

CGC for ultra-peripheral Pb+Pb collisions at the Large Hadron Collider

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Based on JHEP 12 (2022) 077, with Alex Kovner and Vladi Skokov

Intersection of nuclear structure and high-energy nuclear collisions
Institute for Nuclear Theory, 2023

Supported by DOE

Ridge correlation in UPC

Two particle angular correlation observed in UPC measurement at LHC

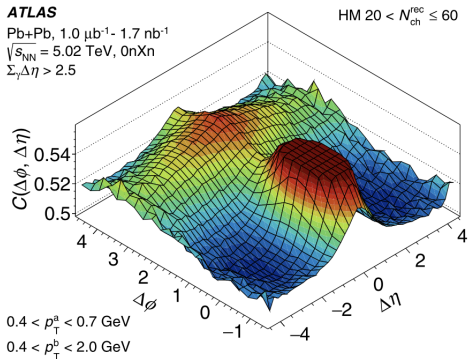
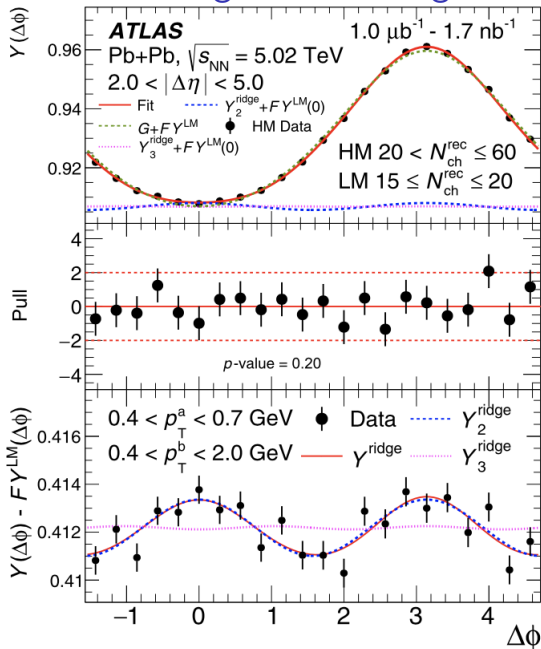
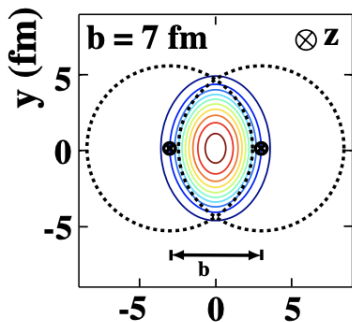


Figure: PHYSICAL REVIEW C 104, 014903 (2021), ATLAS

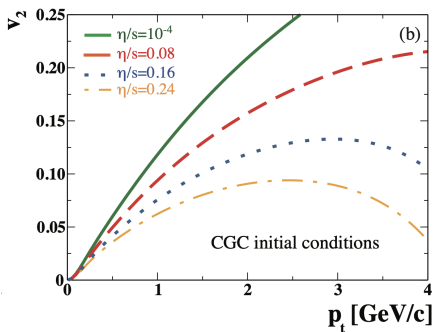
A demonstration of the signal and background



Elliptic flow



(a) Peripheral collision for AA



(b) $v_2 \rightarrow$ viscosity

Small viscosity η/s leads to higher v_2 . (Figures from Raimond Snellings (2011))

$$\frac{dN}{dq_1^2 dq_2^2} \propto 1 + \sum_n 2v_n^2 \cos(n\Delta\theta)$$

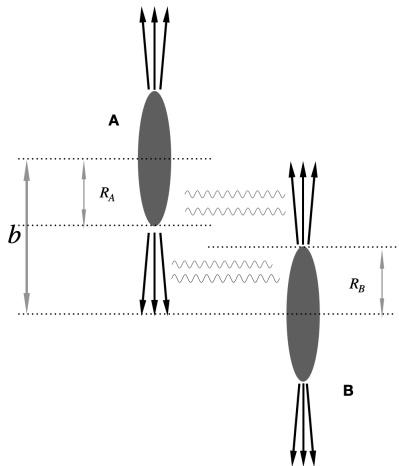
Ridge correlation in small systems ?

