Evgeny Epelbaum, Ruhr University Bochum

Nuclear Hamiltonians for Advancing Nuclear Physics and Beyond
April 27 — May 29, 2026, INT Seattle, USA

Power counting (yet again) and implications for 3-nucleon forces

Part I: A quick tutorial on the renormalization debate in chiral EFT

Based on: EE, Gegelia, EPJA 41 (09); EE, Gegelia, Meißner NPB 925 (17); EE et al. EPJA 55 (19); EE et al. EPJA 56 (20); EE et al. FBS 62 (21); EE et al. FBS 63 (22); Gasparyan, EE, 2022-25; …

Also: KITP Program: Frontiers in Nuclear Physics, Aug 22 - Nov 4, 2016:

— Nuclear EFTs: the crux of the matter, part I (Mike Birse)
— Nuclear EFTs: the crux of the matter, part II (EE)
https://online.kitp.ucsb.edu/online/nuclear16/

Part II: Scrutiny of the new class of 3N forces

EE, Gasparyan, Gegelia, Hog, Krebs, PRC 113 (26)

NON-PERTURBATIVE RENORMALIZATION IN FUNDAMENTAL AND EFFECTIVE THEORIES

30 March - 3 April 2026

ECT* - Villa Tambosi, Villazzano

ORGANIZERS

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MAIN TOPICS

- Renormalization in fundamental and effective theories
- Perturbative versus non-perturbative renormalization
- The role played by the renormalization scale and running couplings
- Symmetry-preserving regularizations

Things we (hopefully) all agree on

- EFT is a model-independent method for analyzing low-energy (scale separation!) phenomena in and beyond the SM

- Renormalization is a key concept in QFT and EFT

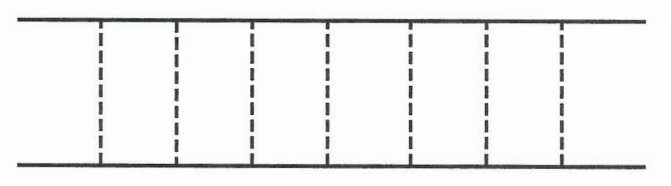
- Heavy-baryon ChPT is an example of a consistent EFT

- In nuclear EFTs, it is not necessary to treat higher-order corrections to potentials non-perturbatively when solving the Schrödinger equation

- NDA for a potential $\neq$ NDA for the amplitude

- The tensor part of the 1π-exchange potential is singular ($\sim 1/r^3$) $\Rightarrow$ an infinite number of UV divergences generated from iterations in every spin-1 partial wave

E.g.:

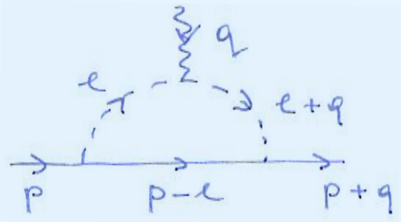


$\longleftarrow$ $(mq)^6 \ln \frac{\Lambda}{M_\pi}, \quad m^2\Lambda^2(mq)^4, \quad \ldots$

see, e.g., Martin Savage, nucl-th/9804034

Things I thought we agree on, but apparently not all do…

- **Lorentz-invariant BChPT (IR, EOMS): A consistent EFT?**



Motivation: The HB expansion for the scalar FFs is known to break down in a vicinity of $t = 4M_\pi^2$


A Primer for Chiral Perturbation Theory


EFFECTIVE FIELD THEORIES

— Infrared Regularization (IR) Tang, Ellis 1996; Becher, Leutwyler 1999

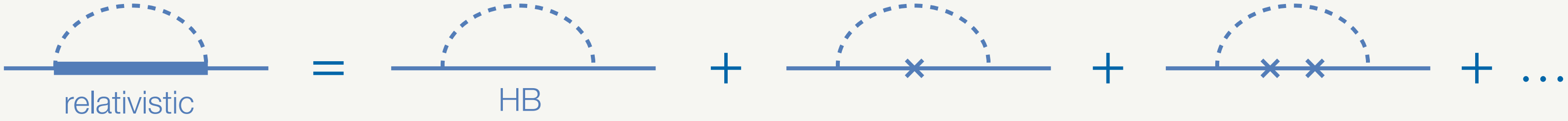
— Extended On-Mass-Shell (EOMS) scheme Gegelia, Japaridze 1999; Fuchs, Gegelia, Japaridze, Scherer 2003

Resummation of 1/m corrections in the IR scheme can be implemented via:

$$i\frac{\not p+m}{p^2-m^2+i\epsilon} = i\frac{\not p+m}{2mv\cdot l+l^2+i\epsilon} = i\frac{\not p+m}{2mv\cdot l+i\epsilon}\,\frac{1}{1+\frac{l^2}{2mv\cdot l+i\epsilon}}$$

use: $p = mv + l$

$$= \frac{\not p+m}{2m}\,\frac{i}{v\cdot l+i\epsilon}\left[1+i\frac{l^2}{2m}\,\frac{i}{v\cdot l+i\epsilon}+\left(i\frac{l^2}{2m}\,\frac{i}{v\cdot l+i\epsilon}\right)^2+\dots\right]$$



Contrary to HB, explicitly scale-depend results, which, however, agree with PC ⇒ consistent

Things we seem to disagree on

- **The meaning of RG invariance beyond perturbation theory (chiral EFT)**

Different opinions, most radical:

$$\frac{\Lambda}{T^{(\mathcal{V})}(Q,\Lambda)}\frac{\mathrm{d}T^{(\mathcal{V})}(Q,\Lambda)}{\mathrm{d}\Lambda} = \mathcal{O}\left(\frac{Q^{\mathcal{V}+1}}{\Lambda_b^{\mathcal{V}}\Lambda}\right)$$

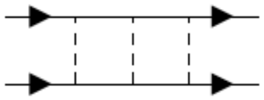
Hammer, König, van Kolck, RMP 92 (2020) 025004

This implies the existence of $\lim_{\Lambda\to\infty} T^{(\nu)}(Q,\Lambda)$ $\Rightarrow$ UV stability of the amplitude for arbitrary large Λ must be required to claim RG invariance.

