Towards J/ψ production off nucleons

"Heavy Flavor Production in Heavy-Ion and Elementary Collisions"

Institute for Nuclear Theory — University of Washington, October 23–28, 2022

Bruno El-Bennich

Laboratório de Física Teórica e Computacional
Universidade Cidade de São Paulo

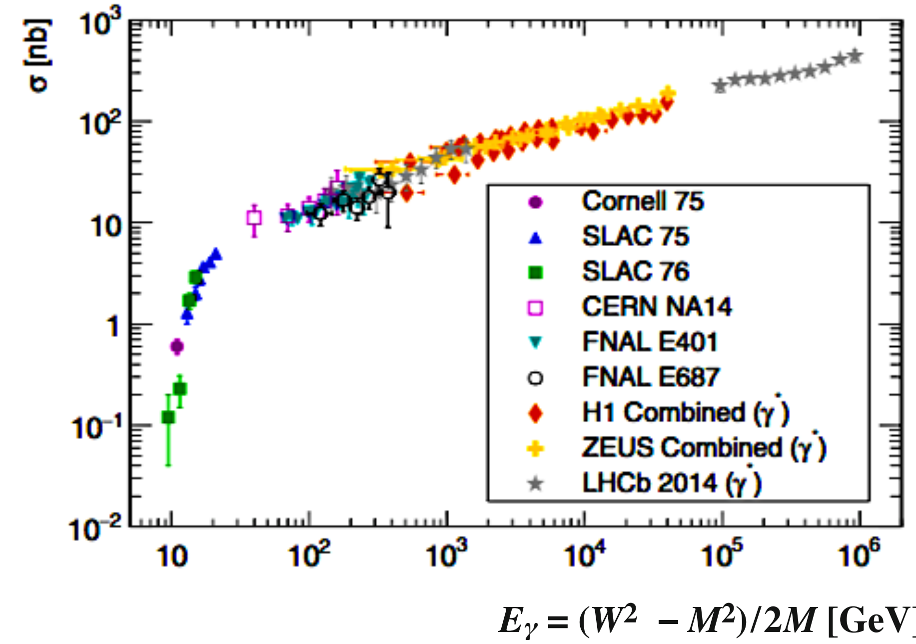
FAPESP
FUNDAÇÃO DE AMPARO À PESQUISA DO ESTADO DE SÃO PAULO

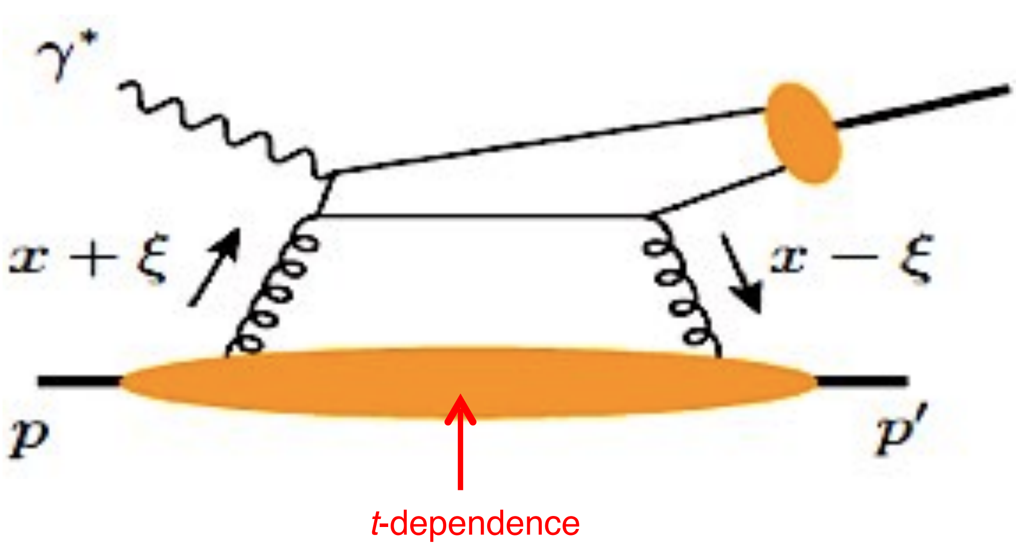
CNPq
Conselho Nacional de Desenvolvimento Científico e Tecnológico

Spatial imaging of glue in a nucleon/nucleus

Exclusive electroproduction: hard-scattering mechanism $E_\gamma > 10$ GeV

J. C. Collins, L. Frankfurt, M. Strikman, Phys.Rev.D 56 (1997)
S.J. Brodsky, E. Chudakov, P. Hoyer, J.M. Laget, Phys. Lett. B (2001).
D.Y. Ivanov, A. Schäfer, L. Szymanowski and G. Krasnikov, Eur. Phys. J. C 34 (2004)





$J/\Psi, \eta_c, \ldots$

$$Q^2 = -\left(p_e^i - p_e^f\right)^2$$
$$W = \sqrt{2\nu \cdot M_p + M_p^2 - Q^2}$$
$$t = \left(p_p^i - p_p^f\right)^2 = (p_\gamma - p_{J/\psi})^2$$
$$\xi = (P' - P) \cdot n/2$$

- Exclusive *charmonium production*: narrow quarkonium exchanges gluons with the nucleon's light quarks.
- Cross section is proportional to the square of the gluon density in the hadron.
- At large virtualities $Q^2 \gg m_Q^2$, electroproduction of heavy mesons is expressed in terms of *meson distribution amplitudes* in the factorization theorem (Collins, Frankfurt & Strikman 1997).

Donnachie and Landshoff model : *VMD* assumption

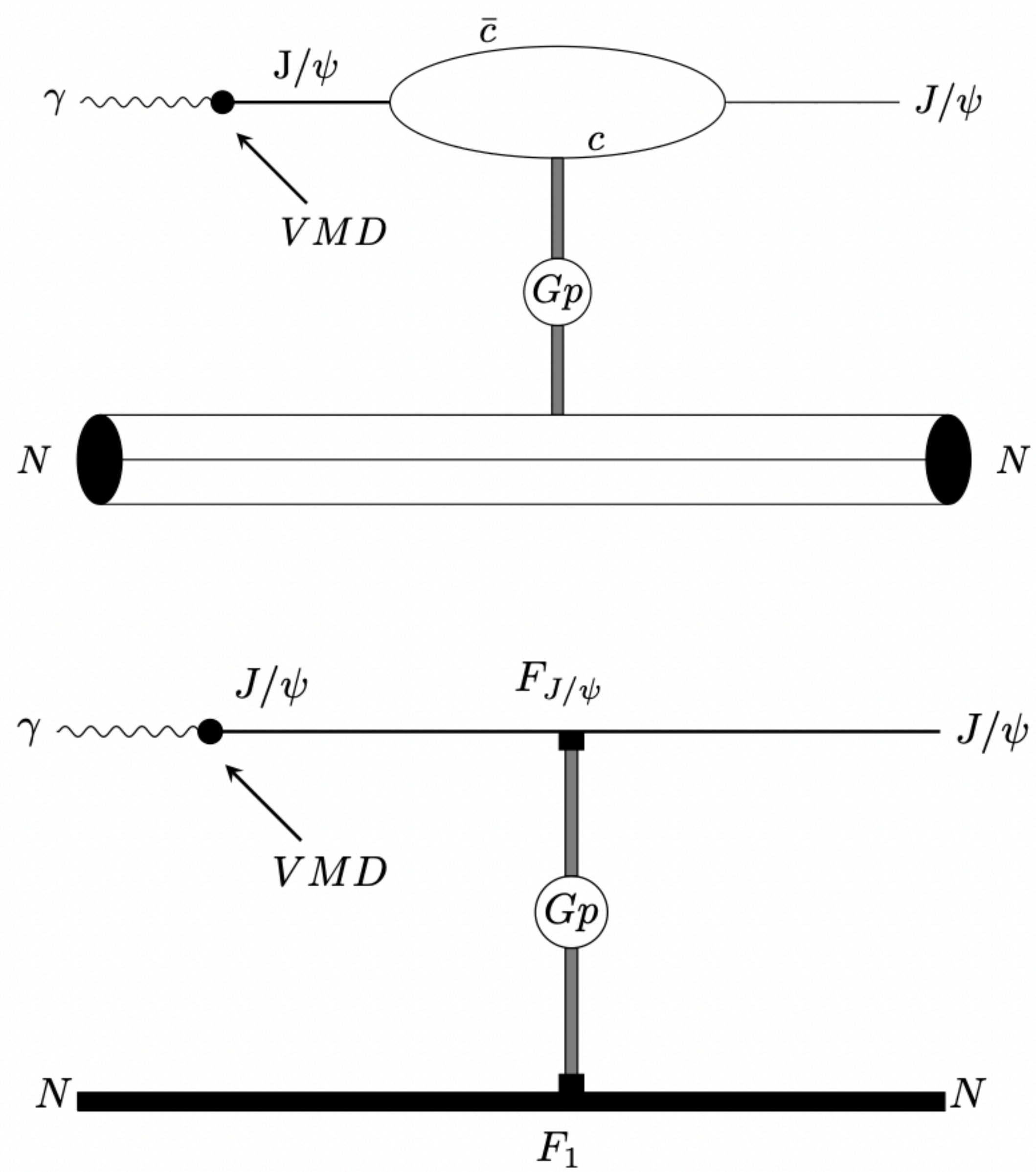


Fig. 2 Pomeron-exchange model of Donnachie and Landshoff (*Pom – DL*). Upper: Pomeron-exchange between quarks in J/Ψ and nucleon, Lower: Pomeron-exchange amplitude Eq. (1) resulted from assuming the Pomeron-photon analogy and using the factorization approximation.

$$t^{\rm Pom} \sim \left[\frac{e}{f_V} F_V(t)\right] \times G_P(s,t) \times F_1(t)$$

Width of V → e^+e^- (*VMD*)

Pomeron propagator

Model can not describe the JLab data at W < 7 GeV

2g + 3g model by Brodsky, E. Chudakov, P. Hoyer, J. M. Laget (2001)