- If ridge correlation indicates fluid behavior, what is the smallest collision system to create QGP?
 - High multiplicity p+p (2010), p+Pb (2012) at LHC
 - p+Au, d+Au, $^3\text{He}+\text{Au}$ at RHIC (2013-2020)

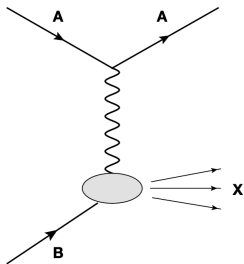
- Is there additional origin of the ridge correlation?
 - Opportunities to probe novel effects

- The smallest projectile is DIS photon!

Ultra-peripheral collisions



- $b > R_A + R_B$
- Equivalent photon approximation
- Weizsäcker-Williams field
- Photon-nuclear interaction



Non-perturbative photon

- Photon emitted by the nucleus coherently
- Resolution bounded by nucleus size

$$\frac{1}{Q} \gtrsim R_A$$

- For $A > 16$

$$Q^2 \lesssim (60\text{MeV})^2$$

Origins of the angular correlation in UPC

- Hydrodynamic

Collectivity in Ultra-Peripheral Pb+Pb Collisions at the Large Hadron Collider

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- Color domain effect in the target

Exploring the Collective Phenomenon at the Electron-Ion Collider

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- Quantum correlations (explored in our work)
 - Bose-Einstein correlation
 - HBT(Hanbury Brown and Twiss) effect
 - Dominated by the correlations in projectile

Bose enhancement

Two particle correlator in a free boson gas,

$$D(\mathbf{x}, \mathbf{y}) = \int_{\mathbf{p}, \mathbf{p}', \mathbf{q}, \mathbf{q}'} e^{-i\mathbf{x}\cdot(\mathbf{p}'-\mathbf{p})} e^{-i\mathbf{y}\cdot(\mathbf{q}'-\mathbf{q})} \langle \hat{a}_a^\dagger(\mathbf{p}) \hat{a}_b^\dagger(\mathbf{q}) \hat{a}_a(\mathbf{p}') \hat{a}_b(\mathbf{q}') \rangle$$

There are three different scenarios

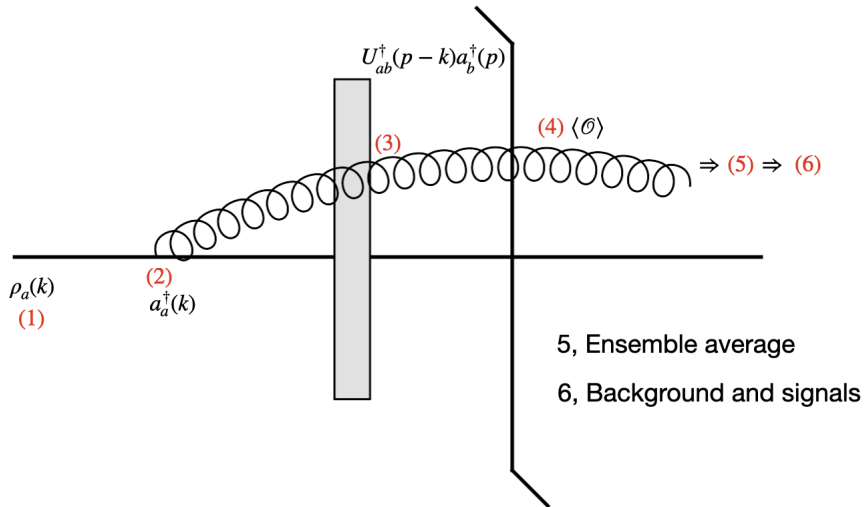
- $\mathbf{p} = \mathbf{p}', \mathbf{q} = \mathbf{q}'$: $\langle \hat{a}_a^\dagger(\mathbf{p}) \hat{a}_b^\dagger(\mathbf{q}) \hat{a}_a(\mathbf{p}') \hat{a}_b(\mathbf{q}') \rangle$, uncorrelated, $\mathcal{O}(1)$
- $\mathbf{p} = \mathbf{q}', \mathbf{q} = \mathbf{p}'$: $\langle \hat{a}_a^\dagger(\mathbf{p}) \hat{a}_b^\dagger(\mathbf{q}) \hat{a}_a(\mathbf{p}') \hat{a}_b(\mathbf{q}') \rangle$, $\mathcal{O}(\frac{1}{N_c^2})$
- $\mathbf{p} = \mathbf{q}' = \mathbf{q} = \mathbf{p}'$, suppressed by $\frac{1}{N_c^2}$ and $\frac{1}{V}$