Appears problematic for several reasons:

- „technically“ impossible to realize beyond LO (exceptional cutoffs) unless one allows renormalization conditions to vary with Λ Gasparyan, EE 2023; Peng, Long, Xu 2024, 2025
- First resum, then $\Lambda \to \infty$ $\neq$ First $\Lambda \to \infty$, then resum
  EE, Gegelia, EPJA 41 (2009); EE et al. EPJA 52 (2019); EPJA 56 (2020); FBS 62 (2021); FBS 63 (2022)

- Spin triplet waves with attractive 1π-exchange tensor potential (singular potential ~-1/r^3) : $^3P_0$, … Need *derivative* contact terms at leading order



Vincenzo's talk

Things we seem to disagree on

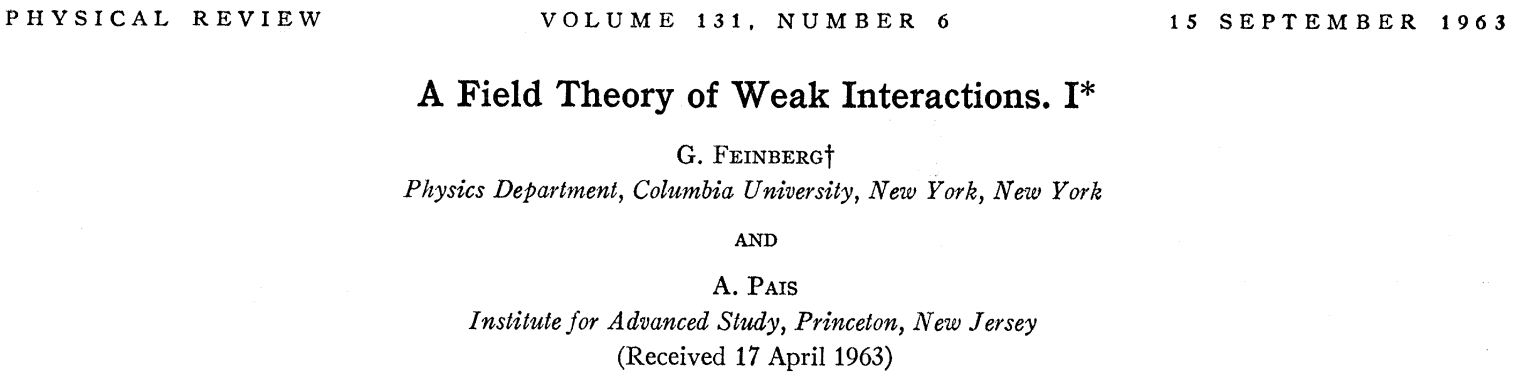
This remark is based on a "theorem", which as far as I know has never been proven, but which I cannot imagine could be wrong. […] This can be put more precisely in the context of perturbation theory: if one writes down the most general possible Lagrangian, including *all* terms consistent with assumed symmetry principles, and then calculates matrix elements with this Lagrangian to any given order of perturbation theory, the result will simply be the most general possible S-matrix consistent with perturbative unitarity, analyticity, cluster decomposition, and the assumed symmetry properties.

Weinberg, Physica 96A (1979) 327; see also Leutwyler, Annals Phys. 235 (1994) 165

Things we seem to disagree on

This remark is based on a "theorem", which as far as I know has never been proven, but which I cannot imagine could be wrong. […] This can be put more precisely ~~in the context of perturbation theory:~~ if one writes down the most general possible Lagrangian, including *all* terms consistent with assumed symmetry principles, and then calculates matrix elements with this Lagrangian ~~to any given order of perturbation theory,~~ by enforcing UV-finiteness with some manifestly nonperturbative prescription the result will simply be 🤔

This is actually pretty much in the spirit of peratization:



