Collisional Flavor Instability in Dense Neutrino Gases

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Flavor Density Matrix

Mean-field approximation

$$\rho = \begin{bmatrix} f_{\nu_e} & S \\ S^* & f_{\nu_x} \end{bmatrix}$$

f_{ν_α} : occupation

S : coherence

See the talks by Patwardhan (7/18), Balentekin (8/3) and Carlson (8/4) for the many-body aspects of neutrino oscillations.

Flavor Transport

Equation of motion

$$(\partial_t + \hat{\mathbf{v}} \cdot \nabla)\rho = -\mathrm{i}[\mathrm{H}, \rho] + \mathscr{C} \leftarrow \text{Collision}$$

mass matrix

electron density

$$\mathrm{H} = \frac{\mathrm{M}^2}{2E} + \sqrt{2}G_\mathrm{F}\,\mathrm{diag}[n_e, 0, 0] + \mathrm{H}_{\nu\nu}$$

neutrino energy

ν-ν forward scattering
(self-coupling)

$$\mathrm{H}_{\nu\nu} = \sqrt{2}G_\mathrm{F} \int \mathrm{d}^3\mathbf{p}'(1 - \hat{\mathbf{v}} \cdot \hat{\mathbf{v}}')(\rho_{\mathbf{p}'} - \bar{\rho}_{\mathbf{p}'})$$

Flavor Instability

Homogeneous and isotropic gas

Electron flavor neutrinos and antineutrinos initially

$$\rho \propto \begin{bmatrix} 1 & S \\ S^* & 0 \end{bmatrix} \qquad \bar{\rho} \propto \begin{bmatrix} 1 & \bar{S} \\ \bar{S}^* & 0 \end{bmatrix} \qquad |S| \ll 1$$

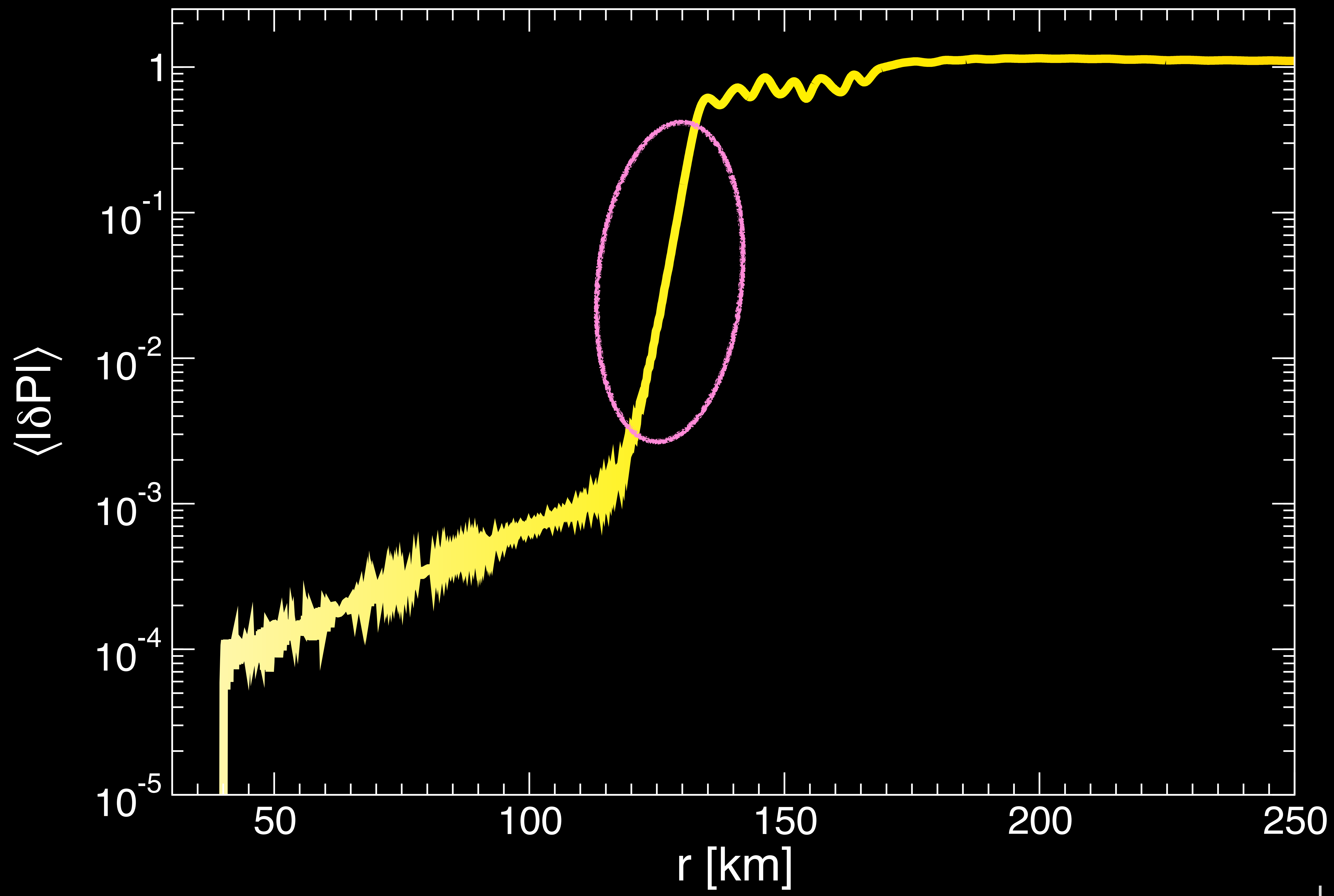
$$\mathrm{i}\begin{bmatrix} \dot{S} \\ \dot{\bar{S}} \end{bmatrix} \approx \begin{bmatrix} -\omega - \alpha\mu & \alpha\mu \\ -\mu & \omega + \mu \end{bmatrix} \begin{bmatrix} S \\ \bar{S} \end{bmatrix}$$