$$\begin{aligned}
D_{\text{HBT}}(\mathbf{k}_1, \mathbf{k}_2) = & \sum_{a,b} \int_{\mathbf{x}_0, \mathbf{x}'_0, \mathbf{y}_0, \mathbf{y}'_0} \int_{\mathbf{x}_1, \mathbf{x}'_1, \mathbf{y}_1, \mathbf{y}'_1} e^{i\mathbf{k}_1 \cdot (\mathbf{x}'_0 - \mathbf{x}_0)} e^{i\mathbf{k}_2 \cdot (\mathbf{y}'_0 - \mathbf{y}_0)} \\
& \times \langle \hat{a}_a^\dagger(\mathbf{x}_0) \hat{a}_b^\dagger(\mathbf{y}_0) \hat{a}_a(\mathbf{x}'_0) \hat{a}_b(\mathbf{y}'_0) \rangle \\
& \times G(\mathbf{x}_0 - \mathbf{x}_1) G(\mathbf{y}_0 - \mathbf{y}_1) G(\mathbf{x}'_0 - \mathbf{x}'_1) G(\mathbf{y}'_0 - \mathbf{y}'_1) \\
& \times \langle J_a(\mathbf{x}_1) J_b(\mathbf{y}_1) J_a(\mathbf{x}'_1) J_b(\mathbf{y}'_1) \rangle
\end{aligned}$$

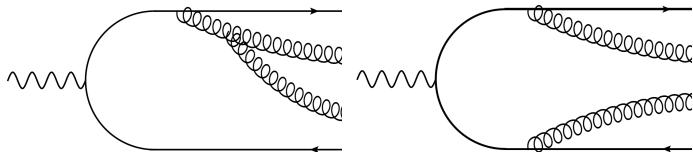
- The "wrong" contraction is enforced by the ensemble average of the source correlator

$$\langle J_a(\mathbf{x}_1) J_b(\mathbf{y}_1) J_a(\mathbf{x}'_1) J_b(\mathbf{y}'_1) \rangle$$

Theoretical set-up



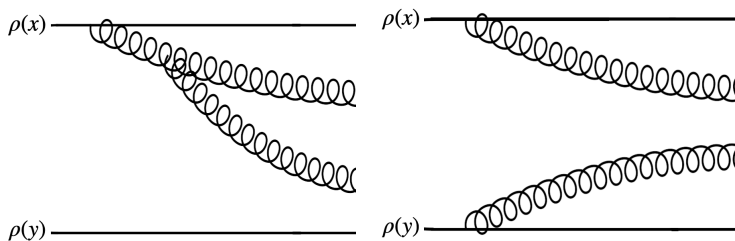
Dipole model ($|Q| < \Lambda_{QCD}$)



- Dipole model to approximate the photon
Small Q^2 suppresses the longitudinal polarization

$$\Psi_{\lambda}^T(z, \mathbf{r}, s_1) = -i \frac{2ee_f}{2\pi} \delta_{s_1, -s_2} (2z - 1 + 2\lambda s_1) \sqrt{z(1-z)} \frac{\mathbf{r} \cdot \boldsymbol{\epsilon}_{\lambda}}{|\mathbf{r}|} \epsilon_f K_1(\epsilon_f |r|)$$

MV model



- Inspired by Vector Meson Dominance Model
- Due to the existence of the high energy fixed point, ρ -meson at asymptotically high energy \equiv nucleus
- Valence degrees of freedom $\rho_a(\mathbf{x})$ follow the distribution defined by McLerran-Venugopalan (MV) model

$$W(\rho_a) = \exp\left\{-\int_{\mathbf{x}} \frac{\rho_a(\mathbf{x})\rho_a(\mathbf{x})}{2\mu^2}\right\}$$

Gluon production

Create gluons within initial states

One account for the emission of the gluons using coherent operators

$$C = \mathcal{P}e^{i\sqrt{2} \int d^2x d\xi \hat{b}_a^i(\xi, \mathbf{x}) [a_{i,a}^\dagger(\xi, \mathbf{x}) + a_{i,a}(\xi, \mathbf{x})]}$$