PHYSICAL REVIEW VOLUME 131, NUMBER 6 15 SEPTEMBER 1963

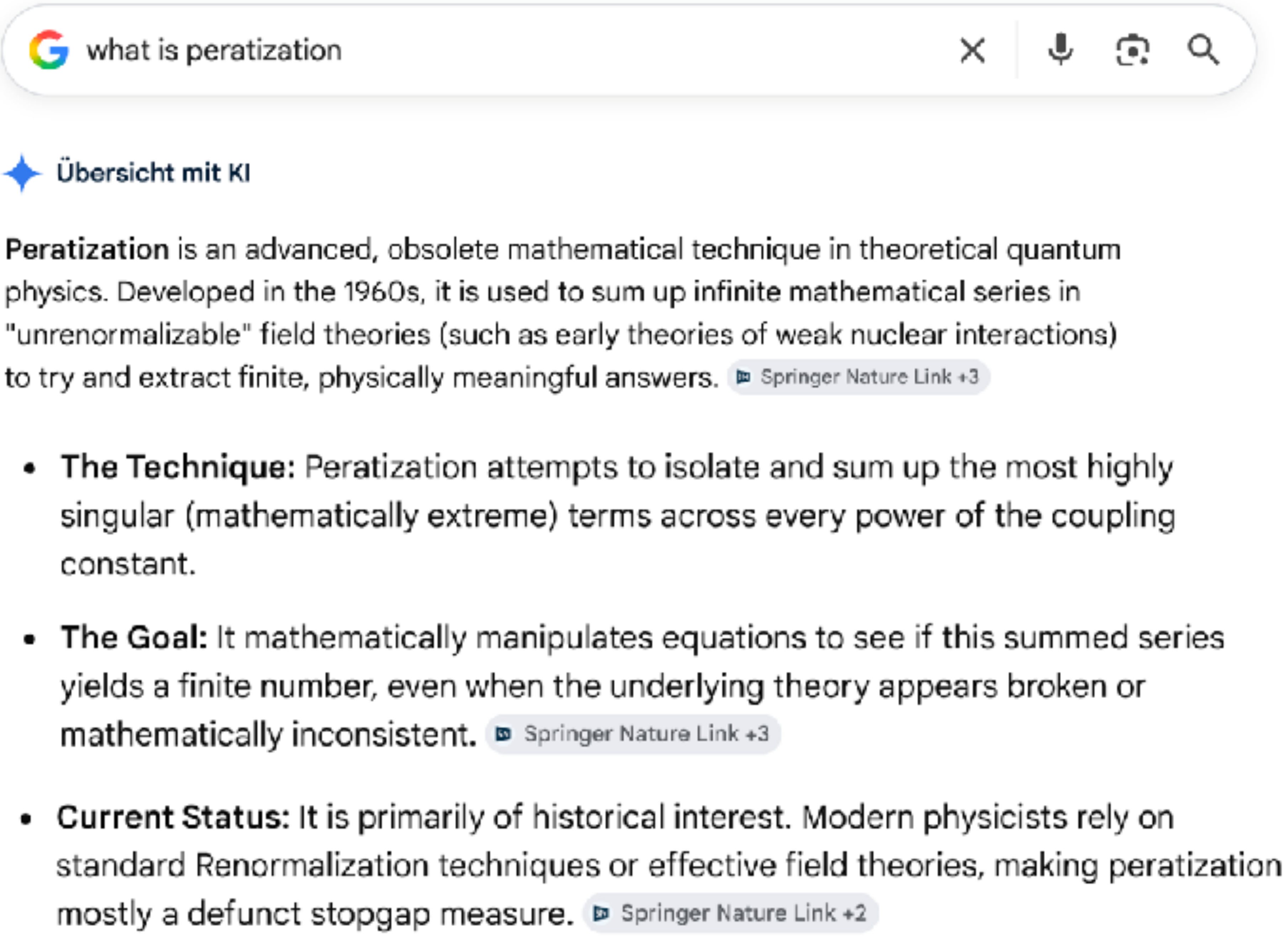
A Field Theory of Weak Interactions. I*

G. FEINBERG†
Physics Department, Columbia University, New York, New York

AND

A. PAIS
Institute for Advanced Study, Princeton, New Jersey
(Received 17 April 1963)

„…We have taken a theory which is unrenormalizable by standard techniques, and shown that a set of graphs which are divergent in the perturbation expansion can be summed to give final results.“



what is peratization

Übersicht mit KI

Peratization is an advanced, obsolete mathematical technique in theoretical quantum physics. Developed in the 1960s, it is used to sum up infinite mathematical series in "unrenormalizable" field theories (such as early theories of weak nuclear interactions) to try and extract finite, physically meaningful answers. Springer Nature Link +3

- **The Technique:** Peratization attempts to isolate and sum up the most highly singular (mathematically extreme) terms across every power of the coupling constant.
- **The Goal:** It mathematically manipulates equations to see if this summed series yields a finite number, even when the underlying theory appears broken or mathematically inconsistent. Springer Nature Link +3
- **Current Status:** It is primarily of historical interest. Modern physicists rely on standard Renormalization techniques or effective field theories, making peratization mostly a defunct stopgap measure. Springer Nature Link +2

Things we seem to disagree on

This remark is based on a "theorem", which as far as I know has never been proven, but which I cannot imagine could be wrong. […] This can be put more precisely ~~in the context of perturbation theory:~~ if one writes down the most general possible Lagrangian, including *all* terms consistent with assumed symmetry principles, and then calculates matrix elements with this Lagrangian ~~to any given order of perturbation theory,~~ by enforcing UV-finiteness with some manifestly nonperturbative prescription the result will simply be 🤔 the most general possible S-matrix consistent with ~~perturbative~~ unitarity, analyticity, cluster decomposition, and the assumed symmetry properties (?)

If true — not allowed to fail !

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the most general possible S-matrix consistent with ~~perturbative~~ unitarity, analyticity, cluster decomposition, and the assumed symmetry properties (?)

If true — not allowed to fail !

Solvable counter-examples:

- „Chiral EFT" with a separable long-range interaction EE, Gegelia, EPJA 41 (2009)
  (see, however, Bira's response in van Kolck, Front. in Phys. 8 (2020))
- π-less EFT for fine-tuned P-wave systems EE, Gegelia, Huesmann, Meißner, Ren, Few Body Syst. 62 (2021)
- „Chiral EFT" with a local long-range force in 5d EE, Gasparyan, Gegelia, Krebs, Few Body Syst. 63 (2022)

See also Frank, Land, Spector, RMP 43 (1971) for a comprehensive discussion of successes and failures of peratization and related techniques.

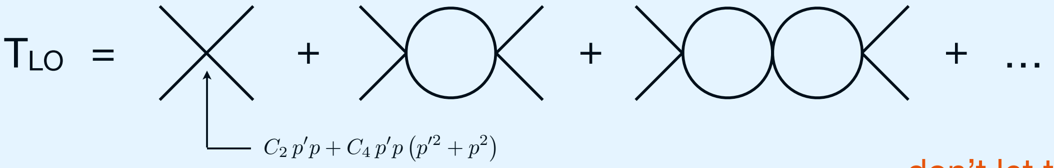
Fine-tuned P-wave systems in halo-EFT

EE, Gegelia, Huesmann, Meißner, Ren, FBS 62 (2021)

$$T(k) \propto \frac{1}{k\cot\delta - ik} \simeq \frac{k^2}{-\frac{1}{a} + \frac{1}{2}rk^2 + v_2k^4 + \ldots - ik^3}$$

with $1/a \sim M_{\rm lo}^2 M_{\rm hi}, \quad r \sim M_{\rm hi}$

Bedaque, Hammer, van Kolck 2003



don't let the artifacts of incomplete renormalization decide for you on the result using $\Lambda \gg \Lambda_b$

Implicit renormalization approach after fixing $C_{2,4}(\Lambda)$ from a, r:

$$k^3\cot\delta = -\frac{1}{a} + \frac{1}{2}rk^2 - \frac{k^4}{2\pi}\underbrace{\boxed{\left(\frac{3(4\Lambda+\pi r)^2}{6\pi a^{-1} - 4\Lambda^3 + 3k^2(4\Lambda + \pi r)} + \frac{2}{k}\ln\frac{\Lambda - k}{\Lambda + k}\right)}}_{\text{only partially renormalized (not all c.t. included…)}}$$

Wigner bound: $C_{2,4} \in \mathbb{R} \quad \Rightarrow \quad r \leq \frac{5\pi}{a^2\Lambda^5} - \frac{20}{3a\Lambda^2} - \frac{16\Lambda}{9\pi} \lesssim -\frac{16\Lambda}{9\pi}$

$\Rightarrow$ choosing $\Lambda \gg M_{\rm hi}$ is incompatible with the assumed scenario!

In contrast, choosing $\Lambda \sim M_{\rm hi}$ works fine.