$$\omega = \Delta m^2/2E$$
$$\alpha = n_{\bar{\nu}}/n_{\nu}$$
$$\mu \propto n_{\nu}$$

- Normal modes —> Collective oscillations ($S, \bar{S} \sim e^{-\mathrm{i}\Omega t}$)
- $\mathrm{Im}(\Omega) > 0$ —> Flavor instabilities

Banerjee+ (2011)

Flavor Instability



HD & Friedland (2010)

Flavor Instability in Dynamic Models

$$S_{\mathbf{p}}(t, \mathbf{r}) \propto e^{-\mathrm{i}(\Omega t - \mathbf{K}\cdot\mathbf{r})}$$

- Collective flavor oscillations are the collective wave modes in the neutrino gas with the dispersion relation $\Omega(\mathbf{K})$.
- $\mathrm{Im}(\Omega) > 0$ —> Flavor instabilities.
- **Slow** oscillations occur on the distance scale of **1 km** ($\sim 10\,\mathrm{MeV}/\Delta m^2_{\mathrm{atm}}$).
- **Fast** oscillations can occur on the distance scale of **1 cm** ($\sim 1/G_{\mathrm{F}} n_\nu$), independent of the neutrino energies.

Electron Lepton Number (ELN) Crossing

$$G(E, \hat{\mathbf{v}}) = \begin{cases} f_{\nu_e} - f_{\nu_x} & \text{if } E > 0 \\ f_{\bar{\nu}_x} - f_{\bar{\nu}_e} & \text{if } E < 0 \end{cases}$$

- **Slow** flavor instability requires crossing in $G(E)$: identical neutrino angular distribution.

- **Fast** flavor instability requires crossing in $G(\hat{\mathbf{v}})$: independent of neutrino energy.

Charged-Current Neutrino Interaction

$$(\partial_t + \hat{\mathbf{v}} \cdot \nabla)\rho = -\,\mathrm{i}[\mathrm{H}, \rho] + \mathscr{C}$$

Consider the charged-current interactions such as

$\nu_e + n \rightleftharpoons e^- + p$ and $\bar{\nu}_e + p \rightleftharpoons e^+ + n$

