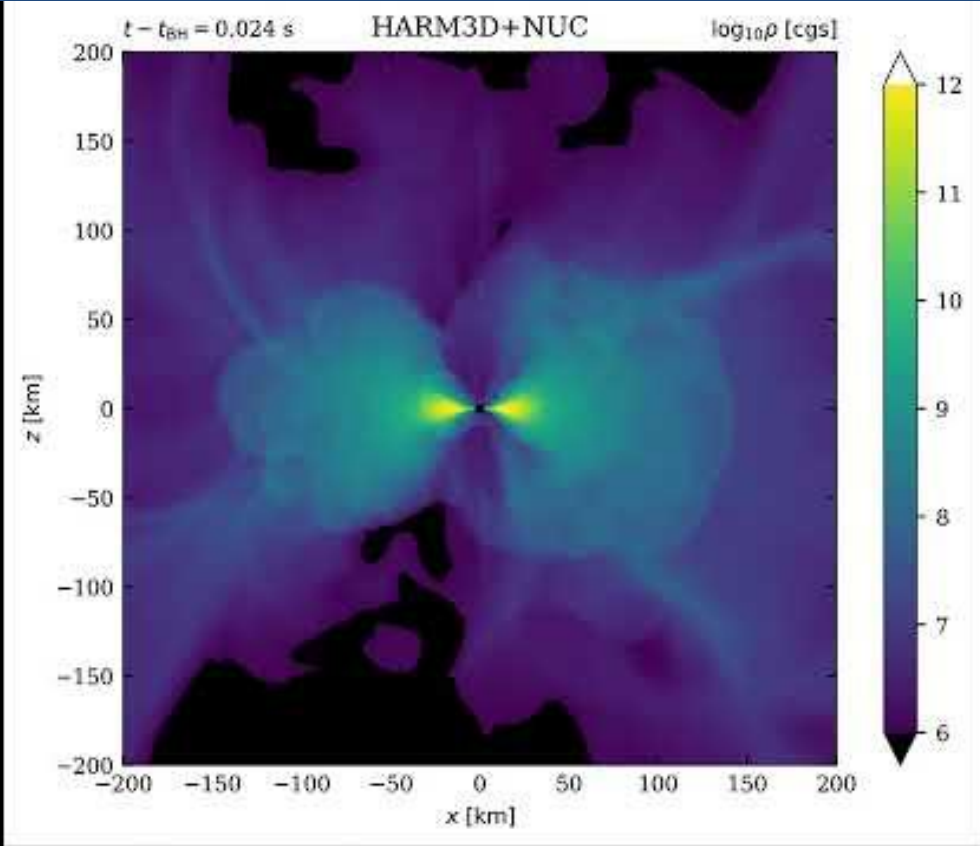
Compact-binaries.org





Improved post-merger BNS simulations:
Inspiral+merger using Cartesian AMR GRMHD
Very long post-merger using spherical GRMHD

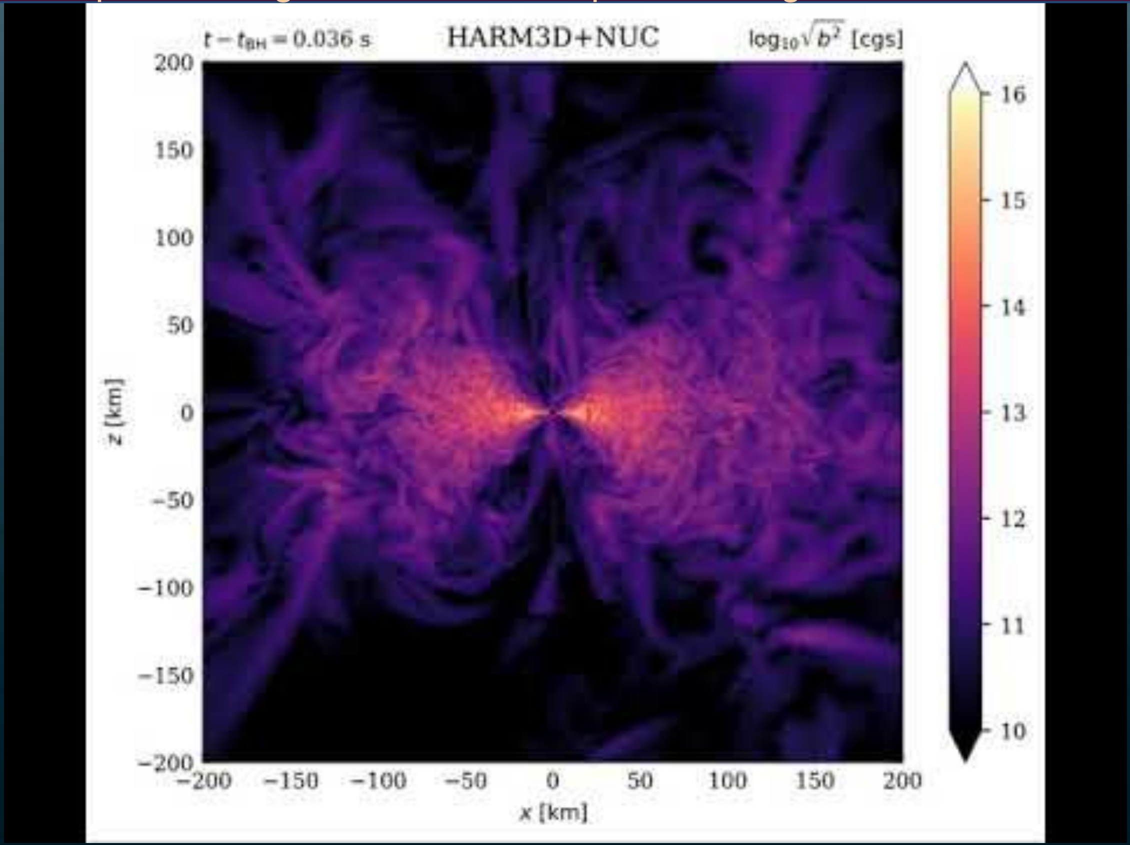
Lopez Armengol, ZBE, *et al.* https://arxiv.org/abs/2112.09817



Trial run:
Magnetized BNS:
q=1
LS220/SLy4 +
postmerger neutrino
leakage

Improved post-merger BNS simulations:
Inspiral+merger using Cartesian AMR GRMHD
Very long post-merger using spherical GRMHD

Lopez Armengol, ZBE, *et al.* https://arxiv.org/abs/2112.09817



Trial run:
Magnetized BNS:
q=1
LS220/SLy4 +
postmerger neutrino
leakage

Improved post-merger BNS simulations:
Inspiral+merger using Cartesian AMR GRMHD
Very long post-merger using spherical GRMHD

Lopez Armengol, ZBE, *et al.* https://arxiv.org/abs/2112.09817

Conclusion:

Remnant BH accretion disks best simulated on spherical grids.

Improved post-merger BNS simulations:
Inspiral+merger using Cartesian AMR GRMHD
Very long post-merger using spherical GRMHD

Lopez Armengol, ZBE, *et al.* https://arxiv.org/abs/2112.09817

Conclusion:

Remnant BH accretion disks best simulated on spherical grids.

… but can inspiral/merger be made more efficient as well?

Improved post-merger BNS simulations:
Inspiral+merger using Cartesian AMR GRMHD
Very long post-merger using spherical GRMHD

Lopez Armengol, ZBE, *et al.* https://arxiv.org/abs/2112.09817

Conclusion:

Remnant BH accretion disks best simulated on spherical grids.

… but can inspiral/merger be made more efficient as well?

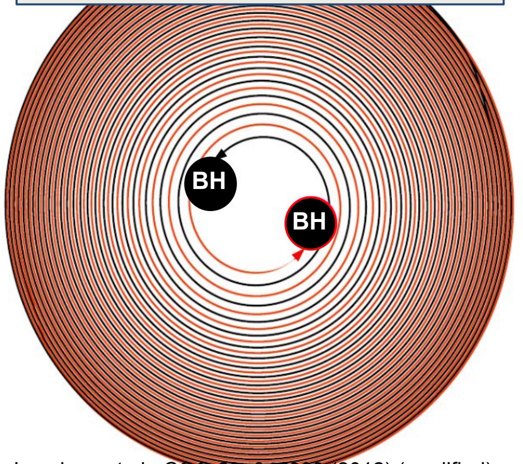
Answer:

