

Precise and Accurate $\beta\beta$ Matrix Elements

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INSTITUTE for
NUCLEAR THEORY

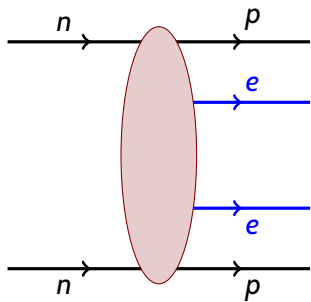
J. Engel

UNIVERSITY of WASHINGTON

June 30, 2026

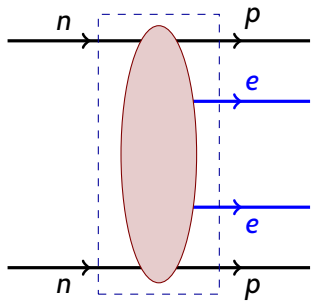


$0\nu\beta\beta$ Decay



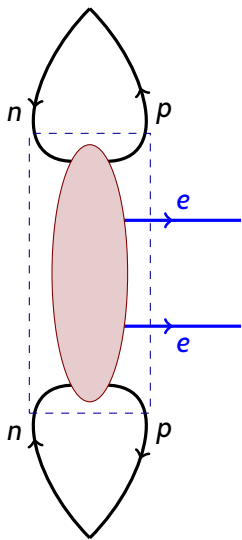
If it's observed ...

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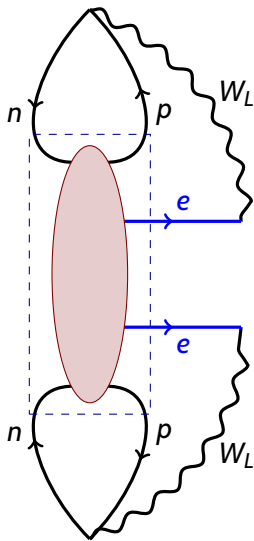
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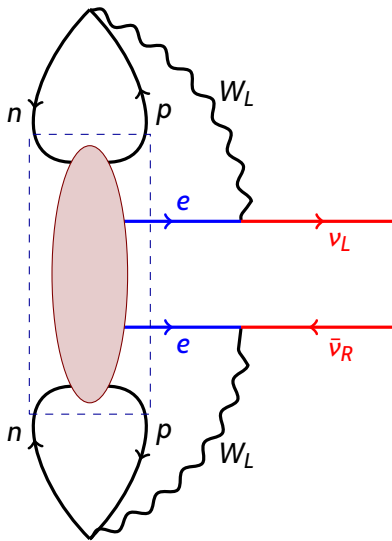
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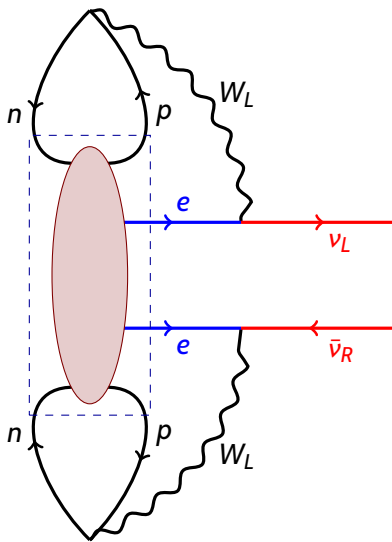
$0\nu\beta\beta$ Decay



If it's observed ...

Majorana neutrino propagator

$0\nu\beta\beta$ Decay

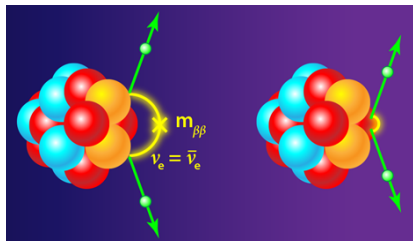


Majorana neutrino propagator

If it's observed ...

...neutrinos are Majorana particles (and lepton number is violated), no matter what physics is responsible.