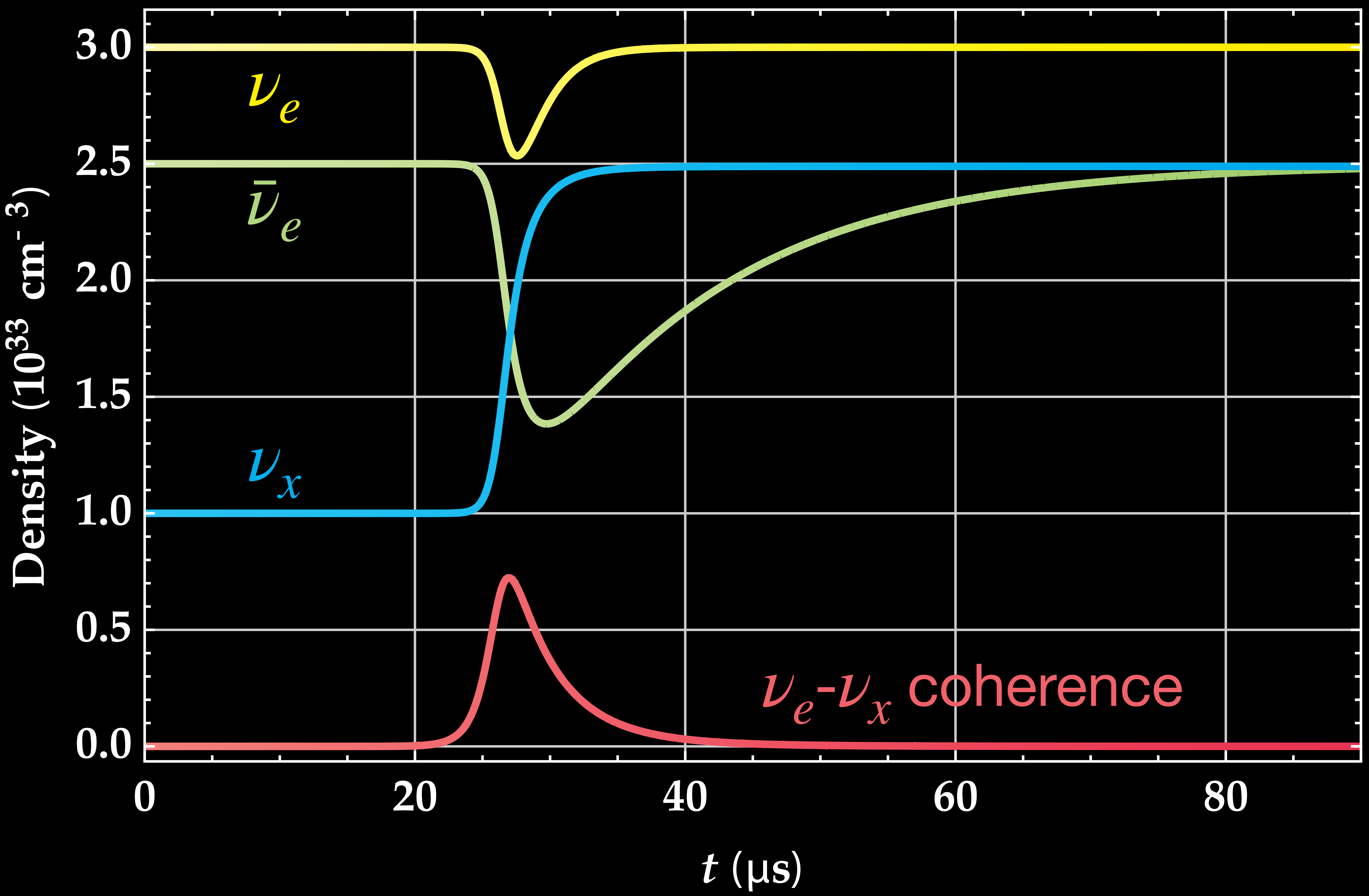
$$\mathscr{C} = \{\Gamma, \rho^{\mathrm{eq}} - \rho\}$$

"collision" rates

equilibrium values

Collisional Flavor Instability

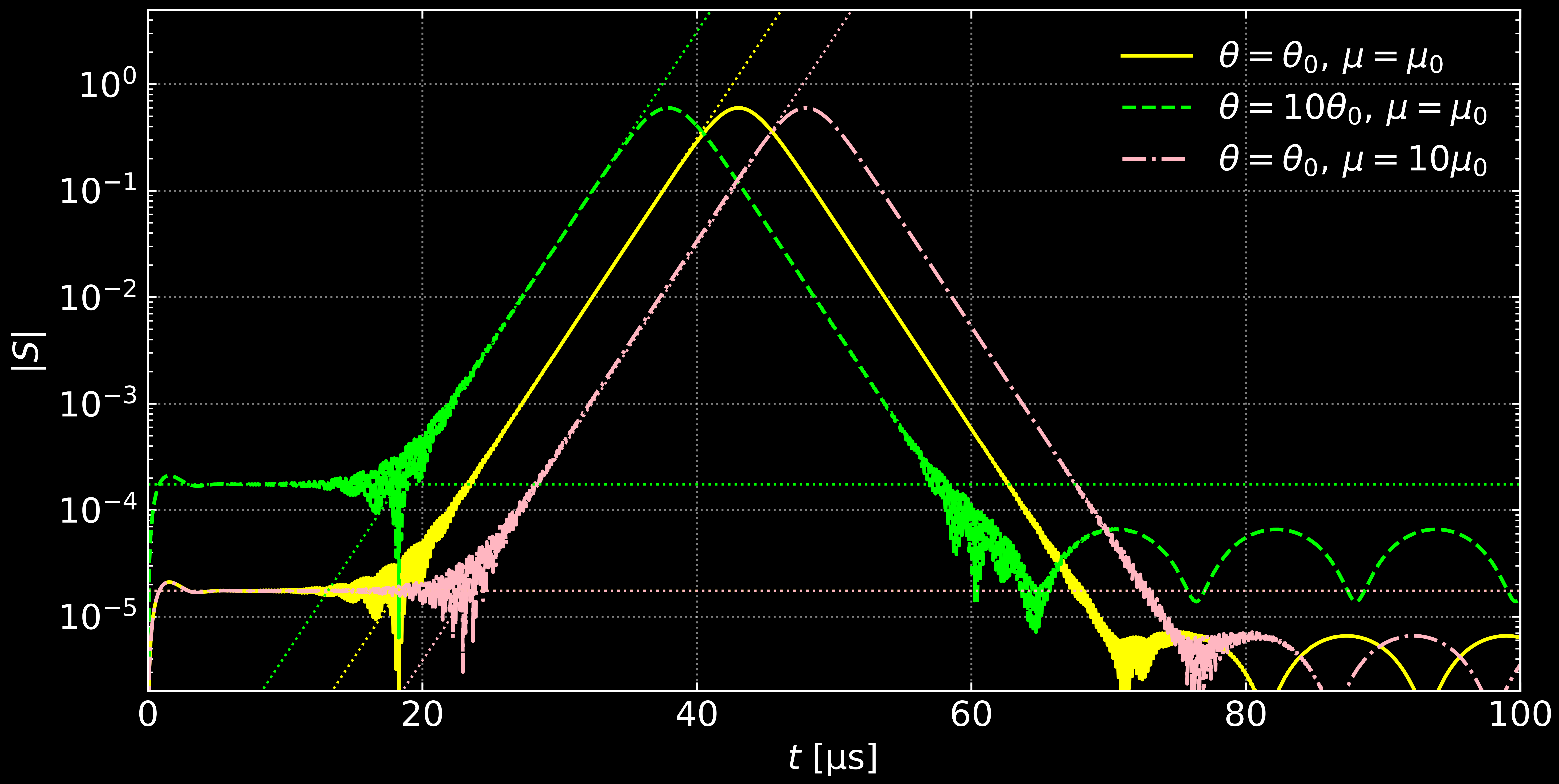
Homogeneous & isotropic gas (single energy)