Notice: Both the dimer-field and explicit renormalization by subtractions have no issues…

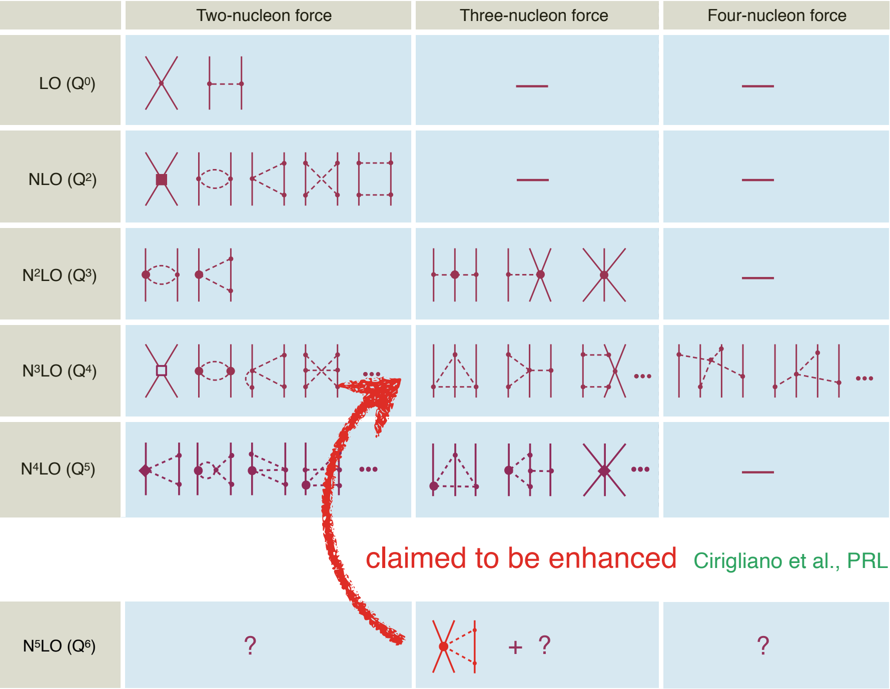
Summary of our point of view

EE, Gasparyan, Gegelia, Huesmann, Krebs, Meißner, Ren

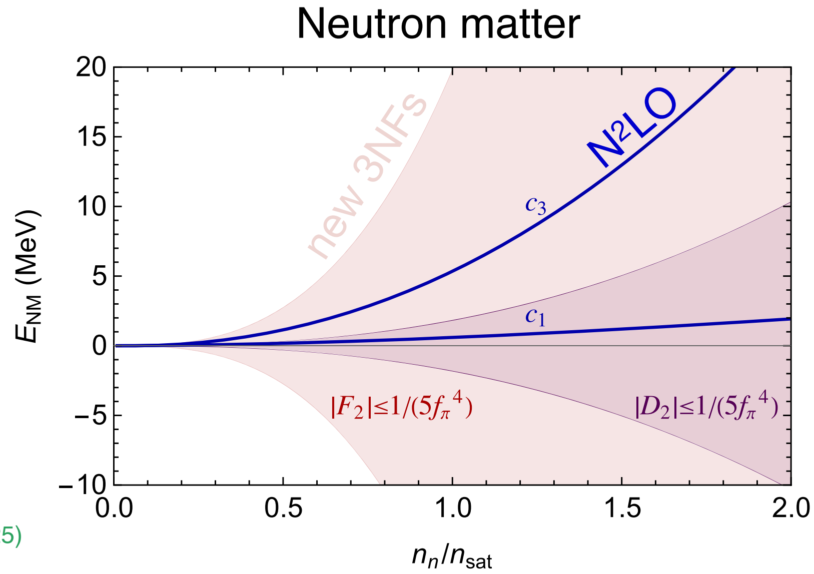
- Singular potentials (like $1/r^3$ OPEP) do not provide a useful concept in chiral EFT as they are not supposed to yield valid approximations for $r \lesssim \Lambda_b^{-1}$. The non-trivial predictive power of chiral EFT comes from chiral symmetry, which governs the $\Lambda_b^{-1} \ll r \sim M_\pi^{-1}$ behavior of the interactions — this is what we are after!
- If not all c.t.'s needed to renormalize perturbative series (e.g., ChEFT) can be included, keep $\Lambda \sim \Lambda_b$ instead of attempting self-adjoint extensions of the Hamiltonian for $\Lambda \gg \Lambda_b$ (for which I see no justification in an EFT).
- Observables must be RG-invariant only up to higher-order corrections in Q/Λ_b. This point of view seems to be shared by some (many?) Grießhammer EPJA 56 (2020); Peng, Long, Xu, PRC 112 (2025); …
- Power counting is controlled by the size of finite pieces in the Lagrangian.
- It would be desirable to have a larger window of accessible cutoff values. May be possible by using partly perturbative approaches. However, don't expect this to improve convergence…
- Appreciate diagnostics using Bayesian statistics.

Scrutiny of the new class of 3NFs

EE, A.M. Gasparyan, J. Gegelia, D. Hog, H. Krebs, PRC 113 (2026) 044005

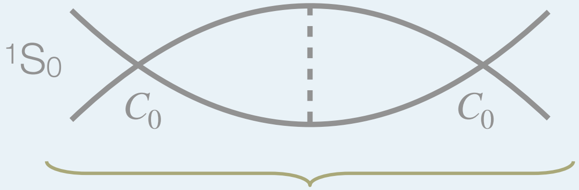


claimed to be enhanced Cirigliano et al., PRL135 (2025)



The arguments of Cirigliano et al.

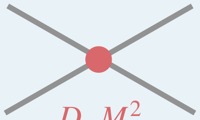
Vincenzo Cirigliano, Maria Dawid, Wouter Dekens, and Sanjay Reddy, PRL 135 (2025) 022501



LO in Weinberg's scheme

← logarithmic divergence $\propto M_\pi^2$

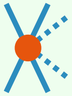
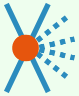
$\Rightarrow$ include contact term at LO →

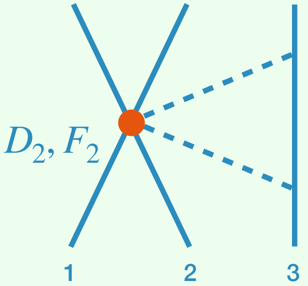


"The above implies that D_2 is needed at LO in approaches to chiral EFT that aim to ensure regulator independence."

$$D_2(\mu) \;=\; \frac{g_A^2 m^2}{64\pi^2 F_\pi^2}\big(C_0(\mu)\big)^2 \ln\frac{\mu}{\mu_0} \;\approx\; 0.27\,\big(C_0(\mu)\big)^2 \;\approx\; 4.2\,\mathrm{fm}^4$$

Cirigliano et al.