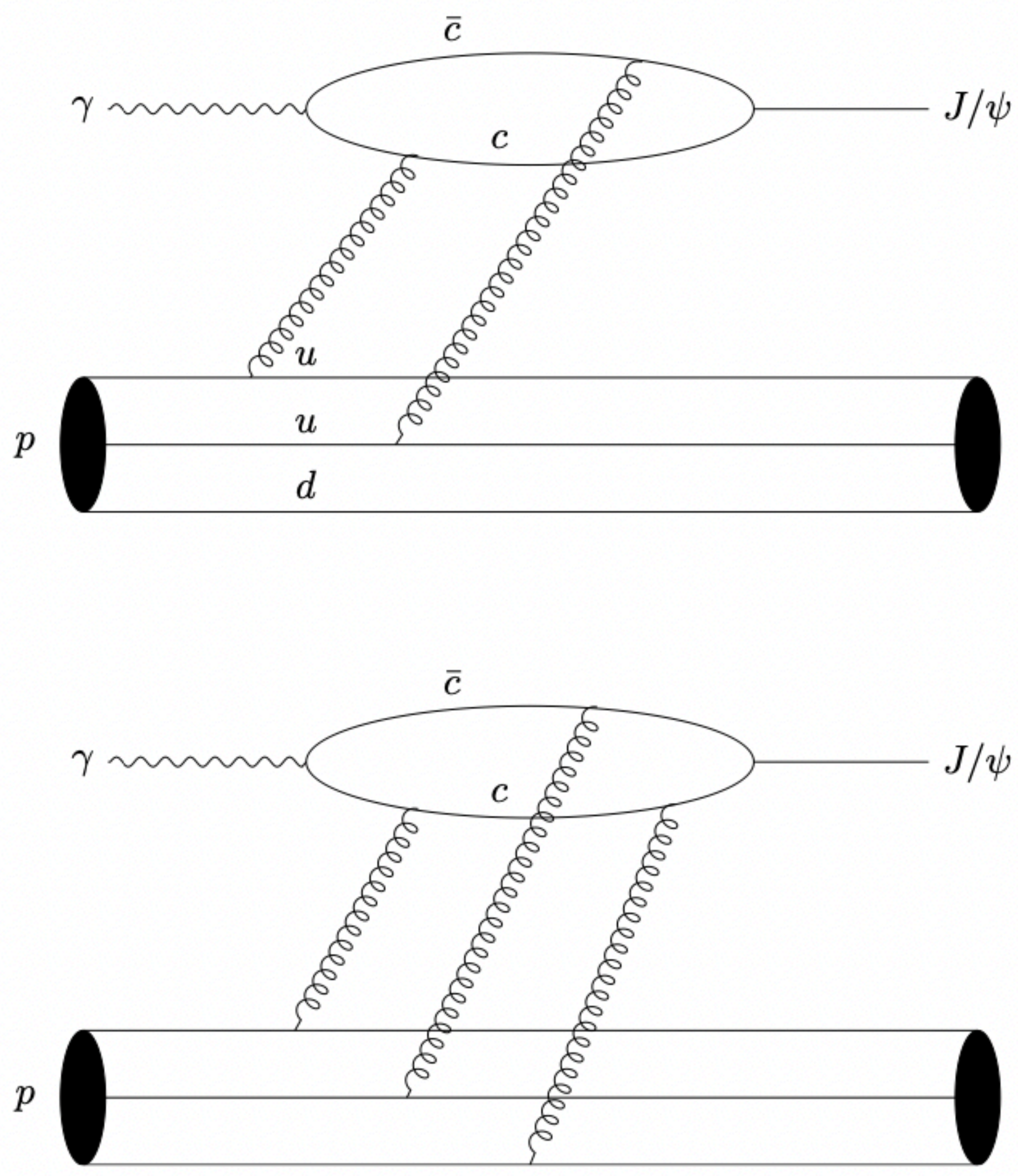


Fig. 5 The $2g+3g$ model. Upper: two-gluon exchange, Lower: three-gluon exchange.

$$\frac{d\sigma_{2g}}{dt} = N_{2g}\frac{(1-x)^2}{R^2 M_{c\bar{c}}^2} F_{2g}^2(t)(s-m_p^2),$$

$$\frac{d\sigma_{3g}}{dt} = N_{3g}\frac{(1-x)^0}{R^4 M_{c\bar{c}}^4} F_{3g}^2(t)(s-m_p^2),$$

Requires form factors to describe how the scattered quarks combine with the spectator quarks to form a proton.

Fails to reproduce experimental data above $W \approx 20$ GeV.

Pomeron exchange + charm-nucleon potential, M. A. Pichowsky & T.-S. H. Lee

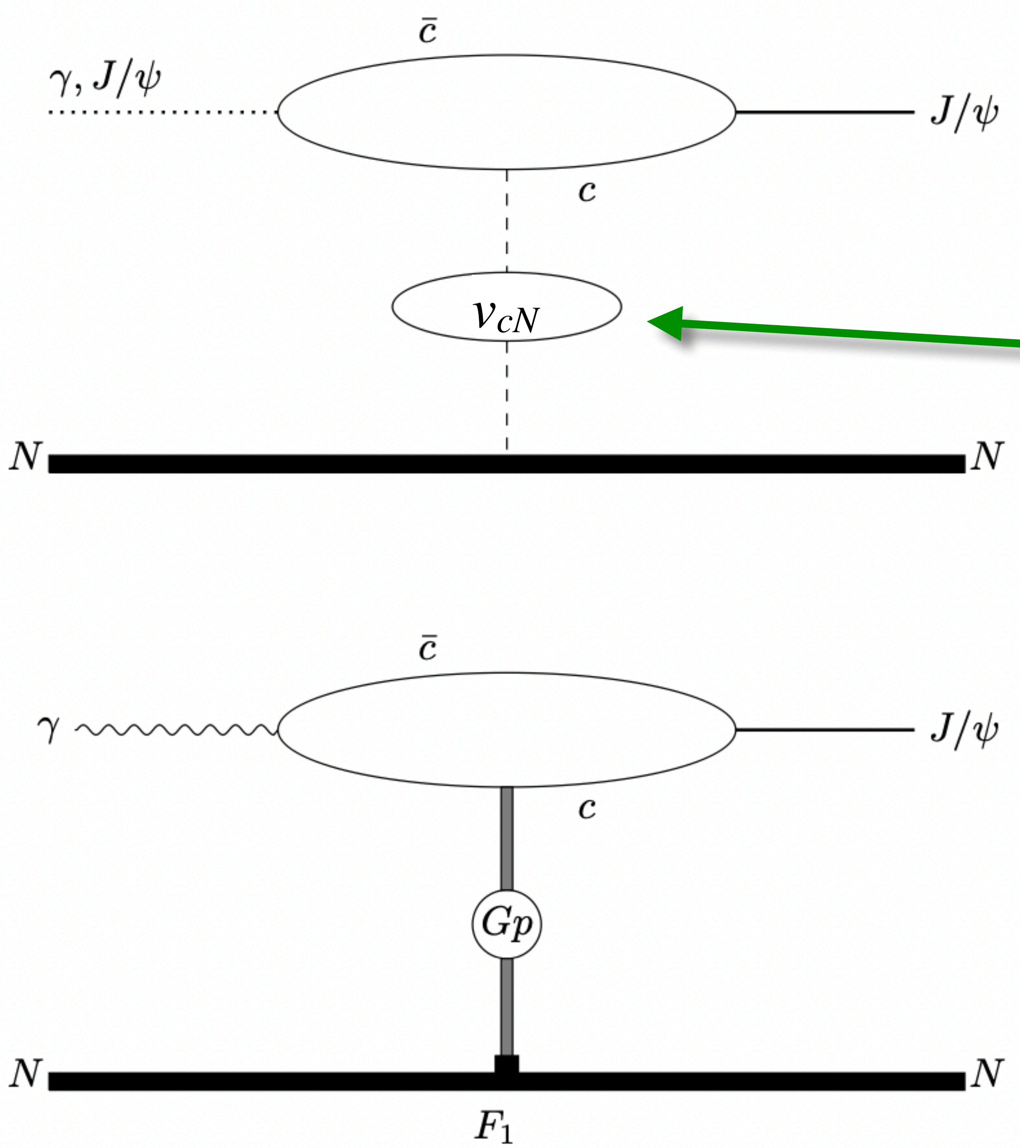
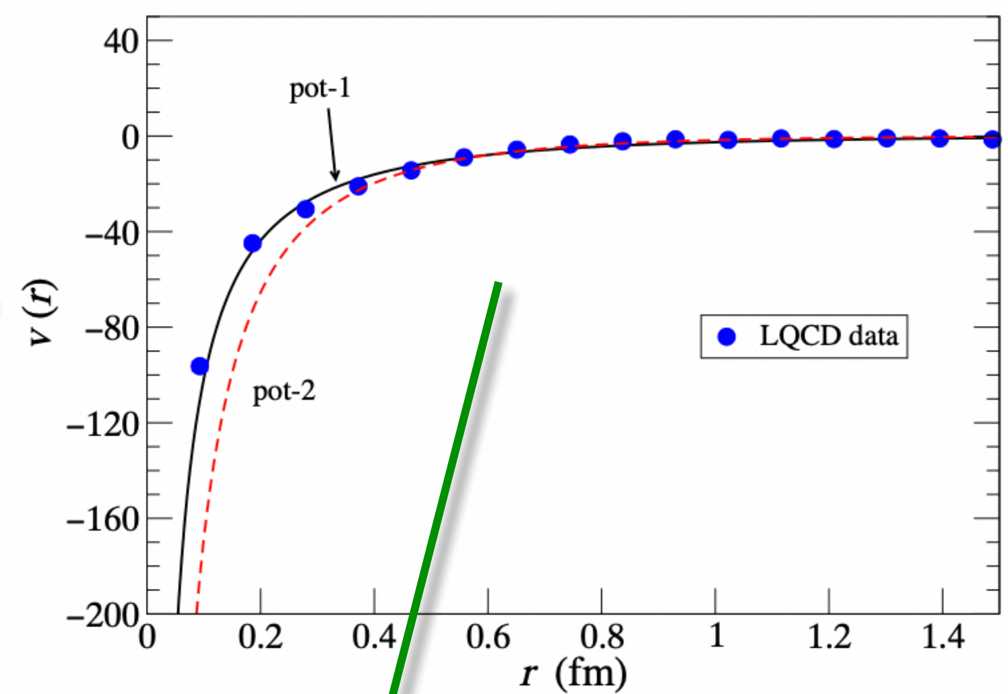


Fig. 7 Models with $c\bar{c}$-loop mechanisms. Upper: calculated from quark-nucleon potential (v_{cN}), Lower: calculated from Pomeron-exchange mechanism.

T. Kawanai, S. Sasaki, Charmonium-nucleon potential from lattice QCD, Phys. Rev. D 82, 091501 (2010)



$$v_{J/\Psi N,J/\Psi N}(r) = v_0 \frac{e^{-\alpha r}}{r}$$

$$t^{\text{Pom+pot}} = t^{\text{Pom}} + t^{\text{pot}}$$

Lippmann-Schwinger equation for the *J/Ψ-N* scattering amplitude:

$$t_{J/\Psi N,J/\Psi N} = v_{J/\Psi N,J/\Psi N} + v_{J/\Psi N,J/\Psi N}\, G_{J/\Psi N}(W)\, t_{J/\Psi N,J/\Psi N}$$

How to improve on this model?

