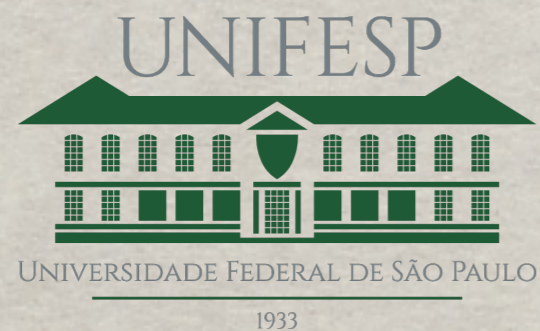


# Light-front tomography of light and heavy pseudoscalar mesons

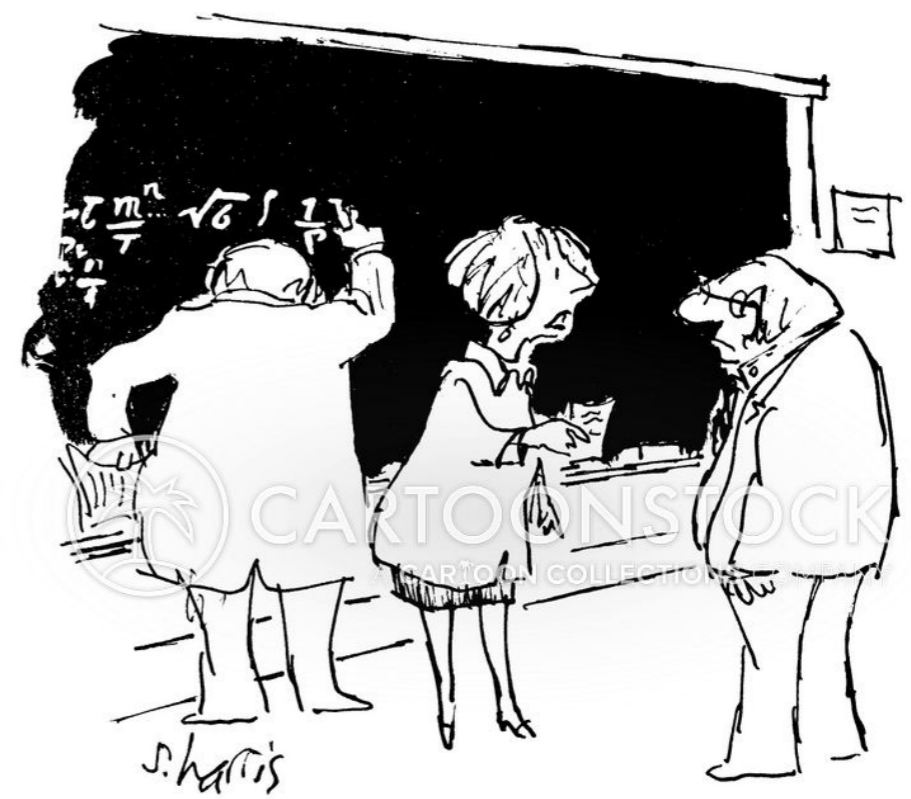
*"Electroweak and Beyond the Standard Model Physics at the EIC"  
INT Workshop, University of Washington, 12-16 September 2024*



Bruno El-Bennich  
Departamento de Física  
Universidade Federal de São Paulo



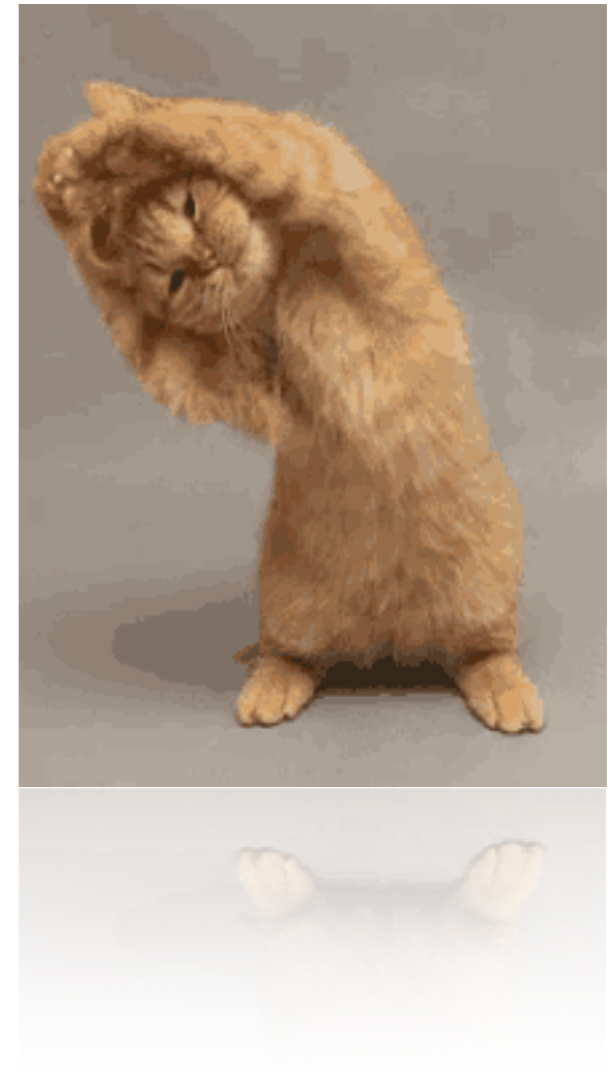




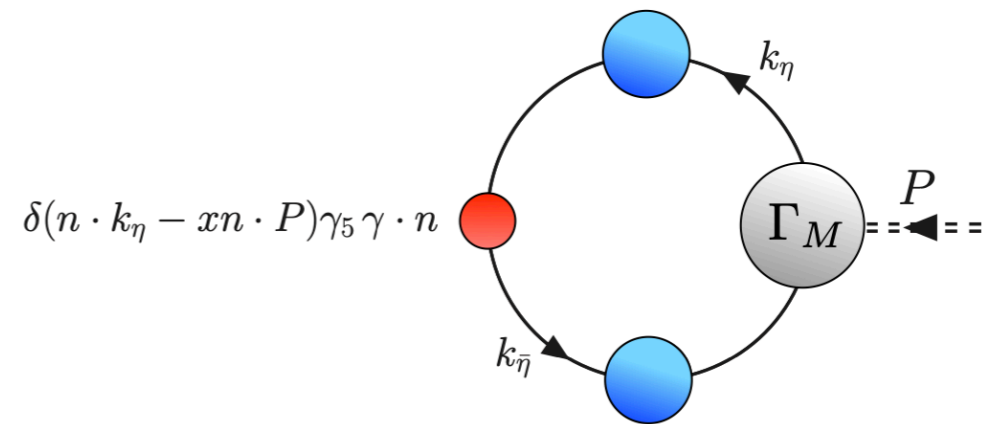
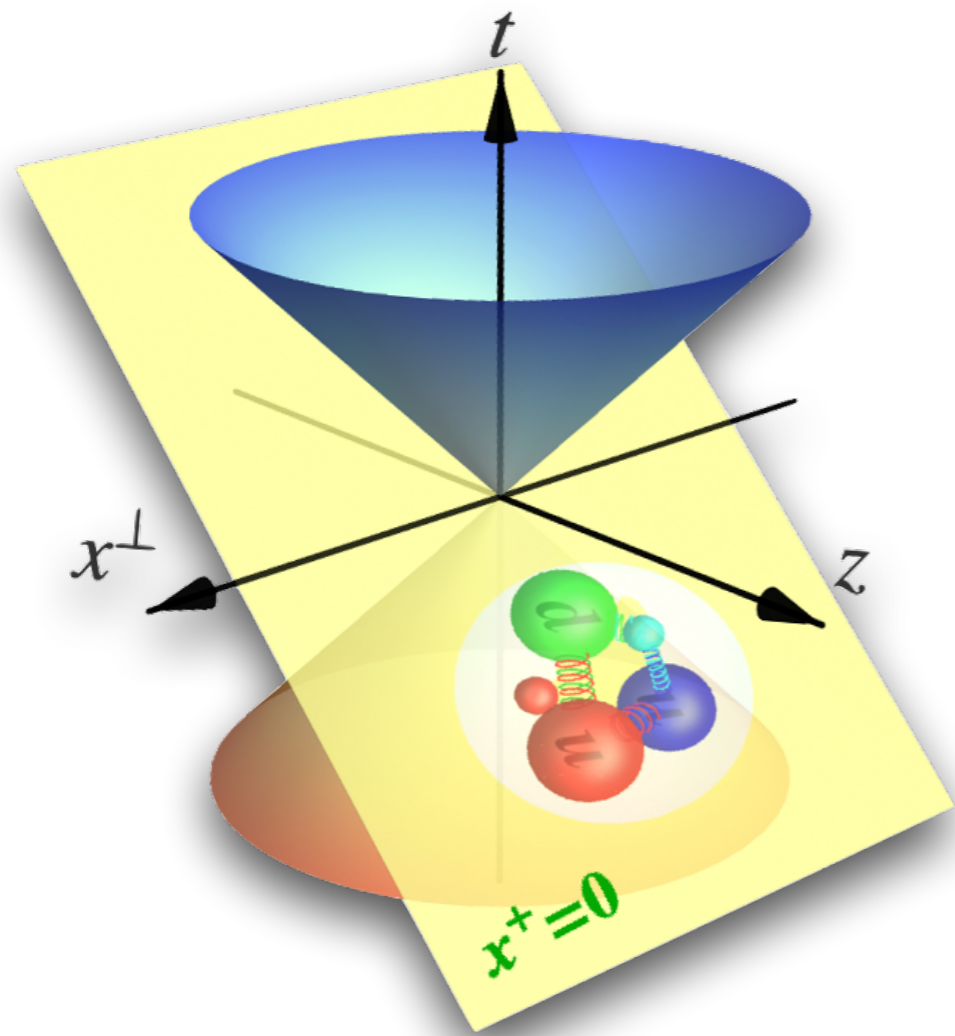
"WE COLLABORATE. I'M AN EXPERT, BUT NOT AN AUTHORITY, AND DR. GELPIS IS AN AUTHORITY, BUT NOT AN EXPERT."

## Collaborators:

Fernando Serna (Sucre, Colombia), Roberto Silveira (São Paulo, Brazil), Ian Cloët (Argonne, USA), Adnan Bashir (Morelia, Mexico & Jlab), Javier Cobos (Sonora, Mexico), Gastão Krein (São Paulo, Brazil)

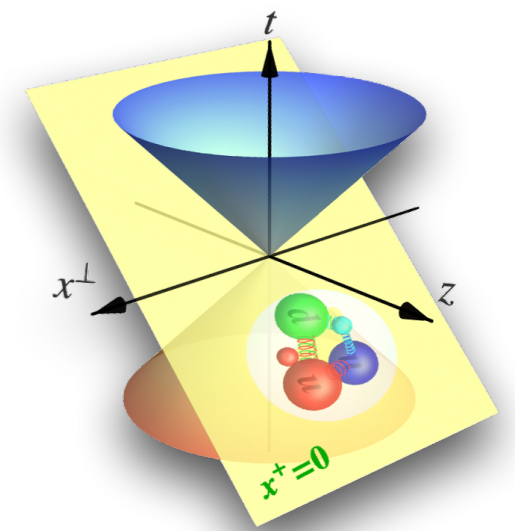


Before coming to the meat (LFWF), let's warm up with simpler distributions ....



# Light-Cone Distribution Amplitudes





# Light-Cone Distribution Amplitudes

- Hadron light-cone distribution amplitudes (LCDAs) have been introduced four decades ago in the context of the QCD description of hard exclusive reactions.
- The LCDAs are scale-dependent nonperturbative functions that can be interpreted as quantum-mechanical amplitudes.
- $\phi_M(x, \mu)$  is a probability amplitude that describes the momentum distribution of a quark and anti-quark in the bound-state's valence Fock state.
- $x$  is the light-front momentum fraction:  $\frac{k^+}{P^+}$  and  $\mu$  is the renormalization scale.

# Light-Front Distribution Amplitudes

QCD factorization in B decays involves matrix elements which are convolution integrals:

$$\langle \pi^+ \pi^- | (\bar{u}b)_{V-A} (\bar{d}u)_{V-A} | \bar{B}_d \rangle \rightarrow \int_0^1 d\xi du dv \Phi_B(\xi) \Phi_\pi(u) \Phi_\pi(v) T(\xi, u, v; m_b)$$

The integrals are over a (hard) scattering kernel  $T(\xi, u, v, m)$  and light-cone distribution amplitudes (LCDA) expanded in Gegenbauer polynomials:

$$\varphi_\pi(x; \tau) = \varphi_\pi^{\text{asy}}(x) \left[ 1 + \sum_{j=2,4,\dots}^{\infty} a_j^{3/2}(\tau) C_j^{(3/2)}(2x-1) \right]$$
$$\varphi_\pi^{\text{asy}}(x) = 6x(1-x)$$

- LCDA were poorly known for light mesons, in recent years improved determinations of the first two Gegenbauer moments of the pion and kaon, [RQCD Collaboration](#), Bali et al. (2019).
- Next to nothing was known about heavy-light mesons, mostly models and asymptotic LCDA used.



# Light-Front Distribution Amplitudes

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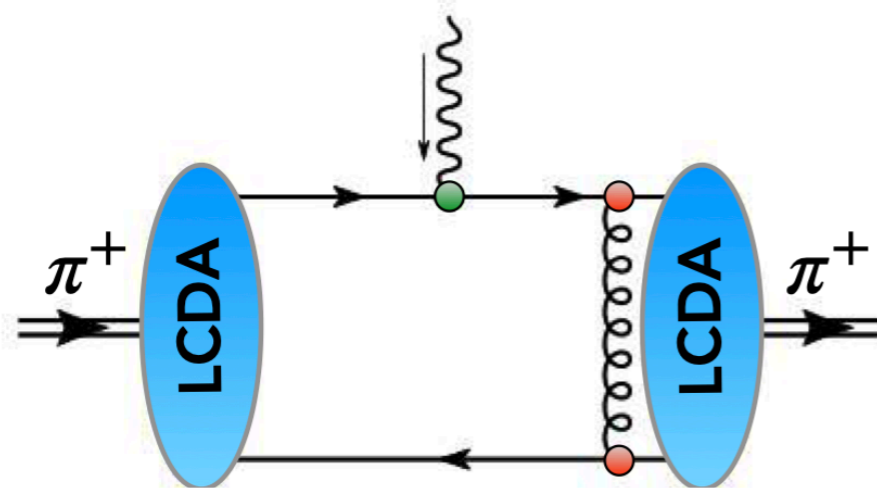
$$\langle \pi^+ \pi^- | (\bar{u}b)_{V-A} (\bar{d}u)_{V-A} | \bar{B}_d \rangle \rightarrow \int_0^1 d\xi du dv \Phi_B(\xi) \Phi_\pi(u) \Phi_\pi(v) T(\xi, u, v; m_b)$$

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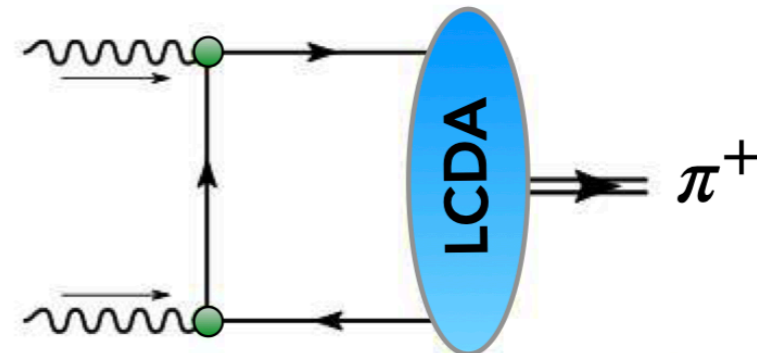
$$\phi_M(x) \neq \phi^{\text{asy}}(x) = 6x(1-x)$$

- LCDA were poorly known for light mesons, in recent years improved determinations of the first two Gegenbauer moments of the pion and kaon, [RQCD Collaboration](#), Bali et al. (2019).
- Next to nothing was known about heavy-light mesons, mostly models and asymptotic LCDA used.

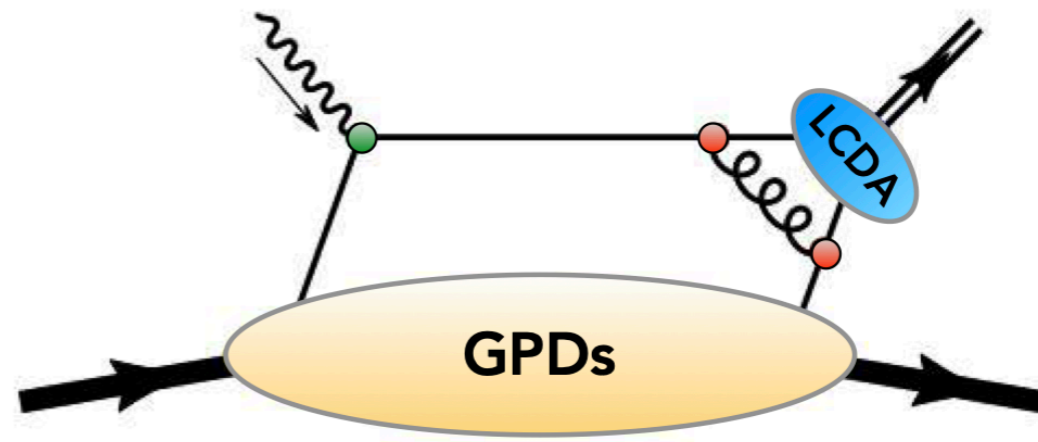
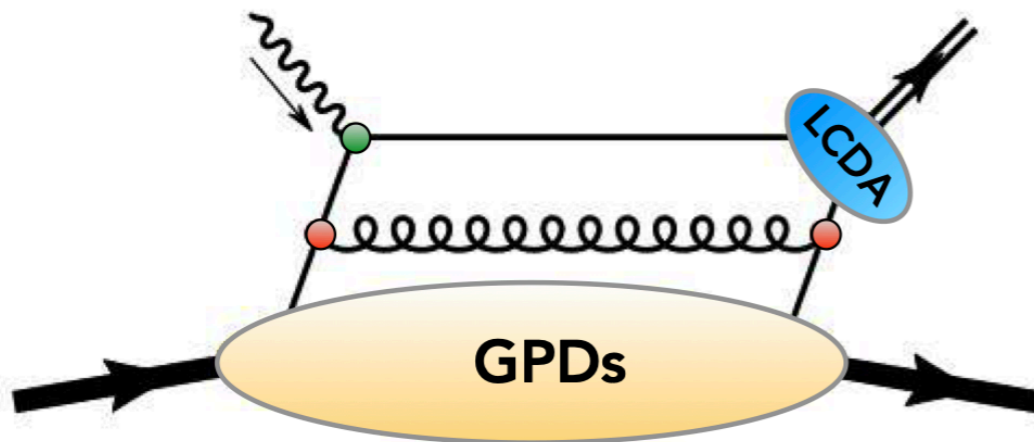
# Hard exclusive scattering processes



$$Q^2 F_\pi(Q^2) \rightarrow 16\pi f_\pi^2 \alpha_s(Q^2)$$



$$Q^2 F_{\gamma^* \gamma \pi}(Q^2) \rightarrow 2f_\pi$$





# Charged current production of charmed mesons

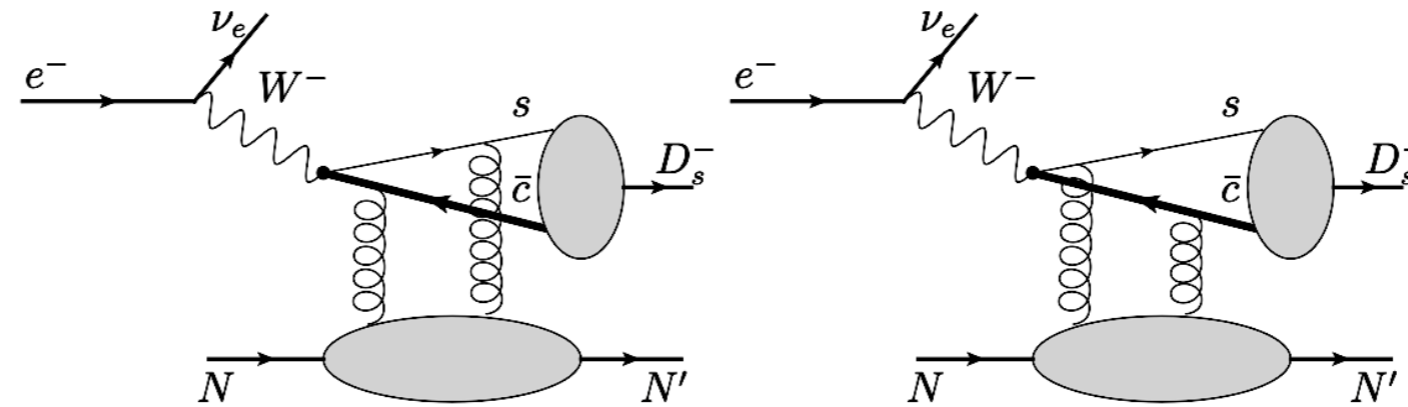


FIG. 1: Feynman diagrams for the factorized amplitude for the  $e^- + N \rightarrow \nu_e + D_s^- + N'$  process involving the gluon GPDs; the thick line represents the heavy anti-quark  $\bar{c}$ .