In chiral EFT, $D_2M_\pi^2$ , comes together with $D_2M_\pi^2$ , $D_2M_\pi^2$ , …. $\Rightarrow$ enhanced many-body forces



$$V \;=\; (1-\vec\sigma_1\cdot\vec\sigma_2)\left[v_{D_2}(q_3)+v_{F_2}(q_3)\right] \;+\; \text{permutations}$$

where
$$v_{D_2}(q) \;=\; \frac{3g_A^2 D_2 M_\pi^2}{256\pi F_\pi^4}(2M_\pi^2+q^2)\,A(q),$$

$$v_{F_2}(q) \;=\; \frac{3g_A^2 F_2}{512\pi F_\pi^4}(2M_\pi^2+q^2)^2\,A(q)$$

calculated using DimReg

← $\frac{1}{2q}\arctan\frac{q}{2M_\pi}$

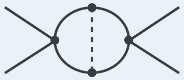
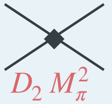
Our response

EE, A.M. Gasparyan, J. Gegelia, D. Hog, H. Krebs, PRC 113 (2026) 044005

- The scaling and values of LECs like C_0 and D_2 are scheme dependent.

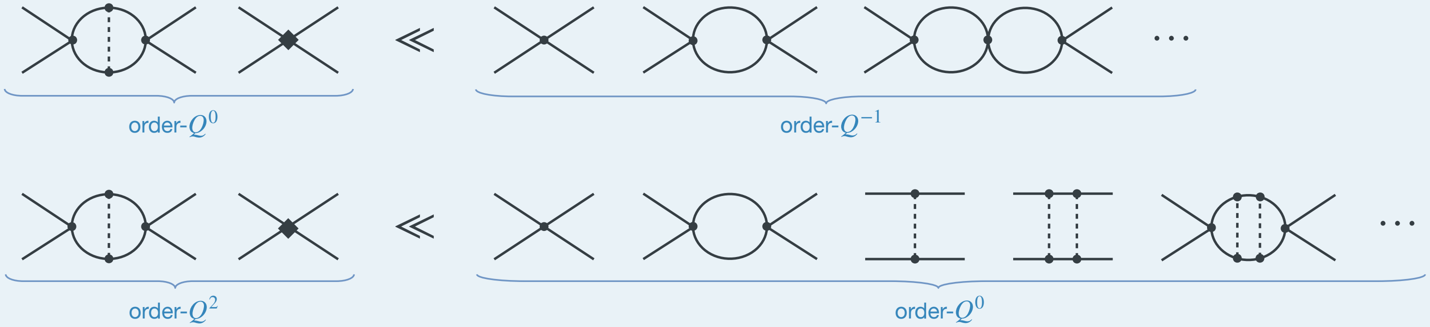
KSW: $C_0 \sim \mathcal{O}(q^{-1}),\ \underbrace{D_2 \sim \mathcal{O}(q^{-2})},\ \ldots$ **Weinberg**: $C_0 \sim \mathcal{O}(1),\ D_2 \sim \mathcal{O}(1),\ \ldots$

$\Rightarrow\ D_2 M_\pi^2 \sim \mathcal{O}(1)$ as assumed by Cirigliano et al.

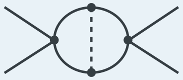
- Removing the log-divergence of  does, of course, require the introduction of 

However, in both KSW & W schemes, these contributions are sub-leading:

KSW:



W:

Therefore, keeping  $\sim \frac{g_A^2}{4(4\pi F_\pi)^2} m^2 M_\pi^2 C_0^2(\mu) \Big[\ln \nu + f\left(p/M_\pi\right) \Big]$ alone does **not** violate RG invariance at LO (unless ν is chosen meaninglessly large) — cf. IR or EOMS BChPT.

Our response

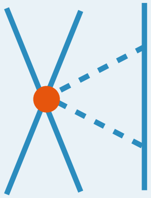
EE, A.M. Gasparyan, J. Gegelia, D. Hog, H. Krebs, PRC 113 (2026) 044005

- To estimate D_2 (and F_2), we follow the approach of Cirigliano et al. and use the coefficient of the Log:

$$|D_2|, |F_2| \sim \frac{g_A^2 m^2}{64\pi^2 F_\pi^2} C_0^2$$

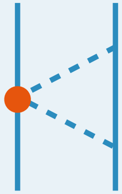
Using the SMS N⁴LO⁺ potentials, we estimate $|D_2|, |F_2| \leq 1\ \mathrm{fm}^4$ significantly smaller than the estimate of $|D_2|, |F_2| \leq 4.2\ \mathrm{fm}^4$ by Cirigliano et al.

- In the 2N case, 2π-exchange contributions are known to suffer from severe enhancements. One then should be careful with DimReg, which may lead to artificially enhanced short-range contributions that are scheme-dependent.



$$V_{3N}^{(Q^6)} = \frac{3g_A^2}{512\pi F_\pi^4}(2M_\pi^2 + q^2)\big[2D_2 M_\pi^2 + F_2(2M_\pi^2 + q^2)\big]A(q) + \ldots$$

Cirigliano et al., PRL 135 (25)



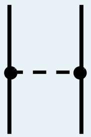
$$V_{2N}^{(Q^3)} = -\frac{3g_A^2}{16\pi F_\pi^4}(2M_\pi^2 + q^2)\big[4c_1 M_\pi^2 - c_3(2M_\pi^2 + q^2)\big]A(q) + \ldots$$

Kaiser et al., NPA 625 (97); EE et al., NPA 637 (98)

It is thus instructive to first look at the well-understood long-range NN force…

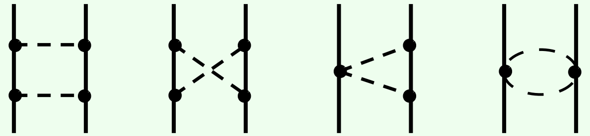
Pion exchange NN potential

Leading order (Q^0)



$$V_{1\pi}^{(0)} = -\frac{g_A^2}{4F_\pi^2}\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2\frac{\vec{\sigma}_1\cdot\vec{q}\,\vec{\sigma}_2\cdot\vec{q}}{q^2+M_\pi^2}$$

Next-to-leading order (Q^2)

