On the Collinear Structure of Hadrons from Ioffe-time Pseudo-distributions

Workshop on Parton Distributions and Nucleon Structure

September 12, 2022

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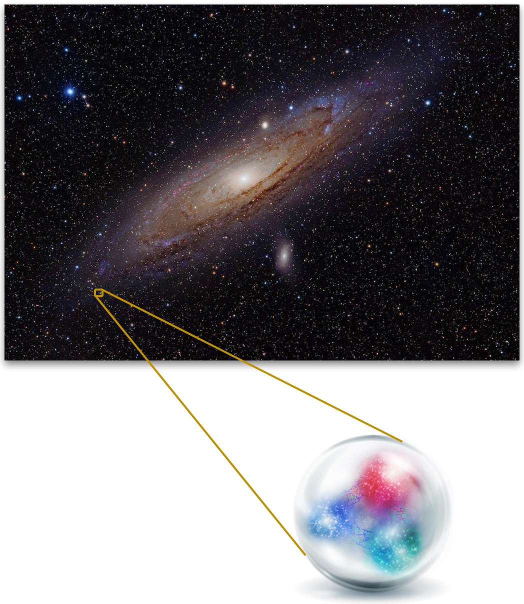
For the **HadStruc** Collaboration

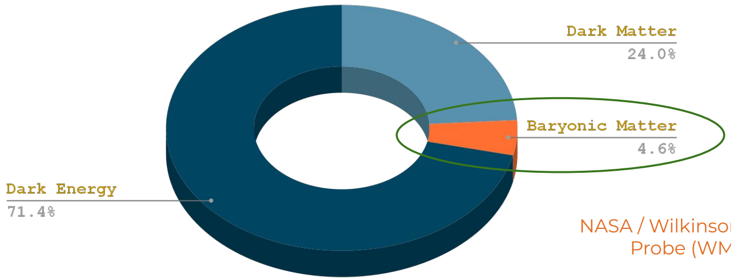
Jefferson Lab

Institute for Nuclear Theory

The Big Picture

Visible Universe suffuse with isotropic distribution of nuclear material





NASA / Wilkinson Microwave Anisotropy Probe (WMAP) Science Team

Planck Collaboration

- How do quarks/gluons and their interactions give rise to global structure of hadrons? (Mass, charge distribution, spin, etc..)
- Internal hadron structure encoded in parton distributions and correlations
  - x-space: longitudinal
  - $\mathbf{k}_\perp, \mathbf{b}_\perp$-spaces: transverse

$$\mathcal{L}_{\text{QCD}} = \sum_f \overline{\psi}^i_f \left(i\not{D}^{ij} - m_f \delta^{ij} \right) \psi^j_f - \frac{1}{4} G^a_{\mu\nu} G^{a\mu\nu}$$

$$\int \mathrm{d}^2\mathbf{k}_\perp \qquad f\left(x, \mathbf{b}_\perp\right) \qquad \int \mathrm{d}^2\mathbf{b}_\perp$$

Impact parameter distributions

$$W\left(x, \mathbf{k}_\perp, \mathbf{b}_\perp\right) \qquad\qquad f\left(x\right)$$

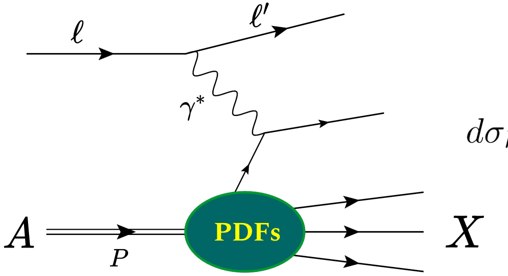
Wigner Distributions

PDF

$$\int \mathrm{d}^2\mathbf{b}_\perp \qquad f\left(x, \mathbf{k}_\perp\right) \qquad \int \mathrm{d}^2\mathbf{k}_\perp$$

TMD

Parton Distributions & Nucleon Structure



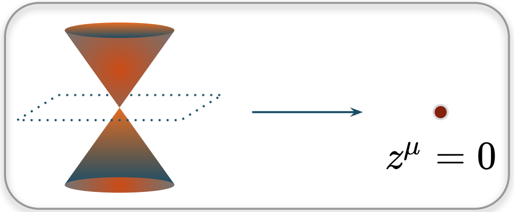
$$d\sigma_h \sim f_{a/h} \otimes d\sigma_q$$

$$F_i\left(x, Q^2\right) = \sum_{a=q,\bar{q},g} f_{a/h}\left(x, \mu^2\right) \otimes H_i^a\left(x, \frac{Q^2}{\mu^2}, \alpha_s\left(\mu^2\right)\right) + h.t.$$

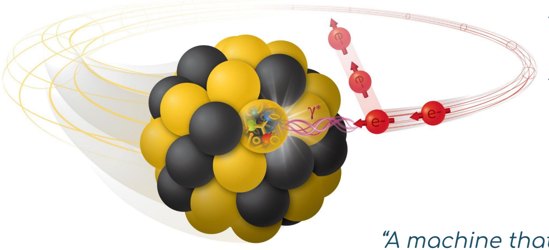
Eg. J. Collins et al., Adv. Ser. Direct. High Energy Phys. 5, 1 (1989)

- ➢ Essential for interpretation of (semi-)inclusive hard scattering processes
- ➢ As process-independent quantities, any uncertainty propagates to new measurements

$$\frac{1}{2}\int \frac{dz^-}{2\pi} e^{ixP^+z^-} \langle h(p)|\,\overline{\psi}\left(\tfrac{z}{2}\right)\gamma^+\Phi^{(f)}_{\hat{z}^-}\left(\{\tfrac{z}{2}, -\tfrac{z}{2}\}\right)\psi\left(-\tfrac{z}{2}\right)|h(p)\rangle$$



EIC will probe hadron structure at unprecedented precision



- ❖ ex. spin-dependent PDFs
- ❖ hadron tomography/ confinement/saturation
  - ➢ some dists. will remain hard to quantify in experiment

"A machine that will unlock the secrets of the strongest force in Nature"

- ➢ Target: leading-twist quark PDFs of the nucleon
  - ○ high fidelity - quantify systematics

Roadmap

From Lattice QCD to PDFs...

- Pseudo-distributions
  - short-distance factorization
  - preface to leading-twist quark PDFs
- Numerical implementation
  - matrix elements with *distillation*
- Pseudo-distributions and distillation applied to nucleon
  - ill-posed inverse & regularization with parametric models
    - [unpolarized quark PDFs]
  - model selection & bias
    - [transversity quark PDFs]
  - disentangling leading/contaminating amplitudes
    - [helicity quark PDFs]
- Global analysis & Lattice QCD
  - a coming synergy
- Towards the off-forward regime

What are Pseudo-Distributions?

This talk: matrix elements of space-like separated parton bilinears

$$M^{[\Gamma]}(p,z) = \langle h(p)|\overline{\psi}(z)\,\Gamma\Phi^{(f)}_{\hat{z}}(\{z,0\})\,\psi(0)\,|h(p)\rangle$$

Standard log. singularities generating perturbative evolution

Additional UV singularities for space-like Wilson line

Quasi-Distributions & LaMET

X. Ji [Mon. 9:00am]
X. Gao [Mon. 9:40am] & A. Hanlon [Thurs. 3:00pm]

X. Ji, Phys. Rev. Lett. 110 (2013) 262002

Pseudo-Distributions & SDF

J. Delmar [Mon. 2:40pm] & R. Sufian [Mon. 3:00pm]

V. Braun and D. Müller, Eur.Phys.J.C 55 (2008) 349-361
A. Radyushkin, PRD 96 (2017) 3, 034025

Lorentz invariance requires $M^{[\Gamma]}(p,z)$ be a function of ν, z^2: $M^{[\Gamma]}(p,z) \sim \mathcal{M}(\nu, z^2)$ $\quad \nu \equiv p\cdot z$

Kinematic setup defining collinear PDFs:

$$p^\alpha = \left(p^+, \frac{m_h^2}{2p^+}, \mathbf{0}_\perp\right) \qquad z^\alpha = \left(0, z^-, \mathbf{0}_\perp\right)$$

$$\mathcal{M}\left(p^+z^-,0\right)_{\mu^2} \equiv Q\left(\nu,\mu^2\right) = \int_{-1}^{1} dx\; e^{i\nu x} f_{q/h}\left(x,\mu^2\right)$$

Ioffe-time distribution (ITD)

V. Braun et al., Eur.Phys.J.C 55 (2008) 349-361

Ioffe-time Pseudo-distribution:

$$\mathcal{M}\left(p_z z_3, z_3^2\right) = \int_{-1}^{1} dx\; e^{i\nu x}\mathcal{P}\left(x, z_3^2\right)$$

Generalize concept of light-cone PDFs onto space-like intervals [Lorentz covariant]

$$\mathcal{P}\left(x, z_3^2\right) = \frac{1}{2\pi}\int_{-\infty}^{\infty} d\nu\; e^{-i\nu x}\mathcal{M}\left(\nu = p_z z_3, z_3^2\right)$$

To map collinear momentum of parton, need only map Ioffe-time dependence

Lorentz invariance is essential!

On Short-Distance Factorization (SDF)

Matrix elements admit a Lorentz decomposition

$\nu \equiv p \cdot z$

$$M^{[\Gamma]}(p,z) = \sum_i \mathcal{K}_i(p^\mu, z^\mu, \cdots)\,\mathcal{M}_i(\nu, z^2)$$

Short-Distance Factorization

Ioffe-time Pseudo-Distribution (pseudo-ITD)

Perturbatively calculable Wilson coefficients

Ioffe-time Distribution (ITD)

Analogous role as momentum for quasi-distributions

$$\mathcal{M}_i(\nu, z^2) = \sum_a C_a(z^2\mu^2, \alpha_s) \otimes \mathcal{Q}_a(\nu, \mu^2) + \mathcal{O}(z^2\Lambda^2_{\mathrm{QCD}})$$

But what about renormalization?

- divergences from space-like Wilson line are indep. of Ioffe-time/hadronic state & renormalize multiplicatively

T. Ishikawa et al., Phys.Rev.D 96 (2017) 9, 094019
X. Ji et al., Phys.Rev.Lett. 120 (2018) 11, 112001
J. Green et al., Phys.Rev.Lett. 121 (2018) 2, 022004

$$\mathfrak{M}(\nu, z^2) \equiv \frac{\mathcal{M}(\nu, z^2)}{\mathcal{M}(0, z^2)}$$

"Reduced pseudo-ITD"

- reduced power corrections expected
- perturbative evolution observed in (first) quenched study

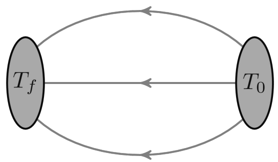
K. Orginos et al., Phys. Rev. D 96, 094503 (2017)

- Key: for which separations do corrections remain small?

	Lorentz Decomposition	Defining Kinematics	Leading-twist Information	Accessing Leading-Amplitude in LQCD
Unpol.	$2p^\alpha\mathcal{M}(\nu,z^2) + 2z^\alpha\mathcal{N}(\nu,z^2)$	$p^\alpha = \left(p^+, \frac{m_h^2}{2p^+}, \mathbf{0}_\perp\right)$ $z^\alpha = (0, z^-, \mathbf{0}_\perp)$ $\alpha = +$	$\mathcal{M}(p^+z^-, 0)_{\mu^2}$	$p^\alpha = (\mathbf{0}_\perp, p_z, E)$ $z^\alpha = (\mathbf{0}_\perp, z_3, 0)$ $\alpha = 4$
Trans.	$2[p^\alpha, S_\perp^\beta]\mathcal{M}(\nu,z^2) + 2im_N^2[z^\alpha, S_\perp^\beta]\mathcal{N}(\nu,z^2) + 2m_N^2[z^\alpha, p^\beta](z\cdot S_\perp)\mathcal{R}(\nu,z^2)$	As Above	$\mathcal{M}(p^+z^-, 0)_{\mu^2}$	As Above $\alpha = 4$ $\beta = \perp$
Helicity	$-2m_N S^\alpha\mathcal{M}(\nu,z^2) - 2im_N p^\alpha(z\cdot S)\mathcal{N}(\nu,z^2) + 2m_N^3 z^\alpha(z\cdot S)\mathcal{R}(\nu,z^2)$	As Above	$\left[\mathcal{M}(p^+z^-, 0) + ip^+z^-\mathcal{N}(p^+z^-, 0)\right]_{\mu^2}$	As Above $\alpha = 3$

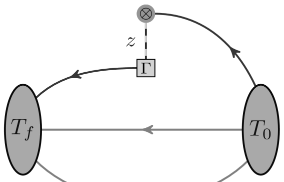
Matrix Elements to Pseudo-Distributions

Require two-point and connected three-point functions:



Operator-state overlaps

$$C_{2\text{pt}}\left(p_z\hat{z},T\right)=\left\langle \mathcal{N}\left(-p_z\hat{z},T\right)\overline{\mathcal{N}}\left(p_z\hat{z},0\right)\right\rangle=\sum_n\frac{\left|\mathcal{Z}_n\left(p_z\right)\right|^2}{2E_n\left(p_z\right)}e^{-E_n\left(p_z\right)T}$$



$$C_{3\text{pt}}^{[\Gamma]}\left(p_z\hat{z},T;z_3,\tau\right)=\left\langle \mathcal{N}\left(-p_z\hat{z},T\right)\overline{\psi}\left(z_3,\tau\right)\Gamma\Phi_{\hat{z}}^{(f)}\left(z_3,0\right)\psi\left(0,\tau\right)\overline{\mathcal{N}}\left(p_z\hat{z},0\right)\right\rangle$$

$$=\sum_{n',n}\frac{\mathcal{Z}_{n'}\left(p_z\right)\mathcal{Z}_n^\dagger\left(p_z\right)}{4E_{n'}\left(p_z\right)E_n\left(p_z\right)}\left\langle n'\right|\mathring{\mathcal{O}}^{[\Gamma]}\left(z_3,\tau\right)\left|n\right\rangle e^{-E_{n'}\left(p_z\right)\left(T-\tau\right)}e^{-E_n\left(p_z\right)T}$$

(Bare) Ground-state matrix element needed

Excited-state contamination + S/N issues

➢ optimize operator/state overlaps - saturate correlation functions at early temporal separations

$$\left\langle 0\right|\hat{\mathcal{O}}\left(\vec{p}\right)\left|h\left(\vec{p}\right)\right\rangle\gg\left\langle 0\right|\hat{\mathcal{O}}\left(\vec{p}\right)\left|h'\left(\vec{p}\right)\right\rangle$$

Distillation: Low-rank and *non-iterative* approximation of a gauge-covariant smearing kernel (typically the Jacobi smearing kernel)

M. Peardon et al., Phys. Rev. D80, 054506 (2009)

$$\Box\left(\vec{x},\vec{y};t\right)_{ab}=\sum_{k=1}^{R_{\mathcal{D}}}\xi_a^{(k)}\left(\vec{x},t\right)\xi_b^{(k)\dagger}\left(\vec{y},t\right)$$

w/ momentum smearing algorithm:

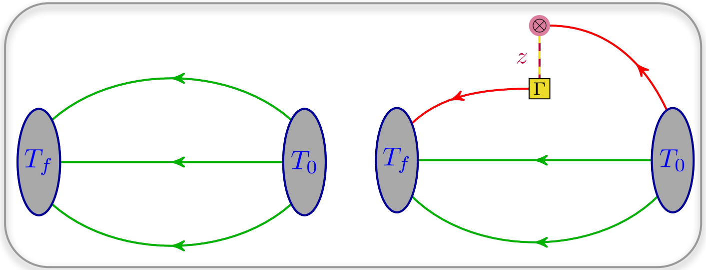
CE et al., PRD 103 (2021) 3, 034502

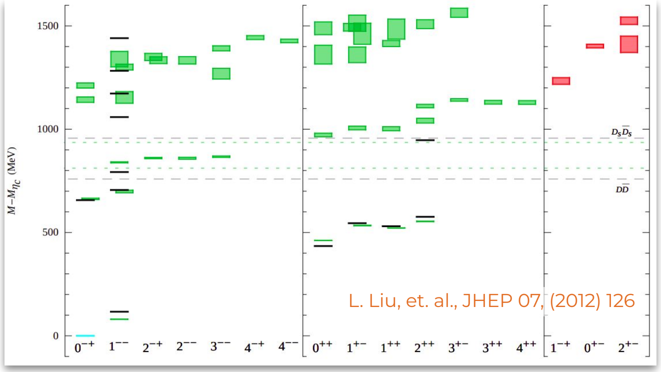
Correlation Functions via Distillation

Wick contractions factorize distillation space

$$C_{mn}(t) = \sum_{\vec{x},\vec{y}} \langle 0| \mathcal{O}_m(t,\vec{x})\, \mathcal{O}_n^\dagger(0,\vec{y})\, |0\rangle$$

$$\equiv \mathrm{Tr}\left[\Phi_m(t) \otimes \tau(t,0)\,\tau(t,0)\,\tau(t,0) \otimes \Phi_n(0)\right]$$





L. Liu, et. al., JHEP 07, (2012) 126

J. Dudek, et. al., PRD 83, 111502 (2011)
J. Dudek et al., PRD 87 (2013) 3, 034505
J. Dudek et. al., PRD 88 (2013) 9, 094505
R. Briceno et al., PRD 97 (2018) 5, 054513
....

