Simulating (2+1)d Lattice Gauge Theories with Fermionic Tensor Networks

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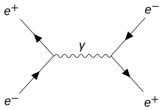
April 04, 2023 | Patrick Emonts | Lorentz Institute, Leiden University

Universiteit Leiden
The Netherlands

Why do we need Lattice Gauge Theories?

QED

$$\mathcal{L}_{QED} = i\overline{\Psi}\gamma^{\mu}\partial_{\mu}\Psi - m\overline{\Psi}\Psi - e\overline{\Psi}\gamma_{\mu}A^{\mu}\Psi - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}$$



Small coupling

$$\alpha_{QED} = \frac{e^2}{4\pi} \approx \frac{1}{137}$$

QCD

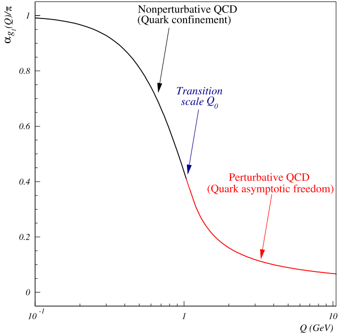
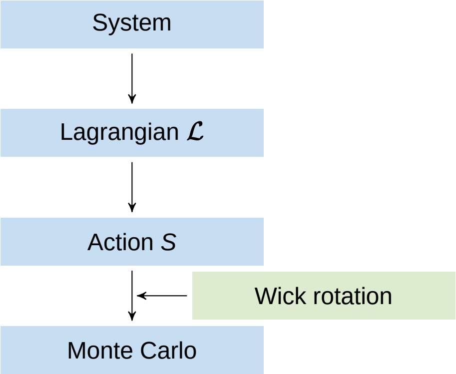


Image adapted from Alexandre Deur, Stanley J. Brodsky, and Guy F. de Téramond (2016) Progress in Particle and Nuclear Physics **90** pp. 1–74

Modeling Nature



Wick rotation

$$t \to -i\tau$$

Problems

- Possibility of a sign-problem
- Time dynamics not accessible

Mari Carmen Bañuls and Krzysztof Cichy (2020) Rep. Prog. Phys. **83** p. 024401; John Kogut and Leonard Susskind (1975) Phys. Rev. D **11** pp. 395–408; Kenneth G. Wilson (1974) Phys. Rev. D **10** pp. 2445–2459

Path integral formalism in QFT

pure QED

$$S_{QED}[A_\mu] = -\frac{1}{4}\int \mathrm{d}x^\alpha F_{\mu\nu}(x_\alpha)F^{\mu\nu}(x^\alpha) = \int \mathrm{d}x^\alpha \partial_\mu A_\nu(x^\alpha)\partial^\nu A^\mu(x^\alpha)$$

vacuum expectation value

$$\langle\Omega|O[A_\mu]|\Omega\rangle = \frac{\int \mathcal{D}A O[A_\mu] e^{iS_{QED}[A_\mu]}}{\int \mathcal{D}A e^{iS_{QED}[A_\mu]}}$$

Problems

- ✖ Numerator oscillating
- ✖ Integration measure ill-defined

Wick rotation

Shift to imaginary time

$$t \longrightarrow -i\tau$$

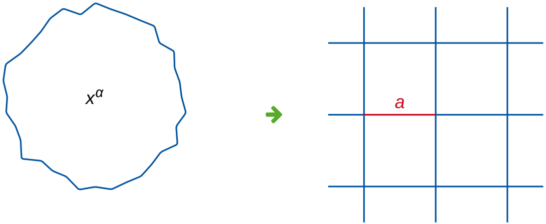
Change of metric from Minkowski to Euclidean

$$e^{iS_M} = e^{i\int \mathrm{d}x_M^\alpha \mathcal{L}(x_M^\alpha)} \longrightarrow e^{-\int \mathrm{d}x_E^\alpha \mathcal{L}(x_E^\alpha)} = e^{-S_E}$$

Problems

- ✔ Numerator converging
- ✖ Integration measure ill-defined

Discretization: Lattice Gauge Theory



$$A_\mu \to U_\mu = e^{iaA_\mu}$$

Find the lattice action $\tilde{S}_E$ that agrees with S_E in the continuum limit of vanishing a

$$\tilde{S}_E[U] \to S_E[A](a \to 0)$$

Vacuum expectation value

Vacuum expectation value

$$\langle O[U] \rangle = \frac{\int \mathcal{D}U O[U] e^{-S_E[U]}}{\int \mathcal{D}U e^{-S_E[U]}} \text{ with } \mathcal{D}U = \prod_{x^\alpha} \mathrm{d}U_\mu(x^\alpha)$$

Problems

- ✔ Numerator converging
- ✔ Integration with the Haar measure

Monte Carlo

Vacuum expectation value

$$\langle O[U] \rangle = \frac{\int \mathcal{D}U O[U] e^{-S_E[U]}}{\int \mathcal{D}U e^{-S_E[U]}} \text{ with } \mathcal{D}U = \prod_{x^\alpha} \mathrm{d}U_\mu(x^\alpha)$$

Monte Carlo

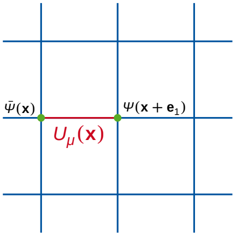
Sampling from the distribution $p(U) = \frac{e^{-S_E[U]}}{\int \mathcal{D}U e^{-S_E[U]}}$ and calculate the expactation value as

$$\langle O[U] \rangle \approx \frac{1}{N} \sum_{i=1}^{N} O[U(i)]$$

Fermions in lattice gauge theories

Action of the fermions

$$S_f[\Psi, \bar{\Psi}, U] = \sum_{x,\mu} \bar{\Psi}(x)\gamma^\mu U_\mu(x)\Psi(x+e_\mu) - h.c. + m\bar{\Psi}(x)\Psi(x) = \bar{\Psi}_x M_{xy}[U]\Psi_y$$