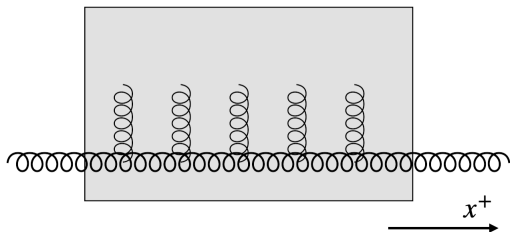
with the background field

$$\hat{b}_a^i(\xi, \mathbf{x}) = \frac{g}{2\pi} \int d^2y \frac{(\mathbf{x} - \mathbf{y})^i}{|\mathbf{x} - \mathbf{y}|^2} \hat{\rho}_P^a(\xi, \mathbf{y})$$

- MV model classical source ρ_a
- $\hat{\rho}_D^a(\mathbf{x}) = b_{\alpha\sigma}^\dagger(\mathbf{x}_1) t_{\alpha\beta}^a b_{\beta\sigma}(\mathbf{x}_1) \delta^{(2)}(\mathbf{x} - \mathbf{x}_1) - d_{\alpha\sigma}^\dagger(\mathbf{x}_2) t_{\beta\alpha}^a d_{\beta\sigma}(\mathbf{x}_2) \delta^{(2)}(\mathbf{x} - \mathbf{x}_2)$
- $\hat{\rho}_g^a(\zeta, \mathbf{x}) = a_b^{i\dagger}(\eta, \mathbf{x}) T_{bc}^a a_c(\eta, \mathbf{x})$



Eikonal scattering through the shock wave



$$U(\mathbf{x}) = \mathcal{P} \exp \left\{ ig \int_{-\infty}^{\infty} dx^+ T^a A_a^-(x^+, \mathbf{x}) \right\}$$

The strong gluon field $A_a^-(x^+, \mathbf{x})$ is a functional of the valance source in the target.

$$\frac{1}{N_c^2 - 1} \langle \text{Tr} (U^\dagger(r) U(0)) \rangle_T = \exp \left[-\frac{1}{4} Q_s^2 r^2 \ln \left(\frac{1}{\Lambda^2 r^2} + e \right) \right].$$

The cross section

$$\frac{d\mathcal{N}}{d\eta dq_1^2 d\xi dq_2^2} = \frac{1}{(2\pi)^4} \int d^2 u_1 d^2 u_2 d^2 \bar{u}_1 d^2 \bar{u}_2 e^{-i\mathbf{q}_1(\mathbf{u}_1 - \bar{\mathbf{u}}_1)} e^{-i\mathbf{q}_2(\mathbf{u}_2 - \bar{\mathbf{u}}_2)} \Sigma$$

and

$$\Sigma = \langle \gamma^* | C^\dagger \hat{S}^\dagger C a_{i,a}^\dagger(\eta, \mathbf{u}_1) a_{j,b}^\dagger(\xi, \mathbf{u}_2) a_{i,a}(\eta, \bar{\mathbf{u}}_1) a_{j,b}(\xi, \bar{\mathbf{u}}_2) C^\dagger \hat{S} C | \gamma^* \rangle$$

where $C = C_\xi C_\eta$, and $\eta \gg \xi$,

$$C_\eta \simeq 1 + i\sqrt{2} \int d^2 v_1 \hat{b}_{Da}^i(\mathbf{v}_1) \left[a_a^{i\dagger}(\eta, \mathbf{v}_1) + a_a^i(\eta, \mathbf{v}_1) \right]$$

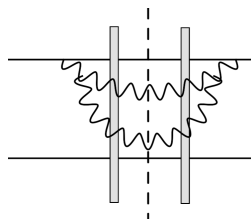
$$C_\xi \simeq 1 + i\sqrt{2} \int d^2 v_2 \left(\hat{b}_{Db}^j(\mathbf{v}_2) + \delta \hat{b}_b^j(\eta, \mathbf{v}_2) \right) \left[a_b^{j\dagger}(\xi, \mathbf{v}_2) + a_b^j(\xi, \mathbf{v}_2) \right]$$

- $C|\gamma^*\rangle$ Initial state
- \hat{S} S-matrix
- $C a_{j,b}(\xi, \bar{\mathbf{u}}_2) C^\dagger$ dressed gluons in the final state

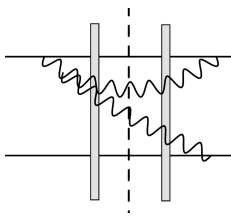
Organize the cross section

Organize the cross section Σ according to the order of ρ

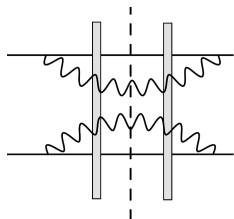
$$\Sigma = \Sigma_2 + \Sigma_3 + \Sigma_4$$