Why Distillation?
- ➢ efficiently realizes variational method
- ➢ explicit momentum projections - all times
- ➢ reusability → factorization of correlators

For Gluon PDFs

R. Sufian [Mon 3:00pm]

Perambulators $\tau^{kl}_{\alpha\beta}(t_f,t_0) = \xi^{(k)\dagger}(t_f)\, M^{-1}_{\alpha\beta}(t_f,t_0)\, \xi^{(l)}(t_0)$

Elementals $\Phi^{(i,j,k)}_{\mu\nu\sigma}(t) = \epsilon^{abc}\left(\mathcal{D}_1\xi^{(i)}\right)^a \left(\mathcal{D}_2\xi^{(j)}\right)^b \left(\mathcal{D}_3\xi^{(k)}\right)^c (t)\, S_{\mu\nu\sigma}$

$$\Xi^{(l,k)}_{\alpha\beta}(T_f,T_0;\tau,z_3) = \sum_{\vec{y}} \xi^{(l)\dagger}(T_f)\, D^{-1}_{\alpha\sigma}(T_f,\tau;\vec{y}+z_3\hat{z})\, [\Gamma]_{\sigma\rho}\, \Phi^{(f)}_{\hat{z}}(\{\vec{y}+z_3\hat{z},\vec{y}\})\, D^{-1}_{\rho\beta}(\tau,T_0;\vec{y})\, \xi^{(k)}(T_0)$$

PDF Selection

Space-like Wilson line

→

Amortization of inversion cost

All Dirac structures and Wilson line lengths realizable with single inversion overhead

Mapping any momentum dep. requires only contractions

Nucleon Interpolators with Distillation

Regularization of QCD by
lattice spacing (UV) and volume (IR)

- ➢ consequence: $O(3) \mapsto O_h^D$
- ➢ symmetry broken further for non-zero momenta
  "Little groups" (subgroups) according to $^*(\vec{p})$

D. Moore and G.T. Fleming, PRD 73 (2006) & PRD 74 (2006)
R.C. Johnson, Phys.Lett.B 114 (1982)

patterns of subduction
classify energy eigenstates

Irreps Λ of O_h^D

J	$1/2$	$3/2$	$5/2$	$\cdots$
Λ	G_1	H	$H \oplus G_2$	$\cdots$

<u>A Way Forward</u>:
Build interpolators of def. J^P & flavor
that transform irreducibly under O_h^D and LGs

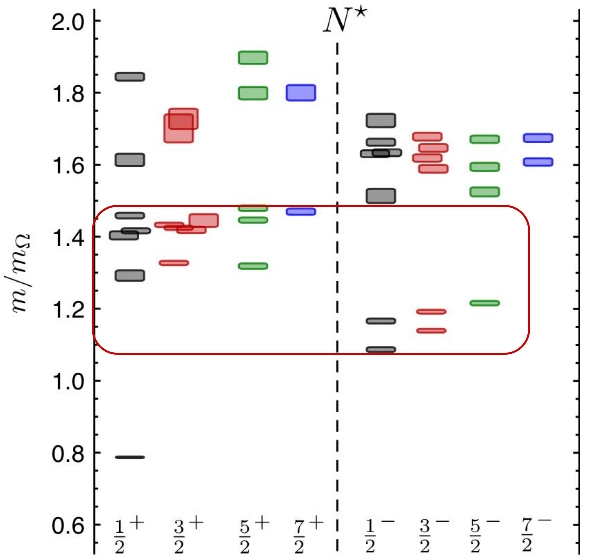
1. continuum ops. at rest with definite flavor and J^P quantum numbers

$$\left\langle \vec{p}=\vec{0}; J'^{P'}, m' \right| \left[\mathcal{O}^{J^P,m}\left(\vec{p}=\vec{0}\right)\right]^\dagger \left|0\right\rangle = Z^{[J]}\delta_{J'J}\delta_{P'P}\delta_{m'm}$$

2. continuum helicity ops. : $\left[\mathbb{O}^{J^P,\lambda}(\vec{p})\right]^\dagger = \sum_m \boxed{\mathcal{D}^{(J)}_{m\lambda}(R)} \left[\mathcal{O}^{J^P,m}(|\vec{p}|\,\hat{z})\right]^\dagger$

   Wigner-D enforcing rotation

3. subduction - interpolators transforming irreducibly under lattice irreps



R. Edwards, et. al., Phys. Rev. D84, 074508 (2011)

Numerical Implementation

ID	a (fm)	m_π (MeV)	β	c_{SW}	$L^3 \times T$	N_{cfg}
E1	0.094(1)	358(3)	6.3	1.205	$32^3 \times 64$	349

(isovector combination only herein)

Parameters/Statistics

ID	N_{vec}	N_{srcs}	T/a	$p_z \times \left(\frac{2\pi}{L}\right)$	z/a
E1	64	4	$4, 6, \cdots, 14$ $0.38, \cdots, 1.32$ fm	$0, \pm 1, \cdots, \pm 6$ $0, 0.41, \cdots, 2.47$ GeV	$0, \pm 1, \cdots, \pm 12, \cdots$ $0, 0.094, \cdots, 1.13$ fm

With precision of distillation, we seek to explore region of applicability of factorization to reduced pseudo-ITD

Unpolarized Quark PDF

- ➢ feasibility study
- ➢ framework for isolating leading-twist signal and systematic corrections

JHEP Published for SISSA by Springer

Towards high-precision parton distributions from lattice QCD via distillation

Colin Egerer,[a,b] Robert G. Edwards,[b] Christos Kallidonis,[b] Kostas Orginos,[a,b] Anatoly V. Radyushkin,[b,c] David G. Richards,[b] Eloy Romero[b] and Savvas Zafeiropoulos[d] on behalf of the HadStruc collaboration

Transversity Quark PDF

- ➢ model selection & data cuts

PHYSICAL REVIEW D **105,** 034507 (2022)

Transversity parton distribution function of the nucleon using the pseudodistribution approach

Colin Egerer,[1,2] Christos Kallidonis,[2] Joseph Karpie,[3] Nikhil Karthik,[1,2] Christopher J. Monahan,[1,2] Wayne Morris,[4,2] Kostas Orginos,[1,2] Anatoly Radyushkin,[4,2] Eloy Romero,[2] Raza Sabbir Sufian,[1,2] and Savvas Zafeiropoulos[5]

Helicity Quark PDF

- ➢ novel source of contamination in space-like matrix element
- ➢ exploit group theory of lattice

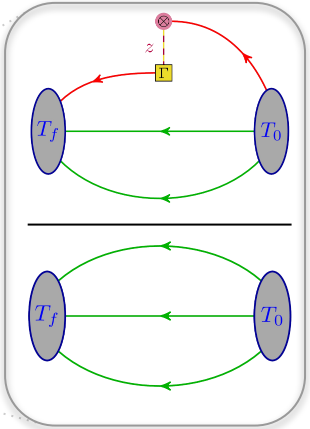
Matrix Elements to Pseudo-Distributions

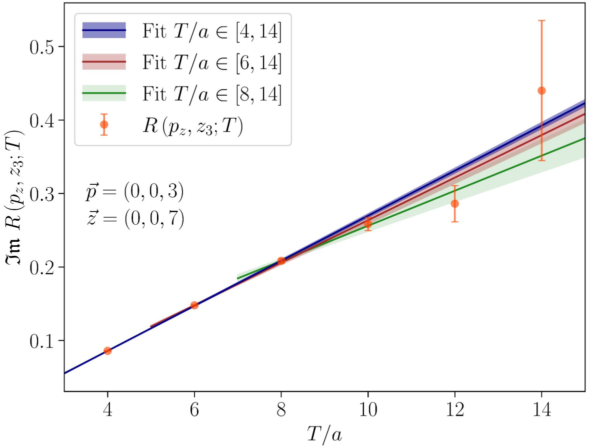
Summation method - further excited-state suppression

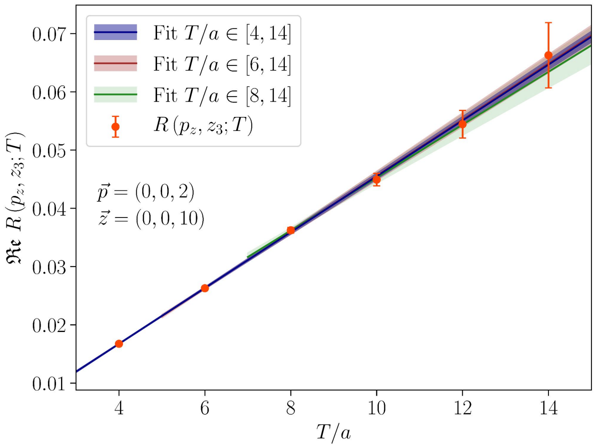
L. Maiani et al., Nucl. Phys. B293 (1987)
C. Bouchard et al., PRD 96, no. 1, 014504 (2017)

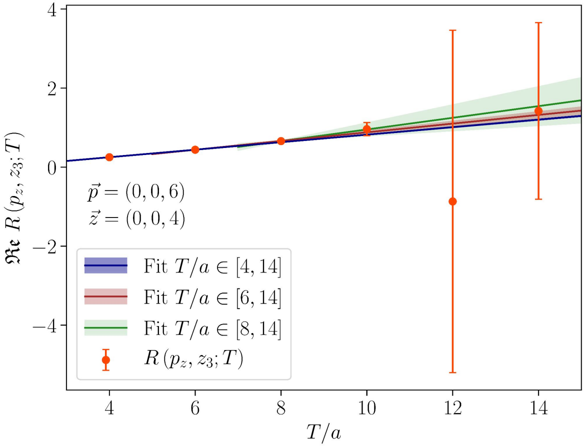
$$R(p_z, z_3; T) = \sum_{\tau/a=1}^{T-1} \frac{C_3(p_z, T, \tau; z_3)}{C_2(p_z, T)}$$

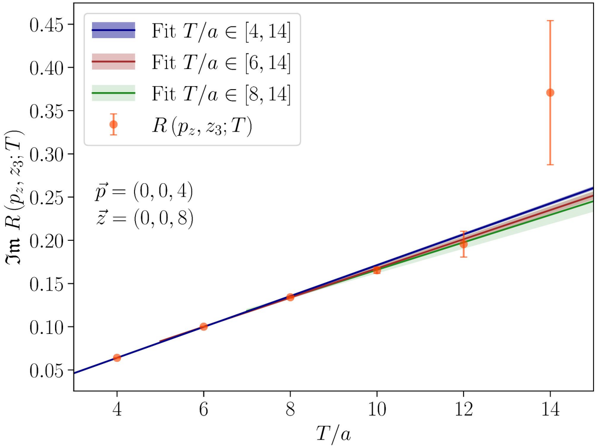
$$R_{\text{fit}}(p_z, z_3; T) = \mathcal{A} + M_4(p_z, z_3)\, T + \mathcal{O}\left(e^{-\Delta E T}\right)$$



