From current quarks to constituent-quark and hadron masses in a functional QCD approach

$$S(p) = \frac{Z(p^2)}{i\gamma \cdot p + M(p^2)}$$

Origin of the Visible Universe: Unraveling the Proton Mass

Workshop @ the *Institute for Nuclear Theory*,
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CNPq
Conselho Nacional de Desenvolvimento Científico e Tecnológico

"WE COLLABORATE. I'M AN EXPERT, BUT NOT AN AUTHORITY, AND DR. GELBIS IS AN AUTHORITY, BUT NOT AN EXPERT."

Work in collaboration with

- Luis Albino, Instituto de Física Teórica, Unesp, Brazil
- Adnan Bashir, Universidad de Michoacán, Mexico
- José Roberto Lessa, Instituto Tecnológico de Aeronáutica, Brazil
- Orlando Oliveira, Universidade de Coimbra, Portugal
- Eduardo Rojas, Universidad de Nariño, Colombia
- Fernando Serna, Universidade Cidade de São Paulo, Brazil
- Roberto Correa da Silveira, Universidade Cidade de São Paulo, Brazil

Quantum ChromoDynamics

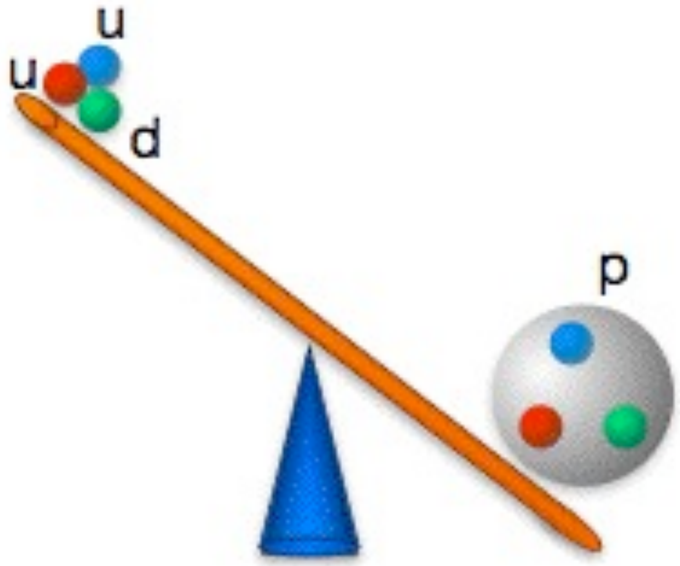
- We strive for a description of interactions between quarks and gluons which form hadrons as observed in *Nature*.

- The key issue is: while the Brout-Englert-Higgs mechanism has been established as the essential explicit source of elementary particle's masses, the same cannot be said of the atoms and their nuclei.

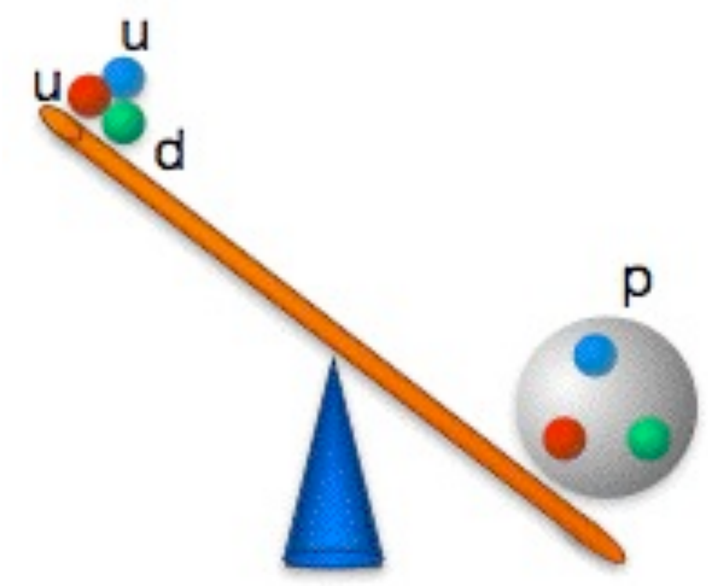
- The lightest Nambu-Goldstone mode of QCD, the pion, is more than an order of magnitude heavier than the sum of two light current quarks

- The formation of hadronic and nuclear bound states via its fundamental constituents is an inherently *nonperturbative* problem.

- It involves precise knowledge of the infrared (long distance) regime of QCD and the dynamical generation of a constituent quark mass.



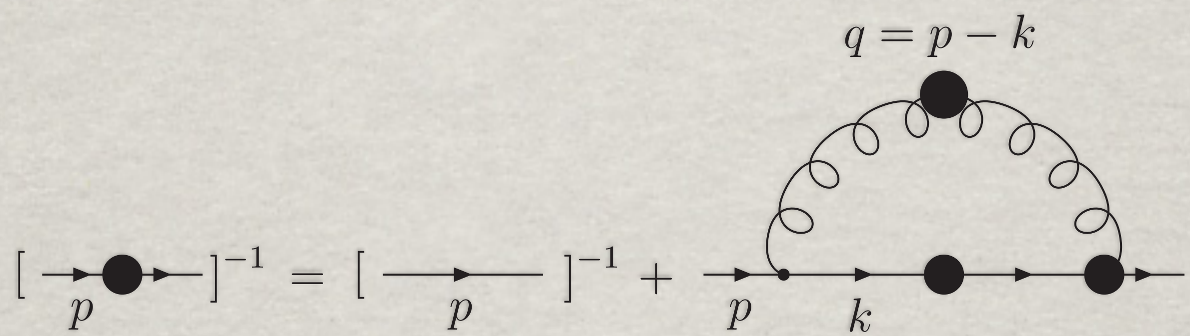
So where do the Hadron's masses come from after all?! The Higgs boson isn't doing the job alone!



Hint: the gluons interact with each other and have infinite ways to interact with the quark and “dress it”.

Dyson-Schwinger equation in QCD

The propagator can be obtained from QCD's gap equation: the Dyson-Schwinger equation (DSE) for the dressed-fermion self-energy, which involves the set of infinitely many coupled equations.



$$
\begin{aligned}
S^{-1}(p) &= Z_2(i\gamma\cdot p + m^{\rm bm}) + \Sigma(p) := i\gamma\cdot p\, A(p^2) + B(p^2) \\
\Sigma(p) &= Z_1 \int^{\Lambda} \frac{d^4 q}{(2\pi)^4}\, g^2 D_{\mu\nu}(p-q) \frac{\lambda^a}{2} \gamma_\mu S(q) \Gamma^a_\nu(q,p)
\end{aligned}
$$

with the *running* mass function $M(p^2) = B(p^2)/A(p^2)$.

$D_{\mu\nu}$	:	dressed-gluon propagator
$\Gamma^a_\nu(q,p)$	:	dressed quark-gluon vertex
Z_2	:	quark wave function renormalization constant
Z_1	:	quark-gluon vertex renormalization constant

Each satisfies it's own DSE !

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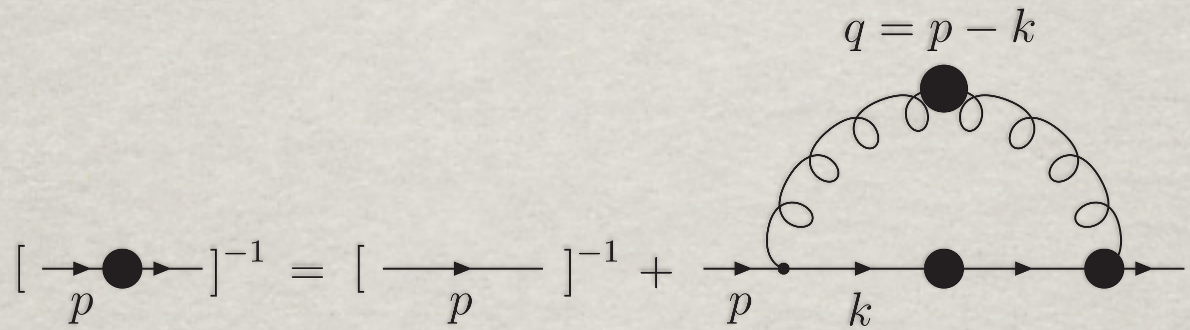

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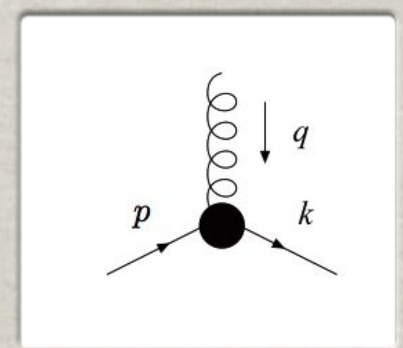

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Dyson-Schwinger equation in QCD

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Running Quark Mass

$$S(p) = \frac{Z(p^2)}{i\gamma \cdot p + M(p^2)}$$

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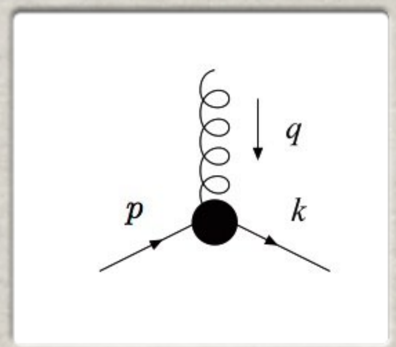

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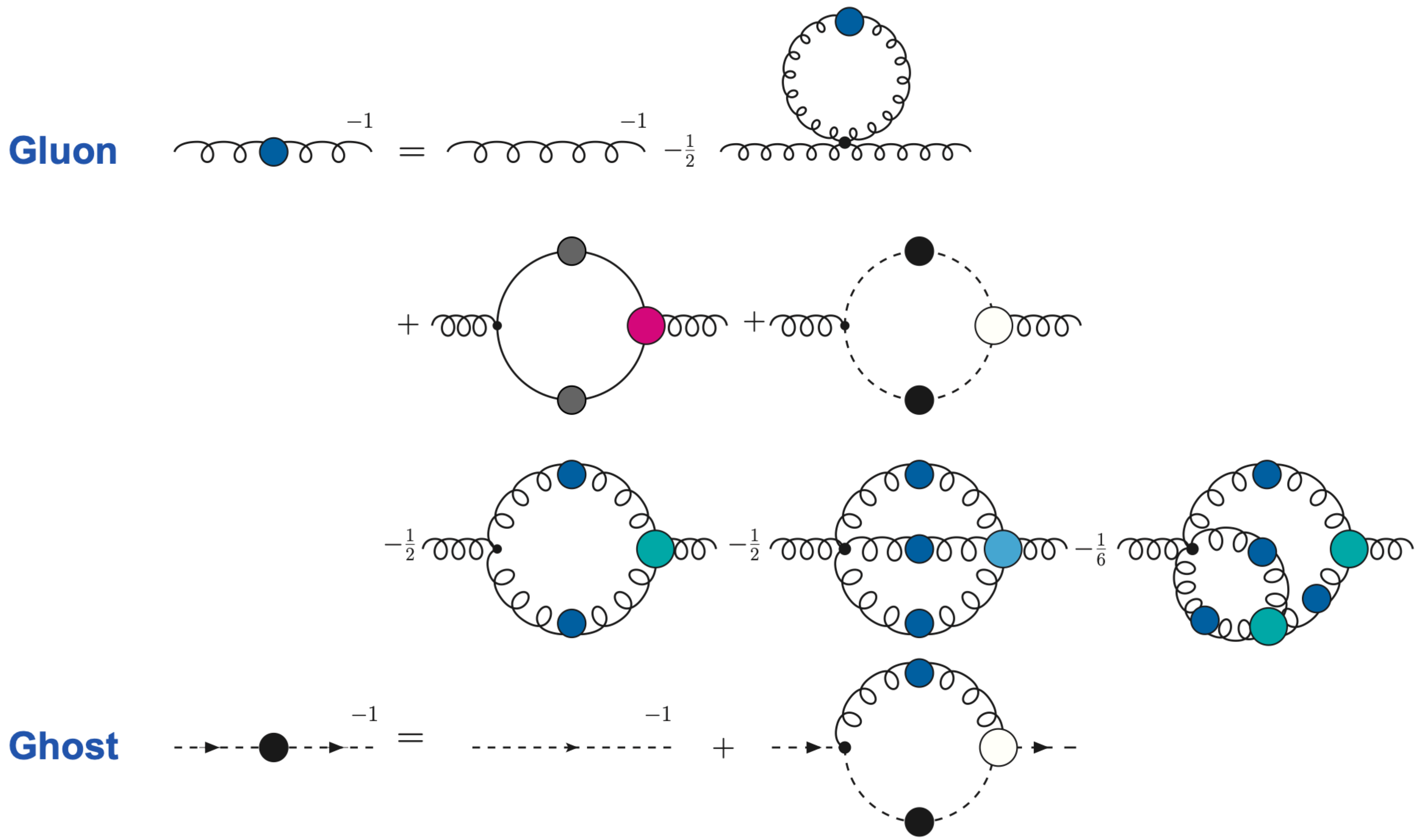
More DSE in QCD: Gluon and Ghost Propagators



However, in this work we employ dressed gluon and ghost propagators from lattice QCD.