Yes, BlackHoles@Home grids provide one path forward

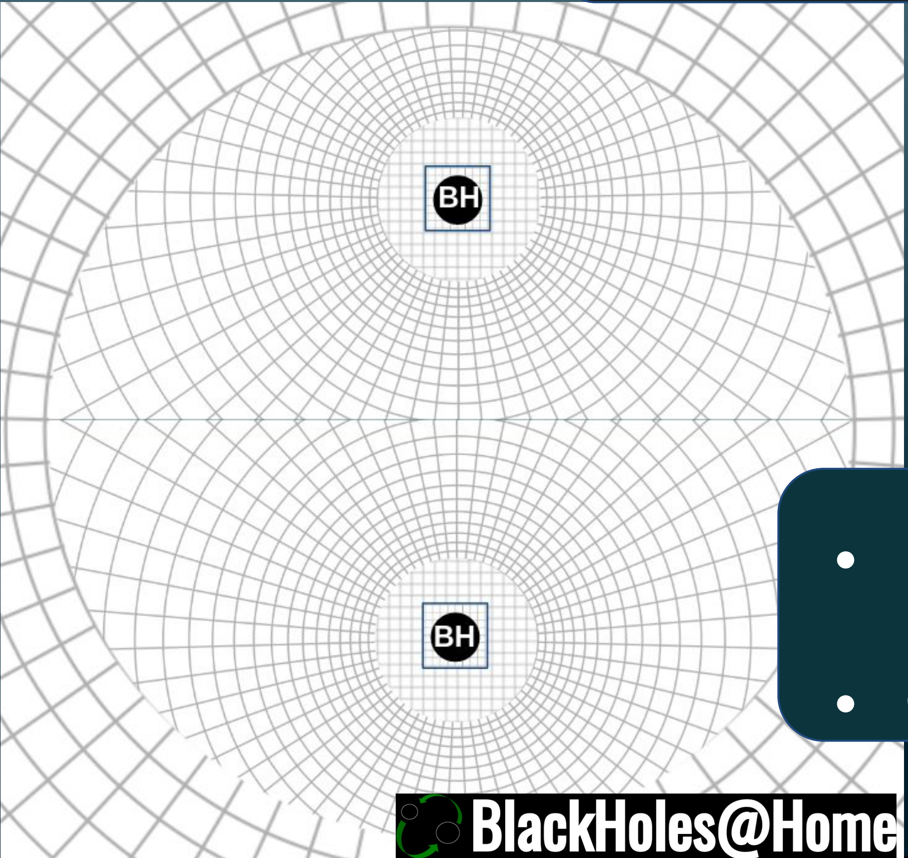
BlackHoles@Home

https://blackholesathome.net

Binary Black Hole (BBH): Orbiting, colliding black holes



Lovelace et al., CQG 29, **045003** (2012) (modified)



BlackHoles@Home

- Proposed volunteer computing/citizen science project
  a. Fit full-NR BBH sims on consumer desktops
  b. Generate large-scale GW follow-ups & catalogs
- Code behind the volunteer computing project

BlackHoles@Home

What is it?

- A NRPy+-based numerical relativity code

NRPy+: Python-based C code generation framework for NR

Tensorial expressions in Einstein-like notation ⇒ Highly optimized C-code kernels (with FDs)

"Nerpy", the NRPy+ mascot. Photo CC2.0 Pacific Environment (modified).

https://nrpyplus.net

BlackHoles@Home

What is it?

- A NRPy+-based numerical relativity code
- NRPy+: builds on "NR in spherical coordinates" work

NRPy+: Python-based C code generation framework for NR

Tensorial expressions in Einstein-like notation ⇒ Highly optimized C-code kernels (with FDs)

"Nerpy", the NRPy+ mascot. Photo CC2.0 Pacific Environment (modified).

https://nrpyplus.net

NR in Spherical coordinates

- Brown (PRD 79, 104029, **2009**)
- Baumgarte, Montero, Cordero-Carrión, Müller (PRD 87, 044026, **2012**)

NR in spherical-like, cylindrical-like, etc.

- Ruchlin, **Etienne**, Baumgarte (PRD 97, 064036, **2018**)

BlackHoles@Home

What is it?

- A NRPy+-based numerical relativity code
- NRPy+: builds on "NR in spherical coordinates" work
  - NRPy+: spherical-like, cylindrical-like, & Cartesian-like coordinates/grids

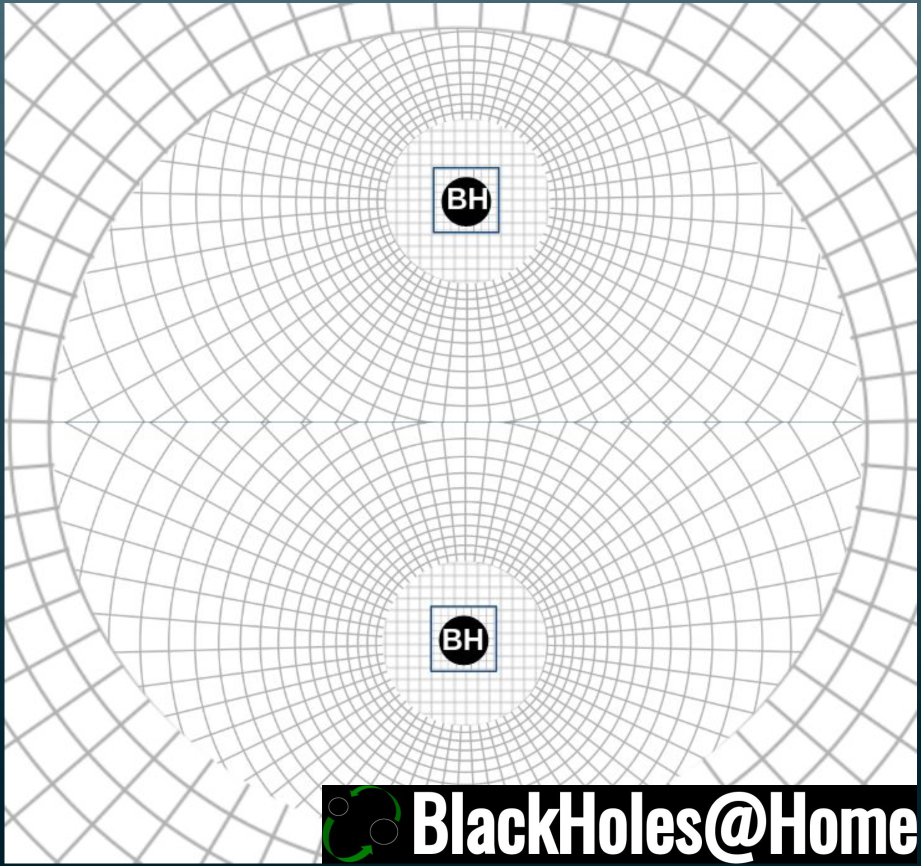
NR in Spherical coordinates
- Brown (PRD 79, 104029, **2009**)
- Baumgarte, Montero, Cordero-Carrión, Müller (PRD 87, 044026, **2012**)

NR in spherical-like, cylindrical-like, etc.
- Ruchlin, **Etienne**, Baumgarte (PRD 97, 064036, **2018**)

BlackHoles@Home

What is it?

- A NRPy+-based numerical relativity code
- NRPy+: builds on “NR in spherical coordinates” work
  - NRPy+: spherical-like, cylindrical-like, & Cartesian-like coordinates/grids
- BlackHoles@Home: Multiple overlapping coordinate/grid patches



NR in Spherical coordinates

- Brown (PRD 79, 104029, **2009**)
- Baumgarte, Montero, Cordero-Carrión, Müller (PRD 87, 044026, **2012**)

NR in spherical-like, cylindrical-like, etc.

- Ruchlin, **Etienne**, Baumgarte (PRD 97, 064036, **2018**)

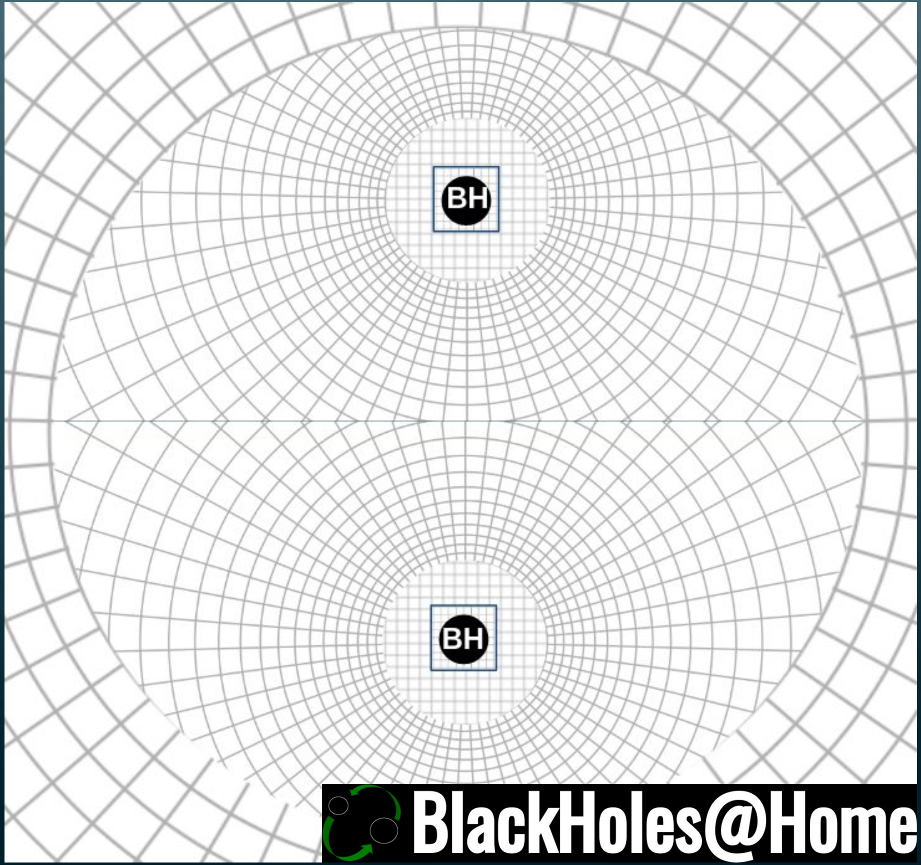
BlackHoles@Home

What is it?