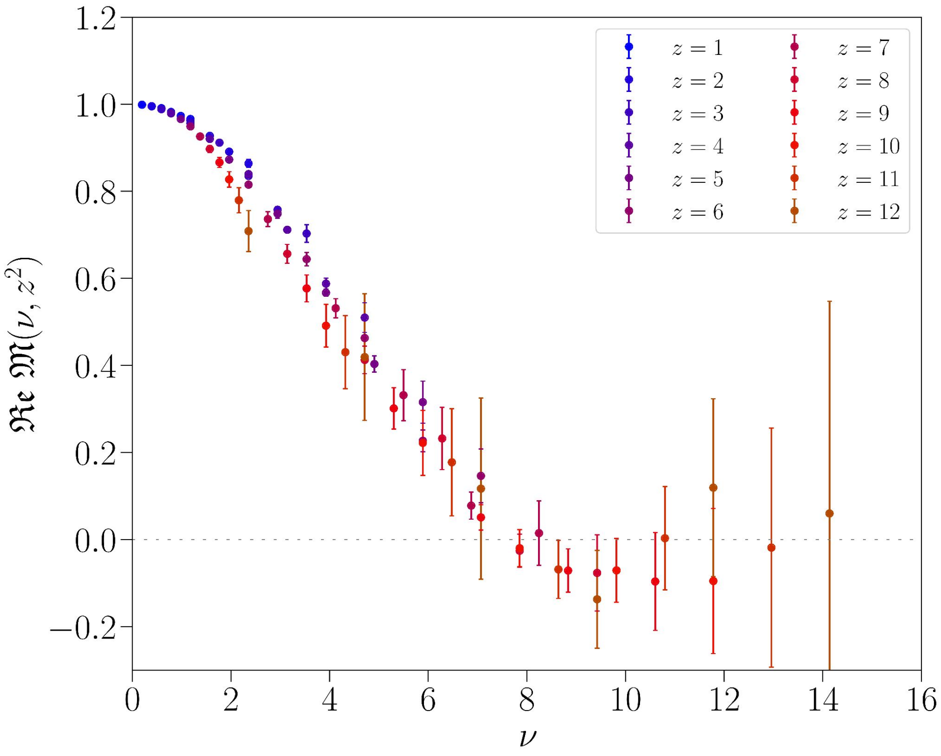


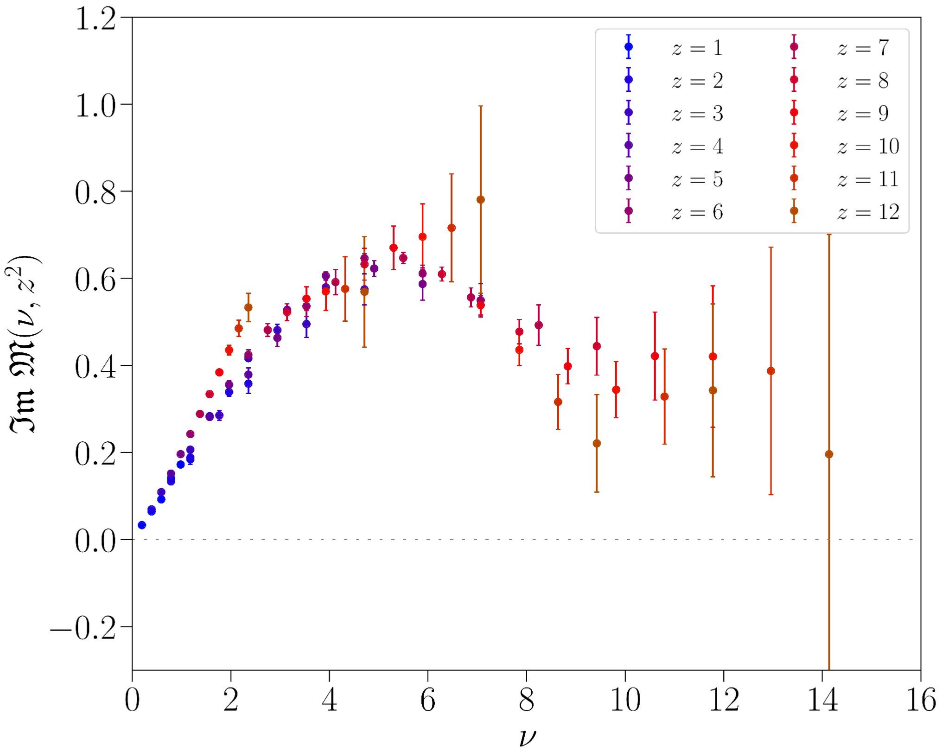






Unpolarized Reduced Pseudo-ITD



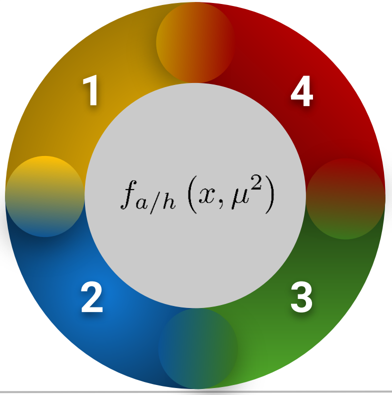


CE, R. Edwards, C. Kallidonis et al., JHEP 11 (2021) 148

Determining the Unknown PDFs

Matrix Element

- pseudo-distributions
- quasi-distributions



Inverse Problem

A grossly ill-posed convolutional relationship connecting lattice data to desired structure function

A serious systematic that must be confronted

Analogous challenge faced by global fitting community!

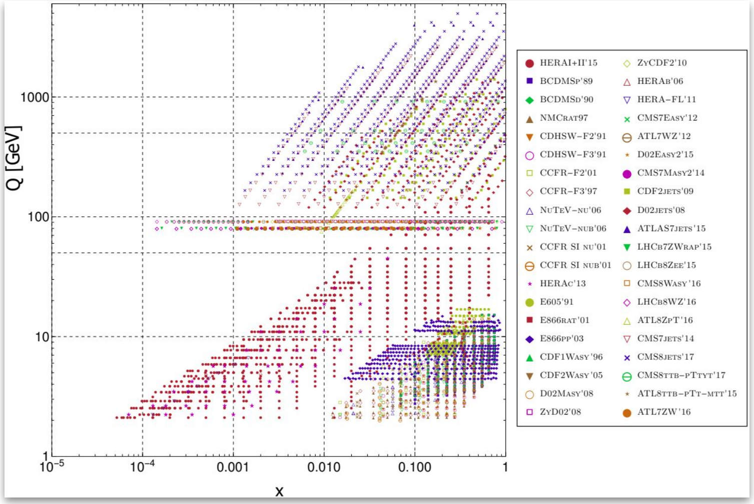
$$F_i\left(x,Q^2\right)=\sum_j\int_x^1 \mathrm{d}\xi\ C_j\left(\xi,\frac{\mu^2}{Q^2}\right)q_j\left(\frac{x}{\xi},\mu^2\right)$$

Renormalization

- RI-MOM/hybrid schemes
- Reduced distribution

Evolution/Matching

Coordinate-space factorization; perturbative matching kernels



How to Proceed?

A) Parametric fits

$$xf_{a/h}\left(x,Q_0^2\right)=Ax^{\alpha}\left(1-x\right)^{\beta}P\left(x\right)$$

MMHT: $1+\sum_{k=1}^{4}c_kT_k^{\mathrm{Ch}}\left(1-2\sqrt{x}\right)$

CJ/MSTW: $1+\gamma\sqrt{x}+\delta x$

CT: $\sum_{k=1}^{4}c_kb_k^4\left(\sqrt{x}\right)$

B) Advanced reconstructions

a) Bayesian reconstruction, Backus-Gilbert, Maximum Entropy, etc J. Karpie, et al., JHEP 04, 057 (2019)

b) Deep-learning methods

L. D. Debbio, et al., JHEP 02 (2021) 138
K. Cichy, L. D. Debbio, T. Giani, JHEP 10 (2019) 137

A Parametric Strategy for Isolating PDFs

Ill-posed (pseudo-)ITD/PDF matching relation:

$$\mathfrak{M}\left(\nu, z^2\right) = \int_{-1}^{1} \mathrm{d}x \, \mathcal{K}\left(x\nu, z^2\mu^2\right) f_{q/h}\left(x, \mu^2\right) + \sum_{k=1}^{\infty} \mathcal{B}\left(\nu\right)\left(z^2\right)^k$$

One choice: model parameterization

$$\int_{-1}^{1} dz \left(1-z\right)^{\alpha}\left(1+z\right)^{\beta} J_n^{(\alpha,\beta)}\left(z\right) J_m^{(\alpha,\beta)}\left(z\right) = \delta_{n,m} h_n\left(\alpha,\beta\right)$$

Change of variables

- ➢ polynomials span support interval of PDFs

$$f_{q/h}\left(x\right) = x^{\alpha}\left(1-x\right)^{\beta} \sum_{n=0}^{\infty} C_{q,n}^{(\alpha,\beta)} \boxed{\Omega_n^{(\alpha,\beta)}\left(x\right)}$$

J. Karpie, K. Orginos, A. Radyushkin et al., JHEP 11 (2021) 024

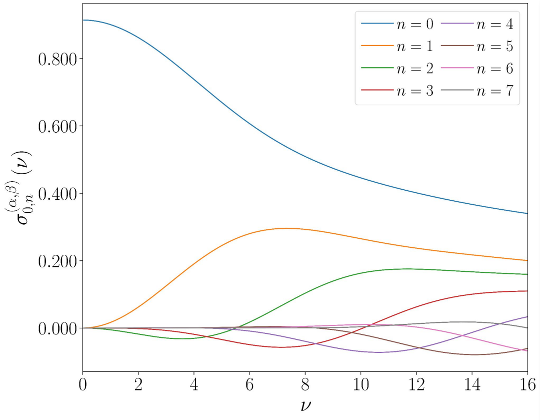
$$\mathfrak{Re}\,\mathfrak{M}_{\text{fit}}\left(\nu, z^2\right) = \sum_{n=0}^{\infty} \sigma_n^{(\alpha,\beta)}\left(\nu, z^2\mu^2\right) C_{\text{v},n}^{lt\,(\alpha,\beta)} + \Delta_{\text{corr}} \sum_{n=1}^{\infty} \sigma_{0,n}^{(\alpha,\beta)}\left(\nu\right) C_{\text{v},n}^{\Delta\,(\alpha,\beta)}$$

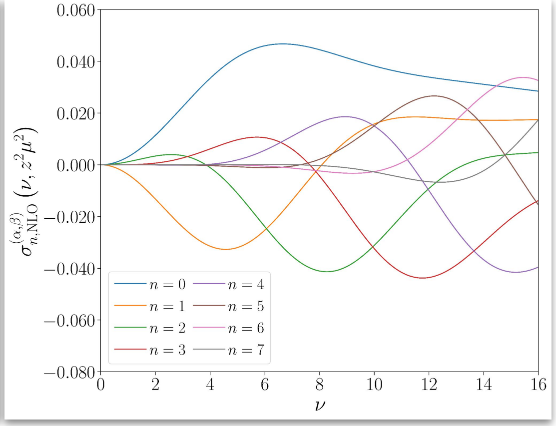
$$\mathfrak{Im}\,\mathfrak{M}_{\text{fit}}\left(\nu, z^2\right) = \sum_{n=0}^{\infty} \eta_n^{(\alpha,\beta)}\left(\nu, z^2\mu^2\right) C_{+,n}^{lt\,(\alpha,\beta)} + \Delta_{\text{corr}} \sum_{n=0}^{\infty} \eta_{0,n}^{(\alpha,\beta)}\left(\nu\right) C_{+,n}^{\Delta\,(\alpha,\beta)}$$

$\frac{a}{|z|}, z^2\Lambda_{\text{QCD}}^2, z^4\Lambda_{\text{QCD}}^4$

$$\sigma_{0,n}^{(\alpha,\beta)}\left(\nu\right) = \sigma_n^{(\alpha,\beta)}\left(\nu, z^2\mu^2\right)\big|_{\alpha_s=0}$$

$$\sigma_{n,\text{NLO}}^{(\alpha,\beta)}\left(\nu, z^2\mu^2\right) = \sigma_n^{(\alpha,\beta)}\left(\nu, z^2\mu^2\right) - \sigma_{0,n}^{(\alpha,\beta)}\left(\nu\right)$$



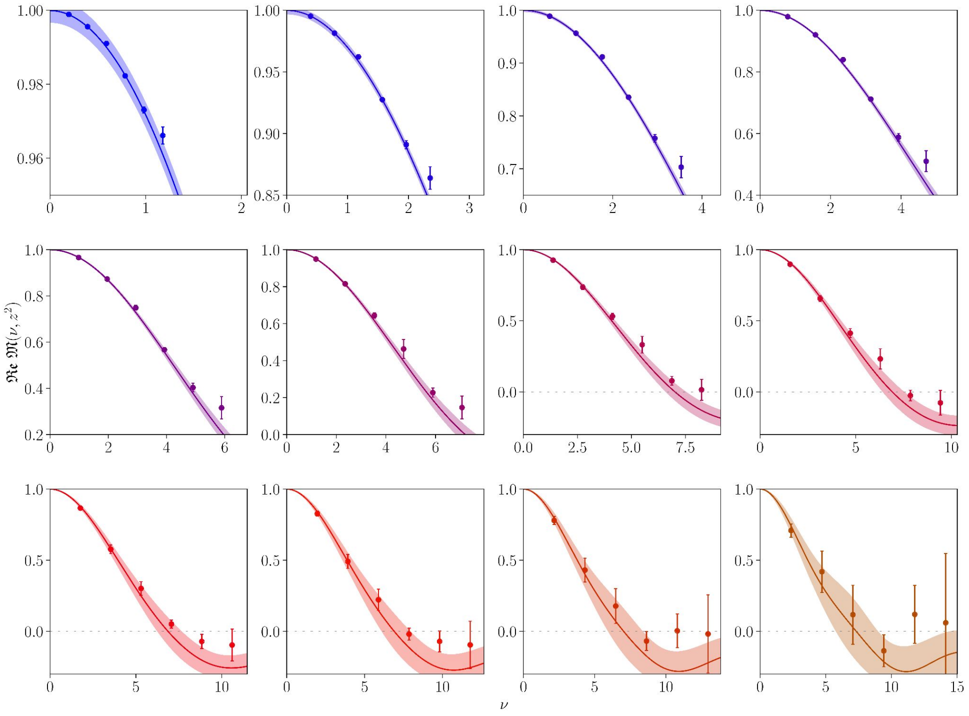


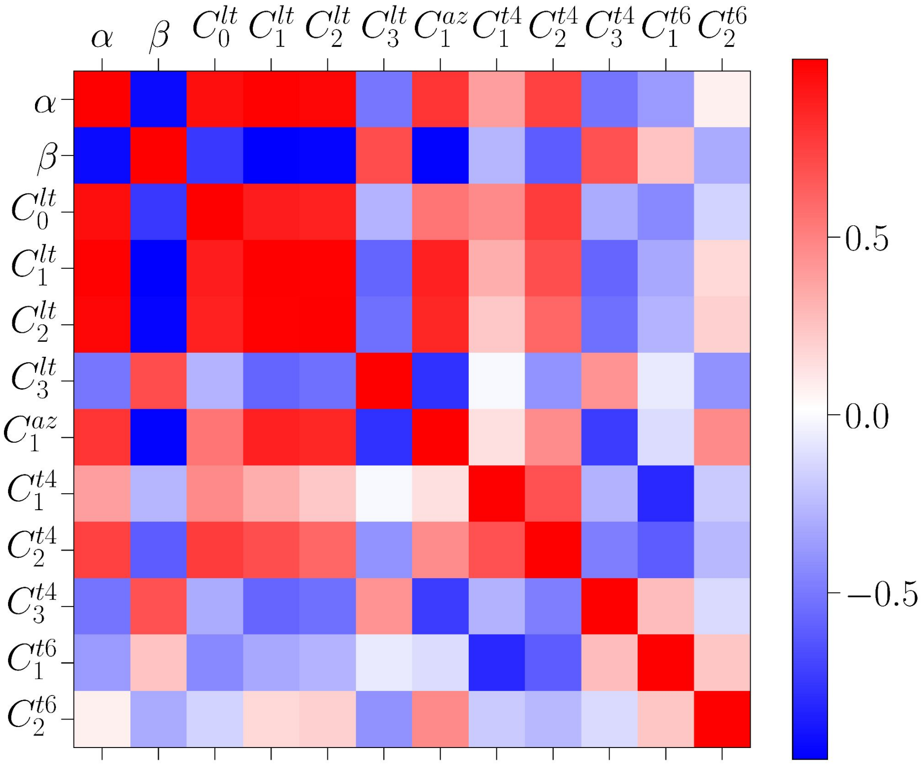
Objective: determine most likely parameters given data & prior information

- ➢ Bayes' Theorem
- ➢ maximize posterior distribution
- ➢ stabilize optimization: `Variable Projection`