Rainbow Truncation

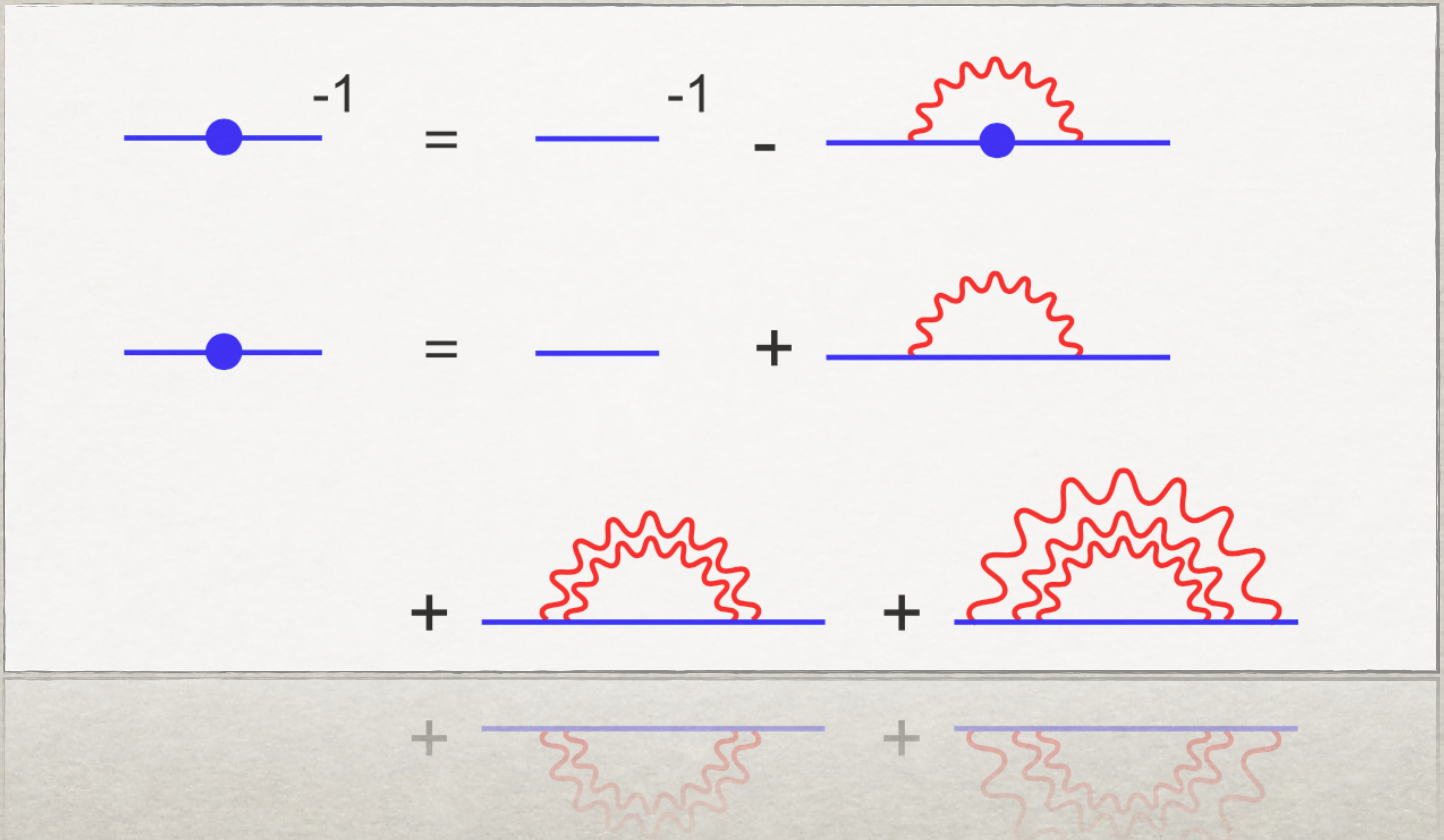
Since the Dyson-Schwinger Equation for QCD imply an infinite tower of non-linear integral equations, a symmetry preserving truncation scheme must be employed. The leading term in such a scheme is the Rainbow-Ladder (RL) truncation (Abelian approach).

$$\Gamma_\nu \;\rightarrow\; \gamma_\nu$$

RL truncation satisfies flavor non-singlet axial-vector Ward-Takahashi identities (chiral symmetry!) but has bad gauge dependence.

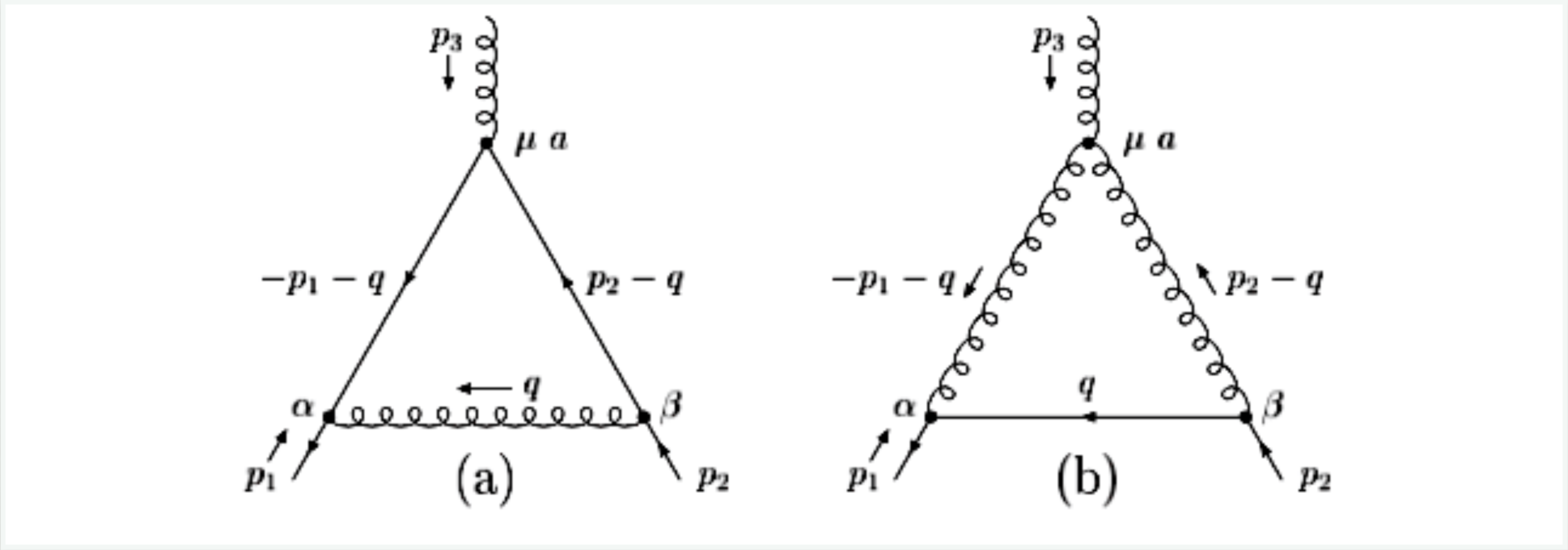
$\Rightarrow$ Landau gauge!

Rainbow Truncation



Here the bare gauge-boson propagator is used, can also be dressed.

The Quark-Gluon Vertex in QCD



(a) *Abelian* correction at one loop

(b) *Non-Abelian* correction at one loop

- The quark-gluon vertex in a tree-order is just $i\,\frac{\lambda_i}{2}\,\gamma_\mu$.
- However, already at one loop the Dirac-tensor structure is very complex.

Davydychev, *Osland and Saks* (2000)

Nonperturbative quark-gluon vertex: *restrictions*

- Must satisfy Slavnov-Taylor identities.
- $\Gamma_\mu(k,p)$ must be free of kinematic singularities for $k^2 \to p^2$.
- Must transform as bare vertex γ_μ under C, P and T transformations.
- Correct weak-coupling limit: must reduce to the perturbative limit when coupling is small.
- Vertex ansatz should lead to gauge independent physical observables, i.e. condensates, hadron masses, decay constants, form factors etc.

Nonperturbative quark-gluon vertex: tensor structure

The fermion-gauge-boson vertex can be decomposed into "longitudinal" and transverse components: $\Gamma_\mu(k,p) = \Gamma^L_\mu(k,p) + \Gamma^T_\mu(k,p)$

$$\Gamma^L_\mu(k,p) = \sum_{i=1}^{4} \lambda_i(k^2,p^2)\, L^i_\mu(k,p)$$

$$\Gamma^T_\mu(k,p) = \sum_{i=1}^{8} \tau_i(k^2,p^2)\, T^i_\mu(k,p)$$

$$\Gamma_\mu(k,p)\big|_{k^2=p^2=q^2=\mu^2} = \gamma_\mu$$

$$q \cdot \Gamma^T_\mu(k,p) = 0$$

Nonperturbative quark-gluon vertex: tensor structure

Which independent tensor structures to specify the longitudinal and transverse vertex?
Following Ball and Chiu (1980), one can write:

RL approximation

$$
\begin{aligned}
L^1_\mu(k,p) &= \gamma_\mu \\
L^2_\mu(k,p) &= \tfrac{1}{2}(k+p)_\mu\, \gamma\cdot(k+p) \\
L^3_\mu(k,p) &= -i\,(k+p)_\mu \\
L^4_\mu(k,p) &= -\sigma_{\mu\nu}\,(k+p)_\mu \\
\\
T^1_\mu(k,p) &= i\,[p_\mu(k\cdot q) - k_\mu(p\cdot q)] \\
T^2_\mu(k,p) &= [p_\mu(k\cdot q) - k_\mu(p\cdot q)]\,\gamma\cdot t \\
T^3_\mu(k,p) &= q^2\gamma_\mu - q_\mu\,\gamma\cdot q \\
T^4_\mu(k,p) &= -\,[p_\mu(k\cdot q) - k_\mu(p\cdot q)]\,p^\nu k^\rho \sigma_{\nu\,\rho} \\
T^5_\mu(k,p) &= \sigma^{\mu\,\nu} q_\nu \\
T^6_\mu(k,p) &= -\gamma_\mu\,\left(k^2 - p^2\right) + t_\mu\,\gamma\cdot q \\
T^7_\mu(k,p) &= \tfrac{i}{2}(k^2 - p^2)\,[\gamma_\mu\gamma\cdot t - t_\mu] + t_\mu\, p^\nu k^\rho \sigma_{\nu\rho} \\
T^8_\mu(k,p) &= -i\gamma_\mu\, p^\nu k^\rho \sigma_{\nu\,\rho} - p_\mu\,\gamma\cdot k + k_\mu\,\gamma\cdot p
\end{aligned}
$$

Nonperturbative quark-gluon vertex: symmetries

Which steps to remedy a gauge dependence? Clearly must be beyond rainbow truncation as bare vertex violates gauge variance:

$$q_\mu i\Gamma_\mu(k,p) \quad = \quad S^{-1}(k) - S^{-1}(p)$$

Best "prepared" with Landau gauge to minimize dependence.

First step is the proposal for the longitudinal vertex by Ball and Chiu:

$$\Gamma^L_\mu(k,p) = \sum_{i=1}^{4} \lambda_i(k^2,p^2)\, L^i_\mu(k,p)$$

$$\lambda_1(k^2,p^2) = \frac{1}{2}\left[A(k^2)+A(p^2)\right] \qquad \lambda_2(k^2,p^2) = \frac{A(k^2)-A(p^2)}{k^2-p^2}$$

$$\lambda_3(k^2,p^2) = \frac{B(k^2)-B(p^2)}{k^2-p^2} \qquad \lambda_4(k^2,p^2) = 0$$

Widely employed in phenomenology though transverse part remains undetermined.

What about gauge covariance, does it satisfy Landau-Khalatnikov-Fradkin transformations?

What about multiplicative renormalizability?

Nonperturbative quark-gluon vertex: symmetries

Which steps to remedy a gauge dependence? Clearly must be beyond rainbow truncation as bare vertex violates gauge variance:

$$iq^\mu \gamma_\mu \neq i\gamma\cdot k\, A(k^2) + B(k^2) - i\gamma\cdot p\, A(p^2) - B(p^2)$$

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Abelian Ward-Takahashi identities: divergence and curl

Ward-Takahashi identity:

$$q_\mu i\Gamma_\mu(k,p) = S^{-1}(k) - S^{-1}(p)$$

Transverse Ward-Takahashi identities:

$$\begin{aligned} q_\mu \Gamma_\nu(k,p) - q_\nu \Gamma_\mu(k,p) &= S^{-1}(p)\sigma_{\mu\nu} + \sigma_{\mu\nu} S^{-1}(k) \\ &+ 2im\Gamma_{\mu\nu}(k,p) + t_\lambda \varepsilon_{\lambda\mu\nu\rho}\Gamma^A_\rho(k,p) + A^V_{\mu\nu}(k,p) \end{aligned}$$

$$\begin{aligned} q_\mu \Gamma^A_\nu(k,p) - q_\nu \Gamma^A_\mu(k,p) &= S^{-1}(p)\sigma^5_{\mu\nu} - \sigma^5_{\mu\nu} S^{-1}(k) \\ &+ t_\lambda \varepsilon_{\lambda\mu\nu\rho}\Gamma_\rho(k,p) + V^A_{\mu\nu}(k,p) \end{aligned}$$

What is the origin of these transverse identities?