- A NRPy+-based numerical relativity code
- NRPy+: builds on "NR in spherical coordinates" work
  - NRPy+: spherical-like, cylindrical-like, & Cartesian-like coordinates/grids
- BlackHoles@Home: Multiple overlapping coordinate/grid patches

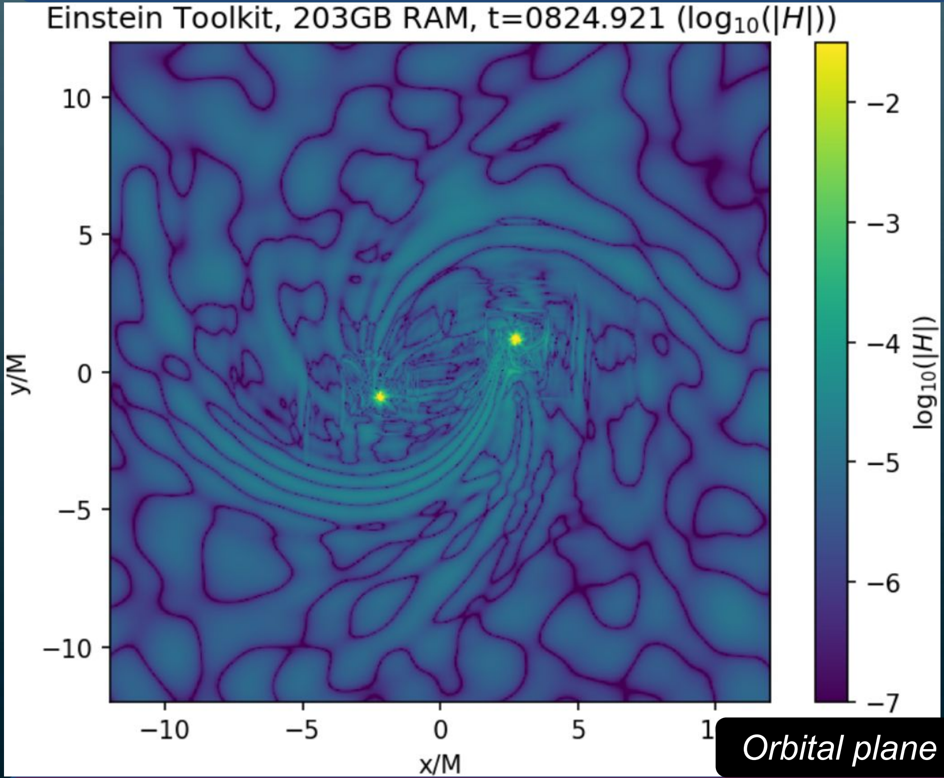
Why use it?

- **Super efficient**: Number of gridpoints: BH@H < **~1%** Cartesian AMR
  - Binary BHs on *desktop computers*
- **Super accurate**: better grids -> less numerical noise than *any other code*; far cleaner convergence than Cart AMR

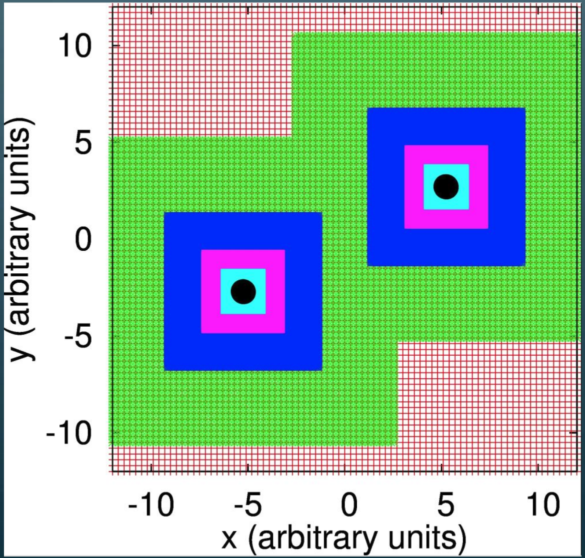


BH@H Code Validation against Einstein Toolkit (ET): GW150914 Gallery Example

Num. Errors ($\log_{10}$|Ham. constraint|) @ 4.8 orbits



Cartesian AMR grids: 203GB of RAM



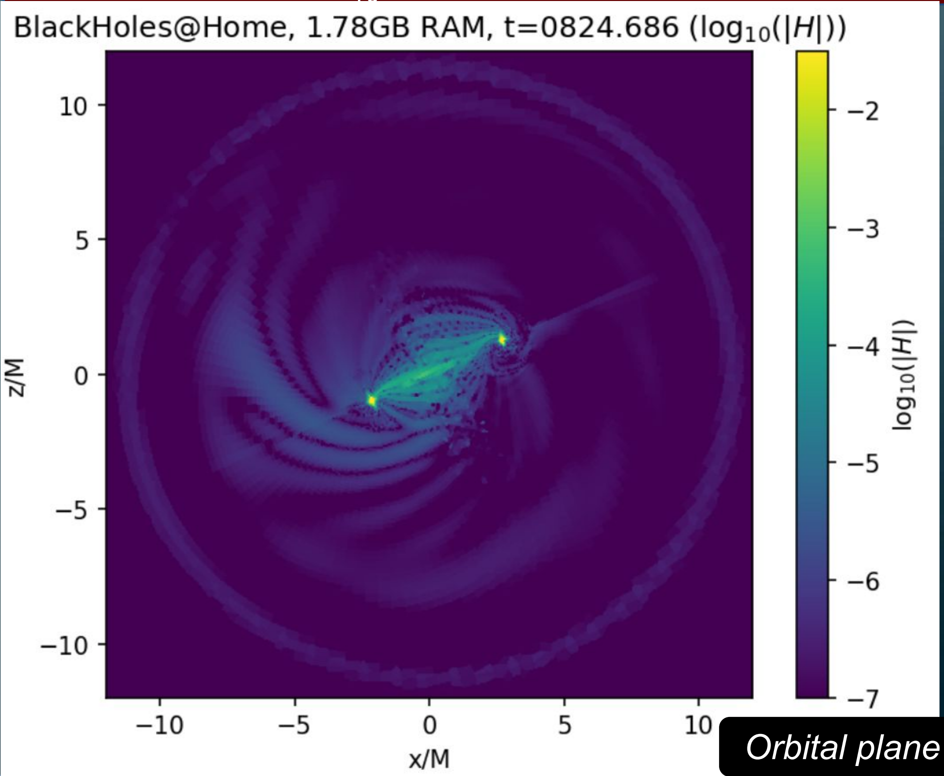
Simulation details

Physical parameters

Initial separation D	10 M
Mass ratio $q = m_1/m_2$	36/29 ~ 1.24
Spin $\chi_1 = a_1/m_1$	0.31
Spin $\chi_2 = a_2/m_2$	-0.46

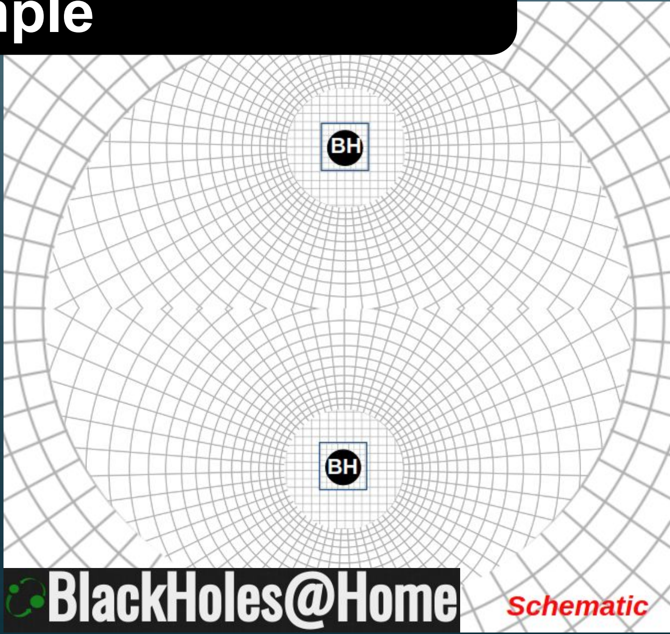
BH@H Code Validation against Einstein Toolkit (ET): GW150914 Gallery Example