G. Golub and V. Pereyra, SIAM Journal on Numerical Analysis 10, 413 (1973)

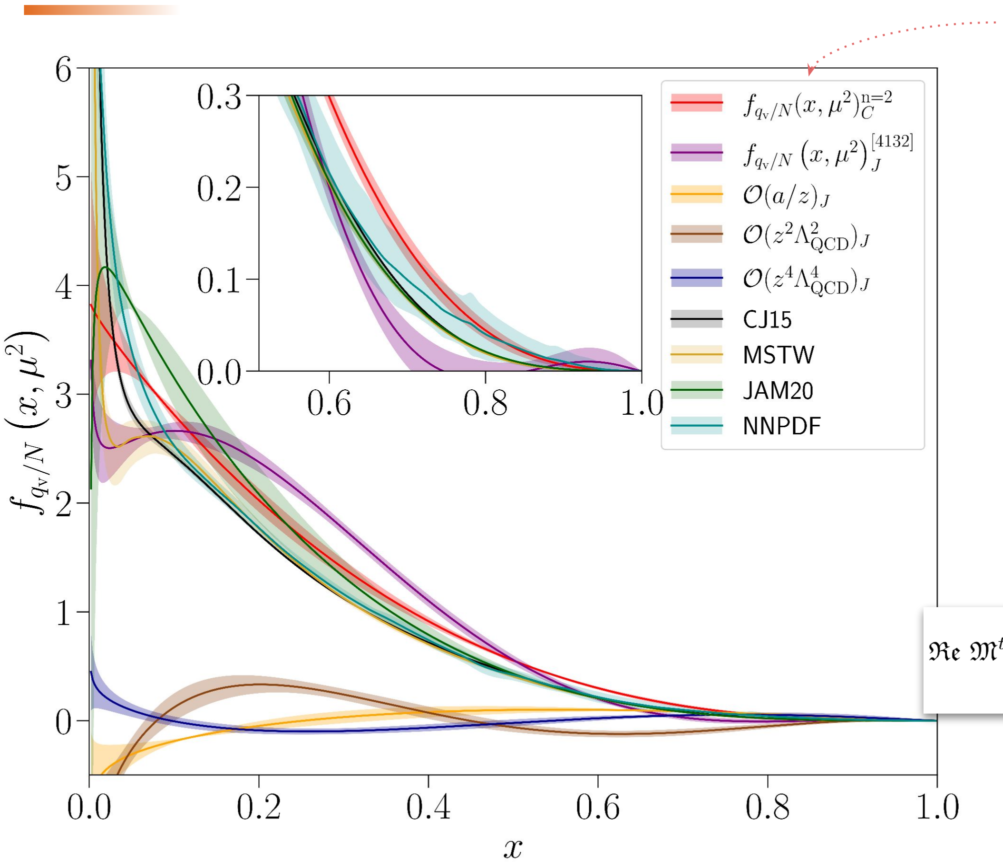
Optimal Fit for Unpolarized Valence Quark PDF

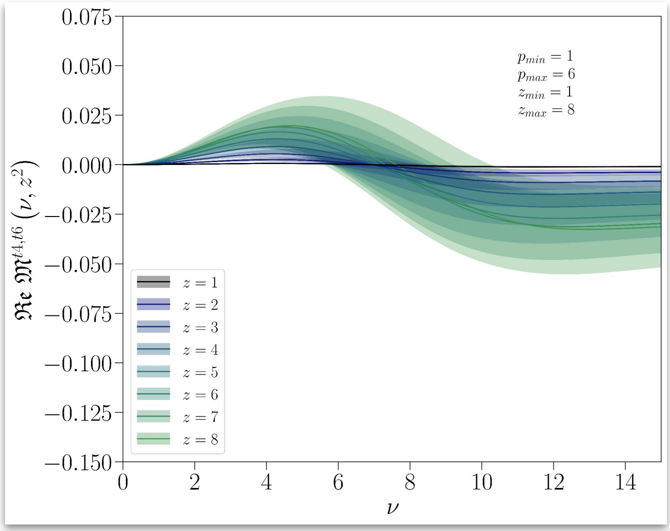




CE, R. Edwards, C. Kallidonis et al., JHEP 11 (2021) 148

Unpolarized Valence Quark PDF + Observations





$$\mathfrak{Re}\ \mathfrak{M}^{t4,t6}\left(\nu,z^2\right) \equiv z^2\Lambda^2_{\rm QCD}\sum_{n=1}^{N_{t4}} C^{t4\ (\alpha,\beta)}_{{\rm v},n}\sigma^{(\alpha,\beta)}_{0,n}\left(\nu\right) + z^4\Lambda^4_{\rm QCD}\sum_{n=1}^{N_{t6}} C^{t6\ (\alpha,\beta)}_{{\rm v},n}\sigma^{(\alpha,\beta)}_{0,n}\left(\nu\right)$$

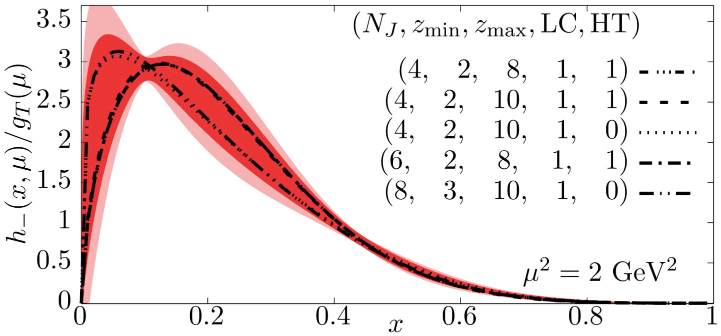
Power corrections in reduced pseudo-ITD (ratio) are numerically small within range of data

CE, R. Edwards, C. Kallidonis et al., JHEP 11 (2021) 148

Model Averaging & Data Cuts

Despite parameterization via Jacobi polynomials...

➢ bias introduced with any choice of truncation/cuts on data/inclusion of systematic effect



Akaike Information Criterion (AIC)

H. Akaike, IEEE Transactions on Automatic Control, vol.19, no.6, 716-723 (1974)

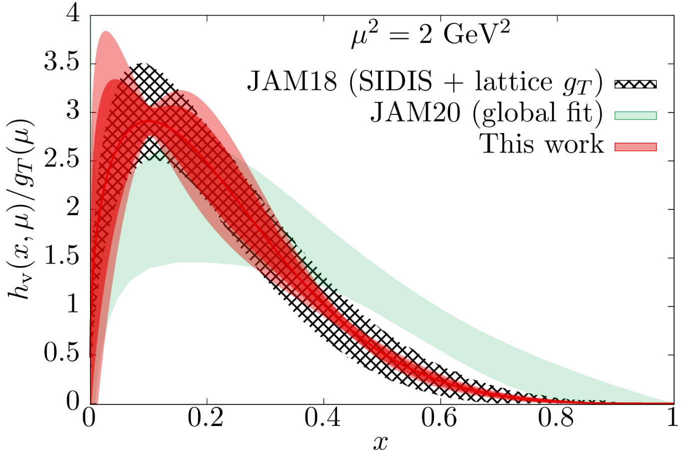
➢ weights assigned based on quality of fit, number of datapoints and parameters

$$w^{(m)} = \frac{e^{-\frac{1}{2}\mathrm{AIC}(m)}}{\sum_{n\in \mathrm{fit}} e^{-\frac{1}{2}\mathrm{AIC}(n)}}$$

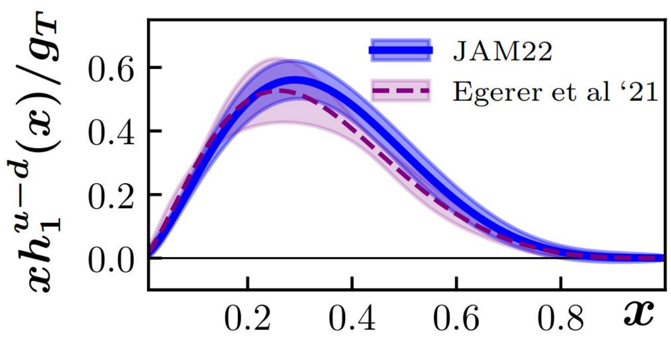
➢ ideally, averages away model biases for large number of models

$$\mathrm{AIC}(n) = \mathcal{L}_n + 2p_n + \frac{2p_n\,(p_n+1)}{(d_n - p_n - 1)}$$

$$h_\pm^{\mathrm{AIC}}(x) = \sum_{m\in \mathrm{fit}} w^{(m)} h_\pm^{(m)}(x)$$



[HadStruc] **CE**, C. Kallidonis, J. Karpie, N. Karthik et al., Phys. Rev. D 105 (2022) 3, 034507



L. Gamberg et al., Phys.Rev.D 106 (2022) 3, 034014

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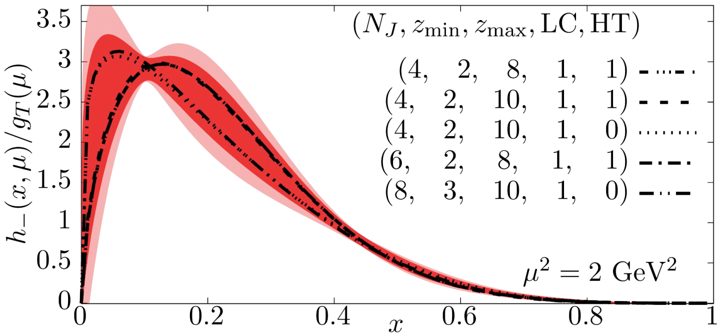

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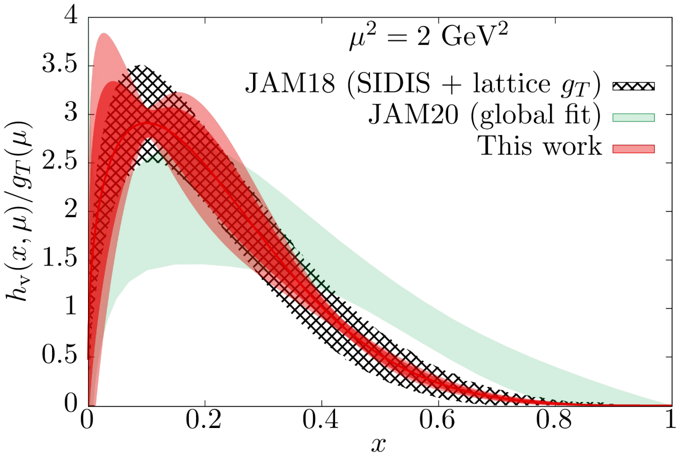
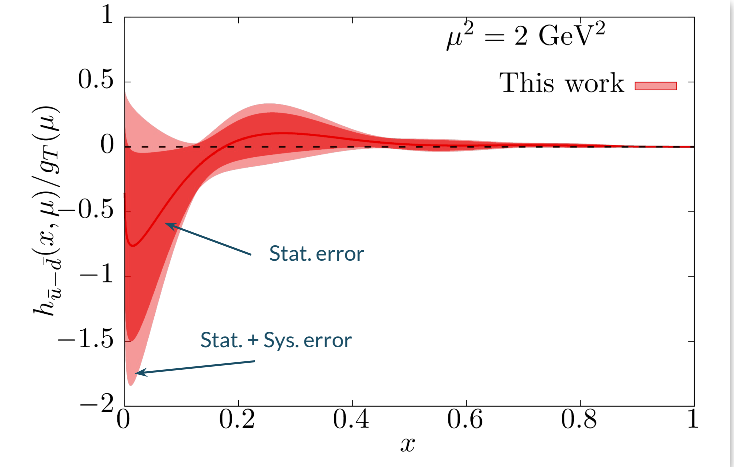
$$h_\pm^{\mathrm{AIC}}(x) = \sum_{m\in\mathrm{fit}} w^{(m)} h_\pm^{(m)}(x)$$

- Non-singlet antiquark distribution found to be consistent with an isospin-symmetric intrinsic sea

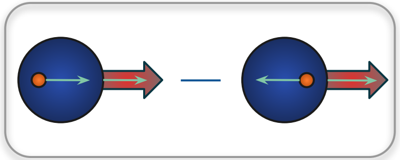
[HadStruc] **CE**, C. Kallidonis, J. Karpie, N. Karthik et al., Phys. Rev. D 105 (2022) 3, 034507

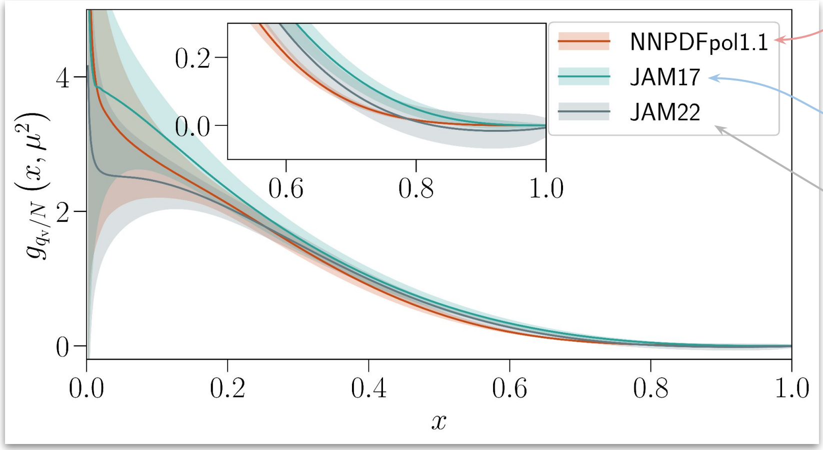




Towards the Quark Helicity PDF

Helicity asymmetry of partons within hadronic state of definite helicity





[First such global analysis]
All long. polarized DIS + jet/W-production @ STAR/PHENIX
E.R. Nocera et al., Nucl.Phys.B 887 (2014)

Simultaneous fit of PDFs/FFs in polarized (semi-)inclusive DIS
N. Sato et al., PRD 93 (2016) 7, 074005

Includes jet production @ STAR/PHENIX; PDF positivity relaxed
C. Cocuzza et al., Phys.Rev.D 106 (2022) 3, L031502

Matrix element defining helicity:

$$M^\alpha(p,z) = \langle N(p,S)|\,\overline{\psi}(z)\gamma^\alpha\gamma^5\Phi_{\hat{z}}^{(f)}(\{z,0\})\psi(0)\,|N(p,S)\rangle$$

$$= -2m_N S^\alpha \mathcal{M}(\nu,z^2) - 2im_N p^\alpha (z\cdot S)\mathcal{N}(\nu,z^2) + 2m_N^3 z^\alpha (z\cdot S)\mathcal{R}(\nu,z^2)$$

$$\xrightarrow{\text{“+” component defines helicity}} -2m_N S^+\left[\mathcal{M}(\nu,0)+i\nu\mathcal{N}(\nu,0)\right]|_{\mu^2} \equiv -2m_N S^+\mathcal{Y}(\nu,0)$$

For the kinematic setup $\boldsymbol{p^\mu = (0_\perp, p_z, E(p_z))\,,\; z^\mu = (0_\perp, z_3, 0)\;\&\;\mu = 3}$

$$M^3(p_z,z_3) = -2m_N S^3\left[p_z\hat{z}\right]\left\{\mathcal{Y}(\nu,z_3^2) + m_N^2 z_3^2\mathcal{R}(\nu,z_3^2)\right\}$$

$$\widetilde{\mathfrak{M}}(\nu,z_3^2) = \left(\frac{\widetilde{\mathcal{Y}}(\nu,z_3^2)}{\widetilde{\mathcal{Y}}(0,z_3^2)\,|_{p_z=0}}\right)\Big/\left(\frac{\widetilde{\mathcal{Y}}(\nu,0)\,|_{z_3=0}}{\widetilde{\mathcal{Y}}(0,0)\,|_{p_z=0,z_3=0}}\right)$$

Standard
Reduced pseudo-ITD

1
- Ignore presence of contamination
- Scale-dependence modified, but fitting corrections should capture effect

2
- Demand separation of leading twist amplitude from contamination
- Overconstrained system - SVD

Amplitudes from Subduced Matrix Elements

Summed ratios expose *subduced* matrix elements

- ➢ relation to invariant amplitudes: $\langle \vec{p}, \Lambda, \mu_f | \mathcal{J}^{\Lambda_\Gamma, \mu_\Gamma} | \vec{p}, \Lambda, \mu_i \rangle = \sum_l \sum_{\lambda_f, \lambda_\Gamma, \lambda_i} S^{\Lambda, J}_{\mu_f, \lambda_f} \left[S^{\Lambda_\Gamma, J_\Gamma}_{\mu_\Gamma, \lambda_\Gamma} \right]^* \left[S^{\Lambda, J}_{\mu_i, \lambda_i} \right]^* \mathcal{K}_l \left(\lambda_f [J, \vec{p}] ; \lambda_i [J, \vec{p}] \right) \mathcal{A}_l \left(\nu, z^2 \right)$