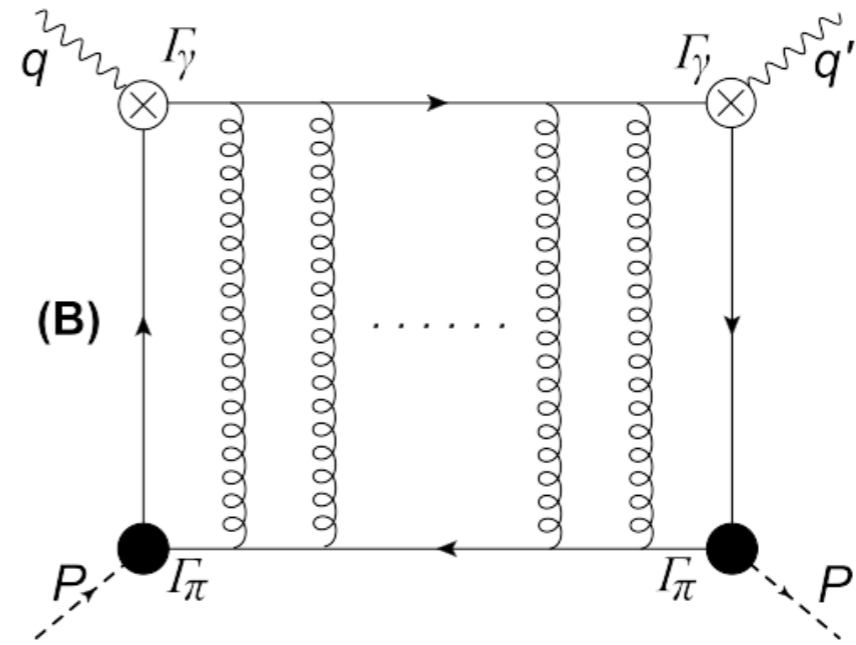
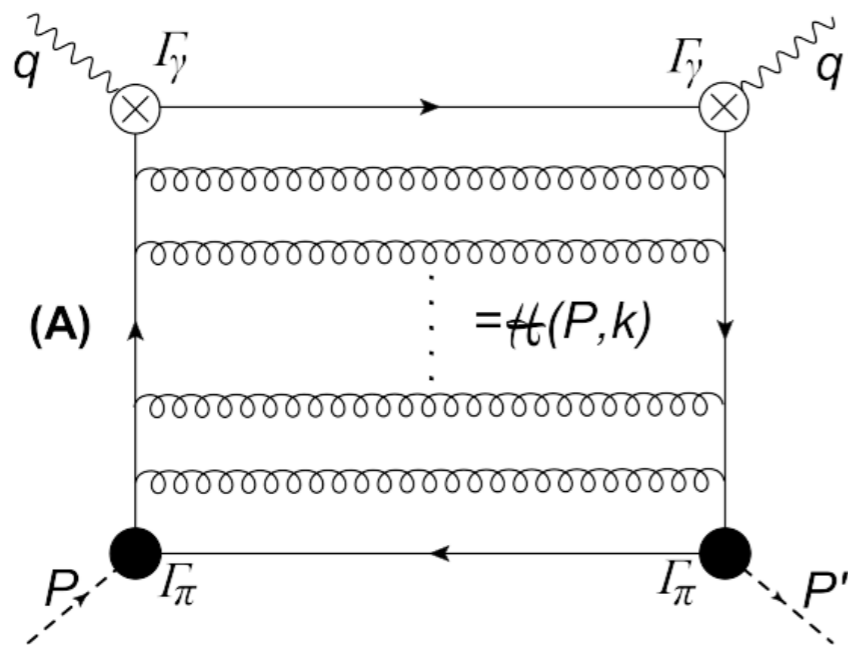
B. Pire, L. Szymanowski, and J. Wagner, Phys. Rev. D 104 (2021)

- Initial work by Siddikov and Schmidt for  $\pi$  and  $\rho$  production, Phys. Rev. D 99 (2019).
- Claim that their numerical estimates show that production could be studied electron-induced processes at EIC.
- Recently extended to  $D_s$  and  $D_s^*$  mesons in collinear QCD where generalized gluon distributions factorize from perturbatively calculable coefficient functions.
- Required input: pseudoscalar and vector meson *distribution amplitudes*.

# Parton Distribution Functions

- Contrary to Light-Cone Distribution Functions, these are measurable objects.
- Model approaches to calculation commonly based on “handbag” diagrams for photon-pion forward Compton scattering in Bjorken limit.

$$q_f(x) = \frac{1}{4\pi} \int d\lambda e^{-ixP \cdot n\lambda} \langle \pi(P) | \bar{\psi}_f(\lambda n) \not{n} \psi_f(0) | \pi(P) \rangle$$

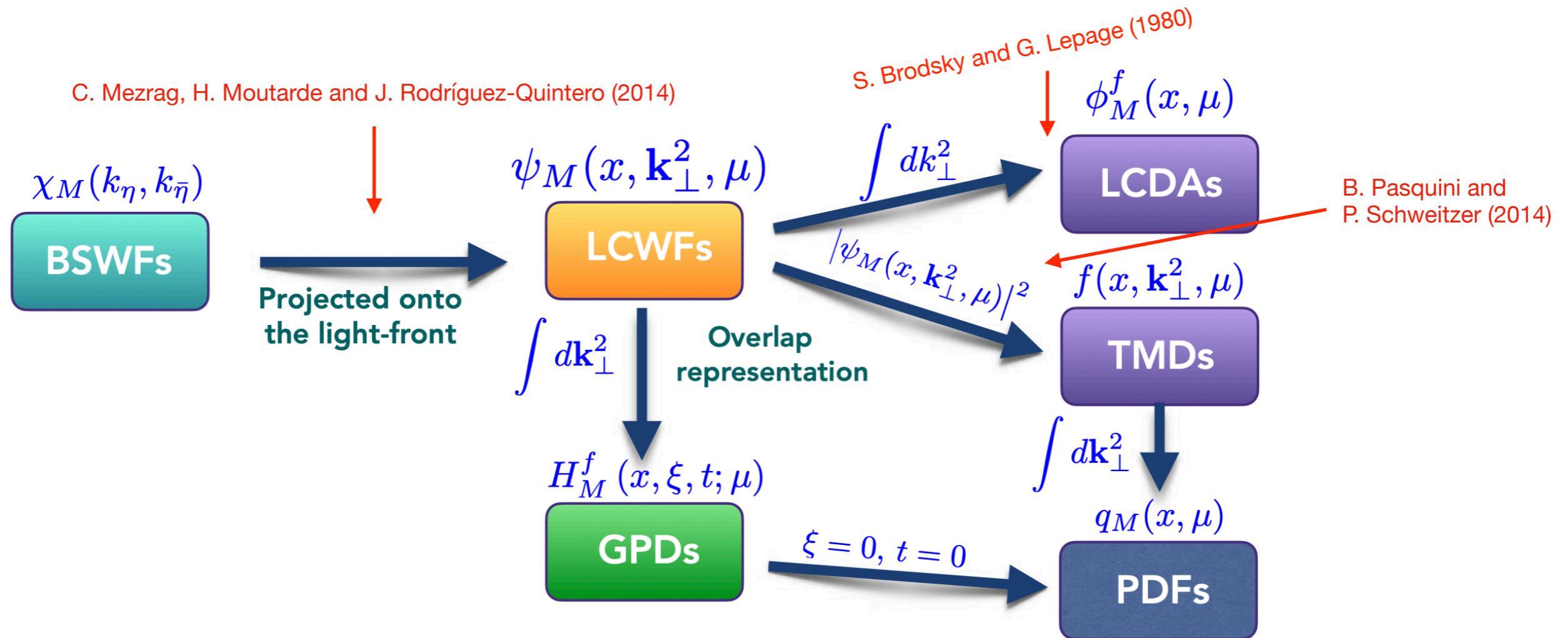




What if we could obtain all kind  
of distribution functions from  
one unified calculation ?

# Light-Front Wave Functions

With a particular projection of the Bethe-Salpeter wave functions we arrive the light-front wave functions, a more general object to describe probability amplitudes.



# Light-Front Wave Functions

- M. Burkardt, X.-D. Ji, and F. Yuan (2002) showed that for pseudoscalar mesons there are two independent light front wave functions for the leading Fock state, with  $l_z = 0$  and  $l_z = 1$ .
- Two-particle Fock-state configuration is given by:  $|M\rangle = |M\rangle_{l_z=0} + |M\rangle_{|l_z|=1}$

$$|M\rangle_{l_z=0} = i \int \frac{d^2 \mathbf{k}_T}{2(2\pi)^3} \frac{dx}{\sqrt{x\bar{x}}} \psi_0(x, \mathbf{k}_T^2) \frac{\delta_{ij}}{\sqrt{3}} \frac{1}{\sqrt{2}} \left[ b_{f\uparrow i}^\dagger(x, \mathbf{k}_T) d_{h\downarrow j}^\dagger(\bar{x}, \bar{\mathbf{k}}_T) - b_{f\downarrow i}^\dagger(x, \mathbf{k}_T) d_{h\uparrow j}^\dagger(\bar{x}, \bar{\mathbf{k}}_T) \right] |0\rangle$$

$$|M\rangle_{|l_z|=1} = i \int \frac{d^2 \mathbf{k}_T}{2(2\pi)^3} \frac{dx}{\sqrt{x\bar{x}}} \psi_1(x, \mathbf{k}_T^2) \frac{\delta_{ij}}{\sqrt{3}} \frac{1}{\sqrt{2}} \left[ k_T^- b_{f\uparrow i}^\dagger(x, \mathbf{k}_T) d_{h\uparrow j}^\dagger(\bar{x}, \bar{\mathbf{k}}_T) + k_T^+ b_{f\downarrow i}^\dagger(x, \mathbf{k}_T) d_{h\downarrow j}^\dagger(\bar{x}, \bar{\mathbf{k}}_T) \right] |0\rangle$$

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- Two-particle Fock-state configuration is given by:  $|M\rangle = |M\rangle_{l_z=0} + |M\rangle_{|l_z|=1}$

The LFWFs are obtained from the Bethe-Salpeter wave function via the light front projections:

$$\psi_0(x, \mathbf{k}_\perp^2) = \sqrt{3}i \int \frac{dk^+ dk^-}{\pi} \delta(xP^+ - k^+) \text{Tr}_D [\gamma^+ \gamma_5 \chi(k, P)]$$

$$\psi_1(x, \mathbf{k}_\perp^2) = -\frac{\sqrt{3}i}{\mathbf{k}_\perp^2} \int \frac{dk^+ dk^-}{\pi} \delta(xP^+ - k^+) \text{Tr}_D [i\sigma_{+i} k_T^i \gamma_5 \chi(k, P)]$$

C. Mezrag, H. Moutarde, and J. Rodriguez-Quintero, Few Body Syst. 57 (2016)

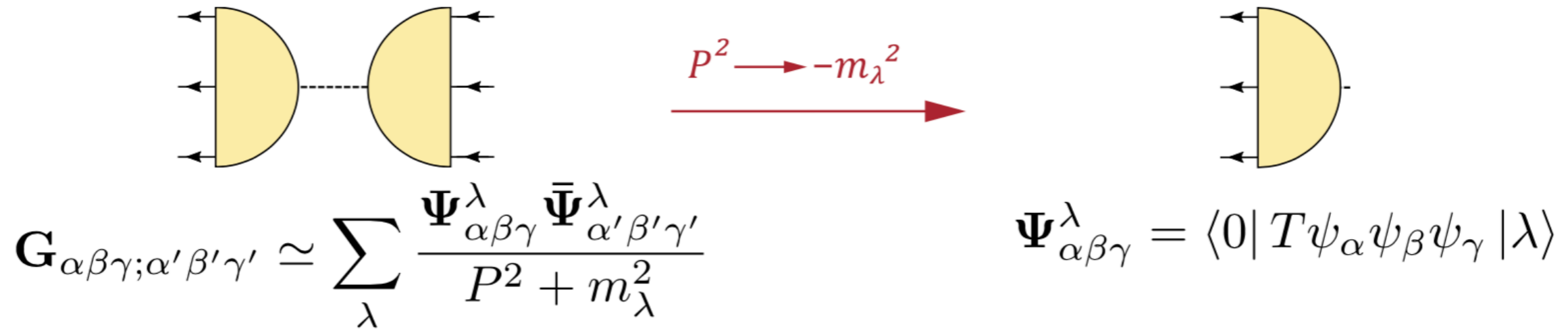
C. Shi and I. C. Cloët, Phys. Rev. Lett. 122, 082301 (2019)



$$(\square_x + m^2) G(x, y) = -\delta(x - y)$$

Green functions in  
functional approaches to QCD

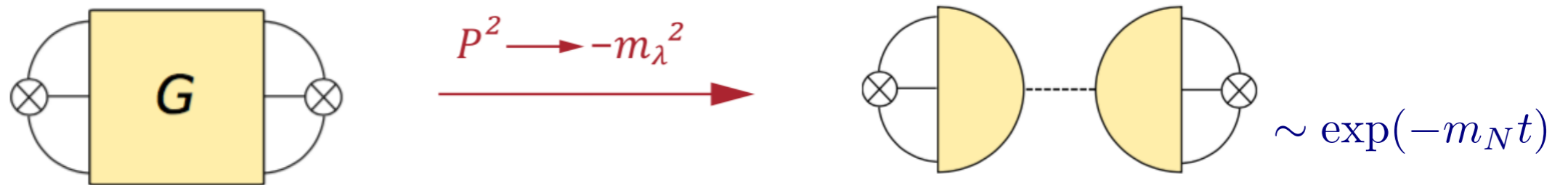
# Everything we need to know is encoded in $n$ -point Green functions



**Spectral decomposition:** extract gauge-invariant baryon poles from gauged-fixed quark 6-point functions



Residue at pole: Bethe-Salpeter wave function, contains all information about baryon



**Lattice:** extract baryon poles from gauge-invariant correlators



exponential Euclidean time decay

# Dyson-Schwinger equation in QCD

The propagator can be obtained from QCD's gap equation: the Dyson-Schwinger equation (DSE) for the dressed-fermion self-energy, which involves the set of infinitely many coupled equations.

$$[\text{fermion with self-energy}]^{-1} = [\text{bare fermion}]^{-1} + \text{loop diagram}$$

$$S^{-1}(p) = Z_2(i\gamma \cdot p + m^{\text{bm}}) + \Sigma(p) := i\gamma \cdot p A(p^2) + B(p^2)$$

$$\Sigma(p) = Z_1 \int^{\Lambda} \frac{d^4 q}{(2\pi)^4} g^2 D_{\mu\nu}(p-q) \frac{\lambda^a}{2} \gamma_{\mu} S(q) \Gamma_{\nu}^a(q, p)$$

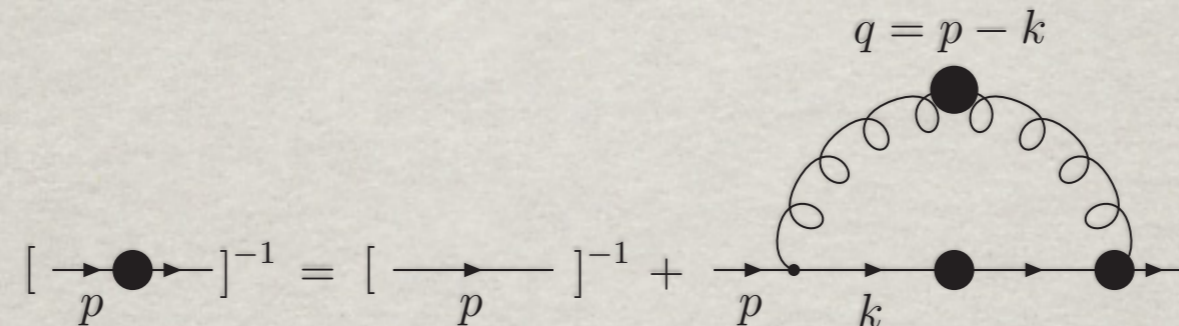
with the *running* mass function  $M(p^2) = B(p^2)/A(p^2)$ .

- $D_{\mu\nu}$  : dressed-gluon propagator
- $\Gamma_{\nu}^a(q, p)$  : dressed quark-gluon vertex
- $Z_2$  : quark wave function renormalization constant
- $Z_1$  : quark-gluon vertex renormalization constant



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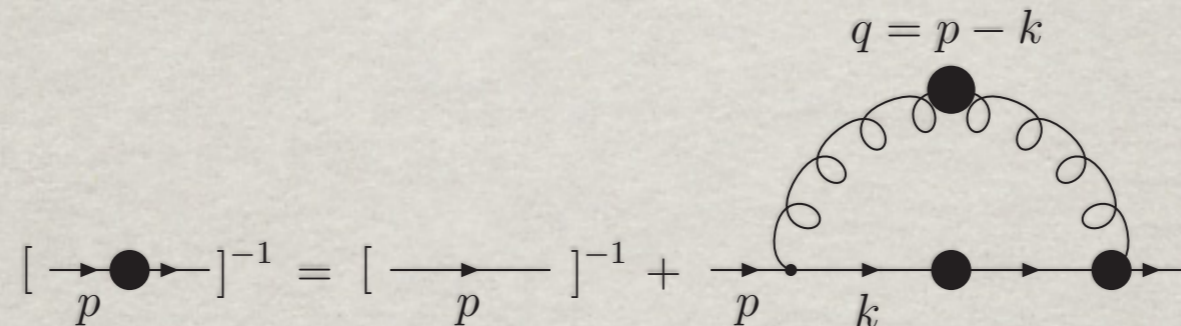
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Each satisfies its own DSE



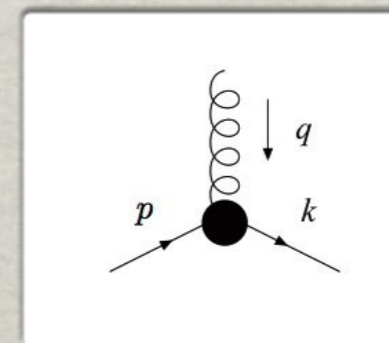
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# Dyson-Schwinger equation in QCD

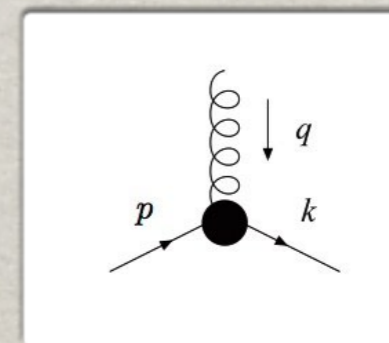
The propagator can be obtained from QCD's gap equation: the Dyson-Schwinger equation (DSE) for the dressed-fermion, which is coupled equations.

