### **Gravitational Laboratories for Nuclear Physics**

Nonparametric Astrophysical Inference of the Nuclear Equation of State and Connections to Nuclear Experiment and Theory

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Perimeter Institute for Theoretical Physics

### Key References

- P. Landry and R. Essick, *Nonparametric Inference of the Neutron Star Equation of State from gravitational Wave Observations*, PRD 99, 084049 (2019)
- R. Essick, P. Landry, and D. E. Holz, *Nonparametric Inference of Neutron Star Composition, Equation of State, and Maximum Mass with GW170817*, PRD 101, 063007 (2020)
- P. Landry, R. Essick, and K. Chatziioannou, *Nonparametric Constraints on Neutron Star Matter with Existing and Upcoming Gravitational Wave and Pulsar Observations*, PRD 101, 123007 (2020)
- I. Legred, K. Chatziioannou, R. Essick, S. Han, and P. Landry, *Impact of the PSR J0740+6620 Radius Constraint on the Properties of High-Density Matter*, PRD 104, 063003 (2021)
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- R. Essick, I. Tews, P. Landry, S. Reddy, D. E. Holz, *Direct Astrophysical tests of Chiral Effective Field Theory at Supranuclear Densities*, PRC 102, 055803 (2020)
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Nonparametric Equation of State Inference getting the EoS right

Comparisons with *ab initio* Chiral Effective Field Theory Calculations *connecting astrophysical observations with terrestrial experiments* 

# Nonparametric Equation of State Inference getting the EoS right

Comparisons with *ab initio* Chiral Effective Field Theory Calculations *connecting astrophysical observations with terrestrial experiments* 

### Parametric vs. Nonparametric Inference

Parametric constructions:

- Typically, include a small number of parameters and claim they reproduce proposed EOS reasonably well
- Think of fitting a collection of data with a fixed function

Parametric analyses require the function to be of a specific form which may or may not faithfully represent the data.



### Parametric vs. Nonparametric Inference

Parametric constructions

- only allow for certain types of behavior (set of measure zero), and all expected behavior must be built into the model from the start
- If true EOS is not exactly described by the parameterized model, it can never be exactly recovered



### Parametric vs. Nonparametric Inference

Nonparametric constructions:

- Do not assume a functional form for the EOS a prior
- Think of making a histogram or kernel density estimate instead of using a fixed functional form

Nonparametric analyses assume things about the type of correlations within a function but do not require the function to have any specific form!













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### Nonparametric EoS Inference



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Comparisons with ab initio Chiral Effective Field Theory Calculations