Integrating out the fermions

$$\int \mathcal{D}\bar{\Psi}\mathcal{D}\Psi e^{-\bar{\Psi}M[U]\Psi} = \det(M[U])$$

Sign Problem

$$\langle O[U]\rangle = \frac{\int \mathcal{D}U\mathcal{D}\Psi\mathcal{D}\bar{\Psi} O[U] e^{-S_E[U]-\bar{\Psi}M[U]\Psi}}{\int \mathcal{D}U\mathcal{D}\Psi\mathcal{D}\bar{\Psi} e^{-S_E[U]-\bar{\Psi}M[U]\Psi}}$$

$$= \frac{\int \mathcal{D}U O[U] \det(M[U]) e^{-S_E[U]}}{\int \mathcal{D}U \det(M[U]) e^{-S_E[U]}}$$

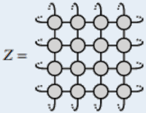
Probability distribution

$$p(U) = \frac{\det(M[U]) e^{-S_E[U]}}{\int \mathcal{D}U \det(M[U]) e^{-S_E[U]}}$$

But for finite potential $\mu > 0$: $\det(M[U]) \ngeq 0$

(Incomplete) Overview of fTN Approaches

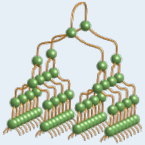
TRG



Nouman Butt et al. (2020) Phys. Rev. D **101** p. 094509

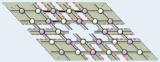
Shinichiro Akiyama and Daisuke Kadoh (2021) J. High Energ. Phys. **2021** p. 188

Tree TN



Timo Felser et al. (2020) Phys. Rev. X **10** p. 041040

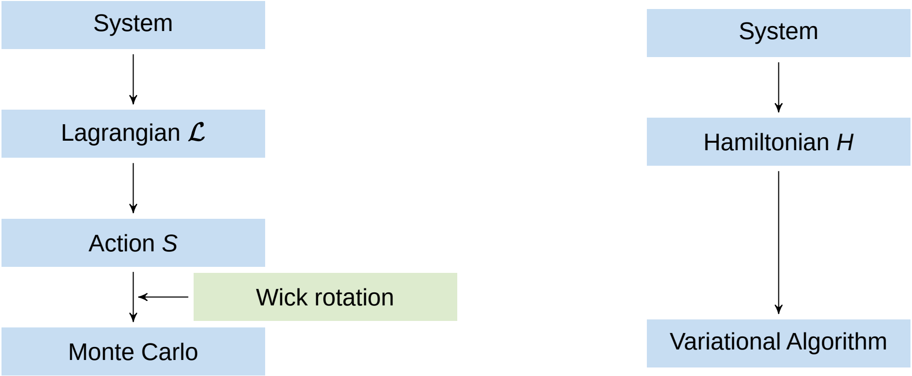
fPEPS/fMERA



Philippe Corboz et al. (2010) **81** p. 165104; Manuel Schneider et al. (2021) **104** p. 155118

Mari Carmen Bañuls and Krzysztof Cichy (2020) Rep. Prog. Phys. **83** p. 024401

Modeling Nature



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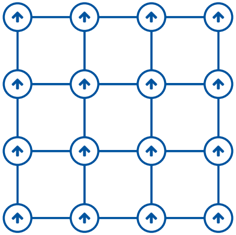
Many-body physics – How hard can it be?

We take a system that can take two states ↑ and ↓



Number of possibilities

$$Z = 2^N$$

Storage of minimal configuration (classical)

$$|\psi_0\rangle = 0101101010$$

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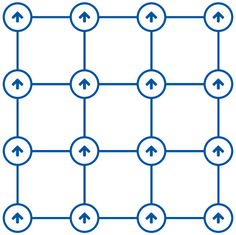


Number of possibilities

$$Z = 2^N$$

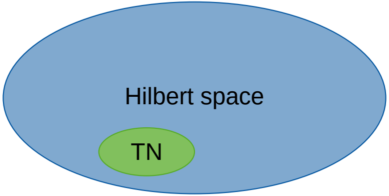
Storage of minimal configuration (quantum)

$$|\psi_0\rangle = \sum_{\{i\}} c_{i_0,\ldots,i_N} |i_1, i_2, \ldots i_N\rangle$$

Finding an Ansatz

Idea

Use an Ansatz with polynomially many parameters although the Hilbert space has exponentially many states



We explore only a small part of the Hilbert space

M. Fannes, B. Nachtergaele, and R. F. Werner (1992) Commun.Math. Phys. **144** pp. 443–490

D. Perez-Garcia et al. (2007)

Tensor Networks: A motivation

General superposition state

$$|\psi\rangle = \sum_{i_1,\cdots,i_N} c^{i_1,\cdots,i_N} \left|i_1,\cdots,i_N\right\rangle$$

An example system: Ising spins

$\uparrow\downarrow\downarrow\uparrow\uparrow$

How to get to polynomial scaling?