Recall operator-state overlaps:

$$\left\langle \vec{p}; J'^{P'}, \lambda' \right| \Big[\underbrace{\mathbb{O}^{J^P, \lambda} (\vec{p})}_{\text{Continuum helicity ops.}} \Big]^\dagger \left| 0 \right\rangle = Z^{[J', J, P', P, \lambda]} \delta_{\lambda' \lambda} \xrightarrow[\text{complete sets of states modified}]{\text{subduction}} \langle n' | \mathring{\mathcal{O}}^{[\Gamma]} (z_3, \tau) | n \rangle$$

projected onto cont. helicity states that subduce into Λ

Algorithm - recast canonical matrix elements in terms of subduced matrix elements

1 Canonical spinor (continuum)

Construct relativistic spin-½ spinor, with all momentum along quantization direction

- (generally unallowed lattice rotation)

2 Helicity spinor (continuum)

Rotations of four-component relativistic spin-½ states, composed of rotations on upper/lower spinors

$$\mathfrak{D}[R] u (|\vec{p}| \hat{z}, m) = \begin{pmatrix} U[R(\alpha, \beta, \gamma)] & \mathbf{0} \\ \mathbf{0} & U[R(\alpha, \beta, \gamma)] \end{pmatrix} u (|\vec{p}| \hat{z}, m)$$

Unitary transformation enforcing rotation of two-component spinors

S. U. Chung, CERN-71-08 (1971), CERN Yellow Reports

3 Subduced spinor

Enforce irrep symmetry of double-cover octahedral group and its subgroups ("little groups")

$$u (\vec{p}; \Lambda, \mu) = \sum_\lambda S^{\Lambda, J}_{\mu, \lambda} \mathfrak{D}[R] u (|\vec{p}| \hat{z}, m)$$

Exploiting Broken Rotational Symmetry

For the kinematic setup $p^\mu = (\mathbf{0}_\perp, p_z, E(p_z)),\ z^\mu = (\mathbf{0}_\perp, z_3, 0)\ \&\ \mu = 3$

$\overline{u}(p,s)\gamma^3\gamma^5 u(p,s)$

$$M^3(p_z, z_3) \propto -2m_N \langle\langle\gamma^3\gamma^5\rangle\rangle \mathcal{Y}(\nu, z_3^2) - 2m_N^3 z_3^2 \langle\langle\gamma^3\gamma^5\rangle\rangle \mathcal{R}(\nu, z_3^2)$$

Matching canonical and subduced matrix elements

- ➢ subduced spinors to compute Lorentz covariant structures for each interpolator "row":

Polarization vector

$$\underbrace{\begin{pmatrix} \langle p_z\hat{z}, \Lambda, \mu_f = 1 | \mathring{\mathcal{O}}^{[\gamma_3\gamma_5]}(z_3) | p_z\hat{z}, \Lambda, \mu_i = 1\rangle \\ \langle p_z\hat{z}, \Lambda, \mu_f = 1 | \mathring{\mathcal{O}}^{[\gamma_3\gamma_5]}(z_3) | p_z\hat{z}, \Lambda, \mu_i = 2\rangle \\ \langle p_z\hat{z}, \Lambda, \mu_f = 2 | \mathring{\mathcal{O}}^{[\gamma_3\gamma_5]}(z_3) | p_z\hat{z}, \Lambda, \mu_i = 1\rangle \\ \langle p_z\hat{z}, \Lambda, \mu_f = 2 | \mathring{\mathcal{O}}^{[\gamma_3\gamma_5]}(z_3) | p_z\hat{z}, \Lambda, \mu_i = 2\rangle \end{pmatrix}}_{\text{(Subduced) bare matrix elements from summed ratio fits}} = -2m_N \begin{pmatrix} S_3[p_z\hat{z}]_{11} & m_N^2 z_3^2 S_3[p_z\hat{z}]_{11} \\ S_3[p_z\hat{z}]_{12} & m_N^2 z_3^2 S_3[p_z\hat{z}]_{12} \\ S_3[p_z\hat{z}]_{21} & m_N^2 z_3^2 S_3[p_z\hat{z}]_{21} \\ S_3[p_z\hat{z}]_{22} & m_N^2 z_3^2 S_3[p_z\hat{z}]_{22} \end{pmatrix} \begin{pmatrix} \mathcal{Y}(\nu, z^2) \\ \mathcal{R}(\nu, z^2) \end{pmatrix}$$

μ_f, μ_i

$$\mathfrak{Y}(\nu, z_3^2) = \left(\frac{\mathcal{Y}(\nu, z_3^2)}{\mathcal{Y}(0, z_3^2)\,|_{p_z=0}} \right) \Big/ \left(\frac{\mathcal{Y}(\nu, 0)\,|_{z_3=0}}{\mathcal{Y}(0,0)\,|_{p_z=0, z_3=0}} \right)$$

$$= \frac{\mathcal{Y}(\nu, z_3^2)\,\mathcal{Y}(0,0)}{\mathcal{Y}(0, z_3^2)\,\mathcal{Y}(\nu, 0)}$$

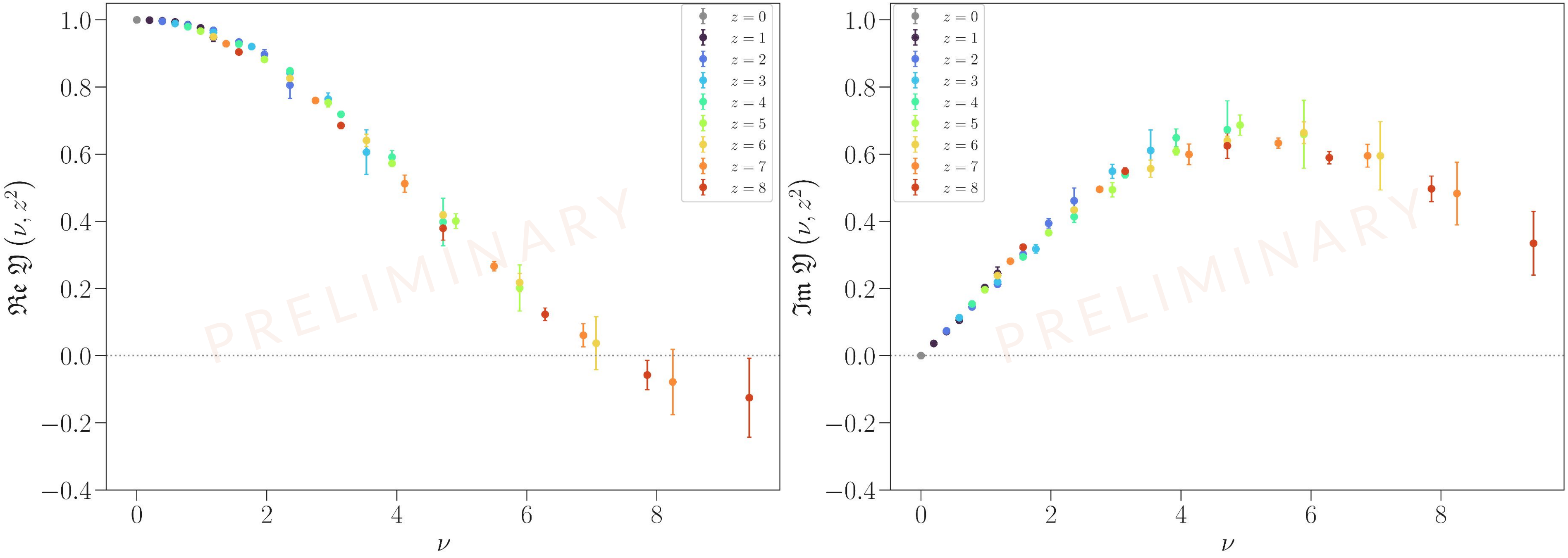
Separated amplitudes have same UV factor as $M^3(p_z, z_3)$

- ➢ state independence

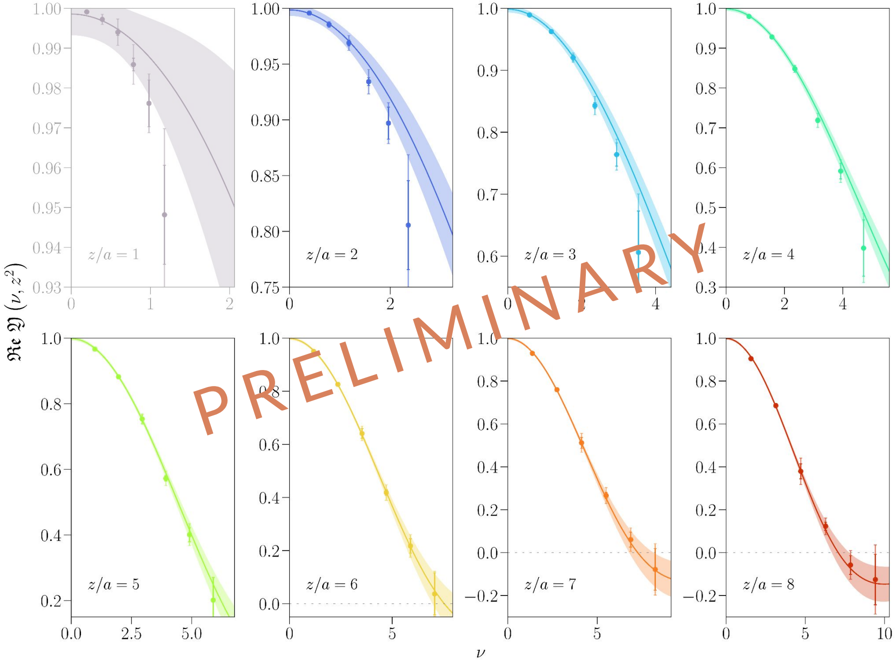
Reduced pseudo-ITD

- ★ by construction, no link-related UV divergences & RG invariant

Leading Helicity Pseudo-ITD from SVD



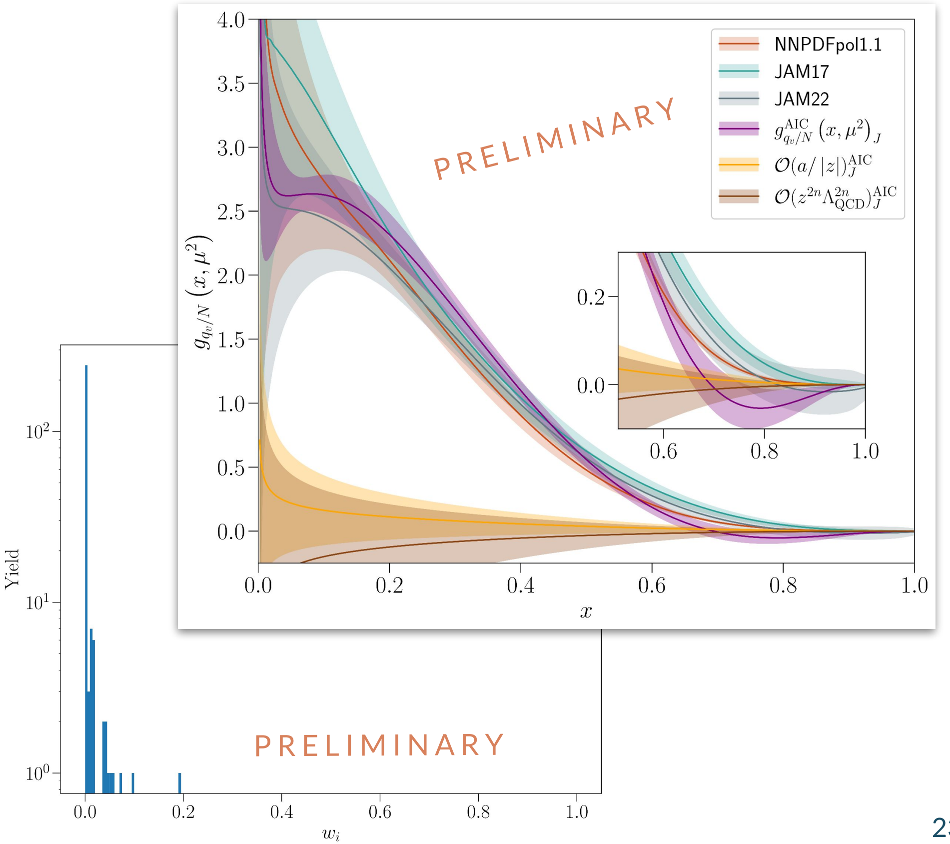
Select Fit for Isovector Valence Quark Helicity PDF



$$p_{\rm latt} \in [1, 6] \qquad z/a \in [2, 8]$$

$$[N_J, N_{a/|z|}, N_{\rm ht}, \cdots] = [3, 1, 0, \cdots]$$

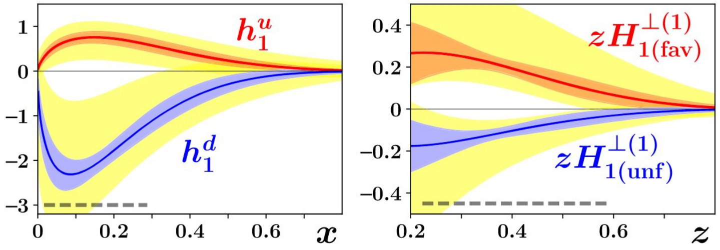
- ➢ discretization effect prominent in AIC average; net higher-twist effects consistent with zero

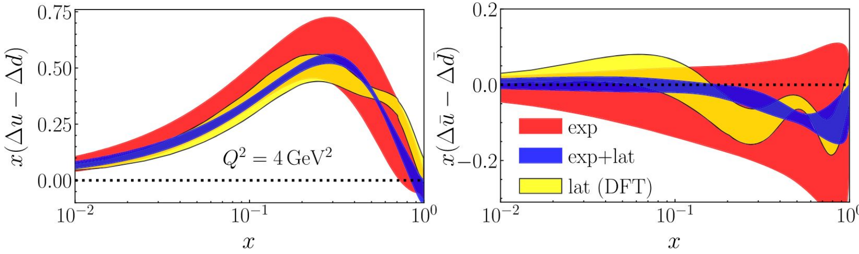


Towards a Synergy between Lattice QCD and Global Analyses

Complementarity with Global Analyses

Constraints provided by lattice QCD (LQCD):





J. Bringewatt, N. Sato, W. Melnitchouk et al., Phys. Rev. D 103 (2021) 1, 016003

H.-W. Lin, W. Melnitchouk, A. Prokudin et al., Phys.Rev.Lett.120 152502 (2018)