Quantity	$\rm QMC^{(2018)}_{N^2LO,l} \lesssim n_{sat}$		$\boxed{\qquad \text{QMC}_{\text{N}^2\text{LO},l}^{(2018)} \lesssim 2n_{\text{sat}}}$		Marginalized $QMC^{(2018)}_{N^2LO,l}$		Completely Agnostic	
	PSRs+GWs	+NICER	PSRs+GWs	+NICER	PSRs+GWs	+NICER	PSRs+GWs	+NICER
$M_{ m max} \; [M_\odot]$	$2.20^{+0.25}_{-0.20}$	$2.25_{-0.25}^{+0.32}$	$2.18^{+0.20}_{-0.17}$	$2.21^{+0.28}_{-0.22}$	$2.19^{+0.24}_{-0.18}$	$2.24^{+0.31}_{-0.23}$	$2.20^{+0.24}_{-0.18}$	$2.22_{-0.20}^{+0.30}$
$R_{1.4}$ [km]	$11.59^{+1.23}_{-1.19}$	$12.59_{-0.59}^{+0.64}$	$11.21^{+1.23}_{-0.91}$	$12.43_{-0.77}^{+0.53}$	$11.40^{+1.38}_{-1.04}$	$12.54_{-0.63}^{+0.71}$	$10.95^{+2.00}_{-1.37}$	$12.32^{+1.09}_{-1.47}$
$\Lambda_{1.4}$	$291^{+264}_{-175}$	$518^{+208}_{-163}$	$227^{+218}_{-108}$	$465^{+125}_{-177}$	$260^{+270}_{-140}$	$494^{+201}_{-166}$	$228^{+319}_{-134}$	$451^{+241}_{-279}$
$p(n_{ m sat}) \; [{ m MeV/fm^3}]$	$2.13\substack{+0.49\\-0.46}$	$2.27\substack{+0.58 \\ -0.46}$	$2.19\substack{+0.49\\-0.47}$	$2.54^{+0.60}_{-0.49}$	$2.15_{-0.53}^{+0.64}$	$2.42_{-0.66}^{+0.75}$	$1.38^{+2.72}_{-1.31}$	$2.68^{+2.37}_{-2.48}$
$p(2n_{\rm sat}) \ [10 { m MeV/fm^3}]$	$1.62^{+1.94}_{-0.99}$	$3.12^{+2.03}_{-1.42}$	$1.33^{+0.99}_{-0.51}$	$2.61^{+0.94}_{-1.23}$	$1.42^{+1.81}_{-0.84}$	$2.87^{+1.53}_{-1.50}$	$1.10^{+1.88}_{-1.09}$	$2.38^{+1.66}_{-1.83}$
$p(3n_{\rm sat}) [10^2 \mathrm{MeV/fm^3}]$	$0.86\substack{+0.61\\-0.57}$	$1.12\substack{+0.59 \\ -0.40}$	$0.68\substack{+0.51\\-0.45}$	$1.05^{+0.49}_{-0.37}$	$0.79\substack{+0.56\\-0.55}$	$1.08\substack{+0.56 \\ -0.38}$	$0.71^{+0.60}_{-0.56}$	$0.98\substack{+0.59 \\ -0.56}$
$p(4n_{\rm sat}) [10^2 \mathrm{MeV/fm^3}]$	$2.03\substack{+0.90\\-0.74}$	$2.12^{+1.07}_{-0.88}$	$2.00\substack{+0.78\\-0.71}$	$2.08\substack{+0.92\\-0.64}$	$2.02^{+0.87}_{-0.73}$	$2.11^{+1.08}_{-0.69}$	$2.01^{+0.95}_{-0.84}$	$2.11^{+1.11}_{-0.73}$
$p(6n_{\rm sat}) [10^2 \mathrm{MeV/fm^3}]$	$5.35^{+2.57}_{-2.78}$	$4.95_{-3.18}^{+3.05}$	$5.41^{+2.20}_{-2.15}$	$4.99^{+2.86}_{-2.58}$	$5.36^{+2.39}_{-2.53}$	$4.97^{+2.96}_{-2.98}$	$5.71^{+2.57}_{-2.48}$	$5.38^{+3.30}_{-2.66}$

## Comparisons with Terrestrial Nuclear Experiments connecting astrophysical observations with terrestrial experiments

- P. Landry and R. Essick, *Nonparametric Inference of the Neutron Star Equation of State from gravitational Wave Observations*, PRD 99, 084049 (2019)
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We can also extract "nuclear parameters" directly from nonparametric EoS Without the need for "parametrized EoS models"



Essick+ PRL (2021) Essick+ PRC (2021)



astro data can distinguish between

nuclear experiments probe lower densities

## Comparisons with Terrestrial Nuclear Experiments $${\rm Prior}$$



Essick+ PRL (2021) Essick+ PRC (2021)

### Comparisons with Terrestrial Nuclear Experiments $_{\rm Prior}$



Essick+ PRC (2021)

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### **Extra Slides**

### Nonparametric EoS Inference

consider a toy model:

 $\rightarrow$  fitting a 1D function (pressure vs. energy density) without constraints

#### point+slope

$$p(\varepsilon) = p_a + c_s^2(\varepsilon - \varepsilon_a)$$

$$p(\varepsilon) = p_a + \frac{p_b - p_a}{\varepsilon_b - \varepsilon_a} (\varepsilon - \varepsilon_a)$$

 $\vec{p} \sim \mathcal{N}(\vec{\mu}, \Sigma)$ 

$$\Sigma_{ij} = \operatorname{Cov}(p_i, p_j)$$
  
=  $K_{se}(\varepsilon_i, \varepsilon_j) = \sigma^2 \exp\left(-\frac{(\varepsilon_i - \varepsilon_j)^2}{l^2}\right)$ 



only the GP has independent marginal distributions for all pressures

Legred+(2022)

### Nonparametric EoS Inference

consider a toy model:

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#### point+slope

$$p(\varepsilon) = p_a + c_s^2(\varepsilon - \varepsilon_a)$$

$$p(\varepsilon) = p_a + \frac{p_b - p_a}{\varepsilon_b - \varepsilon_a} (\varepsilon - \varepsilon_a)$$