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$$\psi(x) \longrightarrow \psi'(x) = \psi(x) + ig\alpha(x)\psi(x), \;\; \bar{\psi}(x) \longrightarrow \bar{\psi}'(x) = \bar{\psi}(x) - ig\alpha(x)\bar{\psi}(x),$$

$$\downarrow$$

$$q_\mu \Gamma^\mu_V(p_1,p_2) = S_F^{-1}(p_1) - S_F^{-1}(p_2),$$

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$$q_\mu \Gamma_V^\mu(p_1, p_2) = S_F^{-1}(p_1) - S_F^{-1}(p_2),$$

$$\delta_T \psi(x) = \frac{1}{4} g\alpha(x)\epsilon^{\mu\nu}\sigma_{\mu\nu}\psi(x), \quad \delta_T \bar{\psi}(x) = \frac{1}{4} g\alpha(x)\epsilon^{\mu\nu}\bar{\psi}(x)\sigma_{\mu\nu},$$

Infinitesimal Lorentz transformation

Kei-Ichi Kondo,Int.J.Mod.Phys.A12(1996)

$$\int D[\psi, \bar{\psi}, A] e^{i\int d^4x L_{\mathrm{QED}}[\psi, \bar{\psi}, A]} \psi(x_1)\bar{\psi}(x_2)$$
$$= \int D[\psi', \bar{\psi}', A'] e^{i\int d^4x L_{\mathrm{QED}}[\psi', \bar{\psi}', A']} \psi'(x_1)\bar{\psi}'(x_2).$$

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$$\begin{aligned} & iq^\mu \Gamma_V^\nu(p_1,p_2) - iq^\nu \Gamma_V^\mu(p_1,p_2) \\ = \ & S_F^{-1}(p_1)\sigma^{\mu\nu} + \sigma^{\mu\nu} S_F^{-1}(p_2) + 2m\Gamma_T^{\mu\nu}(p_1,p_2) \\ & + (p_{1\lambda} + p_{2\lambda})\varepsilon^{\lambda\mu\nu\rho}\Gamma_{A\rho}(p_1,p_2) - \int \frac{d^4k}{(2\pi)^4} 2k_\lambda \varepsilon^{\lambda\mu\nu\rho}\Gamma_{A\rho}(p_1,p_2;k), \end{aligned}$$

Non-Abelian Ward-Takahashi identities: *divergence and curl*

Slavnov-Taylor identity:

$$i\, q \cdot \Gamma^a(k,p) = G(q^2)\Big[S^{-1}(-k)\, H^a(k,p) - \bar{H}^a(p,k) S^{-1}(p)\Big]$$

Transverse Slavnov-Taylor identities:

H.-X. He, PRD 80, 016004 (2009)

$$\begin{aligned} q_\mu \Gamma^a_\nu(k,p) - q_\nu \Gamma^a_\mu(k,p) &= G(q^2)\left[S^{-1}(p)\sigma_{\mu\nu}\, H^a(k,p) + \bar{H}^a(p,k)\, \sigma_{\mu\nu} S^{-1}(k)\right] \\ &+ 2im\, \Gamma^a_{\mu\nu}(k,p) + t_\alpha \epsilon_{\alpha\mu\nu\beta}\, \Gamma^{5a}_\beta(k,p) + A^a_{\mu\nu}(k,p) \end{aligned}$$

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Vector and axialvector vertices must be decoupled !

Slavnov-Taylor Identity

One can relate the longitudinal form factors λ_i to the quark propagator's scalar and vector pieces, $B(p^2)$ and $A(p^2)$ via an ST

$$i\, q \cdot \Gamma^a(k,p) = G(q^2)\Big[S^{-1}(-k)\, H^a(k,p) - \bar{H}^a(p,k) S^{-1}(p)\Big]$$

Ghost dressing function — Quark-ghost scattering kernel

Decomposition of $H(k,p)$ and its conjugate in terms of Lorentz covariants:

$$H(p_1,p_2,p_3) = X_0\, \mathbb{I}_D + i\, X_1\, \gamma \cdot p_1 + i\, X_2\, \gamma \cdot p_2 + i\, X_3\, \sigma_{\alpha\beta}\, p_1^\alpha p_2^\beta$$

$$\overline{H}(p_2,p_1,p_3) = \overline{X}_0\, \mathbb{I}_D - i\, \overline{X}_2\, \gamma \cdot p_1 - i\, \overline{X}_1\, \gamma \cdot p_2 + i\, \overline{X}_3\, \sigma_{\alpha\beta}\, p_1^\alpha p_2^\beta$$

$X_i \equiv X_i(p_1,p_2,p_3)$ $\qquad$ $X_i(p,k,q) = \bar{X}_i(k,p,q)$

Davydychev, Osland & Saks (2001)
A .C. Aguilar and J. Papavassiliou (2011)
A. C. Aguilar, J. C. Cardona, M. N. Ferreira and J.~Papavassiliou (2016, 2018)

Decoupling the transverse STIs

$$
\begin{aligned}
q_\mu \Gamma^a_\nu(k,p) - q_\nu \Gamma^a_\mu(k,p) &= G(q^2)\left[S^{-1}(p)\sigma_{\mu\nu}\, H^a(k,p) + \bar{H}^a(p,k)\,\sigma_{\mu\nu} S^{-1}(k)\right] \\
&+ 2im\,\Gamma^a_{\mu\nu}(k,p) + t_\alpha \epsilon_{\alpha\mu\nu\beta}\,\Gamma^{5a}_\beta(k,p) + A^a_{\mu\nu}(k,p)
\end{aligned}
$$

$$
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&+ t_\alpha \epsilon_{\alpha\mu\nu\beta}\,\Gamma^a_\beta(k,p) + V^a_{\mu\nu}(k,p)
\end{aligned}
$$

- The decoupling of the vector and axialvector vertices can be achieved by appropriate projections with two tensors which lead to two independent equations for each vertex ! S.-x. Qin, L. Chang, Y.-x. Liu, C.D. Roberts & S. Schmidt (2013)

- Using the two identities for the vector vertex, we can use another set of projections to isolate the 8 tensor structures of the transverse vertex as functions of the *quark propagator, the ghost dressing function, the quark-ghost scattering form factors and an hitherto undetermined nonlocal tensor structure.*

The unknown ingredient …

The cumbersome nonlocal tensor structure originates in the Fourier transform of a 4-point function with a vector-vertex insertion and a Wilson line.

$$V_{\mu\nu} = \int \frac{d^4k}{(2\pi)^4}\, 2k_\lambda \varepsilon^{\lambda\mu\nu\rho}\, \Gamma_{A\rho}\,(p_1, p_2; k)$$

H.-X. He, PRD 80, 016004 (2009)

$$\int d^4x d^4x' d^4x_1 d^4x_2 e^{i\left(p_1\cdot x_1 - p_2\cdot x_2 + (p_2-k)\cdot x - (p_1-k)\cdot x'\right)}$$
$$\times \left\langle 0 \left| T\bar{\psi}\,(x')\,\gamma_\rho\gamma_5\, U_P\,(x',x)\psi(x)\psi\,(x_1)\,\bar{\psi}\,(x_2)\right| 0\right\rangle$$
$$= (2\pi)^4\delta^4\,(p_1 - p_2 - q)\, iS_F\,(p_1)\,\Gamma_{A\rho}\,(p_1, p_2; k) iS_F\,(p_2)$$

The unfamiliar, complicated components in these identities can be decomposed:

$$i\,T^1_{\mu\nu}V_{\mu\nu} = Y_1(k,p)\mathbf{I}_D + Y_2(k,p)\gamma\cdot q + Y_3(k,p)\gamma\cdot t + Y_4(k,p)\,[\gamma\cdot q, \gamma\cdot t\,]$$

$$i\,T^2_{\mu\nu}V_{\mu\nu} = Y_5(k,p)\mathbf{I}_D + Y_6(k,p)\gamma\cdot q + Y_7(k,p)\gamma\cdot t + Y_8(k,p)\,[\gamma\cdot q, \gamma\cdot t\,]$$

Transverse form factors of the quark-gluon vertex from transverse STIs

$$\tau_3^{\rm QCD} = \frac{1}{2}G(q^2)X_0(q^2)\left[\frac{A(k^2)-A(p^2)}{k^2-p^2}\right] + \frac{Y_2}{4\nabla(k,p)} - \frac{(k+p)^2(Y_3-Y_5)}{8(k^2-p^2)\nabla(k,p)}$$

$$\tau_5^{\rm QCD} = -G(q^2)X_0(q^2)\left[\frac{B(k^2)-B(p^2)}{k^2-p^2}\right] - \frac{2Y_4+Y_6^A}{2(k^2-p^2)}$$

$$\tau_8^{\rm QCD} = -G(q^2)X_0(q^2)\left[\frac{A(k^2)-A(p^2)}{k^2-p^2}\right] - \frac{2Y_8^A}{k^2-p^2}$$

No dependence on A(p²) and B(p²) functions !

$$\tau_1^{\rm QCD} = -\frac{Y_1}{2\left(k^2-p^2\right)\left(k^2p^2-(k\cdot p)^2\right)}$$

$$\tau_2^{QCD} = -\frac{Y_5-3Y_3}{4\left(k^2-p^2\right)\left(k^2p^2-(k\cdot p)^2\right)},$$

$$\tau_4^{QCD} = \frac{Y_1-(6Y_4+Y_6)\left(k^2-p^2\right)-Y_7\left(k+p\right)^2}{2\left(k^2-p^2\right)^2\left(k^2p^2-(k\cdot p)^2\right)},$$

$$\tau_6^{QCD} = \frac{2Y_2\left(k-p\right)^2-(Y_3-Y_5)(k^2-p^2)}{8\left(k^2-p^2\right)\left(k^2p^2-(k\cdot p)^2\right)},$$

$$\tau_7^{QCD} = \frac{Y_1\left(k-p\right)^2-4Y_7(k^2p^2-(k\cdot p)^2)}{4\left(k^2-p^2\right)\left(k^2p^2-(k\cdot p)^2\right)}$$

The unknown ingredient …

We constrain the Y_i form factors with a known ansatz for transverse vertex based on *perturbation theory, symmetry considerations and multiplicative renormalizability* in a given limit $k^2 \gg p^2$.

Bashir, Bermudez, Chang & Roberts (2012)

Use an ansatz based on perturbation theory, symmetry considerations and multiplicative renormalizability

Guided by perturbation theory; draws on comparison with structural dependence of the Ball-Chiu vertex on $A(p^2)$ and $B(p^2)$. The perturbative limit of the transverse vertex conforms with its one loop expansion in the asymptotic limit of $k^2 \gg p^2$.

Bashir, Bermúdez, Chang & Roberts (2012)

$$
\begin{aligned}
\tau_1(k^2,p^2) &= \frac{a_1\,\Delta_B(k^2,p^2)}{(k^2+p^2)} \\
\tau_2(k^2,p^2) &= \frac{a_2\,\Delta_A(k^2,p^2)}{(k^2+p^2)} \\
\tau_3(k^2,p^2) &= a_3\,\Delta_A(k^2,p^2) \\
\tau_4(k^2,p^2) &= \frac{a_4\,\Delta_B(k^2,p^2)}{[k^2+M^2(k^2)[p^2+M^2(p^2)]} \\
\tau_5(k^2,p^2) &= a_5\,\Delta_B(k^2,p^2) \\
\tau_6(k^2,p^2) &= \frac{a_6(k^2+p^2)\,\Delta_A(k^2,p^2)}{[(k^2-p^2)^2+(M^2(k^2)+M^2(p^2))^2]} \\
\tau_7(k^2,p^2) &= \frac{a_7\,\Delta_B(k^2,p^2)}{(k^2+p^2)} \\
\tau_8(k^2,p^2) &= a_8\Delta_A(k^2,p^2)
\end{aligned}
$$

Comparing the transverse vertex derived from the STIs with this ansatz

$$
\begin{aligned}
\tau_1(k^2,p^2) &= \frac{a_1\,\Delta_B(k^2,p^2)}{(k^2+p^2)}\\
\tau_2(k^2,p^2) &= \frac{a_2\,\Delta_A(k^2,p^2)}{(k^2+p^2)}\\
\tau_3(k^2,p^2) &= a_3\,\Delta_A(k^2,p^2)\\
\tau_4(k^2,p^2) &= \frac{a_4\,\Delta_B(k^2,p^2)}{[k^2+M^2(k^2)][p^2+M^2(p^2)]}\\
\tau_5(k^2,p^2) &= a_5\,\Delta_B(k^2,p^2)\\
\tau_6(k^2,p^2) &= \frac{a_6(k^2+p^2)\,\Delta_A(k^2,p^2)}{[(k^2-p^2)^2+(M^2(k^2)+M^2(p^2))^2]}\\
\tau_7(k^2,p^2) &= \frac{a_7\,\Delta_B(k^2,p^2)}{(k^2+p^2)}\\
\tau_8(k^2,p^2) &= a_8\Delta_A(k^2,p^2)
\end{aligned}
$$