Can we just skip the small coefficients?

$$c^{0,0,1,0,1} = 0.3623$$
$$c^{0,1,1,0,1} = 0.0003$$
$$c^{1,0,0,0,0} = -0.0004$$
$$c^{0,1,0,0,1} = 0.5203$$

Tensor Networks: A motivation

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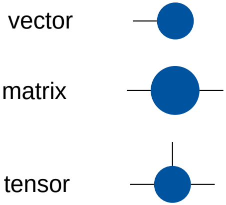
Tensor Network Notation

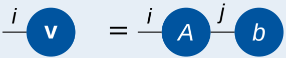


- The number of legs determines the number of indices of the object
- A connection $\Leftrightarrow$ Contraction of indices

Calculations with pictures

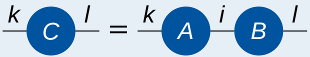
Matrix-Vector Multiplication

$$\mathbf{v}_i = \sum_{i,j} A_{ij} \mathbf{b}_j$$



Matrix-Matrix Multiplication

$$C_{kl} = \sum_{i,k,l} A_{ki} B_{il}$$



SVD – Splitting a state

Singular value decomposition

$$M = USV^\dagger$$

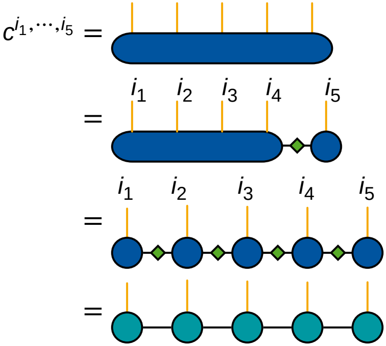
Tensor Network Notation

Properties

U, V unitaries

S diagonal, real, non-negative

Construction of a state



General state

$$|\psi\rangle = \sum_{\{i\}} c^{i_1,\cdots,i_N} |i_1,\cdots,i_N\rangle$$

Matrix Product State

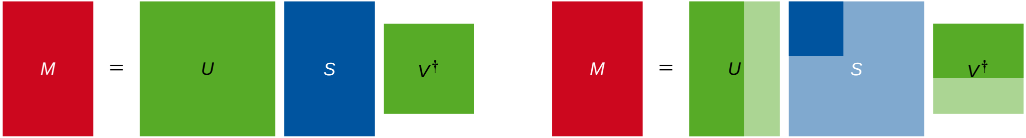
$$|\psi\rangle = \sum_{\{i\}} \sum_{\{\alpha\}} A^{i_1}_{1,\alpha_1} A^{i_2}_{\alpha_1,\alpha_2} \cdots A^{i_N}_{\alpha_{N-1},1} |i_1,\cdots,i_N\rangle$$

Jacob C. Bridgeman and Christopher T. Chubb (2017) **50** p. 223001

SVD – Selecting the important bits

Best approximation

The best approximation with rank r in the Frobenius norm for a given matrix M is to truncate to r singular values.



Carl Eckart and Gale Young (1936) Psychometrika **1** pp. 211–218

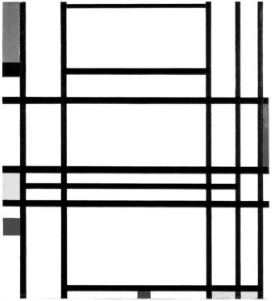
SVD – An example

Original Image

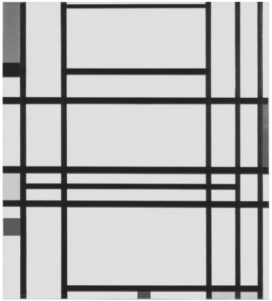
Truncated Image (20 SV)

SVD – An example

Original Image



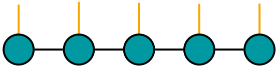
Truncated Image (20 SV)



Putting it all together – Approximating the state

Reducing to polynomially many parameters

Truncate to a virtual bond dimension D to reduce to polynomially many parameters.

$|\psi\rangle =$ 

$$= \sum_{\{i\}} \sum_{\{\alpha\}} A^{i_1}_{1,\alpha_1} A^{i_2}_{\alpha_1,\alpha_2} \cdots A^{i_N}_{\alpha_{N-1},1} |i_1, \cdots, i_N\rangle$$

Notation

$A^i_{\alpha,\beta}$: $d \times D \times D$ tensors
Physical index: i_j
Virtual index: α_j

What did we gain? – Counting Parameters

Full state

$$|\psi\rangle = \sum_{\{i\}} c_{i_0,\ldots,i_N} \left|i_1, i_2, \ldots i_N\right\rangle$$

MPS

$$|\psi\rangle = \sum_{\{i\}} \sum_{\{\alpha\}} A^{i_1}_{1,\alpha_1} A^{i_2}_{\alpha_1,\alpha_2} \cdots A^{i_N}_{\alpha_{N-1},1} \left|i_1, \cdots, i_N\right\rangle$$

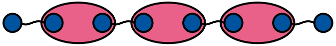
Number of parameters

$$d^N$$

Number of parameters

$$N(D \times D \times d)$$

MPS as 1D PEPS



Entangled pairs

$$|\Phi\rangle = \sum_{j=0}^{D-1} |jj\rangle$$

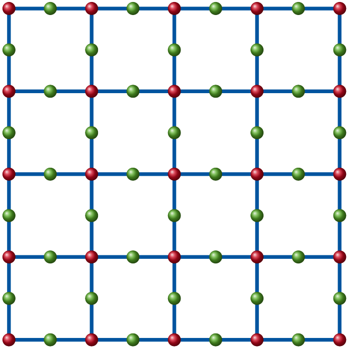
Example: Bell state

$$|\Phi\rangle = |00\rangle + |11\rangle$$

Projector

$$\omega = \sum_{i,\alpha,\beta} A^{i}_{\alpha,\beta} |i\rangle \langle \alpha\beta|$$