 $\operatorname{GP}$ 

 $\vec{p} \sim \mathcal{N}(\vec{\mu}, \Sigma)$  $\Sigma_{ij} = \text{Cov}(p_i, p_j)$  $= K_{\text{se}}(\varepsilon_i, \varepsilon_j) = \sigma^2 \exp\left(-\frac{(\varepsilon_i - \varepsilon_j)^2}{l^2}\right)$ 



Legred+(2022)

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Map from nonparametric EoS in  $\beta$ -equilibrium to nuclear params describing the energy per particle near nuclear saturation ( $n_0$ : minimum of  $E_{SNM}$ )

 $x = n_p/n$ proton fraction  $E_{\rm nuc}(n,x) = E_{\rm SNM}(n) + (1-2x)^2 S_0(n) + \mathcal{O}(x^4)$ nuclear energy per particle  $=\frac{\varepsilon_{\beta}(n)-\varepsilon_{e}(n,x)}{n}-m_{N}$  $E_{\rm SNM}(n) = E_0 + \frac{1}{2}K_0\left(\frac{n-n_0}{3n_0}\right)^2 + \cdots$ 

 $\mu_n = \mu_p + \mu_e$ 

symmetric-nuclear-matter energy per particle (local min at  $n_0$ )

condition for  $\beta$ -equilib

Map from nonparametric EoS in  $\beta$ -equilibrium to nuclear params describing the energy per particle near nuclear saturation ( $n_0$ : minimum of  $E_{SNM}$ )

 $x = n_p/n$ proton fraction  $E_{\rm nuc}(n,x) = E_{\rm SNM}(n) + (1-2x)^2 S_0(n) +$  $\mathcal{O}(x^4)$ nuclear energy per particle  $= \frac{\varepsilon_{\beta}(n) - \varepsilon_{e}(n, x)}{\varepsilon_{\beta}(n) - \varepsilon_{e}(n, x)}$  $E_{\rm SNM}(n) = E_0 + \frac{1}{2} K_0 \left( \frac{n - n_0}{3n_0} \right)^2 + \cdots$ symmetric-nuclear-matter energy per particle (local min at  $n_{0}$ ) condition for  $\beta$ -equilib

constrained by astro observations (input from nonparametric analysis) measured in the lab (input from terrestrial experiment) modeled as degenerate Fermi gas (input from theory) expressed in terms of derivatives of E<sub>pue</sub>

Essick+ PRL (2021) Essick+ PRC (2021)

$$\mu_i = \frac{dE}{dN_i}$$

Map from nonparametric EoS in  $\beta$ -equilibrium to nuclear params describing the energy per particle near nuclear saturation ( $n_0$ : minimum of  $E_{SNM}$ )

$$\begin{aligned} x &= n_p/n & \text{proton fraction} \\ E_{\text{nuc}}(n,x) &= E_{\text{SNM}}(n) + (1-2x)^2 \widehat{S_0(n)} + \mathcal{O}(x^4) & \text{nuclear energy per particle} \\ &= \frac{\varepsilon_\beta(n) - \varepsilon_e(n,x)}{n} - m_N & \text{symmetric-nuclear-matter} \\ E_{\text{SNM}}(n) &= E_0 + \frac{1}{2} K_0 \left(\frac{n-n_0}{3n_0}\right)^2 + \cdots & \text{symmetric-nuclear-matter} \\ \mu_n &= \mu_p + \mu_e & \text{condition for } \beta\text{-equilib} \end{aligned}$$

constrained by astro observations (input from nonparametric analysis) measured in the lab (input from terrestrial experiment) modeled as degenerate Fermi gas (input from theory) expressed in terms of derivatives of  $E_{nuc}$ 

$$\mu_i = \frac{dE}{dN_i}$$

current  $R_{skin}$  uncertainty

R<sub>skin</sub> uncertainty improved by a factor of 2

hypothetical perfect  $R_{skin}$  measurement

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 $\begin{array}{l} \mbox{nonparametric prior} \\ \mbox{nonparametric astro-only posterior} \\ \chi EFT + astro posterior \\ \mbox{nonparametric astro+R}_{skin} \mbox{ posterior} \\ \chi EFT + astro+R_{skin} \mbox{ posterior} \end{array}$ 

improved precision in nuclear experiments is unlikely to affect our knowledge of NS radii without improved theoretical calculations