The Slide I Really Don't Need to Show Here

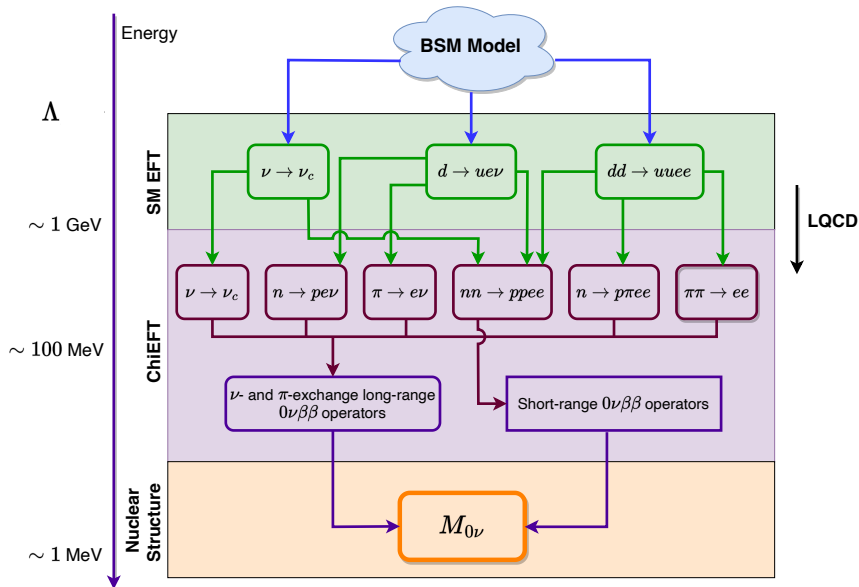


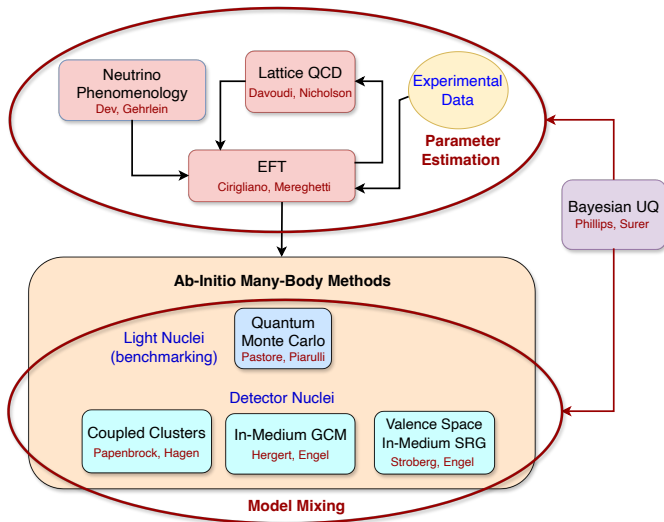
Rate depends on squares of unknown nuclear matrix elements. We need to compute them and **assign a believable (and not too large) uncertainty** so that experimentalists can better

1. plan their experiments
2. draw conclusions from their results

A Tower of EFTs

How we understand all the physics producing $\beta\beta$ decay





Tower's Top: Beyond-Standard Models

- ▶ **J. Gehrlein:** Working with **E. Mereghetti** at LANL to map interesting, well-motivated models onto SM-EFT coefficients.

Currently using decays of mesons (M) such as $M_1^- \rightarrow M_2^+ \ell_1^- \ell_2^-$ and τ decays of the such as $\tau^- \rightarrow \ell^+ M_1^- M_2^-$ to constrain lepton-violating operators and the models that generate them (along with $0\nu\beta\beta$ decay).

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- ▶ **UW group:** Looking at implications of other experiments for $0\nu\beta\beta$ decay in 3+3 seesaw model.

SM EFT: High Mass Scale Λ for New Physics

For Λ at GUT scale, lepton-number violation occurs only through dimension-5 “Weinberg operator,” leading to ν masses of

$$m \approx w \frac{v^2}{\Lambda} \approx 6 \times 10^{-3} w \text{ eV}.$$

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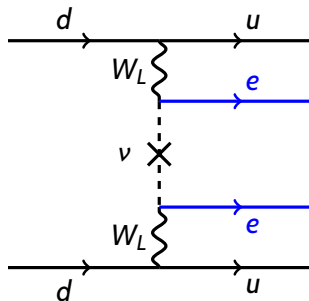


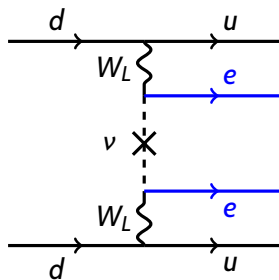
Diagram is proportional to weighted “Majorana mass” of light neutrinos,

$$m_{\beta\beta} = \sum_i U_{ei}^2 m_i$$

U_{ei} is amount of mixing of electron flavor with i^{th} mass eigenstate.

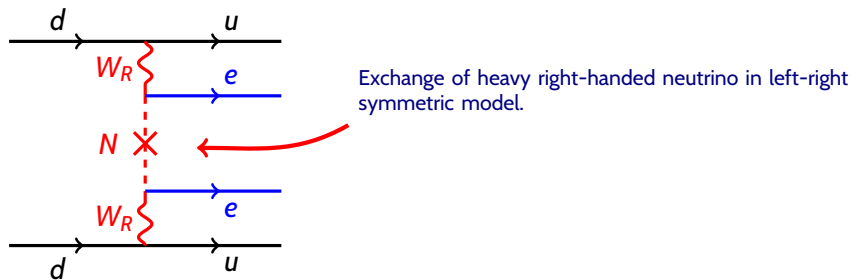
SM EFT: New Physics at Lower Scales

If $\Lambda = \mathcal{O}(1 - 100)$ TeV then presence of other particles at the weak scale means that higher-dimensional operators can be important.



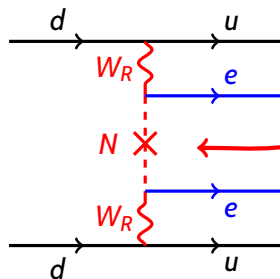
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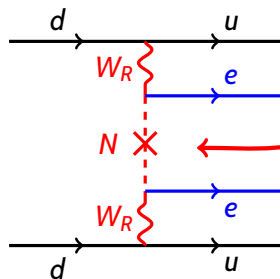
Exchange of heavy right-handed neutrino in left-right symmetric model.

Corresponds to dimension-9 operator below new-physics scale

This process can occur at the same rate as light- ν exchange (or even a larger rate) if $m_N \approx m_{W_R} \approx 1$ TeV.

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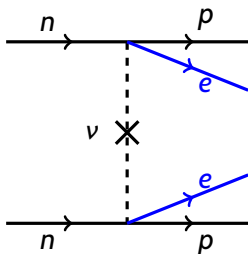
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Light sterile ν 's can also be exchanged with comparable rates.