Lattice Systems



Hilbert space

$$\mathcal{H} \subset \mathcal{H}_{\text{gauge fields}} \otimes \mathcal{H}_{\text{fermions}}$$

A general state

$$|\Psi\rangle = \int \mathcal{D}\mathcal{G}\, |\mathcal{G}\rangle \left|\Psi_F(\mathcal{G})\right\rangle$$

with $\mathcal{D}\mathcal{G} = \prod_{\mathbf{x},k} dg(\mathbf{x}, k)$

Erez Zohar and J. Ignacio Cirac (2018) Phys. Rev. D **97** p. 034510
Patrick Emonts and Erez Zohar (2020) SciPost Phys. Lect. Notes p. 12

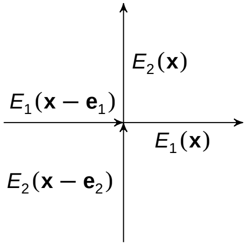
Gauss law

Gauss law

$$\sum_k \left(E_k(\mathbf{x}) - E_k(\mathbf{x}-\mathbf{e}_i)\right)|\text{phys}\rangle = 0 \quad \forall \mathbf{x}$$

Classical analogue in (cont.) electrodynamics

$$\nabla \cdot \mathbf{E} = 0$$



Expectation value of an Observable

Assume that O acts only on the gauge field and is diagonal in the group element basis:

$$
\begin{aligned}
\langle O \rangle &= \frac{\langle \Psi | O | \Psi \rangle}{\langle \Psi | \Psi \rangle} \\
&= \frac{\int \mathcal{DG} \, \langle \mathcal{G} | \, O \, | \mathcal{G} \rangle \, \langle \Psi_F(\mathcal{G}) | \Psi_F(\mathcal{G}) \rangle}{\int \mathcal{DG}' \, \langle \Psi_F(\mathcal{G}') | \Psi_F(\mathcal{G}') \rangle} \\
&= \int \mathcal{DG} \mathcal{F}_O(\mathcal{G}) p(\mathcal{G})
\end{aligned}
$$

with $p(\mathcal{G}) = \frac{\langle \Psi_F(\mathcal{G}) | \Psi_F(\mathcal{G}) \rangle}{\int \mathcal{DG}' \langle \Psi_F(\mathcal{G}') | \Psi_F(\mathcal{G}') \rangle} = \frac{\langle \Psi_F(\mathcal{G}) | \Psi_F(\mathcal{G}) \rangle}{Z}$

The rest of this talk

Expectation value

$$\langle O \rangle = \int \mathcal{D}\mathcal{G}\mathcal{F}_O(\mathcal{G})p(\mathcal{G})$$

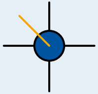
$$\text{with } p(\mathcal{G}) = \frac{\langle \Psi_F(\mathcal{G}) | \Psi_F(\mathcal{G}) \rangle}{Z}$$

TODO List

1. How do we construct $|\Psi_F(\mathcal{G})\rangle$?
2. How do we efficiently calculate $p(\mathcal{G})$?
3. Are those states useful?

Let’s use a tensor network

The Tensor

$\mathcal{A}(\mathbf{x}) =$ 

The Projector

$\omega_\ell =$ 

$$\mathcal{A}(\mathbf{x}) = \exp\left(T_{ij} a_i^\dagger a_j^\dagger\right)$$

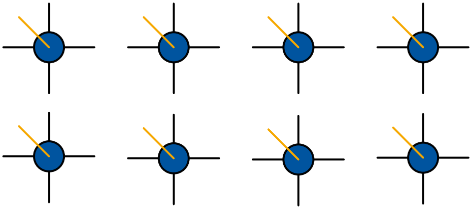
with

$$a \in \{r, u, l, d, \Psi\}$$

$$\omega(\mathbf{x}, k) = \omega_k(\mathbf{x}) \Omega_k(\mathbf{x}) \omega_k^\dagger(\mathbf{x})$$

$$\omega_0(\mathbf{x}) = \exp\left(l^\dagger(\mathbf{x} + \mathbf{e}_1) r^\dagger(\mathbf{x})\right)$$

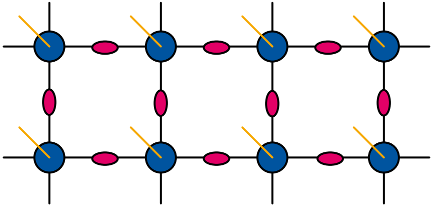
Building a state



Formula

$$\prod_{\mathbf{x}} \mathcal{A}(\mathbf{x}) \left|\Omega\right\rangle$$

Building a state



Formula

$$|\psi_0\rangle = \langle\Omega_v| \prod_{\ell} \omega_\ell \prod_{\mathbf{x}} \mathcal{A}(\mathbf{x}) \, |\Omega\rangle$$

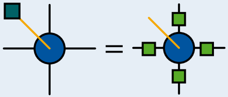
Global Invariance

Global Gauge Transformation

$$e^{i\Lambda \sum_{\mathbf{x}} Q(\mathbf{x})} |\psi_0\rangle$$

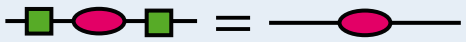
Gauge Invariance of the tensors

Acting on a physical degree of freedom with a gauge transformation is equivalent to acting on all auxiliary degrees of freedom.



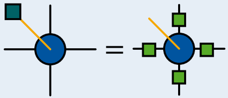
Gauge Invariance of the projectors

The projectors are invariant under auxiliary gauge transformations.