Num. Errors ($\log_{10}$|Ham. constraint|) @ 4.8 orbits



Orbital plane

BlackHoles@Home grids: 1.78 of RAM

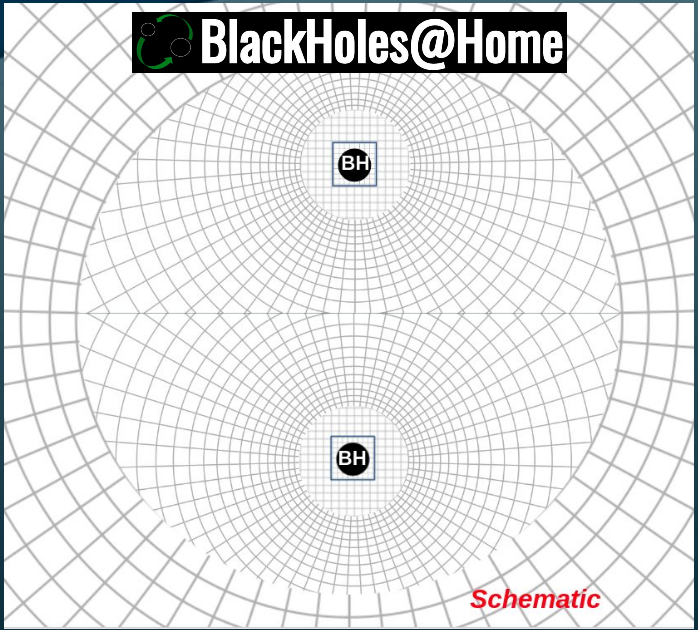


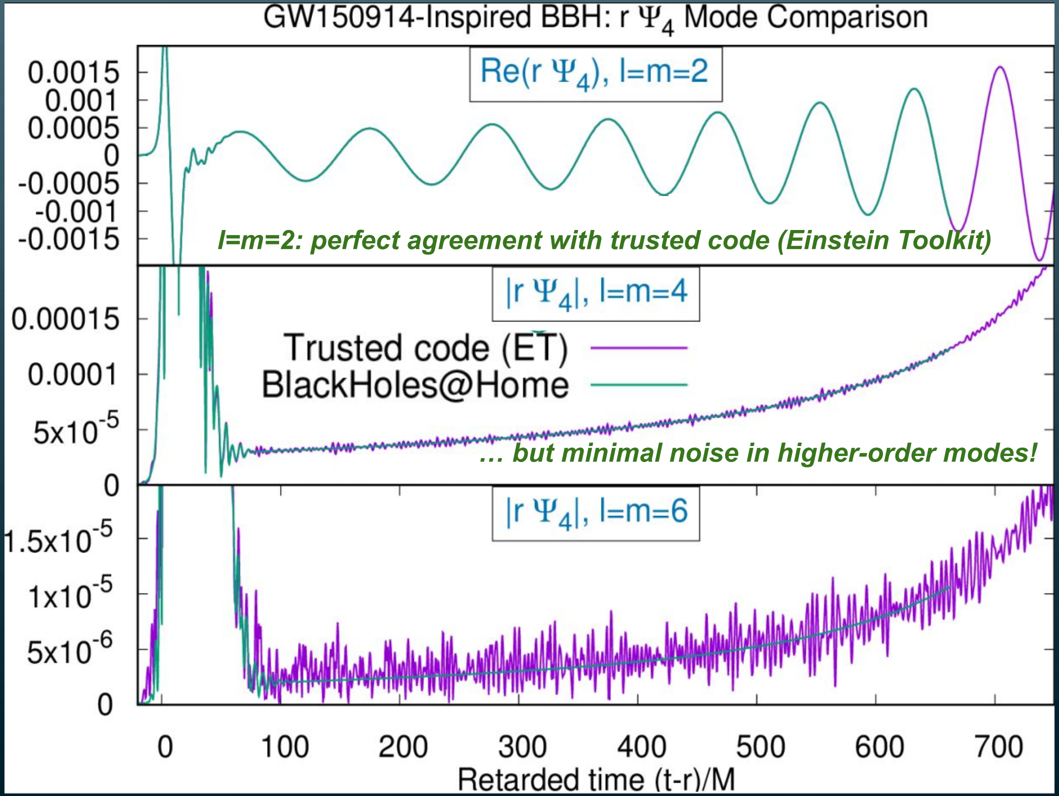
Simulation details

Physical parameters

Initial separation D	10 M
Mass ratio $q = m_1/m_2$	36/29 ~ 1.24
Spin $\chi_1 = a_1/m_1$	0.31
Spin $\chi_2 = a_2/m_2$	-0.46

BlackHoles@Home

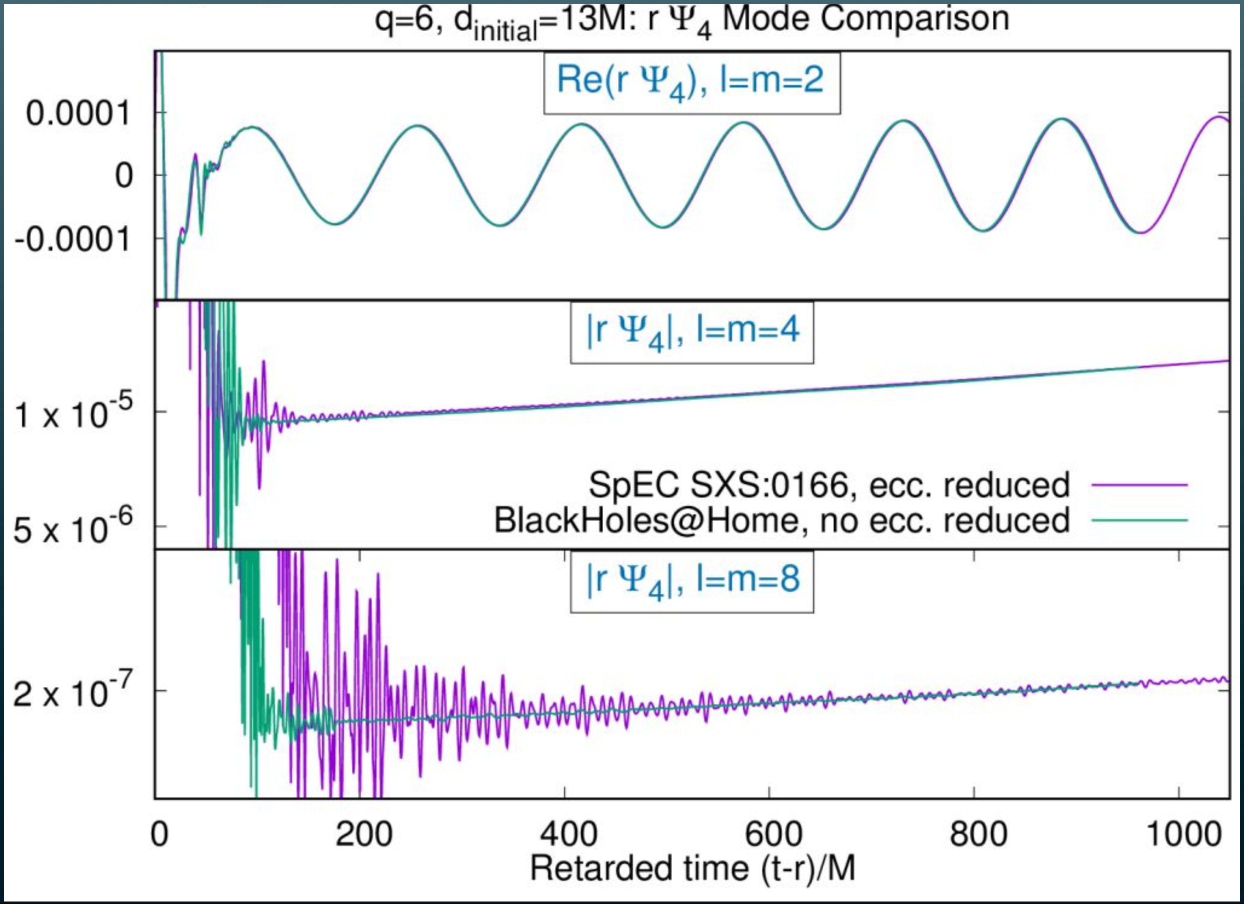




GW noise in AMR references:

1. Zlochower, Ponce, & Lousto, PRD 86, 104056 (2012)
2. Etienne, Baker, Paschalidis, Kelly, Shapiro, Phys. Rev. D 90, 064032 (2014)

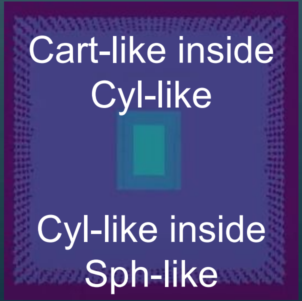
BH@H Code Validation against SpEC: q=6 Example