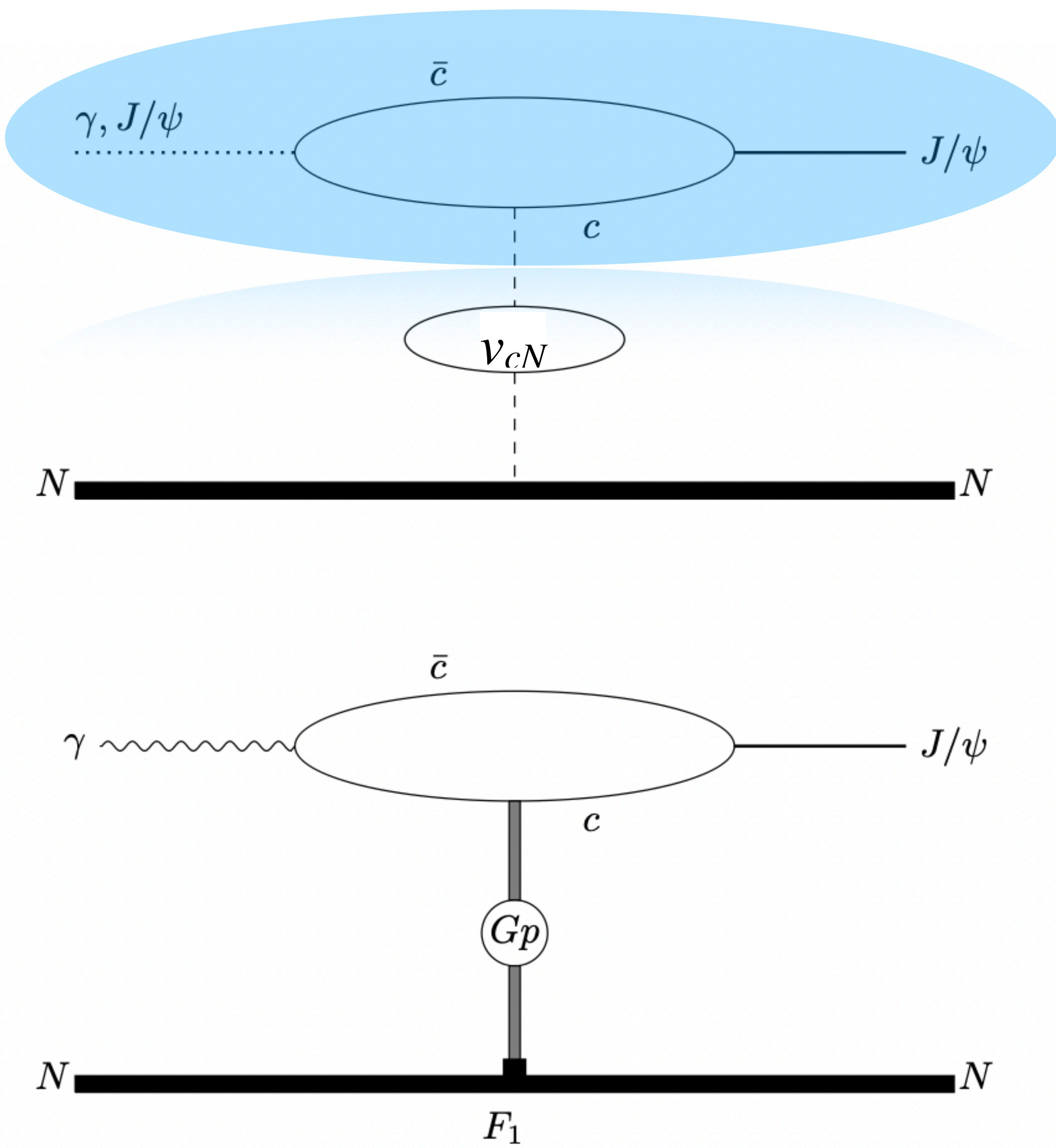


Fig. 7 Models with $c\bar{c}$-loop mechanisms. Upper: calculated from quark-nucleon potential (v_{cN}), Lower: calculated from Pomeron-exchange mechanism.

- *Nonperturbative quark-photon $\gamma \to \bar{c}c$ vertex that satisfies Ward identity. No VMD !!*
- *Realistic Bethe-Salpeter wave functions/ distribution amplitudes for heavy quarkonia.*
- *Quark propagators as numerical solutions of gap equation consistent with wave functions instead of constituent quark propagators.*
- *This implies the calculation of $\gamma \to J/\psi$ transition form factor that describes a Pomeron exchange with the nucleon.*
- *Employ nucleon GPD to describe data near threshold.*

$$< P\lambda_V; p_2 m_s' | J_\mu | p_1, m_s > = 2 t_{\mu\alpha\nu}(q, P)\epsilon_\nu^{\lambda_V}(P) \times [G(s,t) 3\beta_u F_1(t)] \bar{u}_{m_s'}(p_2)\gamma_\alpha u_{m_s}(p_1)$$

$$t_{\mu\alpha\nu}(q, P) = \beta_c N_c\, e\, tr \int \frac{d^4k}{(2\pi)^4} S_c(k_-)\gamma_\mu S_c(k_- + q))\gamma_\alpha S_c(k_+) V_\nu(k_+, k_-)$$

Let’s focus on the $\bar{c}c \to J/\psi$ hadronization …

Green functions in functional approaches to QCD

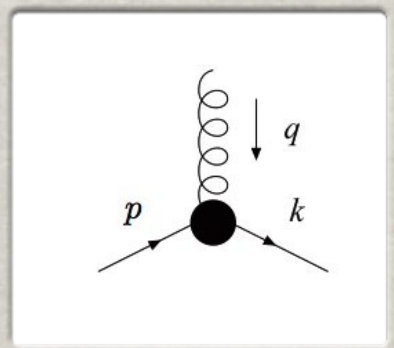
Dyson-Schwinger equation in QCD

The propagator can be obtained from QCD's gap equation: the Dyson-Schwinger equation (DSE) for the dressed-fe… coupled equations.

Running Quark Mass

$$S(p) = \frac{Z(p^2)}{i\gamma \cdot p + M(p^2)}$$

$$\begin{aligned} S^{-1}(p) &= Z_2(i\gamma \cdot p + m^{\rm bm}) + \Sigma(p) := i\gamma \cdot p\, A(p^2) + B(p^2) \\ \Sigma(p) &= Z_1 \int^{\Lambda} \frac{d^4 q}{(2\pi)^4}\, g^2 D_{\mu\nu}(p-q) \frac{\lambda^a}{2} \gamma_\mu S(q) \Gamma^a_\nu(q,p) \end{aligned}$$

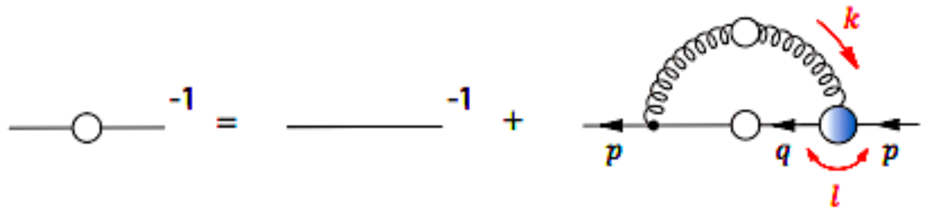


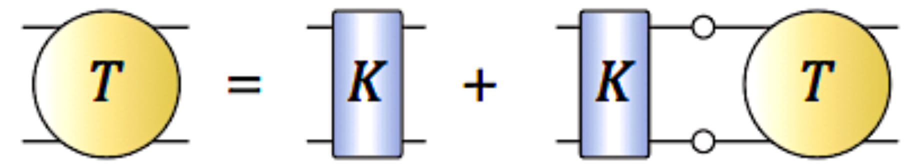
with the *running* mass function $M(p^2) = B(p^2)/A(p^2)$.

$D_{\mu\nu}$	:	dressed-gluon propagator
$\Gamma^a_\nu(q,p)$	:	dressed quark-gluon vertex
Z_2	:	quark wave function renormalization constant
Z_1	:	quark-gluon vertex renormalization constant

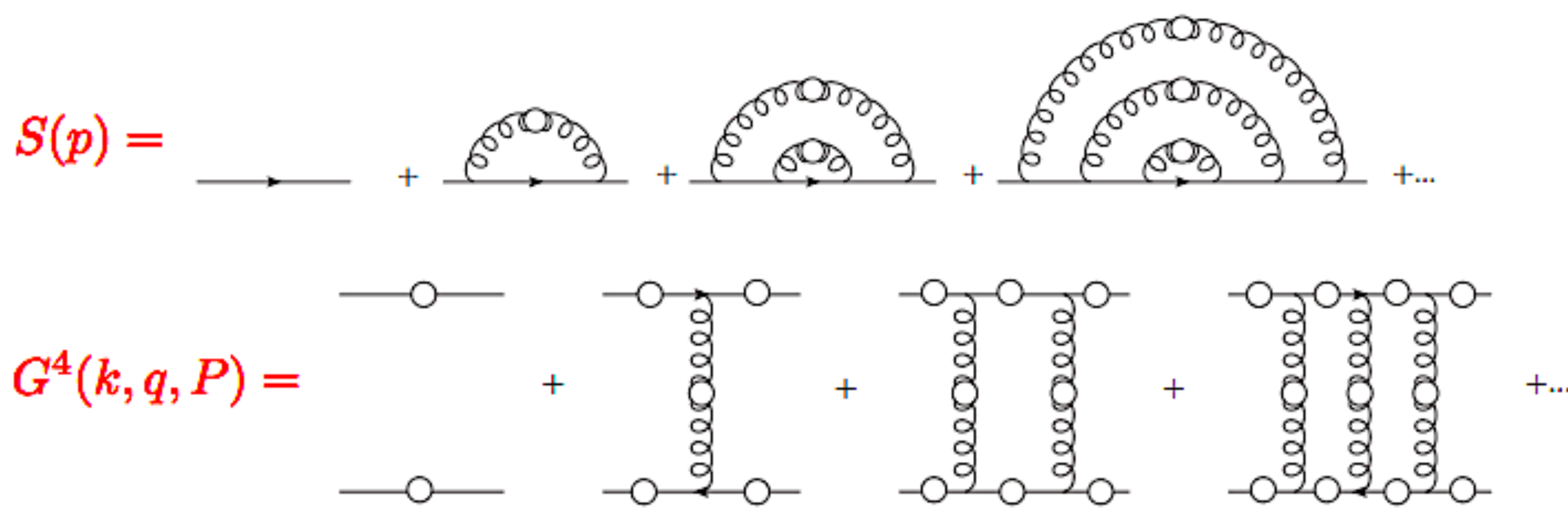
Each satisfies its own DSE

Bethe-Salpeter Equation for QCD Bound States





Rainbow-ladder truncation (leading symmetry-preserving approximation)



- Quark propagators are obtained by solving the gap equation (DSE) for space-like momenta.
- In solving the BSE in Euclidean space, the propagators are functions of $(k+P)^2,\ P=(0,0,0,iM)$.
- Extension to complex plane via Cauchy's integral theorem.