(a) $\Sigma_2(\rho^2)$



(b) $\Sigma_3(\rho^3)$



(c) $\Sigma_4(\rho^4)$

Continue the calculation of Σ

Use Σ_2 as example, in coordinate space,

$$\Sigma_2 = 4 \int d^2 \mathbf{x} \int d^2 \bar{\mathbf{x}} f^i(\bar{\mathbf{u}}_1 - \mathbf{x}) f^i(\mathbf{u}_1 - \bar{\mathbf{x}}) f^j(\bar{\mathbf{u}}_2 - \bar{\mathbf{u}}_1) f^j(\mathbf{u}_2 - \mathbf{u}_1) \langle \rho_{d'}(\bar{\mathbf{x}}) \rho_d(\mathbf{x}) \rangle_P \\ \left\langle \left[[U^\dagger(\mathbf{u}_1) T^a U(\mathbf{u}_1)] [U^\dagger(\mathbf{u}_2) - U^\dagger(\mathbf{u}_1)] [U(\bar{\mathbf{u}}_2) - U(\bar{\mathbf{u}}_1)] [U^\dagger(\bar{\mathbf{u}}_1) T^a U(\bar{\mathbf{u}}_1)] \right]_{d'd} \right\rangle_T$$

where $f^i(\mathbf{x}) = \frac{g}{(2\pi)^2} \frac{x_i}{x^2}$.

- Kinematic factors (Eikonal emission vertices)
- Projectile (photon)
- Target (nucleus)

Expectation values for projectile and target

Dipole expectation values

- Expectation values for $q\bar{q}$

$$\langle q\bar{q} | \hat{\rho}_{d'}(\bar{\mathbf{x}}) \hat{\rho}_d(\mathbf{x}) | q\bar{q} \rangle = \frac{\delta^{dd'}}{2} (\delta^2(\bar{\mathbf{x}} - \mathbf{z}_1) - \delta^2(\bar{\mathbf{x}} - \mathbf{z}_2)) (\delta^2(\mathbf{x} - \mathbf{z}_1) - \delta^2(\mathbf{x} - \mathbf{z}_2))$$

$$\begin{aligned} & \langle q\bar{q} | \hat{\rho}^a(\mathbf{x}_1) \hat{\rho}^b(\mathbf{x}_2) \hat{\rho}^c(\mathbf{x}_3) | q\bar{q} \rangle \\ &= \frac{if_{abc}}{4} (\delta^{(2)}(\mathbf{x}_2 - \mathbf{z}_1) + \delta^{(2)}(\mathbf{x}_2 - \mathbf{z}_2)) \prod_{i=1,3} (\delta^{(2)}(\mathbf{x}_i - \mathbf{z}_1) - \delta^{(2)}(\mathbf{x}_i - \mathbf{z}_2)) \end{aligned}$$

$\mathbf{z}_1, \mathbf{z}_2$ are the transverse coordinates of quark and anti-quark.

- Average over different dipole size $\mathbf{r} = \mathbf{z}_1 - \mathbf{z}_2$

$$\langle \rho_{d'}(\bar{\mathbf{x}}) \rho_d(\mathbf{x}) \rangle_P \approx \sum_{s_1} \int_z \int d^2\mathbf{r} \Psi_\lambda^{T*}(z, r, s_1) \Psi_\lambda^T(z, r, s_1) \langle q\bar{q} | \rho_{d'}(\bar{\mathbf{x}}) \rho_d(\mathbf{x}) | q\bar{q} \rangle$$

MV model projectile average

- MV model describes the distribution of classical color source not quantum operators.

$$W(\rho_a) = \exp\left\{-\int_{\mathbf{x}} \frac{\rho_a(\mathbf{x})\rho_a(\mathbf{x})}{2\mu^2}\right\}$$

-

$$\mu^2(\mathbf{x}) = \mathcal{N} \exp\left\{-\frac{\mathbf{x}^2}{R^2}\right\}.$$