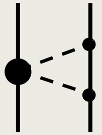


$$L(q) = \frac{\sqrt{q^2+4M_\pi^2}}{q}\ln\frac{\sqrt{q^2+4M_\pi^2}+q}{2M_\pi}$$

$$\begin{aligned}V_{2\pi}^{(2)}(q) = & -\frac{1}{384\pi^2F_\pi^4}\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2\left[4M_\pi^2(5g_A^4-4g_A^2-1)+q^2(23g_A^4-10g_A^2-1)+\frac{48g_A^4M_\pi^4}{4M_\pi^2+q^2}\right]L(q)\\ & -\frac{3g_A^4}{64\pi^2F_\pi^4}\left(\vec{\sigma}_1\cdot\vec{q}\,\vec{\sigma}_2\cdot\vec{q}-\vec{\sigma}_1\cdot\vec{\sigma}_2\,q^2\right)L(q)\end{aligned}$$

Next-to-next-to-leading order (Q^3)

strong numerical enhancement

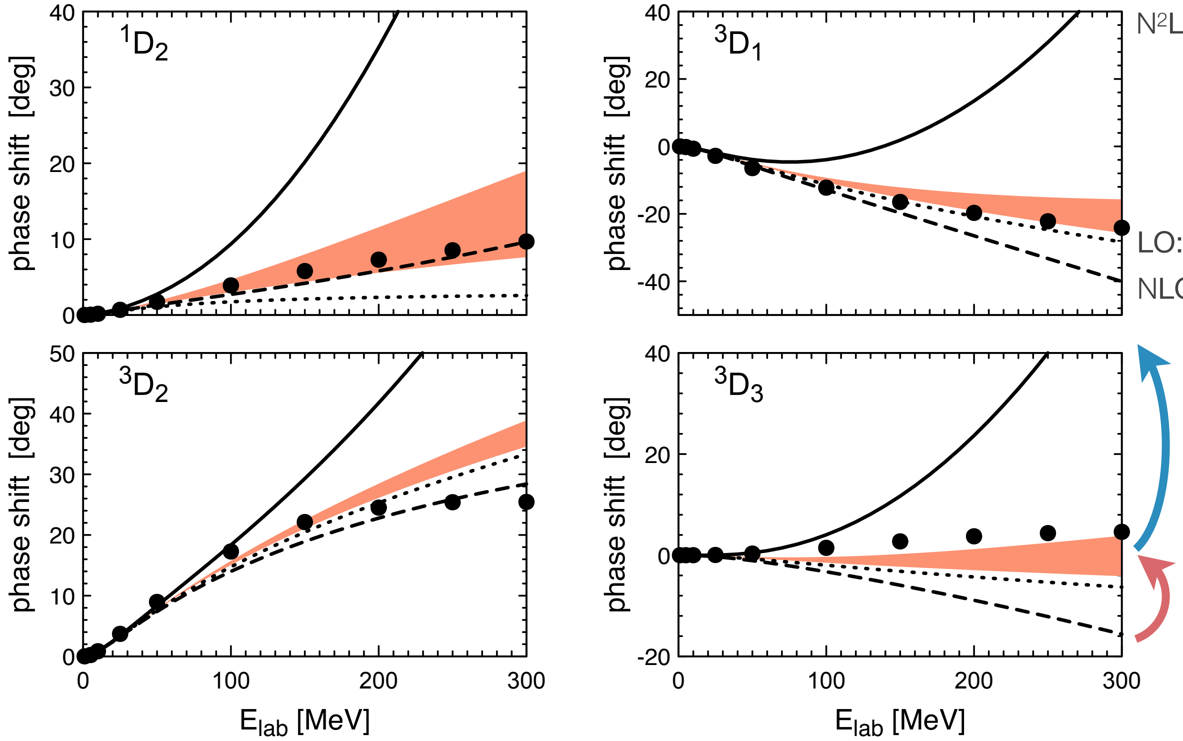


$$\begin{aligned}V_{2\pi}^{(3)} = & -\frac{3g_A^2}{16\pi F_\pi^4}\left[2M_\pi^2(2c_1-c_3)-c_3q^2\right](2M_\pi^2+q^2)A(q)\\ & -\frac{g_A^2c_4}{32\pi F_\pi^4}\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2\left(\vec{\sigma}_1\cdot\vec{q}\,\vec{\sigma}_2\cdot\vec{q}-\vec{\sigma}_1\cdot\vec{\sigma}_2\,q^2\right)(4M_\pi^2+q^2)\,A(q)\end{aligned}$$

$$A(q) = \frac{1}{2q}\arctan\frac{q}{2M_\pi}$$

The two-pion exchange NN potential

Testing chiral EFT predictions in peripheral NN scattering Kaiser, Brockmann, Weise, NPA 625 (97)



N^{2}LO: $V_{1\pi} + V_{2\pi}^{(2)} + V_{2\pi}^{(3)}$ (DimReg)

LO: $V_{1\pi}$

NLO: $V_{1\pi} + V_{2\pi}^{(2)}$ (DimReg)

enhanced N^{3}LO contact terms

$\Lambda_b = 500\ldots800$ MeV

EE, Glöckle, Meißner, EPJA 19 (04)

Triple enhancement at N^{2}LO: Factor of π, large numerical factor + large c_i's… Breakdown of χEFT?