$+$

$$
\begin{aligned}
\tau_1(k^2,p^2) &= -\frac{Y_1}{2(k^2-p^2)\nabla(k,p)}\\
\tau_2(k^2,p^2) &= -\frac{Y_5-3Y_3}{4(k^2-p^2)\nabla(k,p)}\\
\tau_3(k^2,p^2) &= \frac{1}{2}G(q^2)X_0(q^2)\left[\frac{A(k^2)-A(p^2)}{k^2-p^2}\right]\\
&+ \frac{Y_2}{4\nabla(k,p)} - \frac{(k+p)^2(Y_3-Y_5)}{8(k^2-p^2)\nabla(k,p)}\\
\tau_4(k^2,p^2) &= -\frac{6Y_4+Y_6^A}{8\nabla(k,p)} - \frac{(k+p)^2Y_7^S}{8(k^2-p^2)\nabla(k,p)}\\
\tau_5(k^2,p^2) &= -G(q^2)X_0(q^2)\left[\frac{B(k^2)-B(p^2)}{k^2-p^2}\right]\\
&- \frac{2Y_4+Y_6^A}{2(k^2-p^2)}\\
\tau_6(k^2,p^2) &= \frac{(k-p)^2Y_2}{4(k^2-p^2)\nabla(k,p)} - \frac{Y_3-Y_5}{8\nabla(k,p)}\\
\tau_7(k^2,p^2) &= \frac{q^2(6Y_4+Y_6^A)}{4(k^2-p^2)\nabla(k,p)} + \frac{Y_7^S}{4\nabla(k,p)}\\
\tau_8(k^2,p^2) &= -G(q^2)X_0(q^2)\left[\frac{A(k^2)-A(p^2)}{k^2-p^2}\right] - \frac{2Y_8^A}{k^2-p^2}
\end{aligned}
$$

Comparing the transverse vertex derived from the STIs with this ansatz

$$
\begin{aligned}
Y_1(k^2,p^2) &= -2a_1\left[B(k^2)-B(p^2)\right]\frac{\Delta(k,p)}{k^2+p^2}\\
Y_2(k^2,p^2) &= \tfrac{1}{2}\left[A(k^2)-A(p^2)\right]\\
&\times \left\{(k^2-p^2)\left(G(q^2)X_0(q^2)-2a_3\right)\right.\\
&- \left.2\left(\frac{k^2+p^2}{k^2-p^2}\right)(k+p)^2a_6\right\}\\
Y_3(k^2,p^2) &= \tfrac{1}{2}\left[A(k^2)-A(p^2)\right]\\
&\times \left\{-(k-p)^2\left(G(q^2)X_0(q^2)-2a_3\right)\right.\\
&+ \left.4\,\frac{\Delta(k,p)}{k^2+p^2}a_2+2(k^2+p^2)a_6\right\}\\
Y_4(k^2,p^2) &= -\frac{B(k^2)-B(p^2)}{4k^2p^2(k^2+p^2)}\left\{2(k^2+p^2)\Delta(k,p)a_4\right.\\
&+ 2k^2p^2(k^2+p^2)\left[a_5-G(q^2)X_0\right]\\
&+ \left.k^2p^2(k+p)^2a_7\right\}\\
Y_5(k^2,p^2) &= \tfrac{3}{2}\left[A(k^2)-A(p^2)\right]\\
&\times \left\{-(k-p)^2\left[G(q^2)X_0-2a_3\right]\right.\\
&+ \left.\frac{4}{3}\frac{\Delta(k,p)}{k^2+p^2}a_2+2(k^2+p^2)a_6\right\}\\
Y_6^A(k^2,p^2) &= \frac{B(k^2)-B(p^2)}{2k^2p^2(k^2+p^2)}\left\{2(k^2+p^2)\Delta(k,p)a_4\right.\\
&+ 6k^2p^2(k^2+p^2)\left(a_5-G(q^2)X_0\right)\\
&+ \left.k^2p^2(k+p)^2a_7\right\}\\
Y_7^S(k^2,p^2) &= a_7\left[B(k^2)-B(p^2)\right]\frac{k^2-p^2}{k^2+p^2}\\
Y_8^A(k^2,p^2) &= -\tfrac{1}{2}\left[A(k^2)-A(p^2)\right]\left(a_8+G(q^2)X_0\right)
\end{aligned}
$$

Comparing the transverse vertex derived from the STIs with this ansatz

With this ansatz for the Y_i functions we come closer to a gauge covariant vertex and include additional mass terms that enhance DCSB.

The parameters a_i are constrained by multiplicative renormalizability and only certain combinations are allowed.

$$
\begin{aligned}
Y_1(k^2,p^2) &= -2a_1\left[B(k^2)-B(p^2)\right]\frac{\Delta(k,p)}{k^2+p^2} \\
Y_2(k^2,p^2) &= \tfrac{1}{2}\left[A(k^2)-A(p^2)\right] \\
&\times \left\{(k^2-p^2)\left(G(q^2)X_0(q^2)-2a_3\right)\right. \\
&- \left. 2\left(\frac{k^2+p^2}{k^2-p^2}\right)(k+p)^2 a_6\right\} \\
Y_3(k^2,p^2) &= \tfrac{1}{2}\left[A(k^2)-A(p^2)\right] \\
&\times \left\{-(k-p)^2\left(G(q^2)X_0(q^2)-2a_3\right)\right. \\
&+ \left. 4\frac{\Delta(k,p)}{k^2+p^2}a_2+2(k^2+p^2)a_6\right\} \\
Y_4(k^2,p^2) &= -\frac{B(k^2)-B(p^2)}{4k^2p^2(k^2+p^2)}\left\{2(k^2+p^2)\Delta(k,p)a_4\right. \\
&+ 2k^2p^2(k^2+p^2)\left[a_5-G(q^2)X_0\right] \\
&+ \left. k^2p^2(k+p)^2a_7\right\} \\
Y_5(k^2,p^2) &= \tfrac{3}{2}\left[A(k^2)-A(p^2)\right] \\
&\times \left\{-(k-p)^2\left[G(q^2)X_0-2a_3\right]\right. \\
&+ \left. \frac{4}{3}\frac{\Delta(k,p)}{k^2+p^2}a_2+2(k^2+p^2)a_6\right\} \\
Y_6^A(k^2,p^2) &= \frac{B(k^2)-B(p^2)}{2k^2p^2(k^2+p^2)}\left\{2(k^2+p^2)\Delta(k,p)a_4\right. \\
&+ 6k^2p^2(k^2+p^2)\left(a_5-G(q^2)X_0\right) \\
&+ \left. k^2p^2(k+p)^2a_7\right\} \\
Y_7^S(k^2,p^2) &= a_7\left[B(k^2)-B(p^2)\right]\frac{k^2-p^2}{k^2+p^2} \\
Y_8^A(k^2,p^2) &= -\tfrac{1}{2}\left[A(k^2)-A(p^2)\right]\left(a_8+G(q^2)X_0\right)
\end{aligned}
$$

Gluon and ghost dressing functions

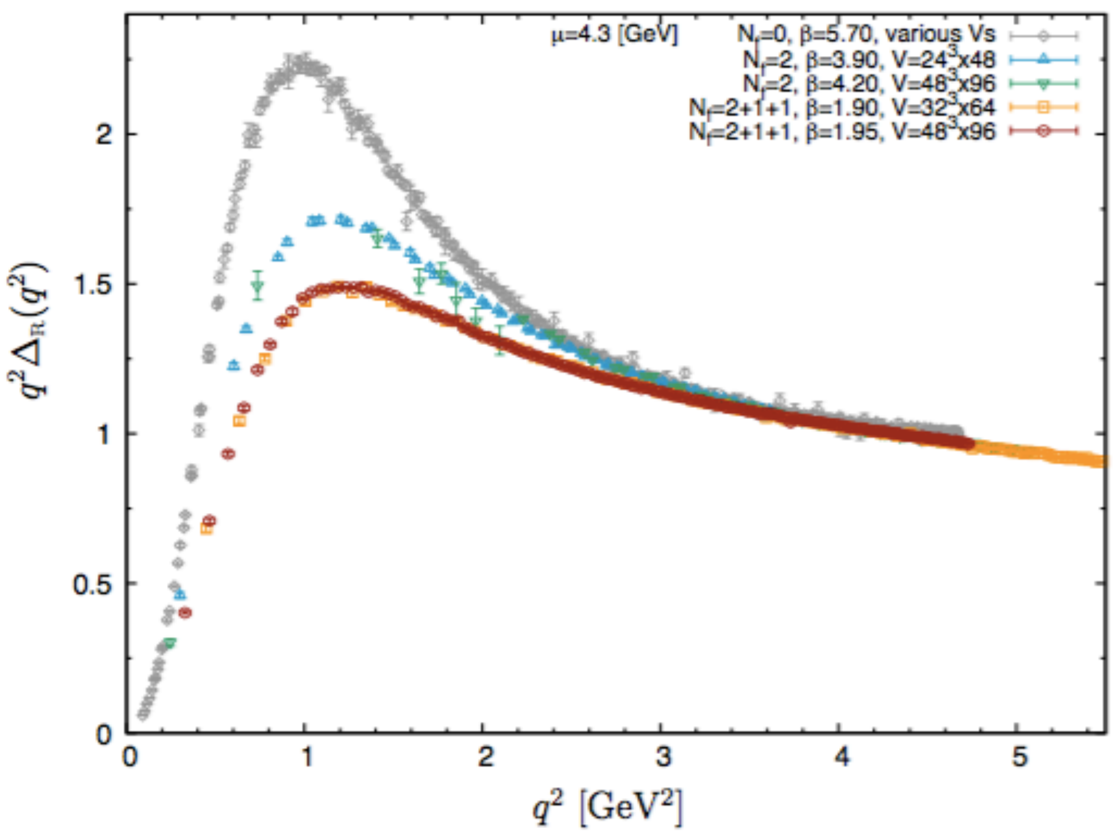
The gluon propagator in Landau gauge is:

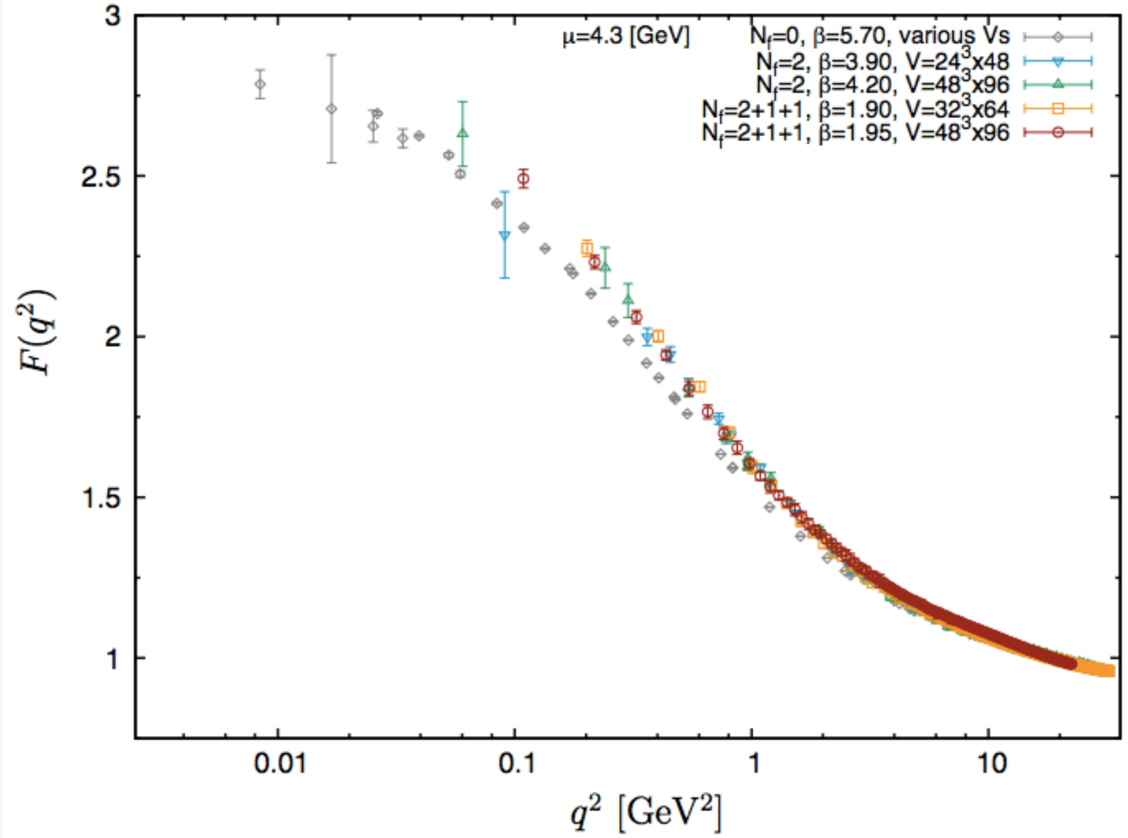
$$\Delta^{ab}_{\mu\nu}(q) = \delta^{ab}\left(\delta_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}\right)\Delta(q^2) \qquad \Delta(q^2) \stackrel{q^2\to\infty}{\longrightarrow} \frac{1}{q^2}$$

The ghost propagator is:

$$D^{ab}(q^2) = -\,\delta^{ab}\,\frac{G(q^2)}{q^2} \qquad G(q^2) \stackrel{q^2\to\infty}{\longrightarrow} 1$$