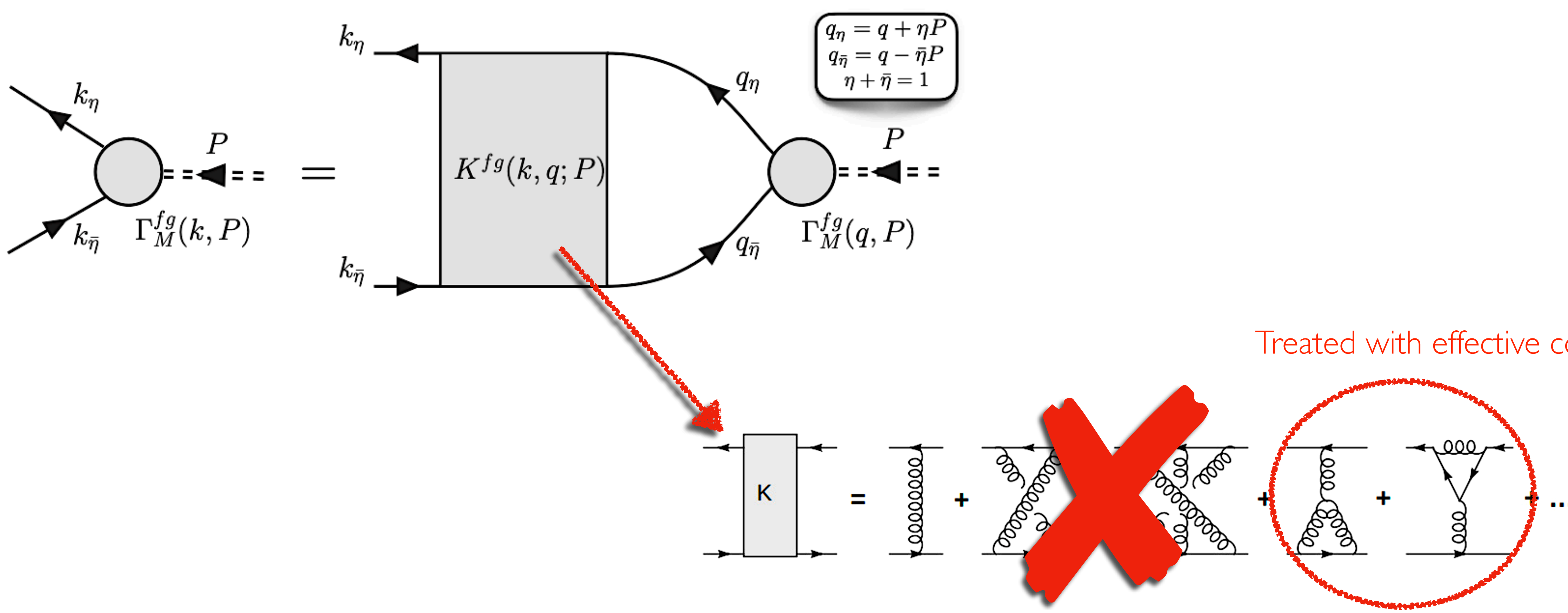
Bethe-Salpeter Equation for QCD Bound States

$$\Gamma_M^{fg}(k,P) = \int^{\Lambda} \frac{d^4q}{(2\pi)^4} K_{fg}(k,q;P) S_f(q_\eta) \Gamma_M^{fg}(q,P) S_g(q_{\bar{\eta}})$$

$K_{fg}(q,k;P)$ = Quark-antiquark scattering kernel

$S_f(q_\eta)$ = Dressed quark propagator

$\Gamma_M^{fg}(k,P)$ = Meson's Bethe-Salpeter Amplitude (BSA)



Bethe-Salpeter Amplitudes

- The general form of $\Gamma_M(k;P)$ is given by

$$\Gamma(k;P) = \sum_{i=1}^{N} \mathcal{T}^i(k,P)\mathcal{F}_i(k^2,z_k,P^2), \quad z_k = k.P/|k||P|,$$

- where $\mathcal{T}^i$(k,P) are Dirac's covariants;
- $\mathcal{F}_i(k^2,z_k,P^2)$ are Lorentz invariant amplitudes;
- N denotes the number of covariants which are different for different meson's channel.
- For the case of pseudoscalar mesons we have $N = 4$ and for vector mesons one has $N = 8$

Pseudoscalar Mesons

- **Pseudoscalar channel**

$$\mathcal{T}^1_{ps}(k,P) = i\gamma_5$$
$$\mathcal{T}^2_{ps}(k,P) = \gamma_5\gamma.P$$
$$\mathcal{T}^3_{ps}(k,P) = \gamma_5\gamma.k(k.P)$$
$$\mathcal{T}^4_{ps}(k,P) = \gamma_5\sigma_{\mu\nu}k_\mu P_\nu$$

- For the pseudoscalar mesons, we employ the tensor structures that are not orthogonal with respect to the dirac trace

$$\Gamma_{ps}(k,P) = \gamma_5[iE_{ps}(k,P) + \gamma.PF_{ps}(k,P) + \gamma.kk.PG_{ps}(k,P) + \sigma_{\mu\nu}k\mu P\nu H_{ps}(k,P)]$$

Beyond the Quark Model

non-relativistic $q\bar{q}$

S	L	J^{PC}
0	0	0^{-+}
1	0	1^{--}
0	1	1^{+-}

$$P : (-1)^{L+1}$$

relativistic $q\bar{q}$

$$\Gamma_\pi(P,p) = \gamma_5 \left[F_1(P,p) + F_2(P,p) i\not{P} + F_3(P,p) pP i\not{p} + F_4(P,p) [\not{p}, \not{P}] \right]$$

s-wave: $F_1(P,p) + F_2(P,p) i\not{P}$

p-wave: $F_3(P,p) pP i\not{p} + F_4(P,p) [\not{p}, \not{P}]$

~~$P : (-1)^{L+1}$~~

Llewellyn-Smith, Annals Phys. 53 (1969) 521–558

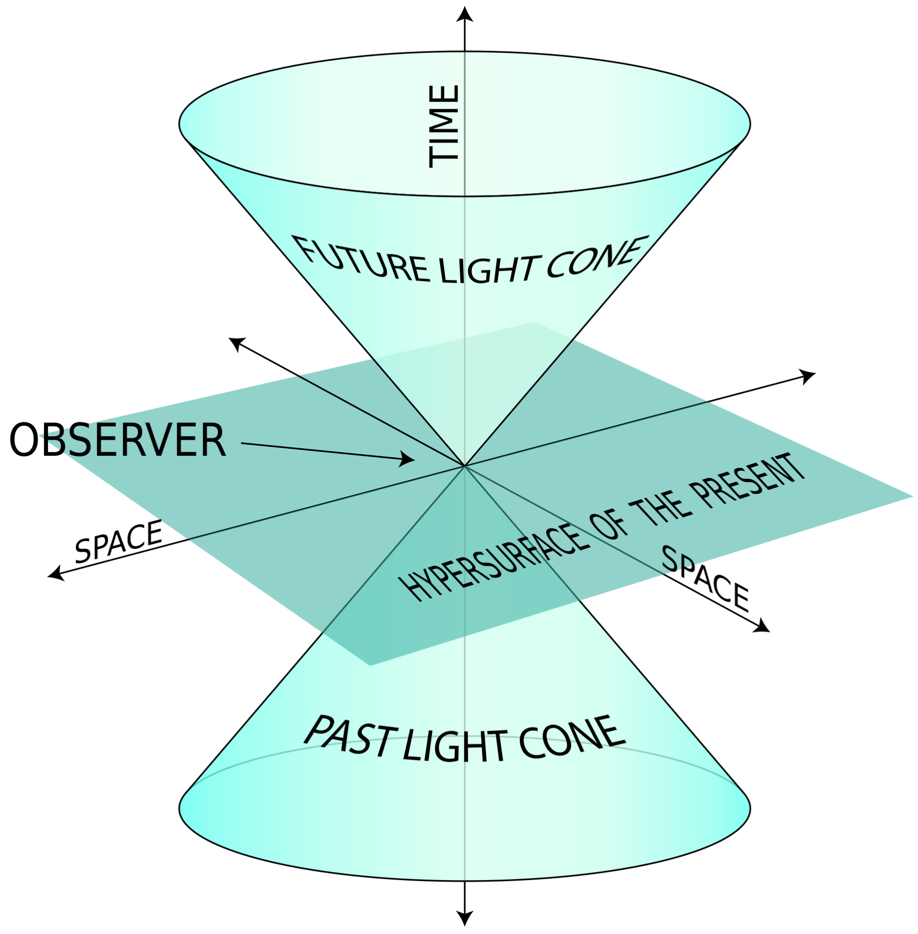
Vector Mesons

- **Vector channel**

$$\mathcal{T}^1_\mu(k,P) = i\gamma^T_\mu$$
$$\mathcal{T}^2_\mu(k,P) = i[3k^T_\mu(\gamma.k_\mu) - \gamma^T_\mu(k^T)^2]$$
$$\mathcal{T}^3_\mu(k,P) = i(k.P)k^T_\mu\gamma.P$$
$$\mathcal{T}^4_\mu(k,P) = i[\gamma^T_\mu\gamma.P(\gamma.k^T) + k^T_\mu\gamma.P]$$
$$\mathcal{T}^5_\mu(k,P) = k^T_\mu$$
$$\mathcal{T}^6_\mu(k,P) = (k.P)[\gamma^T_\mu(\gamma.k^T) - (\gamma.k^T)\gamma^T_\mu]$$
$$\mathcal{T}^7_\mu(k,P) = \gamma^T_\mu\gamma.P - \gamma.P\gamma^T_\nu - 2\hat{k}^T_\mu(\gamma.\hat{k}^T)\gamma.P$$
$$\mathcal{T}^8_\mu(k,P) = \hat{k}^T_\mu(\gamma.\hat{k}^T)\gamma.P$$

The base is transverse to the meson's momentum (transverse polarization):

$$V^T_\nu = V_\nu - P_\nu(V\cdot P)/P^2 \text{ with } P\cdot V^T = 0$$



Meson Distribution Amplitudes on the Light Front

Light-Cone Distribution Amplitudes

- Hadron light-cone distribution amplitudes (LCDAs) have been introduced four decades ago in the context of the QCD description of hard exclusive reactions.