## Running Quark Mass

$$S(p) = \frac{Z(p^2)}{i\gamma \cdot p + M(p^2)}$$

$$S^{-1}(p) = Z_2(i\gamma \cdot p + m^{\text{bm}}) + \Sigma(p) := i\gamma \cdot p A(p^2) + B(p^2)$$

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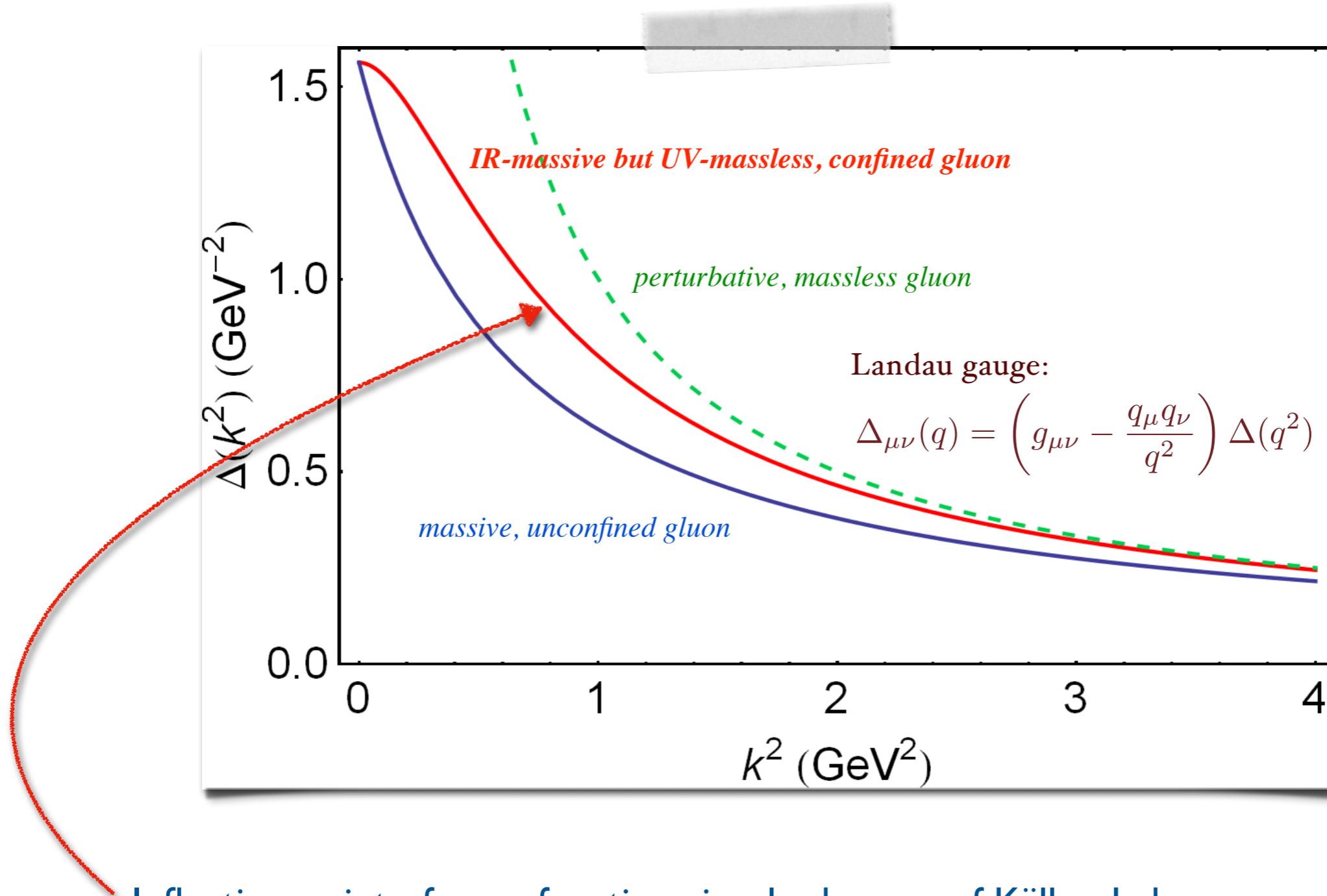


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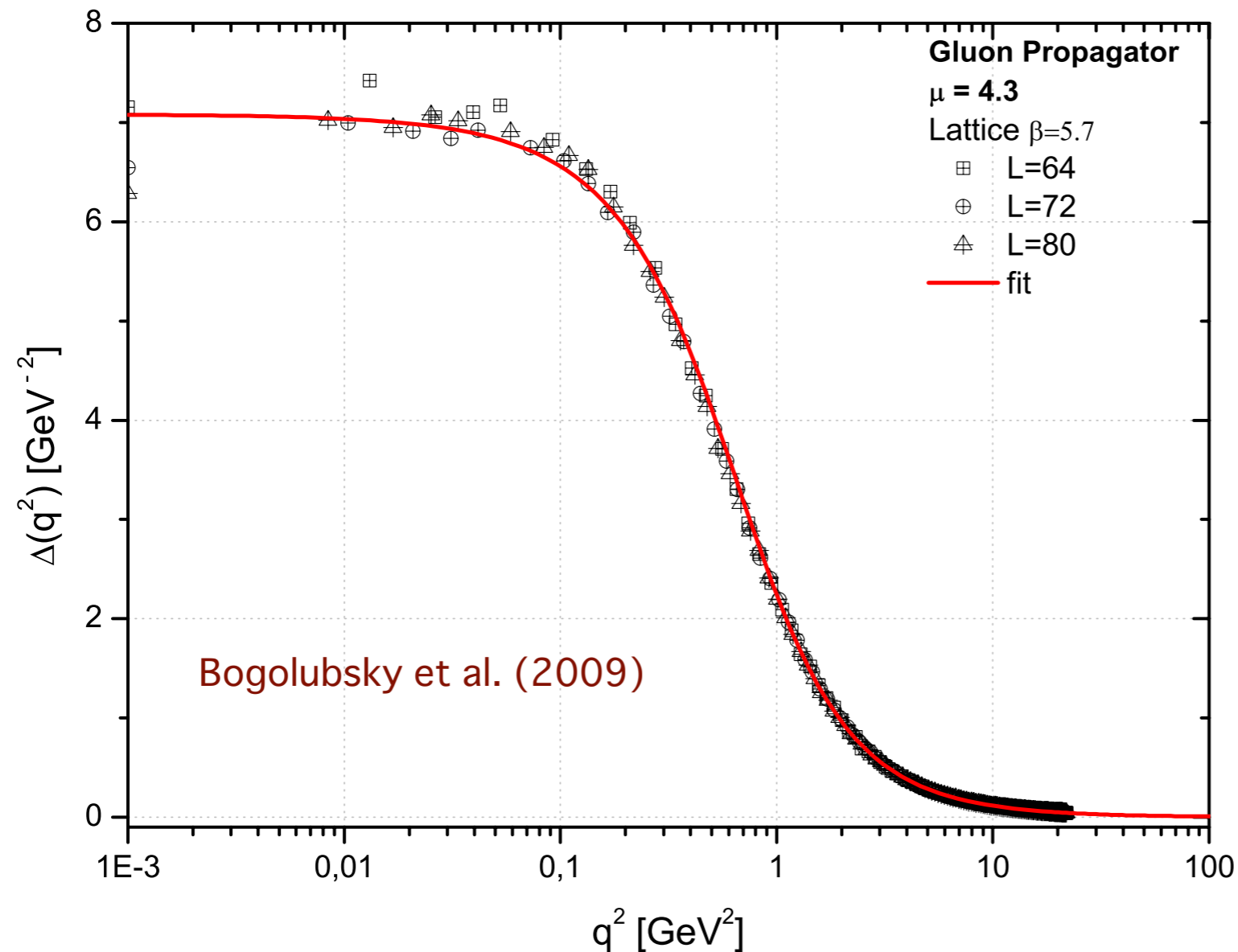
Each satisfies its own DSE

# Gluon infrared behavior



Inflection point of mass function signals absence of Källen-Lehmann spectral representation  $\Rightarrow$  violation of reflection positivity  $\Rightarrow$  **Confinement**

# Lattice QCD gluon dressing function



Landau gauge:

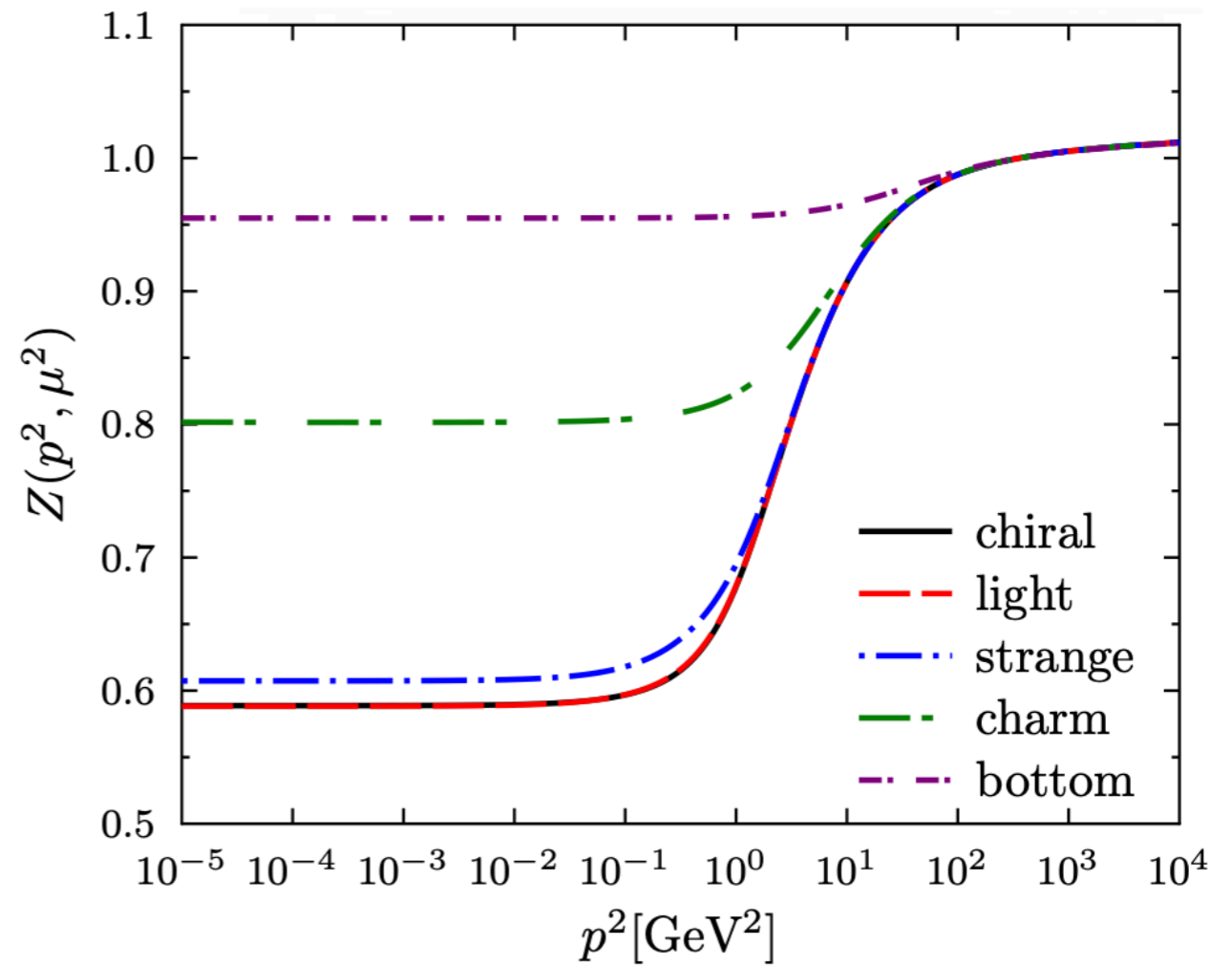
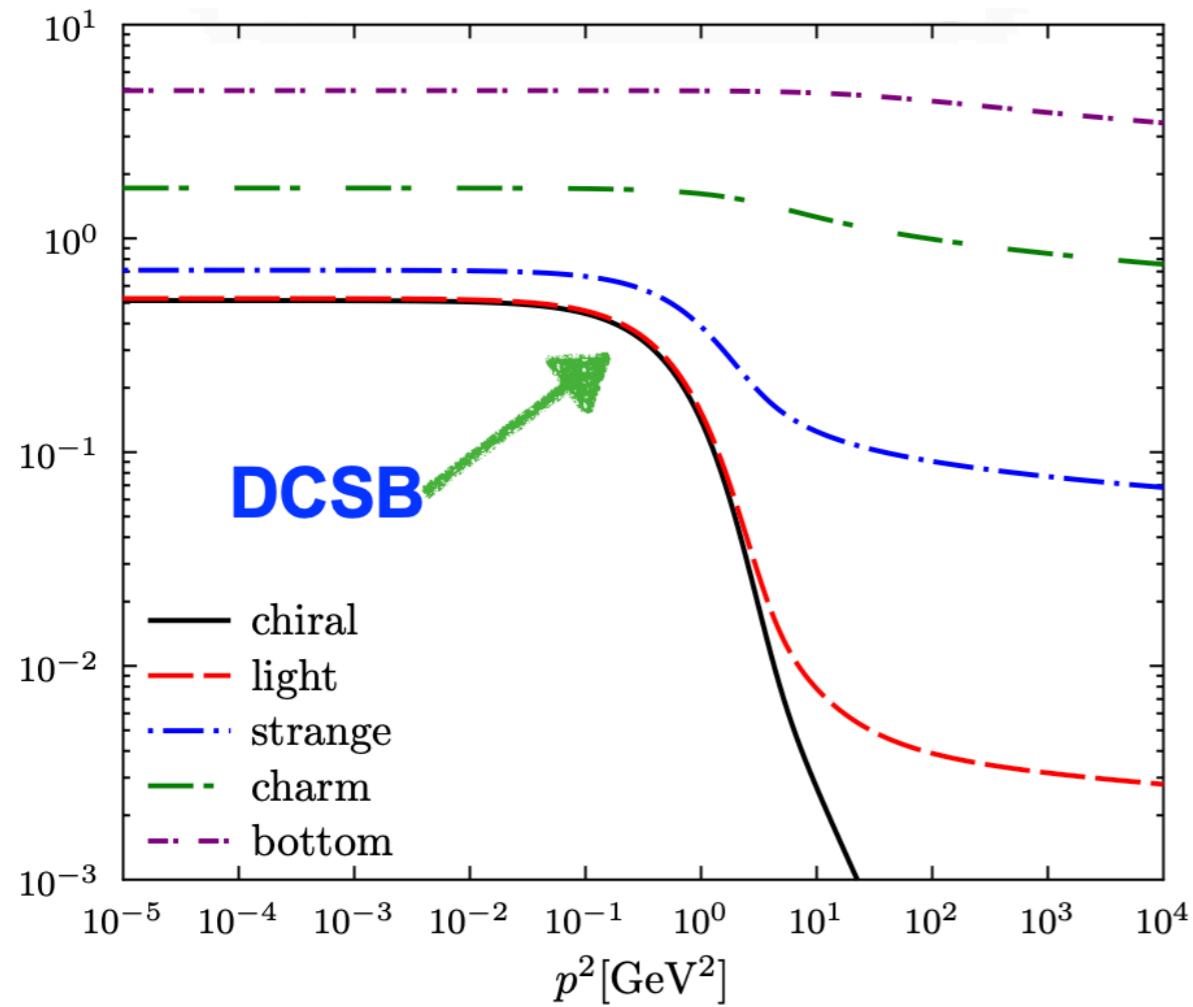
$$\Delta_{\mu\nu}(q) = \left( g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2} \right) \Delta(q^2)$$

Use effective interaction which reproduces Lattice QCD and DSE results for gluon-dressing function: *infrared massive fixed point; ultraviolet massless propagator.*

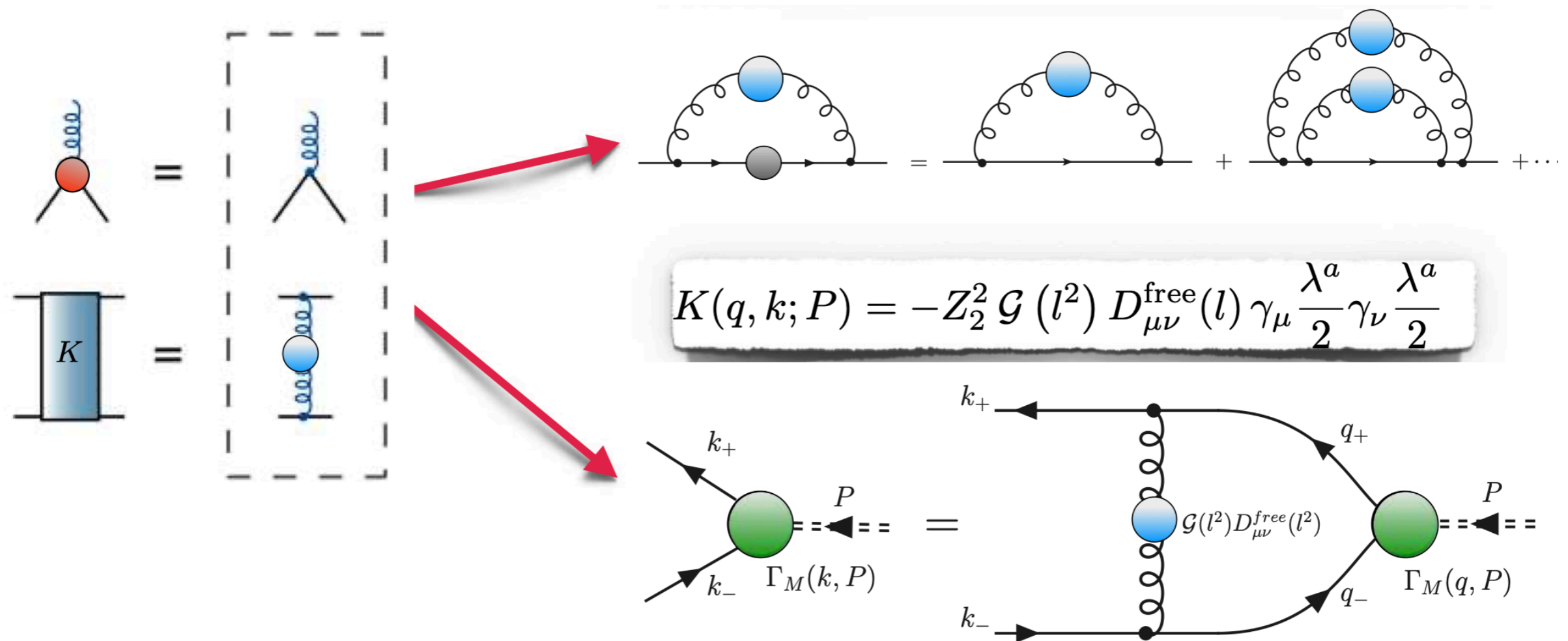


# DSE numerical solutions

$$S(p) = \frac{Z(p^2)}{i\gamma \cdot p + M(p^2)}$$



# Bethe-Salpeter equation for QCD bound states



- Quark propagators are obtained by solving the gap equation (DSE) for space-like momenta.
- In solving the BSE in Euclidean space, the propagators are functions of  $(k+P)^2$ ,  $P = (0,0,0,iM)$ .
- Extension to complex plane via Cauchy's integral theorem.

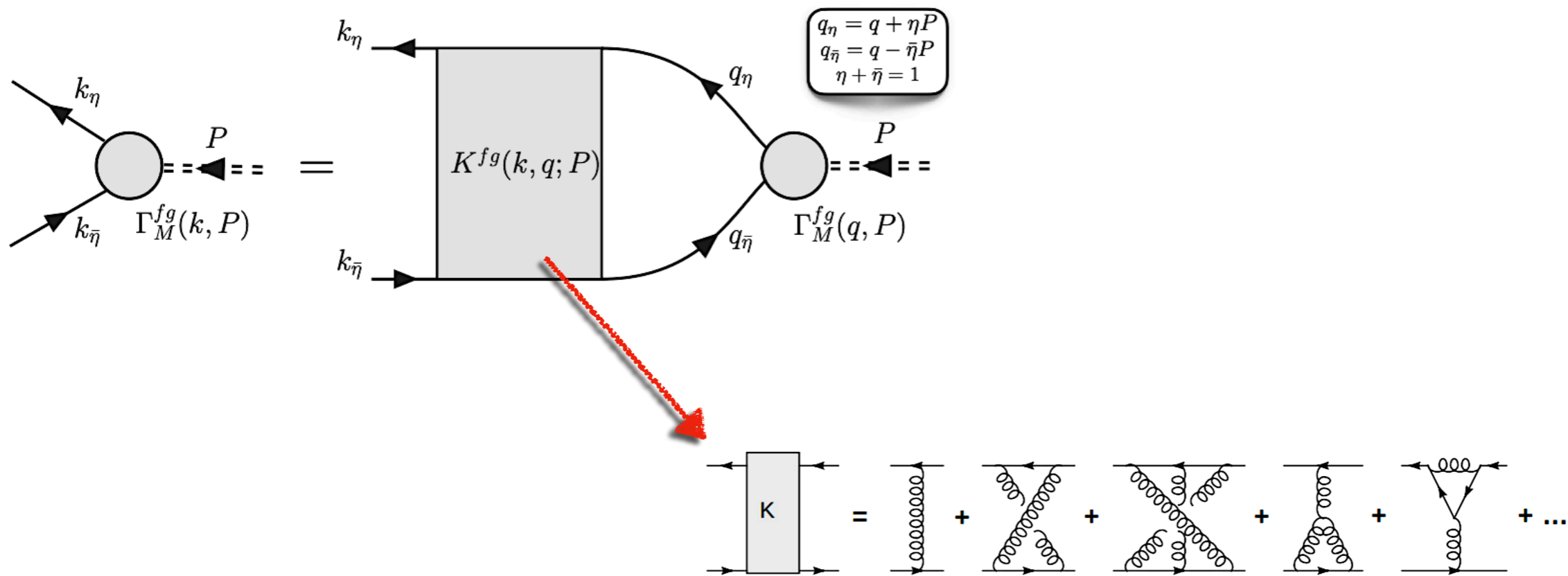
# Bethe-Salpeter Equation for QCD Bound States

$$\Gamma_M^{fg}(k, P) = \int^{\Lambda} \frac{d^4q}{(2\pi)^4} K_{fg}(k, q; P) S_f(q_\eta) \Gamma_M^{fg}(q, P) S_g(q_{\bar{\eta}})$$