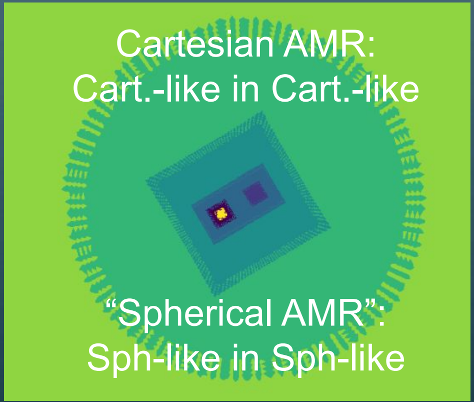


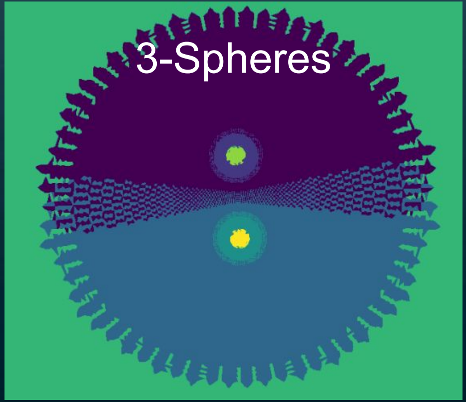
BlackHoles@Home

12-month update

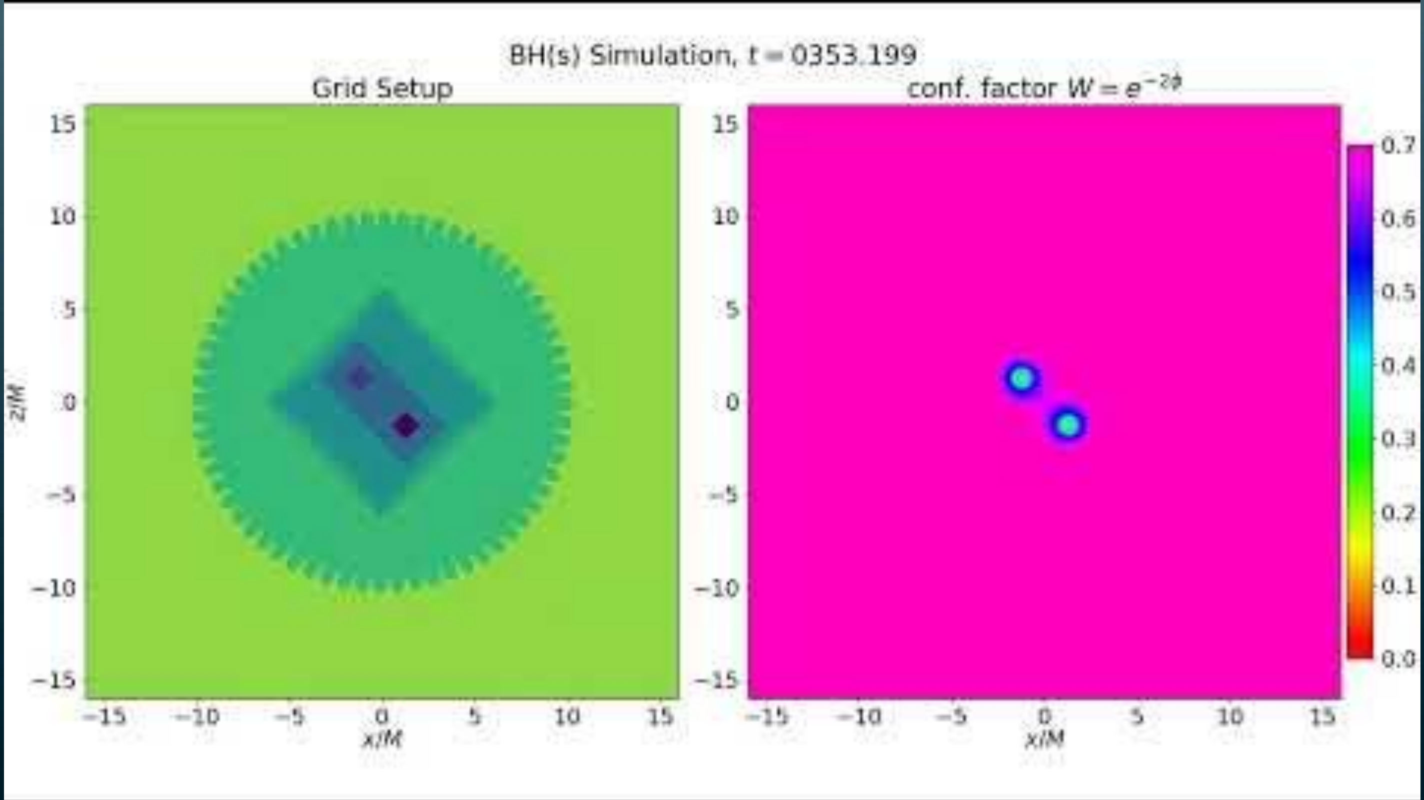
- BH@H completely rewritten
  - Can do mergers!
    - New grids: Months → ***minutes***



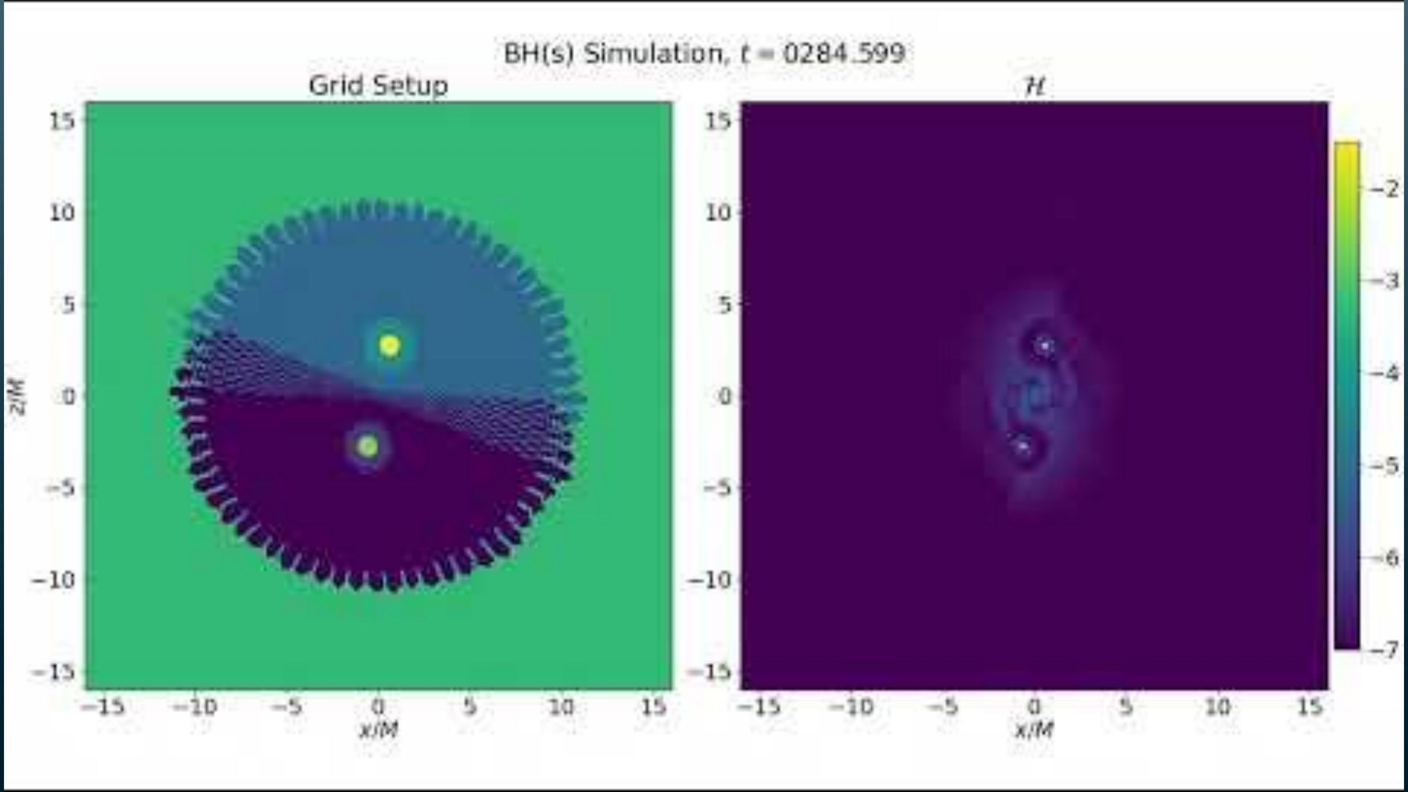




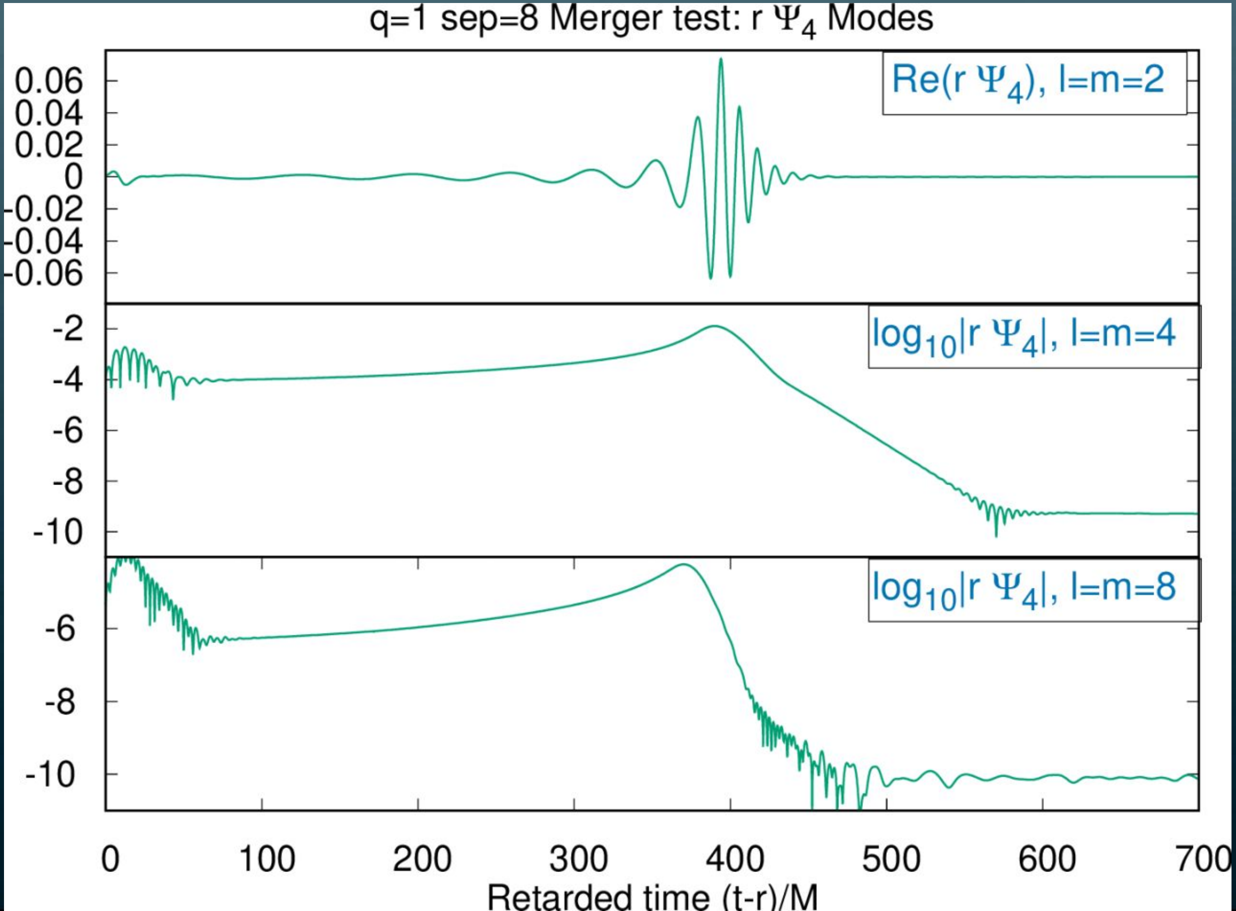
New grid structures example
q=1 nonspinning, initial sep=8M BBH