Johns (2021)

Collisional Flavor Instability

Homogeneous & isotropic gas (single energy, damping only)



$\theta_0 = 10^{-5}$

$\mu_0 = 10^5\,\mathrm{km}^{-1}$

$\mu \propto n_\nu$

Lin & HD (2022)

Collisional Flavor Instability

Homogeneous & isotropic gas

$$\mathrm{i}\dot{S}_E(t) \approx -\mathrm{i}\Gamma_E S_E(t) - \mu G(E)\int_{-\infty}^{\infty} S_{E'}(t)\,\mathrm{d}E'$$

Negative $E \Longrightarrow$ antineutrino

$$S_E(t) = Q_E\, e^{-\mathrm{i}\Omega t}$$

$\mu \propto n_\nu$

$$\int_{-\infty}^{\infty} \frac{G(E)\,\mathrm{d}E}{\Omega + \mathrm{i}\Gamma_E} \approx -\frac{1}{\mu}$$

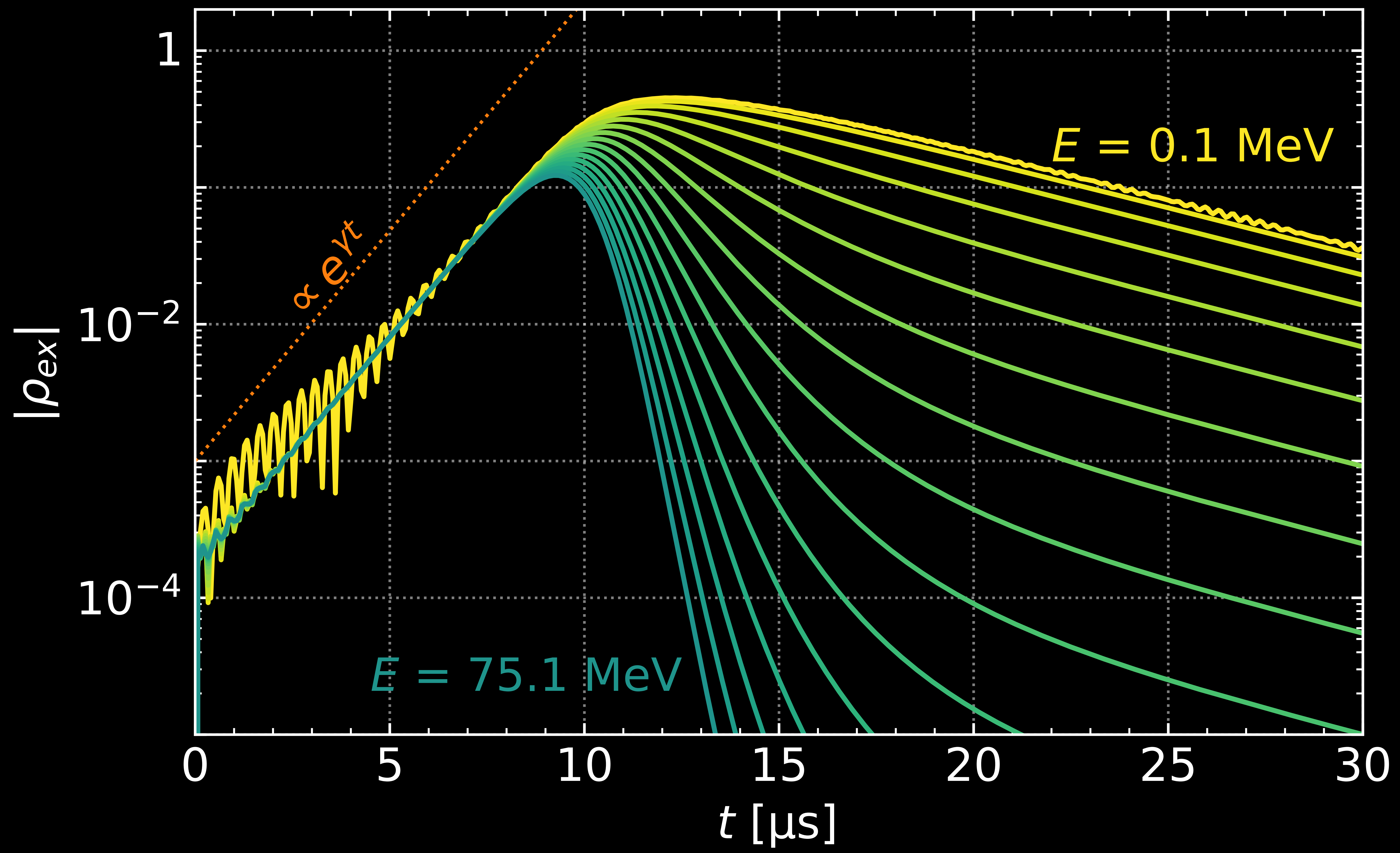
$$|\Omega| \sim \mu \gg \Gamma_E$$

$$\Omega \approx -\mu\int_{-\infty}^{\infty} G(E)\,\mathrm{d}E + \mathrm{i}\left(-\frac{\int_{-\infty}^{\infty}\Gamma_E\, G(E)\,\mathrm{d}E}{\int_{-\infty}^{\infty} G(E)\,\mathrm{d}E}\right)$$

Lin & HD (2022)

Collisional Flavor Instability

Homogeneous & isotropic gas (damping only)

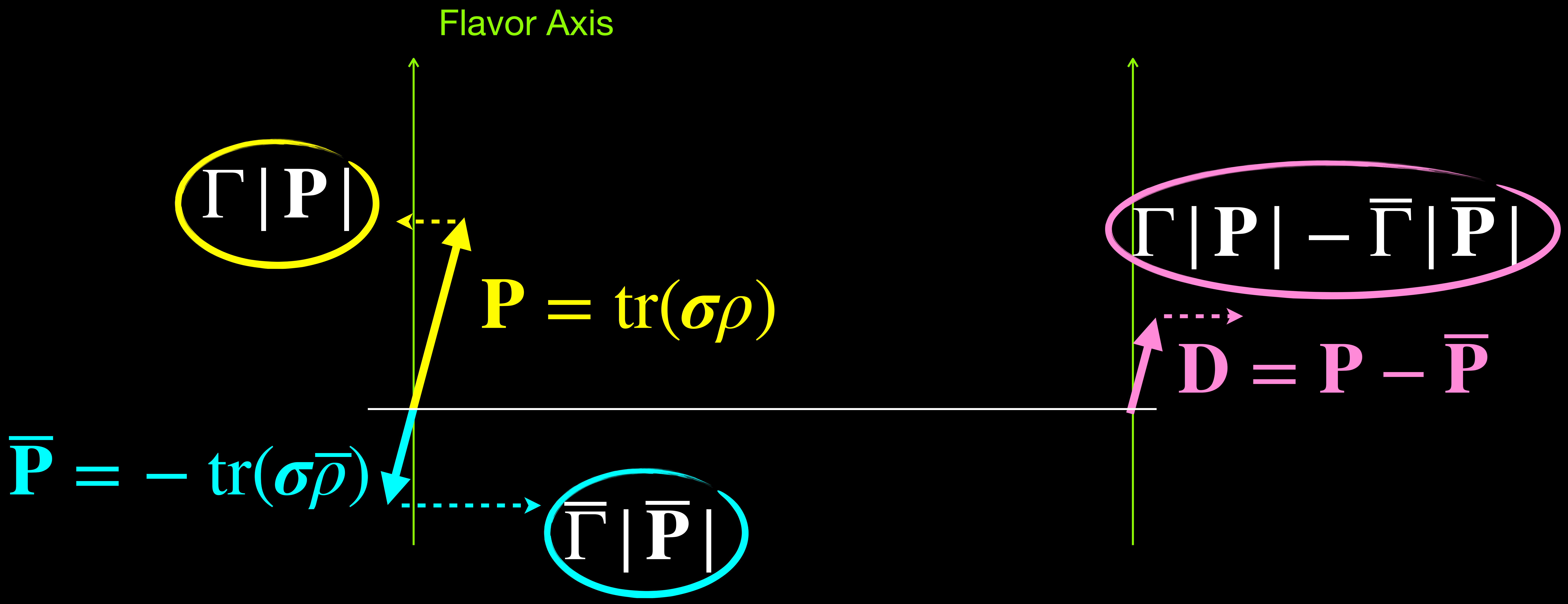


$$\gamma = -\frac{\int_{-\infty}^{\infty} \Gamma_E \, G(E) \, \mathrm{d}E}{\int_{-\infty}^{\infty} G(E) \, \mathrm{d}E}$$

Negative $E \Longrightarrow$ antineutrino

Lin & HD (2022)