$K_{fg}(q, k; P)$  = Quark-antiquark scattering kernel

$S_f(q_\eta)$  = Dressed quark propagator

$\Gamma_M^{fg}(k, P)$  = Meson's Bethe-Salpeter Amplitude (BSA)



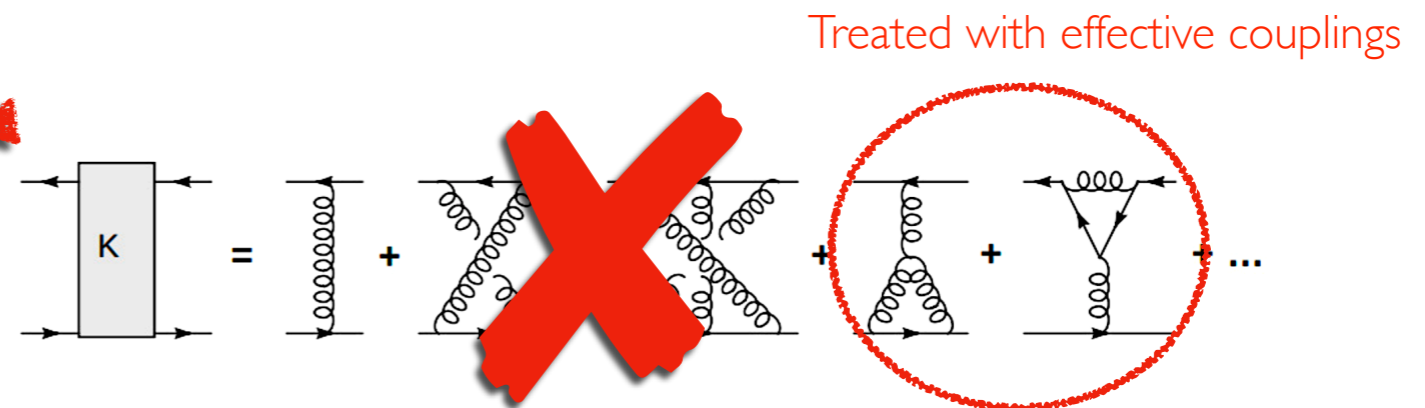
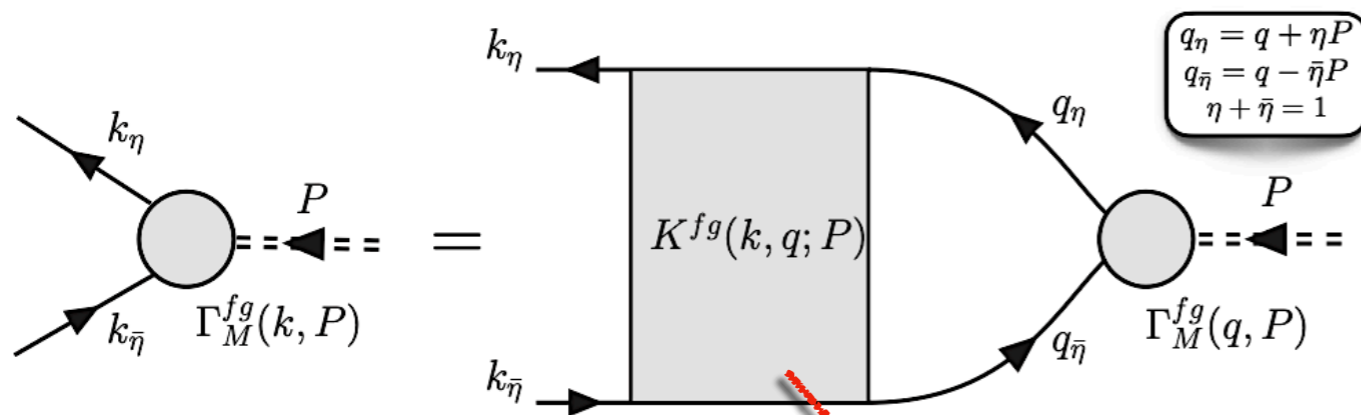
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# Bethe-Salpeter Amplitudes

- The general form of  $\Gamma_M(k; P)$  is given by

$$\Gamma(k; P) = \sum_{i=1}^N \mathcal{T}^i(k, P) \mathcal{F}_i(k^2, z_k, P^2), \quad z_k = k \cdot P / |k| |P|,$$

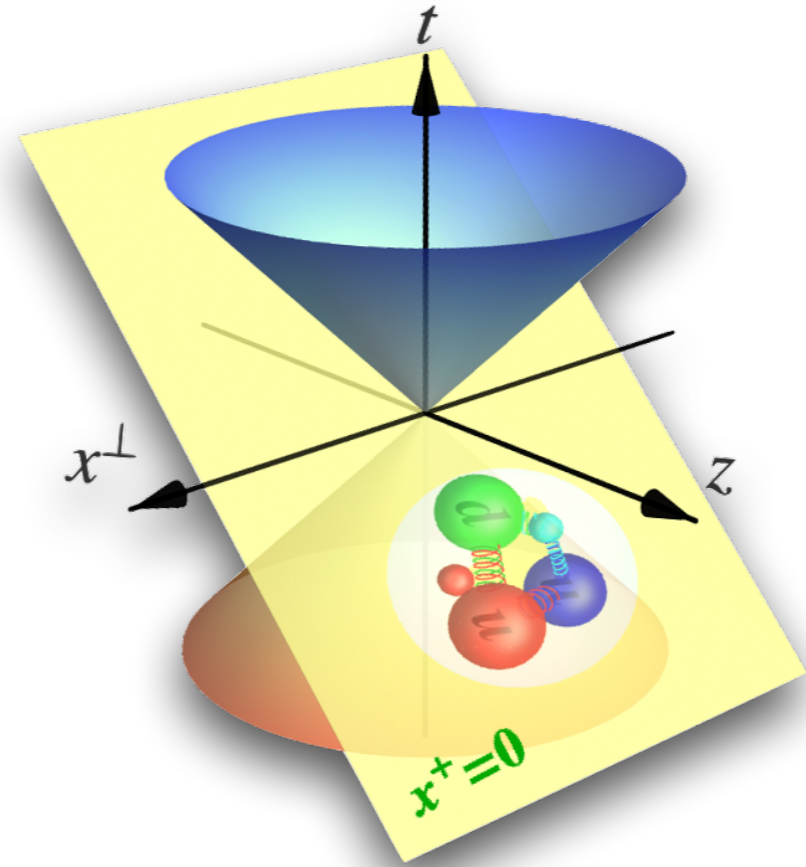
- where  $\mathcal{T}^i(k, P)$  are Dirac's covariants;
- $\mathcal{F}_i(k^2, z_k, P^2)$  are Lorentz invariant amplitudes;
- $N$  denotes the number of covariants which are different for different meson's channel.
- For the case of pseudoscalar mesons we have  $N = 4$  and for vector mesons one has  $N = 8$

# Pseudoscalar meson spectrum

	$M_P$	$M_P^{\text{exp}}$	$\epsilon_{M_P}$ [%]	$f_P$	$f_P^{\text{exp/lQCD}}$	$\epsilon_{f_P}$ [%]
$\pi(u\bar{d})$	0.140	0.138	1.45	$0.094^{+0.001}_{-0.001}$	0.092(1)	2.17
$K(u\bar{s})$	0.494	0.494	0	$0.110^{+0.001}_{-0.001}$	0.110(2)	0
$D(c\bar{d})$	$1.867^{+0.008}_{-0.004}$	1.864	0.11	$0.144^{+0.001}_{-0.001}$	0.150 (0.5)	4.00
$D_s(c\bar{s})$	$2.015^{+0.021}_{-0.018}$	1.968	2.39	$0.179^{+0.004}_{-0.003}$	0.177(0.4)	1.13
$\eta_c(c\bar{c})$	$3.012^{+0.003}_{-0.039}$	2.984	0.94	$0.270^{+0.002}_{-0.005}$	0.279(17)	3.23
$\eta_b(b\bar{b})$	$9.392^{+0.005}_{-0.004}$	9.398	0.06	$0.491^{+0.009}_{-0.009}$	0.472(4)	4.03
$B(u\bar{b})$	$5.277^{+0.008}_{-0.005}$	5.279	0.04	$0.132^{+0.004}_{-0.002}$	0.134(1)	4.35
$B_s(s\bar{b})$	$5.383^{+0.037}_{-0.039}$	5.367	0.30	$0.128^{+0.002}_{-0.003}$	0.162(1)	20.5
$B_c(c\bar{b})$	$6.282^{+0.020}_{-0.024}$	6.274	0.13	$0.280^{+0.005}_{-0.002}$	0.302(2)	10.17

# Pseudoscalar meson spectrum

	$M_V$	$M_V^{\text{exp}}$	$\epsilon_{M_V}$ [%]	$f_V$	$f_V^{\text{exp/lQCD}}$	$\epsilon_{f_V}$ [%]
$\rho(u\bar{u})$	0.730	0.775	5.81	0.145	0.153(1)	5.23
$\phi(s\bar{s})$	1.070	1.019	5.20	0.187	0.168(1)	11.31
$K^*(u\bar{s})$	0.942	0.896	5.13	0.177	0.159(1)	11.32
$D^*(c\bar{d})$	2.021	2.009	0.60	0.165	0.158(6)	4.43
$D_s^*(c\bar{s})$	2.169	2.112	2.70	0.205	0.190(5)	7.90
$J/\psi(c\bar{c})$	3.124	3.097	0.87	0.277	0.294(5)	5.78
$\Upsilon(b\bar{b})$	9.411	9.460	0.52	0.594	0.505(4)	17.62



# Light-Front Wave Functions



# Light-Front Wave Functions

- The **LCWFs** are obtained from the **BSWF** via the light front projections:

$$\psi_M^{\uparrow\downarrow}(x, \mathbf{k}_\perp^2) = \sqrt{3}i \int \frac{dk^+ dk^-}{\pi} \delta(xP^+ - k^+) \text{Tr}_D[\gamma^+ \gamma_5 \chi(k, P)],$$
$$\psi_M^{\uparrow\uparrow}(x, \mathbf{k}_\perp^2) = -\frac{\sqrt{3}i}{\mathbf{k}_\perp^2} \int \frac{dk^+ dk^-}{\pi} \delta(xP^+ - k^+) \text{Tr}_D[i\sigma_{+i} k_T^i \gamma_5 \chi(k, P)]$$

- With the **LCWF** one can readily derive two distributions:

- The leading-twist **TMD** [B. Pasquini and P. Schweitzer (2014)]

$$f_M(x, \mathbf{k}_\perp^2, \mu) = \frac{1}{(2\pi)^3} \left| \psi_M^{\uparrow\downarrow}(x, \mathbf{k}_\perp^2, \mu) + \mathbf{k}_\perp^2 \psi_M^{\uparrow\uparrow}(x, \mathbf{k}_\perp^2, \mu) \right|^2$$

- The **PDF**

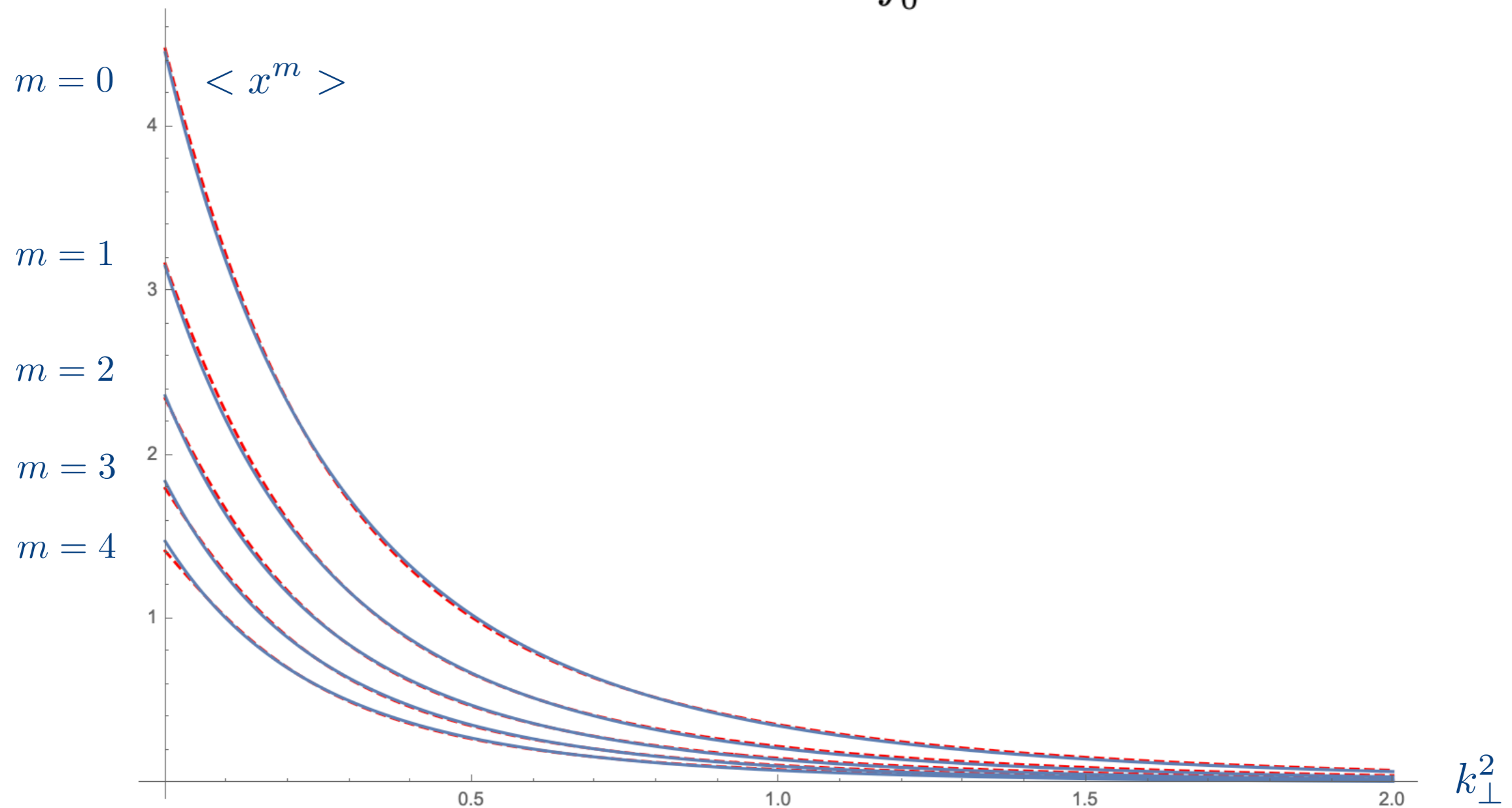
$$q_M(x, \mu) = \int d\mathbf{k}_\perp^2 f_M(x, \mathbf{k}_\perp^2, \mu)$$

Normalization condition:  $\int_0^1 dx q_M(x, \mu) = 1.$

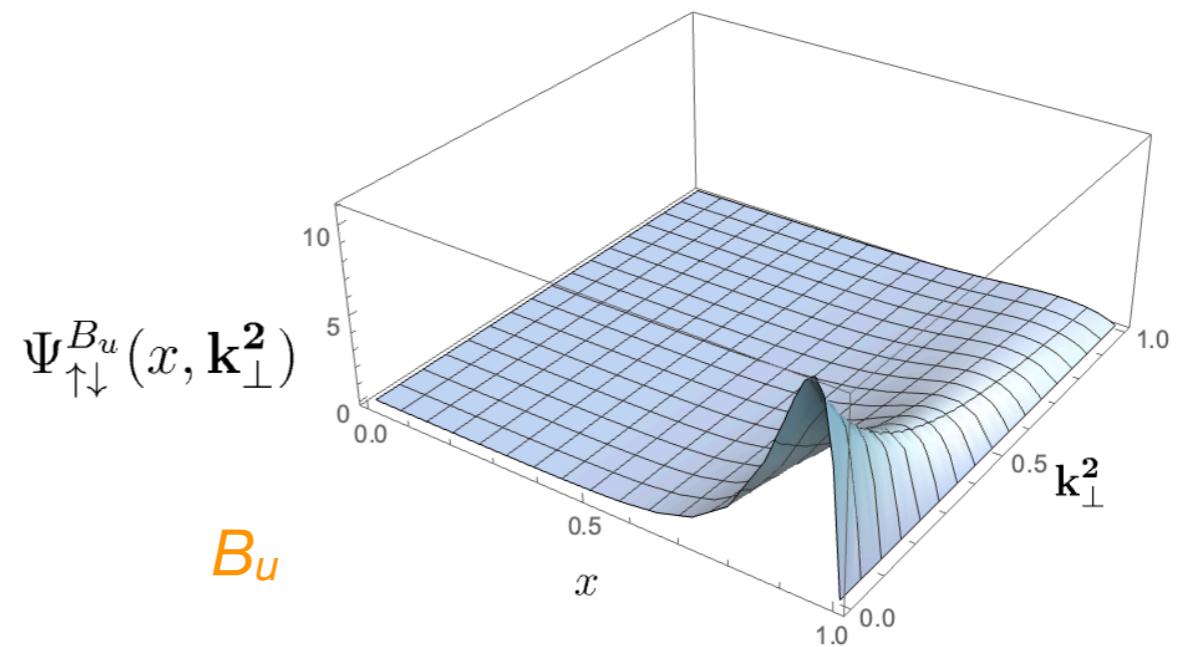
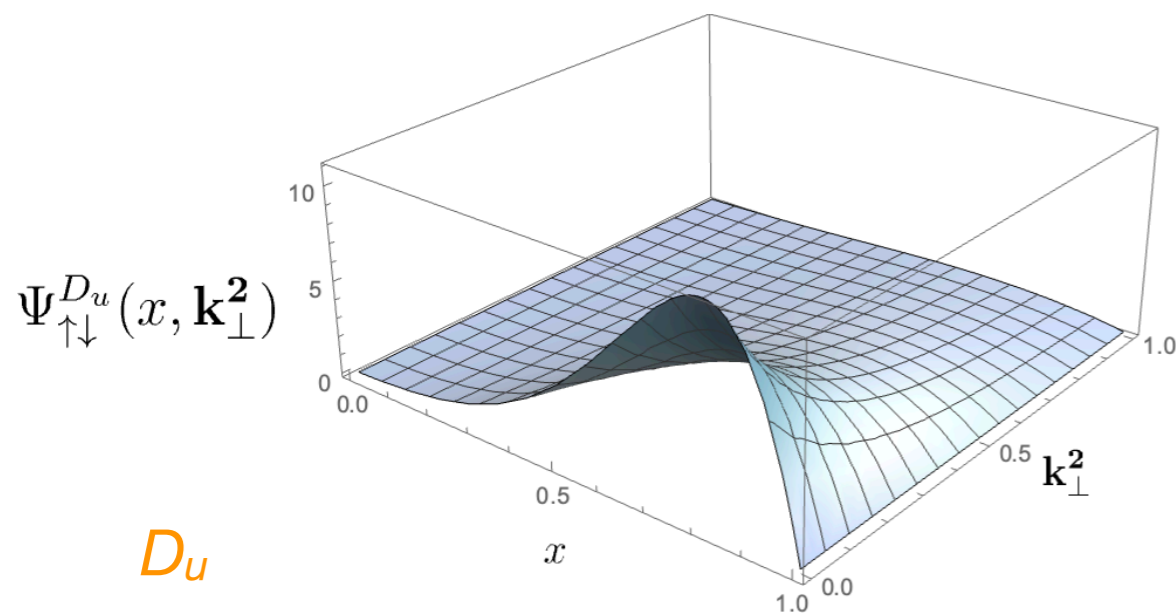
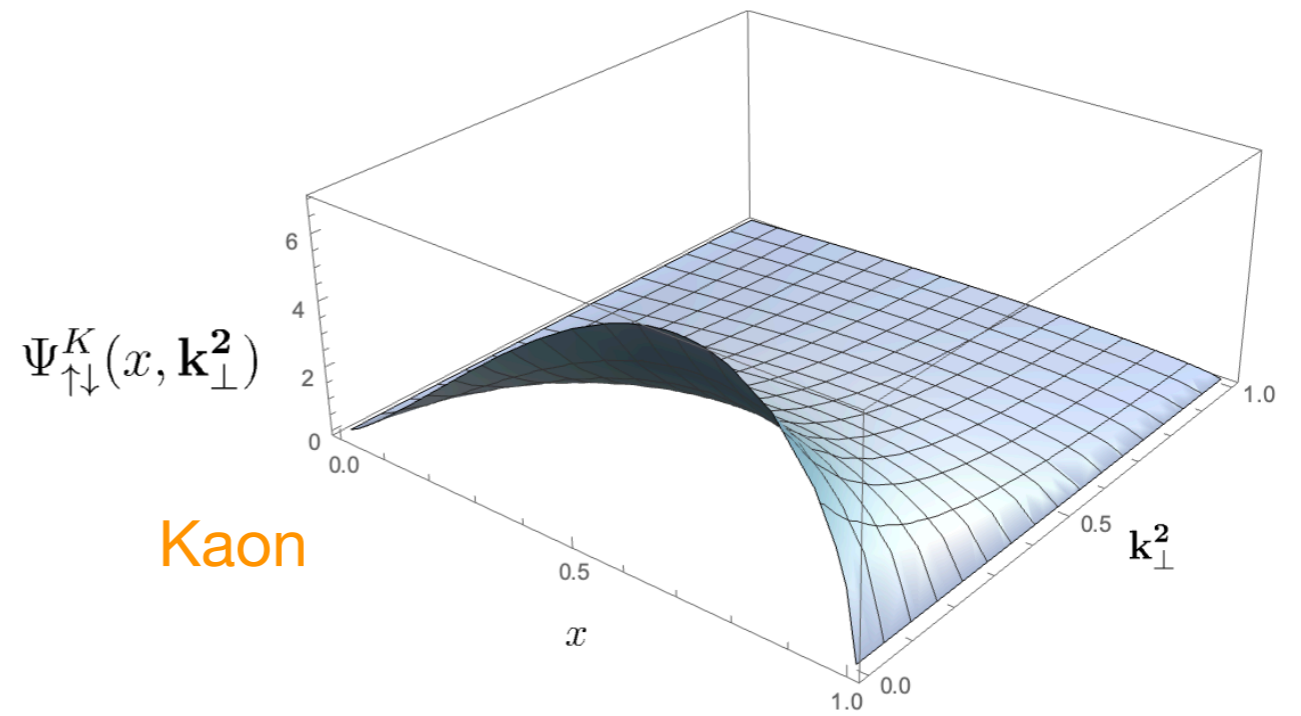
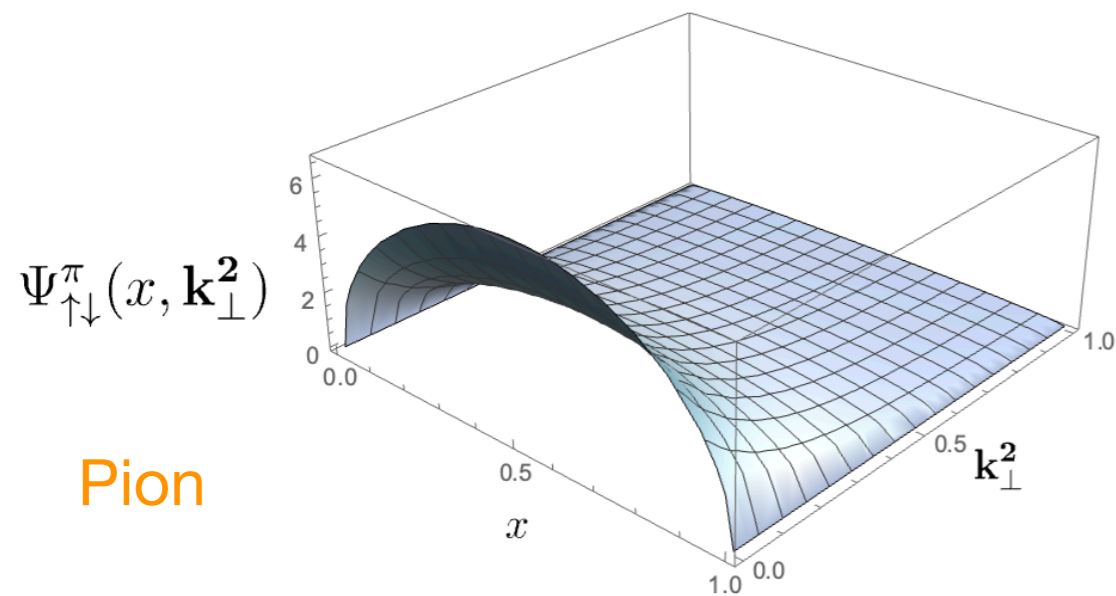
# Light-Front Wave Functions