Moving to nucleons: Chiral EFT

Light- ν Exchange at Leading Order

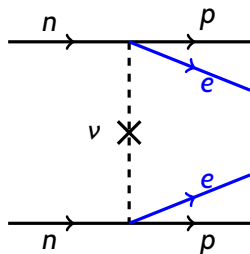


Usual long-range exchange

Older models typically include only this.

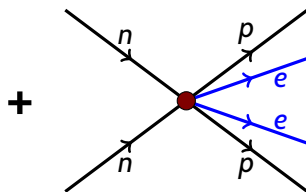
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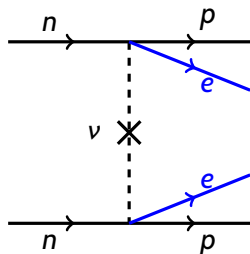
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Contact counter term from short short-range exchange at energies beyond breakdown scale of EFT.

Moving to nucleons: Chiral EFT

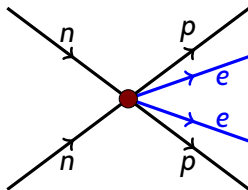
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+



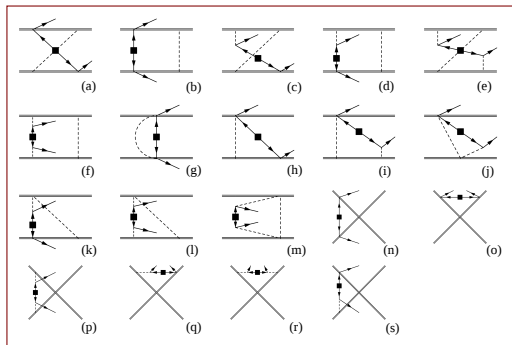
Contact counter term from short short-range exchange at energies beyond breakdown scale of EFT.

At present, coefficient of contact term estimated from large- N QCD or via sum rules.

Values of coefficients can be improved with lattice QCD.

Chiral EFT at Higher Order

$\approx 10\%$ effects



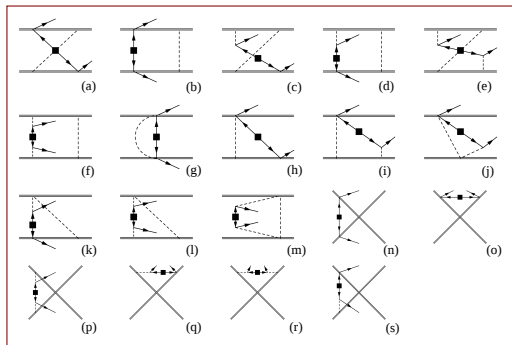
Some N^2 LO diagrams for $O_{\nu\beta\beta}$ operator in chiral EFT

Based on Cirigliano, Dekens, Mereghetti, and Walker-Loud, PRC 97 065501 (2018)

Some of these require contact counterterms with still unknown coefficients.

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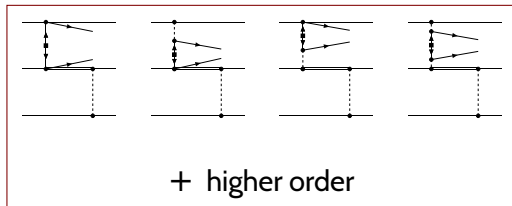
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Three-nucleon diagrams for $O_{\nu\beta\beta}$ operator in chiral EFT with Δ 's

From G. Chambers-Wall, based on Chambers-Wall, Lieffers, King, Mereghetti, Pastore, Piarulli, and Wiringa, Physical Review C 113, 025502 (2026).

Effects of all subleading operators have been evaluated in light nuclei.

Chambers-Wall et al., Pastore et al., PRC 97, 014606 (2018).

V. Cirigliano, E. Mereghetti still working on

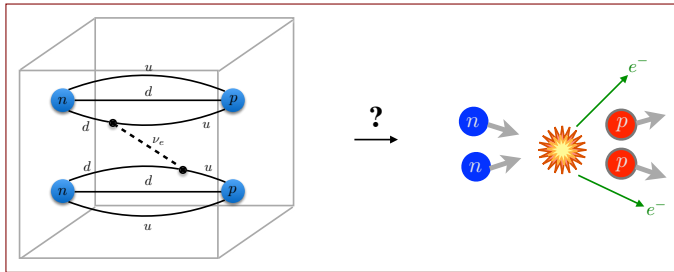
- ▶ Three-body operators in Δ -less EFT
- ▶ N3LO operators
- ▶ Coefficients of higher-order contact terms?

J. Engel working with K. Hebeler and T Miyagi to compute matrix elements of the three-body operators in heavy nuclei.

Modifying their code NuHamil, which works in oscillator basis, for charge-changing $\beta\beta$ operators instead of interactions.

Lattice QCD

Deriving chiral-EFT coefficients

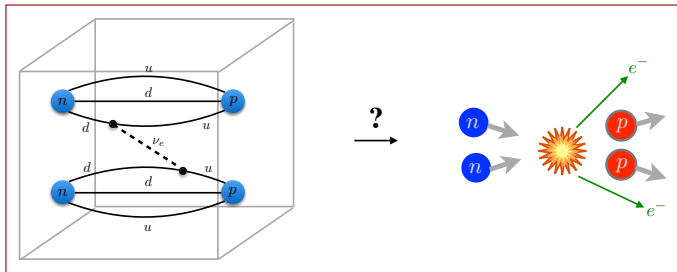


Chiral-EFT $O_{\nu\beta\beta}$
operator from
LQCD.