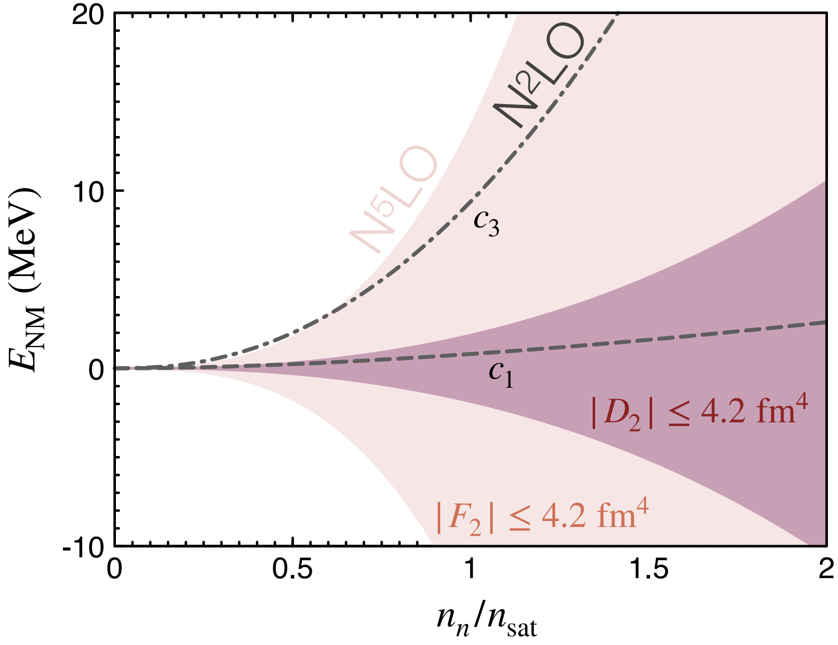
$$V_{2\pi}(q) = \frac{2}{\pi}\int_{2M_\pi}^{\infty} \mu d\mu \, \frac{\rho(\mu)}{q^2+\mu^2} + \ldots = \frac{2}{\pi}\int_{2M_\pi}^{\Lambda_b} \mu d\mu \, \frac{\rho(\mu)}{q^2+\mu^2} \underbrace{+ \frac{2}{\pi}\int_{\Lambda_b}^{\infty} \mu d\mu \, \frac{\rho(\mu)}{q^2+\mu^2} + \ldots}_{\textit{purely short-range contributions}}$$

⇒ the ***long-range*** part of the TPEP enhancement improves agreement with the data

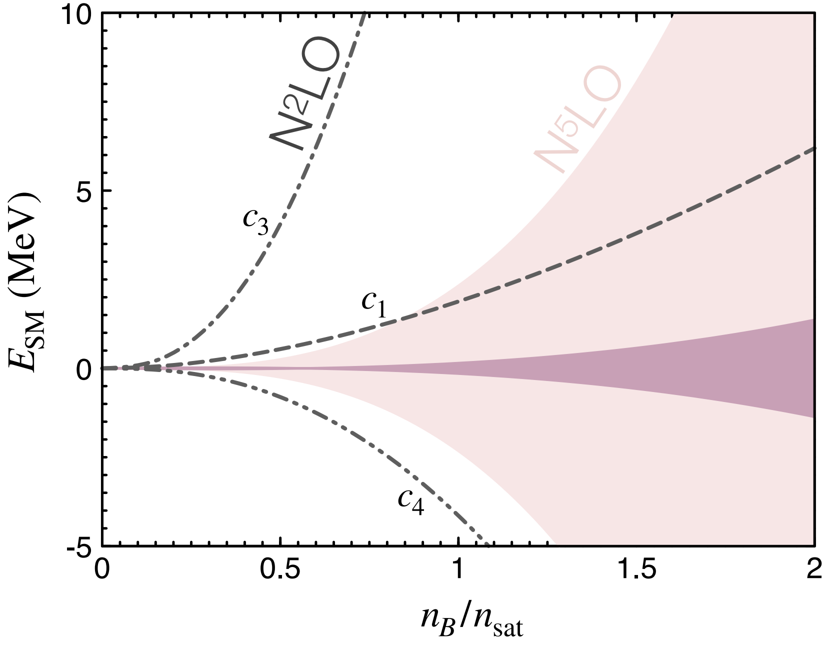
Neutron and symmetric nuclear matter

EE, A.M. Gasparyan, J. Gegelia, D. Hog, H. Krebs, PRC 113 (2026) 044005

Neutron matter



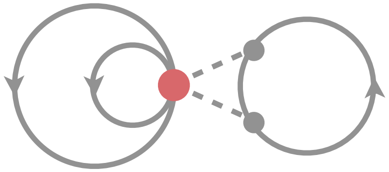
Symmetric nuclear matter



We start with reproducing the results of Cirigliano et al.,:

- using their (large) values for D_2, F_2
- without imposing a regulator

$\langle V_{3N} \rangle_{\text{Fermi gas}} =$



$+ \; \dots$

Neutron and symmetric nuclear matter

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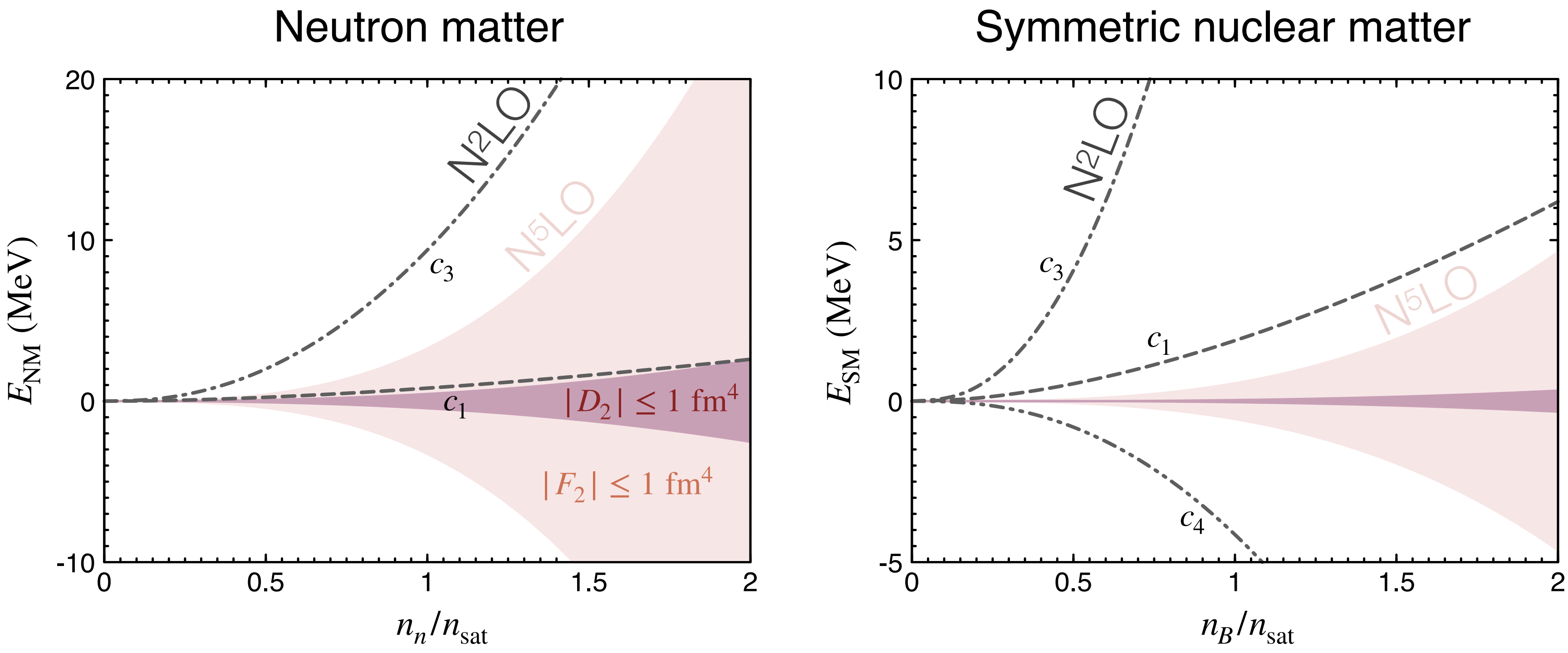