- The LCDAs are scale-dependent nonperturbative functions that can be interpreted as quantum-mechanical amplitudes.

- $\phi_M(x, \mu)$ is a probability amplitude that describes the momentum distribution of a quark and anti-quark in the bound-state's valence Fock state.
- x is the light-front momentum fraction: $\frac{k^+}{P^+}$ and μ is the renormalization scale.

Light-Cone Distribution Amplitudes

- We do not obtain the LCDA directly from the light-front wave functions:

$$f_M \, \phi_M(x,\mu) = \int^{\mu^2} \frac{d^2 k_\perp}{16\pi^2} \, \psi_M(x, k_\perp)$$

- Instead, we compute the Mellin moments:

$$\langle x^m \rangle = \int_0^1 dx \, x^m \phi_M(x,\mu) \qquad \langle x^0 \rangle = \int_0^1 dx \, \phi_M(x,\mu) = 1$$

$$f_M \phi_M(x,\mu) = Z_2 \, \mathrm{tr}_{\mathrm{CD}} \int \frac{d^4 k}{(2\pi)^4} \, \delta(n\cdot k - x\, n\cdot P) \, \gamma_5 \, \gamma \cdot n \, S(k) \Gamma_\pi(k;P) S(k-P)$$

$$f_M (n\cdot P)^{m+1} \int_0^1 dx \, x^m \phi_M(x,\mu) = Z_2 \, \mathrm{tr}_{\mathrm{CD}} \int \frac{d^4 k}{(2\pi)^4} \, (n\cdot k)^m \, \gamma_5 \, \gamma \cdot n \, \chi_\pi(k;P)$$

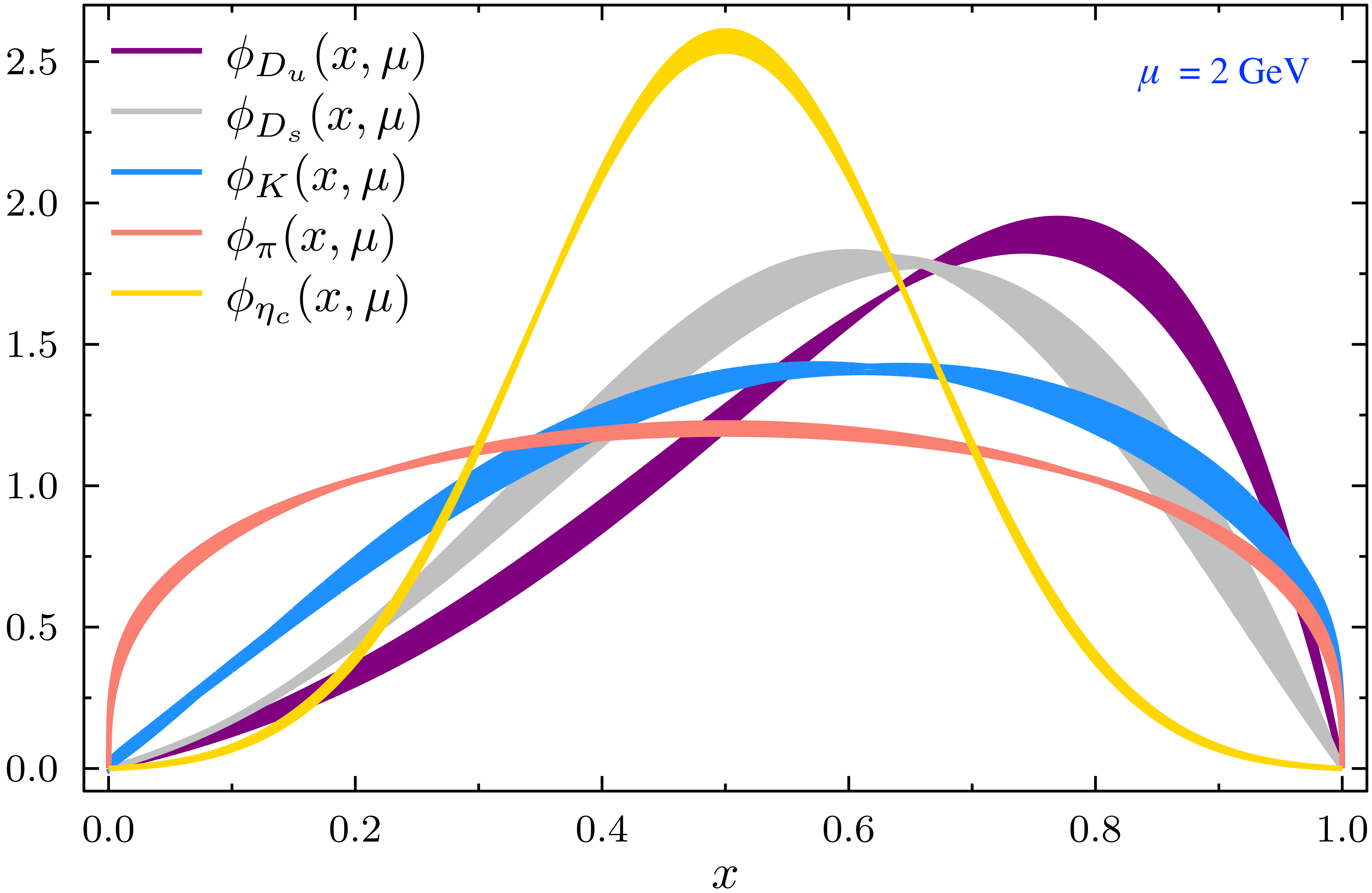
Bethe-Salpeter wave function

- Distribution amplitudes are reconstructed with a Gegenbauer-like expansion or other functional forms:

$$\phi_\pi^{\mathrm{rec.}}(x,\mu) = \mathcal{N}(\alpha) \, [x\bar{x}]^{\alpha - 1/2} \, [1 + a_2 C_2^\alpha(2x-1)]$$

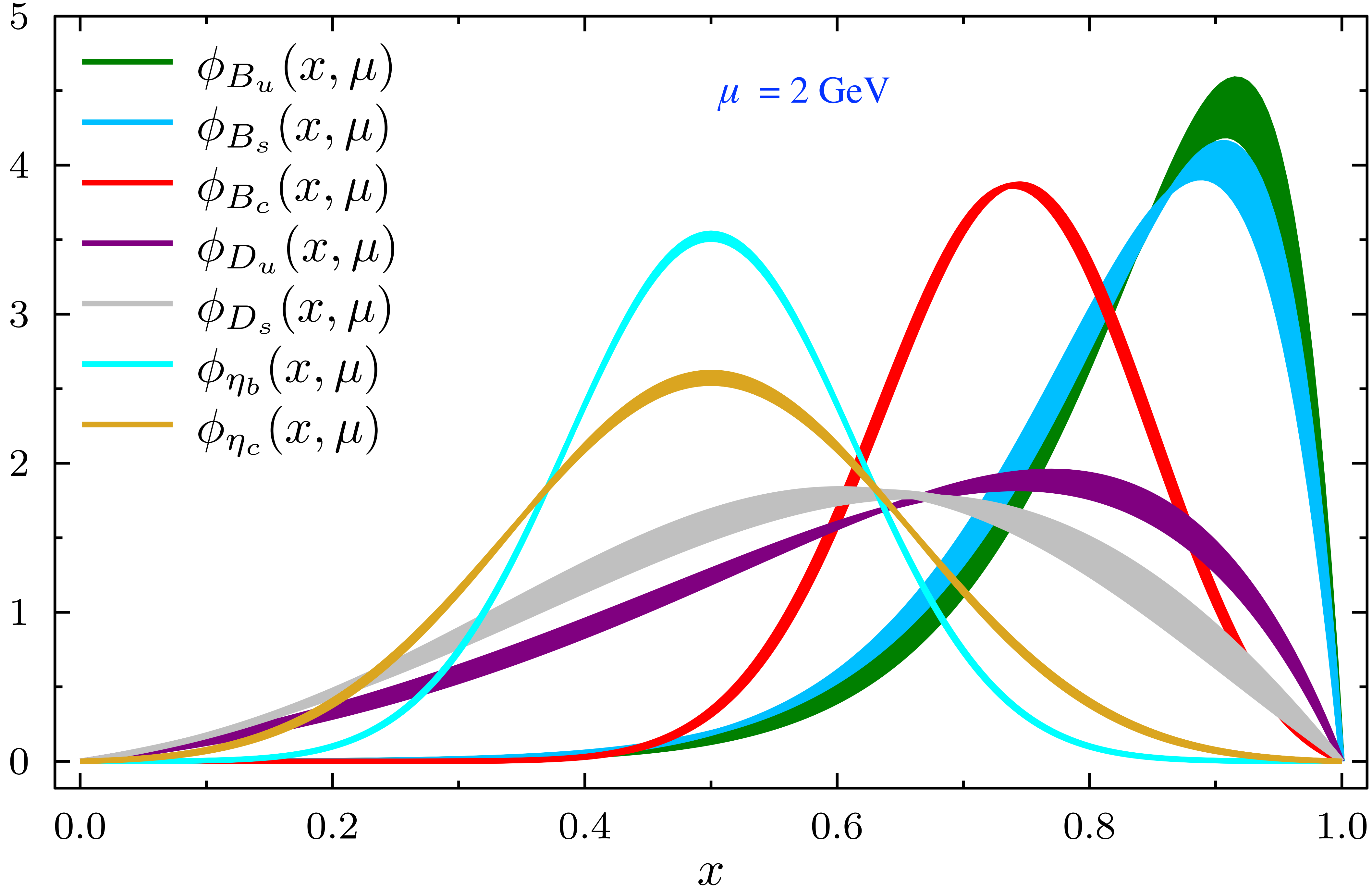
$$\phi_H^{\mathrm{rec.}}(x,\mu) = \mathcal{N}(\alpha,\beta) \, 4x\bar{x} \, e^{4\,\alpha x\bar{x} + \beta(x-\bar{x})}$$