We calculate transverse momentum dependent moments:

$$\langle x^m \rangle_{\uparrow\downarrow,\uparrow\uparrow}(\mathbf{k}_\perp^2, \mu) = \int_0^1 dx x^m \Psi_{\uparrow\downarrow,\uparrow\uparrow}(x, \mathbf{k}_\perp^2, \mu)$$

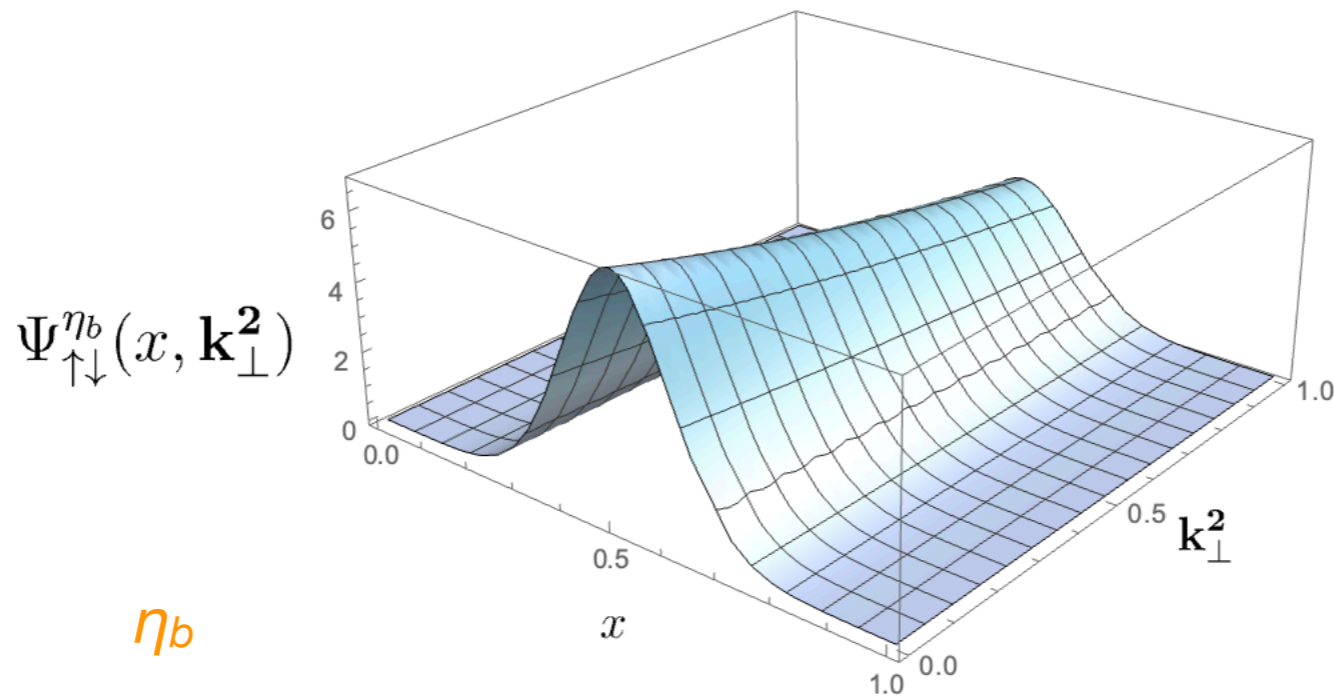
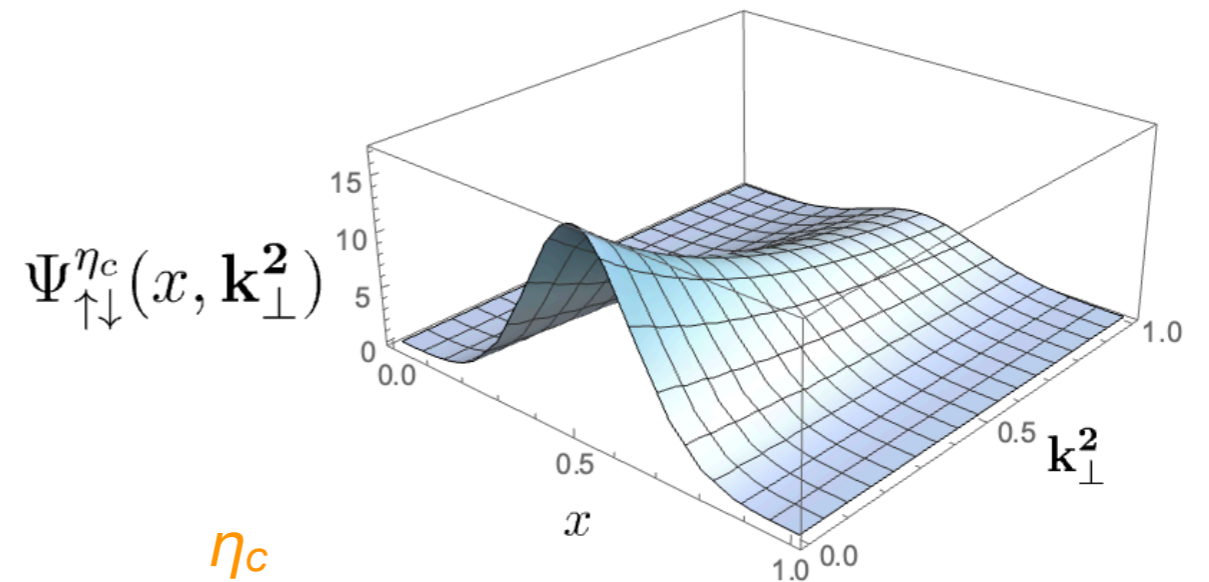
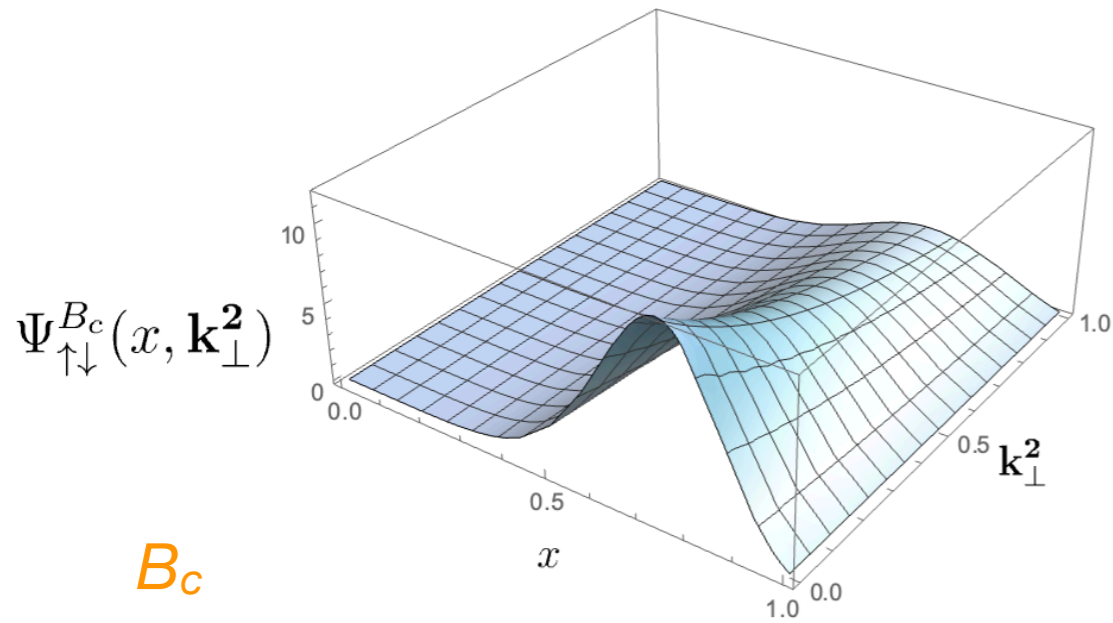


# Light-Front Wave Functions

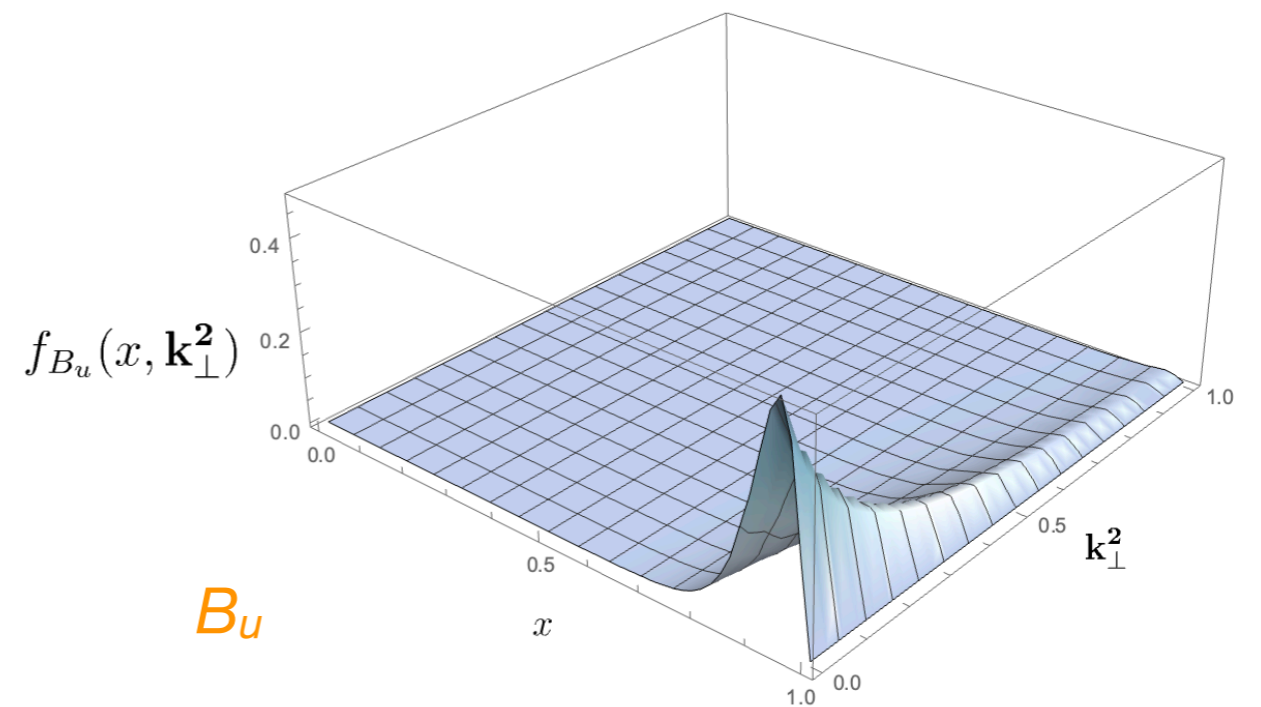
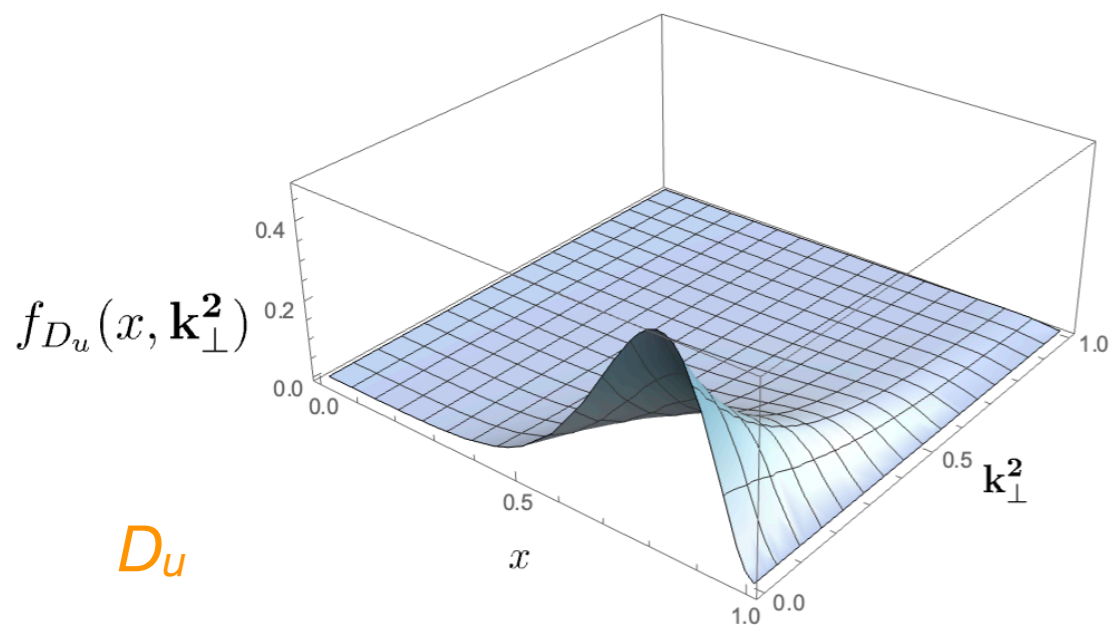
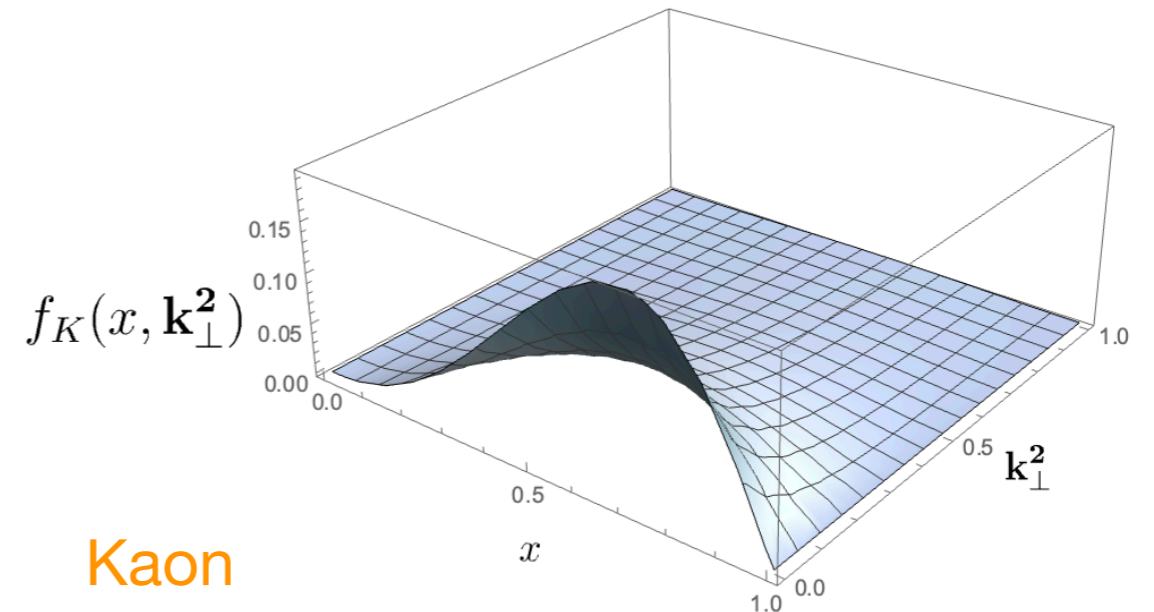
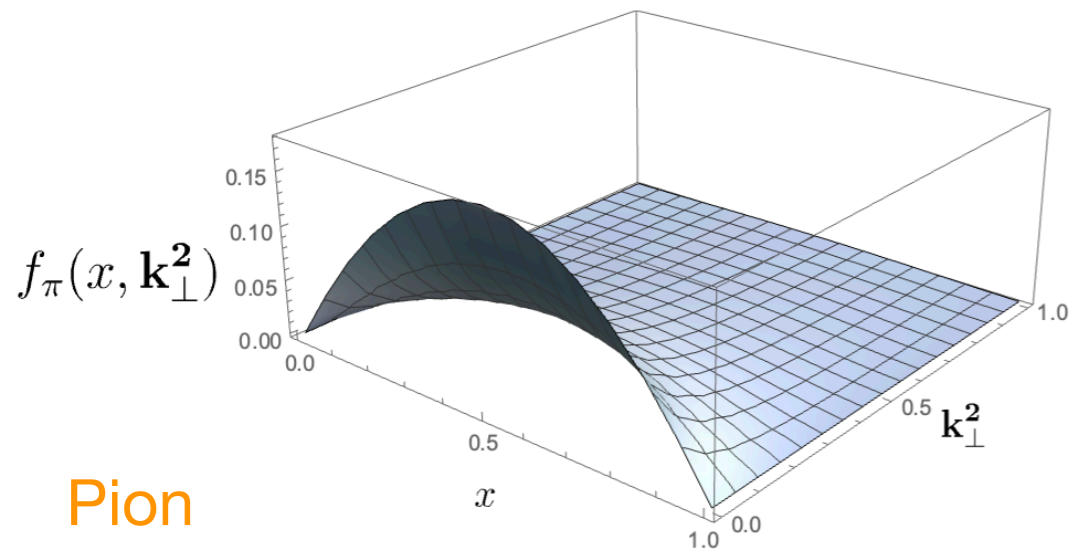