Pion PDFs extracted in a MC global QCD analysis

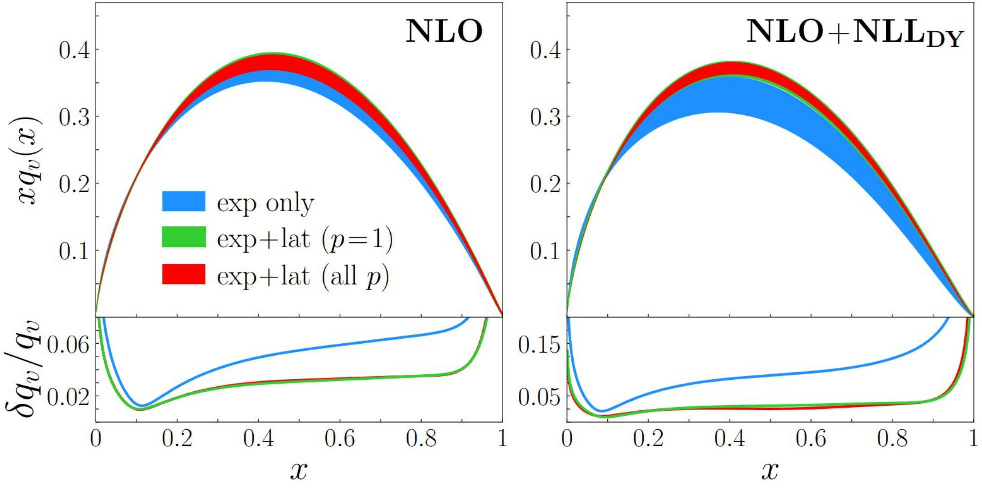
- experimental data
- reduced Ioffe-time pseudo-distributions & current-current (CC) matrix elements
  - CC systematics limit impact

Each dataset can inform errors of counterpart

- PDF uncertainties guided by matrix elements
- systematics inherent to LQCD calculation guided by experiment

Pseudo-distributions dramatically affect PDF

$\sim (1-x)^{\beta_{\mathrm{eff}}}$ $\qquad \beta_{\mathrm{eff}} \simeq 1.0 - 1.2$



[JAM/HadStruc] P.C. Barry et al., Phys.Rev.D 105 (2022) 11, 114051

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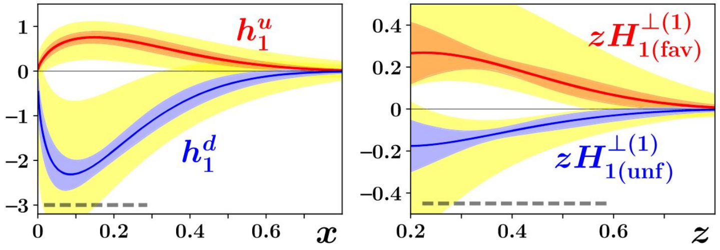

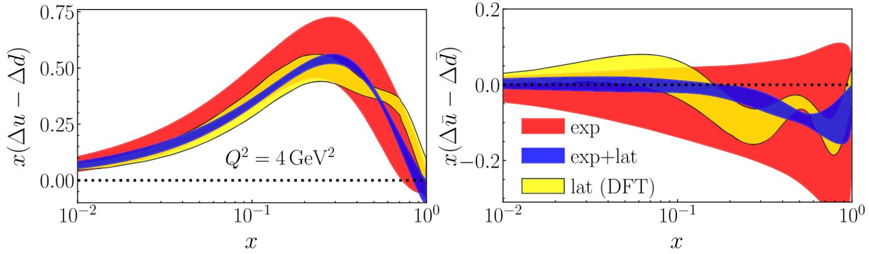

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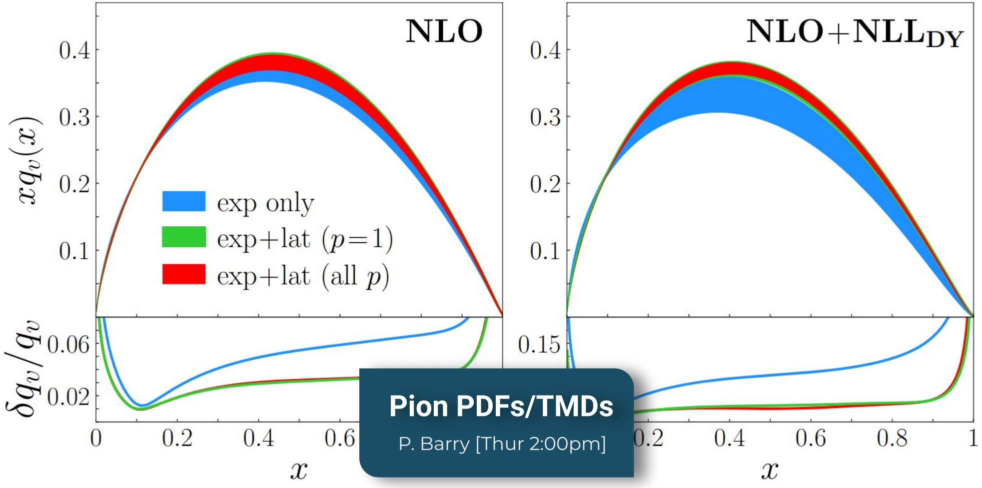


Pion PDFs/TMDs

P. Barry [Thur 2:00pm]

[JAM/HadStruc] P.C. Barry et al., Phys.Rev.D 105 (2022) 11, 114051

Phenomenological Insight

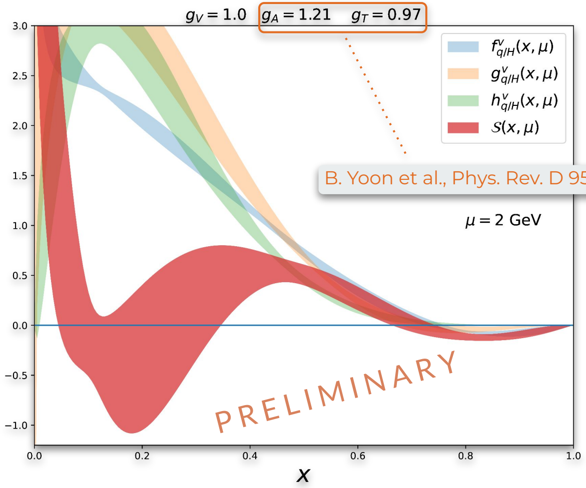
Transversity is only chiral-odd twist-2 collinear PDF

➢ chiral-odd process needed [eg. transverse SSAs in SIDIS]

(Often used) Theory constraint: Soffer bound J. Soffer, Phys. Rev. Lett. 74, 1292 (1995)

$$\left|h_{q/h}\left(x,\mu^2\right)\right| \leq \frac{1}{2}\left[f_{q/h}\left(x,\mu^2\right)+g_{q/h}\left(x,\mu^2\right)\right]$$

[E.g] Z.-B. Kang et al., Phys. Rev. D 93, 014009 (2016); M. Radici et al., JHEP 05, 123 (2015)
U. D'Alesio, C. Flore and A. Prokudin, Phys. Lett. B 803, 135347 (2020)
M. Anselmino et al., Phys. Rev. D 87, 094019 (2013)



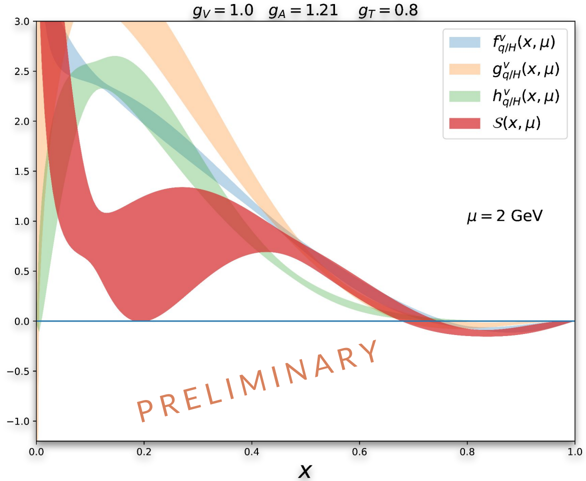
B. Yoon et al., Phys. Rev. D 95 (2017) 7, 074508

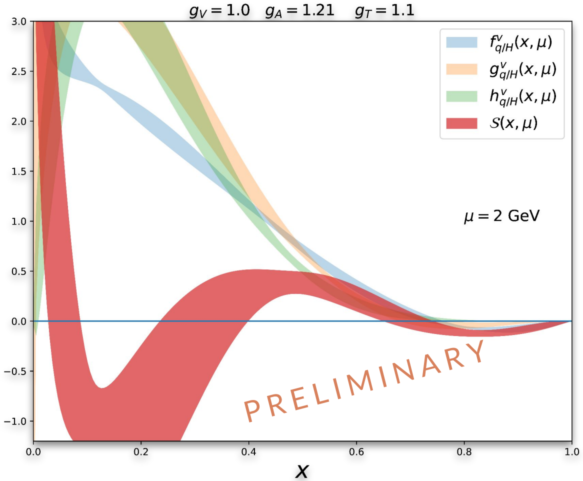
Do PDFs computed from LQCD support or challenge this bound?

"Soffer-PDF"

$$\boxed{\mathcal{S}_{q/h}\left(x,\mu^2\right)} \equiv f_{q/h}\left(x,\mu^2\right)+\boxed{g_{q/h}\left(x,\mu^2\right)}-2\boxed{h_{q/h}\left(x,\mu^2\right)}$$

Must rescale with renormalized charges for correct normalization

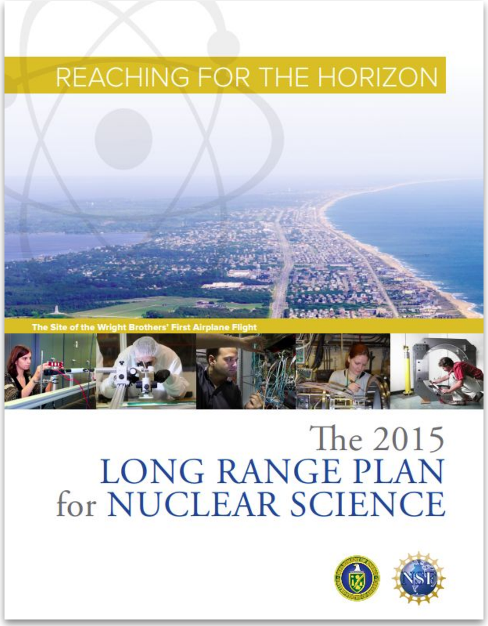




- ➢ shape of "Soffer-PDF" driven by g_T^{u-d}
- ➢ caveat - model bias (no AIC here)
- ➢ (Soffer bound as a prior): PDFs have potential to constrain, or provide precise upper bound on g_T^{u-d}

Towards a Multidimensional Image of Hadrons

Nuclear Science Long-Range Plans



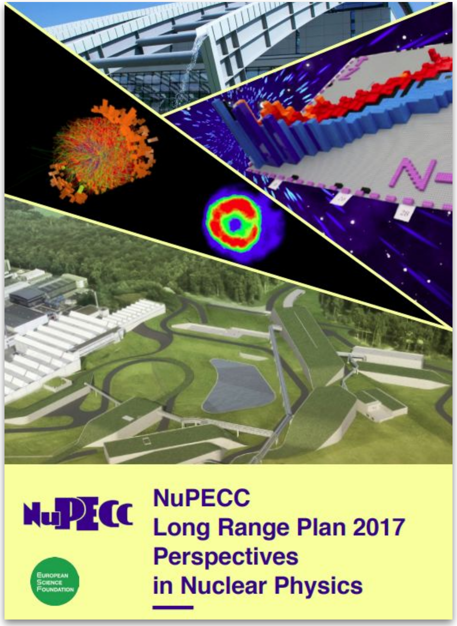
DOE Office of Science & NSF Directorate of Mathematical and Physical Sciences charge to Nuclear Science Advisory Committee (NSAC)

1. "How does subatomic matter organize itself and what phenomena emerge?"
2. "...What are the static and dynamical properties of hadrons?"

"To meet challenges and realize full scientific potential of current/future experiments, we require new investments in theoretical/computational nuclear physics."

- NSAC 2015 recommended new investments in computational nuclear theory that leverage the U.S. leadership in *high-performance computing*, to supplement experimental programs

"... a multidimensional description of nucleon structure is emerging that is providing profound new insights"

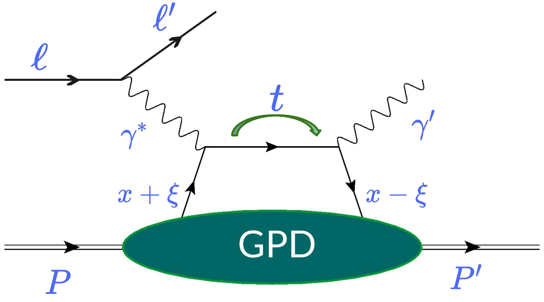


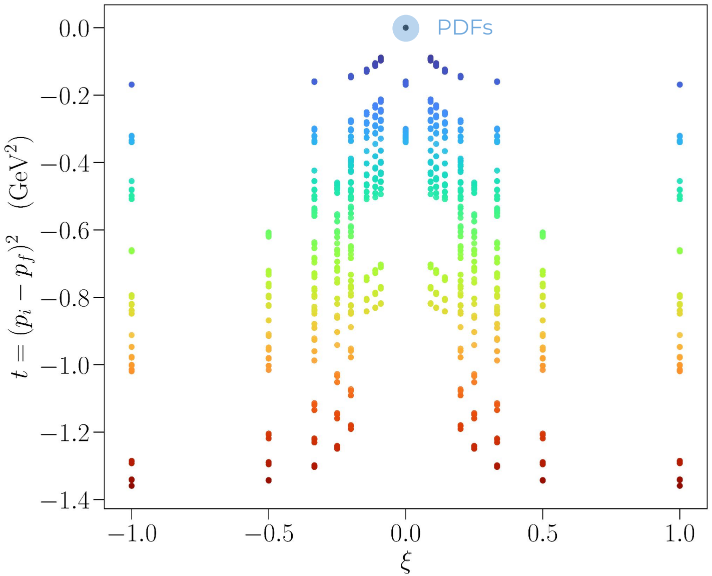
Off-Forward Matrix Elements

Relevant in variety of exclusive channels

- DVCS/DVMP: (e.g. E12-06-113 [HRS] & E12-11-003 [CLAS12])

Eur.Phys.J.A 52 (2016) 6, 157; Eur.Phys.J.A 52 (2016) 6, 151
Eur.Phys.J.A 52 (2016) 6, 158





Matching for pseudo-distributions in off-forward regime

A. Radyushkin, Phys. Rev. D100, 116011 (2019)
A. Radyushkin, arXiv: Int.J.Mod.Phys.A 35 (2020) 05, 2030002

- Expansive coverage with single inversion overhead

$$\mathbb{M}^{\mu}(p_f, p_i, z) \equiv \langle N(p_f)|\,\overline{\psi}(-z/2)\,\frac{\tau^3}{2}\gamma^{\mu} W(-z/2, z/2; A)\,\psi(z/2)\,|N(p_i)\rangle$$

$$= \langle\langle\gamma^{\mu}\rangle\rangle M(\nu_f, \nu_i, t; z^2) + \langle\langle\mathbb{1}\rangle\rangle z^{\mu} N(\nu_f, \nu_i, t; z^2) - \frac{i}{2m_N}\langle\langle\sigma^{\mu\nu}\rangle\rangle (p_i - p_f)_{\nu} L(\nu_f, \nu_i, t; z^2)$$

Invoke subduced spinors and SVD...