Using realistic values for the LECs D_2, F_2 and Dimensional Regularization

Neutron and symmetric nuclear matter

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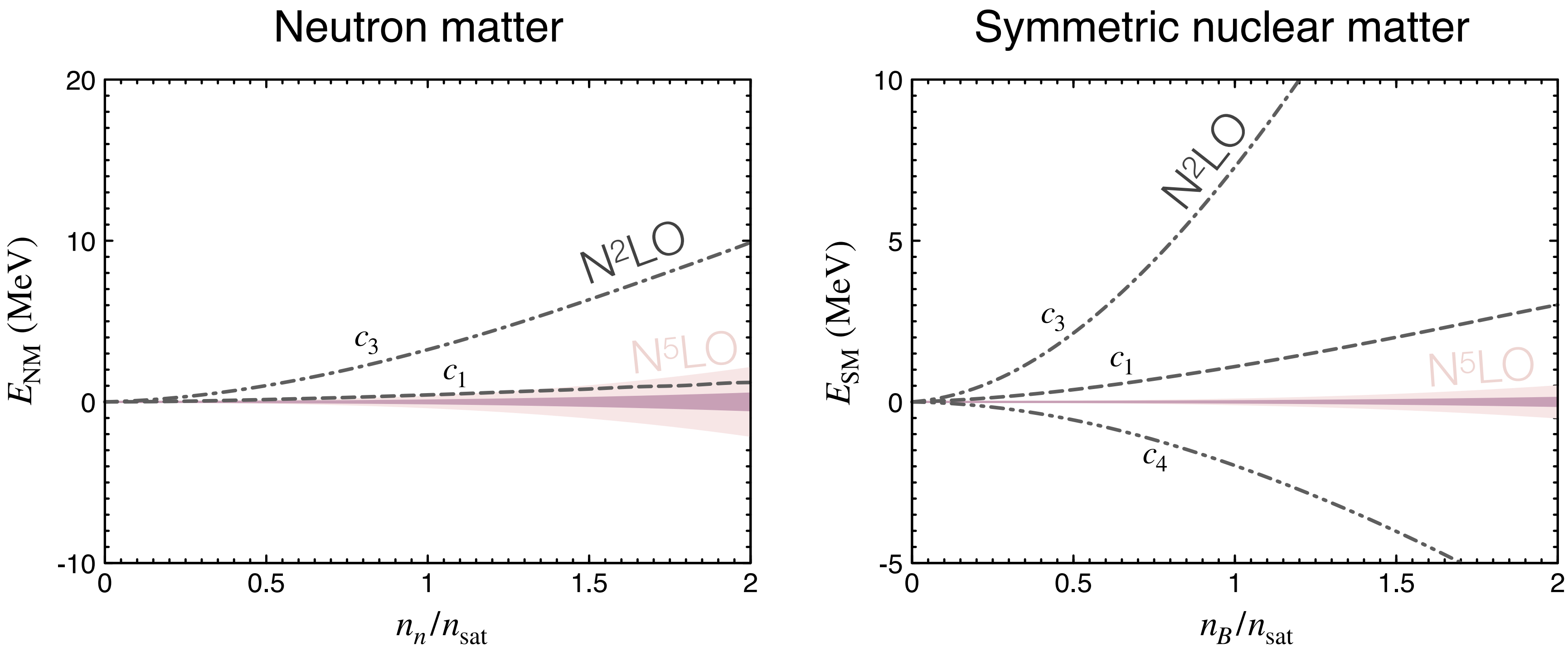

Now, realistic values for the LECs D_2, F_2 + SMS regulator with $\Lambda = 500$ MeV:

N²LO (c_i's): $\frac{1}{(q_1^2+M_\pi^2)(q_3^2+M_\pi^2)} \quad \to \quad \frac{e^{-\frac{q_1^2+M_\pi^2}{\Lambda^2}}\, e^{-\frac{q_3^2+M_\pi^2}{\Lambda^2}}}{(q_1^2+M_\pi^2)(q_3^2+M_\pi^2)}$

N³LO: $v_i(q) \quad \to \quad -\frac{2q^2}{\pi}\int_{2M_\pi}^{\infty}\frac{d\mu}{\mu}\,\frac{\rho_i(\mu)}{q^2+\mu^2}\, e^{-\frac{q^2+M_\pi^2}{2\Lambda^2}}$

N⁴˒⁵LO: $v_i(q) \quad \to \quad \frac{2q^4}{\pi}\int_{2M_\pi}^{\infty}\frac{d\mu}{\mu^3}\,\frac{\rho_i(\mu)}{q^2+\mu^2}\, e^{-\frac{q^2+M_\pi^2}{2\Lambda^2}}$

Conclusions from part II

- RG arguments do **NOT** justify the need to promote $D_2 M_\pi^2$ to LO, and thus do Not require promoting the new-class 3N forces from N^{5}LO to N^{3}LO

- In the approach we are using, estimate $|D_2| \sim 1$ fm^4. Of course, one cannot exclude that its actual value is larger — wait for lattice results…

- Within our regularization scheme (SMS), found up to 40 (!) times smaller contributions of the D_2, F_2 3N forces to nuclear matter than Cirigliano et al. No indication for the need to promote from phenomenological perspective.

- On the other hand, chiral expansion of π-exchange 3N forces known to suffer from enhancements beyond NDA Krebs, Gasparyan, EE 2012, 2013, 2015, 2018. Also slow convergence for weak MECs ⇒ implications for ^{3}H β-decay (in preparation)

Thank you for your attention