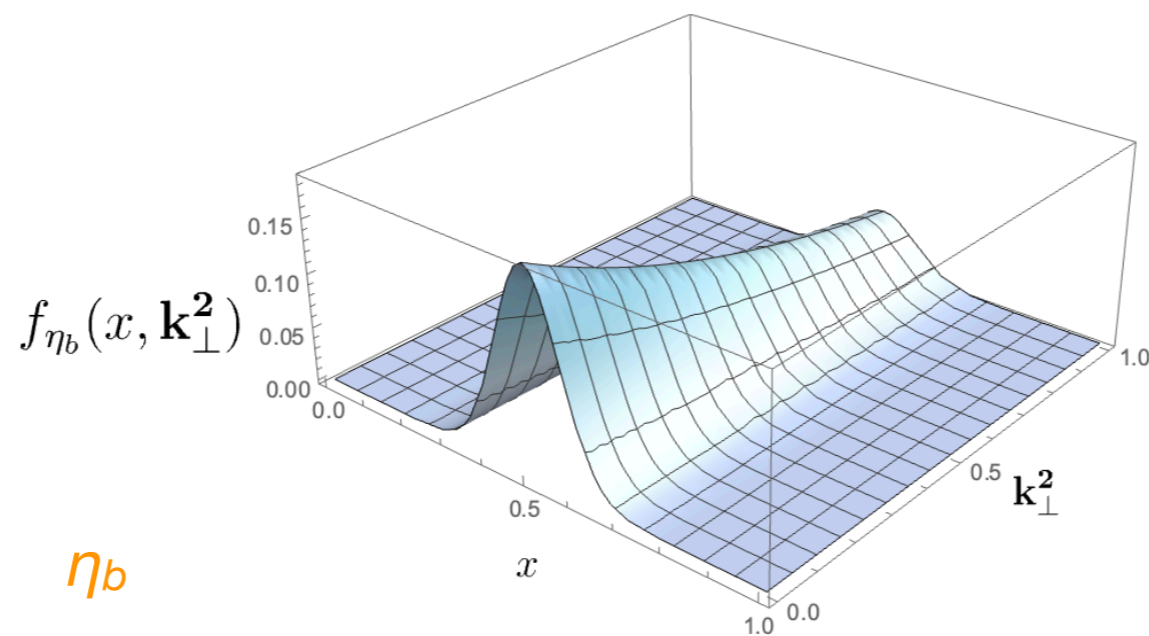
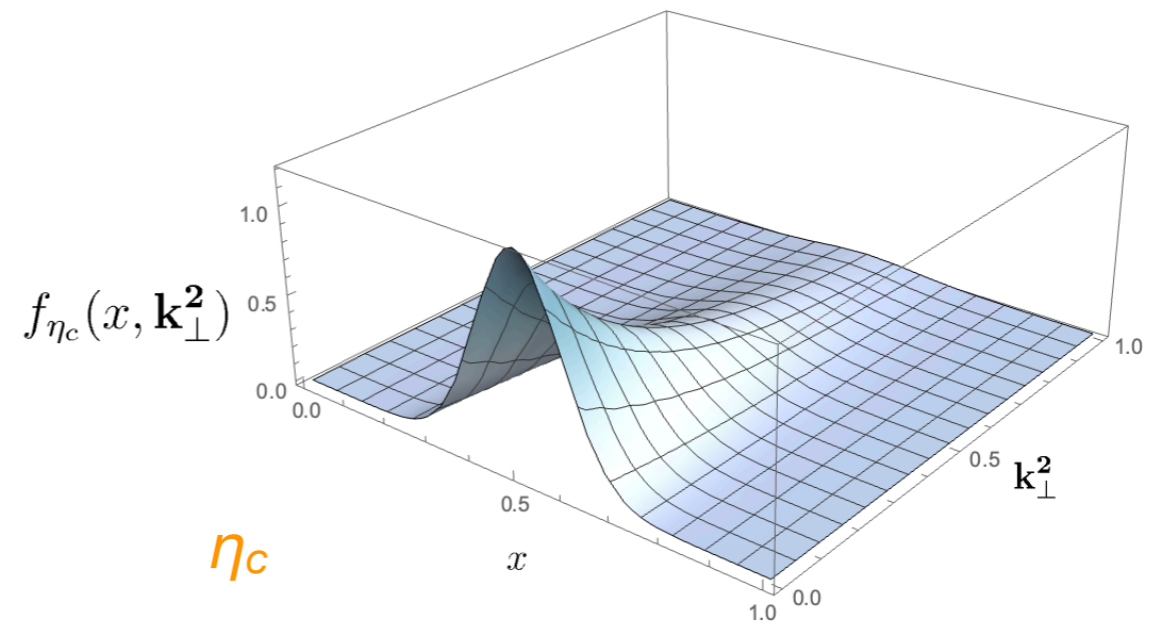
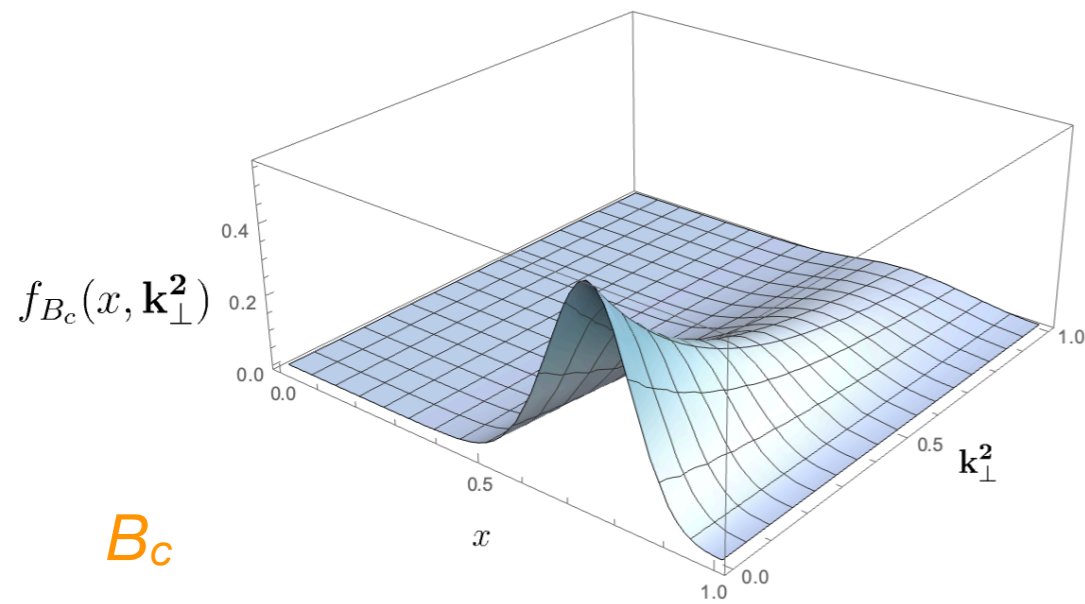
# Light-Front Wave Functions



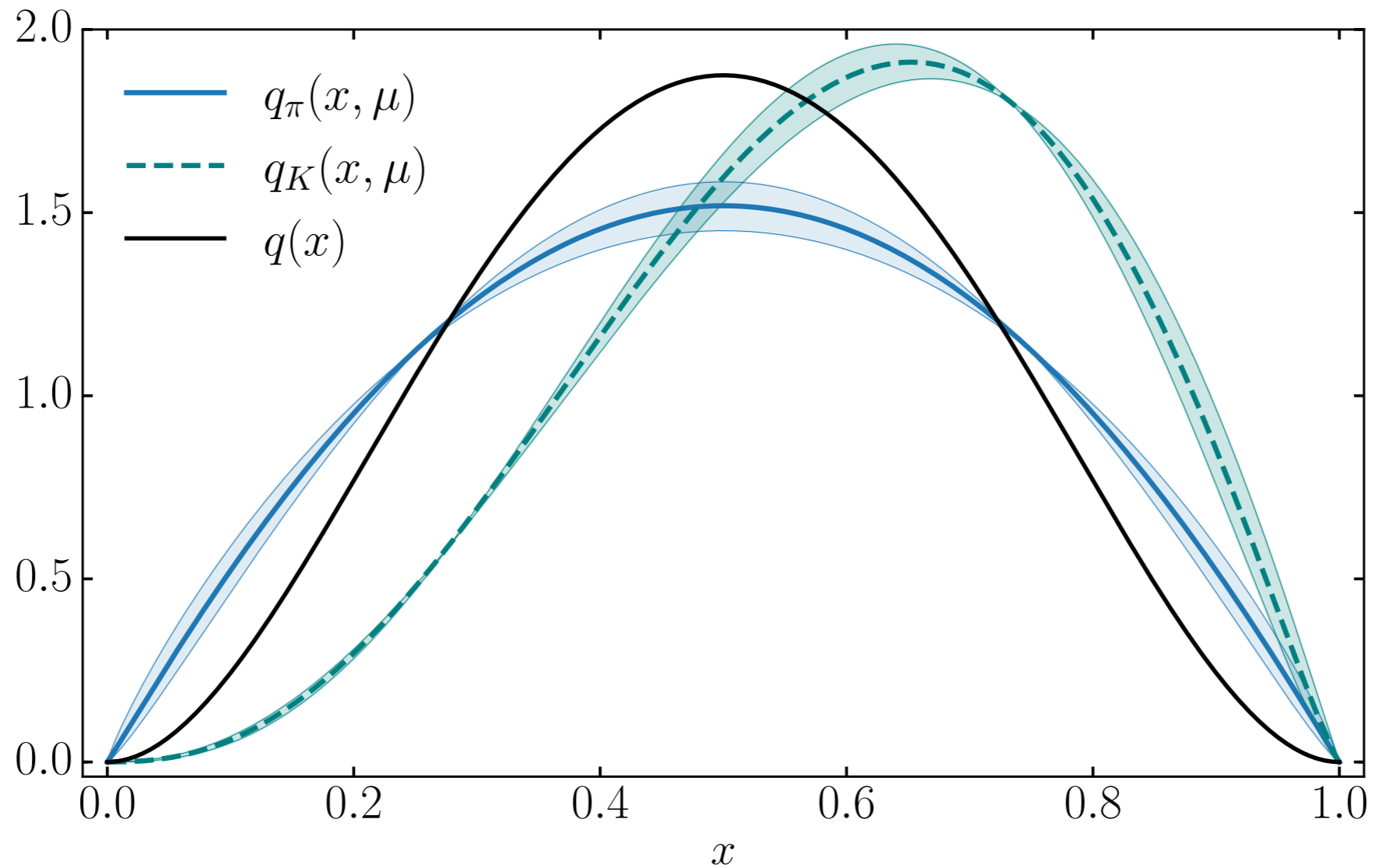
# Transverse Distribution Functions



# Transverse Distribution Functions



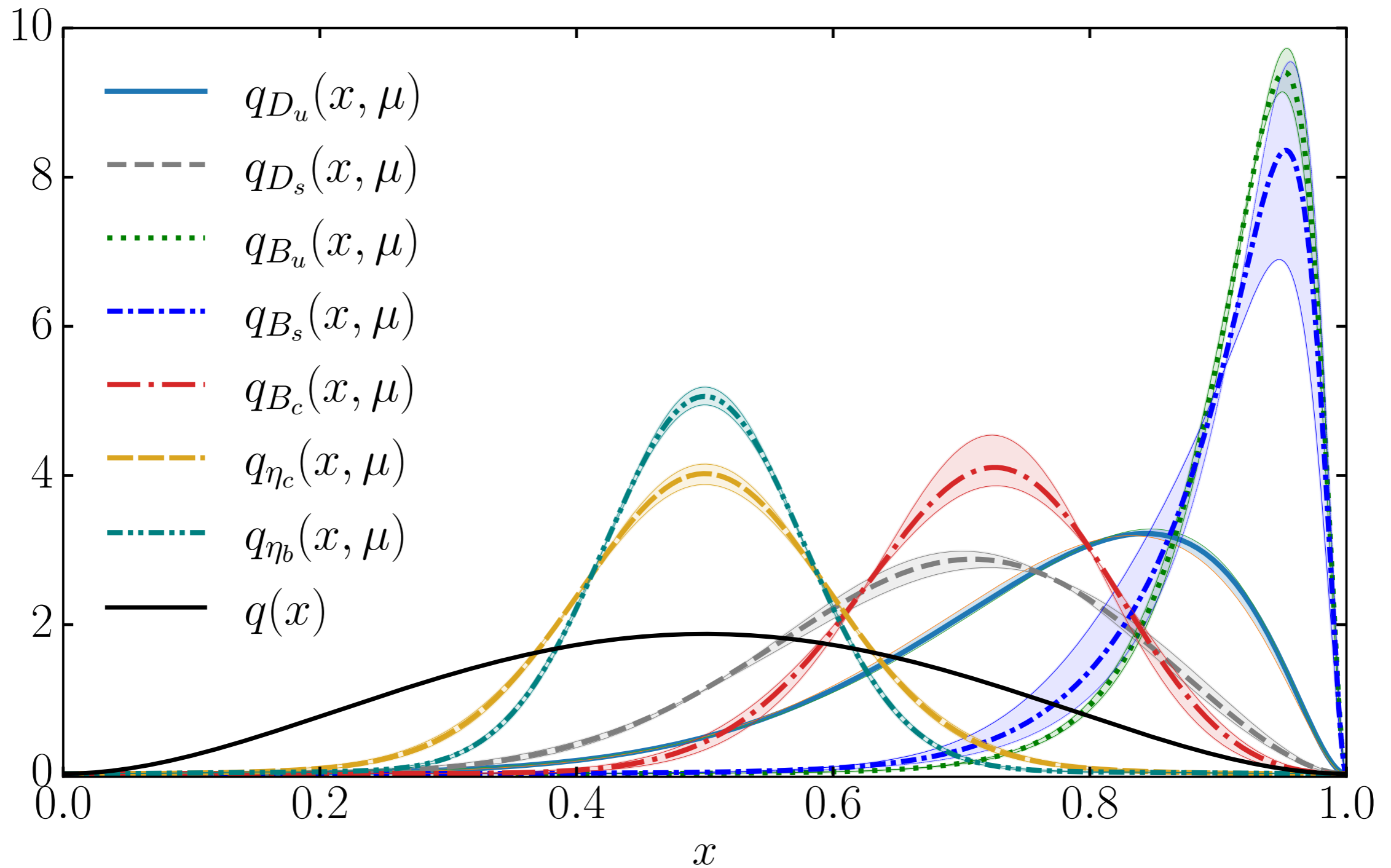
# Parton Distribution Functions

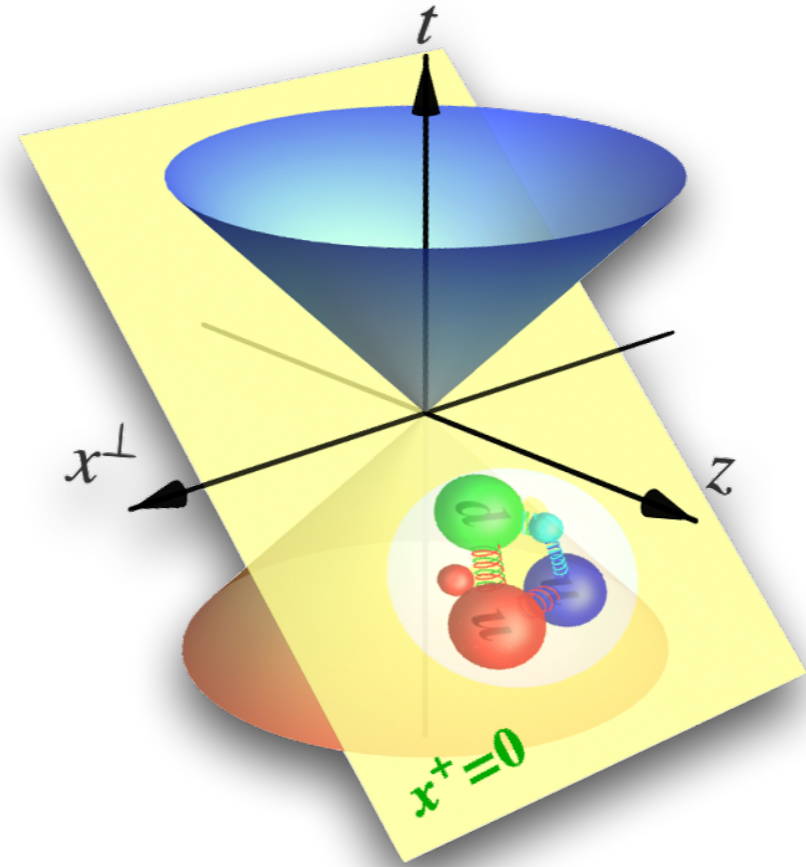


The PDFs can be parametrized by: 
$$q(x; \mu_0) = 30[x(1-x)]^2 \left[ 1 + \sum_{j=1}^{j_m} a_j C_j^{5/2} (2x-1) \right]$$