Virtual vs Physical Gauge Invariance

Virtual Gauge Invariance



- ✔ Physical charge, virtual electric fields
- ✘ Physical symmetry is global

Physical Gauge Invariance

?

- ? Physical charge, physical electric fields
- ? Physical symmetry is local

Global Gauge Transformation

$$e^{i\Lambda \sum_{\mathbf{x}} Q(\mathbf{x})} |\psi_0\rangle = |\psi_0\rangle$$

Local Gauge transformation

$$e^{i\Lambda \mathcal{G}(\mathbf{x})} |\psi\rangle = |\psi\rangle$$

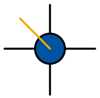
Minimal coupling of a state

Goal

Couple the gauge field to the state such that it is locally invariant under gauge transformations.

Substitution procedure

$$|\psi_0\rangle = \langle \Omega_v| \prod_{\ell} \omega_{\ell} \prod_{\mathbf{x}} \mathcal{A}(\mathbf{x}) \, |\Omega\rangle$$



Erez Zohar and Michele Burrello (2016) New J. Phys. **18** p. 043008; Erez Zohar and J. Ignacio Cirac (2018) Phys. Rev. D **97** p. 034510

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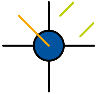
$$\rightarrow \langle\Omega_v| \prod_\ell \omega_\ell \prod_{\mathbf{x}} \mathcal{A}(\mathbf{x}) |0\rangle_{\mathbf{x},1} |0\rangle_{\mathbf{x},2} |\Omega\rangle$$



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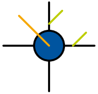
$$|\psi_0\rangle = \langle\Omega_v| \prod_\ell \omega_\ell \prod_{\mathbf{x}} \mathcal{A}(\mathbf{x}) |\Omega\rangle$$

$$\to \langle\Omega_v| \prod_\ell \omega_\ell \prod_{\mathbf{x}} \mathcal{A}(\mathbf{x}) |0\rangle_{\mathbf{x},1} |0\rangle_{\mathbf{x},2} |\Omega\rangle$$

$$|\psi\rangle = \langle\Omega_v| \prod_\ell \omega_\ell \prod_{\mathbf{x}} \mathcal{U}_G(\mathbf{x},1)\mathcal{U}_G(\mathbf{x},2) \prod_{\mathbf{x}} \mathcal{A}(\mathbf{x}) |0\rangle_{\mathbf{x},1} |0\rangle_{\mathbf{x},2} |\Omega\rangle$$



Erez Zohar and Michele Burrello (2016) New J. Phys. **18** p. 043008; Erez Zohar and J. Ignacio Cirac (2018) Phys. Rev. D **97** p. 034510

Where are we now?

TODO List

1. How do we construct $|\Psi_F(\mathcal{G})\rangle$? ✔
2. How do we efficiently calculate $p(\mathcal{G})$?
3. Are those states useful?

The probability

$$p(\mathcal{G}) = \frac{\langle \Psi_F(\mathcal{G}) | \Psi_F(\mathcal{G}) \rangle}{\int \mathcal{D}\mathcal{G}' \langle \Psi_F(\mathcal{G}') | \Psi_F(\mathcal{G}') \rangle}$$

Who is scared of norms?

Matrix Product State
Efficient computation of
▪ Norm
▪ Expectation values
▪ Correlation functions
(canonical form)

PEPS
No efficient way to compute exactly
▪ Norm
▪ Expectation values
▪ Correlation functions

Norbert Schuch et al. (2007) Phys. Rev. Lett. **98** p. 140506

Is $|\Psi_F(\mathcal{G})\rangle$ special?

The fermionic state $|\Psi_F(\mathcal{G})\rangle$

$$|\Psi_F(\mathcal{G})\rangle = \langle\Omega_v| \prod_{\mathbf{x}} \omega(\mathbf{x}) \prod_{\mathbf{x}} \mathcal{U}_{\Phi(\mathbf{x})} \prod_{\mathbf{x}} \mathcal{A}(\mathbf{x}) |\Omega\rangle$$

$$\mathcal{A}(\mathbf{x}) = \exp\left(\sum_{ij} T_{ij} a_i^\dagger(\mathbf{x}) a_j^\dagger(\mathbf{x})\right)$$

$$\omega(x) = \omega_0(\mathbf{x})\omega_1(\mathbf{x})\Omega(\mathbf{x})\omega_1^\dagger(\mathbf{x})\omega_0^\dagger(\mathbf{x})$$

$$\omega_0(\mathbf{x}) = \exp\left(l^\dagger(\mathbf{x}+\mathbf{e}_1) r^\dagger(\mathbf{x})\right)$$

$$\omega_1(\mathbf{x}) = \exp\left(d^\dagger(\mathbf{x}+\mathbf{e}_2) u^\dagger(\mathbf{x})\right)$$

Gaussian States

Definition

Fermionic Gaussian states are represented by density operators that are exponentials of a quadratic form in Majorana operators.

$$\rho = K \exp\left(-\tfrac{i}{4}\gamma^T G \gamma\right)$$

Covariance matrix

Covariance matrix for a state Φ:

$$\Gamma_{ab} = \tfrac{i}{2}\left\langle [\gamma_a, \gamma_b] \right\rangle = \tfrac{i}{2}\frac{\langle \Phi | [\gamma_a, \gamma_b] | \Phi \rangle}{\langle \Phi | \Phi \rangle}$$

Sergey Bravyi (2005) Quantum Inf. and Comp. **5** pp. 216–238

What is actually on the computer?