Gluon and ghost dressing functions

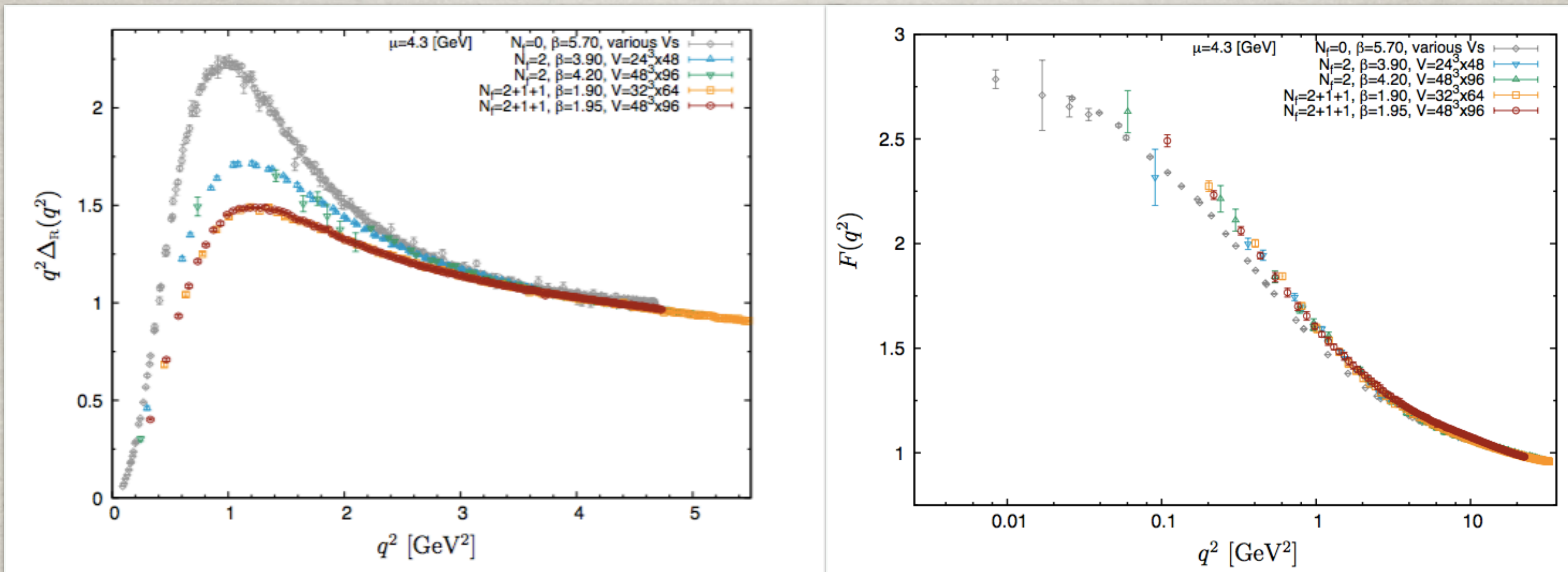




DSE solutions with three sets of propagators from different collaborations:

- Set I: Bogolubsky *et al.*, Phys. Lett. B 676, 69 (2009)
- Set II: Dudal *et al.*, Annals Phys. 397, 351-364 (2018)
  Duarte *et al.*, Phys. Rev. D 94 (2016)
- Set III: A. Ayala et al., Phys. Rev. D 86, 074512 (2012)

Gluon and ghost dressing functions



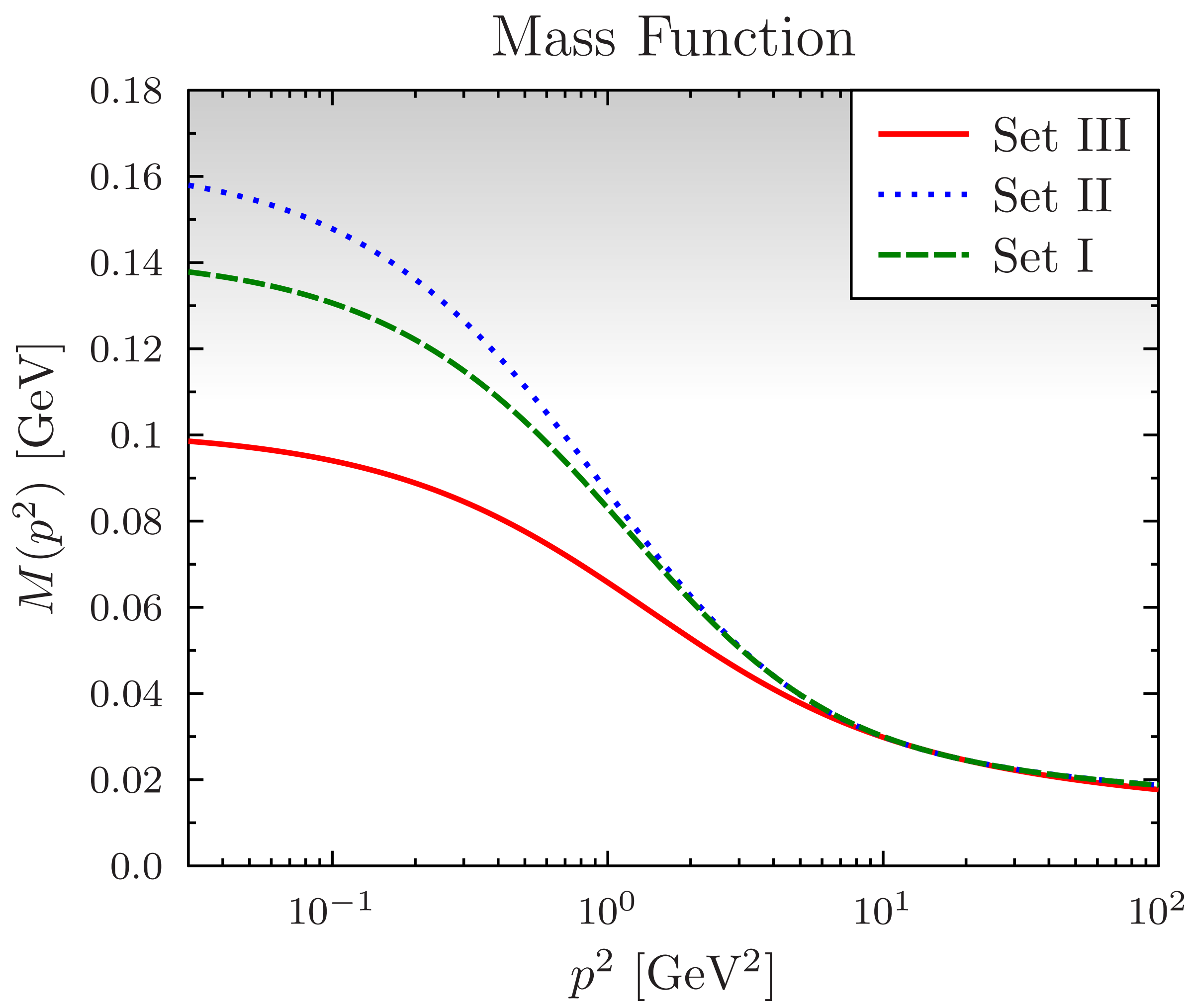
$$\Delta(q^2) = Z\,\frac{q^2+M_1^2}{q^4+M_2^2q^2+M_3^4}\left[1+\omega\ln\left(\frac{q^2+M_0^2}{\Lambda_{\rm QCD}^2}\right)\right]^{\gamma_{\rm gl}}$$

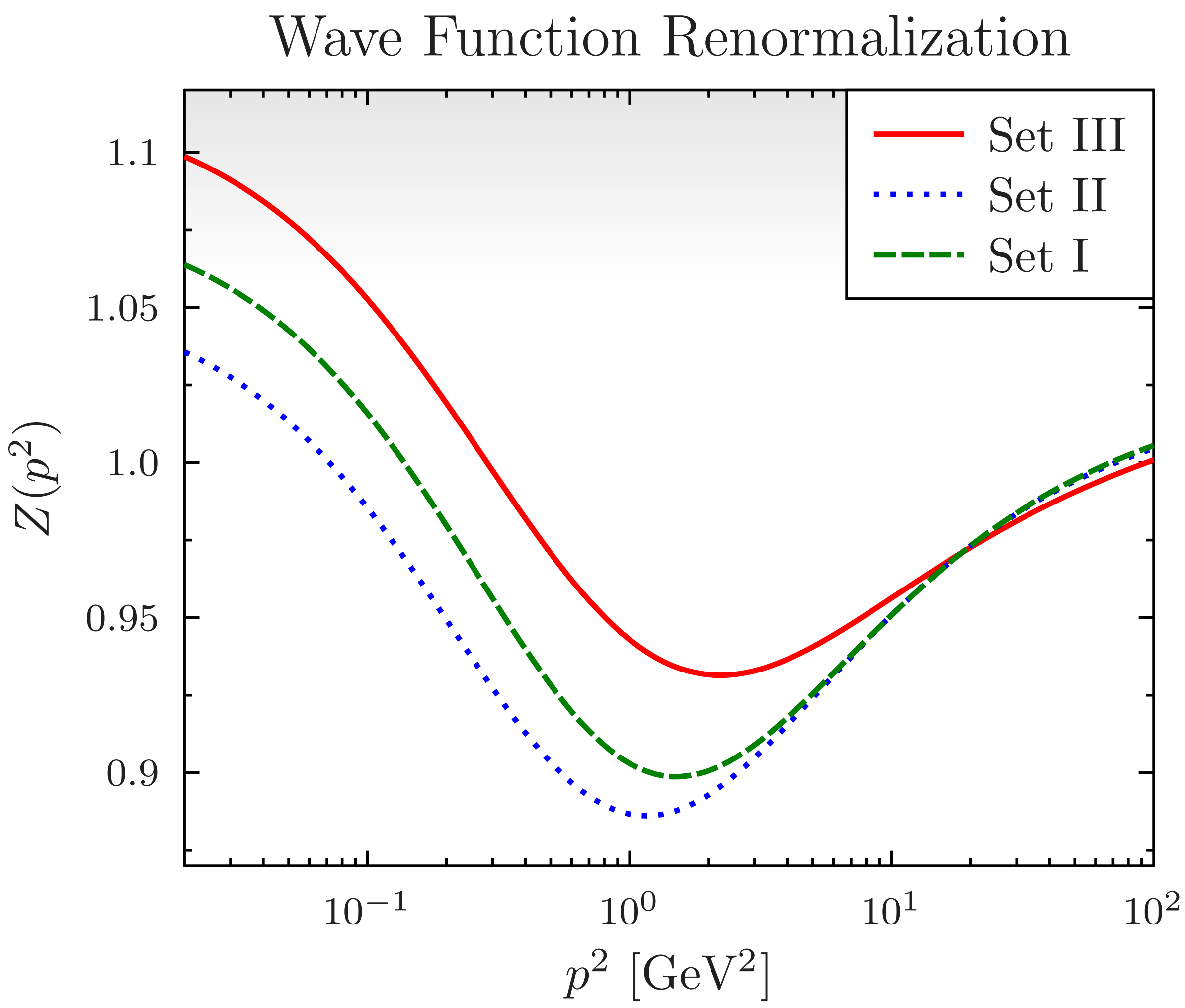
$$G(q^2) = Z\,\frac{q^4+M_2^2q^2+M_1^4}{q^4+M_4^2q^2+M_3^4}\left[1+\omega\ln\left(\frac{q^2+\frac{m_1^4}{q^2+m_0^2}}{\Lambda_{\rm QCD}^2}\right)\right]^{\gamma_{\rm gh}}$$