Light-Cone Distribution Amplitudes



F. Serna, R. Correa da Silveira, J.J. Cobos Martínez, B.E., E. Rojas, Eur. Phys. J. C 80 (2020)

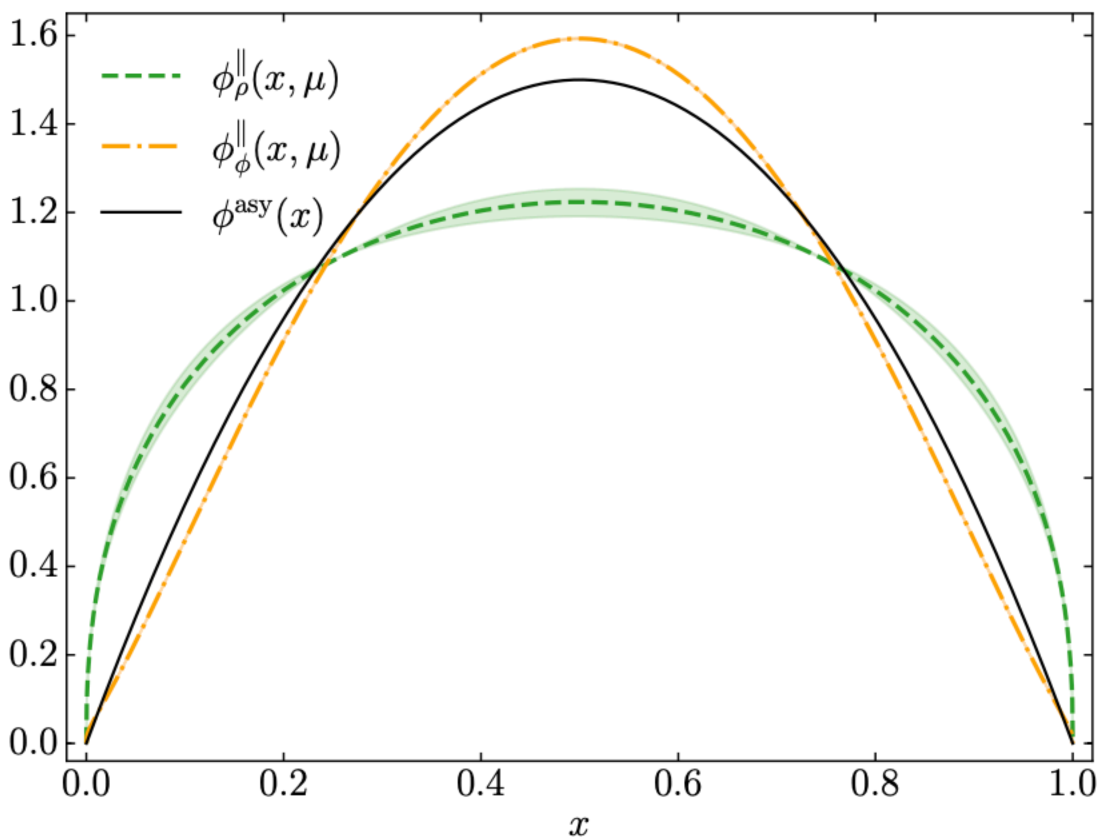
Light-Cone Distribution Amplitudes



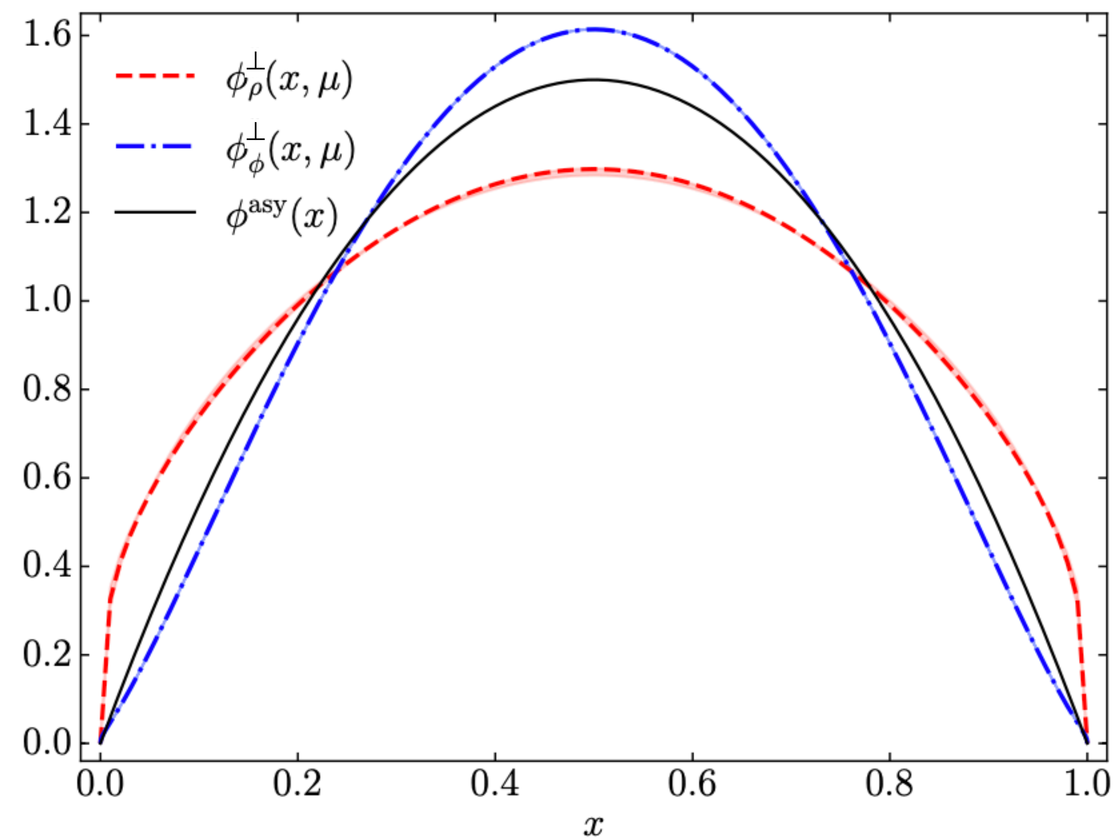
F. Serna, R. Correa da Silveira, J.J. Cobos Martínez, B.E., E. Rojas, Eur. Phys. J. C 80 (2020)

Light-Cone Distribution Amplitudes

ρ and ϕ mesons: longitudinal LCDA



ρ and ϕ mesons: transverse LCDA

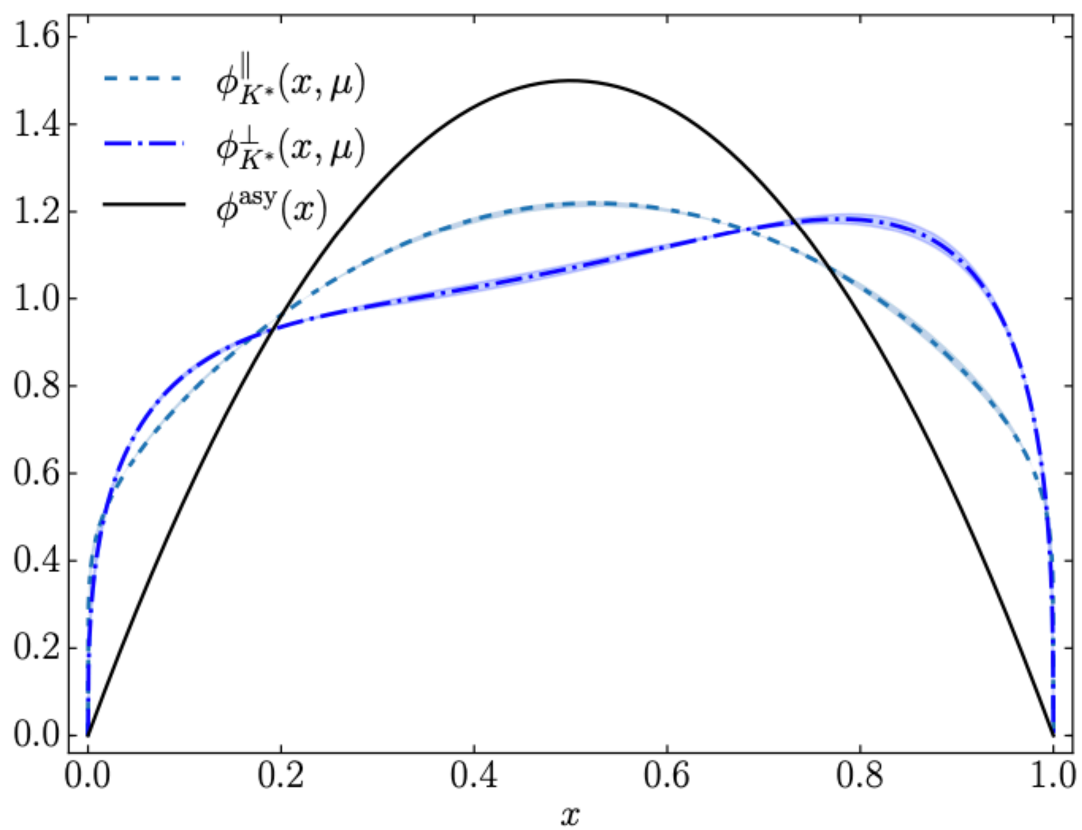


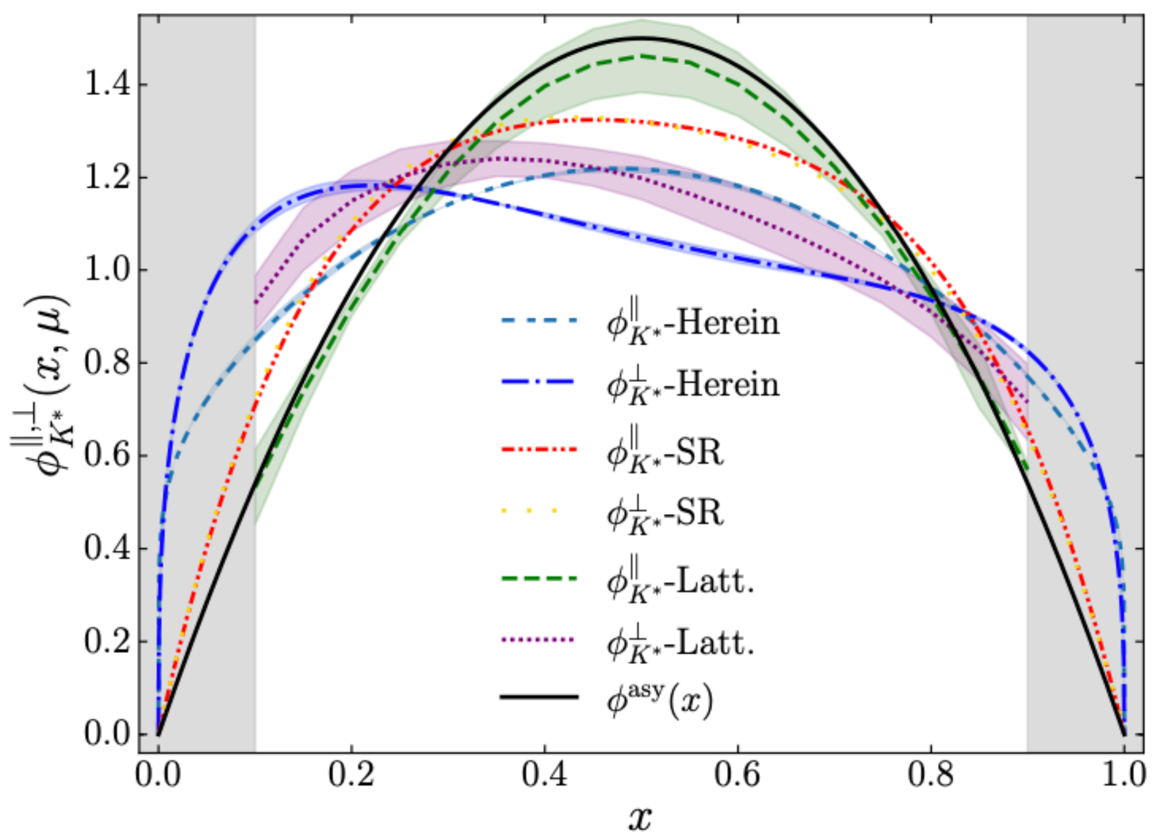
F. Serna, R. Correa da Silveira, B.E., PRD Lett. (2022)