A Simple Explanation



Lin & HD (2022)

Two Types of Instability

$$\int_{-\infty}^{\infty} \frac{G(E)\,\mathrm{d}E}{\Omega + \mathrm{i}\Gamma_E} \approx -\frac{1}{\mu}$$

$$|\Omega| \gg \Gamma_E$$

$$\Omega^2 + \mu D \Omega - \mu D \langle \Gamma_E \rangle \approx 0$$

$$\Omega_- \approx -\mu D - \mathrm{i}\langle \Gamma_E \rangle$$

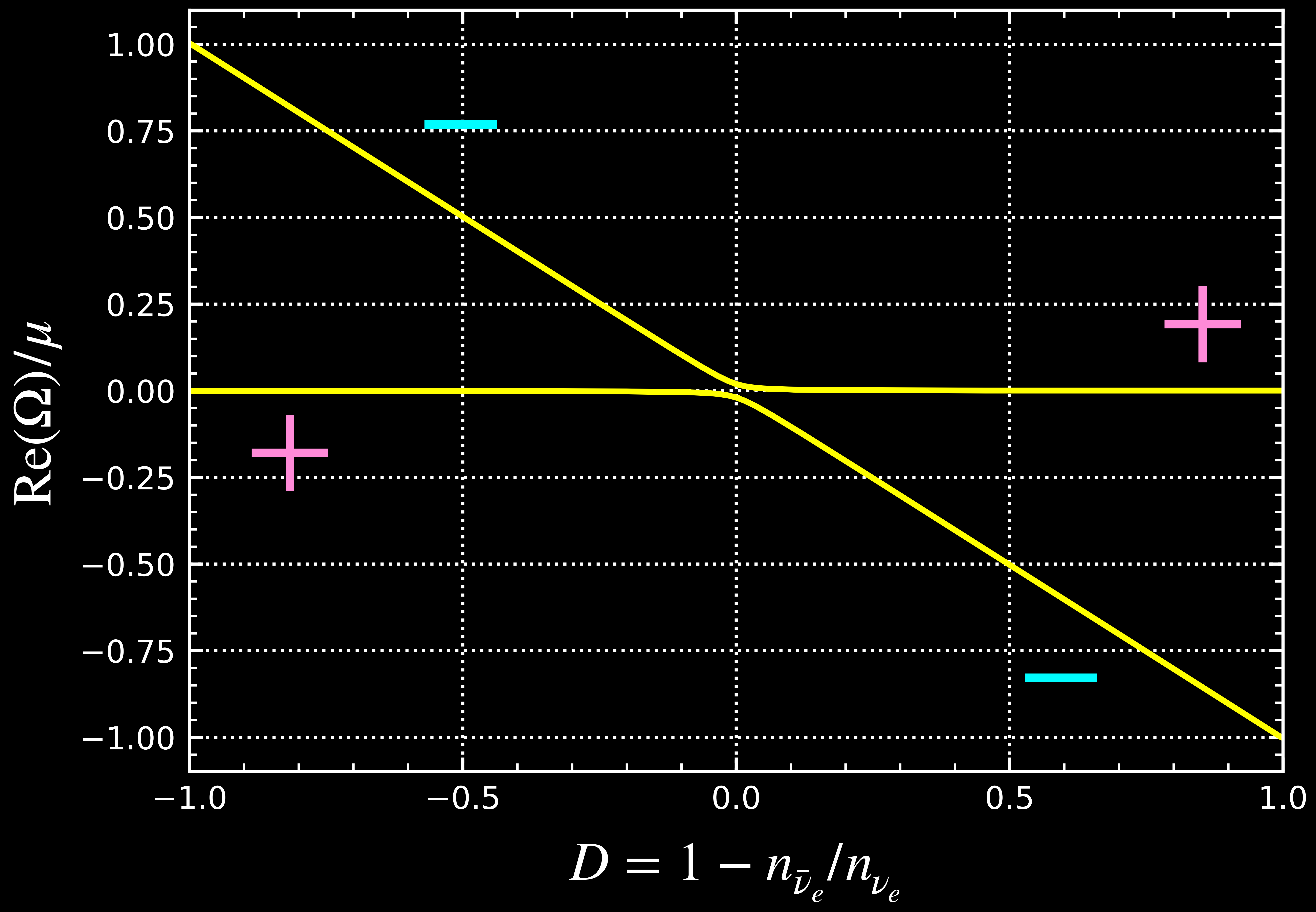
$$\Omega_+ \approx \mathrm{i}\langle \Gamma_E \rangle$$

$$D = \int_{-\infty}^{\infty} G(E)\,\mathrm{d}E$$

$$\langle \Gamma_E \rangle = \frac{\int_{-\infty}^{\infty} \Gamma_E G(E)\,\mathrm{d}E}{\int_{-\infty}^{\infty} G(E)\,\mathrm{d}E}$$

Xiong, Johns, Wu & HD (2022)