From Z. Davoudi

Lattice QCD

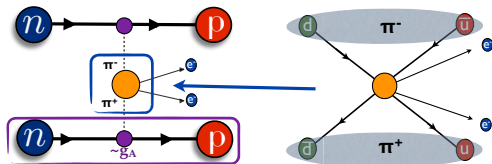
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Treated already: Heavy-particle exchange between pions



From A. Nicholson

Lattice QCD

First: Good NN Spectroscopy, Form Factors, Scattering Amplitudes

Z. Davoudi and A. Nicholson both working hard on these problems.

Two @NDB postdocs currently at Maryland:

- ▶ A. Grebe working on quantifying uncertainties/providing bounds in energies of low-lying NN states (right now at large pion mass).
- ▶ J. Mocosco, former Nicholson student, working on form factors in bound two-nucleon systems (also still at large pion mass).

Together with Davoudi and M. Wagman, these two are also working on finite-volume effects and EFT matching of current matrix elements. Still need to move to physical pion mass.

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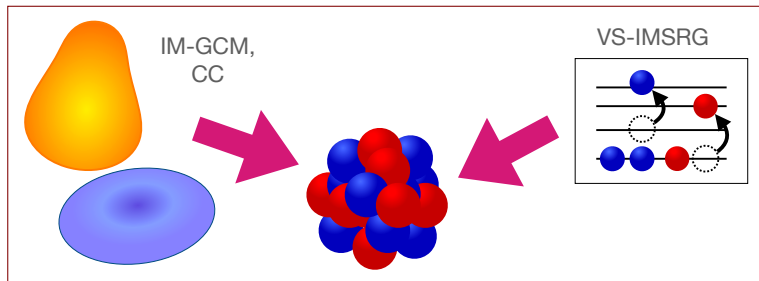
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And the QCD and EFT members of the Hub are still figuring out how to ultimately do the O_v matching to chiral EFT at finite volume.

- ▶ Lüscher-based matching formalism for any EFT?
- ▶ Chiral EFT in a box?
- ▶ \vdots

Tower's Base: *Ab Initio* Nuclear-Structure Theory



Ab initio nuclear many-body methods – coupled clusters and two variants of IMSRG – schematically

Why *ab initio*? Because you can control uncertainty ...

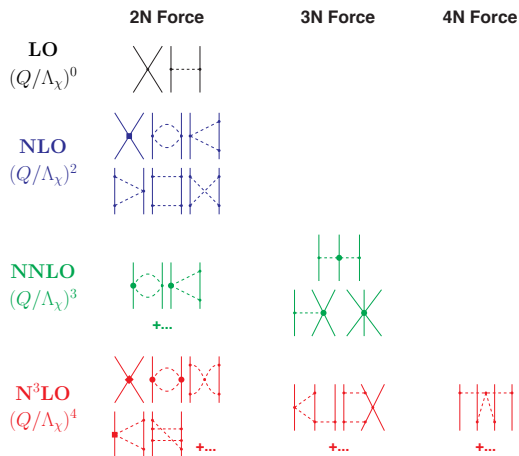
...at least in principle.

Interactions

From chiral EFT

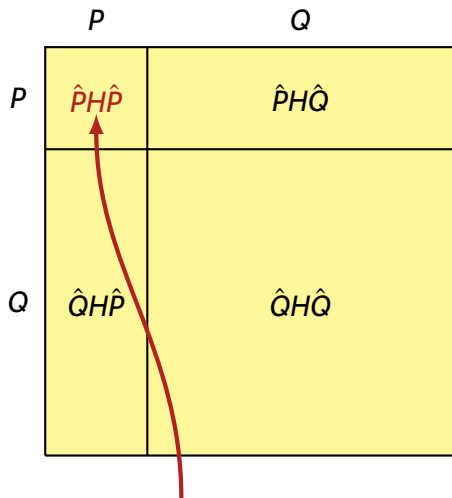
Expand interaction in powers of Q/Λ_χ .

$Q = m_\pi$ or typical nucleon momentum.



Ab Initio Nuclear-Structure for Heavy Nuclei

Partition of Full Hilbert Space



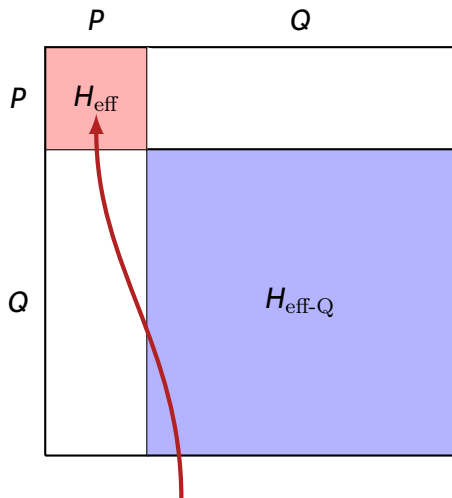
P = subspace you want
 Q = the rest

Task: Find unitary transformation to make H block-diagonal in P and Q , with H_{eff} in P reproducing most important eigenvalues.

Simpler calculation done here.