Covariance matrices all the way

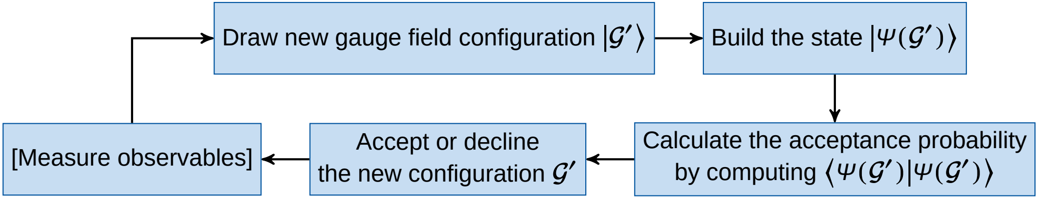
$$|\psi(\mathcal{G})\rangle = \langle\Omega_v| \underbrace{\prod_{\mathbf{x}} \omega(x) \prod_{\mathbf{x}} \mathcal{U}_{\Phi(\mathbf{x})}}_{\rightsquigarrow \Gamma_{\text{in}}(\mathcal{G})} \underbrace{\prod_{\mathbf{x}} A(\mathbf{x})}_{\rightsquigarrow \Gamma_M} |\Omega\rangle$$

Norm of the state

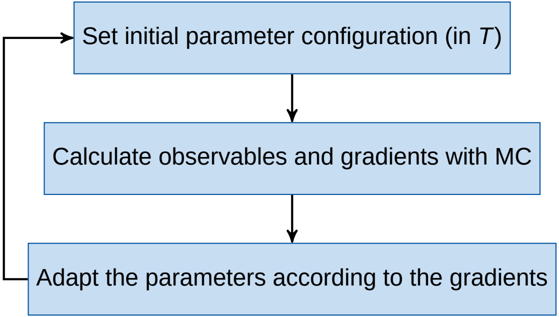
$$\langle \Psi_F(\mathcal{G}) | \Psi_F(\mathcal{G}) \rangle = \sqrt{\det\left(\frac{1 - \Gamma_{\text{in}}(\mathcal{G}) D}{2}\right)}$$

$$\Gamma_M = \begin{pmatrix} A & B \\ -B^T & D \end{pmatrix}$$

The whole framework for a given set of parameters



Variational Monte Carlo



Where are we now?

TODO List

1. How do we construct $|\Psi_F(\mathcal{G})\rangle$? ✔
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3. Are those states useful?

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Application to pure $\mathbb{Z}_2$

Properties

1. No physical fermions
2. Abelian gauge theory

Kogut Susskind Hamiltonian

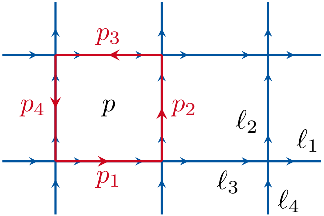
$$
\begin{aligned}
H &= H_E + H_B \\
&= \frac{g^2}{2} \sum_{\ell} \left[2 - (P_\ell + P_\ell^\dagger) \right] + \frac{1}{2g^2} \sum_p \left[2 - (Q_{p_1}^\dagger Q_{p_2}^\dagger Q_{p_3} Q_{p_4} + \text{H.c.}) \right]
\end{aligned}
$$

John Kogut and Leonard Susskind (1975) Phys. Rev. D **11** pp. 395–408
D. Horn, M. Weinstein, and S. Yankielowicz (1979) Phys. Rev. D **19** pp. 3715–3731

Application to pure $\mathbb{Z}_2$

Kogut Susskind Hamiltonian

$$H = H_E + H_B$$
$$= \lambda \sum_{\ell} \left[1 - \sigma^z_{\ell}\right] + \frac{1}{\lambda} \sum_{p} \left[1 - \sigma^x_{p_1}\sigma^x_{p_2}\sigma^x_{p_3}\sigma^x_{p_4}\right],$$



$$P_\ell^2 = Q_\ell^2 = 1 \qquad P_\ell^\dagger P_\ell = Q_\ell^\dagger Q_\ell = 1$$
$$P_\ell^\dagger Q_\ell P_\ell = e^{i\pi} Q_\ell.$$

What changes in the Ansatz

The full state

$$|\Psi\rangle = \int \mathcal{D}\mathcal{G} \left|\Psi_F(\mathcal{G})\right\rangle |\mathcal{G}\rangle \longrightarrow |\Psi\rangle = \sum_{\mathcal{G}} \Psi_F(\mathcal{G}) |\mathcal{G}\rangle$$

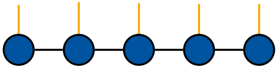
Ansatz with physical fermions

$$\left|\psi_F(\mathcal{G})\right\rangle = \langle\Omega_V| \prod_{\mathbf{x}} \omega(\mathbf{x}) \prod_{\mathbf{x}} \mathcal{U}_{\Phi(\mathbf{x})} \prod_{\mathbf{x}} A(\mathbf{x}) |\Omega\rangle$$

Ansatz without physical fermions

$$\psi_F(\mathcal{G}) = \langle\Omega_V| \prod_{\mathbf{x}} \omega(\mathbf{x}) \prod_{\mathbf{x}} \mathcal{U}_{\Phi(\mathbf{x})} \prod_{\mathbf{x}} \tilde{A}(\mathbf{x}) \left|\Omega_V\right\rangle$$

Talking about bond dimension



MPS

$$|\psi\rangle = \sum_{\{i\}} \sum_{\{\alpha\}} A^{i_1}_{1,\alpha_1} A^{i_2}_{\alpha_1,\alpha_2} \cdots A^{i_N}_{\alpha_{N-1},1} \left|i_1, \cdots, i_N\right\rangle$$