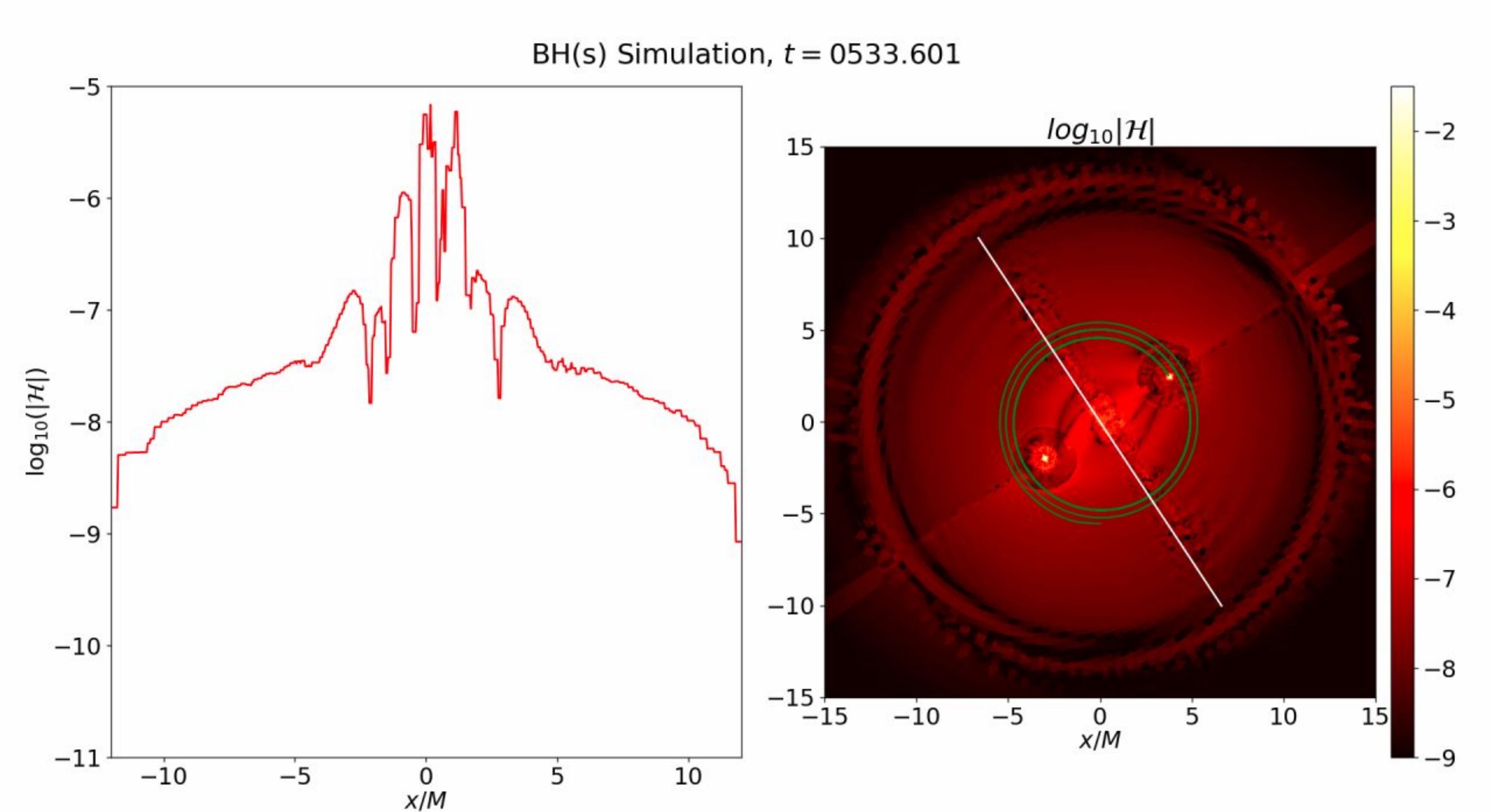
New grid structures example
q=1 nonspinning, initial sep=8M BBH