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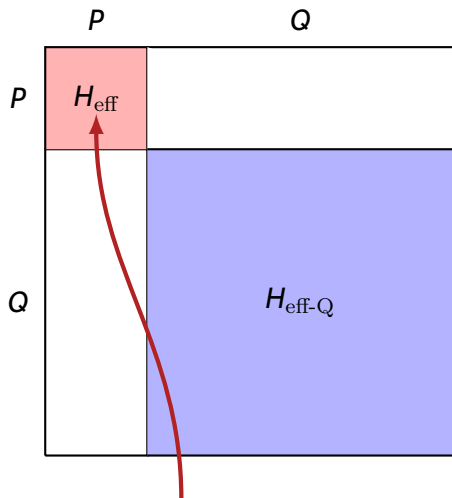
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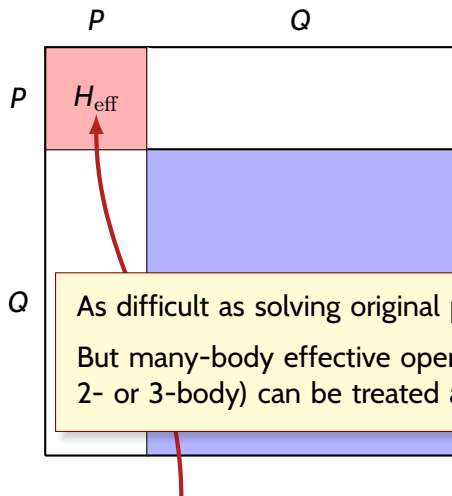
Task: Find unitary transformation to make H block-diagonal in P and Q , with H_{eff} in P reproducing most important eigenvalues.

Must must apply same unitary transformation to transition operator.

Simpler calculation done here.

Ab Initio Nuclear-Structure for Heavy Nuclei

Partition of Full Hilbert Space



P = subspace you want
 Q = the rest

Task: Find unitary transformation to make H block-diagonal in P and Q , with H_{eff} in P reproducing most values.

As difficult as solving original problem.

But many-body effective operators (beyond 2- or 3-body) can be treated approximately.

By same unitary to transition

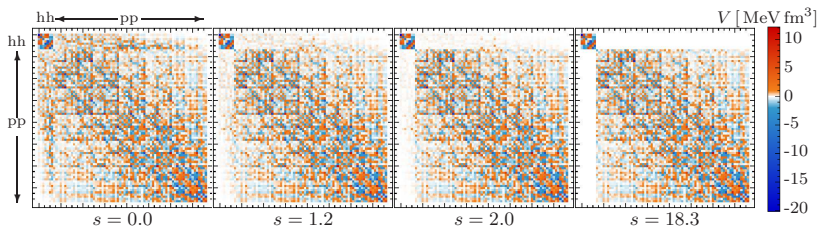
operator.

Simpler calculation done here.

In-Medium Similarity Renormalization Group

One way to determine the transformation

Flow equation for effective Hamiltonian.
Gradually decouples selected set of states.



from H. Hergert

Trick is to keep all 1- and 2-body terms in effective Hamiltonian at each step (**IMSRG-2**, also includes “coherent” 3, 4-body ... terms).

If selected set is one state, end up with ground-state energy. If it's a valence space, get effective shell-model interaction and operators.

Coupled-Cluster Theory

Ground state in closed-shell nucleus:

$$|\Psi_0\rangle = e^T |\varphi_0\rangle \quad T = \sum_{i,m} t_i^m a_m^\dagger a_i + \sum_{ij,mn} \frac{1}{4} t_{ij}^{mn} a_m^\dagger a_n^\dagger a_i a_j + \dots$$

m,n > F *i,j < F*

Slater determinant

Here the Hamiltonian is transformed in a non-unitary way:

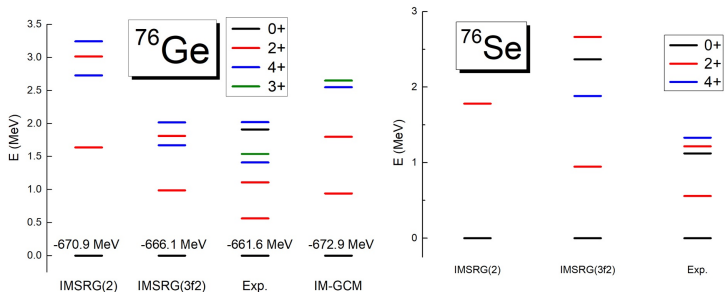
$$H \longrightarrow \tilde{H} \equiv e^{-T} H e^T$$

so that $|\varphi_0\rangle$ is its ground state. Must solve algebraic equations for the t 's.

Excited states, states in closed-shell + a few nucleons, constructed from simple excitations of $|\varphi_0\rangle$.