Two Types of Instability

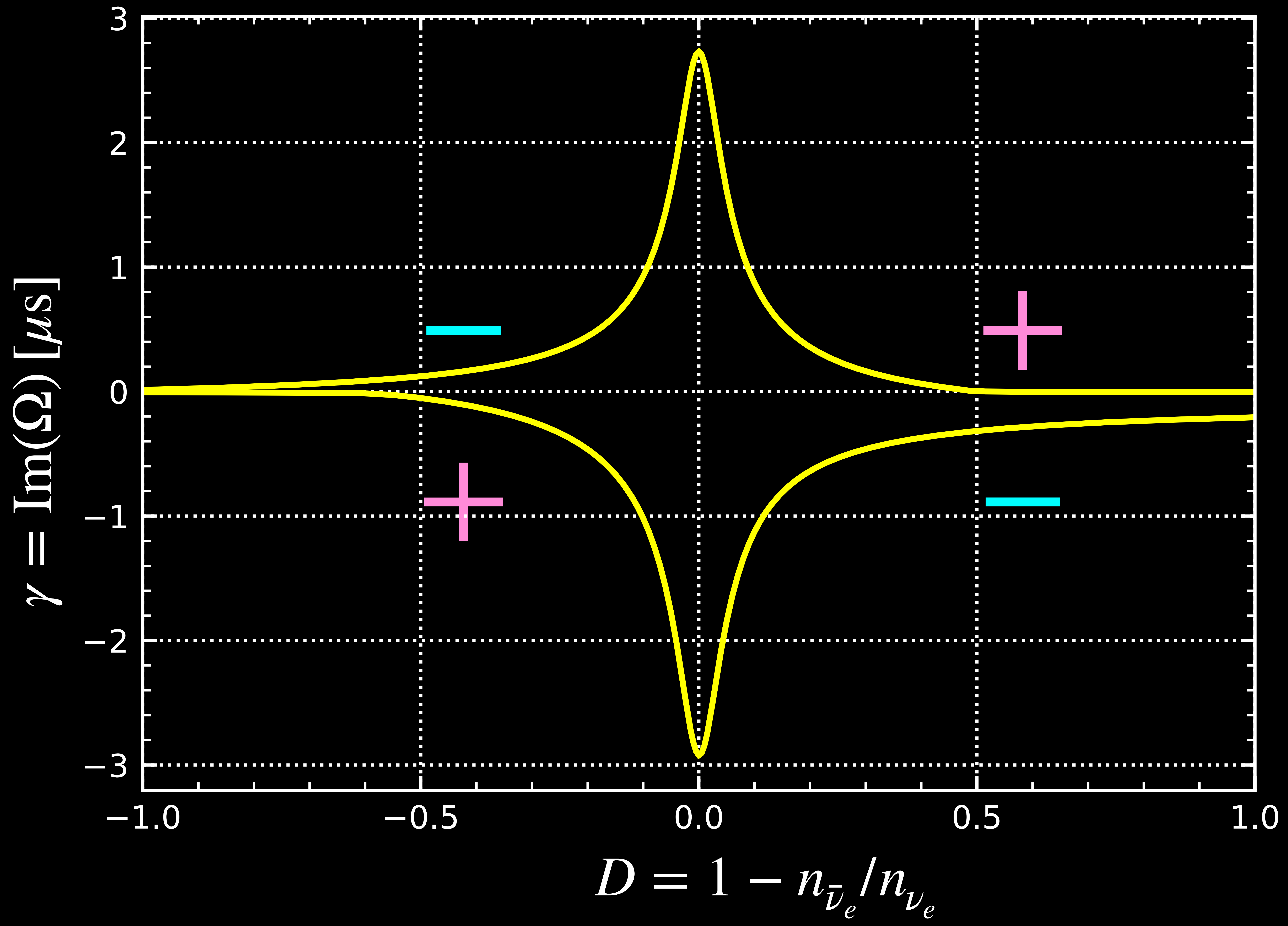


$\Omega_+ \approx \mathrm{i}\langle\Gamma_E\rangle$

$\Omega_- \approx -\mu D - \mathrm{i}\langle\Gamma_E\rangle$

Xiong, Johns, Wu & HD (2022)

Two Types of Instability



$\Omega_+ \approx \mathrm{i}\langle\Gamma_E\rangle$

$\Omega_- \approx -\mu D - \mathrm{i}\langle\Gamma_E\rangle$

Xiong, Johns, Wu & HD (2022)