We find: $\phi_\phi^{\parallel}(x,\mu) \approx \phi_\phi^{\perp}(x,\mu)$

Light-Cone Distribution Amplitudes

K^* meson longitudinal and transverse LCDA





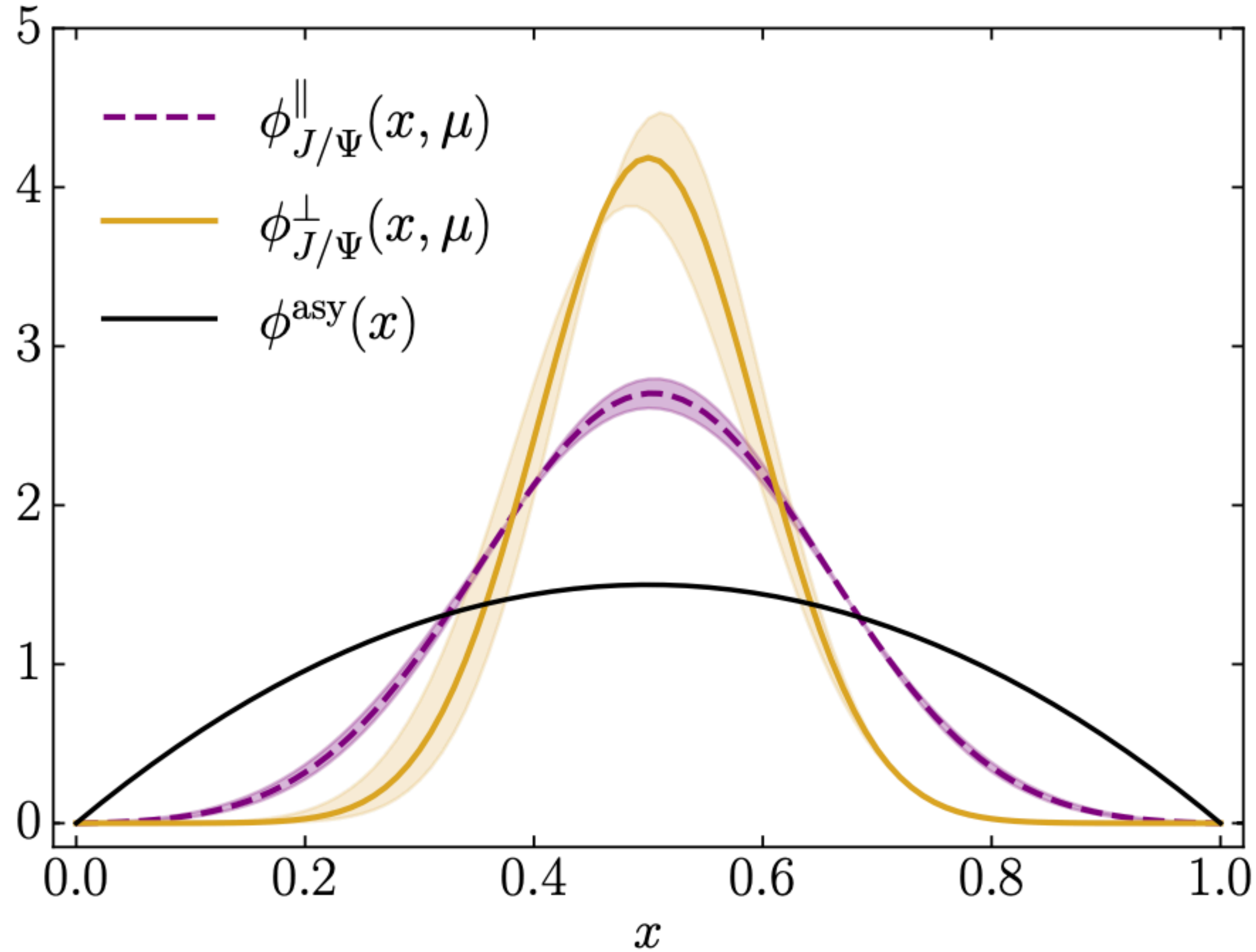
F. Serna, R. Correa da Silveira, B.E., PRD Lett. (2022)

Lattice QCD: Lattice Parton Collaboration, PRL 127 (2021)

QCD Sum Rule : P. Ball, V. M. Braun and A. Lenz, JHEP 08 (2007)

Light-Cone Distribution Amplitudes

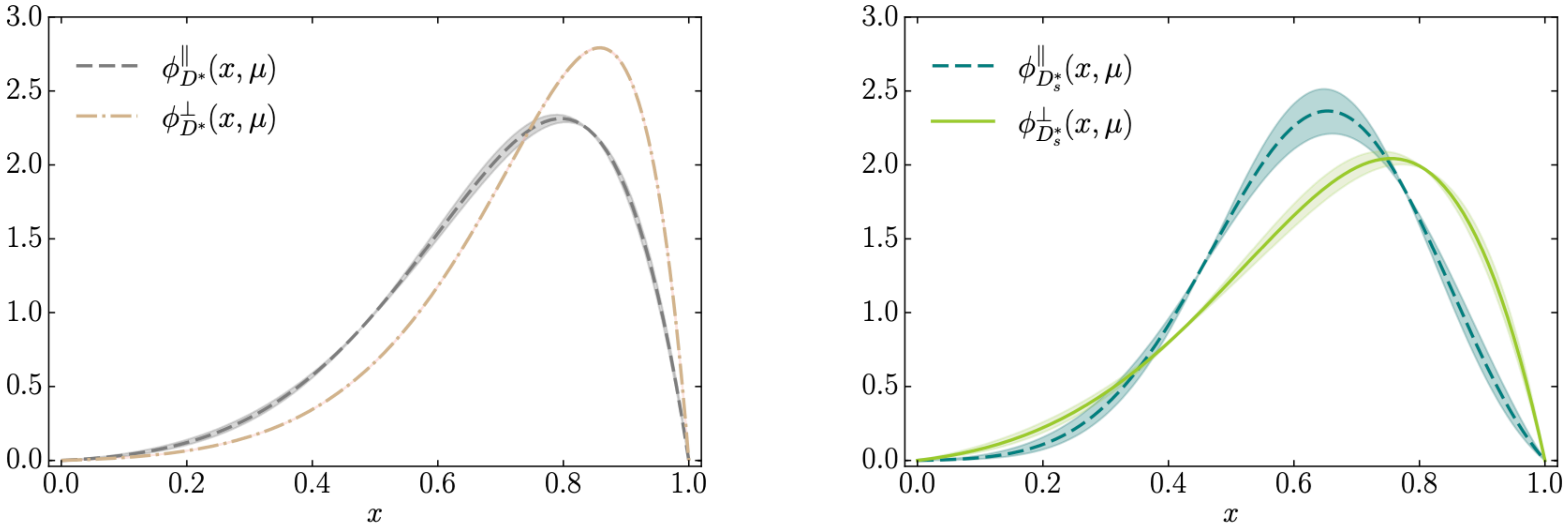
J/ψ meson longitudinal and transverse LCDA



F. Serna, R. Correa da Silveira, B.E., PRD Lett. (2022)

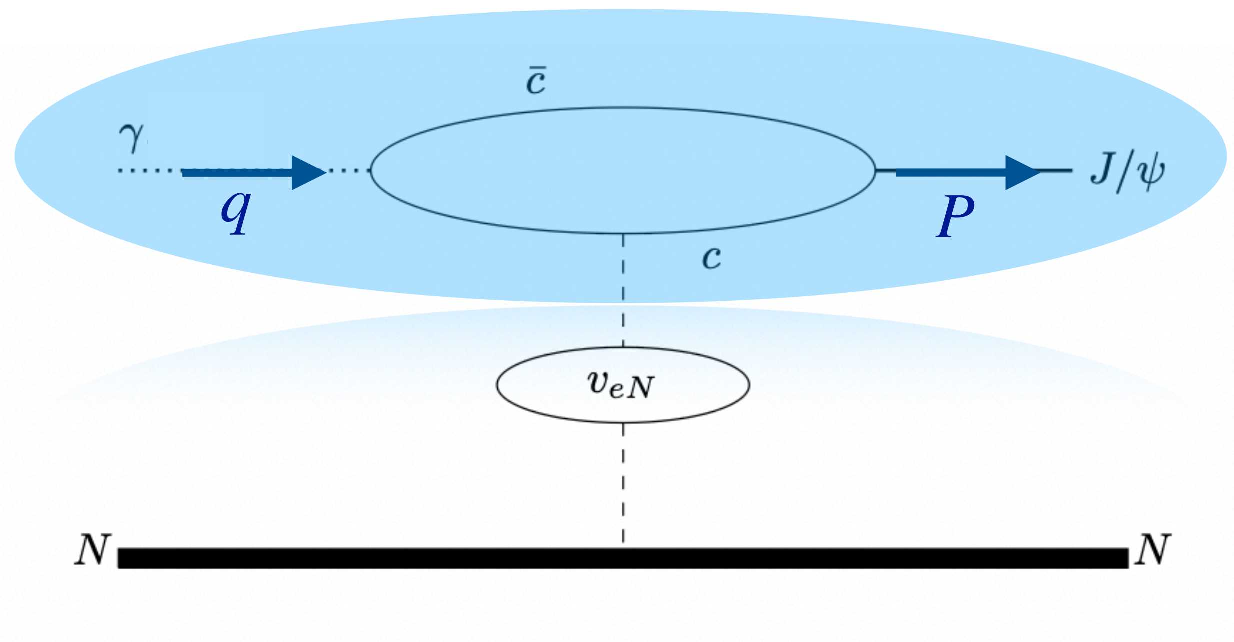
Light-Cone Distribution Amplitudes

D and D_s** mesons longitudinal and transverse LCDA



F. Serna, R. Correa da Silveira, B.E., PRD Lett. (2022)

Let's go back to the transition form factor ....