IMSRG in Ge-Se System

Spectrum of ^{76}Ge and ^{76}Se

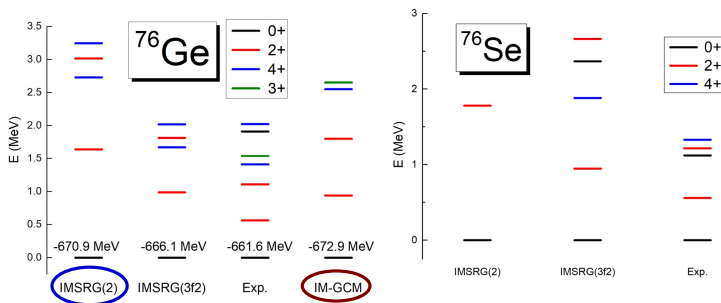


Belley et al., Phys. Rev. Lett. 132, 182502 (2024)

From talk by B. He

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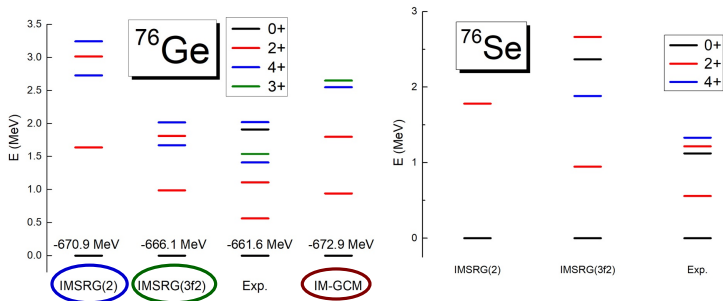
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Two variants of IMSRG: valence-space version and “In-Medium Generator Coordinate Method,” with deformed reference state.

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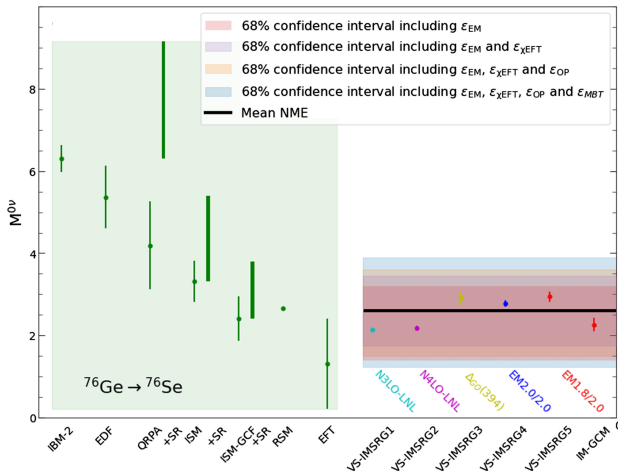
Two variants of IMSRG: **valence-space version** and **“In-Medium Generator Coordinate Method,”** with deformed reference state.

For the valence version, have approximation to **IMSRG-3**.

IMSRG Matrix Elements: ^{76}Ge

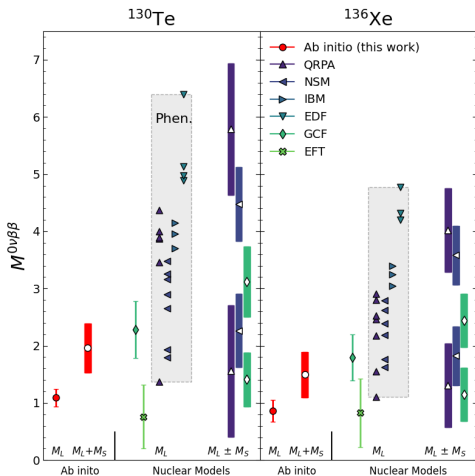
^{76}Ge with Two Versions of In-Medium Similarity Renormalization Group

A. Belley et al. PRL 132, 182502 (2024)



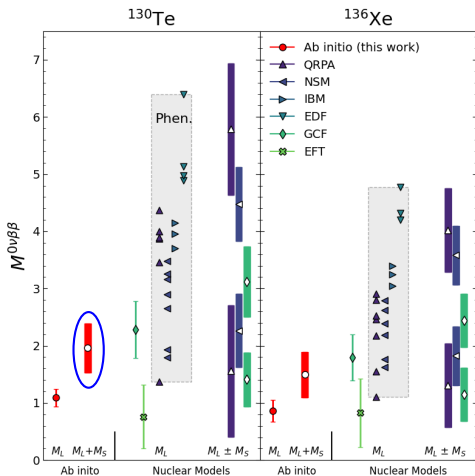
Matrix elements are on the small side (compared to phenomenological ones) but still quite uncertain.

IMSRG Matrix Elements: Heavier Nuclei



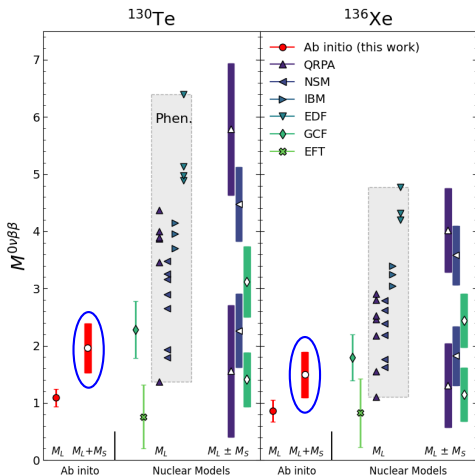
A. Belley, T. Miyagi, S.R. Stroberg, and J.D. Holt, arXiv:2307:15156

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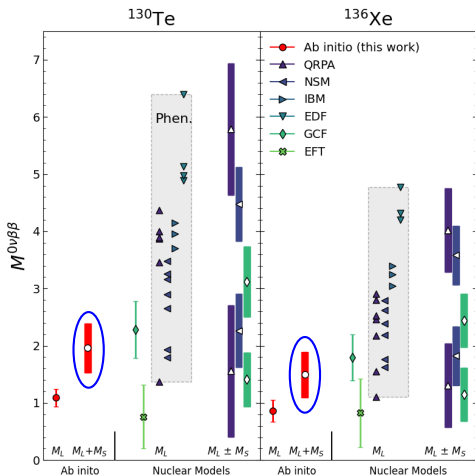
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A. Belley, T. Miyagi, S.R. Stroberg, and J.D. Holt, arXiv:2307:15156

Again, results are on low end of those produced by models.