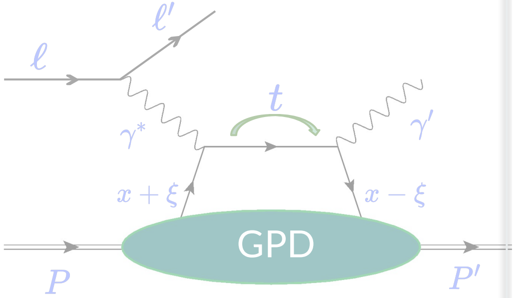
Off-Forward Matrix Elements

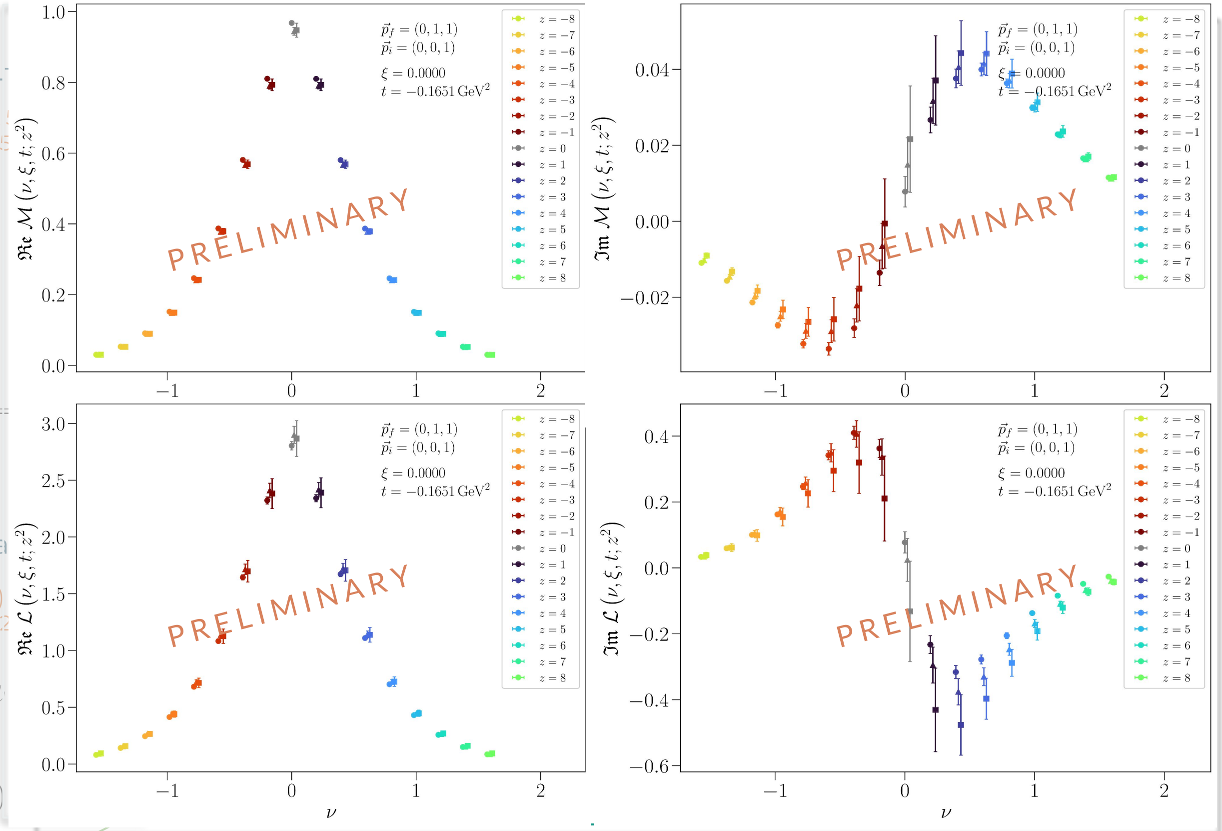
Relevant in variety of exclusive channels

➢ DVCS/DVMP: (e.g. E12-06-113 [HRS] & E12-

Eur.Phys.J.A 52 (2016) 6, 157; Eur.Phys.J.A

Eur.Phys.J.A 5





Matching for pseudo-distributions in off-forwa

A. Radyushkin, Phys. Rev. D100, 116011 (2019)
A. Radyushkin, arXiv: Int.J.Mod.Phys.A 35 (2020) 05, 2

$$\mathbb{M}^\mu(p_f,p_i,z) \equiv \langle N(p_f)|\overline{\psi}(-z/2)\frac{\tau^3}{2}\gamma^\mu W(-z/2,z/2;A)\psi$$

$$= \langle\langle\gamma^\mu\rangle\rangle M(\nu_f,\nu_i,t;z^2) + \langle\langle\mathbb{1}\rangle\rangle z^\mu N(\nu_f,\nu_i,t;z^2)$$

Invoke subduced spinors and SVD...

Off-Forward Matrix Elements

Relevant in variety of exclusive channels

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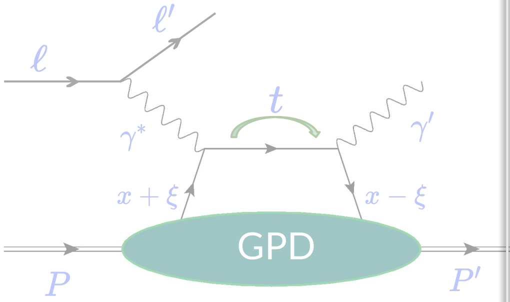
Eur.Phys.J.A 52 (2016) 6, 157; Eur.Phys.J.A

Eur.Phys.J.A



Matching for pseudo-distributions in off-forwa

A. Radyushkin, Phys. Rev. D100, 116011 (2019)

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$$\mathbb{M}^{\mu}\left(p_f,p_i,z\right)\equiv\langle N\left(p_f\right)|\overline{\psi}\left(-z/2\right)\frac{\tau^3}{2}\gamma^{\mu}W\left(-z/2,z/2;A\right)\psi$$

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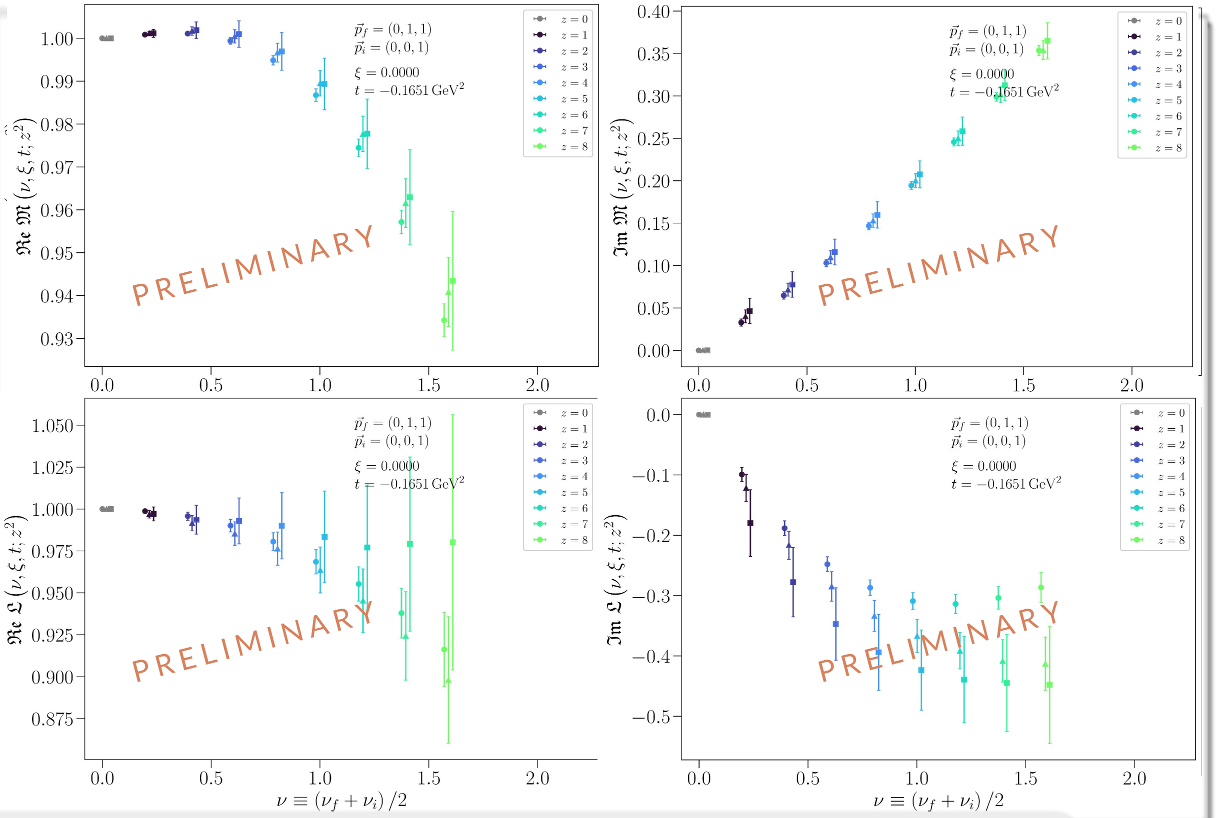


Renormalize (ratio) → match onto x-dependence of GPD $\left\{\ \widetilde{\mathfrak{M}}\left(\nu,\xi,t,z^2\right)=\mathcal{K}\left(x\nu,\xi\nu,z^2\mu^2;\alpha_s\right)\otimes H\left(x,\xi,t,\mu^2\right)\right.$

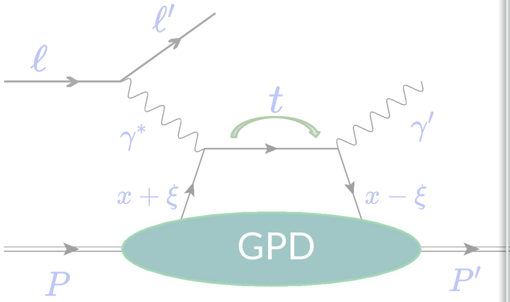
Off-Forward Matrix Elements

Relevant in variety of exclusive channels

➢ DVCS/DVMP: (e.g. E12-06-113 [HRS] & E12-

Eur.Phys.J.A 52 (2016) 6, 157; Eur.Phys.J.A

Eur.Phys.J.A

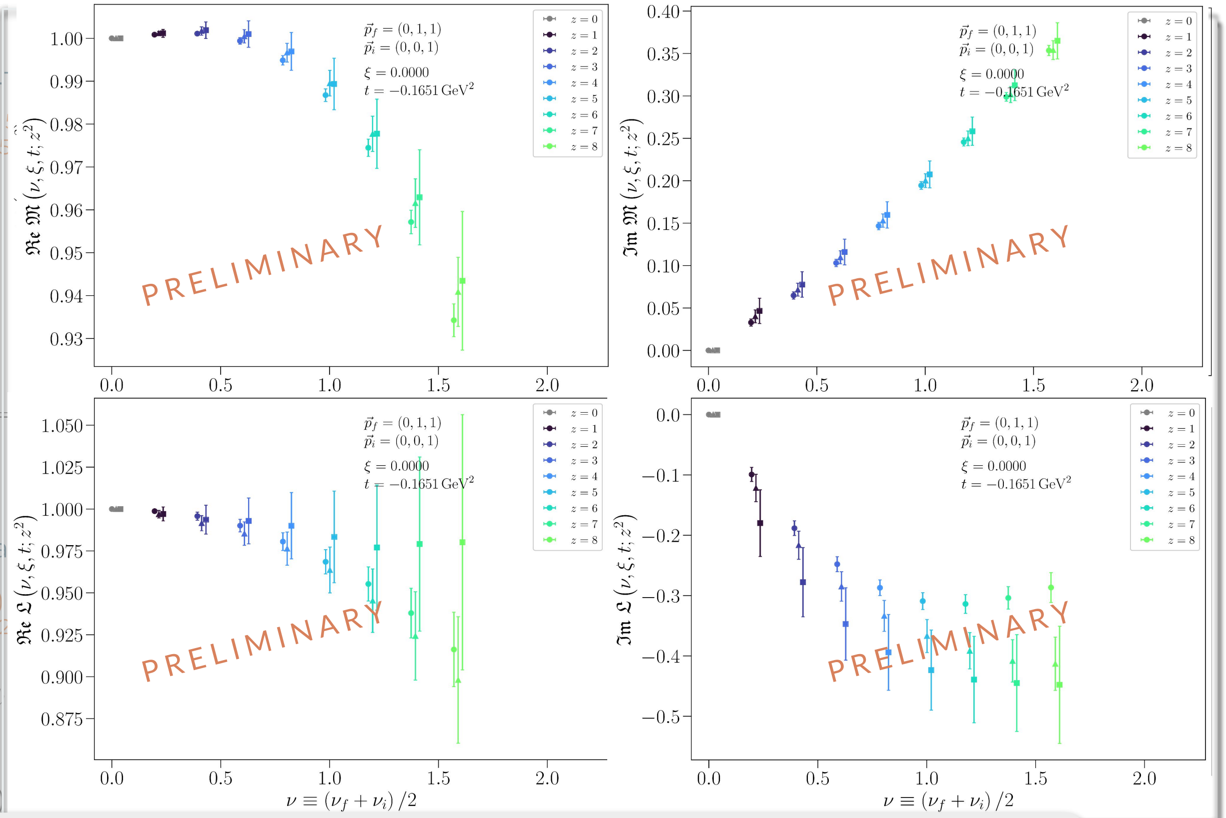


Matching for pseudo-distributions in off-forwa

$= \langle\langle\gamma^\mu\rangle\rangle M\left(\nu_f,\nu_i,t;z^2\right) + \langle\langle\mathbb{1}\rangle\rangle z^\mu N\left(\nu_f,\nu_i,t;z^2\right)$

GPDs from Lattice QCD [Friday]

J. Zhang [9:00am] K. Cichy [9:40am]
R. Young [2:00pm] S. Batthacharya [2:40pm]
J. Dodson [3:00pm]



Renormalize (ratio) → match onto x-dependence of GPD $\left\{ \widetilde{\mathfrak{M}}\left(\nu,\xi,t,z^2\right) = \mathcal{K}\left(x\nu,\xi\nu,z^2\mu^2;\alpha_s\right) \otimes H\left(x,\xi,t,\mu^2\right) \right.$

Closing Remarks

Hadronic structure accessible from certain lattice calculable matrix elements

- ➢ short-distance factorization
- ➢ considerable progress in factorizable methods

K. Cichy & M. Constantinou, Adv.High Energy Phys. (2019), 3036904
K. Cichy, PoS LATTICE2021 (2022) 017; M. Constantinou, Eur. Phys. J. A 57, 77 (2021)

Isovector twist-2 quark PDFs of Nucleon

m_π [MeV] / a[fm]	$f_{q_\pm/N}\left(x,\mu^2\right)$	$g_{q_\pm/N}\left(x,\mu^2\right)$	$h_{q_\pm/N}\left(x,\mu^2\right)$
358(3) / 0.094(1)	Published	Forthcoming	Published
278(4) / 0.094(1)	Preliminary	Preliminary	Preliminary
170(5) / 0.091(2)	Ongoing	Ongoing	Ongoing

- ➢ statistical precision afforded by use of distillation and its union with momentum smearing idea
- ➢ systematic effects can be reliably addressed

Stay tuned:

- ➢ towards a continuum extrapolation
- ➢ Distillation in the off-forward regime

HadStruc Collaboration

Robert Edwards, **CE**, Nikhil Karthik,
Jianwei Qiu, David Richards, Eloy Romero, Frank Winter [1]

Balint Joó[2]

Carl Carlson, Chris Chamness, Tanjib Khan,
Christopher Monahan, Kostas Orginos, Raza Sufian[3]

Wayne Morris, Anatoly Radyushkin [4]

Joe Karpie [5]

Savvas Zafeiropoulos [6]

Yan-Qing Ma [7]

Jefferson Lab [1], Oak Ridge [2], William and Mary [3], Old Dominion University [4],
Columbia University [5], Aix Marseille University [6], Peking University [7]

Euclidean Methods Towards Hadron Structure

Many types of Euclidean correlations can be used to understand hadronic structure!