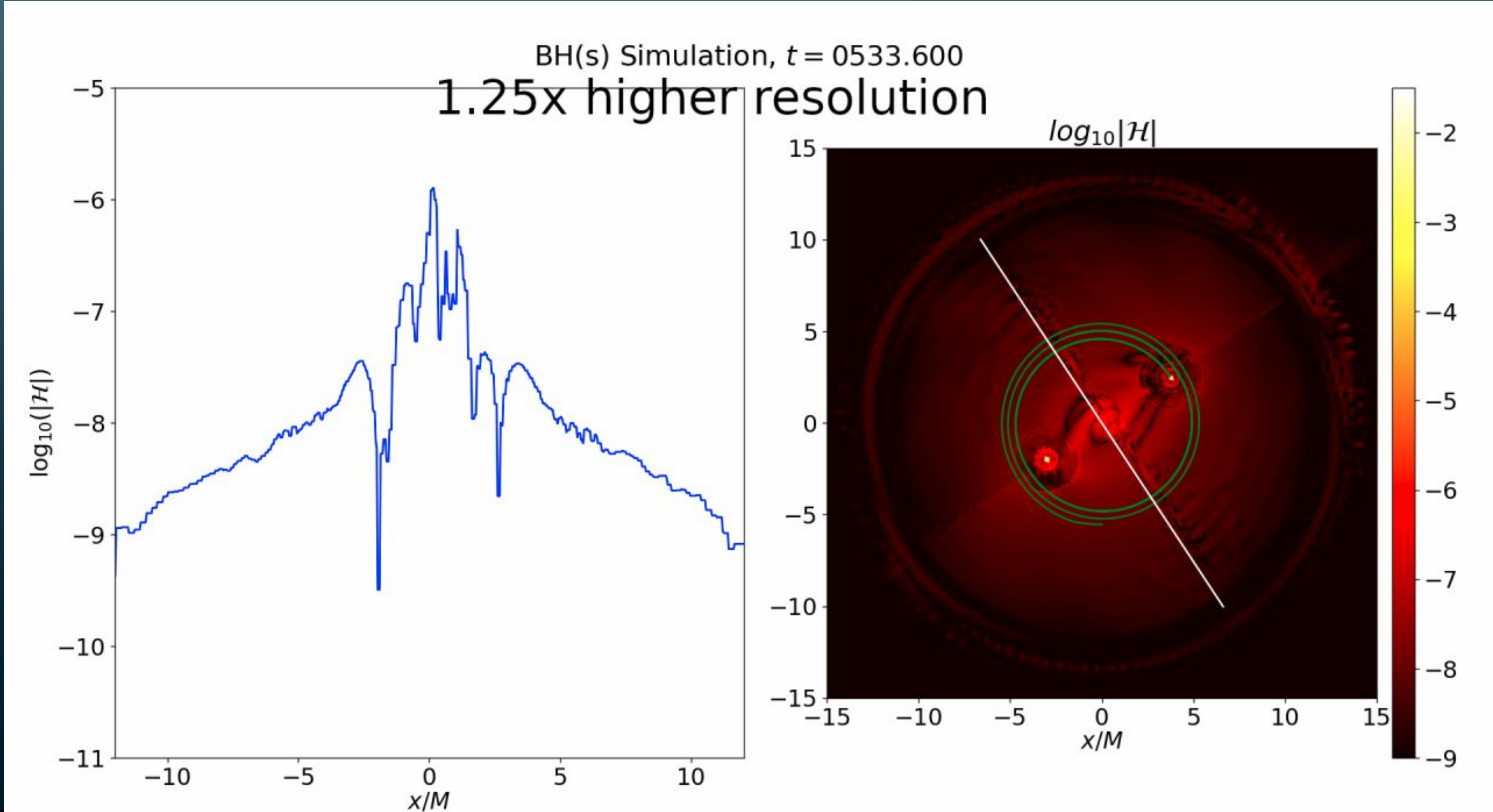
q=1 nonspinning, initial separation=8M



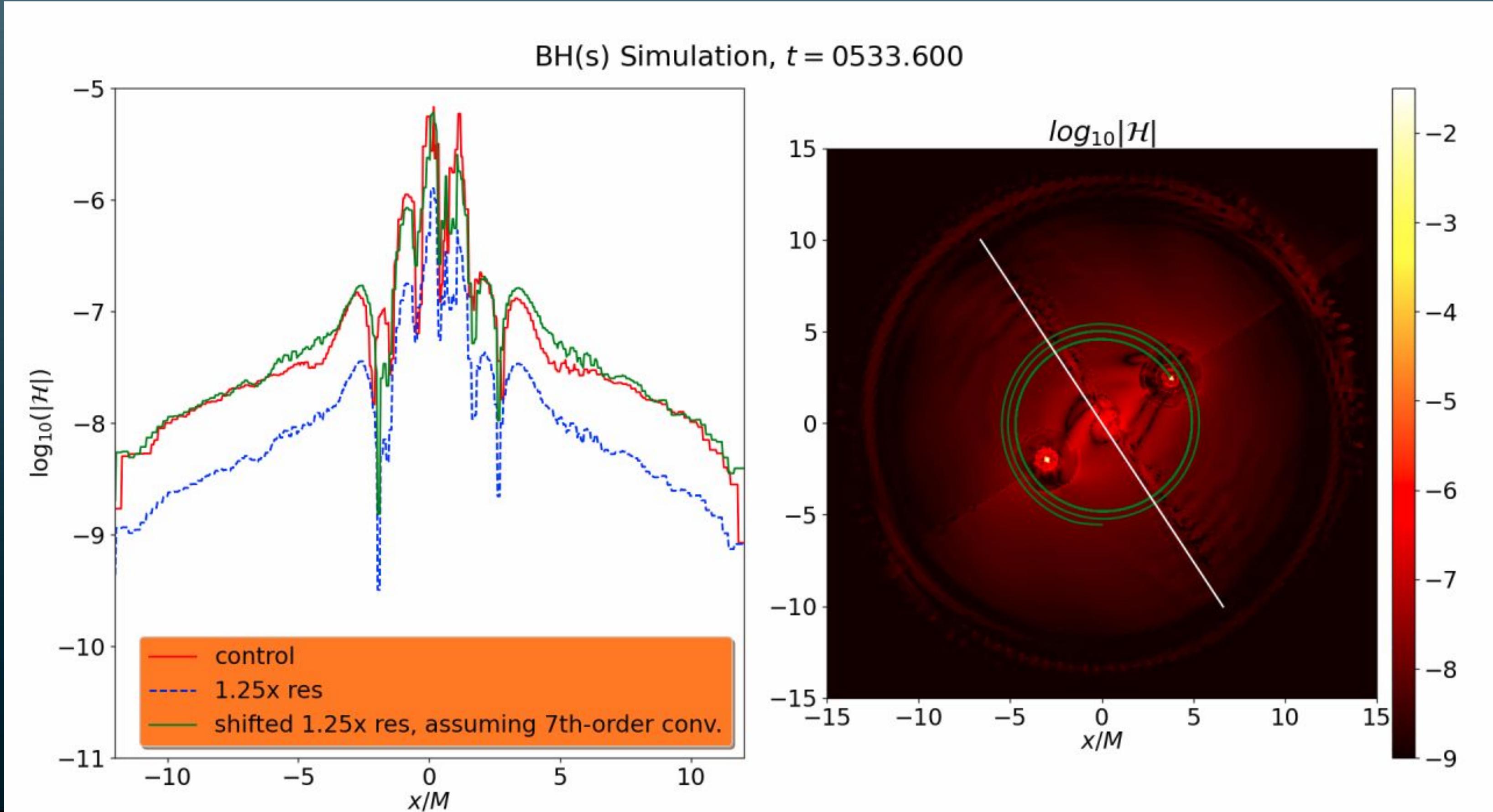
GW150914 ET Gallery example: BlackHoles@Home convergence study



GW150914 ET Gallery example: BlackHoles@Home convergence study



GW150914 ET Gallery example: BlackHoles@Home convergence study



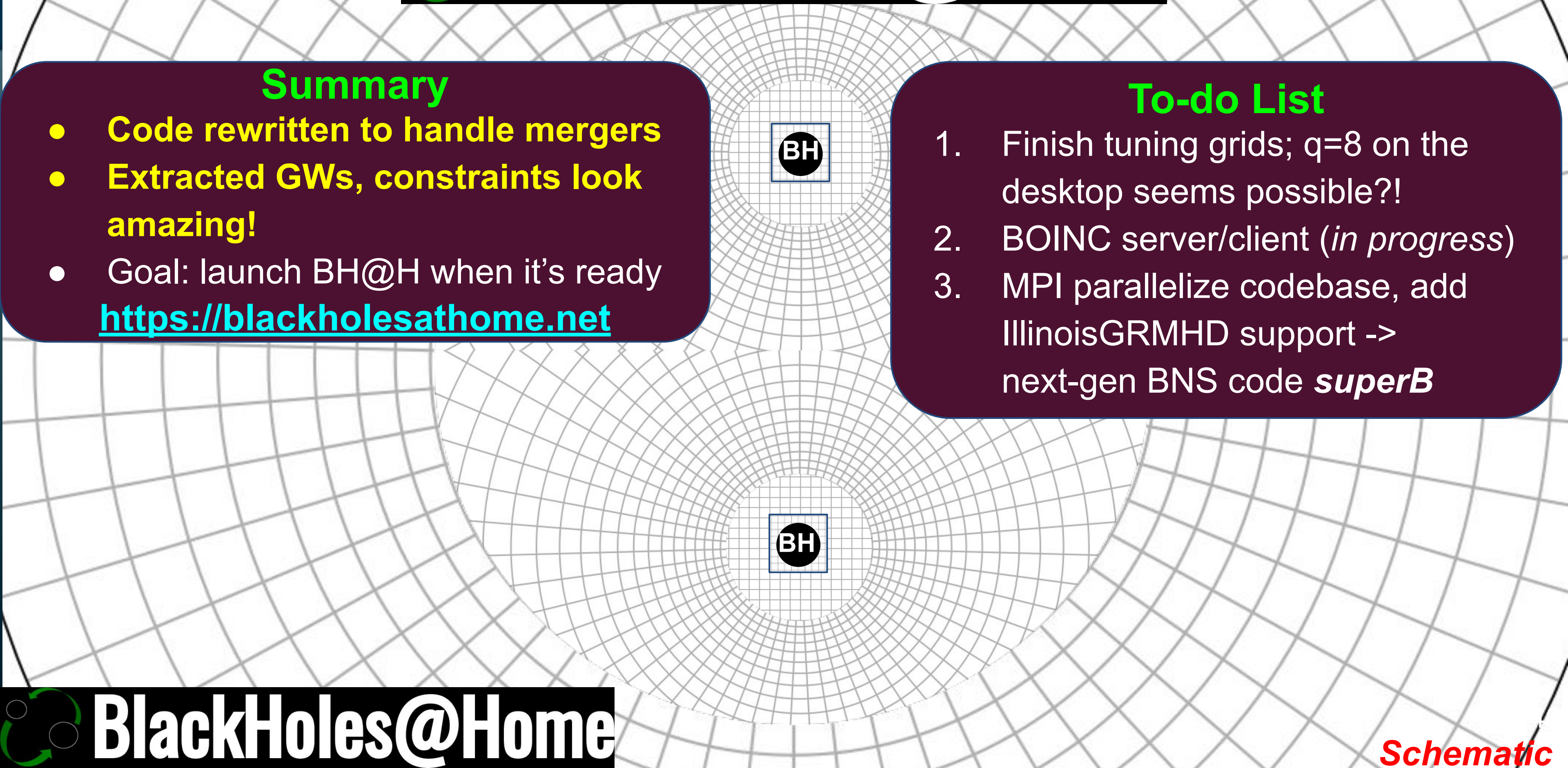
BlackHoles@Home

Summary

- **Code rewritten to handle mergers**
- **Extracted GWs, constraints look amazing!**
- Goal: launch BH@H when it's ready https://blackholesathome.net

To-do List

1. Finish tuning grids; q=8 on the desktop seems possible?!
2. BOINC server/client (*in progress*)
3. MPI parallelize codebase, add IllinoisGRMHD support -> next-gen BNS code ***superB***



BlackHoles@Home

Lessons for the multimessenger era

1. **Diversity in numerical techniques is essential**

   … to building confidence in predictions

Lessons for the multimessenger era

1. **Diversity in numerical techniques is essential**

   … to building confidence in predictions

2. **Diversity in physics means interoperability between codes is essential**

   Just as no person can be an expert in everything, no code can/should do everything

Lessons for the multimessenger era

1. **Diversity in numerical techniques is essential**

   … to building confidence in predictions

2. **Diversity in physics means interoperability between codes is essential**

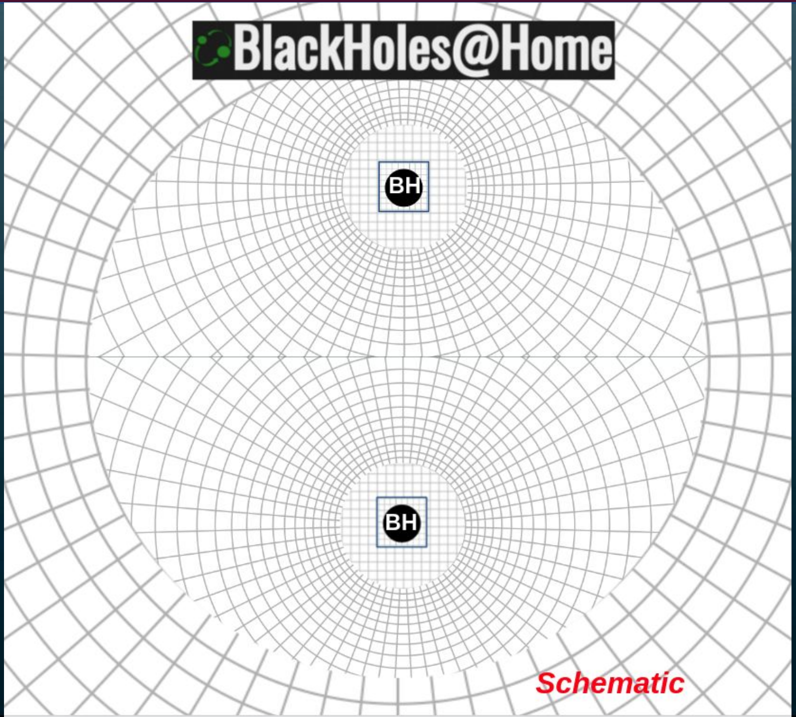
   Just as no person can be an expert in everything, no code can/should do everything

BlackHoles@Home + MANGA

BlackHoles@Home

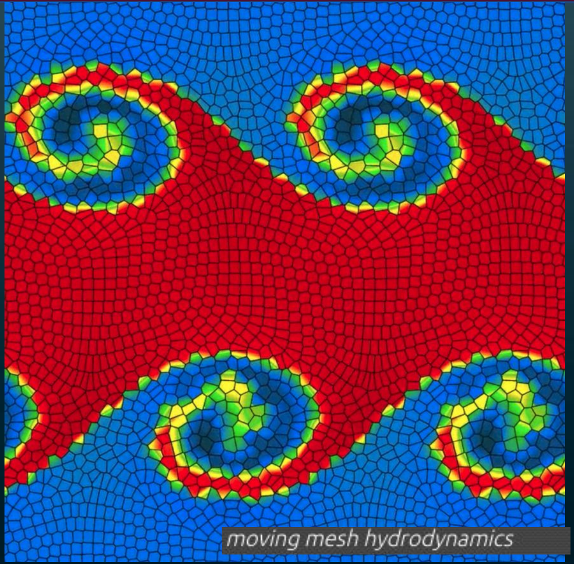
MANGA

NR code with highly efficient grids
(NR = the cheap part)



Moving-mesh Voronoi code

- Has been used to study
  - Common envelope evolution
  - Tidal disruptions
- Supports advanced EOSs, radiation hydro, GRHD in progress!