Minimal Ansatz

1 virtual fermion per link

Optimized Ansatz

2 virtual fermions per link

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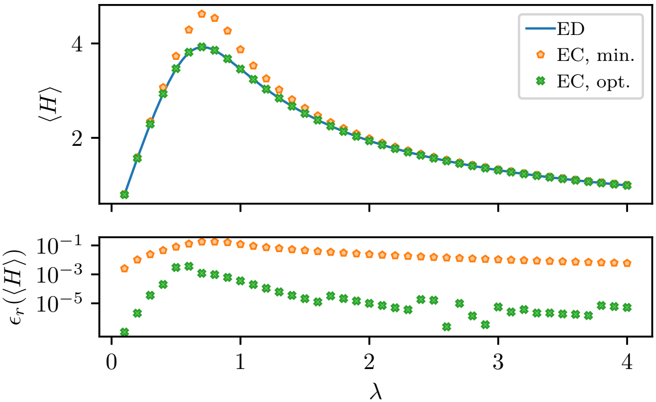
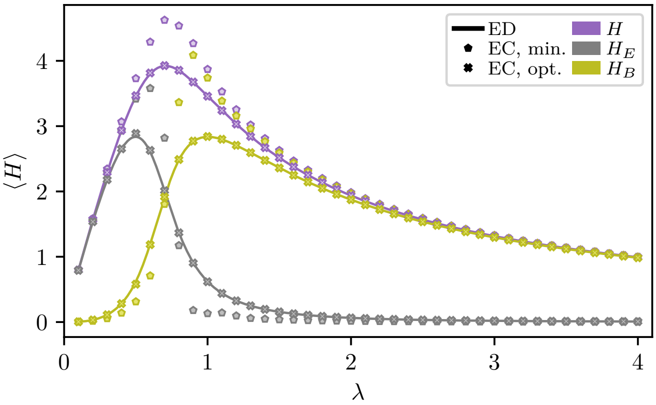
Results for an exact contraction calculation (L=2)



Patrick Emonts et al. (2023) Phys. Rev. D **107** p. 014505

Magnetic and electric energy



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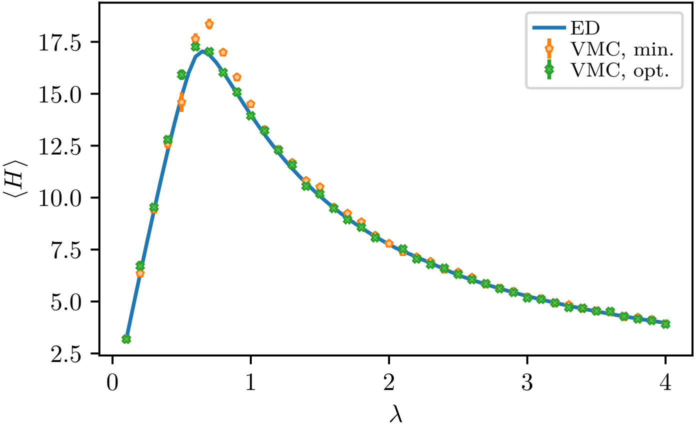
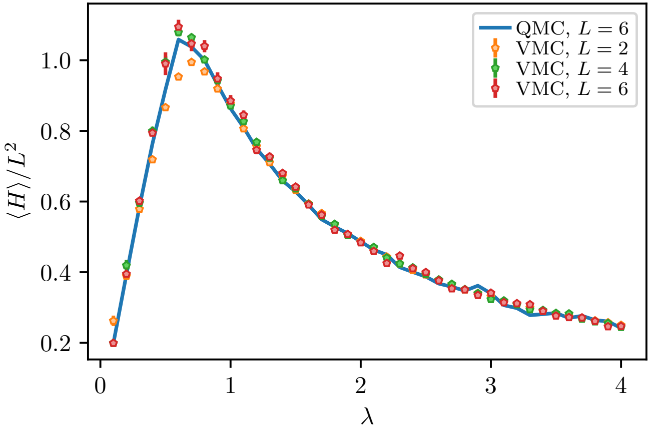
Energy minimization with Monte Carlo calculations



Patrick Emonts et al. (2023) Phys. Rev. D **107** p. 014505

Energy minimization with Monte Carlo calculations



Patrick Emonts et al. (2023) Phys. Rev. D **107** p. 014505

Science is a team effort

Ariel Kelman

Umberto Borla

Sergej Moroz

Snir Gazit

Erez Zohar

Summary and Outlook

Summary

- A Hamiltonian approach shows promising possibilities (time evolution, finite chemical potential)
- Carefully designing the Ansatz is important (analytical guarantee on gauge-symmetry)
- Good results for the energy minimization across the whole spectrum for $\mathbb{Z}_2$

Open Questions

- What happens in three space dimensions?
- What happens if we add physical fermions?
- And if we do not use tensor networks?

Advertisement

A variational Monte Carlo algorithm for lattice gauge theories with continuous gauge groups: a study of (2+1)-dimensional compact QED with dynamical fermions at finite density

Julian Bender, PE, Ignacio Cirac
arxiv:2304.*

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The Netherlands

Simulating (2+1)d Lattice Gauge Theories with Fermionic Tensor Networks

Institute for Nuclear Theory, Seattle

April 04, 2023 | Patrick Emonts | Lorentz Institute, Leiden University

Universiteit Leiden
The Netherlands

References I

[1] Shinichiro Akiyama and Daisuke Kadoh. “More about the Grassmann Tensor Renormalization Group”. In: Journal of High Energy Physics 2021.10 (Oct. 22, 2021), p. 188. ISSN: 1029-8479. DOI: 10.1007/JHEP10(2021)188.