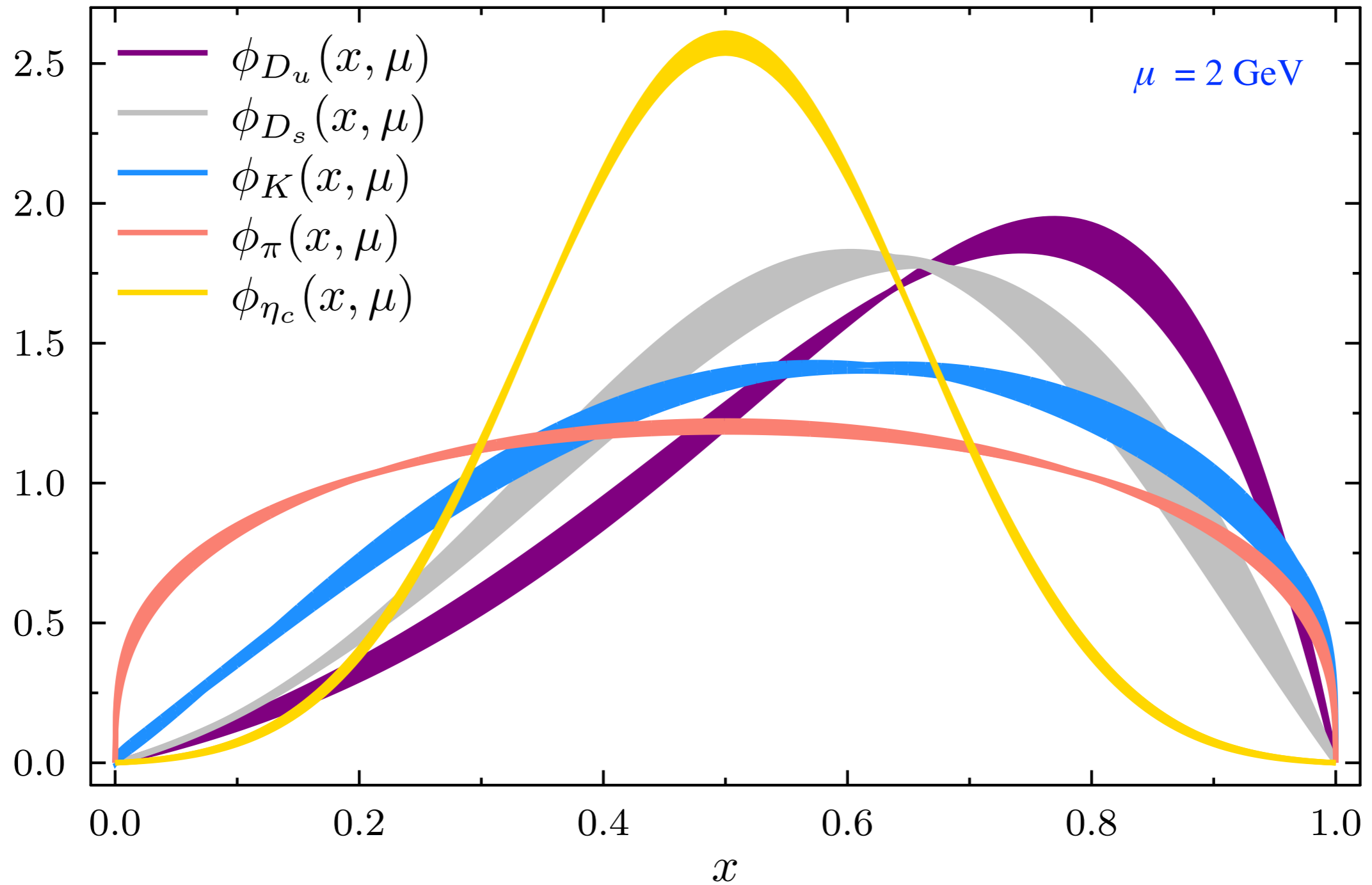
# Parton Distribution Functions



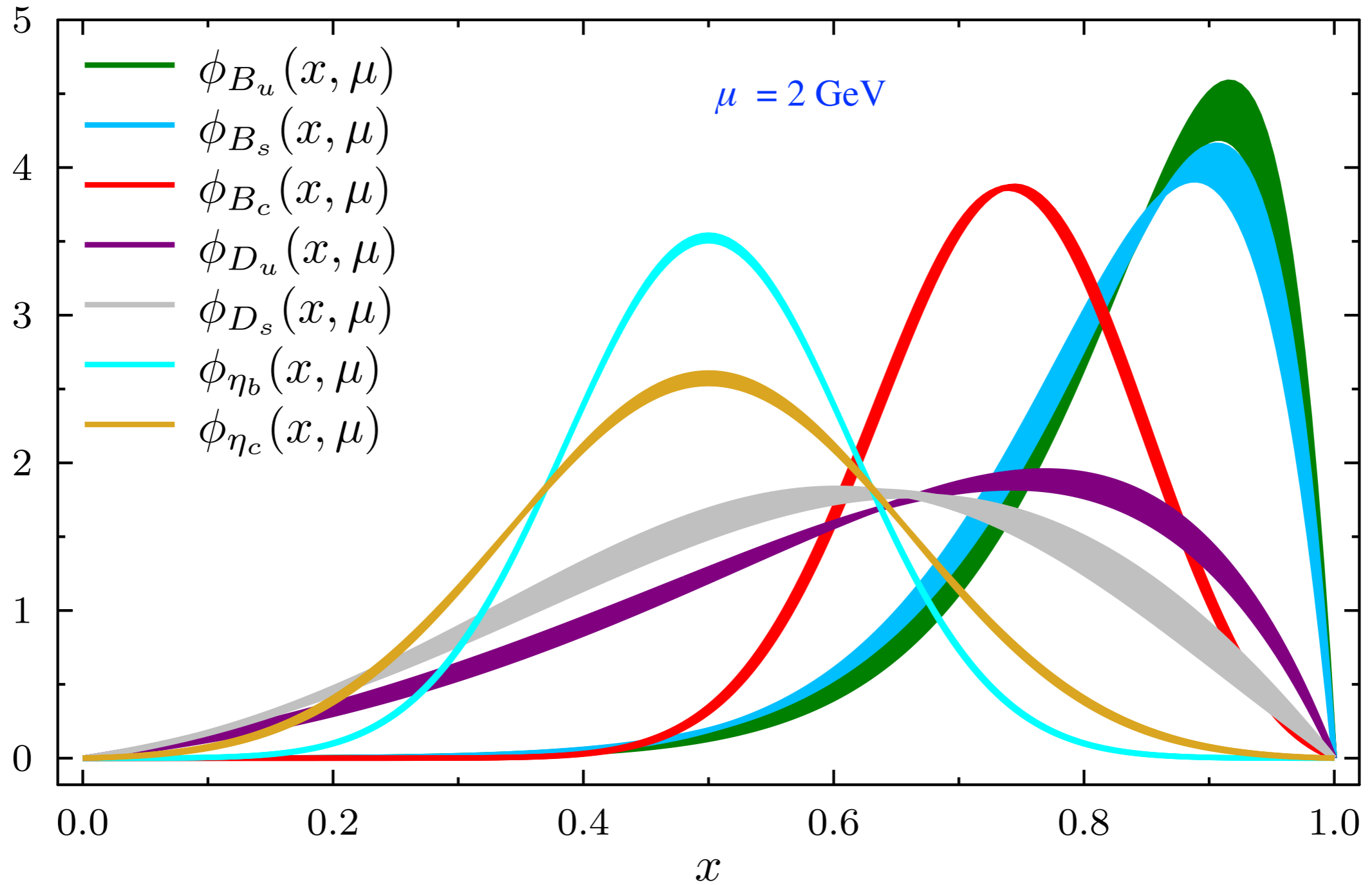


# Meson Distribution Amplitudes on the Light Front

# Light-Cone Distribution Amplitudes



# Light-Cone Distribution Amplitudes





# Vector Meson LCDAs

- Vector mesons are described by **two** distribution amplitudes  $\phi_V^{\parallel}(x; \mu)$  and  $\phi_V^{\perp}(x; \mu)$
- The distributions are obtained via the projections :

$$(n \cdot P) f_V \phi_V^{\parallel}(x; \mu) = \frac{m_V N_c \mathcal{Z}_2}{\sqrt{2}} \text{Tr}_D \int^{\Lambda} \frac{d^4 k}{(2\pi)^4} \delta(n \cdot k_{\eta} - x n \cdot P) \gamma \cdot n n_{\nu} \chi_{V\nu}^{fg}(k; P),$$

$$f_V^{\perp} \phi_V^{\perp}(x; \mu) = -\frac{N_c \mathcal{Z}_T}{2\sqrt{2}} \text{Tr}_D \int^{\Lambda} \frac{d^4 k}{(2\pi)^4} \delta(n \cdot k_{\eta} - x n \cdot P) n_{\mu} \sigma_{\mu\rho} \mathcal{O}_{\rho\nu}^{\perp} \chi_{V\nu}^{fg}(k; P),$$

with

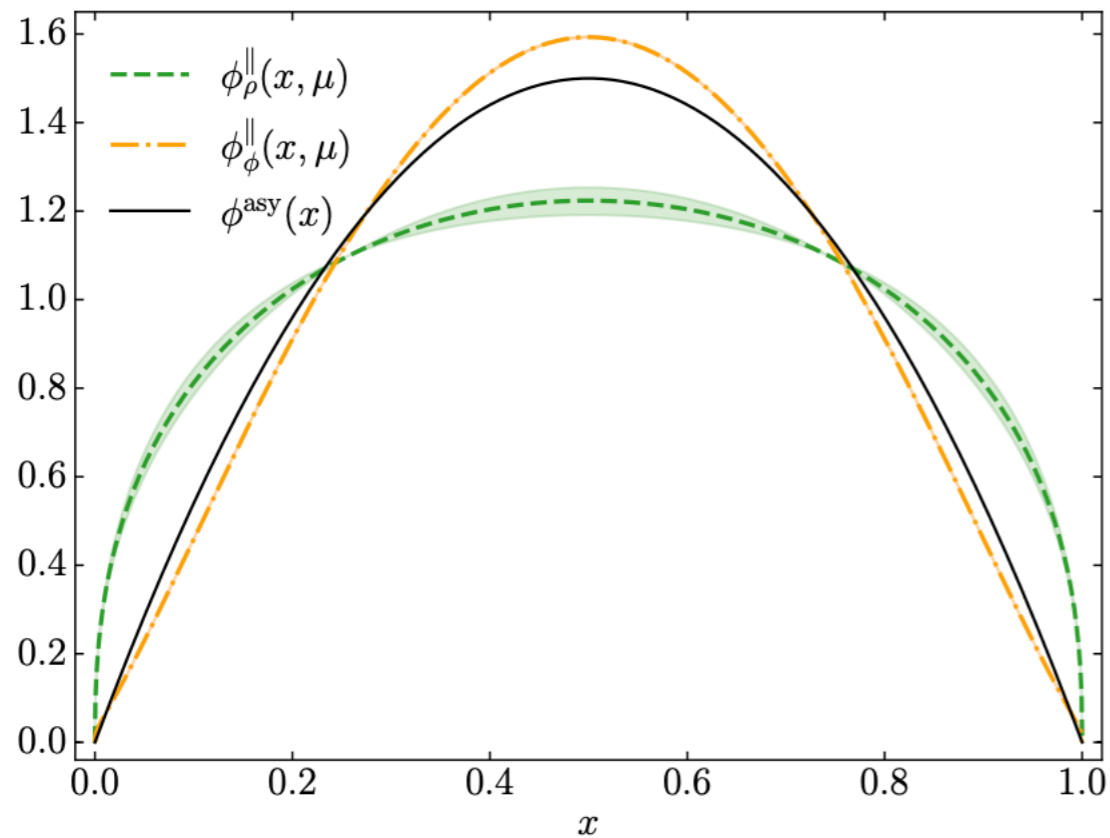
$$\chi_{V\nu}^{fg}(k; P) = S_f(k_{\eta}) \Gamma_{V\nu}^{fg}(k; P) S_g(k_{\bar{\eta}}), \quad \text{and} \quad \mathcal{O}_{\rho\nu}^{\perp} = \delta_{\rho\nu} + n_{\rho} \bar{n}_{\nu} + \bar{n}_{\rho} n_{\nu},$$

$n^2 = 0$ ;  $P^2 = -m_V^2$  and  $n \cdot P = -m_V$ ;  $\bar{n}$  is a conjugate light-like four-vector,  $\bar{n}^2 = 0$ ,

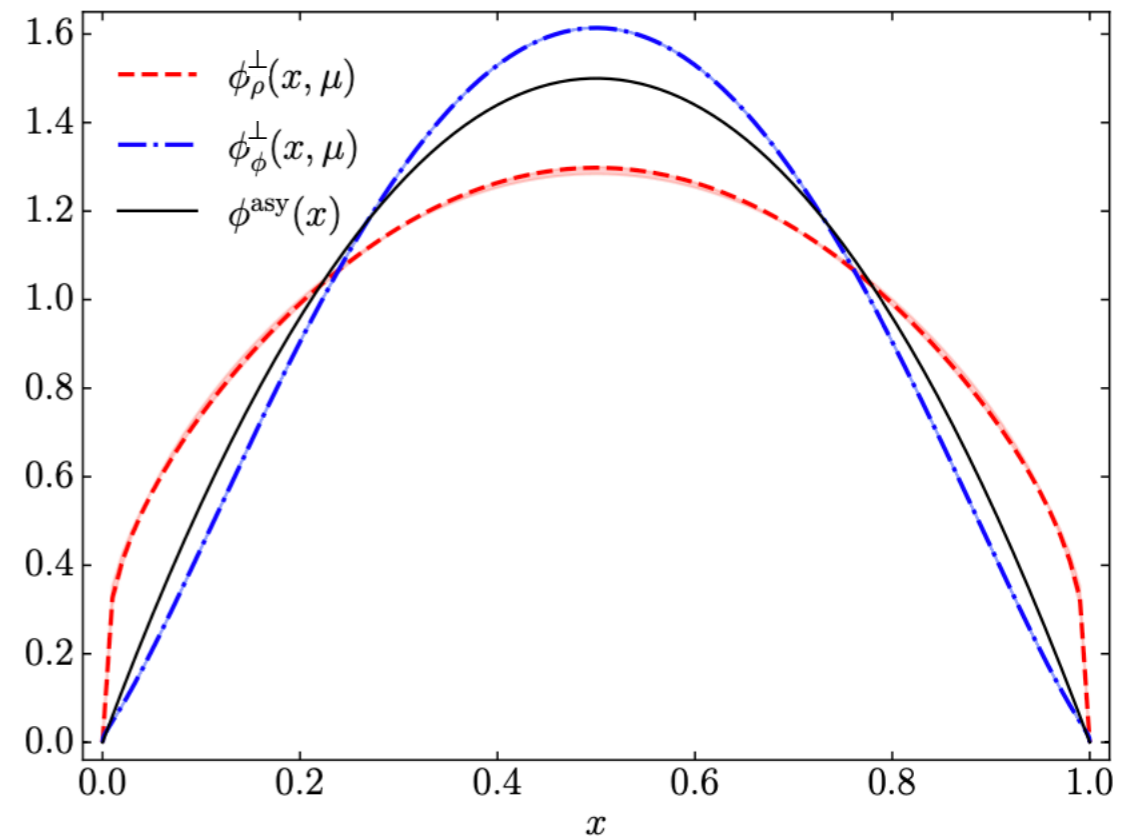
$$n \cdot \bar{n} = -1, \quad \bar{n} \cdot P = -m_V/2$$

# Vector Meson LCDAs

$\rho$  and  $\phi$  mesons: longitudinal LCDA



$\rho$  and  $\phi$  mesons: transverse LCDA

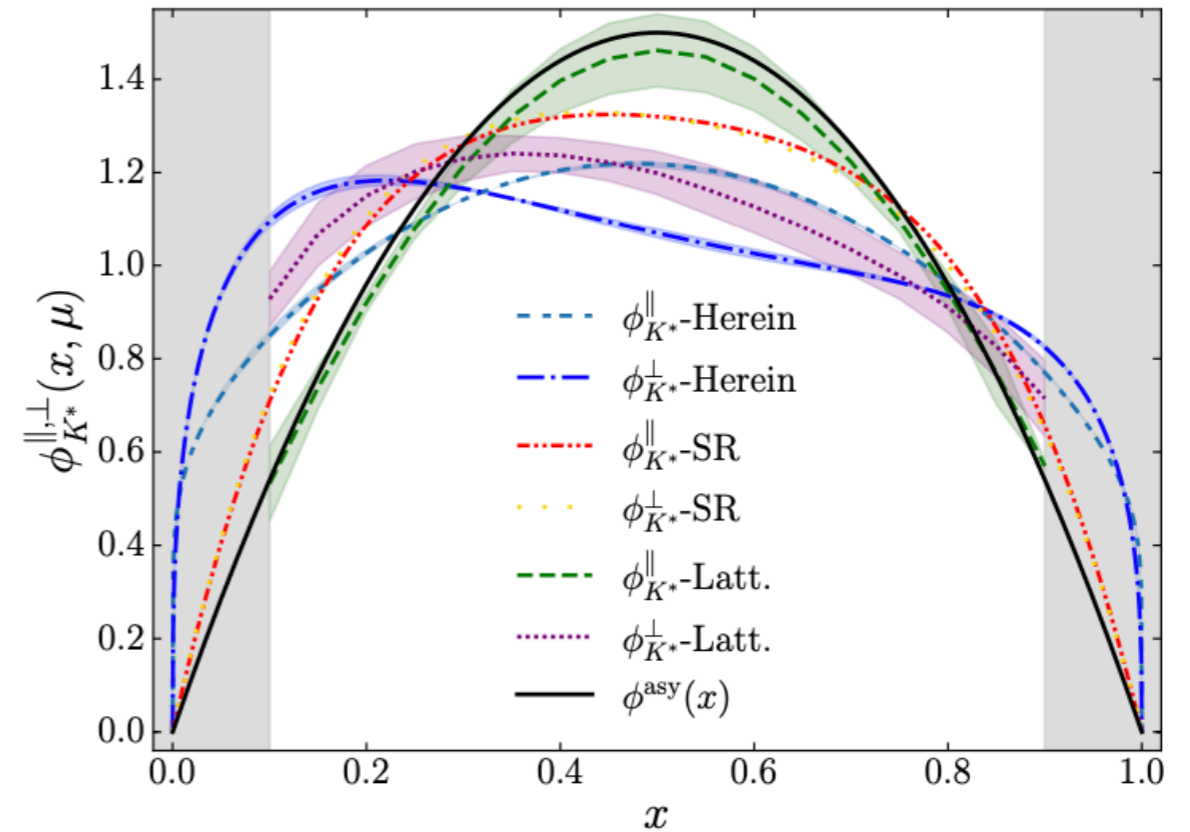
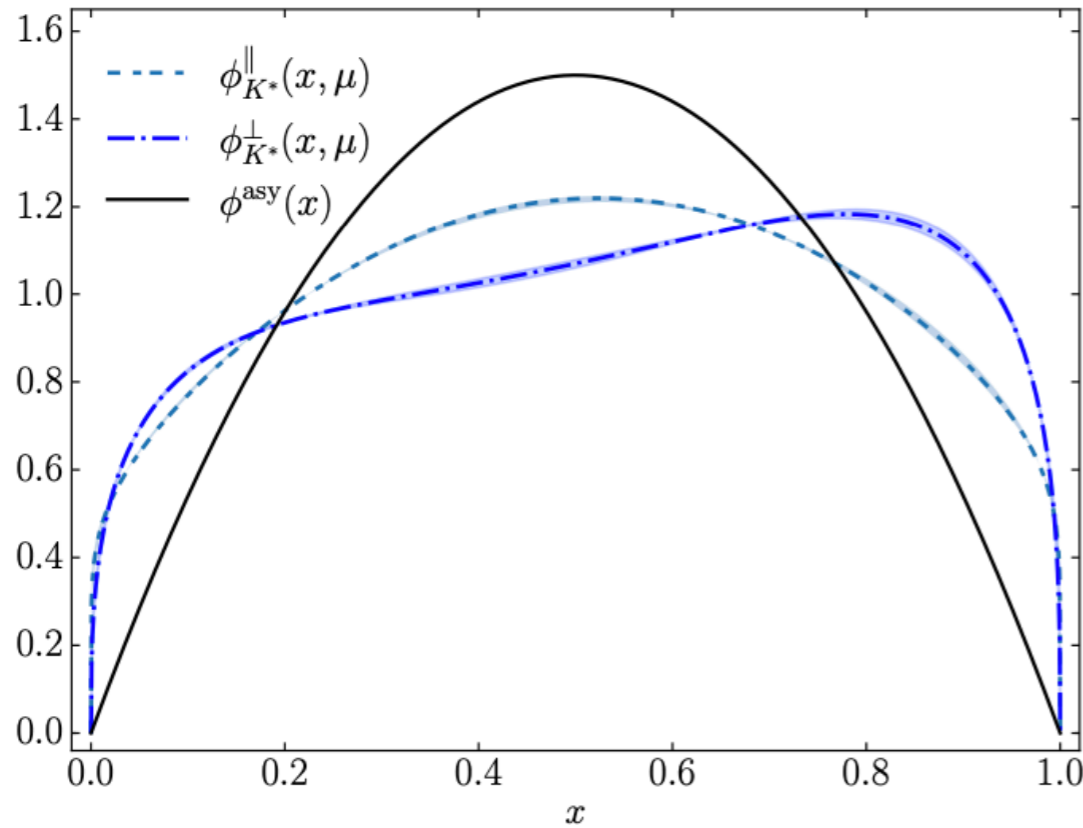