Lots of UQ still to do here.

At least two useful version exist: valence-space IMSRG and IMGCM, which uses correlated state (with deformation, pairing) as starting point.

Initial calculations already exist for ^{48}Ca , ^{76}Ge , ^{130}Te , ^{136}Xe , etc.

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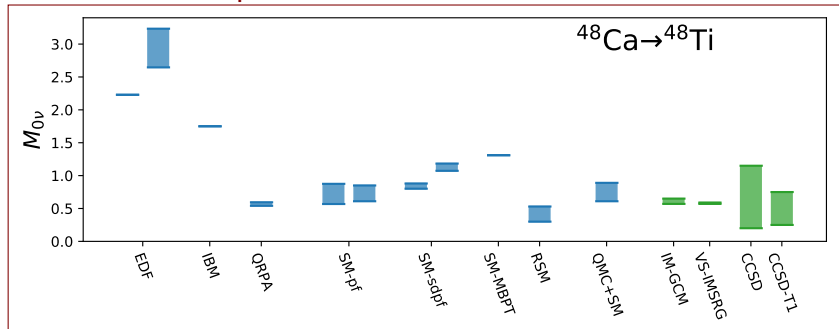
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 - ▶ Valence-space version already has an efficient one.
 - ▶ H. Hergert and collaborators have developed similar emulator for IMGCM.
- ▶ Postdoc Z. Wang implementing equations-of-motion method for treating 2ν decay (much harder than 0ν) or μ capture.
Important for judging quality of model, which enters UQ.

Coupled-Clusters Matrix Element for ^{48}Ca

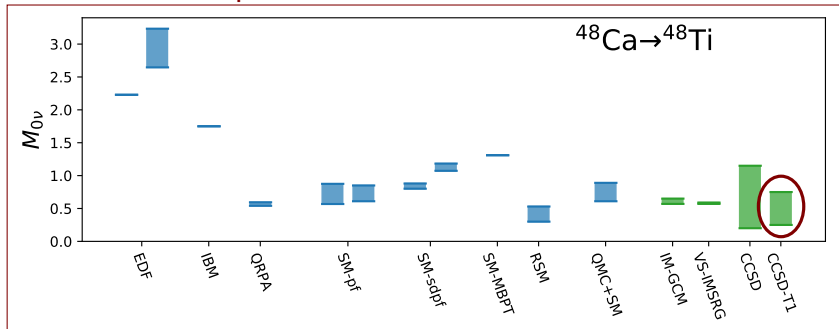
Comparison of all methods: no contact



Ab initio results are leading order in χEFT

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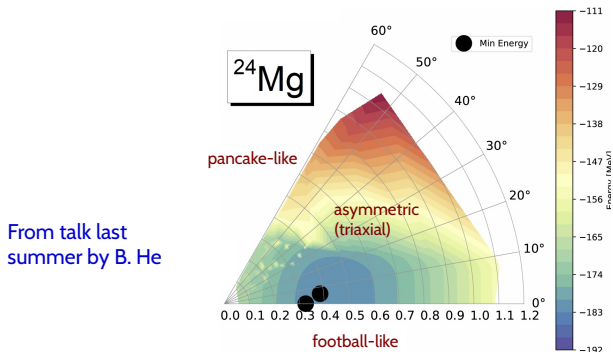


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Coupled Clusters for ^{76}Ge

^{76}Ge is triaxial.

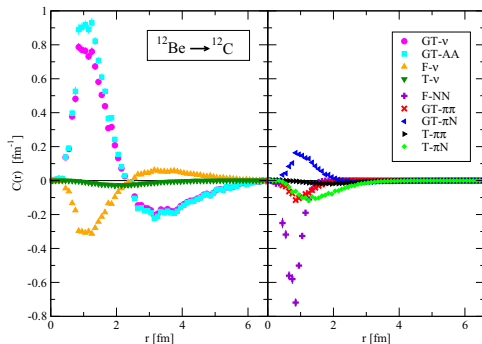
First triaxial-nucleus structure calculations have been done:



Now scaling up to heavier nuclei, moving to $\beta\beta$ decay.

Benchmarking

Contributions to matrix elements as function of internucleon separation

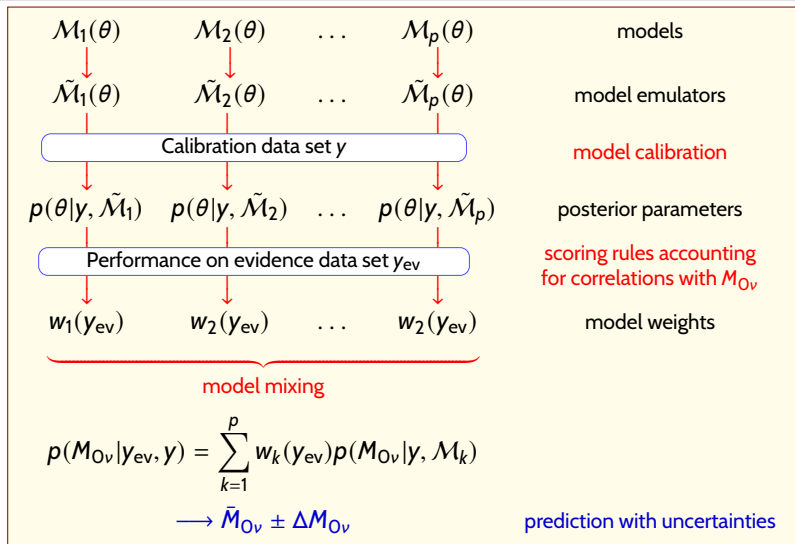


From S. Pastore et al., PRC 97, 014606 (2018).