- Two and three point correlators

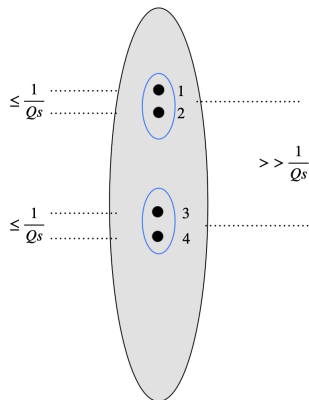
$$\langle \hat{\rho}_a(\mathbf{x})\hat{\rho}_b(\mathbf{y}) \rangle_{MV} = \langle \rho_a(\mathbf{x})\rho_b(\mathbf{y}) \rangle_{MV} = \mu^2 \delta^{(2)}(\mathbf{x} - \mathbf{y}) \delta_{ab}$$

$$\langle \hat{\rho}_a(\mathbf{x})\hat{\rho}_b(\mathbf{y})\hat{\rho}_c(\mathbf{z}) \rangle_{MV} = -\frac{1}{2} \delta^{(2)}(\mathbf{x} - \mathbf{y}) \delta^{(2)}(\mathbf{y} - \mathbf{z}) T_{bc}^a \mu^2$$

- Symmetrization of $\hat{\rho}_S$

$$\begin{aligned} \hat{\rho}_a(x)\hat{\rho}_b(y) &= \frac{1}{2} \{\hat{\rho}_a(\mathbf{x}), \hat{\rho}_b(\mathbf{y})\} + \frac{1}{2} [\hat{\rho}_a(\mathbf{x}), \hat{\rho}_b(\mathbf{y})] \\ &= \rho_a(\mathbf{x})\rho_b(\mathbf{y}) - \frac{1}{2} \delta^{(2)}(x - y) T_{ab}^c \rho_c(\mathbf{x}) \end{aligned}$$

Target average(I)



- Factorized Dipole Approximation

Phys. Rev. D 96, 074018, Kovner, Rezaeian

- Dense target \rightarrow Saturated

- $\frac{1}{Q_s}$ serves the role of correlation length in transverse plane

- For the example configuration

$$\text{Tr} [U(x_1)U^\dagger(x_2)U(x_3)U^\dagger(x_4)]$$

\approx

$$\frac{1}{N_c^2-1} \text{Tr} [U(x_1)U^\dagger(x_2)] \text{Tr} [U(x_3)U^\dagger(x_4)] +$$

...

Target average (II)

We only have one type of Wilson line correlator in momentum space

$$\begin{aligned} & \left\langle \text{Tr} \left[U(k_1) T^a U^\dagger(k_2) U(k_3) T^a U^\dagger(k_4) \right] \right\rangle_T \\ &= T_{bc}^a T_{de}^a \left\langle \left[U^{fb}(k_1) U^{\dagger cg}(k_2) U^{gd}(k_3) U^{\dagger ef}(k_4) \right] \right\rangle_T \\ &\approx T_{bc}^a T_{de}^a \left(\frac{(2\pi)^2}{N_c^2 - 1} \right)^2 \left\{ (N_c^2 - 1) \delta^{bc} \delta^{de} \delta^{(2)}(k_1 - k_2) D(k_1) \delta^{(2)}(k_3 - k_4) D(k_3) \right. \\ &\quad + (N_c^2 - 1) \delta^{bd} \delta^{ce} \delta^{(2)}(k_1 + k_3) D(k_1) \delta^{(2)}(k_2 + k_4) D(-k_2) \\ &\quad \left. + (N_c^2 - 1)^2 \delta^{be} \delta^{cd} \delta^{(2)}(k_1 - k_4) D(k_1) \delta^{(2)}(k_2 - k_3) D(-k_2) \right\} \end{aligned}$$

here the dipole $D(p)$ is defined as

$$D(p) = \frac{1}{N_c^2 - 1} \int d^2x e^{ipx} \langle \text{Tr} \left(U^\dagger(x) U(0) \right) \rangle_T$$

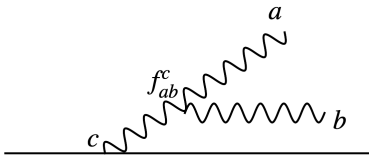
Isolating the signal

- Symmetrization of $\hat{\rho}_S$ (MV model)

$$\begin{aligned}\hat{\rho}_a(\mathbf{x})\hat{\rho}_b(\mathbf{y}) &= \frac{1}{2} \{ \hat{\rho}_a(\mathbf{x}), \hat{\rho}_b(\mathbf{y}) \} + \frac{1}{2} [\hat{\rho}_a(\mathbf{x}), \hat{\rho}_b(\mathbf{y})] \\ &= \rho_a(\mathbf{x})\rho_b(\mathbf{y}) - \frac{1}{2} \delta^{(2)}(x-y) T_{ab}^c \rho_c(\mathbf{x})\end{aligned}$$