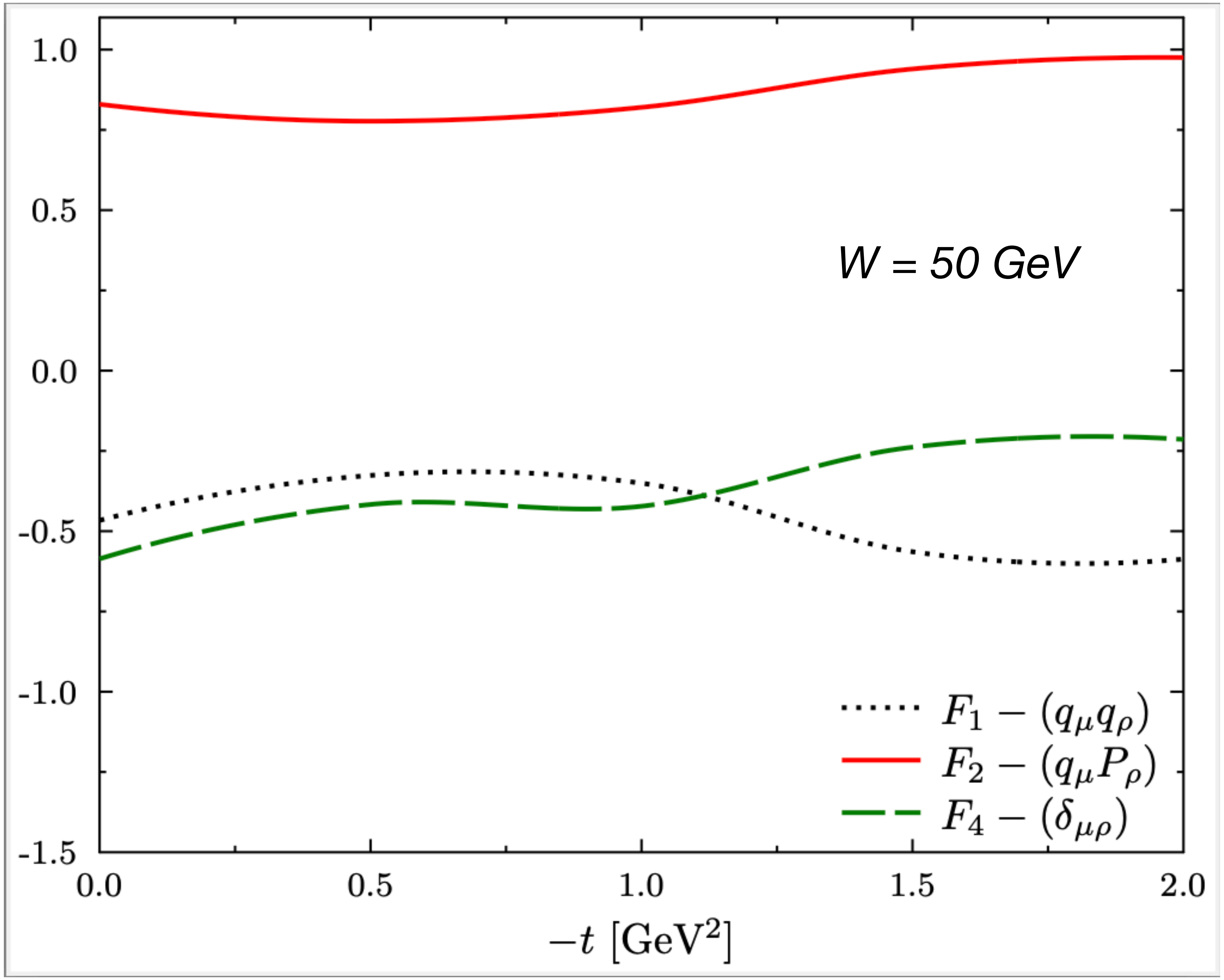


Assuming a scalar Pomeron (most likely better use a vector one), we can decompose the form factor/three-point Green function:

$$t_{\mu\nu}(q,P) = F_1(q,P)q_\mu q_\nu + F_2(q,P)q_\mu P_\nu + F_3(q,P)P_\mu P_\nu + F_4(q,P)\delta_{\mu\nu}$$

Beware, very preliminary !!!

Doesn't use full Bethe-Salpeter wave function of J/ψ



Conclusions & Progress

- Much progress was made from QCD based modeling toward nonperturbative numerical solutions of quark propagators and quark-antiquark bound states for flavored mesons satisfying chiral symmetry and Poincaré covariance.

- Good reproduction of charmonium and bottonium as well as D and B meson mass spectrum and their weak decay constants.

- Improvements in Bethe-Salpeter kernels beyond ladder truncation underway …
  $\Longrightarrow$ needed for scalar and axialvector channels and their higher radially excited states, as well as better control of quark correlation functions on complex plane.

- First predictions for LCDAs of vector D and D_s mesons.

- Concluding the calculation of the J/ψ production cross sections with form factors

- For all LCDAs we can readily provide functional parametrized expression.

Backup Slides

Pseudoscalar Meson Spectrum

Mesons/Observables	m_M	$m_M^{\text{exp.}}$	ϵ_r^m [%]	f_M	$f_M^{\text{exp./lQCD}}$	ϵ_r^f [%]
$\pi(u\bar{d})$	0.136	0.140	2.90	$0.094^{+0.001}_{-0.001}$	0.092(1)	2.17
$K(s\bar{u})$	0.494	0.494	0.0	$0.110^{+0.001}_{-0.001}$	0.110(2)	0.0
$D_u(c\bar{u})$	$1.867^{+0.008}_{-0.004}$	1.870	0.11	$0.144^{+0.001}_{-0.001}$	0.150(0.5)	4.00
$D_s(c\bar{s})$	$2.015^{+0.021}_{-0.018}$	1.968	2.39	$0.179^{+0.004}_{-0.003}$	0.177(0.4)	1.13
$\eta_c(c\bar{c})$	$3.012^{+0.003}_{-0.039}$	2.984	0.94	$0.270^{+0.002}_{-0.005}$	0.279(17)	3.23
$\eta_b(b\bar{b})$	$9.392^{+0.005}_{-0.004}$	9.398	0.06	$0.491^{+0.009}_{-0.009}$	0.472(4)	4.03

Mesons/Observables	m_M	$m_M^{\text{exp.}}$	ϵ_r^m [%]	f_M	f_M^{lQCD}	ϵ_r^f [%]
$B_u(b\bar{u})$	$5.277^{+0.008}_{-0.005}$	5.279	0.04	$0.132^{+0.004}_{-0.002}$	0.134(1)	4.35
$B_s(b\bar{s})$	$5.383^{+0.037}_{-0.039}$	5.367	0.30	$0.128^{+0.002}_{-0.003}$	0.162(1)	20.50
$B_c(b\bar{c})$	$6.282^{+0.020}_{-0.024}$	6.274	0.13	$0.280^{+0.005}_{-0.002}$	0.302(2)	7.28
$\eta_b(b\bar{b})$	$9.383^{+0.005}_{-0.004}$	9.398	0.16	$0.520^{+0.009}_{-0.009}$	0.472(4)	10.17

F. Serna, R. Correa da Silveira, J.J. Cobos Martínez, B.E., E. Rojas, Eur. Phys. J. C 80 (2020)

Vector Meson Spectrum

Mesons/Properties	m_M	m_M^{exp}	ϵ_r^m %	f_M	$f_M^{exp/\text{lQCD}}$	ϵ_r^f %
$\rho(u\bar{d})$	0.730	0.775	5.810	0.145	0.153	5.229
$\phi(s\bar{s})$	1.070	1.019	5.197	0.187	0.168	11.309
$K^*(s\bar{u})$	0.942	0.896	5.134	0.177	0.159	11.321
$J/\Psi(c\bar{c})$	3.124	3.097	0.872	0.277	0.294	5.782
$\Upsilon(b\bar{b})$	9.411	9.460	0.518	0.594	0.505	17.624

Mesons/Properties	m_M	m_M^{exp}	ϵ_r^m %	f_M	$f_M^{exp/\text{lQCD}}$	ϵ_r^f %
$D_u^*(c\bar{u})$	2.021	2.009	0.597	0.165	0.158	4.430
$D_s^*(c\bar{s})$	2.169	2.112	2.699	0.205	0.190	7.895

F. Serna, B.E., *PoS CHARM2020* (2021)

Light-Cone Distribution Amplitudes

- As for the pseudoscalar mesons, one reconstructs the LCDA from Mellin moments:

$$\langle x^m \rangle_{\parallel} = \int_0^1 dx\, x^m\, \varphi_V^{\parallel}(x,\mu)\ , \qquad \langle x^m \rangle_{\perp} = \int_0^1 dx\, x^m\, \varphi_V^{\perp}(x,\mu),$$

- which are normalized as,

$$\langle x^0 \rangle_{\parallel} = \int_0^1 dx\, \varphi_V^{\parallel}(x,\mu) = 1, \qquad \langle x^0 \rangle_{\perp} = \int_0^1 dx\, \varphi_V^{\perp}(x,\mu) = 1.$$

- and are given by:

$$\langle x^m \rangle_{\parallel} = \frac{m_V N_c \mathcal{Z}_2}{\sqrt{2} f_V} \mathrm{Tr}_D \int^{\Lambda} \frac{d^4k}{(2\pi)^4} \frac{(n\cdot k_\eta)^m}{(n\cdot P)^{m+2}} \gamma\cdot n\, n_\nu \chi_{V\nu}^{fg}(k;P),$$

$$\langle x^m \rangle_{\perp} = -\frac{N_c \mathcal{Z}_T}{2\sqrt{2} f_V^{\perp}} \mathrm{Tr}_D \int^{\Lambda} \frac{d^4k}{(2\pi)^4} \frac{(n\cdot k_\eta)^m}{(n\cdot P)^{m+1}} n_\mu \sigma_{\mu\rho} \mathcal{O}^{\perp}_{\rho\nu} \chi_{V\nu}^{fg}(k;P).$$

- Again, we can determine a large number of moments and fit a Gegenbauer expansion:

$$\varphi_V^{\parallel,\perp,\mathrm{rec.}}(x,\mu) = \mathcal{N}(\alpha)[x\bar{x}]^{\alpha-1/2}\left[1 + \sum_{n=1}^{N} a_n C_n^{\alpha}(2x-1)\right]$$