[2] Mari Carmen Bañuls and Krzysztof Cichy. “Review on Novel Methods for Lattice Gauge Theories”. In: Reports on Progress in Physics 83.2 (Feb. 1, 2020), p. 024401. ISSN: 0034-4885, 1361-6633. DOI: 10.1088/1361-6633/ab6311.

[3] Sergey Bravyi. “Lagrangian Representation for Fermionic Linear Optics”. In: Quantum Inf. and Comp. 5.3 (2005), pp. 216–238.

[4] Jacob C. Bridgeman and Christopher T. Chubb. “Hand-Waving and Interpretive Dance: An Introductory Course on Tensor Networks”. In: Journal of Physics A: Mathematical and Theoretical 50.22 (June 2, 2017), p. 223001. ISSN: 1751-8113, 1751-8121. DOI: 10.1088/1751-8121/aa6dc3.

References II

[5] Nouman Butt et al. “Tensor Network Formulation of the Massless Schwinger Model with Staggered Fermions”. In: Physical Review D 101.9 (May 26, 2020), p. 094509. ISSN: 2470-0010, 2470-0029. DOI: 10.1103/PhysRevD.101.094509.

[6] Philippe Corboz et al. “Simulation of Strongly Correlated Fermions in Two Spatial Dimensions with Fermionic Projected Entangled-Pair States”. In: Phys. Rev. B 81.16 (Apr. 2010), p. 165104. DOI: 10.1103/PhysRevB.81.165104.

[7] Alexandre Deur, Stanley J. Brodsky, and Guy F. de Téramond. “The QCD Running Coupling”. In: Progress in Particle and Nuclear Physics 90 (Sept. 2016), pp. 1–74. ISSN: 01466410. DOI: 10.1016/j.ppnp.2016.04.003.

[8] Carl Eckart and Gale Young. “The Approximation of One Matrix by Another of Lower Rank”. In: Psychometrika 1.3 (Sept. 1936), pp. 211–218. ISSN: 0033-3123, 1860-0980. DOI: 10.1007/BF02288367.

References III

[9] Patrick Emonts and Erez Zohar. “Gauss Law, Minimal Coupling and Fermionic PEPS for Lattice Gauge Theories”. In: SciPost Physics Lecture Notes (Jan. 17, 2020), p. 12. ISSN: 2590-1990. DOI: `10.21468/SciPostPhysLectNotes.12`.

[10] Patrick Emonts et al. “Finding the Ground State of a Lattice Gauge Theory with Fermionic Tensor Networks: A $2+1d$ $\mathbb{Z}_2$ Demonstration”. In: Physical Review D 107.1 (Jan. 4, 2023), p. 014505. ISSN: 2470-0010, 2470-0029. DOI: `10.1103/PhysRevD.107.014505`.

[11] M. Fannes, B. Nachtergaele, and R. F. Werner. “Finitely Correlated States on Quantum Spin Chains”. In: Communications in Mathematical Physics 144.3 (Mar. 1992), pp. 443–490. ISSN: 0010-3616, 1432-0916. DOI: `10.1007/BF02099178`.

References IV

[12] Timo Felser et al. “Two-Dimensional Quantum-Link Lattice Quantum Electrodynamics at Finite Density”. In: Physical Review X 10.4 (Nov. 25, 2020), p. 041040. DOI: 10.1103/PhysRevX.10.041040.

[13] D. Horn, M. Weinstein, and S. Yankielowicz. “Hamiltonian Approach to $Z(N)$ Lattice Gauge Theories”. In: Physical Review D 19.12 (June 15, 1979), pp. 3715–3731. ISSN: 0556-2821. DOI: 10.1103/PhysRevD.19.3715.

[14] John Kogut and Leonard Susskind. “Hamiltonian Formulation of Wilson’s Lattice Gauge Theories”. In: Physical Review D 11.2 (Jan. 15, 1975), pp. 395–408. ISSN: 0556-2821. DOI: 10.1103/PhysRevD.11.395.

[15] D. Perez-Garcia et al. “Matrix Product State Representations”. May 14, 2007.

[16] Manuel Schneider et al. “Simulating Both Parity Sectors of the Hubbard Model with Tensor Networks”. In: Physical Review B 104.15 (2021), p. 155118.

References V

[17] Norbert Schuch et al. “Computational Complexity of Projected Entangled Pair States”. In: Physical Review Letters 98.14 (Apr. 4, 2007), p. 140506. ISSN: 0031-9007, 1079-7114. DOI: 10.1103/PhysRevLett.98.140506.

[18] Kenneth G. Wilson. “Confinement of Quarks”. In: Physical Review D 10.CLNS-262 (8 Oct. 15, 1974), pp. 2445–2459. ISSN: 0556-2821. DOI: 10.1103/PhysRevD.10.2445.

[19] Erez Zohar and Michele Burrello. “Building Projected Entangled Pair States with a Local Gauge Symmetry”. In: New Journal of Physics 18.4 (Apr. 8, 2016), p. 043008. ISSN: 1367-2630. DOI: 10.1088/1367-2630/18/4/043008.

[20] Erez Zohar and J. Ignacio Cirac. “Combining Tensor Networks with Monte Carlo Methods for Lattice Gauge Theories”. In: Physical Review D 97.3 (Feb. 23, 2018), p. 034510. ISSN: 2470-0010, 2470-0029. DOI: 10.1103/PhysRevD.97.034510.