D. Dudal, O. Oliveira and P. J. Silva, Annals Phys. **397** (2018)

DSE Solutions with non-transverse vertex

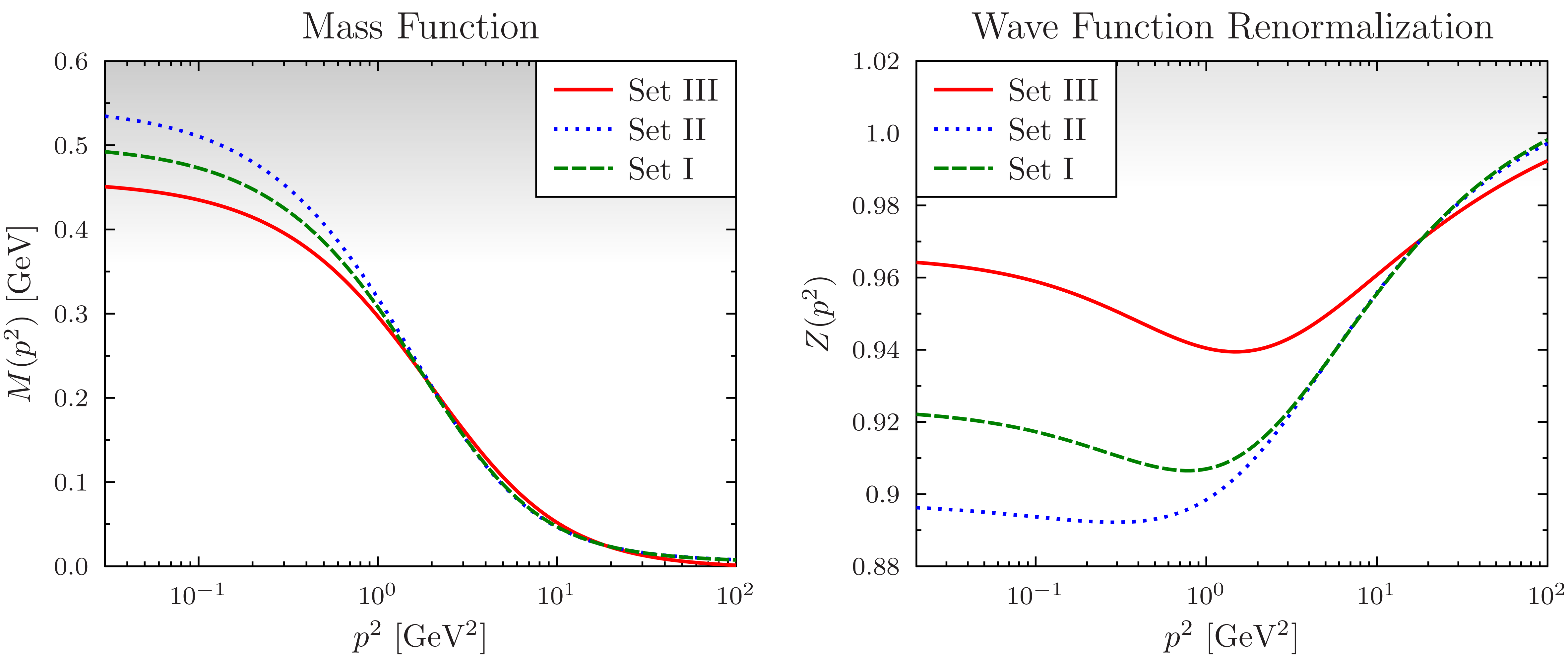
$$\Gamma^L_\mu(k,p) = \sum_{i=1}^{4} \lambda_i(k^2,p^2)\, L^i_\mu(k,p)$$



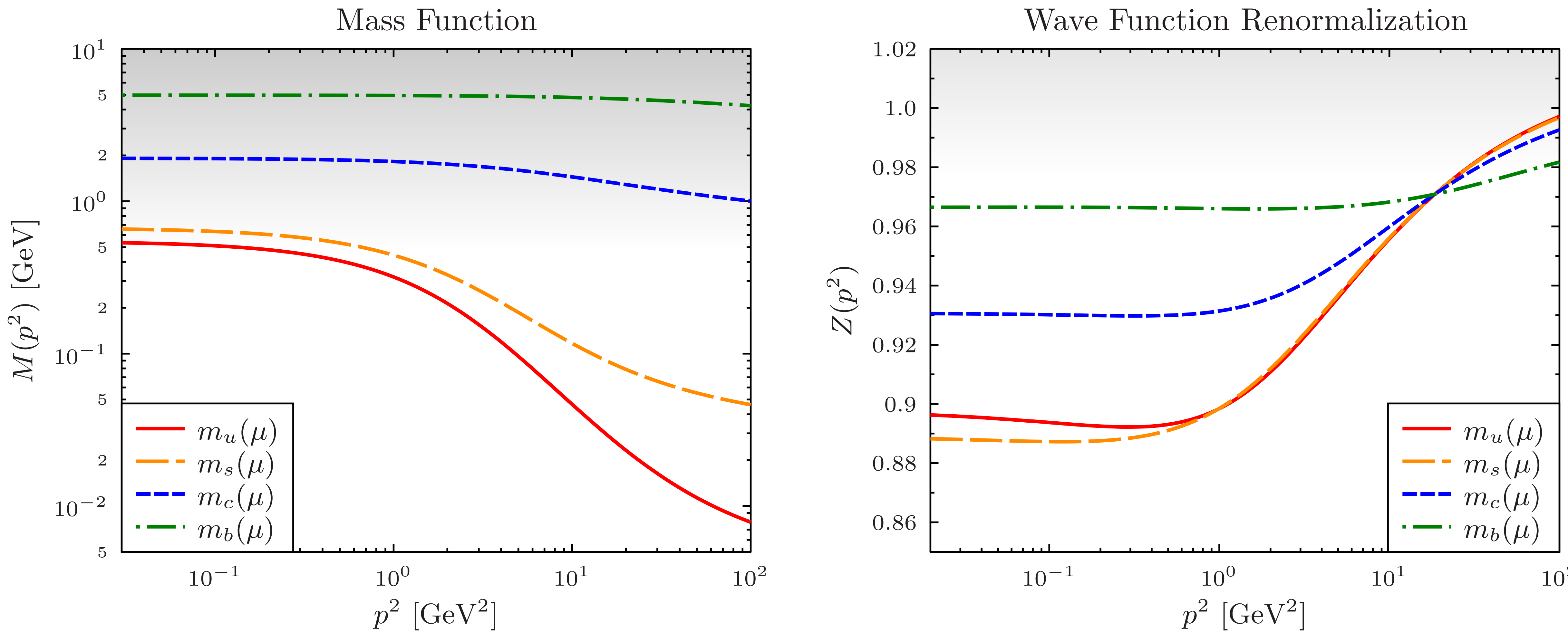


DSE solutions with full quark-gluon vertex

$$\Gamma_\mu(k,p) = \Gamma^L_\mu(k,p) + \Gamma^T_\mu(k,p)$$



Flavor dependence of DSE solutions



Quark Sigma Term and Constituent Quark Mass

Study the effect of DCSB with the renormalization-point invariant ratio: $\zeta := \dfrac{\sigma_f}{M_f^E}$

Define in analogy with nucleon's sigma term a measure of the contribution from CSB to the constituent quark mass: $\sigma_f \;:=\; m_f(\mu)\langle Q|\bar{q}_f q_f|Q\rangle$

Hellmann-Feynman theorem relates this matrix element to the constituent-quark mass: $\sigma_f = m_f(\mu)\,\dfrac{\partial M_f^E}{\partial m_f(\mu)}$

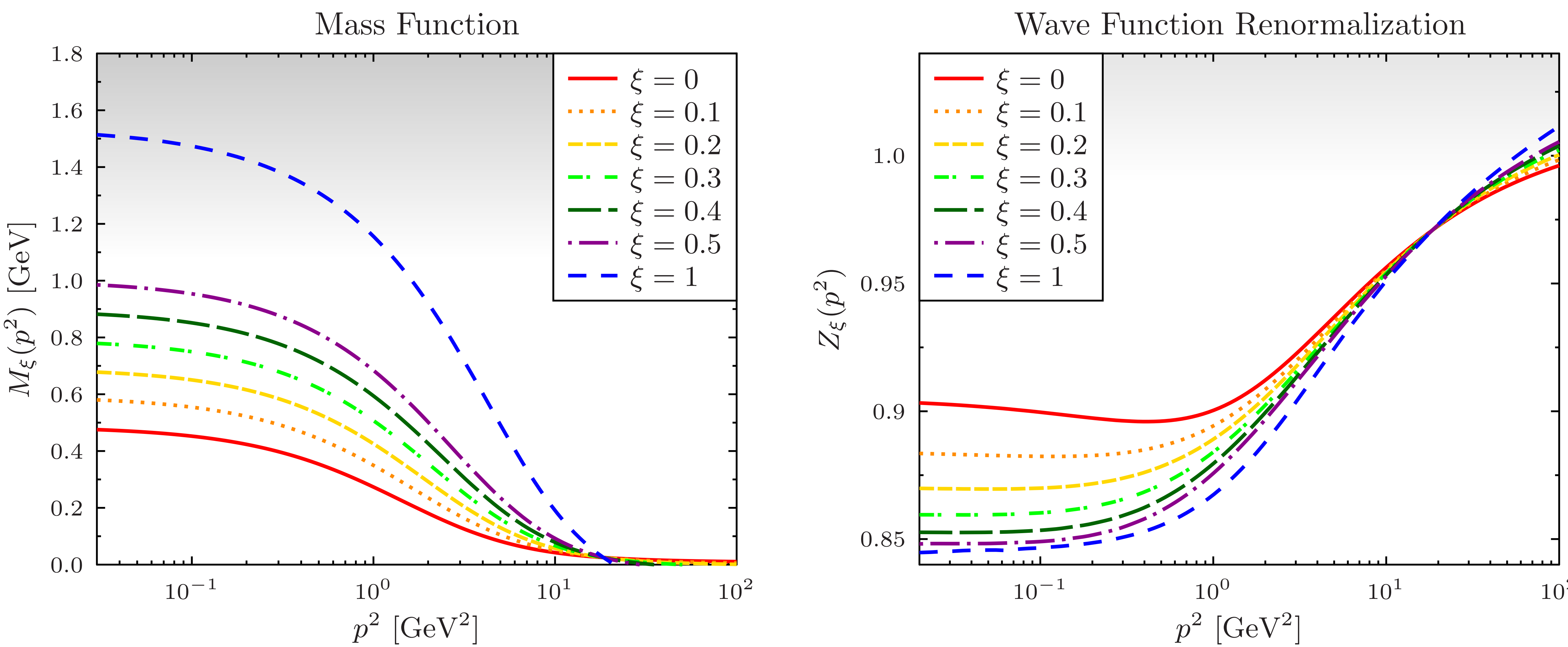
where the Euclidean mass is defined as: $(M_f^E)^2 := \{p^2 | p^2 = M^2(p^2)\}$

f	u, d	s	c	b
$(M_f^E)^{\rm Latt+STI}$	0.390	0.514	1.530	4.687
$\zeta^{\rm Latt+STI}$	0.019	0.224	0.678	0.852

DSE with gluon propagators in R_ξ gauge

Lattice QCD input for gluon: P. Bicudo, D. Binosi, N. Cardoso, O. Oliveira and P. J. Silva, Phys. Rev. D 92, 114514 (2015)

Extrapolation to Feynman Gauge using Padé parametrization of lattice gluon and ghost dressing function.