- ➢ Hadronic tensor

  K.-F. Liu PRL. 72 (1994) 1790-1793
  J. Liang et al., PRD 101 (2020) 11, 114503

- ➢ "OPE without OPE"

  K.U. Can et al., PRD 102 (2020) 114505
  A.J. Chambers et al., PRL 118 (2017) 24, 242001

- ➢ Auxiliary quark methods (Moments & DAs)

  HOPE Collab., PRD 105 (2022) 3, 034506
  W. Detmold and C.J.D. Lin, PRD 73 (2006) 014501
  G. Bali et al., Eur.Phys.J.C 78 (2018) 3, 217
  G. Bali et al., PRD 98 (2018) 9, 094507

- ➢ Current-current correlators

  R.S. Sufian, J. Karpie, **CE** et al., PRD 99 (2019) 7, 074507
  R.S. Sufian, **CE**, J. Karpie et al., PRD 102 (2020) 5, 054508

On Subductions

Canonical subductions

- ➢ spinors/derivatives combined into object of definite spin/parity

$$\mathcal{O}^{\{J\}}_{{}^n\Lambda,r} = \sum_m S^{J,m}_{{}^n\Lambda,r}\ \mathcal{O}^{\{J,m\}}$$

R. Edwards, et. al., Phys. Rev. D84, 074508 (2011)
J. Dudek and R. Edwards, Phys. Rev. D85, 054016 (2012)

Helicity subductions

C. Thomas, et al., Phys. Rev. D85, 014507 (2012)
C. Thomas, private communication

- ➢ boost breaks (double-cover) octahedral symmetry to little groups

$$\left[\mathbb{O}^{J^P,\lambda}(\vec{p})\right]^\dagger = \sum_m \mathcal{D}^{(J)}_{m,\lambda}(R)\left[O^{J^P,m}(\vec{p})\right]^\dagger$$

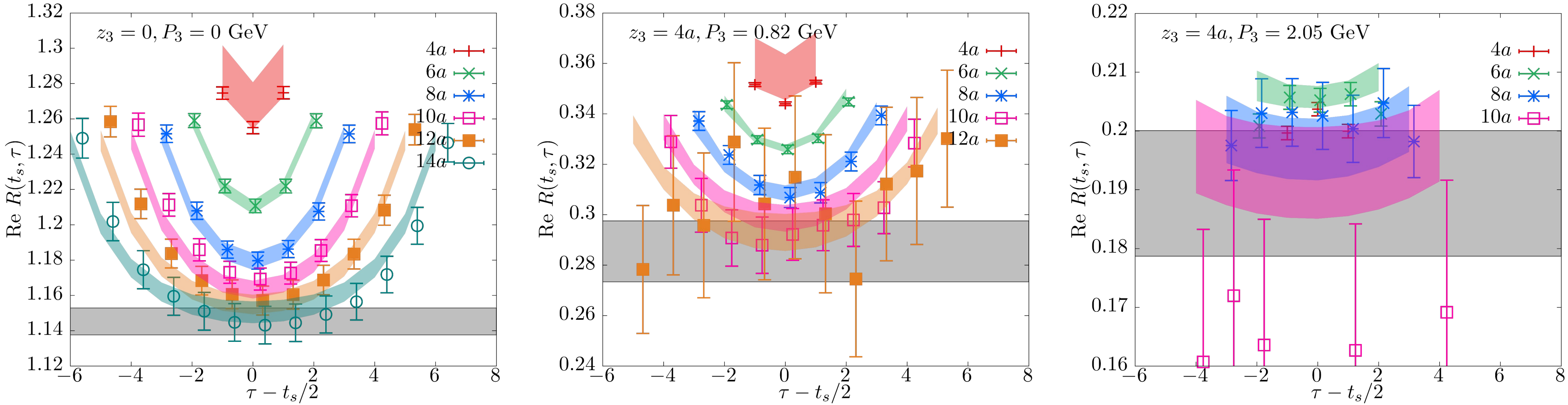
- ➢ subduce into little groups

$$\left[\mathbb{O}^{J^P,|\lambda|}_{\Lambda,\mu}(\vec{p})\right]^\dagger = \sum_{\hat{\lambda}=\pm|\lambda|} S^{\tilde{\eta},\hat{\lambda}}_{\Lambda,\mu}\left[\mathbb{O}^{J^P,\hat{\lambda}}(\vec{p})\right]^\dagger$$

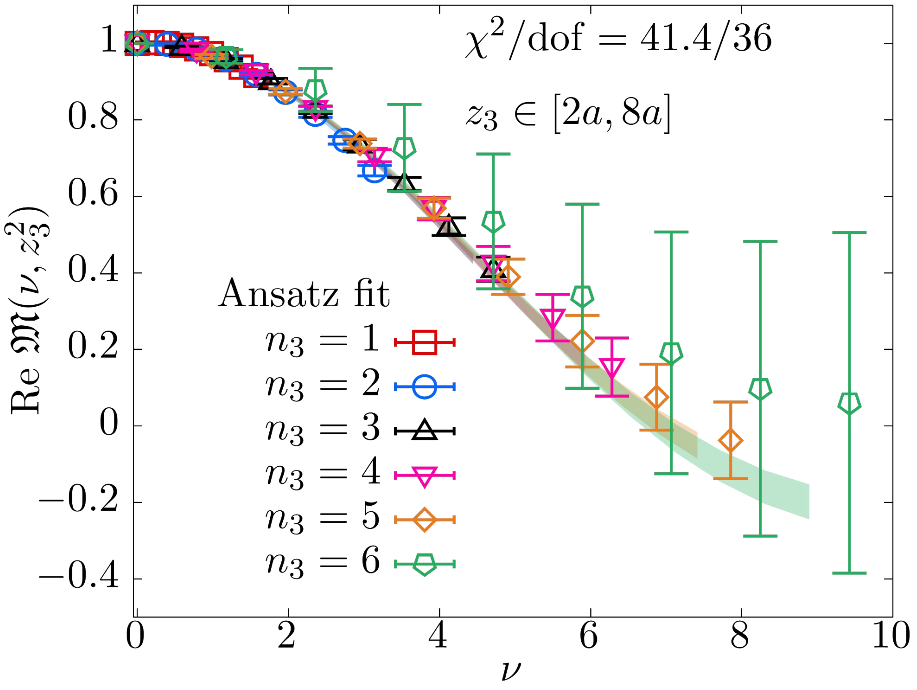
Dirac structure/covariant derivatives

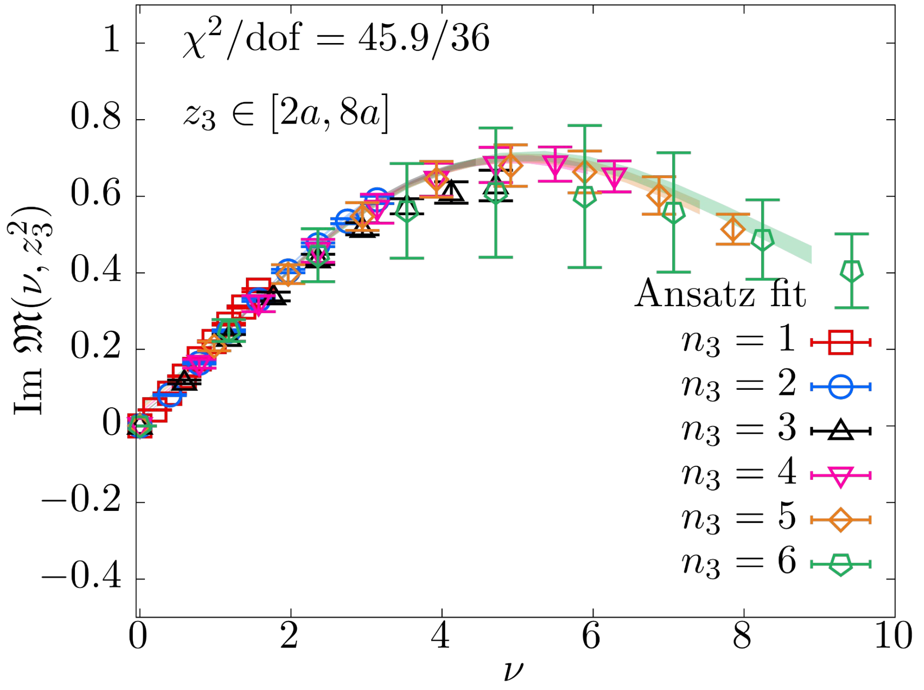
$$\mathcal{O}_i(t) = \epsilon^{abc}\left(\boxed{\mathcal{D}_1}\square u\right)^\alpha_a \left(\boxed{\mathcal{D}_2}\square d\right)^\beta_b \left(\boxed{\mathcal{D}_3}\square u\right)^\gamma_c (t)\, S^{\alpha\beta\gamma}_i$$

Select Transversity Matrix Element Extractions



Transversity Reduced Pseudo-ITD





Pheno.-type parameterization

- ➢ Twist-2 OPE - Taylor series in Ioffe-time
- ➢ plus leading discretization/higher-twist

$$g_T^{-1} h_\pm(x) = N_\pm x^{\alpha_\pm} (1-x)^{\beta_\pm} \left(1 + \gamma_\pm \sqrt{x} + \delta_\pm x\right)$$

Quark Helicity Matrix Element

- Light-cone kinematics:

$$M^{+}\left(p,z^{-}\right)_{\mathrm{Reg}_{\mu^2}} = -2m_N S^{+}\left[\mathcal{M}\left(p^{+}z^{-},0\right) + ip^{+}z^{-}\mathcal{N}\left(p^{+}z^{-},0\right)\right]_{\mathrm{Reg}_{\mu^2}}$$

$$= -2m_N S^{+}[\mathcal{M}\left(\nu,0\right) - i\nu\mathcal{N}\left(\nu,0\right)]_{\mathrm{Reg}_{\mu^2}}$$

$$\equiv -2m_N S^{+}\mathcal{I}\left(\nu,\mu^2\right)$$

- Space-like separation with $\gamma^4\gamma^5$:

$$M^{4}\left(p,z_3\right) = -2m_N\left[S^{4}\mathcal{M}\left(\nu,z_3^2\right) - iS^{3}E(p_z)z_3\mathcal{N}\left(\nu,z_3^2\right)\right]$$

$$Z_{\text{link}}\left(z,a\right) \simeq e^{-A|z|/a}$$

Matching Kernels to Quark PDFs

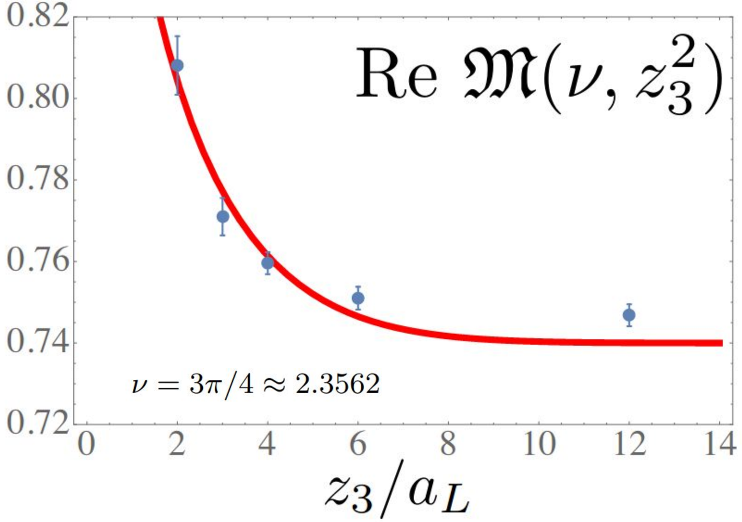
$$\mathfrak{M}\left(\nu, z^2\right) = \left\{\delta\left(1-u\right) - \frac{\alpha_s C_F}{2\pi}\int_0^1 du\left[\ln\left(\frac{e^{2\gamma_E+1}z^2\mu^2}{4}\right)B\left(u\right) + L\left(u\right)\right]\right\}\mathcal{Q}\left(u\nu, \mu^2\right) + \mathcal{O}\left(z^2\Lambda^2_{\mathrm{QCD}}\right)$$

Unpolarized: $B\left(u\right) = \left[\frac{1+u^2}{1-u}\right]_+$ $\quad L\left(u\right) = \left[4\frac{\ln(1-u)}{1-u} - 2\left(1-u\right)\right]_+$

Helicity: $B\left(u\right) = \left[\frac{1+u^2}{1-u}\right]_+$ $\quad L\left(u\right) = \left[4\frac{\ln(1-u)}{1-u} - 4\left(1-u\right)\right]_+$

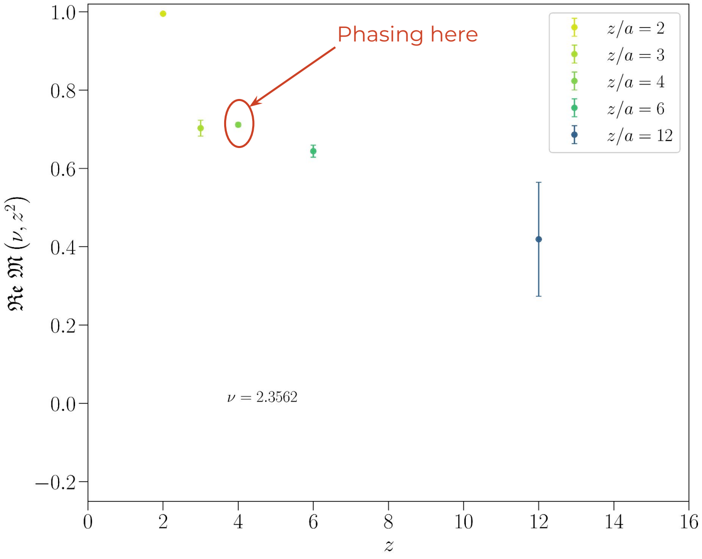
Transversity: $B\left(u\right) = \left[\frac{2u}{1-u}\right]_+$ $\quad L\left(u\right) = 4\left[\frac{\ln(1-u)}{1-u}\right]_+$

Observed Scale Dependence in Pseudo-ITDs



Log-z^2dependence observed in first (quenched) study of pseudo-distributions

K. Orginos et al., Phys. Rev. D 96, 094503 (2017)



Dynamical study of pseudo-distributions with distillation

Higher-Twist Effects in Ratio

$$\mathfrak{M}\left(\nu, z^2\right) = \frac{\mathcal{M}\left(\nu, z^2\right)}{\mathcal{M}\left(0, z^2\right)} = \frac{\int_{-1}^{1} \mathrm{d}x \; e^{i\nu x} \mathcal{P}\left(x, z^2\right)}{\int_{-1}^{1} \mathrm{d}x \; \mathcal{P}\left(x, z^2\right)}$$

$$\int_{-1}^{1} \mathrm{d}x \; \frac{\mathcal{P}\left(x, z^2\right)}{\mathcal{M}\left(0, z^2\right)} = 1$$

In some sense, avg. ht-effects over all x cancel some
@ specific value of Ioffe-time