F. Serna, R. Correa da Silveira, **B.E.**, PRD Letters 106 (2022)

We find:  $\phi_\rho^\parallel(x, \mu) \approx \phi_\phi^\perp(x, \mu)$

# Light-Cone Distribution Amplitudes

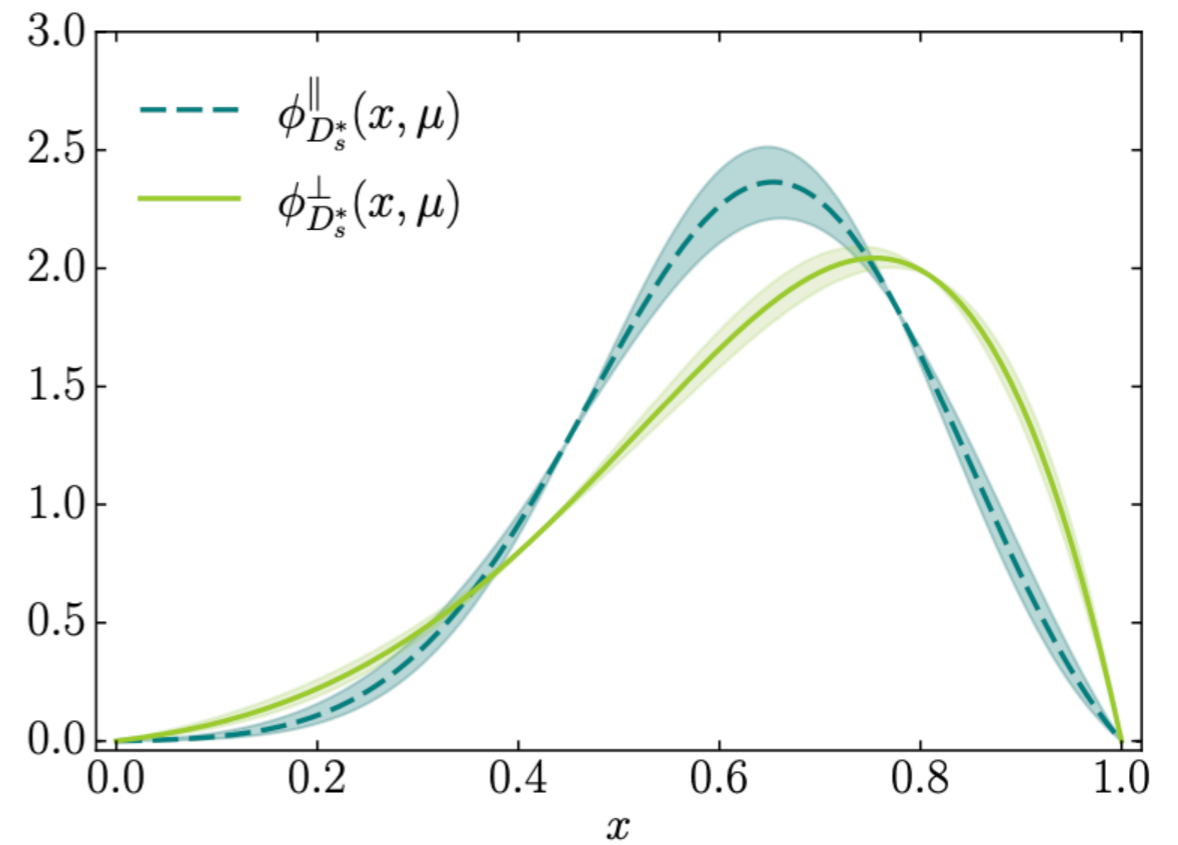
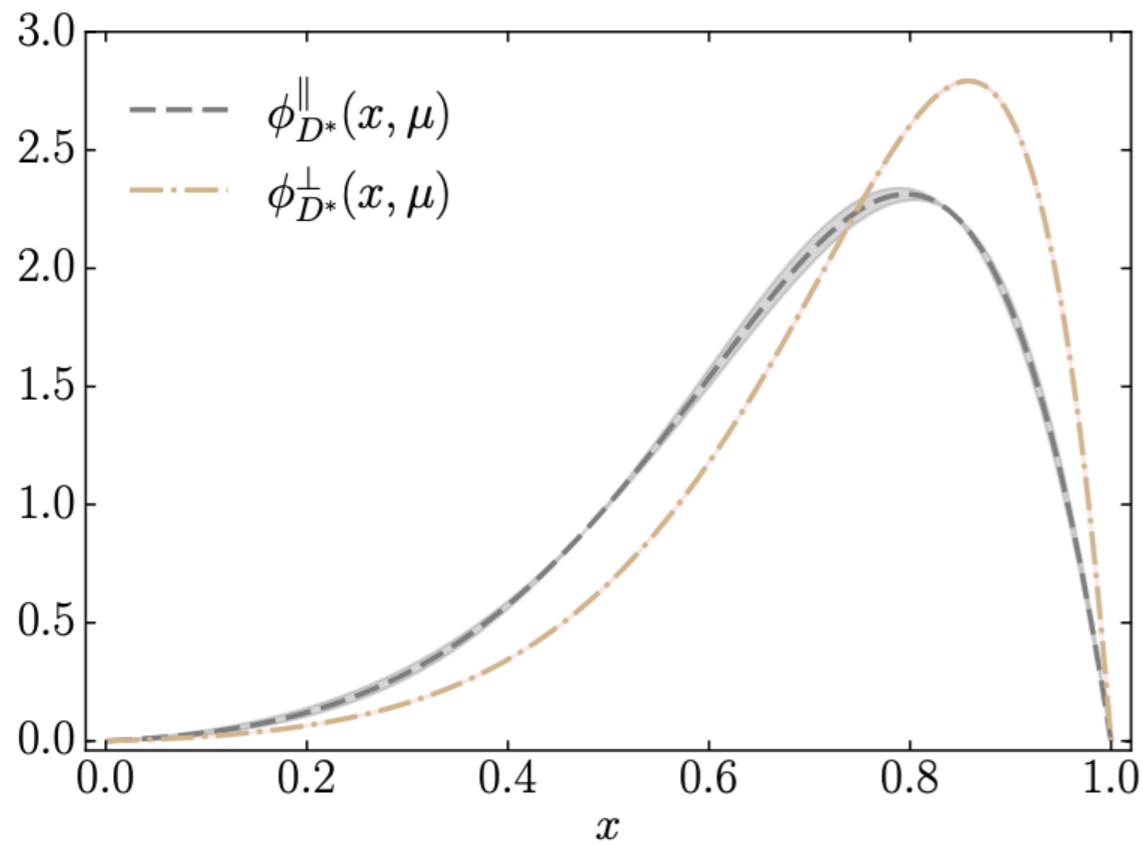
$K^*$  meson longitudinal and transverse LCDA



Comparison between our predictions for the  $K^*$ -LCDAs and Lattice QCD calculations [20]. **Left panel:** Calculated longitudinal and transverse LCDAs:  $\phi_{K^*}^{\parallel}(x, \mu)$  and  $\phi_{K^*}^{\perp}(x, \mu)$ . **Right panel:** Comparison with SR and lattice QCD calculations [16, 20], to compare our results we just replacing  $x \rightarrow 1 - x$  in Eq. (3.37).

# Light-Cone Distribution Amplitudes

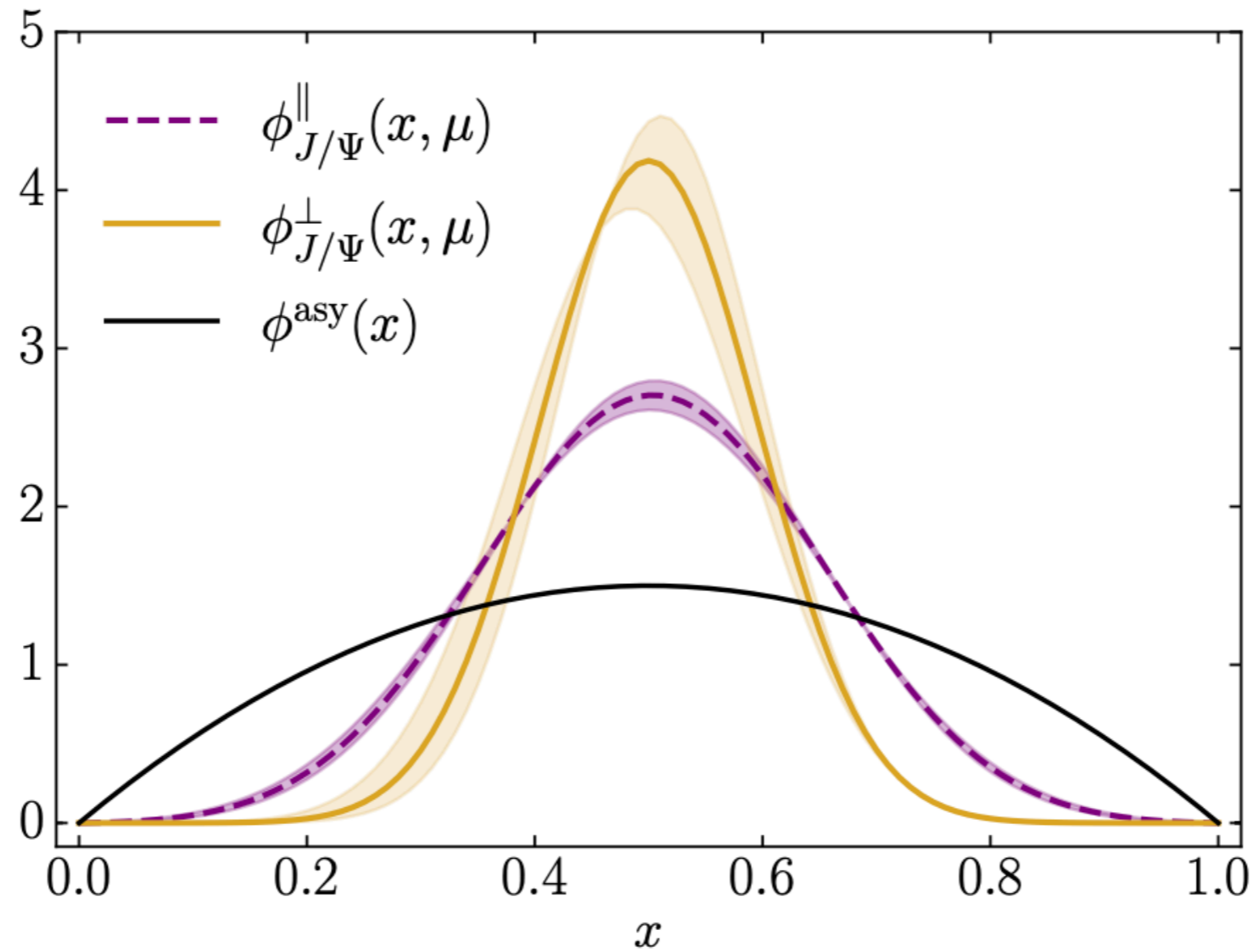
$D^*$  and  $D_s^*$  mesons longitudinal and transverse LCDA





# Light-Cone Distribution Amplitudes

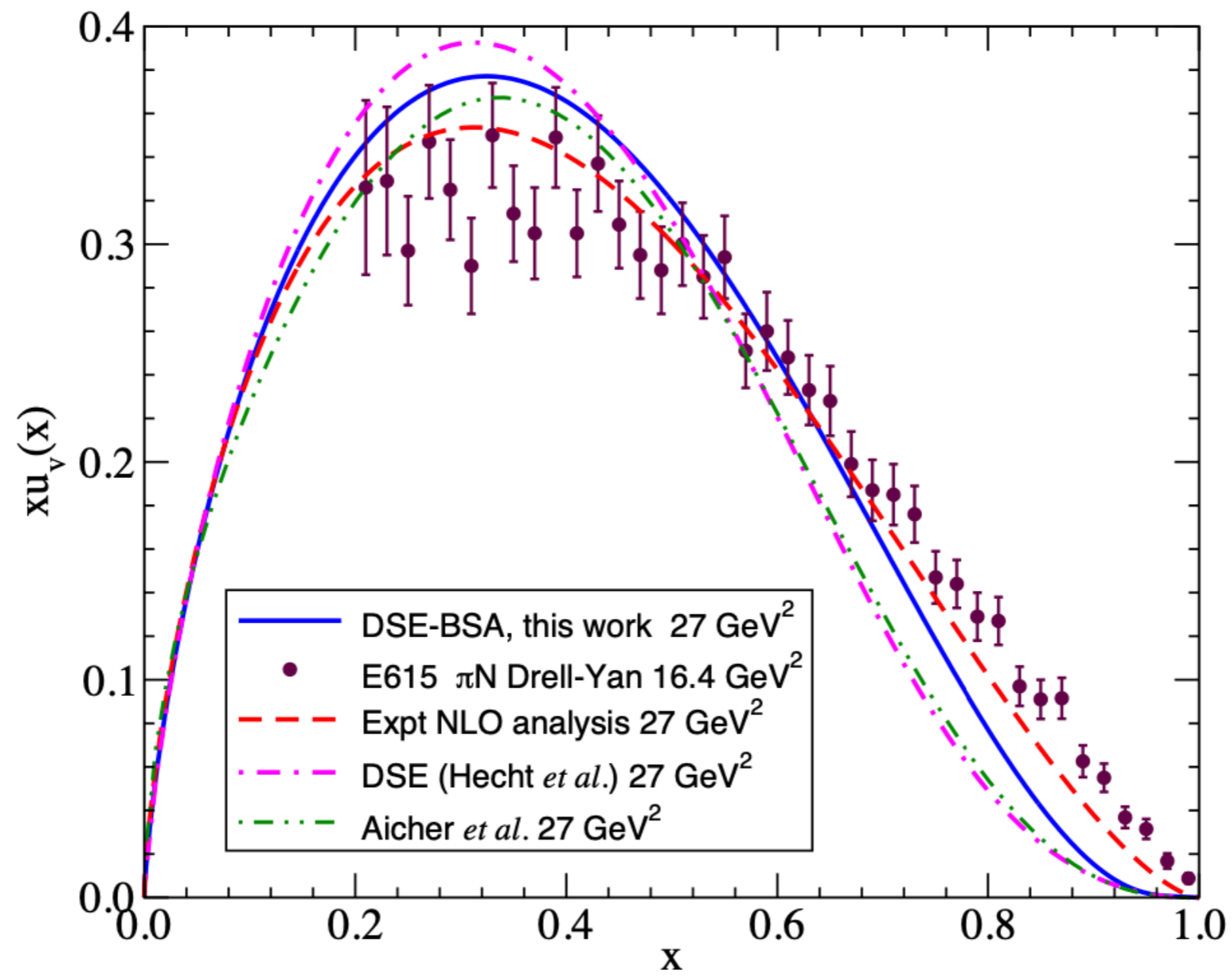
$J/\psi$  meson longitudinal and transverse LCDA



# Conclusions & Progress

- Much progress was made from QCD-based modeling toward nonperturbative numerical solutions of quark propagators and quark-antiquark bound states for flavored mesons satisfying chiral symmetry and Poincaré covariance.
- Reproduce very well the charmonium and bottonium as well as D and B meson mass spectrum and their weak decay constants.
- The three-dimensional momentum landscape of light and heavy mesons is obtained from different light-front projection of their Bethe-Salpeter wave function and don't involve the calculation of diagrams.
- For all LCDA, PDF and TMD we can readily provide functional parametrized expressions.
- Current progress: *improve beyond-leading corrections in BSE kernel for heavy-light mesons; computing GPDs and gravitational form factors; extension to nucleons.*

# Back-up Slides



T. Nguyen, A. Bashir, C. D. Roberts & P. C. Tandy PRC (2011)



# Light-Cone Distribution Amplitudes

One can compute the Mellin moments:  $\langle x^m \rangle = \int_0^1 dx x^m \phi_M(x, \mu)$       $\langle x^0 \rangle = \int_0^1 dx \phi_M(x, \mu) = 1$

$$f_M \phi_M(x, \mu) = Z_2 \text{tr}_{\text{CD}} \int \frac{d^4 k}{(2\pi)^4} \delta(n \cdot k - x n \cdot P) \gamma_5 \gamma \cdot n S(k) \Gamma_\pi(k; P) S(k - P)$$

$$f_M (n \cdot P)^{m+1} \int_0^1 dx x^m \phi_M(x, \mu) = Z_2 \text{tr}_{\text{CD}} \int \frac{d^4 k}{(2\pi)^4} (n \cdot k)^m \gamma_5 \gamma \cdot n \chi_\pi(k; P)$$

**Bethe-Salpeter wave function**

Distribution amplitudes are reconstructed with a Gegenbauer-like expansion:

$$\phi_\pi(x, \mu) = \mathcal{N}(\alpha) [x(1-x)]^{\alpha-1/2} [1 + a_2 C_2^\alpha(2x-1)]$$

# Pseudoscalar Mesons

- Pseudoscalar channel

$$\mathcal{T}_{ps}^1(k, P) = i\gamma_5$$

$$\mathcal{T}_{ps}^2(k, P) = \gamma_5 \gamma \cdot P$$

$$\mathcal{T}_{ps}^3(k, P) = \gamma_5 \gamma \cdot k (k \cdot P)$$

$$\mathcal{T}_{ps}^4(k, P) = \gamma_5 \sigma_{\mu\nu} k_\mu P_\nu$$

- For the pseudoscalar mesons, we employ the tensor structures that are not orthogonal with respect to the dirac trace

$$\Gamma_{ps}(k, P) = \gamma_5 [iE_{ps}(k, P) + \gamma \cdot P F_{ps}(k, P) + \gamma \cdot k k \cdot P G_{ps}(k, P) + \sigma_{\mu\nu} k_\mu P_\nu H_{ps}(k, P)]$$

# Beyond the Quark Model

non-relativistic  $q\bar{q}$

S	L	$J^{PC}$
0	0	$0^{-+}$
1	0	$1^{--}$
0	1	$1^{+-}$

$$P : (-1)^{L+1}$$

relativistic  $q\bar{q}$

$$\Gamma_{\pi}(P, p) = \gamma_5 [F_1(P, p) \quad \text{s-wave} \\ + F_2(P, p)i\not{p} \\ + F_3(P, p)p\not{P}i\not{p} \quad \text{p-wave} \\ + F_4(P, p)[\not{p}, \not{P}]]$$

~~$$P : (-1)^{L+1}$$~~

# Vector Mesons

- Vector channel

$$\mathcal{T}_\mu^1(k, P) = i\gamma_\mu^T$$

$$\mathcal{T}_\mu^2(k, P) = i[3k_\mu^T (\gamma \cdot k^T) - \gamma_\mu^T (k^T)^2]$$

$$\mathcal{T}_\mu^3(k, P) = i(k \cdot P) k_\mu^T \gamma \cdot P$$

$$\mathcal{T}_\mu^4(k, P) = i[\gamma_\mu^T \gamma \cdot P (\gamma \cdot k^T) + k_\mu^T \gamma \cdot P]$$

$$\mathcal{T}_\mu^5(k, P) = k_\mu^T$$

$$\mathcal{T}_\mu^6(k, P) = (k \cdot P) [\gamma_\mu^T (\gamma \cdot k^T) - (\gamma \cdot k^T) \gamma_\mu^T]$$

$$\mathcal{T}_\mu^7(k, P) = \gamma_\mu^T \gamma \cdot P - \gamma \cdot P \gamma_\mu^T - 2\hat{k}_\mu^T (\gamma \cdot \hat{k}^T) \gamma \cdot P$$

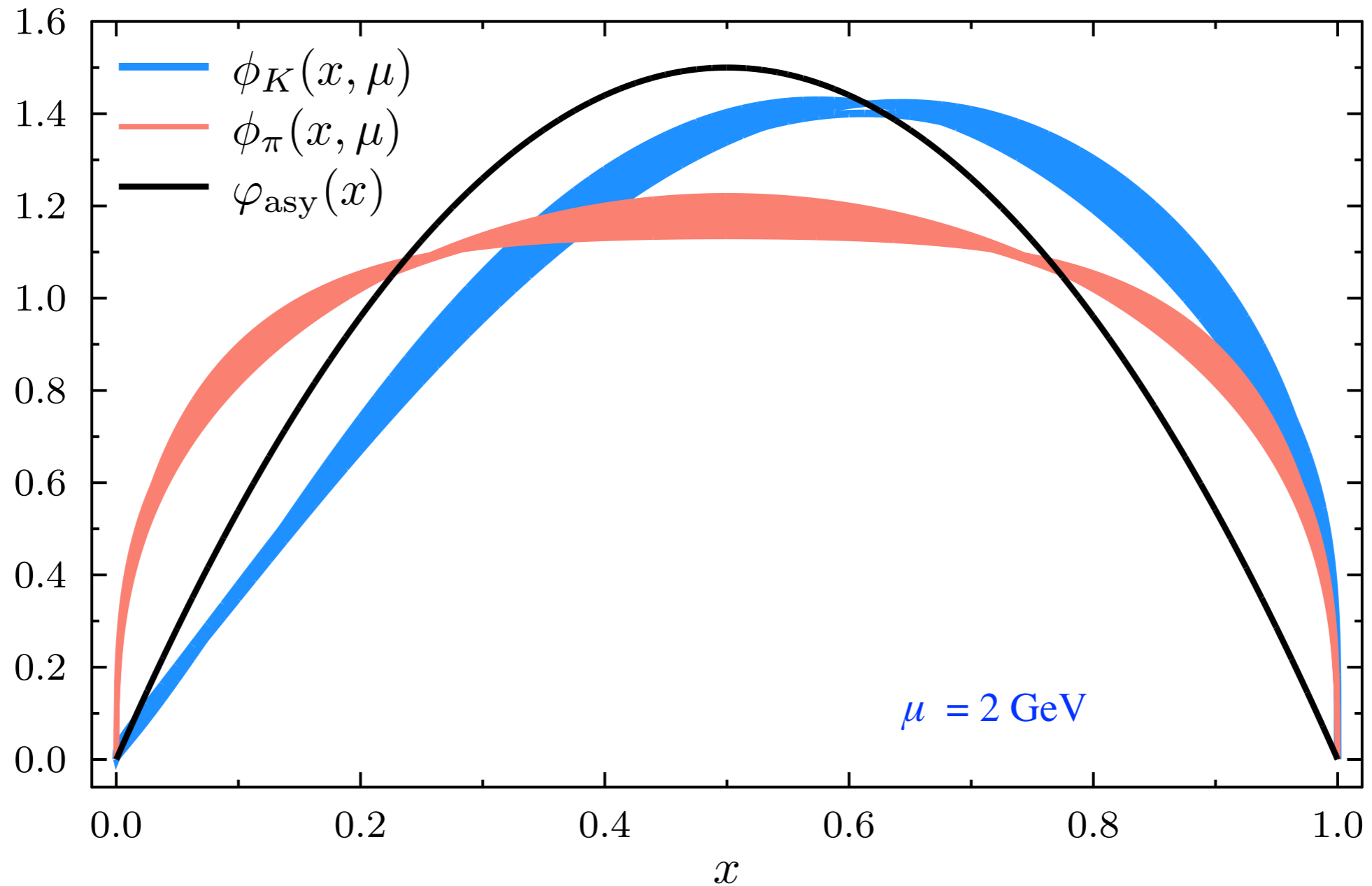
$$\mathcal{T}_\mu^8(k, P) = \hat{k}_\mu^T (\gamma \cdot \hat{k}^T) \gamma \cdot P$$

The base is transverse to the meson's momentum (transverse polarization):

$$V_\nu^T = V_\nu - P_\nu (V \cdot P) / P^2 \text{ with } P \cdot V^T = 0$$



# Light-Cone Distribution Amplitudes



L. Chang, I.C. Cloët, J.J. Cobos Martínez, C.D. Roberts, S.M. Schmidt, P. C. Tandy, Phys. Rev. Lett. 110 (2013)

F. Serna, R. Correa da Silveira, J.J. Cobos Martínez, **B.E.**, E. Rojas, Eur. Phys. J. C 80 (2020)