- Symmetrization of color factors (Dipole model)

$$t^a t^b = \frac{1}{2} \{ t^a, t^b \} + \frac{1}{2} i f_{ab}^c t^c$$



Angular correlation from the cross section

From the cross section of the two gluon production

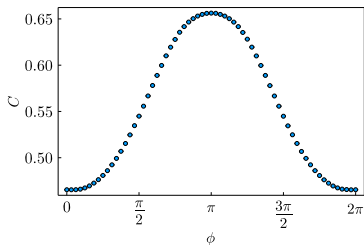
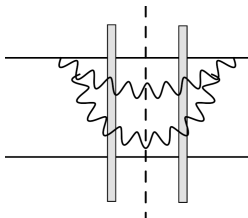
$$\Sigma = \frac{d\mathcal{N}}{d\eta dq_1^2 d\xi dq_2^2}$$

one can extract the angular correlation function

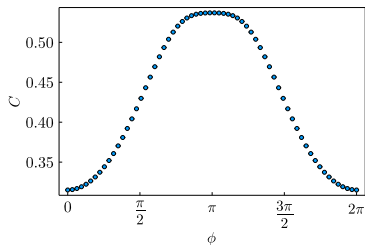
$$C(q, \theta) = \frac{\Sigma(q, \theta)}{\frac{1}{2\pi} \int_0^{2\pi} \Sigma(q, \theta) d\theta}$$

set $|q_1| = |q_2| = q$, and θ is the angle between the two particles

$$\Sigma_2, q = Q_s$$

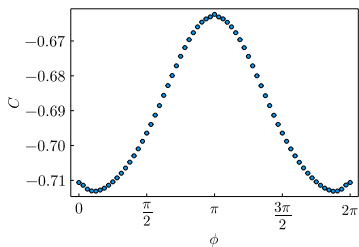
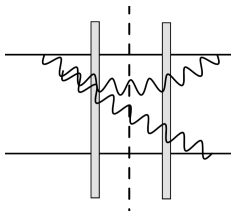


(a) Dipole

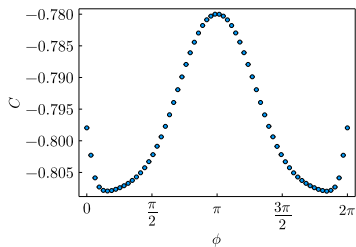


(b) MV

$$\Sigma_3, q = Q_s$$

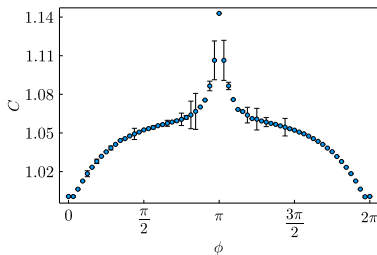


(a) Dipole

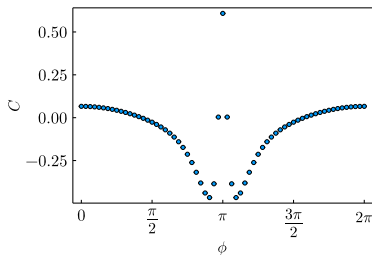


(b) MV

Σ_4^{nsym} , non-symmetric part, $q = Q_s$



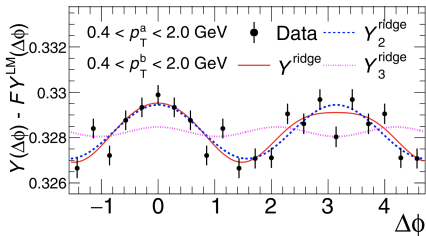
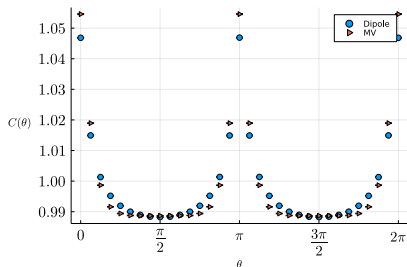
(a) Dipole



(b) MV

Also gives us back-to-back correlation. large error bar comes from the that monstrous dipole Σ_4^{nsym} is not Monte Carlo friendly.