Pros and Cons of Voronoi Hydrodynamics

Pros

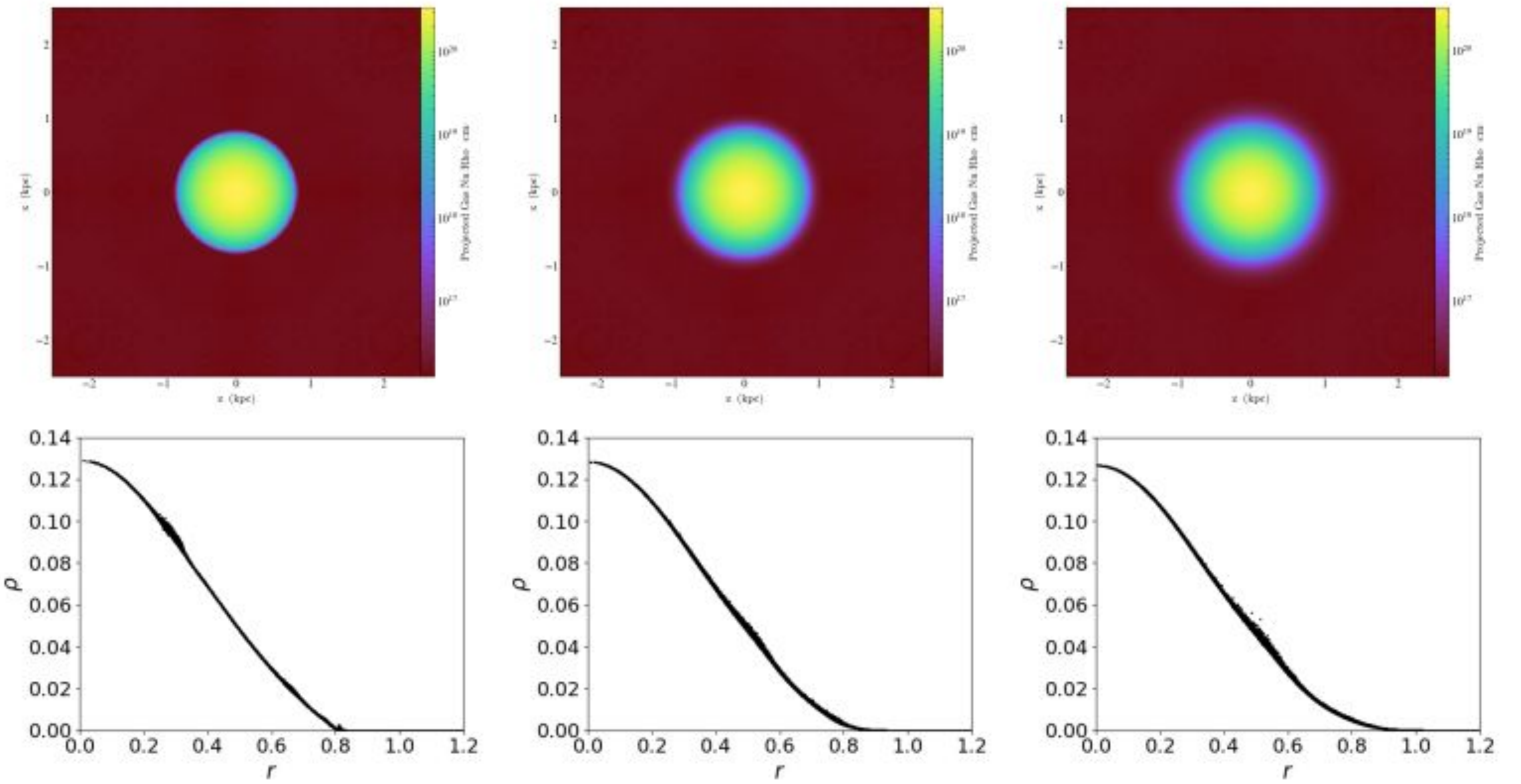
- Far better advection than Eulerian.
- Superior conservation of momentum and angular momentum compared to Eulerian schemes
- Superior shock capturing compared to SPH.
- Better capture of interface instabilities in principle.
- Continuously varying resolution – no factor of 2 or 4 jumps as in AMR.
- Almost anything solvable on Eulerian grids map to Voronoi methods.

Cons

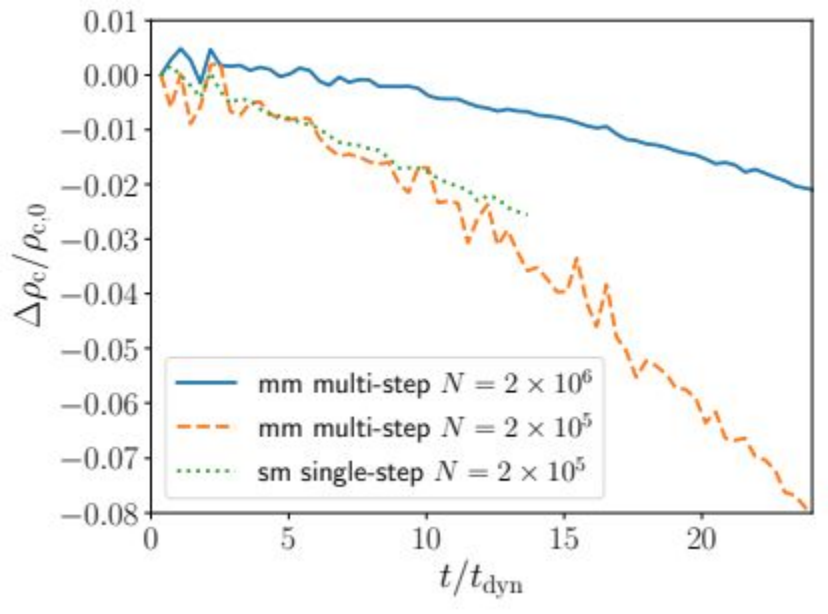
- Much more complex – combination of SPH and Eulerian + computational geometry
- Have to think about the grid (on top of everything else).
- "slower"
- MHD is divergence cleaning or vector potential based – no "staggered" CT scheme exists.
- Might be overkill for many problems

Slide courtesy Phil Chang, lead author of MANGA

General Relativistic Hydrodynamics on a Moving-mesh I: Static Spacetimes

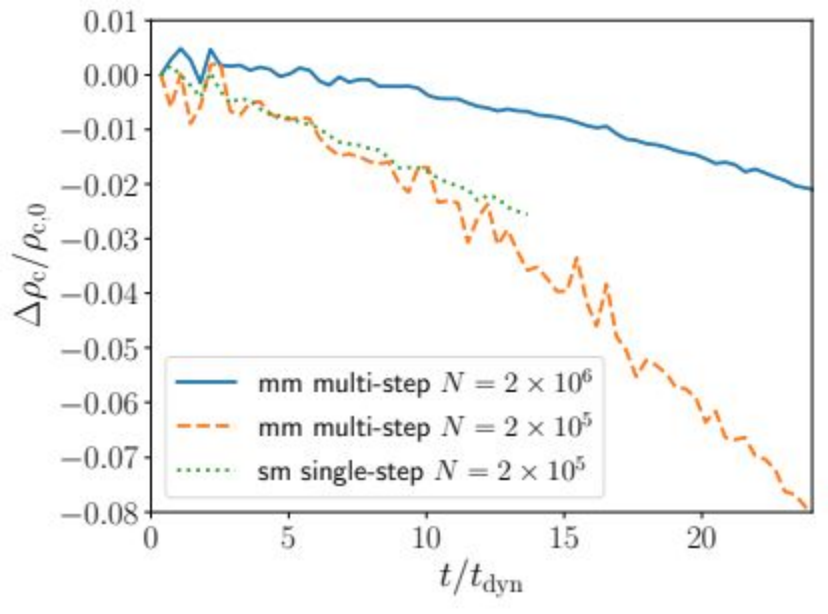


Chang & ZBE: https://arxiv.org/abs/2002.09613 ; MNRAS 496, 1, 2020



- **Code converges as expected with increased number of mesh generating points**
  - **conv order of ~1.75**

Chang & ZBE: https://arxiv.org/abs/2002.09613 ; MNRAS 496, 1, 2020



- Code converges as expected with increased number of mesh generating points
    - conv order of ~1.75

- Current work:
    - Evolve spacetime with NRPy+
- Future work:
    - Couple to BH@H & perform BNS evolutions on moving mesh!
    - BNS with GRMHD, radiation, & advanced EOS

Chang & ZBE: https://arxiv.org/abs/2002.09613 ; MNRAS 496, 1, 2020