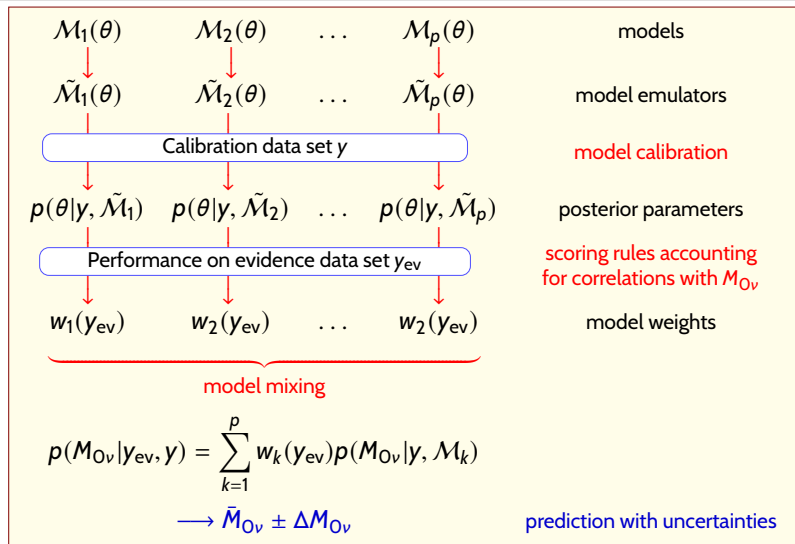
Pastore and Piarulli leading efforts to compute matrix elements in light nuclei. We will try to use their results to benchmark IMSRG and coupled-cluster methods.

Need oscillator-basis matrix elements of Norfolk interaction and associated $\beta\beta$ operators.

Quantifying Uncertainty



Quantifying Uncertainty



- Repeat for all $\beta\beta$ nuclei
- Repeat with calibration data set refined for $Ov\beta\beta$.

Quantifying Uncertainty

Important UQ work within this plan:

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Design of procedure for choosing y_{ev} and scoring models a major challenge.

Timeline

		Y1	Y2	Y3	Y4	Y5
Phenomenology	Analysis of neutrino mass models and matching onto SMEFT.	→		
	Study of complementary probes of neutrino mass models, e.g. colliders, astrophysics.	→		
	UQ: Establishment of precision needed in nuclear matrix elements to disentangle signatures of different models of LNV.	→	
EFT	Calculation of higher chiral order two-nucleon transition operators for light Majorana neutrino exchange.	→				
	Analysis of three-body operators.	...	→			
	Operators made available for nuclear structure calculations.			→		
	Beginning of testing of two- and three-body operators at chiral N^2LO in light nuclei.			→		
	UQ: Calibration of EFT and validation of power counting.			→		

Timeline

	Y1	Y2	Y3	Y4	Y5
LQCD	<p>Variational NN calculations at a pion mass below 200 MeV.</p> <p>Variational algorithms for local and non-local NN matrix elements.</p> <p>Finite-volume EFT matching of NLO operators.</p> <p>Analysis of low-energy NN spectra and scattering amplitudes at a pion mass below 200 MeV.</p> <p>UQ: Application of Bayesian tools to design of LQCD studies of double-beta decay, given precision target.</p> <p>Benchmarking of variational matrix elements at larger pion mass to understand systematic uncertainties.</p> <p>Initial studies of local and non-local matrix elements related to $nn \rightarrow pp$ at a pion mass below 200 MeV, and EFT matching.</p> <p>Use of UQ to combine and propagate uncertainties to LECs.</p>				
Nuclear Structure	<p>Triaxiality in CC reference state. Initial $\beta\beta$ matrix elements.</p> <p>Approximate IMSRG(3) for $0\nu\beta\beta$ and other relevant operators.</p> <p>Emulators for all three <i>ab initio</i> methods.</p> <p>Benchmarking with QMC in light systems.</p> <p>UQ: Error model for many-body methods.</p> <p>UQ: Scheme for weighting methods in model mixing.</p> <p>Two- and three-body matrix elements for NNLO $0\nu\beta\beta$ operator.</p> <p>UQ: Error model for many-body methods.</p>				

Timeline

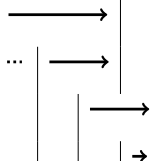
| Y1 | Y2 | Y3 | Y4 | Y5 |

	Y1	Y2	Y3	Y4	Y5
Overall				X	X
LQCD					
Nuclear Structure					

Preliminary nuclear matrix elements for all important isotopes.
Final matrix elements, with quantified uncertainty

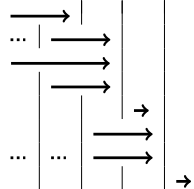
LQCD

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To Conclude ...

Lots done already to go from underlying theories of lepton-number violation to nuclear matrix elements.

Still a lot to do, but matrix elements in the important nuclei, with real uncertainty estimates, are coming

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That's all.
Thanks for listening!