Σ_4^{sym} , symmetric part, $q = Q_s$



As what was done in experimental analysis, we subtract backgrounds and normalize the signal. The results show similar correlations in CGC calculation.

v_2 and v_2^2

Recall,

$$\frac{dN}{d\mathbf{q}_1^2 d\mathbf{q}_2^2} \propto 1 + \sum_n 2v_n^2 \cos(n\Delta\theta)$$

One first define,

$$V_n(q_1) = \int d\theta_1 \int_0^{p_\perp^{\max}} d^2\mathbf{q}_2 \exp(in\Delta\theta) \frac{dN}{d\mathbf{q}_1^2 d\mathbf{q}_2^2 d\eta d\xi}$$

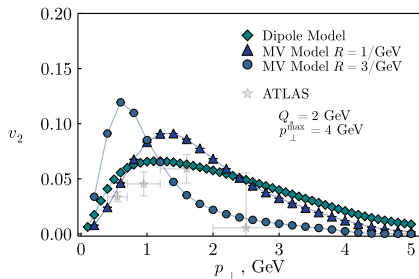
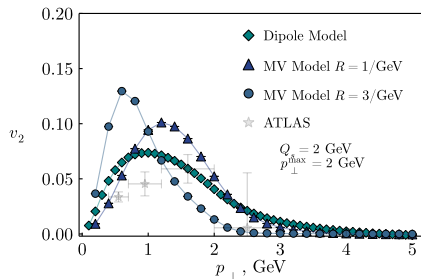
by definition,

$$v_2^{(2)}(p_\perp) = \sqrt{\frac{V_2(p_\perp)}{V_0(p_\perp)}}$$

assuming factorization,

$$v_2(p_\perp) = \frac{V_2(p_\perp)/V_0(p_\perp)}{\sqrt{V_2/V_0}}.$$

v_2 results



- Different behavior above 2 GeV due to the lack of HBT contribution on the left.
- In the ATLAS analysis, $P_{\text{Max}} = 2$ GeV

Factorization test

Theoretical calculation

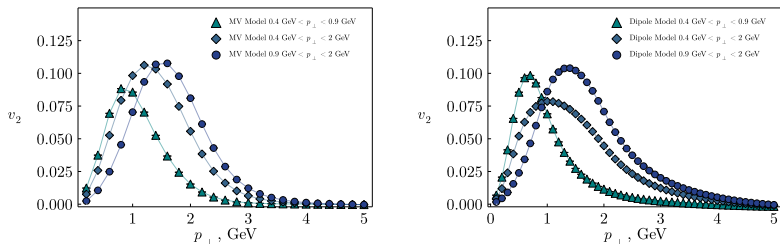


Figure: The elliptic flow v_2 for three different kinematic ranges of the trigger particle. Here as in the previous figure, $Q_s = 2 \text{ GeV}$. The size of the projectile is set by $R = 1/\text{GeV}$.

Average in momentum bins

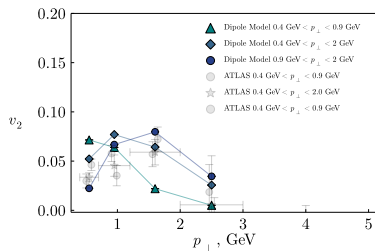
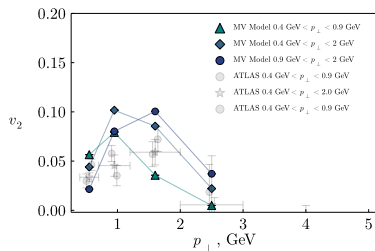


Figure: Parameters are the same as previous slides but binned with the same bin choice as the ATLAS analysis.

Binning the particles decreases the differences between the models.

Summary and outlook

- We analytically derived inclusive two gluon production in UPC at mid-rapidity.
- To estimate systematic uncertainty originated from the poor knowledge of the real photon wave function, we studied two limiting cases.
- Both models result in qualitatively similar correlation. Quantitatively, the amplitude of azimuthal anisotropy for MV model is about two times the dipole model.
- Our results show similar correlation as experimental data.
- Further developments
 - Phenomenology
 - To extend to EIC physics (large Q^2 , work in progress)
 - To incorporate rapidity dependence