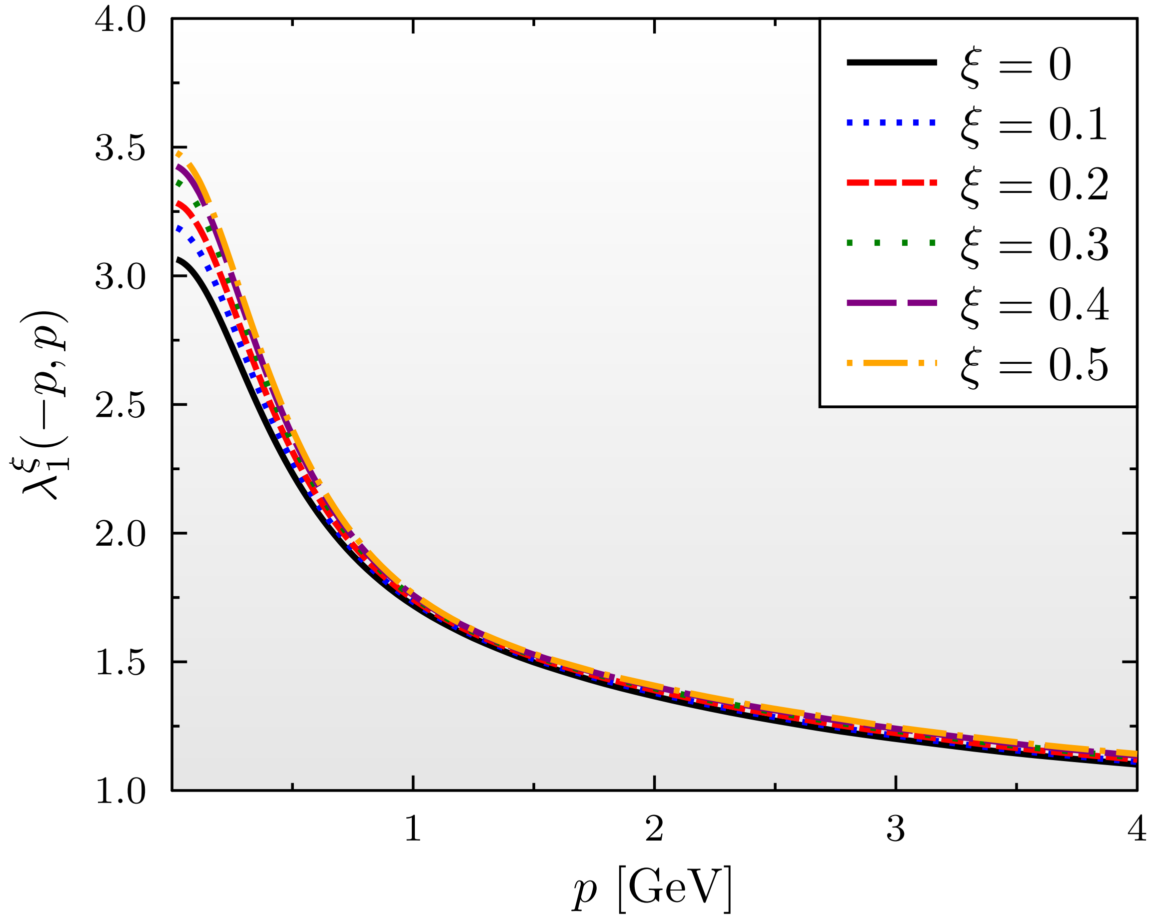


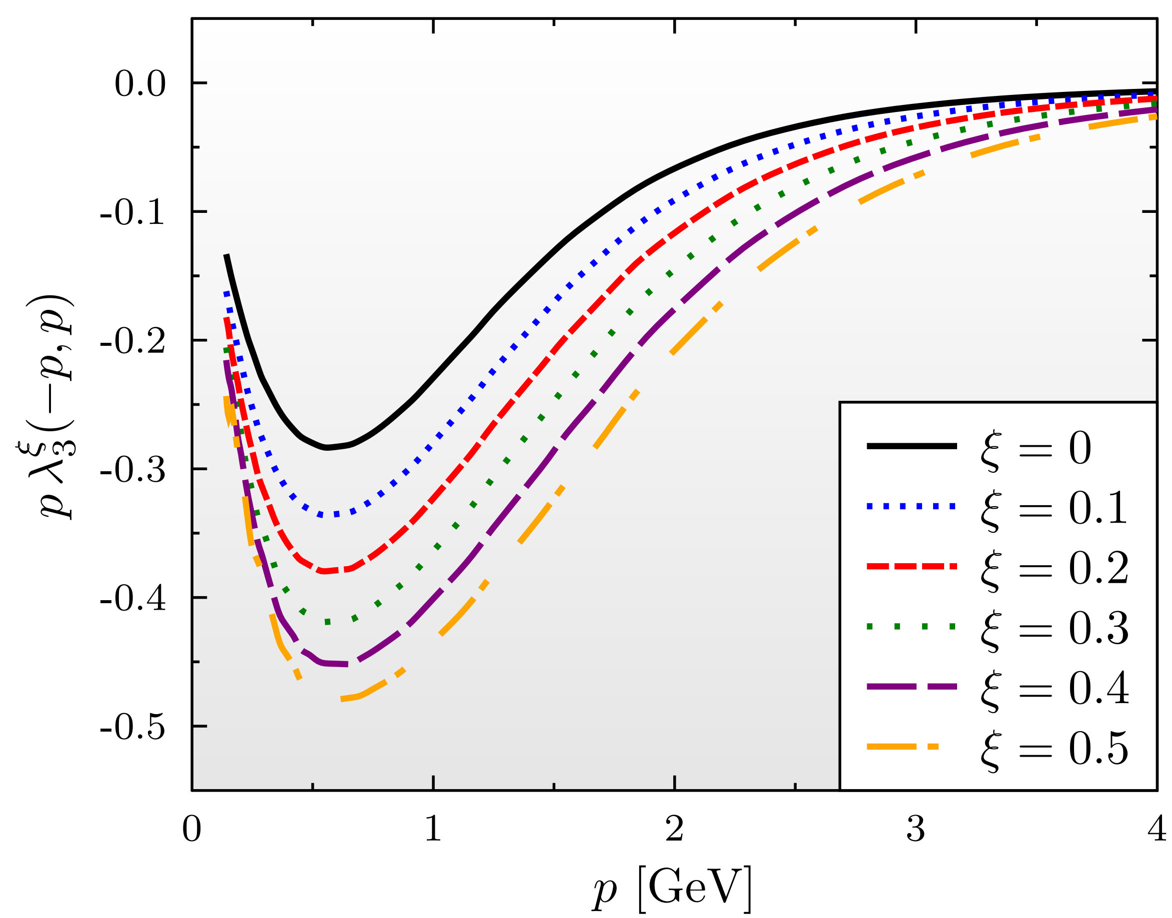
$$\alpha_s^\xi = 0.29 + 0.098\xi - 0.064\xi^2$$

A. C. Aguilar, D. Binosi and J. Papavassiliou, Phys. Rev. D95, 034017 (2017)

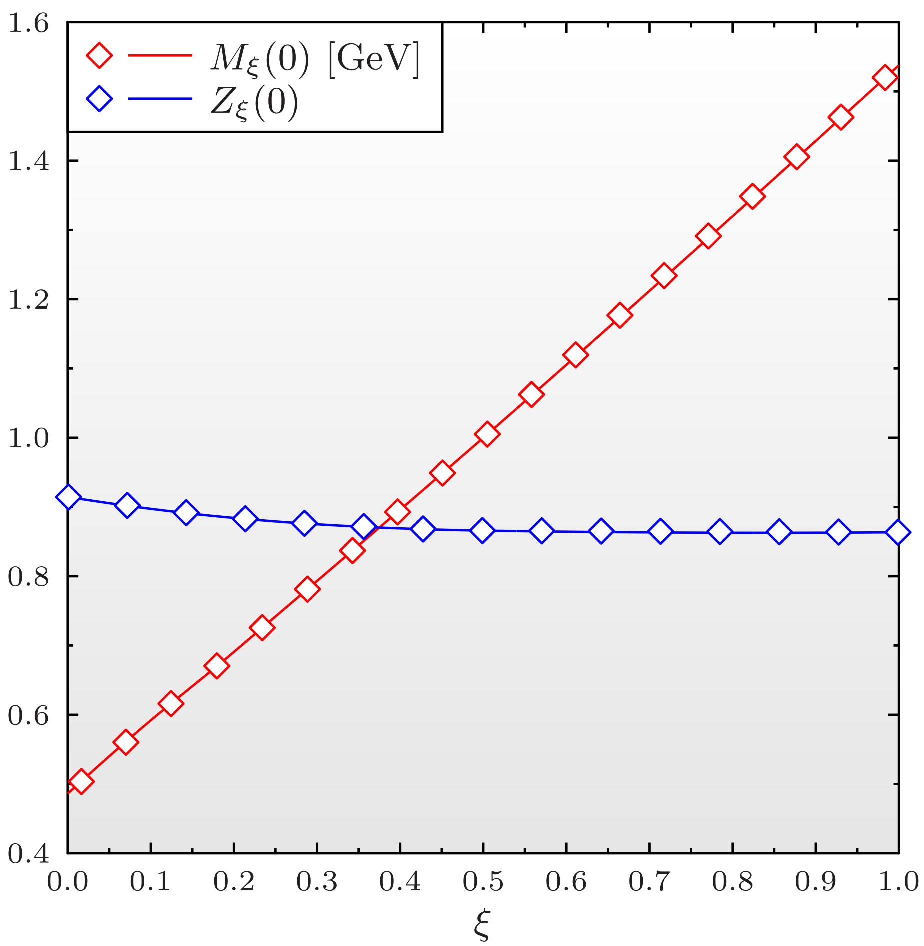
DSE with gluon propagators in R_ξ gauge: quark-gluon vertex

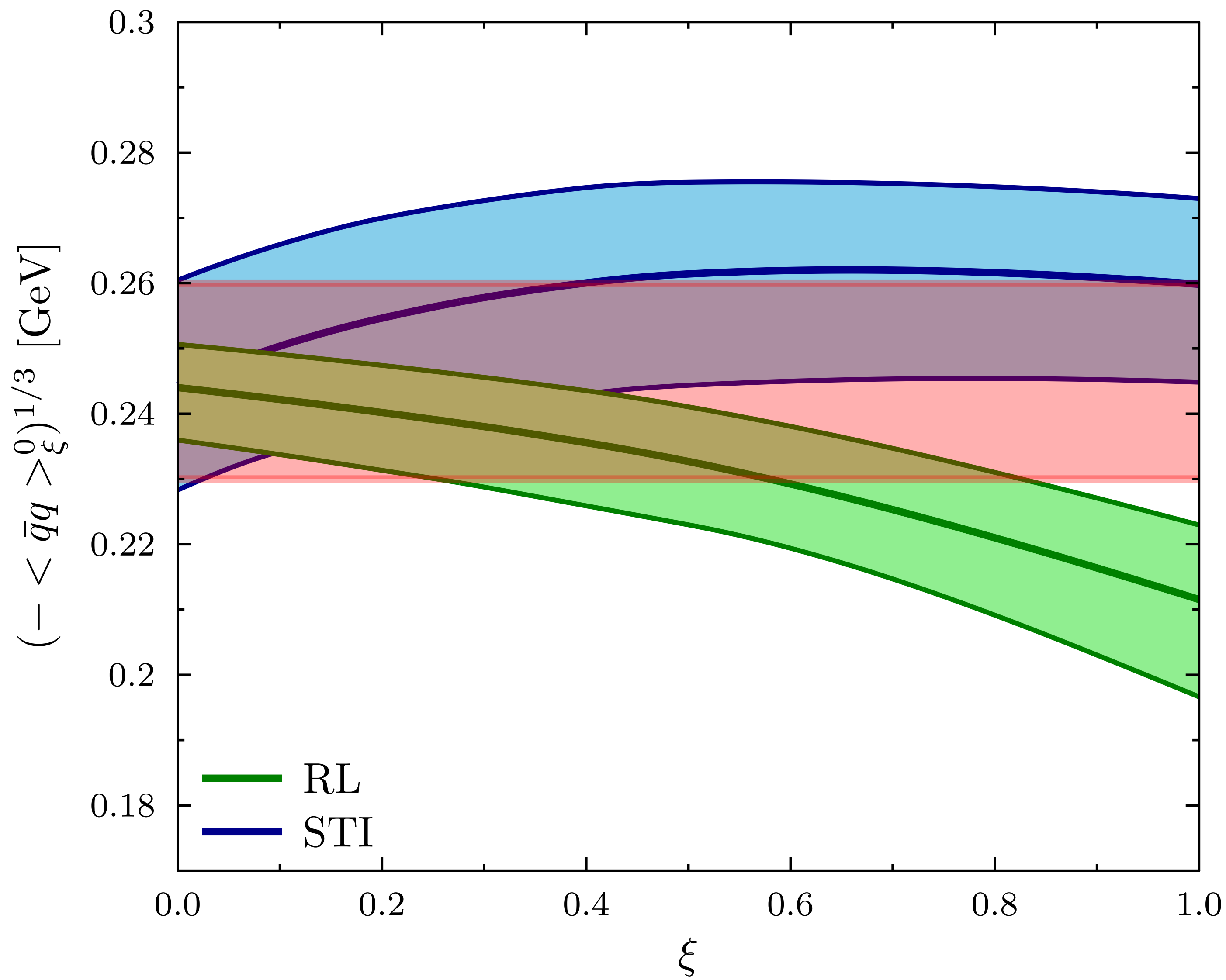
Lattice QCD input for gluon: P. Bicudo, D. Binosi, N. Cardoso, O. Oliveira and P. J. Silva, Phys. Rev. D 92, 114514 (2015)





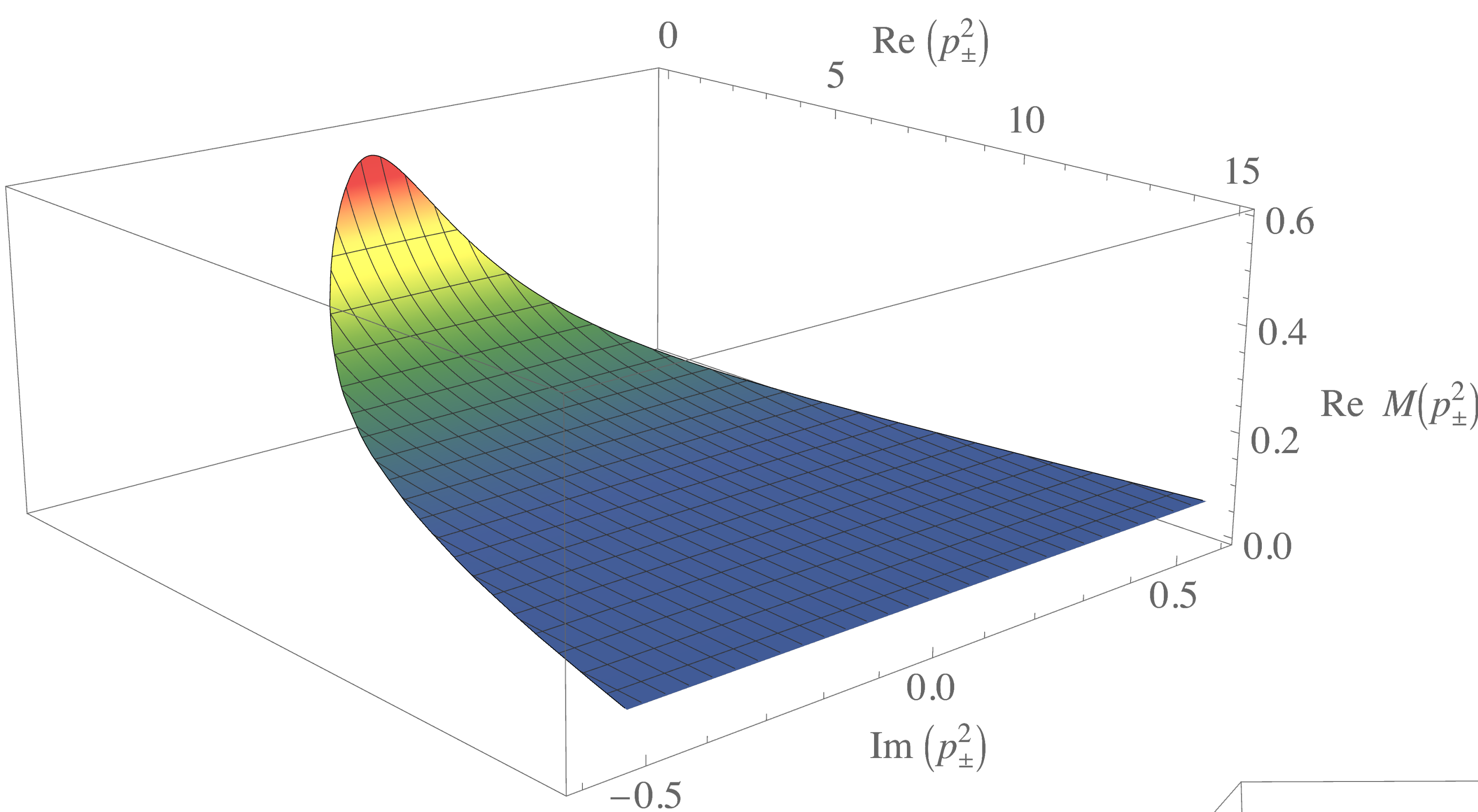
DSE with gluon propagators in R_ξ gauge: constituent mass and quark condensate