Criteria for CFI

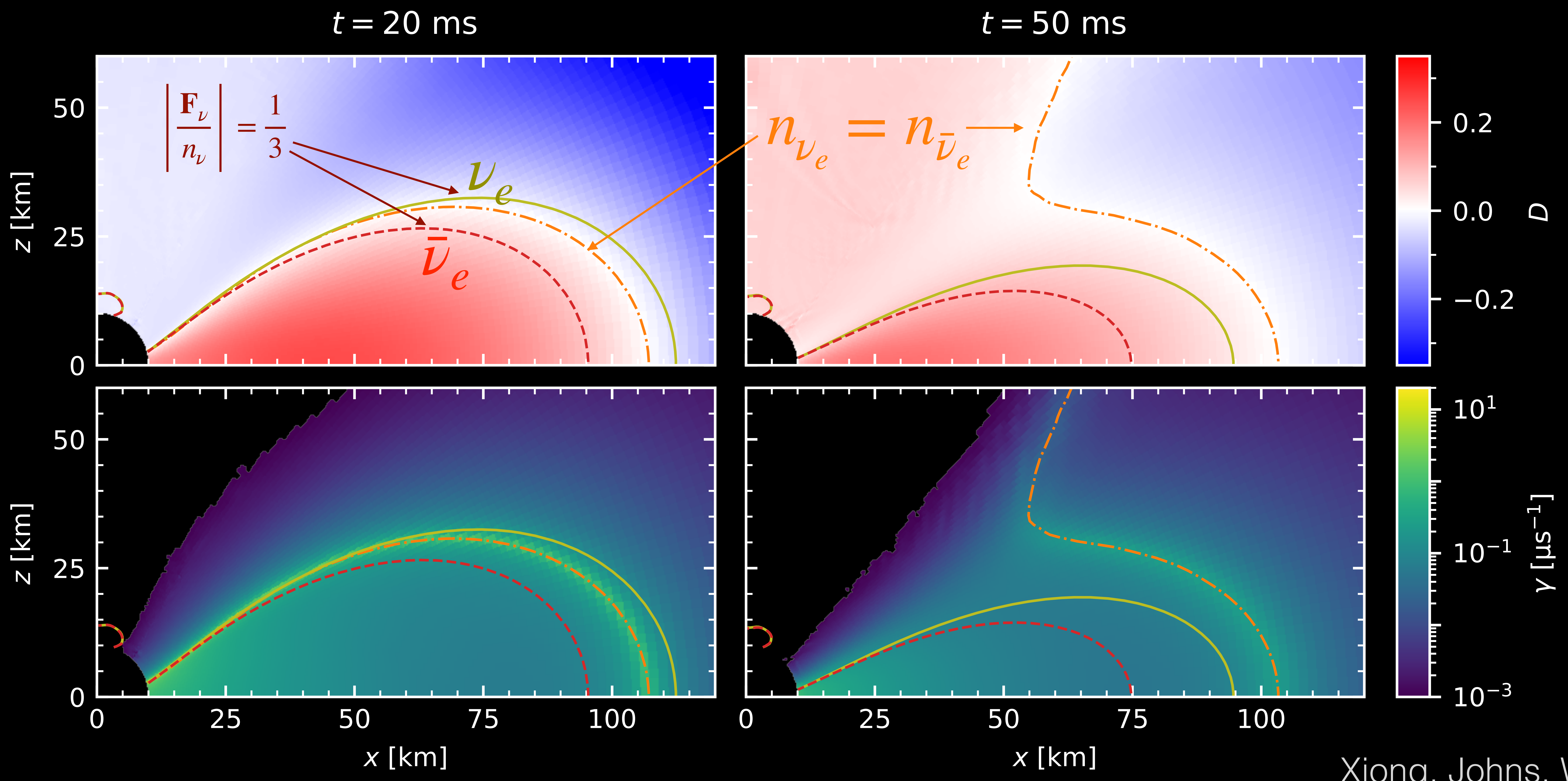
$$\Gamma = \frac{\int_0^\infty \Gamma_E \, G(E) \, \mathrm{d}E}{\int_0^\infty G(E) \, \mathrm{d}E}$$

$$\bar{\Gamma} = \frac{\int_{-\infty}^0 \Gamma_E \, G(E) \, \mathrm{d}E}{\int_{-\infty}^0 G(E) \, \mathrm{d}E}$$

$$\min\left(\frac{\Gamma}{\bar{\Gamma}}, \frac{\bar{\Gamma}}{\Gamma}\right) \lesssim \frac{n_{\bar{\nu}_e} - n_{\bar{\nu}_x}}{n_{\nu_e} - n_{\nu_x}} \lesssim \max\left(\frac{\Gamma}{\bar{\Gamma}}, \frac{\bar{\Gamma}}{\Gamma}\right)$$

Xiong, Johns, Wu & HD (2022)

Black Hole Accretion Disk



BH-disk model
M3A8m3a5
(Just+ 2014)

Xiong, Johns, Wu & HD (2022)

Summary

- Charged-current neutrino interaction can trigger neutrino flavor conversions through collisional flavor instability (CFI).
- CFI can exist below the neutrino decoupling surface and without ELN angular crossing $\Longrightarrow$ faster cooling of disk in BNSM (?)
- For homogeneous and isotropic environments, growth rate of CFI is the neutrino “collision” rate averaged over the ELN energy distribution.
    - Largest growth rate at where ELN $\approx 0 \Longrightarrow$ interaction with FFC (?)
    - No qualitative change in anisotropic environments [Liu+ (2023)] (?)