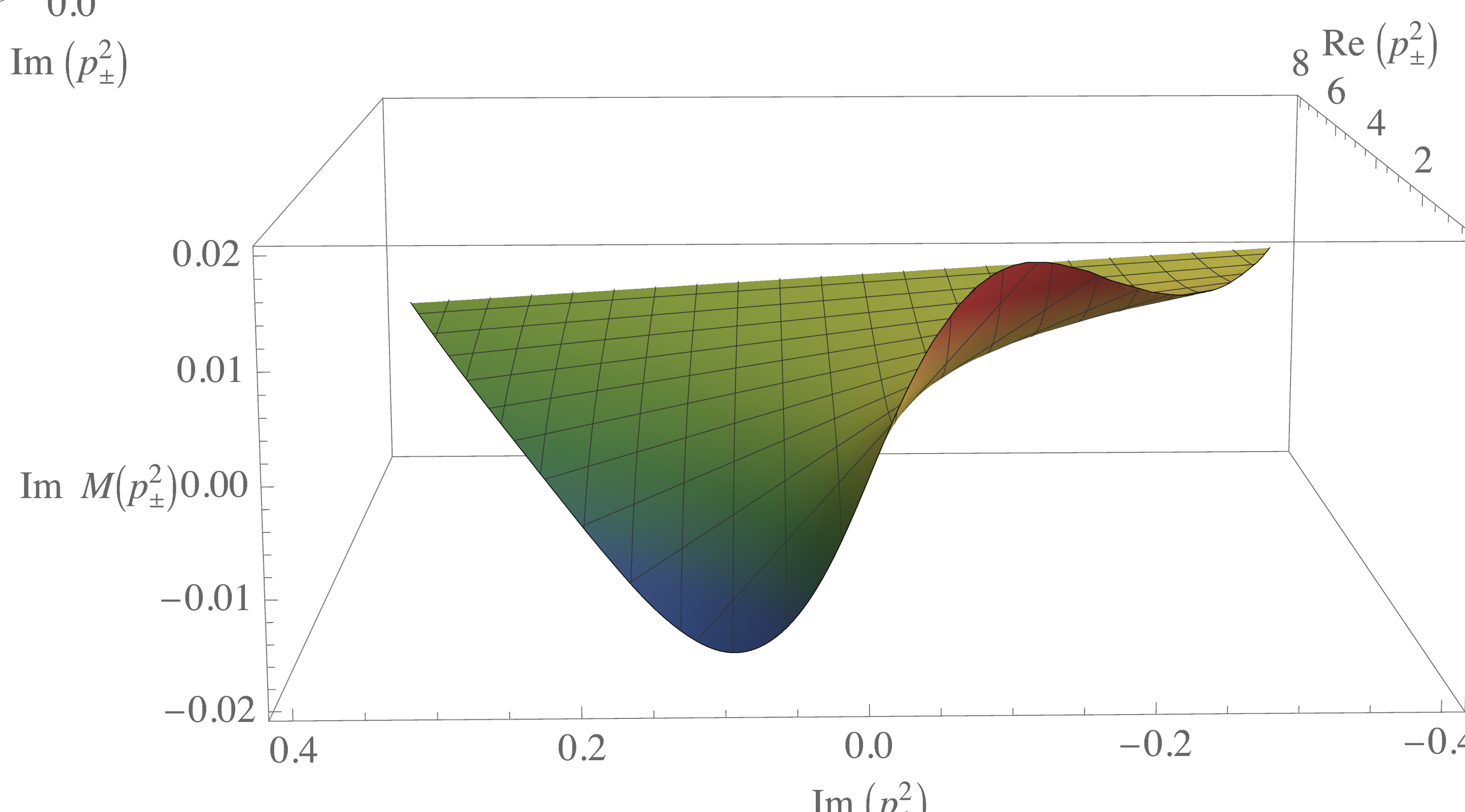




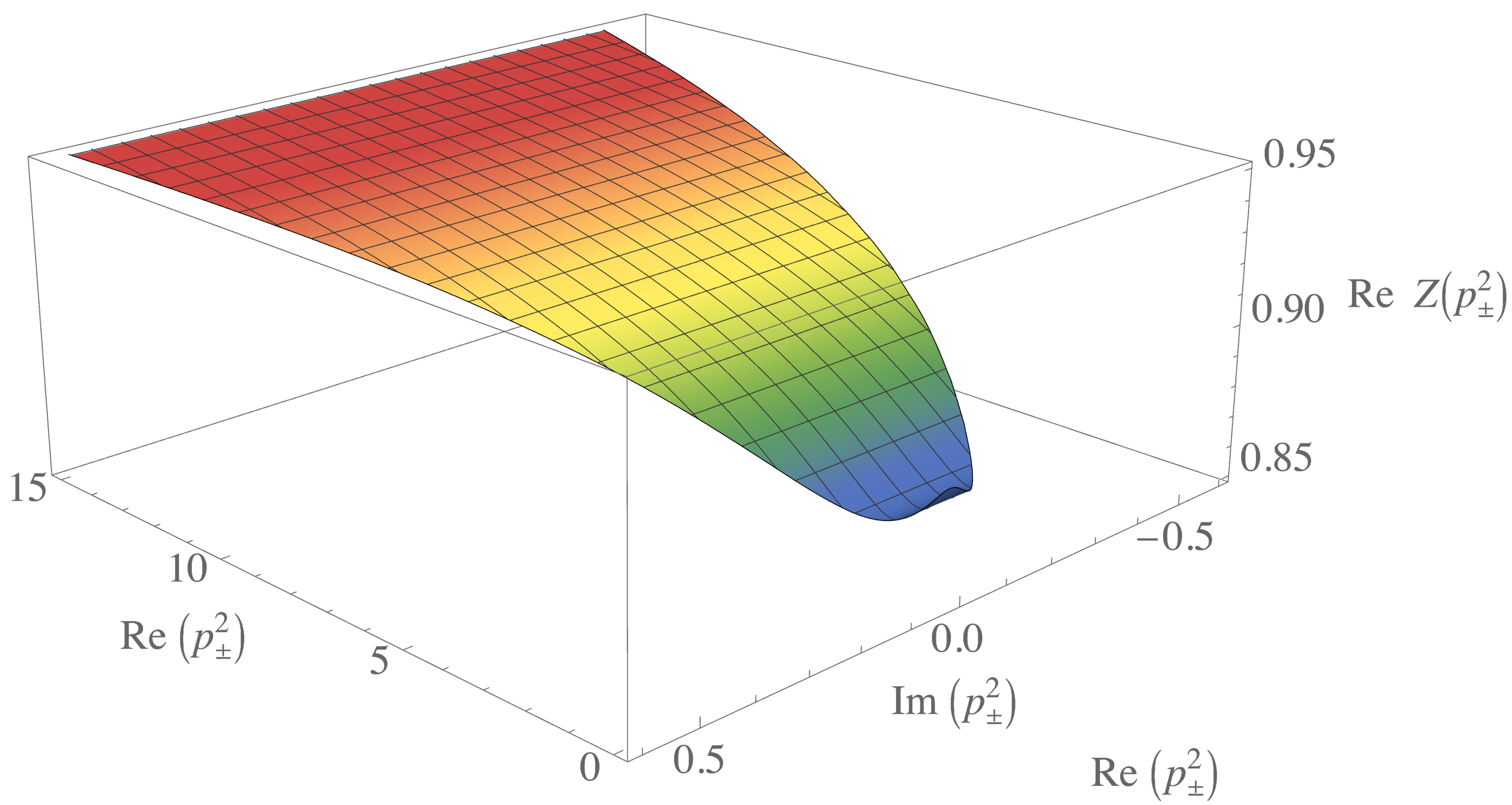
Lattice QCD input for gluon: P. Bicudo, D. Binosi, N. Cardoso, O. Oliveira and P. J. Silva, Phys. Rev. D 92, 114514 (2015)

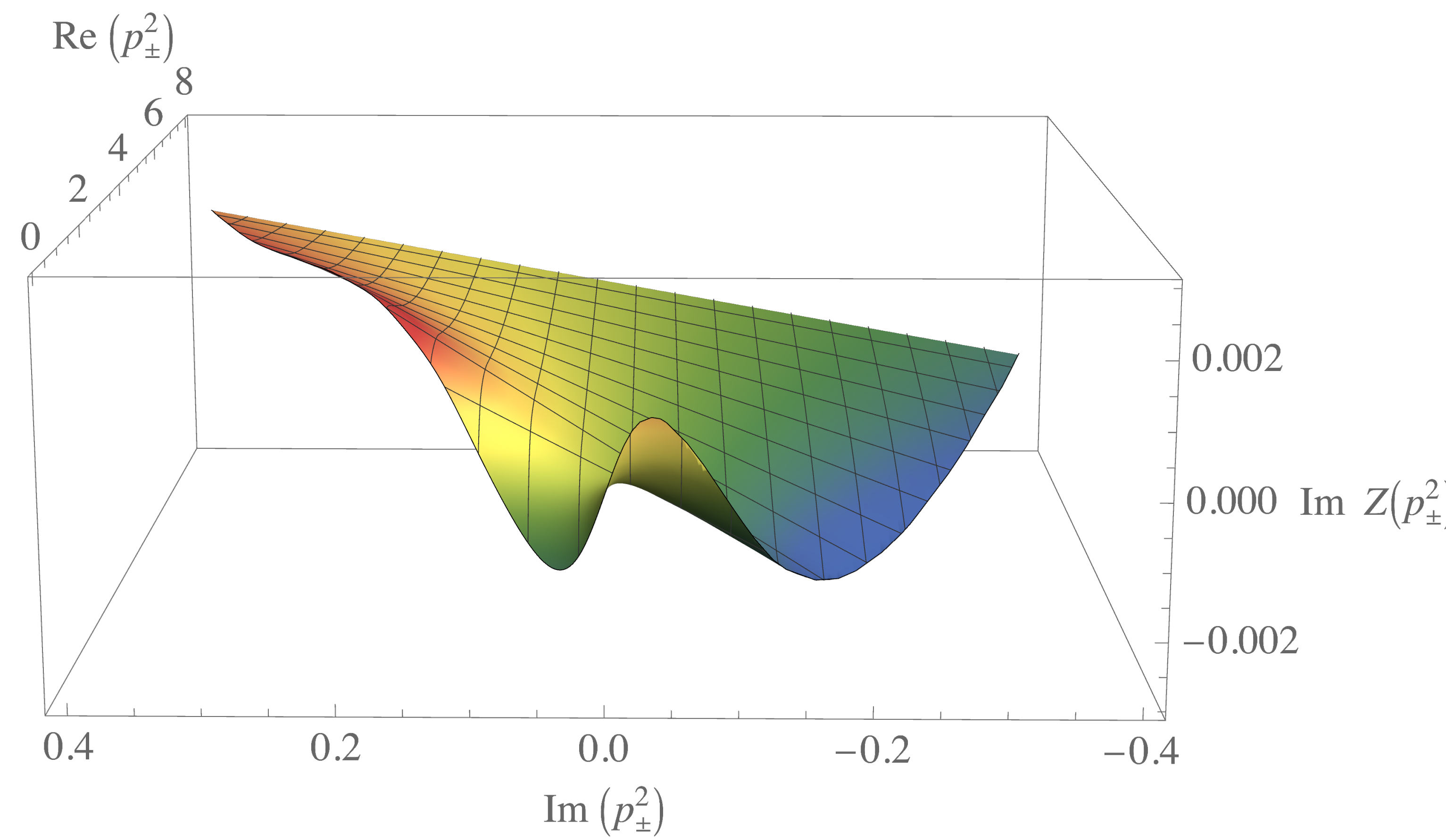
Light quarks on the complex plane: *mass function*





Light quarks on the complex plane: *wave renormalization*





Conclusions & Progress

- We derived a quark-gluon vertex from symmetries (gauge + Lorentz), that is we don't solve the inhomogeneous BSE for the quark-gluon vertex.
- The self-consistent solutions employ as ingredients gluon and ghost propagators from lattice QCD.
- Current status of DCSB still unsatisfying when only known terms are kept and multiplicative renormalizability is not satisfied.
- Next step: calculate the Y_i form factors of the four-point function in transverse STI.
- Underway: deriving the Bethe-Salpeter kernel consistent with this quark-gluon vertex (STIs) that *also satisfies the axialvector Ward identity and thus guarantees a zero pion mass in the chiral limit and the correct DCSB pattern for the meson spectrum.*